# Chapter 1

# Determinacy and Indeterminacy

## CHAPTER HIGHLIGHTS

- *Introduction*
- *Statically determinate structure*
- *Statically indeterminate structure*
- *Degree of indeterminacy or redundancy ($D_s$)*
- *Instability of structures*
- *Degrees of freedom (or) kinematic indeterminacy*
- *Formulation of degree of kinematic indeterminacy ($D_k$)*

## Introduction

In this chapter, the concept of determinate and indeterminate structures are explained and also the formulations for finding the degree of static and kinematic Indeterminacy are explained for beams, plane frames, space frames, plane truss and space truss with and without releases.

## Statically Determinate Structure

Structures which can be fully analysed by the conditions of equilibrium are known as determinate structures.

**Example:** A simply supported beam, a cantilever beam, etc.

### Equations of Equilibrium

- In general, for a space frame, there are six equilibrium equations

$$\Sigma F_x = 0; \quad \Sigma F_y = 0; \quad \Sigma F_z = 0$$
$$\Sigma M_x = 0; \quad \Sigma M_y = 0; \quad \Sigma M_z = 0$$

- For a plane frame and also the loads are in same plane (i.e., coplanar); the above equations are reduced to 3.

$$\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma M_z = 0$$

### *Pin Jointed Plane Frame (Plane Truss)*

- If all the members lie in one plane, it is called a plane frame.
- The simplest stable form of a truss is triangle.
- For a statically determinate plane truss,

$$\boxed{m + r = 2j}$$

Where

$m$ = Number of members
$r$ = Number of reaction components
$j$ = Number of joints

### Rigid Jointed Plane Frame

- For a statically determinate frame,

$$\boxed{\text{Number of unknowns} = 3 \times \text{Number}}$$

**NOTES**

1. For a rigid joint of a plane frame, there are 3 equilibrium equations at each joint.
2. For a pin joint of a plane frame, there are 2 equilibrium equations at each joint.

### *Pin Jointed Space Frame (Space Truss)*

- The basic element of a space truss is tetrahedron with four joints.
- The necessary condition for the space truss to be statically determinate is

$$\boxed{m + r = 3j}$$

Where

$m$ = Number of members;

$r$ = Number of reaction components

$j$ = Number of joints

### Rigid Jointed Space Frame

- The necessary condition for it to be statically determinate is

$$\boxed{\text{Number of unknowns} = 6 \times \text{Number of joints}}$$

**NOTES**

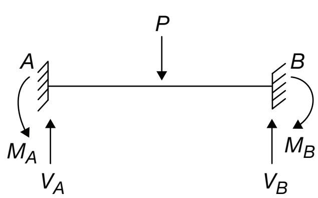
1. For a pin joint of a space frame, there are 3 equilibrium equations.
2. For a rigid joint of a space frame, there are 6 equilibrium equations.

## STATICALLY INDETERMINATE STRUCTURE

- Structures which cannot be fully analysed by the conditions of equilibrium are known as indeterminate structures.

  **Example:** Continuous beam, two hinged arch, etc.
- Also termed as a redundant structure
- Incase of indeterminate structure, the number of unknowns are more than the number of equilibrium equations.
- Additional equations called conditions of compatibility or consistent displacements shall be used for complete analysis.

**Example:** A fixed beam subjected to vertical point load $P$ at centre has 4 unknowns (two at each support—one moment and one vertical reaction). The number of equilibrium equations available for this case are only two ($\Sigma F_y = 0$ and $\Sigma M = 0$). Therefore, the number of unknowns are greater than the number of equilibrium equations comes under indeterminate structure.




Number of unknowns = 4($V_A$, $M_A$; $V_B$, $M_B$)
Number of equilibrium equations = 2
($\Sigma F_y = 0$ and $\Sigma M = 0$)
Number of unknowns > Number of equilibrium equations
⇒ Statically indeterminate.

## DEGREE OF INDETERMINACY OR REDUNDANCY ($D_s$)

- Degree of indeterminacy or redundancy is the number of additional equations needed in addition to the equilibrium equations to determine the various external and internal reactions.
- Statically indeterminate structures may be classified into externally indeterminate structures and internally indeterminate structures.

