03

Kinematics I
(Motion in 1-D)

IN THIS CHAPTER ....

- Frame of Reference
- Basic Terms Related to Motion
- Speed and Velocity
- Uniform and Non-uniform Motion
- Acceleration
- Graphs in One Dimensional Motion
- Relative Velocity

The branch of Physics in which we study the motion of objects or particles and their equilibrium under the action of external forces is known as *mechanics*.

Kinematics is one of the branch of Mechanics. It describes the motion of objects without looking at the cause of the motion. Here, time factor plays an important role.

Frame of Reference

The frame of reference is a suitable coordinate system involving space and time used as a reference to study the motion of different bodies. The most common reference frame is the cartesian frame of reference involving (x, y, z and t).

(i) **Inertial Frame of Reference** A frame of reference which is either at rest or moving with constant velocity is known as inertial frame of reference. In this frame of reference, Newton's first law of motion is applicable.

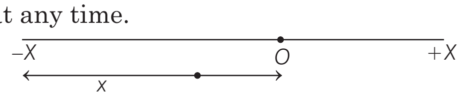
(ii) **Non-Inertial Frame of Reference** A frame of reference moving with some acceleration is known as non-inertial frame of reference. In this frame of reference, Newton's law of motion is not applicable.

Rest and Motion

An object is said to be at rest, if it does not change its position with time and in a state of motion, if it continuously changes its position with time.

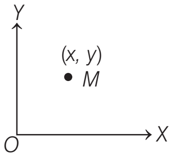
On the basis of number of coordinates required to specify the object, motion of object can be classified as

(i) **One-dimensional motion** The motion of an object is considered as 1-D (one-dimensional), if only one coordinate is needed to specify the position of the object at any time.

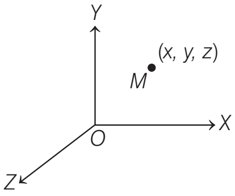


(ii) **Two-dimensional motion** The motion of an object is considered as 2-D (two-dimensional), if two coordinates are needed to specify the position of the object at any time.

In 2-D motion, the object moves in a plane.



(iii) **Three-dimensional motion** The motion of an object is considered as 3-D (three-dimensional), if all the three coordinates are needed to specify the position of the object.



Basic Terms Related to Motion

Here, we will consider the motion of a point object in a straight line in one dimension, therefore, the motion of the point object can be described by specifying the distance x of the point object and the corresponding instant of time t. Mathematically, the position of the object in one dimensional motion can be expressed as follows

$$x = x(t)$$

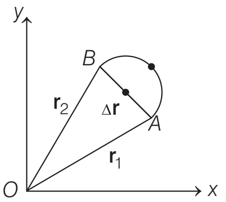
or $$x = f(t)$$

Here, the distance x is the function of the time t.

Position Vector

It describes the instantaneous position of a particle with respect to the chosen frame of reference. It is a vector joining the origin to the particle. If at any time (x, y, z) be coordinates of the particle, then its position vector is given by $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

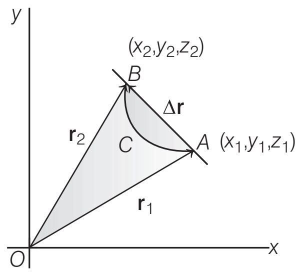
In one dimensional motion, position vector may be given by $\mathbf{r} = x\hat{\mathbf{i}}$, $y = z = 0$ (along X-axis). In two dimensional motion, $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ (in xy-plane $z = 0$).



Distance and Displacement

Distance is the total length of the path travelled by the particle in a given interval of time.

Displacement is a vector joining the initial position of the particle to its final position in a given interval of time. Mathematically, it is equal to the change in position vectors, *i.e.* $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$



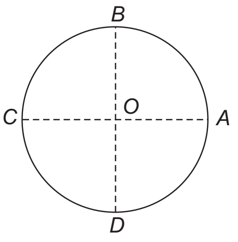
The direction of displacement is directed from A to B through the straight line AB and the magnitude of displacement is

$$|\mathbf{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Distance *vs* Displacement

- Distance is a scalar quantity and displacement is a vector quantity.
- For motion between two points, displacement is single valued while distance depends on actual path and so can have many values.
- Path length or distance is a positive scalar quantity which does not decrease with time and can never be zero for a moving body. Displacement of a body can be zero or negative.
- Magnitude of displacement can never be greater than distance travelled by the body.
- When a body returns to its initial position, its displacement is zero but distance or path length is non-zero.

Example 1. *A particle moves along a circle of radius R. It starts from A and moves in an anti-clockwise direction. What is the distance and displacement of the particle from A to D?*



(a) $\frac{\pi R}{2}, \sqrt{2}R$ (b) $\pi R, 2R$

(c) $\frac{3}{2}\pi R, \sqrt{2}R$ (d) $2\pi R$, zero

Sol. (c) For the motion from A to D,

$$\text{Distance travelled} = \frac{2\pi R \times 3}{4} = \frac{3}{2}\pi R$$

$$\therefore \quad \text{Displacement} = |\mathbf{AD}| = \sqrt{(OA)^2 + (OD)^2}$$
$$= \sqrt{R^2 + R^2} = \sqrt{2}R$$

Speed and Velocity

Speed of an object in motion is defined as the ratio of total path length (*i.e.* actual distance covered) and the corresponding time taken by the object, *i.e.*

$$\text{speed} = \frac{\text{total path length / distance}}{\text{time taken}}$$

Speed is a scalar quantity. It gives an idea about the direction of motion of the object.

