CHAPTER-3

Trigonometric Functions

One mark questions

1. Define radian measure of an angle. (K)

2. Define degree measure of an angle. (K)

3. Define an angle. (K)

4. Define trigonometric functions. (K)

5. Define principal solution of trigonometric functions. (K)

6. Define general solution of trigonometric functions. (K)

7. Write the general solution of the following functions:

(i) $\sin x = 0$ (U) (ii) $\cos x = 0$ (U) (iii) tanx=0 (U)

(iv) $\sin x = \sin y$ (U) (v) $\cos x = \cos y$ (U) (vi) $\tan x = \tan y$ (U)

(vii) $\sin 2\theta = 0$ (U) (viii) $\cos^2 3\theta = 0$ (U) (ix) $\tan\left(\frac{3\theta}{4}\right) = 0$ (U)

8. Write the domain and range of the following function

(i) $\sin x$ (U) (ii) $\cos x$ (U) (iii) $\tan x$ (U)

(iv) $\text{cesec}x$ (U) (v) $\sec x$ (U) (vi) $\cot x$ (U)

9. Convert the following in to degrees.

(i) $\frac{\pi}{3}$ (U) (ii) $\frac{\pi}{4}$ (U) (iii) $\frac{\pi}{6}$ (U)

(iv) $\frac{\pi}{2}$ (U) (v) $\frac{3\pi}{4}$ (U) (vi) $\frac{5\pi}{4}$ (U)

(vii) $\frac{4\pi}{3}$ (U) (viii) $\frac{7\pi}{3}$ (U) (ix) $\frac{2\pi}{3}$ (U)

(x) $\frac{5\pi}{6}$ (U) (xi) $\frac{7\pi}{4}$ (U) (xii) $\frac{5\pi}{3}$ (U)

(xiii) $\frac{11\pi}{3}$ (U)

10. Convert the following in to radians

(i) 45^0 (U) (ii) 60^0 (U) (iii) 30^0 (U)

(iv) 90^0 (U) (v) 135^0 (U) (vi) 120^0 (U)

(vii) 150^0 (U) (viii) 180^0 (U) (ix) 210^0 (U)

(x) 225^0 (U) (xi) 240^0 (U) (xii) 300^0 (U)

(xiii) 330^0 (U)

11. Find the value of $\operatorname{cosec} 1305^0$ (U)

12. If $x\cos^2\left(\frac{\pi}{6}\right)=2$, find x. (U)

13. If $\sin\theta=\frac{5}{13}$ and θ is in the second quadrant find $\tan\theta, \operatorname{cosec}\theta$, $\sec\theta$ (U)

14. If $x=a\sec^4\theta, y=a\tan^4\theta$ prove that $\sqrt{x}-\sqrt{y}=\sqrt{a}$ (A)

15. If $\tan\theta=\frac{5}{13}$ and $\pi<\theta<\frac{3\pi}{2}$, find the values of $\sec\theta$ and $\operatorname{cosec}\theta$ (A)

Two marks questions

1. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second. (U)

2. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22cm.(use $\pi=\frac{22}{7}$). (U)

3. If in two circles, the arcs of the same length subtendangles 60^0 and 75^0 at the centre, find the ratio of their radii. (U)

4. A minute hand of watch is 1.5cm long. How for does its tip moves in 40 minutes. (A)

5. Find the angle in radian through which a pendulum swings if its length 75cm and the tip describe an arc of length. (U)

6. Convert 40^020^1 into radian. (A)

7. Convert 6radians into degree. (U)

8. Find the value of $(i)\sin\left(\frac{31\pi}{3}\right)$(U) $(ii)\tan\left(\frac{19\pi}{3}\right)$(U) $(iii)\cot\left(-\frac{15\pi}{4}\right)$ (U)

9. Find the values of the following:

$(i)\sin 765^0$ (A) $(ii)\operatorname{cosec}\left(-1410^0\right)$ (A)

$(iii)\sin 75^0$ (U) $(iv)\cos 105^0$ (U) $(v)\tan 15^0$ (U)

10. Find the principle value of the following:

$(i)\tan x=\frac{1}{\sqrt{3}}$ (K) $(ii)\sin x=-\frac{\sqrt{3}}{2}$ (K) $(iii)\sec x=2$ (K)

$(iv)\ \sin x=\frac{\sqrt{3}}{2}$ (K) $(v)\ \cos x=-\frac{1}{2}$ (K) $(vi)\operatorname{cosec} x=-2$ (K)

$(vii)\cos x=\frac{1}{2}$ (K) $(viii)\tan x=-\sqrt{3}$ (K) $(ix)\cot x=-\sqrt{3}$ (K).

11. Prove that $\cos^4\theta-\sin^4\theta=\cos 2\theta$. (A)

12. Prove that $\sqrt{2+\sqrt{2+2\cos\theta}}=2\cos\frac{\theta}{2}$. (S)

13. Prove the following:

i)$\sin 2x=2\sin x\cos x$ (U) ii) $\tan 2x=\frac{2\tan x}{1-\tan^2 x}$ (U)

ii) $\cos 2x=\cos^2 x-\sin^2 x=1-2\sin^2 x=2\cos^2 x-1$ (U)

iii) $\sin 3x=3\sin x-4\sin^3 x$ (U) iv) $\cos 3x=4\cos^3 x-3\cos x$ (U)

v) $\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$ (U)vi)$\sin 2x=\frac{2\tan x}{1+tan^2 x}$ (U)

vii) $\cos 2x=\frac{1-\tan^2 x}{1+\tan^2 x}$ (U)

viii) $\cot(x+y)=\frac{\cot x\cot y-1}{\cot x+\cot y}$(U)ix) $\cot(x+y)=\frac{\cot x\cot y+1}{\cot y-\cot x}$ (U)

Three marks questions.