### Formulation of $D_s$

$$\boxed{D_s = D_{si} + D_{se}}$$

Where

$D_{si}$ = Internal indeterminacy due to the internal reactions.

$D_{se}$ = External indeterminacy due to external reactions offered by the supports.

### External Indeterminacy ($D_{se}$)

$D_{se} = r - 3$, for a plane frame

$= r - 6$, for a space frame

Where

$r$ = Number of reaction components

### Internal Indeterminacy ($D_{si}$)

$D_{si} = m - (2j - 3)$, for a pin jointed plane frame

$= m - (3j - 6)$, for pin jointed space frame

$= 3c$, for rigid jointed plane frame

$= 6c$, for rigid jointed space frame

Where

$c$ = Number of cuts required for obtaining an open tree configuration

= Number of closed boxes

$m$ = Number of members

$j$ = Number of joints

**Another method:**

$D_s$ = Number of unknowns – Equilibrium equations

**For a pin jointed plane frame:**

- Number of unknowns = $(m + r)$
- Number of equilibrium equations = $2 \times j$

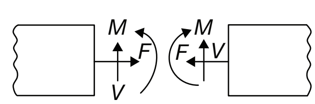
$$\boxed{D_s = (m + r) - 2j}$$

**For a pin jointed space frame:**

$$\boxed{D_s = (m + r) - 3i}$$

**For a rigid jointed plane frame:**

In case of rigid jointed plane frame, there are three unknown member forces at any cut in the member.




Therefore, total number of unknown forces = $3m + r$

$$\boxed{D_s = (3m + r) - 3j}$$

**For a rigid jointed space frame:**

In case of rigid jointed space frame, there are six unknowns at any cut in the member.

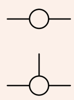
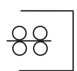
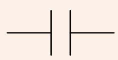
Therefore, total number of unknown forces = $6m + r$

$$\boxed{D_s = (6m + r) - 6j}$$

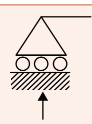
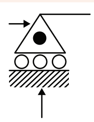
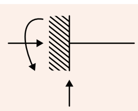
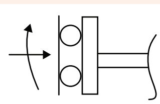
**NOTE**

The above degree of indeterminacy is to be modified in case of moment releases or force releases to account for additional condition equations.

$$\boxed{D_s' = D_s - C}$$

| Releases | Symbol | Number of Additional Equations (*C*) |
|---|---|---|
| Moment release or Internal pin or Internal hinge |  | 1 [$\Sigma M = 0$]<br>2 |
| Axial force release [Horizontal sleeve] |  | 1 [$\Sigma F_x = 0$] |
| Vertical sleeve |  | 1[$\Sigma F_y = 0$] |
| Internal link (short bar with hinges at each end) |  | $2\begin{bmatrix}\Sigma M = & 0\\ \Sigma F = & 0\end{bmatrix}$ |

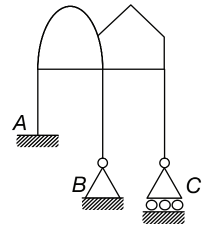
**Various supports and their reactions**

| Type of Support | Symbol | Number of Reactions |
|---|---|---|
| Roller support |  | 1 |
| Hinged support | <br>or<br> | 2 |
| Fixed support |  | 3 |
| Vertical shear release support (or) slider |  | 2 |

## SOLVED EXAMPLES

### Example 1

The static indeterminacy of the structure shown in the following figure is ________.




(A) 3 (B) 6

(C) 9 (D) 12

### Solution

**First method:**

$$D_s = D_{se} + D_{si}$$
$$D_{se} = r - 3 \text{ (for plane frame)}$$

Where

$r$ = Number of reactions

= 3(fixed) + 2(hinge) + 1(roller)

$r = 6$

$D_{se} = 6 - 3 = 3$

$D_{si} = 3 \times c$, for rigid jointed plane frame

$c$ = Number of closed boxes = 2

$\therefore D_{si} = 3 \times 2 = 6$

$\therefore D_s = D_{se} + D_{si}$

$= 3 + 6$

$D_s = 9.$

Hence, the correct answer is option (C).