Velocity of an object in motion is defined as the ratio of displacement and the corresponding time interval taken by the object, *i.e.*

$$\text{velocity} = \frac{\text{displacement}}{\text{time interval}}$$

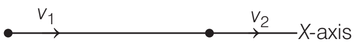
Velocity is a vector quantity as it has both the magnitude (speed) and direction.

Average Velocity & Speed and Instantaneous Velocity & Speed

Average Velocity Average velocity $\bar{v}$ of an object moving through a displacement (Δx) during a time interval (Δt) is given by

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

If the beginning and ending velocities for the given motion are known and the acceleration is constant, then the average velocity can also be expressed as



$$v_{\text{average}} = \bar{v} = \frac{v_1 + v_2}{2}$$

Average Speed Average speed is a measurement of the total distance travelled in a given period of time. It is sometimes referred as the distance per time ratio.

$$\text{Average Speed, } v_{\text{av}} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Note Average velocity in magnitude is always smaller than or equal to average speed of a given particle.

Instantaneous Velocity The instantaneous velocity of an object at an instant of time t, is defined as the limit of average velocity as time interval Δt, around time t becomes infinitesimally small. Thus, instantaneous velocity at instant of time t is

$$v_i = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

where, dx/dt = the differential coefficient of x w.r.t. time.

Instantaneous Speed The instantaneous speed is defined as the limit of the average speed as the time interval (Δt) becomes infinitesimally small or approaches to zero.

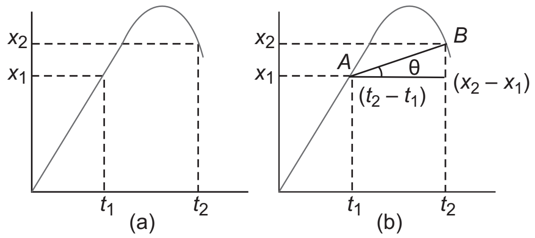
Thus, instantaneous speed at instant of time t is

$$s_i = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

where, ds is distance for time dt.

Important Points about Speed and Velocity

- The velocity in the uniform circular motion does not depend upon the time interval.
- Velocity can be negative, zero or positive, but speed is never negative.
- If motion takes place in the same direction, then the average speed and average velocity are the same.
- If a particle travels equal distances at speeds $v_1, v_2, v_3, \ldots$ etc respectively, then the average speed is harmonic mean of individual speeds.
- If a particle moves a distance at speed v_1 and comes back with speed v_2, then $v_{\text{av}} = \dfrac{2\,v_1 v_2}{v_1 + v_2}$ but $\mathbf{v}_{\text{av}} = 0$.
- If a particle moves in two equal intervals of time at different speeds v_1 and v_2 respectively, then $v_{\text{av}} = \dfrac{v_1 + v_2}{2}$.
- The average velocity between two points in a time interval can be obtained from a position *versus* time graph by calculating the slope of the straight line joining the coordinates of the two points.



The graph, describes the motion of a particle moving along X-axis (along a straight line).

Suppose, we wish to calculate the average velocity between $t = t_1$ and $t = t_2$. The slope of chord AB [shown in Fig. (b)] gives the average velocity.

Mathematically, $\quad v_{\text{av}} = \tan\theta = \dfrac{x_2 - x_1}{t_2 - t_1}$

- If a body moves with a constant velocity, the instantaneous velocity is equal to average velocity. The instantaneous speed is equal to modulus of instantaneous velocity.
- x-component of displacement is $\Delta x = \int v_x \, dt$

  y-component of displacement is $\Delta y = \int v_y \, dt$

  z-component of displacement is $\Delta z = \int v_z \, dt$

  Thus, displacement of particle is

  $$\Delta \mathbf{r} = \Delta x \hat{\mathbf{i}} + \Delta y \hat{\mathbf{j}} + \Delta z \hat{\mathbf{k}}$$
- The magnitude of instantaneous velocity is equal to the instantaneous speed at a given instant.
- If during motion, velocity remains constant throughout a given interval of time, then the motion is said to be uniform.

For uniform motion, instantaneous velocity = average velocity = uniform velocity.

However, converse may or may not be true, *i.e.* if average velocity = instantaneous velocity, the motion may or may not be uniform.

