11. Algebraic Expressions

11.0 Introduction:

Consider the expressions:

(i) $3 + 8 - 9$ (ii) $\frac{1}{3}xy$ (iii) 0 (iv) $3x + 5$ (v) $4xy + 7$ (vi) $15 + 0 - 19$ (vii) $\frac{3x}{y} (y \neq 0)$

(i), (iii) and (vi) are numerical expressions where as (ii), (iv) and (v), (vii) are algebraic expressions.

Do you identify the difference between them?

You can form many more expressions. As you know expressions are formed with variables and constants. In the expression $3x + 5$, x is varaible and 3, 5 are constants. $3x$ is an algebraic term and 5 is a numerical term. The expression $4xy + 7$ is formed with variables x and y and constants 4 and 7.

Now $\frac{1}{3}xy$ has one term and $2xy + pq - 3$ has 3 terms in it.

So you know that terms are formed as a product of constants and one or more variables.

Terms are added or subtracted to form an **expression**.

We know that the value of the expression $3x + 5$ could be any number. If $x = 2$ the value of the expression would be $3(2) + 5 = 6 + 5 = 11$. For different values of x, the expression $3x + 5$ holds different values.

Do This

1. Find the number of terms in following algebraic expressions

   $5xy^2$, $5xy^3 - 9x$, $3xy + 4y - 8$, $9x^2 + 2x + pq + q$.

2. Take different values for x and find values of $3x + 5$.

The sum of all exponents of the variables in a monomial is the degree of the monomial

The highest degree among the degrees of the different terms of an algebraic expression is called the degree of that algebraic expression.

Let us consider some more algebraic expressions, $5xy^2$, $5xy^3 - 9x$, $3xy + 4y - 8$ etc. It is clear that $5xy^2$ is monomial, $5xy^3 - 9x$ is binomial and $3xy + 4y - 8$ is trinomial.

As you know that the degree of a monomial $5x^2y$ is '3'.

Moreover, the degree of the binomial $5xy^3 - 9x$ is '4'.

Similarly, the degree of the trinomial $3xy + 4y - 8$ is '2'.

Expressions that contain exactly one, two and three terms are called **monomials**, **binomials** and **trinomials** respectively. In general, any expression containing one or more terms with non-zero coefficients is called a multinomial.

11.1 Like and unlike terms:

Observe the following terms.

$2x$, $3x^2$, $4x$, $-5x$, $7x^3$

Among these $2x$, $4x$ and $-5x$ have same variable with same exponent. These are called like terms. Like terms may not have same numerical coefficients. Why $8p$ and $8q$ are not like? Why $8p$ and $8pq$ are not like? Why $8p$ and $8p^2$ are not like?

Do This

1. Find the like terms in the following

   ax^2y, $2x$, $5y^2$, $-9x^2$, $-6x$, $7xy$, $18y^2$.

2. Write 3 like terms for $5pq^2$

11.2 Addition and subtraction of algebraic expressions:

Example:1 Add $5x^2 + 3xy + 2y^2$ and $2y^2 - xy + 4x^2$

Solution: Write the expression one under another so that like terms align in columns. Then add

$$\begin{array}{l} 5x^2 + 3xy + 2y^2 \\ \underline{+\ 4x^2 - xy + 2y^2} \\ 9x^2 + 2xy + 4y^2 \end{array}$$

Think, Discuss and Write 

1. Sheela says the sum of $2pq$ and $4pq$ is $8p^2q^2$ is she right ? Give your explanation.
2. Rehman added $4x$ and $7y$ and got $11xy$. Do you agree with Rehman ?

Example:2 Subtract $2xy + 9x^2$ from $12xy + 4x^2 - 3y^2$

Solution: Write the expressions being subtracted (subtrahend) below the expression from which it is being subtracted (minuend) aligning like term in columns.

minuend	$12xy + 4x^2 - 3y^2$
subtrahend	$2xy + 9x^2$
	$(-)\quad(-)$
	$10xy - 5x^2 - 3y^2$

Change the signs of each term in the expression being subtracted then add.

[*Note* : Subtraction of a number is the same as addition of its additive inverse. Thus subtracting -3 is the same as adding $+3$. Similarly subtracting $9x^2$ is the same as adding $-9x^2$, subtracting $-3xy$ is same as adding $+3xy$].