**Another method:**

$$D_s = (3m + r) - 3j$$

Where

$m$ = Number of members = 10

$r$ = Number of reactions = 6

$j$ = Number of joints = 9

$D_s = (3 \times 10 + 6) - (3 \times 9)$

$= 36 - 27$

$D_s = 9.$

- The following are some important differences between statically determinate and statically indeterminate structures

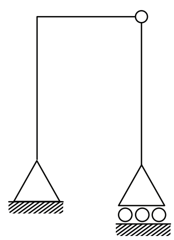
| Statically Determinate Structures | Statically Indeterminate Structures |
|---|---|
| Equilibrium equations are sufficient to analyse. | Equilibrium equations are not sufficient to fully analyse. |
| The bending moment at any section or force in any member (SF) is independent of the cross sectional area and material of the compo-nents of the structure. | The bending moment at a section or the force in any member depends on material and cross sectional areas of components. |
| No stresses are caused due to temperature changes and lack of fit. | Stress are caused. |

## Instability of Structures

- If there are fewer reactive forces than the equations of equilibrium, a structure will be geometrically unstable, i.e., it will move slightly or collapse.
- Instability occurs if the lines of action of the reactive forces intersect at a common point or are parallel to one another even there are enough reactions available.
- If the structure consists of several members or components, local instability of one or more members can generally be determined by inspection.
- In general, if $D_s < 0$, then it is unstable.

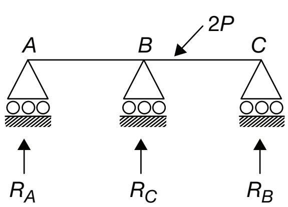
**Examples:**

1. Number of reactions < Equilibrium equations.

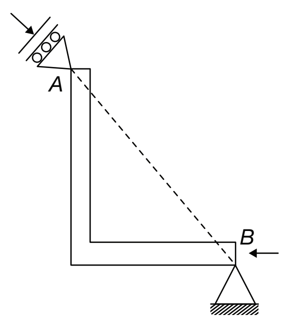


**Geometric instability**

2. Instability due to parallel reactions or concurrent reactions.

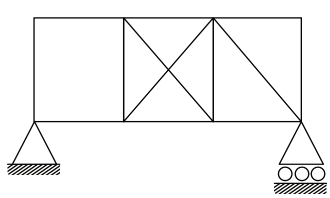



**Parallel reactions**



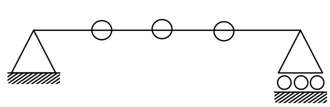

**Concurrent reactions at *B***

3. Improper arrangement of members



**Local instability (Due to no diagonal member in 1st panel)**

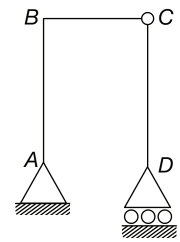
4. Due to the presence of hinges



**Internal instability due to hinges**

### Example 2

The plane frame shown the figure is ______. **[GATE, 1988]**




(A) statically indeterminate but unstable
(B) unstable
(C) determinate and stable
(D) None of these

### Solution

In this case, number of reactions ($r$) = 2(hinges) + 1(roller) = 3.
Number of equilibrium equations = 3 + 1(due to moment release) = 4.
Therefore number of reactions are less than the equilibrium equation leads to Geometric instability.
Hence, the correct answer is option (B).

## Degrees of Freedom (or) Kinematic Indeterminacy

1. When a structure is loaded, specified points on it, called nodes, will undergo unknown displacements known as degrees of freedom.
2. The number of unknown displacement components in addition to the number of compatibility conditions is known as degree of freedom or degree of kinematic indeterminacy of structure.
3. Nodal displacements may be restricted by supports or due to assumption based on behavior of the structure, i.e., inextensibility of members known as compatibility conditions.

**Degree of freedom (DOF) for different types of joints**

| | Type of Joint | DOF | Remarks |
|---|---|---|---|
| **1.** | Pin joint | | No rotation |
| | -plane frame | 2 | 2 translations |
| | -space frame | 3 | 3 translations |
| **2.** | Rigid joint | | |
| | -plane frame | 3 | 1 rotation<br>2 translations |
| | -space frame | 6 | 3 rotations<br>3 translations |