Example 2. *A car travels a distance A to B at a speed of 40 kmh^{-1} and returns to A at a speed of 30 kmh^{-1}. What is the average speed for the whole journey?*

(a) Zero (b) 34.3 kmh^{-1} (c) 68.6 kmh^{-1} (d) 120 kmh^{-1}

Sol. (b) Let $AB = s$, time taken to go from A to B, $t_1 = \frac{s}{40}$ h

and time taken to go from B to A, $t_2 = \frac{s}{30}$ h

$$\therefore \quad \text{Total time taken} = t_1 + t_2 = \frac{s}{40} + \frac{s}{30} = \frac{(3+4)s}{120} = \frac{7s}{120} \text{ h}$$

Total distance travelled = $s + s = 2s$

$$\therefore \quad \text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{2s}{7s/120} = \frac{120 \times 2}{7} = 34.3 \text{ kmh}^{-1}$$

Example 3. *A man walks on a straight road from his home to a market 3 km away with a speed of 6 kmh^{-1} finding the market closed, he instantly turns and walks back with a speed of 9 kmh^{-1}. What is the magnitude of average velocity and average speed of the man, over the interval of time 0 to 40 min ?*

(a) 2.25 kmh^{-1}, 6.75 kmh^{-1} (b) 6.75 kmh^{-1}, 2.25 kmh^{-1}
(c) zero, 2.25 kmh^{-1} (d) zero, 6.75 kmh^{-1}

Sol. (a) Time taken by man to go from his home to market,

$$t_1 = \frac{\text{distance}}{\text{speed}} = \frac{3\text{ km}}{6\text{ kmh}^{-1}} = \frac{1}{2}\text{ h} = 30 \text{ min}$$

Time taken by man to go from market to home,

$$t_2 = \frac{3\text{ km}}{9\text{ kmh}^{-1}} = \frac{1}{3}\text{ h} = 20 \text{ min}$$

Distance moved in 30 min (from home to market) = 3.0 km

Distance moved in 10 min (from market to home) with speed

$$9 \text{ kmh}^{-1} = 9 \times \frac{1}{6} = 1.5 \text{ km}$$

So, displacement = 3.0 − 1.5 = 1.5 km

Total path length travelled = 3.0 + 1.5 = 4.5 km

$$\text{Average velocity} = \frac{1.5\text{ km}}{(40/60)\text{ h}} = 2.25 \text{ kmh}^{-1}$$

$$\text{Average speed} = \frac{4.5\text{ km}}{(40/60)\text{ h}} = 6.75 \text{ kmh}^{-1}$$

Example 4. *A particle travels half the distance with velocity u. The remaining part of the distance is covered with velocity v_1 for the first half time and v_2 for the remaining half time.*

The average velocity of the particle during the complete motion is

(a) $\frac{2u + v_1 + v_2}{2u(v_1 + v_2)}$ (b) $\frac{2u(v_1 + v_2)}{2u + v_1 + v_2}$

(c) $\frac{v_1 + v_2}{v_1 - v_2}$ (d) $\frac{v_1 - v_2}{v_1 + v_2}$

Sol. (b) When time intervals are equal, then distances are equal

$$v_{av} = \frac{2uv_m}{u + v_m} = \frac{2u\left(\frac{v_1 + v_2}{2}\right)}{u + \left(\frac{v_1 + v_2}{2}\right)} = \frac{2u(v_1 + v_2)}{2u + v_1 + v_2}$$

Example 5. *The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by*

(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$

(c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

Sol. (b) The given co-ordinate

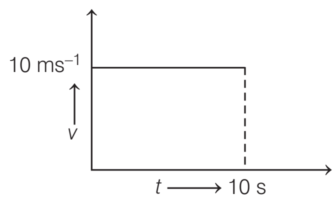
$$x = \alpha t^3, \quad y = \beta t^3$$

Then, $v_x = \frac{dx}{dt} = 3\alpha t^2$ and $v_y = \frac{dy}{dt} = 3\beta t^2$

$$\therefore \text{ Resultant velocity, } v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4} = 3t^2\sqrt{\alpha^2 + \beta^2}$$

Example 6. *A particle is moving according to graph is shown in figure. What is the average velocity in the interval of 3 s to 8 s?*

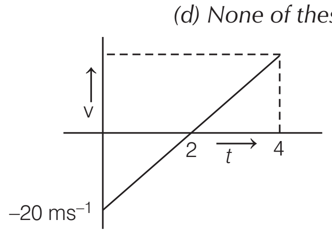
(a) 10 ms^{-1} (b) 10 ms^{-2} (c) 8 ms^{-1} (d) 3 ms^{-1}



Sol. (a) In uniform motion, average velocity equals to instantaneous velocity. Hence, average velocity will be 10 ms^{-1}.

Example 7. *A particle is moving according to the equation $x = 5t^2 - 20t + 4$. What is the average velocity between time $t_1 = 0$ s to $t_2 = 4$ s ? (where, x = displacement, t = time)*

(a) 20t ms^{-1} (b) 20 ms^{-2}
(c) 10 ms^{-1} (d) None of these



Sol. (c) Given that, $x = 5t^2 - 20t + 4$

$$v = \frac{dx}{dt} = 10t - 20$$

At $t = 0$, $v_i = -20\ \text{ms}^{-1}$

$t = 2\ \text{s}$, $v_f = 0\ \text{ms}^{-1}$

$t = 4\ \text{s}$, $v = 20\ \text{ms}^{-1}$

Area of v-t graph gives displacement and distance.