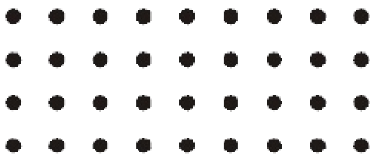
Do This

1. If $A = 2y^2 + 3x - x^2$, $B = 3x^2 - y^2$ and $C = 5x^2 - 3xy$ then find
   (i) A + B (ii) A – B (iii) B + C (iv) B – C (v) A + B + C (vi) A + B – C

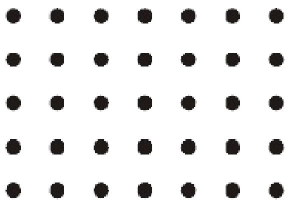
11.3 Multiplication of Algebraic Expressions:

Introduction: (i) Look at the following patterns of dots.

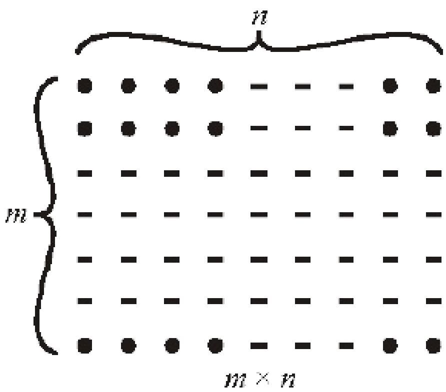
Pattern of dots — Total number of dots Row × Column



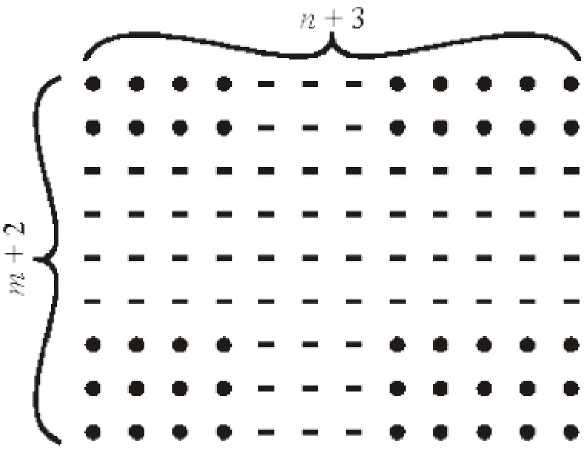
4×9



5×7

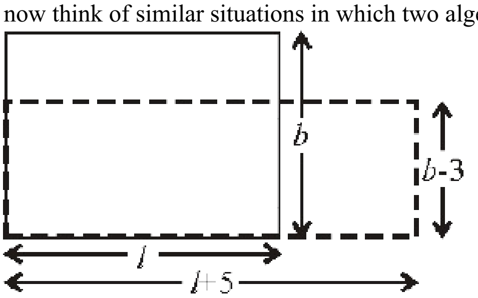


$m \times n$ To find the number of dots we have to multiply the number of rows by the number of columns.



$(m + 2) \times (n + 3)$ Here the number of rows is increased by 2, i.e. m+2 and number of columns increased by 3, i.e. n+3

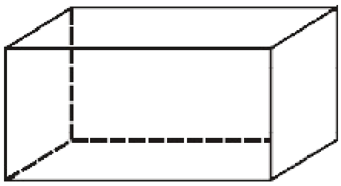
(ii) Can you now think of similar situations in which two algebraic expressions have to be multiplied?



We can think of area of a rectangle. The area of a rectangle is $l \times b$, where l is the length, and b is breadth. If the length of the rectangle is increased by 5 units, i.e., $(l + 5)$ and breadth is decreased by 3 units , i.e., $(b – 3)$ units, then the area of the new rectangle will be $(l + 5) \times (b – 3)$ sq. units.

To find the area of a rectangle. We have to multiply algebraic expression like $l \times b$ and extended as $(l+5) \times (b–3)$.

(iii) Can you think about volume of a cuboid in the form of algebraic expression? (The volume of a rectangular box is given by the product of its length, breadth and height).



(iv) When we buy things, we have to carry out multiplication.
For example, if price of bananas per dozen is ` p
and bananas needed for the school picnic are z dozens,
then we have to pay = ` p × z
Suppose, the price per dozen was less by ` 2 and the bananas needed were less by 4 dozens.
The price of bananas per dozen = ` (p – 2) and
bananas needed = $(z – 4)$ dozens,
Therefore, we would have to pay = ` (p – 2) × (z – 4)

Try These

Write an algebriac expression using speed and time; simple interest to be paid, using principal and the rate of simple interest.

Can you think of two more such situations, where we can express in algebraic expressions?

In all the above examples, we have to carry out multiplication of two or more quantities. If the quantities are given by algebraic expressions, we need to find their product. This means that we should know how to obtain this product. Let us do this systematically. To begin with we shall look at the multiplication of two monomials.