## Formulation of Degree of Kinematic Indeterminacy ($D_K$)

**For pin jointed plane frame:**

$D_k = [2j - r]$, if members are extensible

$D_k = [2j - (m + r)]$, if extension of members are neglected or axially rigid

**For pin jointed space frame:**

$D_k = [3j - r]$, if members are extensible

$D_k = [3j - (m + r)]$, neglecting axial strains of members

**For rigid jointed plane frame:**

$D_k = [3j - r]$, if members are extensible

$D_k = [3j - (m + r)]$, if members are axially rigid

**For rigid jointed space frame:**

$D_k = [6j - r]$, considering axial strains

$D_k = [6j - (m + r)]$, neglecting axial strains

Where

$r$ = Number of reactions
= 3 for fixed support
= 2 for pinned or hinged support
= 1 for roller support

$j$ = Number of joints

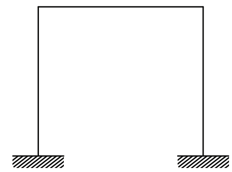
$m$ = Number of (members whole) extension is neglected.

**DOF for different types of supports of plane frame**

| | Type of Support | D | Remarks |
|---|---|---|---|
| **1.** | Fixed | 0 | – |
| **2.** | pinned | 1 | 1 Rotation ($\theta$) |
| **3.** | Roller | 2 | 1 Translation ($\delta_x$)<br>1 Rotation ($\theta$) |
| **4.** | free end | 3 | $\delta_x$, $\delta_y$, $\theta$ |
| **5.** | Vertical shear support (VSS) and Horizontal shear support (HSS) | 1 | $\delta_y$ for VSS<br>$\delta_x$ for HSS |

### Example 3

The kinematic indeterminacy of plane frame shown in the figure is **[GATE, 1994]**



(A) 1 (B) 2
(C) 3 (D) 0

### Solution

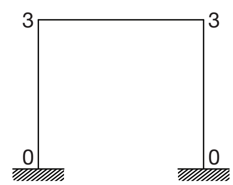
$$D_k = 3j - (m + r)$$
$$= 3(4) - (3 + 6)$$
$$D_k = 3.$$

Hence, the correct answer is option (C).

**Another method:**

Given rigid jointed plane frame

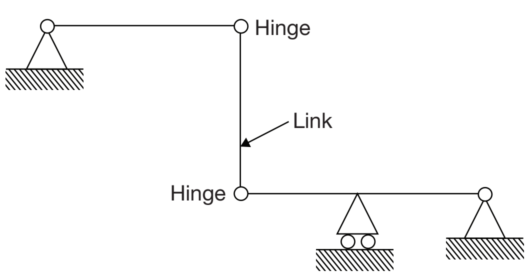
$$\text{DOF} = [0 + 3 + 3 + 0] - m$$
$$= 6 - 3 = 3$$
$$D_k = 3.$$




Hence, the correct answer is option (C).

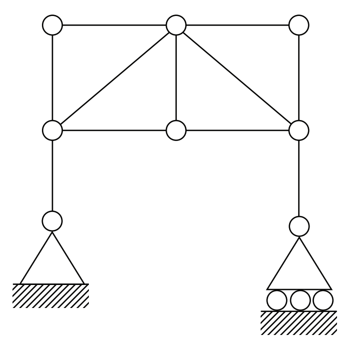
## Exercises

**1.** Degree of static indeterminacy of the structure as shown in the figure is




(A) 0
(B) 1
(C) 2
(D) 3

**2.** The structure shown below is ________.

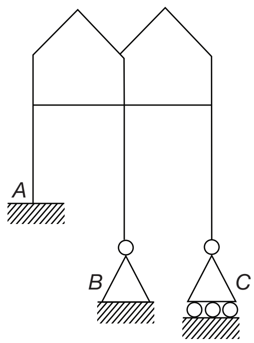


(A) externally indeterminate
(B) internally indeterminate
(C) determinate
(D) mechanism

**3.** A beam fixed at the ends and subjected to lateral loads only is statically indeterminate and the degree of indeterminacy is _______.