Average velocity = 0

Average speed $= \frac{40}{4} = 10\ \text{ms}^{-1}$

Uniform and Non-uniform Motion

Uniform Motion An object is said to be in uniform motion if its velocity is uniform, *i.e.* it undergoes equal displacement in equal intervals of time, howsoever small these intervals may be. For a uniform motion along a straight line in a given direction, the magnitude of displacement is equal to the actual distance covered by the object.

Non-uniform Motion An object is said to be in non-uniform motion if it undergoes equal displacement in unequal intervals of time, howsoever small these intervals may be. Clearly, in non-uniform motion, the velocity of an object is different at different instants.

Acceleration

Acceleration of an object is defined as rate of change of velocity. It is a vector quantity and its SI unit is m/s².

Average acceleration for a given time

$$\mathbf{a}_{\text{av}} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Instantaneous acceleration at a particular instant is defined as

$$\mathbf{a}_{\text{ins}} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} = \left(\frac{dv_x}{dt} \hat{\mathbf{i}} + \frac{dv_y}{dt} \hat{\mathbf{j}} + \frac{dv_z}{dt} \hat{\mathbf{k}} \right)$$

$$= (a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} + a_z \hat{\mathbf{k}})$$

Uniformly Accelerated Motion

In an accelerated motion, if the change in velocity of an object in each unit of time is constant, the object is said to be moving with **constant acceleration** and such a motion is called uniformly accelerated motion.

- If a particle is accelerated for a time t_1 with acceleration a_1 and for time t_2 with acceleration a_2, then average acceleration is

$$a_{\text{av}} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2}$$

- If a body starts from rest and moves with uniform acceleration, then distance travelled by the body in t second is proportional to t^2 (*i.e.* $s \propto t^2$)
  So, we can say that the ratio of distance covered in 1s, 2s, 3s, is $1^2 : 2^2 : 3^2$ or $1 : 4 : 9$.

- A particle moving with a uniform acceleration from A to B along a straight line has velocities v_1 and v_2 at A and B respectively. If C is the mid-point between A and B, then velocity of the particle at C is equal to $v = \sqrt{\frac{v_1^2 + v_2^2}{2}}$.

Example 8. *A particle travels according to the equation $a = A - Bv$, where a is acceleration, A and B are constants and v is velocity of the particle. It's velocity as a function of time is*

(a) $\frac{B}{A}(1 - e^{-At})$ (b) $\frac{B}{A}(1 - e^{-Bt})$ (c) $\frac{A}{B}(1 - e^{-Bt})$ (d) $\frac{A}{B}(1 - e^{-At})$

Sol. (c) Given acceleration, $a = \frac{dv}{dt}$

$$\therefore \quad \frac{dv}{dt} = A - Bv$$

$$\Rightarrow \quad \frac{dv}{A - Bv} = dt$$

$$\int_0^v \frac{dv}{A - Bv} = \int_0^t dt$$

$$-\frac{1}{B} \log_e (A - Bv) \Big|_0^v = t$$

$$\Rightarrow \quad \log_e \frac{A - Bv}{A} = -Bt$$

$$\Rightarrow \quad A - Bv = Ae^{-Bt}$$

$$\therefore \quad v = \frac{A}{B}(1 - e^{-Bt})$$

Example 9. *A body moves along a straight line with an acceleration 3 ms^{-2} for 2 s and then with an acceleration 4 ms^{-2} for 3 s. What is his average acceleration?*

(a) 3.4 ms^{-2} (b) 3.5 ms^{-2}
(c) 3.6 ms^{-2} (d) 3.7 ms^{-2}

Sol. (c) Average acceleration,

$$a_{\text{av}} = \frac{a_1 t_1 + a_2 t_2}{t_1 + t_2} = \frac{3 \times 2 + 4 \times 3}{2 + 3} = 3.6\ \text{ms}^{-2}$$

Kinematics Equations for Uniformly Accelerated Motion

We can establish the relation between velocity, acceleration and the distance travelled by the body in a particular time interval by a set of equations. These equations are known as *kinematics equations* or *equations of motion*.

If u be the initial velocity, v the final velocity and distance covered by the body in time t is s, then the equations of motion are as under

(i) $v = u + at$

(ii) $s = ut + \frac{1}{2}at^2$

(iii) $v^2 - u^2 = 2as$ and

(iv) $s_{n\text{th}} = u + \frac{a}{2}(2n - 1)$

For motion in a plane, we may consider motion of an object along X-axis and Y-axis independently and then combine the two motions so as to get the net motion of the particle. Thus, we have

(i) $\mathbf{v} = \mathbf{u} + \mathbf{a}\,t$,

$$v_x = u_x + a_x t,$$
$$v_y = u_y + a_y t$$

and $|\mathbf{v}| = \sqrt{v_x^2 + v_y^2}$

$$= (v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}) = (u_x\hat{\mathbf{i}} + u_y\hat{\mathbf{j}}) + (a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}})\,t$$

(ii) $\mathbf{s} = \mathbf{u}\,t + \frac{1}{2}\mathbf{a}t^2$,

$$s_x = u_x t + \frac{1}{2}a_x t^2,\quad s_y = u_y t + \frac{1}{2}a_y t^2$$

and $|\mathbf{s}| = \sqrt{s_x^2 + s_y^2}$

(iii) $v_x^2 - u_x^2 = 2a_x \cdot s_x,\ v_y^2 - u_y^2 = 2a_y \cdot s_y$

Equations of motion in free space are similar to those for motion in a plane.