11.4 Multiplying a monomial by a monomial

11.4.1 Multiplying two monomials

We know that

$$4 \times x = x + x + x + x = 4x$$

and $$4 \times (3x) = 3x + 3x + 3x + 3x = 12x$$

Now, observe the following products.

(i) $x \times 3y = x \times 3 \times y = 3 \times x \times y = 3xy$

(ii) $5x \times 3y = 5 \times x \times 3 \times y = 5 \times 3 \times x \times y = 15xy$

(iii) $5x \times (-3y) = 5 \times x \times (-3) \times y$

$= 5 \times (-3) \times x \times y = -15xy$

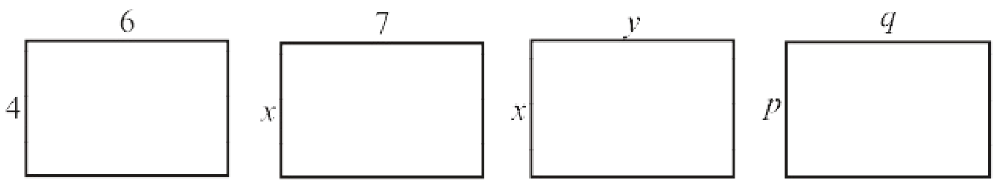
(iv) $5x \times 4x^2 = (5 \times 4) \times (x \times x^2)$

$= 20 \times x^3 = 20x^3$

(v) $5x \times (-4xyz) = (5 \times -4) \times (x \times xyz)$

$= -20 \times (x \times x \times yz) = -20x^2yz$

For finding the product of algebraic terms we add the power of same base variables, we use the rules of exponents.
Observe the following and fill the blanks.



Area = 4 × 6 = 24 square units Area $x \times 7$ = ... Area = $x \times y$ =...... Area = ... ×..... = ...

Observe the following products:-

1. $7x \times 5y = (7 \times 5) \times (x \times y) = 35xy$
2. $3x \times (-2y) = \{3 \times (-2)\} \times (x \times y) = -6xy$
3. $(-4x) \times (-6y) = (-4) \times (-6) \times (x \times y) = 24xy$
4. $3x \times 5x^2 = (3 \times 5) \times (x \times x^2) = 15x^3$
5. $(-2x^2) \times (-4x^2) = (-2) \times (-4) \times x^2 \times x^2 = 8x^4$

Note (i) Product of two positive numbers is a positive number.
(ii) Product of two negative numbers is a positive number.
(iii) Product of a positive and a negative numbers is a negative number.

Do This

1. Complete the table:

1st Monomial	2nd Monomial	Product of two monomials
$2x$	$-3y$	$2x \times (-3y) = -6xy$
$-4y^2$	$-2y$	…….
$3abc$	$5bcd$	…….
mn	$-4m$	…….
$-3mq$	$-3nq$	…….

2. Check whether you always get a monomial when two monomials are multiplied.
3. Product of two monomials is a monomial ? Check

11.4.2 Multiplying three or more monomials

Observe the following examples:-

Example 3: Find the product of $5x$, $6y$ and $7z$

Solution: Method I

$5x \times 6y \times 7z = (5x \times 6y) \times 7z$

$= 30xy \times 7z$

$= 210\ xyz$

Method II

$5x \times 6y \times 7z = 5 \times x \times 6 \times y \times 7 \times z$

$= 5 \times 6 \times 7 \times x \times y \times z$

$= 210\ xyz$

Example 4: Find $3x^2y \times 4xy^2 \times 7x^3y^3$

Solution: $= 3 \times 4 \times 7 \times (x^2y) \times (xy^2) \times (x^3y^3)$

$= 84 \times x^2 \times y \times x \times y^2 \times x^3 \times y^3$

$= 84 \times (x^2 \times x \times x^3) \times (y \times y^2 \times y^3)$

$= 84 \times x^6 \times y^6 = 84x^6y^6$.

Example 5: Find the product of $3x$, $-4xy$, $2x^2$, $3y^2$, $5x^3y^2$

Solution: $3x \times (-4xy) \times 2x^2 \times 3y^2 \times 5x^3y^2$

$= [3 \times (-4) \times 2 \times 3 \times 5] \times (x \times x \times x^2 \times x^3) \times (y \times y^2 \times y^2)$

$= -360x^7y^5$.

Have to observe that the product of any number of monomials is a monomial?