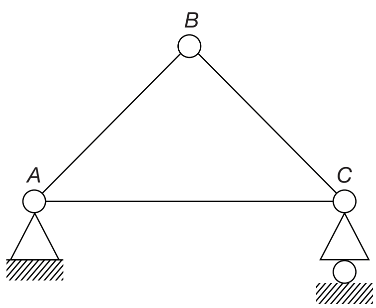
(A) one (B) two
(C) three (D) four

**4.** The static indeterminacy of the structure shown below is _______.

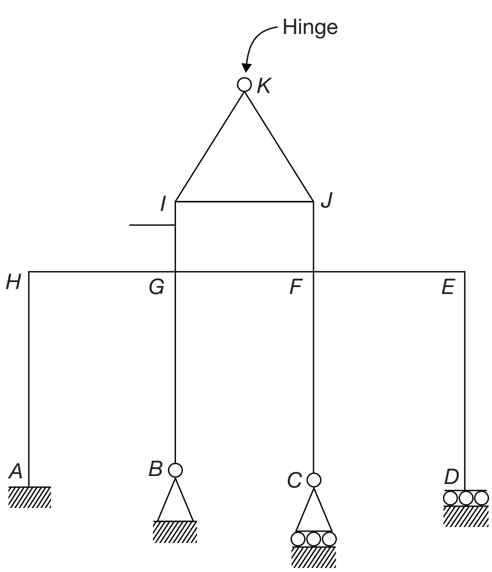



(A) 3 (B) 6
(C) 9 (D) 12

**5.** Member '*AB*' of the truss shown below has a lack of fit of 1 mm at *B*. If $E = 2 \times 10^5$ MPa, area of cross-section $= 20$ mm$^2$, the force in '*AB*' is _______.

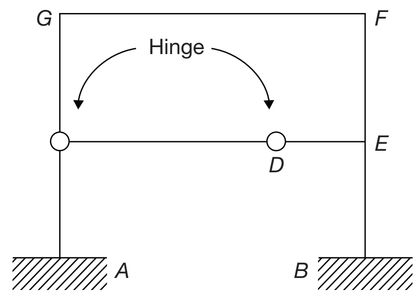



**6.** The static indeterminacy of the structure shown below is _______.



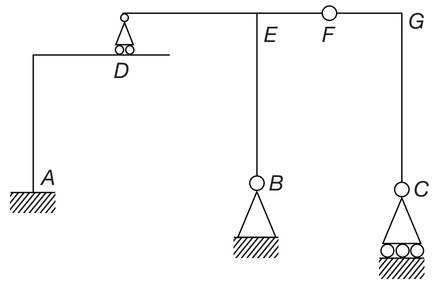

(A) 4
(B) 6
(C) 8
(D) 10

**7.** The static indeterminacy of the structure shown below is _______.



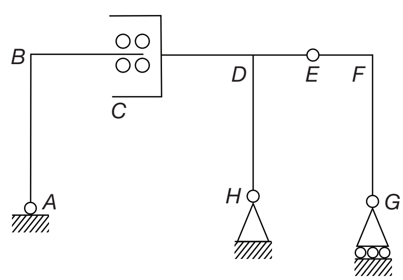

(A) unstable
(B) stable, determinate
(C) stable, 5th degree indeterminate
(D) stable, 3rd degree indeterminate

**8.** The plane structure shown below is _______.



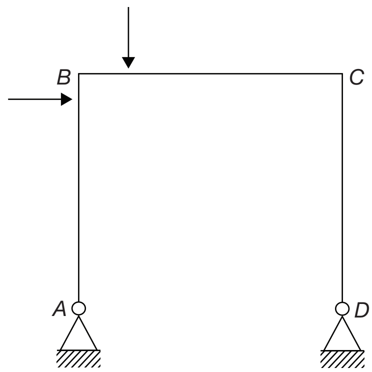

(A) stable and determinate
(B) stable and indeterminate
(C) unstable and determinate
(D) unstable and indeterminate

**9.** A plane frame *ABCDEFGH* shown in given figure has clamp supports at *A* and axial force release (horizontal sleeve) at '*C*' and moment release (hinge) at *E*. The static indeterminacy of the frame is _________.