Example 10. *Starting from the origin at time $t = 0$, with initial velocity $5\hat{\mathbf{j}}$ ms^{-1}, a particle moves in the xy-plane with a constant acceleration of $(10\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ ms^{-2}. At time t, its co-ordinates are (20 m, y_0 m). The values of t and y_0 respectively, are* **[JEE Main 2020]**

(a) 2 s and 18 m (b) 5 s and 25 m
(c) 2 s and 24 m (d) 4 s and 52 m

Sol. (a) $\mathbf{u} = (5\hat{\mathbf{j}})$ m/s $= (0\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$ m/s

$\mathbf{a} = (10\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ m/s^2

$\mathbf{s} = [(20 - 0)\hat{\mathbf{i}} + (y_0 - 0)\hat{\mathbf{j}}]$ m $= (20\hat{\mathbf{i}} + y_0\hat{\mathbf{j}})$ m

Using second equation of motion in x-direction,

$$s_x = u_x t + \frac{1}{2}a_x t^2$$
$$20 = (0)t + \frac{1}{2}(10)(t)^2$$
$$20 = 0 + 5t^2$$
$$t^2 = 4$$
$$t = 2\text{ s}$$

Using second equation of motion in y-direction,

$$s_y = u_y t + \frac{1}{2}a_y t^2 = 5\times 2 + \frac{1}{2}\times 4\times(2)^2 = 10 + 8 = 18\text{ m}$$

Hence, correct option is (a).

Example 11. *A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{\mathbf{i}}$ m/s and moves in the xy-plane with a constant acceleration $(6.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ m/s^2. The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D metres. The value of D is* **[JEE Main 2020]**

(a) 50 (b) 60 (c) 40 (d) 32

Sol. (b) Given,

At $t = 0$, initial velocity of particle is $\mathbf{x} = 3\hat{\mathbf{i}}$ ms^{-1}

Acceleration of particle, $\mathbf{a} = (6\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$ms^{-2}

By $s = ut + \frac{1}{2}at^2$, we have

$$\mathbf{r} = (3\hat{\mathbf{i}})t + \frac{1}{2}(6\hat{\mathbf{i}} + 4\hat{\mathbf{j}})t^2$$
$$\Rightarrow \mathbf{r} = (3t + 3t^2)\hat{\mathbf{i}} + 2t^2\hat{\mathbf{j}}$$

So, at time t, x-coordinate of particle, $x = 3t + 3t^2$

and y-coordinate of particle, $y = 2t^2$

When $y = 32$ m

$\Rightarrow 2t^2 = 32$

$\Rightarrow t^2 = 16$ or $t = 4$s

Value of x-coordinate at $t = 4$s,

$$x = (3t + 3t^2)_{t=4} = 12 + 48 = 60\text{ m}$$

Hence, $D = 60$ m

Motion Under Gravity

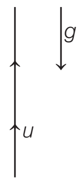
When an object falls towards the earth, an acceleration is involved, this acceleration is due to earth's gravitational pull and is called gravitation acceleration/acceleration due to gravity (g).

The magnitude of gravitational acceleration near the surface of earth is $g = 9.8$ ms^{-2} = 32 fts^{-2}.

Cases of Motion Under Gravity

***Case* I If particle is moving upwards**

In this case, applicable kinematics relations are



$$v = u - gt \quad \text{... (i)}$$
$$h = ut - \frac{1}{2}gt^2 \quad \text{... (ii)}$$
$$v^2 = u^2 - 2gh \quad \text{... (iii)}$$

Here, h is the vertical height of the particle in upward direction.

Note For maximum height attained by a particle

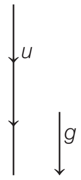
$$h = h_{max}, v = 0$$

i.e. $(0)^2 = u^2 - 2gh_{max}$

$\therefore$ $h_{max} = \dfrac{u^2}{2g}$

***Case* II If particle is moving vertically downwards.**

In this is case,



$$v = u + gt \quad \text{... (i)}$$
$$h = ut + \frac{1}{2}gt^2 \quad \text{... (ii)}$$
$$v^2 = u^2 + 2gh \quad \text{... (iii)}$$

Here, h is the vertical height of particle in downward direction.

Note **Sign Conventions** Normally, vertically upward motion is taken as negative and vertically downward motion is taken as positive. Similarly, for horizontally rightward motion is taken positive and leftward motion is taken negative.