Exercise - 11.1

1. Find the product of the following pairs:

(i) 6, $7k$ (ii) $-3l$, $-2m$ (iii) $-5t^2$ $-3t^2$ (iv) $6n$, $3m$ (v) $-5p^2$, $-2p$

2. Complete the table of the products.

X	$5x$	$-2y^2$	$3x^2$	$6xy$	$3y^2$	$-3xy^2$	$4xy^2$	x^2y^2
$3x$	$15x^2$	….	….	….	….	….	….	….
$4y$	….	….	….	….	….	….	….	….
$-2x^2$	$-10x^3$	$4x^2y^2$	….	….	….	….	….	….
$6xy$	….	….	….	….	….	….	….	….
$2y^2$	….	….	….	….	….	….	….	….
$3x^2y$	….	….	….	….	….	….	….	….
$2xy^2$	….	….	….	….	….	….	….	….
$5x^2y^2$	....	....	....	....	....	....	...	. ....

3. Find the volumes of rectangular boxes with given length, breadth and height in the following table.

S.No.	**Length**	**Breadth**	**Height**	**Volume (v) = $l \times b \times h$**
(i)	$3x$	$4x^2$	5	$v = 3x \times 4x^2 \times 5 = 60x^3$
(ii)	$3a^2$	4	$5c$	v = …………………
(iii)	$3m$	$4n$	$2m^2$	v = …………………
(iv)	$6kl$	$3l^2$	$2k^2$	v = …………………
(v)	$3pr$	$2qr$	$4pq$	v = …………………

4. Find the product of the following monomials

(i) xy, x^2y, xy, x (ii) a, b, ab, a^3b, ab^3 (iii) kl, lm, km, klm

(iv) pq ,pqr, r (v) $-3a$, $4ab$, $-6c$, d

5. If A = xy, B = yz and C = zx, then find ABC = ...

6. If P = $4x^2$, T = $5x$ and R = $5y$, then $\frac{PTR}{100} =$ ....

7. Write some monomials of your own and find their products .

11.5 Multiplying a binomial or trinomial by a monomial

11.5.1 Multiplying a binomial by a monomial

Multiplying a monomial $5x$ and a binomial $6y+3$

The process involved in the multiplication is:

Step	Instruction	Procedure
1.	Write the product of monomial and binomial using multiplication symbol	$5x \times (6y+3)$
2.	**Use distributive law:** Multiply the monomial by the first term of the binomial then multiply the monomial by the second term of the binomial and add their products.	$(5x \times 6y) + (5x \times 3)$
3.	Simplify the terms	$30xy + 15x$

Hence, the product of $5x$ and $6y+3$

$$5x(6y + 3) = 5x \times (6y + 3)$$
$$= (5x \times 6y) + (5x \times 3)$$
$$= 30xy + 15x$$

Example6: Find the product of $(-4xy)(2x - y)$

Solution: $(-4xy)(2x - y) = (-4xy) \times (2x - y)$

$$= (-4xy) \times 2x + (-4xy) \times (-y)$$
$$= -8x^2y + 4xy^2$$

Example7: Find the product of $(3m - 2n^2)(-7mn)$

Solution: $(3m - 2n^2)(-7mn) = (3m - 2n^2) \times (-7mn)$

$$= (-7mn) \times (3m - 2n^2) \quad \because \text{Commutative law}$$
$$= ((-7mn) \times 3m) - ((-7mn) \times 2n^2)$$
$$= -21m^2n + 14mn^3$$

Do This

1. Find the product: (i) $3x(4ax + 8by)$ (ii) $4a^2b(a-3b)$ (iii) $(p + 3q^2)pq$ (iv) $(m^3 + n^3)5mn^2$
2. Find the number of maximum terms in the product of a monomial and a binomial?

11.5.2 Multiplying a trinomial by a monomial

Consider a monomial $2x$ and a trinomial $(3x + 4y - 6)$

Their product = $2x \times (3x + 4y - 6)$

$$= (2x \times 3x) + (2x \times 4y) + (2x \times (-6)) \text{ (by using distributive law)}$$

$= 6x^2 + 8xy - 12x$

How many maximum terms are there in the product of a monomial and a trinomial?

Exercise - 11.2

1. Complete the table:

S.No.	First Expression	Second Expression	Product
1	$5q$	$p+q-2r$	$5q(p+q-2r)=5pq+5q^2-10qr$
2	$kl+lm+mn$	$3k$	………………………………
3	ab^2	$a+b^2+c^3$	………………………………
4	$x-2y+3z$	xyz	………………………………
5	$a^2bc+b^2cd-abd^2$	$a^2b^2c^2$	………………………………

2. Simplify: $4y(3y+4)$
3. Simplify $x(2x^2-7x+3)$ and find the values of it for (i) $x = 1$ and (ii) $x = 0$
4. Add the product: $a(a-b), b(b-c), c(c-a)$
5. Add the product: $x(x+y-r), y(x-y+r), z(x-y-z)$
6. Subtract the product of $2x(5x-y)$ from product of $3x(x+2y)$
7. Subtract $3k(5k-l+3m)$ from $6k(2k+3l-2m)$
8. Simplify: $a^2(a-b+c)+b^2(a+b-c)-c^2(a-b-c)$