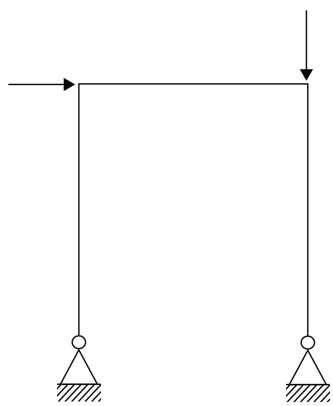

(A) 4
(B) 3
(C) 2
(D) 1

**10.** The plane figure shown below is ________.




(A) stable and statically determinate
(B) unstable and statically determinate
(C) stable and statically indeterminate
(D) unstable and statically indeterminate

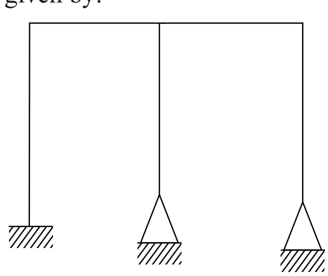
**11.** The degrees of freedom of the following frames is ________.



**12.** The kinematic indeterminacy of single bay portal frame fixed at base is

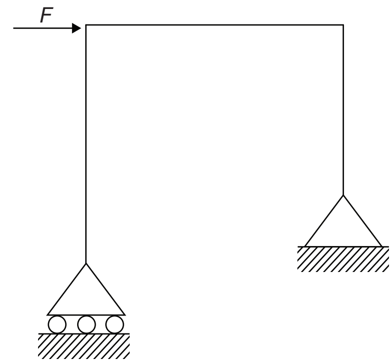
(A) one (B) two
(C) three (D) zero

**13.** The degree of static indeterminacy, $N_s$ and the degree of kinematic indeterminacy, $N_k$ for the plane frame shown below, assuming axial deformations to be negligible, are given by:



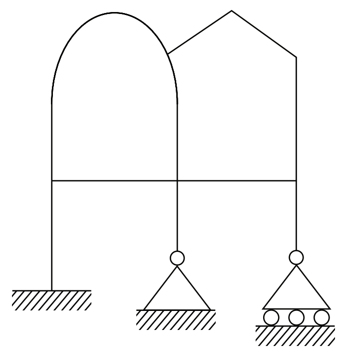
(A) $N_s = 6$ and $N_k = 11$
(B) $N_s = 6$ and $N_k = 6$
(C) $N_s = 4$ and $N_k = 6$
(D) $N_s = 4$ and $N_k = 4$

**14.** Considering beam as axially rigid, the degrees of freedom of a plane frame shown below is

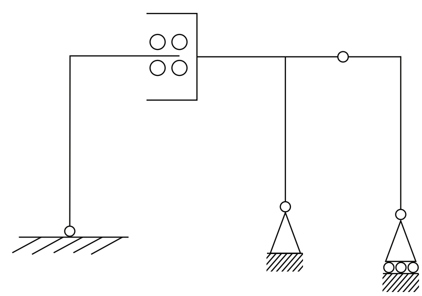



(A) 9 (B) 8
(C) 7 (D) 6

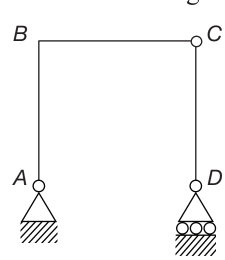
**15.** Determine the degrees of freedom of the following frame:



**16.** Determine the degrees of freedom of the following frame:



**17.** The plane frame shown in the figure is ________.




(A) statically indeterminate but unstable
(B) unstable
(C) determinate and stable
(D) None of these

**18.** Match List I (Type of structure) with List II (Static indeterminacy) and select the correct answer using the codes given below the lists:
Number of members = $m$
Number of joints = $n$
Number of reaction elements = $r$

| | List I | | List II |
|---|---|---|---|
| **a.** | Plane frame | **1.** | $m + r - 3n$ |
| **b.** | Space truss | **2.** | $6m + r - 6n$ |
| **c.** | Space frame | **3.** | $6m + r - 3n$ |
| | | **4.** | $3m + r - 3n$ |

**Codes:**

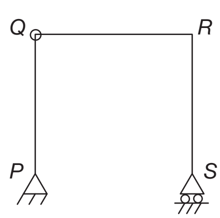
| | a | b | c | | a | b | c |
|---|---|---|---|---|---|---|---|
| (A) | 1 | 2 | 3 | (B) | 4 | 3 | 2 |
| (C) | 2 | 1 | 3 | (D) | 4 | 1 | 2 |

**19.** An statically indeterminate building frame may be converted to a statically determinate one by assuming
(A) hinges at mid-height of columns.
(B) hinges at the mid-span of the beams.
(C) hinges at both mid-height of columns and mid-span of beams.
(D) one support as fixed at base and other support on rollers.