Example 12. *A ball is dropped from a high tower such that distance covered by it in last second of its motion is same as the distance covered by it during first three seconds. Find the height of tower. (Take, g = 10 ms^{-2})*

(a) 125 *m* (b) 125 *cm* (c) 120 *m* (d) 120 *cm*

Sol. (a) Let ball takes t seconds to reach the ground and h be the height of tower. Then,

$$h = 0 + \frac{1}{2}gt^2 \quad \ldots\text{(i)}$$

and $$h_{\text{nth}} = 0 + \frac{g}{2}(2t-1) = 0 + \frac{1}{2}g\,(3)^2$$

or $$2t - 1 = 9 \quad \ldots\text{(ii)}$$

$\Rightarrow$ $$t = 5\text{ s}$$

and $$h = \frac{1}{2}\times 10\times (5)^2 = 125\text{m}$$

Example 13. *A particle is projected vertically upwards with velocity 40 ms^{-1}. Find the displacement and distance travelled by the particle in 6 s. (Take g =10 ms^{-2})*

(a) 60 *m*, 100 *m* (b) 60 *m*, 120 *m*

(c) 40 *m*, 100 *m* (d) 40 *m*, 80 *m*

Sol. (a) Here, u is positive (upwards) and a is negative). So, first we will find t_0, the time when velocity becomes zero, *i.e.* when the particle is the highest point.

$$t_0 = \left|\frac{u}{a}\right| = \frac{40}{10} = 4\text{ s}$$

Here, $t > t_0$

Hence, distance > displacement

$$s = 40\times 6 - \frac{1}{2}\times 10\times 36 = 60\text{ m}$$

While, $$d = \left|\frac{u^2}{2a}\right| + \frac{1}{2}\left|a(t-t_0)^2\right|$$

$$= \frac{(40)^2}{2\times 10} + \frac{1}{2}\times 10\times (6-4)^2 = 100\text{ m}$$

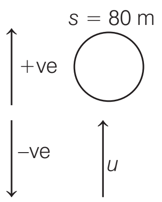
Example 14. *A ball is thrown upwards from the ground with an initial speed u. The ball is at a height of 80 m at two times, the time interval being 6s. Then, the value of u is (Take, g = 10 ms^{-2})*

(a) 20 ms^{-1} (b) 30 ms^{-1} (c) 40 ms^{-1} (d) 50 ms^{-1}

Sol. (d) Here, $a = g = -10\text{ ms}^{-2}$ and $s = 80$ m

Substituting the values in $s = ut + \frac{1}{2}at^2$, we have

$$80 = ut - 5t^2$$



or $$5t^2 - ut + 80 = 0$$

or $$t = \frac{u + \sqrt{u^2 - 1600}}{10}$$

and $$t = \frac{u - \sqrt{u^2 - 1600}}{10}$$

Now, it is given that

$$\frac{u + \sqrt{u^2-1600}}{10} - \frac{u - \sqrt{u^2-1600}}{10} = 6$$

or $$\frac{\sqrt{u^2-1600}}{5} = 6$$

or $$\sqrt{u^2-1600} = 30$$

or $$u^2 - 1600 = 900$$

$\therefore$ $$u^2 = 2500$$

or $$u = \pm 50\text{ ms}^{-1}$$

Ignoring the negative sign, we have

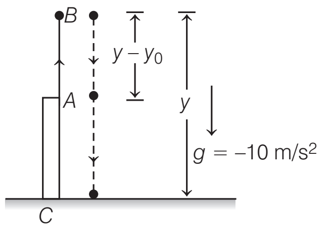
$$u = 50\text{ ms}^{-1}$$

Example 15. *A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25 m from the ground. How long will it be before the ball hits the ground? (Take, g = 10 ms^{-2})*

(a) 1 *s* (b) 3 *s*

(c) 5 *s* (d) 7 *s*

Sol. (c) We split the motion of ball in two parts, the upward motion (A to B) and the downward motion (B to C) and calculate the corresponding time taken t_1 and t_2. Since, the velocity at B is zero, we have



$$v = v_0 + gt$$
$$0 = 20 - 10t_1$$
$$t_1 = 2\text{ s}$$

This is the time in going from A to B. From B or the point of the maximum height the ball falls freely under the acceleration due to gravity. The ball is moving in negative y-direction. We use equation

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

We have, $y_0 = 45$ m, $y = 0, v_0 = 0$, $a = -g = -10\text{ms}^{-2}$

$$0 = 45 + \frac{1}{2}(-10)\,t_2^2$$

$\Rightarrow$ $$t_2 = 3\text{ s}$$

Total time taken by the ball before it hits the ground

$$= t_1 + t_2 = 2\text{ s} + 3\text{ s} = 5\text{ s}$$

Non-Uniformly Accelerated Motion

When motion of a particle is not uniform, *i.e.* acceleration of particle is not constant or acceleration is a function of time, then following relations hold for one dimensional motion.