11.6 Multiplying a binomial by a binomial or trinomial

11.6.1 Multiplying a binomial by a binomial:

Consider two binomials as $5x+6y$ and $3x - 2y$

Now, the product of two binomials $5x+6y$ and $3x - 2y$

The procedure of multiplication is:

Step	Instructions	Procedure
1.	Write the product of two binomials	$(\underline{5x}+\underline{6y})(3x-2y)$
2.	Use distributive law:Multiply the first term of the first binomial by the second binomial, multiply the second term of the first binomial by the second binomial and add the products.	$\underline{5x}(3x-2y)+\underline{6y}(3x-2y)$ $= (5x\times3x)-(5x\times2y) + (6y\times3x)-(6y\times2y)$
3.	Simplify	$(5x\times3x)-(5x\times2y) + (6y\times3x)-(6y\times2y)$ $=15x^2 - 10xy +18xy - 12y^2$
4.	Add like terms	$15x^2+8xy-12y^2$

Hence, the product of $5x+6y$ and $3x - 2y$

$= (5x + 6y)(3x - 2y)$

$= 5x(3x - 2y) + 6y(3x - 2y)$ (by using distribution)

$= (5x \times 3x) - (5x \times 2y) + (6y\times3x)-(6y\times2y)$

$=15x^2- 10xy+ 18xy - 12y^2$

$= 15x^2+8xy - 12y^2$

Do This

1. Find the product:

(i) $(a - b)(2a + 4b)$ (ii) $(3x + 2y)(3y - 4x)$

(iii) $(2m - l)(2l - m)$ (iv) $(k + 3m)(3m - k)$

2. How many number of terms will be there in the product of two binomials?

11.6.2 Multiplying a binomial by a trinomial

Consider a binomial $2x + 3y$ and trinomial $3x + 4y - 5z$.

Now, we multiply $2x + 3y$ by $3x + 4y - 5z$.

The process of the multiplication is:

Step	**Instructions**	**Process**
1.	Write the products of the binomials and trinomial using multiplicative symbol	$(2x+3y)(3x+4y-5z)$
2.	Use distributive law: Multiply the first term of the binomial by the trinomial and multiply the second term of the binomial by the trinomial and then add the products.	$2x(3x+4y-5z)+3y(3x+4y-5z)$

3.	Simplify	$(2x\times3x)+(2x\times4y)-(2x\times5z)+$ $(3y\times3x)+(3y\times4y)-(3y\times5z)$ $6x^2+8xy-10xz+9xy+12y^2-15yz$
4.	Add like terms	$6x^2+17xy-10xz+12y^2-15yz$

Hence, the product *of* $(2x+3y)$ and $(3x+4y-5z)$ can be written as

$= (2x+3y)(3x+4y-5z)$

$= 2x(3x+4y-5z)+3y(3x+4y-5z)$ (by using distributive law)

$= (2x\times3x)+(2x\times4y)-(2x\times5z)+(3y\times3x)+(3y\times4y)-(3y\times5z)$

$=6x^2+8xy-10xz+9xy+12y^2-15yz$

$= 6x^2+17xy-10xz+12y^2-15yz$

How many maximum number of terms we get in the products of a binomial and a trinomial?

Exercise - 11.3

1. Multiply the binomials:

(i) $2a-9$ and $3a+4$ (ii) $x-2y$ and $2x-y$

(iii) $kl+lm$ and $k-l$ (iv) m^2-n^2 and $m+n$

2. Find the product:

(i) $(x+y)(2x-5y+3xy)$ (ii) $(a-2b+3c)(ab^2-a^2b)$

(iii) $(mn-kl+km)(kl-lm)$ (iv) $(p^3+q^3)(p-5q+6r)$

3. Simplify the following :

(i) $(x-2y)(y-3x)+(x+y)(x-3y)-(y-3x)(4x-5y)$

(ii) $(m+n)(m^2-mn+n^2)$

(iii) $(a-2b+5c)(a-b)-(a-b-c)(2a+3c)+(6a+b)(2c-3a-5b)$

(iv) $(pq-qr+pr)(pq+qr)-(pr+pq)(p+q-r)$

4. If a, b, c are positive real numbers such that $\frac{a+b-c}{c}=\frac{a-b+c}{b}=\frac{-a+b+c}{a}$, find the value of $\frac{(a+b)(b+c)(c+a)}{abc}$.