**20.** Which one of the following is true example of a statically determinate beam?
(A) One end is fixed and the other end is simply supported.
(B) Both the ends are fixed.
(C) The beam overhangs over two supports.
(D) The beam is supported on three supports.

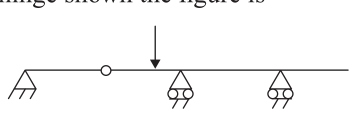
**21.** The number of independent equations to be satisfied for static equilibrium in a space structure is
(A) 3 (B) 6
(C) 4 (D) 2

**22.** The frame shown below is




(A) statically indeterminate but unstable.
(B) unstable.
(C) determinate and stable.
(D) None of these

**23.** The static indeterminacy of a continuous beam with an internal hinge shown the figure is



(A) zero (B) 1
(C) 2 (D) None of these

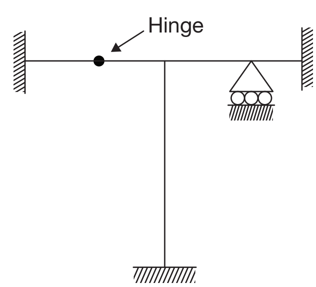
**24.** Which of the following cross-sections has the highest shape factor?
(A) Rectangle (B) Diamond
(C) Triangle (D) Circle

**25.** The stiffness coefficient $K_{ji}$ indicates
(A) force at $j$ due to a unit deformation at $i$.
(B) force at $I$ due to a unit deformation at $j$.
(C) deformation at $j$ due to a unit force at $i$.
(D) deformation at $I$ due to a unit force at $j$.

**26.** Which of the following statements is true with regard to the flexibility method of analysis?
(A) The method is used to analyse determinate structures.
(B) The method is used only for manual analysis of indeterminate structures.
(C) The method is used for analysis of flexible structures.
(D) The method is used for analysis of indeterminate structures with lesser degree of static indeterminacy.

**27.** A suspension bridge with a two hinged stiffening girder is statically
(A) determinate.
(B) indeterminate to 1 degree.
(C) indeterminate to 2 degrees.
(D) indeterminate to 3 degrees.

**28.**



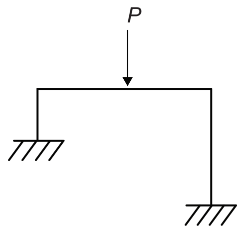

The static indeterminacy of the frame given is _____.
(A) 3 (B) 5
(C) 6 (D) 9

**29.** A prismatic beam is shown in the figure given below.


Consider the following statements:
I. The structure is unstable.
II. The bending moment is zero at supports and internal hinge.
III. It is a mechanism.
IV. It is statically indeterminate.
Which of these statements are correct?

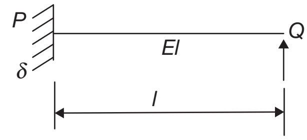
(A) I, II, III and IV (B) I, II and III
(C) I and II (D) III and IV

**30.** The portal frame as shown in the given frame is statically indeterminate to the




(A) third degree (B) second degree
(C) first degree (D) None of these

**31.** Due to settlement of support at $P$ of propped cantilever shown in the figure given below, what is the vertical reaction at $Q$?