(i) $v = \dfrac{ds}{dt}$

(ii) $a = \dfrac{dv}{dt} = v\dfrac{dv}{ds}$

(iii) $ds = v\,dt$ and

(iv) $dv = a\,dt$ or $v\,dv = a\,ds$

Example 16. *A particle is moving with a velocity of*

$$v = (3 + 6t + 9t^2)\ cms^{-1}$$

The displacement of the particle in time interval $t = 5$ s to $t = 8$ s is

(a) 1352 *cm* (b) 1287 *cm*
(c) 1182 *cm* (b) 11000 *cm*

Sol. (*b*) Given, $v = (3 + 6t + 9t^2)$ cms^{-1}

or $\dfrac{ds}{dt} = (3 + 6t + 9t^2)$

or $ds = (3 + 6t + 9t^2)\,dt$

$\therefore$ $\int_0^s ds = \int_0^t (3 + 6t + 9t^2)\,dt$

$\therefore$ $s = [3t + 3t^2 + 3t^3]_5^8$

or $s = 1287$ cm

Example 17. *The motion of a particle along a straight line is described by the function $x = (2t - 3)^2$, where x is in metres and t is in seconds. The acceleration at $t = 2$ s is*

(a) 5 ms^{-2} (b) 6 ms^{-2} (c) 7 ms^{-2} (d) 8 ms^{-2}

Sol. (*d*) Position, $x = (2t - 3)^2$

Velocity, $v = \dfrac{dx}{dt} = 4(2t - 3)\ \text{ms}^{-1}$

and acceleration, $a = \dfrac{dv}{dt} = 8\ \text{ms}^{-2}$

At $t = 2$ s, $a = 8\ \text{ms}^{-2}$

Graphs in One Dimensional Motion

The tabular forms of *s-t* and *v-t* graphs are given for one dimensional motion with uniform velocity or with constant acceleration.

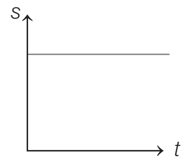
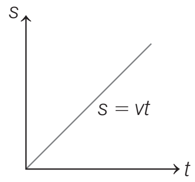
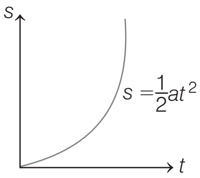
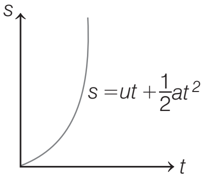
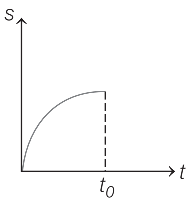
Position-Time Graph

(i) Position-time graph gives instantaneous value of displacement at any instant.

(ii) The slope of tangent drawn to the graph at any instant of time gives the instantaneous velocity at that instant.

(iii) The *s-t* graph cannot make sharp turns.

Different Cases of Position-Time Graph

Different Cases	*s-t* Graph	Features of Graph
At rest		Slope = v = 0
Uniform motion		Slope = constant, v = constant a = 0
Uniformly accelerated motion with u = 0, s = 0 at t = 0		$u = 0$, *i.e.* Slope of *s-t* graph at $t = 0$, should be zero.
Uniformly accelerated motion with $u \neq 0$ but s = 0 at t = 0		Slope of *s-t* graph gradually goes on increasing
Uniformly retarded motion		Slope of graph is decreasing with time so, v is decreasing, a is negative

Velocity-Time Graph

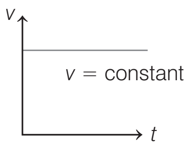
(i) Velocity-time graph gives the instantaneous value of velocity at any instant.

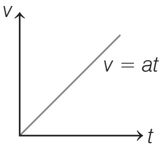
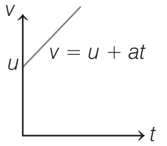
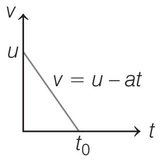
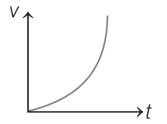
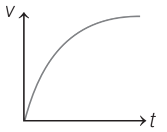
(ii) The slope of tangent drawn on graph gives instantaneous acceleration.

(iii) Area under *v-t* graph with time axis gives the value of displacement covered in given time.

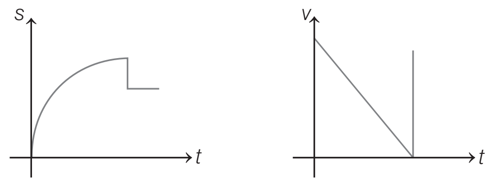
(iv) The *v-t* curve cannot take sharp turns.

Different Cases of Velocity-Time Graph

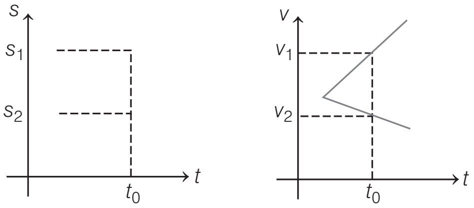
Different Cases	*v-t* Graph	Features of Graph
Uniform motion		(i) $\theta = 0^\circ$ (ii) v = constant (iii) Slope of *v-t* graph = a = 0

Different Cases	*v-t* Graph	Features of Graph
Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$		So slope of *v-t* graph is constant. *i.e.* $u = 0$, so a = constant. Also $v = 0$ at $t = 0$.
Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$		Positive constant acceleration because θ is constant and $<90^\circ$ but the initial velocity of the particle is positive.
Uniformly decelerated motion		Slope of *v-t* graphs $= -a$ (retardation)
Non-uniformly accelerated motion		Slope of *v-t* graph increases with time, so acceleration is increasing.
Non-uniformly decelerating motion		Slope of *v-t* graph is decreasing, so acceleration is decreasing.