11.7 What is an identity?

Consider the equation $a(a-2)=a^2-2a$

Evaluate the both sides of the equation for any value of a

For $a=5$, LHS $= 5(5-2) = 5\times3 = 15$

RHS $= 5^2-2(5) = 25-10 = 15$

Hence, in the equation LHS = RHS for $a=5$.

Similarly for $a=-2$

LHS $= (-2)(-2-2) = (-2)\times(-4) = 8$

RHS $= (-2)^2-2(-2) = 4+4 = 8$

Thus, in the equation LHS = RHS for $a=-2$ *also.*

We can say that the equation is true for any value of a. Therefore, the equation is called an identity.

Consider an equation $a(a+1) = 6$

This equation is true only for $a = 2$ and -3 but it is not true for other values. So, this $a(a+1) = 6$ equation is not an identity.

An equation is called an identity if it is satisfied by any value that replaces its variable(s).

An equation is true for certain values for the variable in it, where as an identity is true for all its variables. Thus it is known as universally true equation.

We use symbol for denoting identity is '$\equiv$' (read as identically equal to)

11.8 Some important Identities:

We often use some of the identities, which are very useful in solving problems. Those identities used in multiplication are also called as special products. Among them, we shall study three important identities, which are products of a binomial.

Consider $(a+b)^2$

Now,

$(a+b)^2 = (a+b)(a+b)$

$= a(a+b) + b(a+b)$

$= a^2 + ab + ba + b^2 = a^2 + ab + ab + b^2$ (since $ab = ba$)

$= a^2 + 2ab + b^2$

Thus $(a+b)^2 = a^2 + 2ab + b^2$ (I)

Now, take a=2, b=3, we obtain (LHS) $= (a+b)^2 = (2+3)^2 = 5^2 = 25$

(RHS) $= a^2 + 2ab + b^2 = 2^2 + 2(2)(3) + 3^2 = 4 + 12 + 9 = 25$

Observe the LHS and RHS. The values of the expressions on the LHS and RHS are equal.

Verify Identity-I for some positive integer, negative integer and fraction

Do This:

Verify the following are identities by taking a, b, c as positive integers.

(i) $(a-b)^2 \equiv a^2 - 2ab + b^2$

(ii) $(a+b)(a-b) \equiv a^2 - b^2$

(iii) $(a+b+c)^2 \equiv a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Consider one more identity, $(x+a)(x+b) \equiv x^2 + (a+b)x + ab$,

$$(x+a)(x+b) = x(x+b) + a(x+b)$$
$$= x^2 + bx + ax + ab$$
$$= x^2 + (a+b)x + ab$$

Do This

Now take $x = 2$, $a = 1$ and $b = 3$, verify the identity.

- What do you observe? Is LHS = RHS?
- Take different values for *x, a and b* for verification of the above identity.
- Is it always LHS = RHS for all values of a and b?
- Consider $(x+p)(x+q) = x^2 + (p+q)x + pq$
  - (i) Put q instead of 'p' what do you observe?
  - (ii) Put p instead of 'q' what do you observe?
  - (iii) What identities you observed in your results?

11.9 Application of Identities:

Example 8: Find $(3x + 4y)^2$

Solution: $(3x + 4y)^2$ is the product of two binomial expressions, which have the same terms $(3x + 4y)$ and $(3x + 4y)$. It can be expanded by the method of multiplying a binomial by a binomial. Compare the identities with this product. In this product $a = 3x$ *and* b $= 4y$. We can get the result of this product by substituting $3x$ and $4y$ terms in the place of a and b respectively in the first identity $(a+b)^2 = a^2+2ab+b^2$

Hence, $(3x + 4y)^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$ *Where a = 3x and b = 4y* identity $(a+b)^2 = a^2 + 2ab + b^2$

$$= 9x^2 + 24xy + 16y^2$$

Example 9: Find 204^2

$$204^2 = (200+4)^2$$
$$= (200)^2 + 2(200)(4) + 4^2$$ *Where a = 200 and b = 4* identity $(a+b)^2 = a^2 + 2ab + b^2$

$$= 40000 + 1600 + 16$$
$$= 41616$$

Do This

Find: (i) $(5m + 7n)^2$ (ii) $(6kl + 7mn)^2$ (iii) $(5a^2 + 6b^2)^2$ (iv) 302^2
(v) 807^2 (vi) 704^2

(vii) Verify the identity : $(a-b)^2 = a^2 - 2ab + b^2$, where $a = 3m$ *and* $b = 5n$