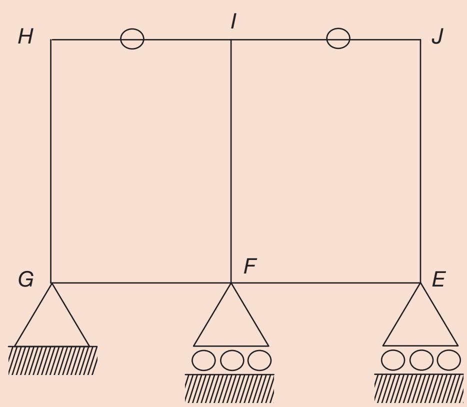



(A) $\frac{6EI\delta}{l^3}$ (B) $\frac{12EI\delta}{l^3}$

(C) $\frac{3EI\delta}{l^3}$ (D) $\frac{4EI\delta}{l^3}$

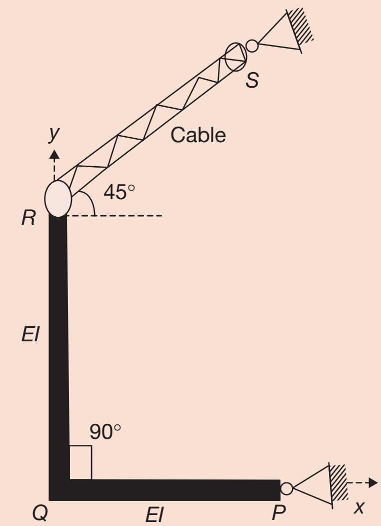
## Previous Years' Questions

**1.** The degree of static indeterminacy of the rigid frame having two internal hinges as shown in the figure below is ________. **[GATE, 2008]**



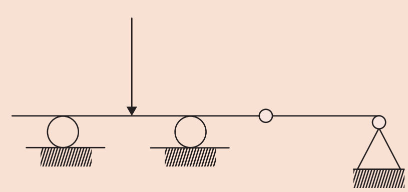

(A) 8
(B) 7
(C) 6
(D) 5

**2.** The degree of static indeterminacy of a rigid jointed frame $PQR$ supported as shown in the figure is ________. **[GATE, 2014]**

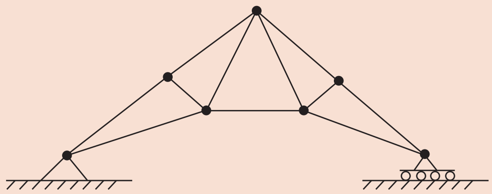



(A) zero (B) one
(C) two (D) unstable

**3.** The static indeterminacy of the two span continuous beam with an internal hinge, shown below, is ________. **[GATE, 2014]**

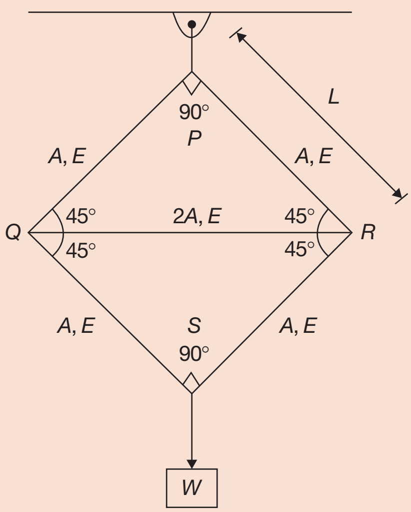


**4.** The kinematic indeterminacy of the plane truss shown in the figure is **[GATE, 2016]**



(A) 11 (B) 8
(C) 3 (D) 0

**5.** Consider the structural system shown in the figure under the action of weight $W$. All the joints are hinged. The properties of the members in terms of length ($L$), area ($A$) and the modulus of elasticity ($E$) are also given in the figure. Let $L$, $A$ and $E$ be 1 m, 0.05 m$^2$ and 30 × 10$^6$ N/m$^2$, respectively, and $W$ be 100 kN. **[GATE, 2016]**




Which one of the following sets gives the correct values of the force, stress and change in length of the horizontal member $QR$?

(A) Compressive force = 25 kN; Stress = 250 kN/m$^2$; Shortening = 0.0118 m
(B) Compressive force = 14.14 kN; Stress = 141.4 kN/m$^2$; Extension = 0.0118 m
(C) Compressive force = 100 kN; Stress = 1000 kN/m$^2$; Shortening = 0.0417 m
(D) Compressive force = 100 kN; Stress = 1000 kN/m$^2$; Extension = 0.0417 m

## Answer Keys

### Exercises

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** A | **2.** D | **3.** B | **4.** C | **5.** Zero | **6.** D | **7.** D | **8.** A | **9.** D | **10.** A |
| **11.** 5 | **12.** C | **13.** C | **14.** B | **15.** 11 | **16.** 14 | **17.** B | **18.** D | **19.** C | **20.** C |
| **21.** B | **22.** B | **23.** A | **24.** C | **25.** A | **26.** D | **27.** B | **28.** C | **29.** B | **30.** A |
| **31.** C | | | | | | | | | |

### Previous Years' Questions

| | | | | |
|---|---|---|---|---|
| **1.** D | **2.** A | **3.** Zero | **4.** A | **5.** C |