1. Prove that for any real numbers x and y $\sin x=\sin y\Rightarrow x=n\pi+(-1)^n y$, $\forall n\in Z$ (U)

2. Prove that for any real numbers x and y $\cos x=\cos y\Rightarrow x=2n\pi\pm y$, $\forall n\in Z$ (U)

3. If x and y are not odd multiple of $\frac{\pi}{2}$, then $\tan x=\tan y\Rightarrow x=n\pi+y$, $\forall n\in Z$ (U)

4. Find the general solution of the following:

$(i)\cos 4x=\cos 2x$ (U) $(ii)2\cos^2 x+3\sin x=0$ (U)

$(iii)\sin 2x+\cos x=0$ (U) $(iv)\sec^2 2x=1-\tan 2x$ (U)

5. Prove the following:

$(i)\cos 4x=1-8\sin^2 x\cos^2 x$ (A)

(ii) $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13}+\cos\frac{3\pi}{13}+\cos\frac{5\pi}{13}=0$ (A)

(iii) $\tan3x\tan2x\tan x=\tan3x-\tan2x-\tan x$ (S)

(iv) $\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)=\sqrt{2}\cos x$ (U)

(v) $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x$ (A)

6. Prove the following:

(i) $\frac{\sin x+\sin3x}{\cos x+\cos3x}=\tan2x$ (U) (ii) $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\left(\frac{x-y}{2}\right)$ (U)

(iii) $\frac{\sin5x+\sin3x}{\cos5x+\cos3x}=\tan4x$ (U) (iv) $\frac{\cos7x+\cos5x}{\sin7x-\sin5x}=\cot x$ (U)

(v) $\frac{\cos9x-\cos5x}{\sin17x-\sin3x}=-\frac{\sin2x}{\cos10x}$ (U)

7. If $A+B=\frac{\pi}{4}$, prove that (i) $(1+\tan A)(1+\tan B)=2$ (A) (ii) $(\cot A-1)(\cot B-1)=2$ (A)

8. Prove that $\cos^6\theta+\sin^6\theta=1-\frac{3}{4}\sin^2 2\theta$ (A)

9. Prove the following:

(i) $\frac{\sin A+\sin2A+\sin3A}{\cos A+\cos2A+\cos3A}=\tan2A$. (A)

(ii) $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$. (A)

(iii) $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$. (A)

10. If θ is acute , then prove that $\sin\theta+\cos\theta>1$. (A)

Five marks questions

1. Prove the following:

(i) $\sin x+\sin3x+\sin5x+\sin7x=4\cos x\cos2x\sin4x$ (A)

(ii) $\sin3x+\sin2x-\sin x=4\sin x\cos\frac{x}{2}\cos\frac{3x}{2}$ (A)

(iii) $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$ (A)

(iv) $\frac{\sin 9x + \sin 7x + \sin 5x + \sin 3x}{\cos 9x + \cos 7x + \cos 5x + \cos 3x} = \tan 6x$ (A)

2. Solve the following:

(i) $\cos 3x + \cos x - \cos 2x = 0$ (A)

(ii) $\sin x + \sin 3x + \sin 5x = 0$ (A)

(iii) $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x = 0$ (A)

3. Prove the following:

(i) $\sin 20^0 \sin 40^0 \sin 60^0 \sin 80^0 = \frac{3}{16}$ (A)

(ii) $\cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0 = \frac{1}{16}$ (A)

(iii) $\tan 20^0 \tan 40^0 \tan 60^0 \tan 80^0 = 3$ (A)

(iv) $\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0 = \frac{1}{16}$ (S)

4. Prove that $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 4\theta}$. (A)

5. Prove that $\frac{\cos 8A \cos 5A - \cos 12A \cos 9A}{\sin 8A \cos 5A + \cos 12A \sin 9A} = \tan 4A$. (A)

6. Prove the following:

(i) $(\cos\alpha + \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\cos^2\left(\frac{\alpha+\beta}{2}\right)$ (S)

(ii) $(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 = 4\sin^2\left(\frac{\alpha-\beta}{2}\right)$ (S)

7. Prove the following:

(i) $\sin A + \sin B + \sin C = 4\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$ (S)

(ii) $\cos A + \cos B + \cos C = 1 + 4\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$ (S)

(iii) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A \cos B \cos C$ (S)

(iv) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A \cos B \cos C$ (S)

8. Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ for the following

(i) $\tan x = -\frac{4}{3}$, Where x is in second quadrant (S)

(ii) $\cos x = -\frac{1}{3}$, Where x is in fourth quadrant, (S)

(iii) $\sin x = \frac{1}{4}$, Where x is in second quadrant, (S)

(iv) $\tan x = \frac{3}{4}, \pi < x < \frac{3\pi}{2}$ (S)

9. Prove geometrically that: $\cos(x + y) = \cos x \cos y - \sin x \sin y$ and hence find $\cos 75°$ (A)

10. $\tan 4x = \frac{4\tan x - 4\tan^3 x}{1 - 6\tan^2 x + \tan^4 x}$ (A)

11. $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ (A)

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