Example10: Find $(3m - 5n)^2$

Solution: $(3m - 5n)^2 = (3m)^2 - 2(3m)(5n) + (5n)^2$ *Where a = 3m and b = 5n* identity: $(a-b)^2 = a^2 - 2ab + b^2$

$$= 9m^2 - 30mn + 25n^2$$

Example11: Find 196^2

Solution: $196^2 = (200 - 4)^2$

$= 200^2 - 2(200)(4) + 4^2$ *Where a = 200 and b = 4* identity: $(a-b)^2 = a^2 - 2ab + b^2$

$= 40000 - 1600 + 16$

$= 38416$

Do This

Find: (i) $(9m - 2n)^2$ (ii) $(6pq - 7rs)^2$ (iii) $(5x^2 - 6y^2)^2$
(iv) 292^2 (v) 897^2 (vi) 794^2

Example:12: Find $(4x + 5y)(4x - 5y)$

Solution: $(4x + 5y)(4x - 5y) = (4x)^2 - (5y)^2$ Where $a = 4x$ and $b = 5y$ identity: $(a + b)(a - b) = a^2 - b^2$

$= 16x^2 - 25y^2$

Example:13: Find 407 × 393

Solution: 407 × 393 = (400 + 7)(400 – 7)

$= 400^2 - 7^2$ Where $a = 400$ and $b = 7$ in the identity: $(a + b)(a - b) = a^2 - b^2$

= 160000 – 49

= 159951

Example:14: Find $987^2 - 13^2$

Solution: $987^2 - 13^2 = (987 + 13)(987 - 13)$ Where $a = 987$ and $b = 13$ in the identity: $a^2 - b^2 = (a + b)(a - b)$

= 1000 × 974 = 974000

Do These

Find: (i) $(6m + 7n)(6m - 7n)$ (ii) $(5a + 10b)(5a - 10b)$
(iii) $(3x^2 + 4y^2)(3x^2 - 4y^2)$ (iv) 106 × 94 (v) 592 × 608 (vi) $92^2 - 8^2$
(vii) $984^2 - 16^2$

Example15: Find 302 × 308

Solution: 302 × 308 = (300 + 2)(300 + 8)

$= 300^2 + (2 + 8)(300) + (2)(8)$ Where $x = 300$, $a = 2$ and $b = 8$ in the identity: $(x + a)(x + b) = x^2 + (a + b)x + ab$

= 90000 + (10 × 300) + 16

= 90000 + 3000 + 16 = 93016

Example16: Find 93 × 104

Solution: 93 × 104 = (100 + (−7))(100 + 4)

93 × 104 = (100 − 7)(100 + 4)

$= 100^2 + (-7 + 4)(100) + (-7)(4)$ Where $x = 100$ $a = -7$ and $b = 4$ in the identity: $(x + a)(x + b) = x^2 + (a+b)x + ab$

= 10000 + (−3)(100) + (−28)

= 10000 – 300 – 28

= 10000 – 328 = 9672

Do you notice? Finding the products by using identities is much easier than finding by direct multiplication.

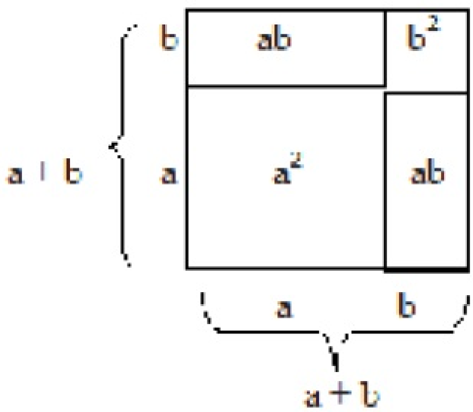
Exercise - 11.4

1. Select a suitable identity and find the following products
   (i) $(3k + 4l)(3k + 4l)$ (ii) $(ax^2 + by^2)(ax^2 + by^2)$
   (iii) $(7d - 9e)(7d - 9e)$ (iv) $(m^2 - n^2)(m^2 + n^2)$
   (v) $(3t + 9s)(3t - 9s)$ (vi) $(kl - mn)(kl + mn)$
   (vii) $(6x + 5)(6x + 6)$ (viii) $(2b - a)(2b + c)$
2. Evaluate the following by using suitable identities:
   (i) 304^2 (ii) 509^2 (iii) 992^2 (iv) 799^2
   (v) 304 × 296 (vi) 83 × 77 (vii) 109×108 (viii) 204×206

11.10 Geometrical Verification of the identities

11.10.1 Geometrical Verification of the identity $(a + b)^2 = a^2 + 2ab + b^2$

Observe the following square:



Consider a square with side $(a + b)$

Its area = square of the side = $(\text{side})^2$ = $\mathbf{(a + b)^2}$

Divided the square into four regions as shown in figure.

It consists of two squares with sides ‘a’ and ‘b’ respectively and two rectangles with length and breadth as ‘a’ and ‘b’ respectively.
Clearly, the area of the given square is equal to sum of the area of four regions.
Area of the given square

= Area of the square with side a + area of rectangle with sides a and b + area of rectangle with sides b and a + area of square with side b

$= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$

Therefore, $(a + b)^2 = a^2 + 2ab + b^2$

Example17: Verify the identity $(a + b)^2 = a^2 + 2ab + b^2$ geometrically by taking $a = 3$ and $b = 2$

Solution:

$(a + b)^2 = a^2 + 2ab + b^2$

Draw a square with the side $a + b$, i.e., 3 + 2

L.H.S. Area of whole square

$= (3 + 2)^2 = 5^2 = 25$

R.H.S. = Area of square with side 3 units +
Area of square with side 2 units +
Area of rectangle with sides 3, 2 units +
Area of rectangle with sides 2, 3 units

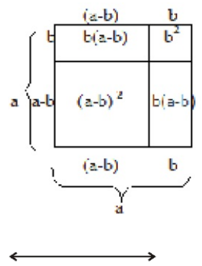
$= 3^2 + 2^2 + 3 \times 2 + 3 \times 2$

$= 9 + 4 + 6 + 6 = 25$

L.H.S. = R.H.S.

∴ Hence the identity is verified.

11.10.2 Geometrical Verification of the identity $(a - b)^2 = a^2 - 2ab + b^2$



a − b + b = a

Consider a square with side a.

- The area of the square = side × side = a^2
- The square is divided into four regions.
- It consists of two squares with sides a − b and b respectively and two rectangles with length and breadth as ‘a − b’ and ‘b’ respectively.

Now Area of figure I = Area of whole square with side ‘a’ −
Area of figure II − Area of figure III − Area of figure IV

$(a–b)^2 = a^2 – b\,(a–b) – b\,(a–b) – b^2$

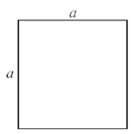
$= a^2 – ab + b^2 – ab + b^2 – b^2$

$= a^2 – 2ab + b^2$

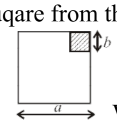
11.10.3 Geometrical Verification of the identity $(a + b)(a – b) = a^2 - b^2$

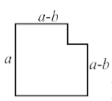
$a^2 – b^2$ = (Area of square where the side is ‘a’) – (Area of square where the side is ‘b’)

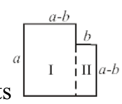
Observe the following square:



Remove suqare from this whose side is b ($b < a$)



We get


It consist of two parts


So $a^2 – b^2$ = Area of figure I + area of figure II

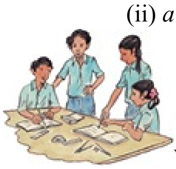
$= a\,(a – b) + b\,(a – b)$

$= (a – b)\,(a + b)$

Thus $a^2 – b^2 = (a – b)\,(a + b)$

Exercise - 11.5

1. Verify the identity $(a + b)^2 = a^2 + 2ab + b^2$ geometrically by taking
   (i) $a = 2$ units, $b = 4$ units
   (ii) $a = 3$ units, $b = 1$ unit
   (iii) $a = 5$ units, $b = 2$ unit
2. Verify the identity $(a - b)^2 = a^2 - 2ab + b^2$ geometrically by taking
   (i) $a = 3$ units, $b = 1$ unit
   (ii) $a = 5$ units, $b = 2$ units
3. Verify the identity $(a + b)(a - b) = a^2 - b^2$ geometrically by taking
   (i) $a = 3$ units, $b = 2$ units
   (ii) $a = 2$ units, $b = 1$ unit



What we have discussed

1. There are number of situations in which we need to multiply algebraic expressions.
2. A monomial multiplied by a monomial always gives a monomial.
3. While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
4. In carrying out the multiplication of an algebraic expression with another algebraic expression (monomial / binomial / trianomial etc.) we multiply term by term i.e. every term of the expression is multiplied by every term in the another expression.
5. An **identity** is an equation, which is true for all values of the variables in the equation. On the other hand, an equation is true only for certain values of its variables. An equation is not an identity.
6. The following are identities:
   I. $(a + b)^2 = a^2 + 2ab + b^2$
   II. $(a - b)^2 = a^2 - 2ab + b^2$
   III. $(a + b)(a - b) = a^2 - b^2$
   IV. $(x + a)(x + b) = x^2 + (a + b)x + ab$
7. The above four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.