Note (i) Slope of *s-t* or *v-t* graphs can never be infinite at any point, because infinite slope of *s-t* graphs means infinite velocity. Similarly infinity slope of *v-t* graph means infinite acceleration. Hence, the following graphs are not possible.

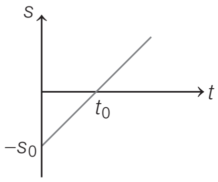


(ii) At one time, two values of velocity or displacement are not possible. Hence, the following graphs are not acceptable



(iii) Different values of displacements in *s-t* graph corresponding to given *v-t* graphs can be calculated just by calculating areas under *v-t* graph. There is no need of using equations like $v = u + at$, etc.

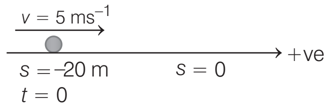
Example 18. *Position-time graph of a particle moving in a straight line is as shown in figure. State the type of motion. (Given, $s_0 = 20$ m and $t_0 = 4$ s)*



(a) *Accelerated* (b) *Decelerated*
(c) *Non-accelerated* (d) *None of these*

Sol. (c) Slope of *s-t* graph is constant. Hence, velocity of particle is constant. Further at time $t = 0$, displacement of the particle from the mean position is $-s_0$ or –20 m. Velocity of particle,

$$v = \text{slope} = \frac{s_0}{t_0} = \frac{20}{4} = 5\ \text{ms}^{-1}$$

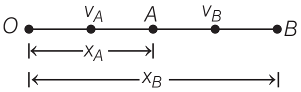


Motion of the particle is as shown in figure. At $t = 0$, particle is at –20 m and has a constant velocity of 5 ms^{-1}. At $t_0 = 4$ s, particle will pass through its mean position. Hence, motion is non-accelerated or uniform.

Relative Velocity

The time rate of change of relative position of one object with respect to another is called relative velocity.

Let two objects A and B are moving along the +ve direction of X-axis. At time t, their displacement from the origin be x_A and x_B.



$\therefore$ Their velocities are $v_A = \dfrac{dx_A}{dt}$ and $v_B = \dfrac{dx_B}{dt}$

The displacement of B relative to A,

$$x_{BA} = x_B - x_A$$

Rate of change of relative displacement w.r.t. time is

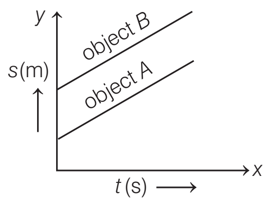
$$\frac{d(\mathbf{x}_{BA})}{dt} = \frac{d}{dt}(\mathbf{x}_B - \mathbf{x}_A)$$

$$\Rightarrow \quad \frac{d\mathbf{x}_{BA}}{dt} = \frac{d\mathbf{x}_B}{dt} - \frac{d\mathbf{x}_A}{dt}$$

$$\therefore \quad \mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A$$

Different Cases

***Case* I** When the two objects move with equal velocities, *i.e.* $v_A = v_B$ or $v_B - v_A = 0$. It means, the two objects stay at constant distance apart during the whole journey.



In this case, the position-time graph of two objects are parallel straight lines.

***Case* II** If both objects A and B move along parallel straight lines in the opposite direction, then relative velocity of B w.r.t. A is given as

$$\mathbf{v}_{BA} = \mathbf{v}_B - (-\mathbf{v}_A) = \mathbf{v}_B + \mathbf{v}_A$$

and the relative velocity of A w.r.t. B is given by

$$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B$$

Examples of Relative Motion

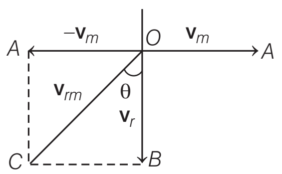
1. Relative Velocity of Rain

Consider a man walking East with velocity $\mathbf{v}_m$, represented by **OA**. Let the rain be falling vertically downwards with velocity $\mathbf{v}_r$, represented by **OB**.

To find the relative velocity of rain w.r.t. man (*i.e.*, $\mathbf{v}_{rm}$) bring the man at rest by imposing a velocity $-\mathbf{v}_m$ on man and apply this velocity on rain also.

Now the relative velocity of rain w.r.t. man will be the resultant velocity of $\mathbf{v}_r\,(= OB)$ and $-\mathbf{v}_m\,(= OA)$, which will be represented by diagonal **OC** of rectangle $OACB$

$$\therefore \qquad \mathbf{v}_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$



If θ is the angle which $\mathbf{v}_{rm}$ makes with the vertical direction, then

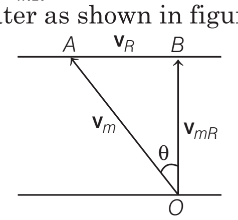
$$\tan\theta = \frac{BC}{OB} = \frac{\mathbf{v}_m}{\mathbf{v}_r} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{\mathbf{v}_m}{\mathbf{v}_r}\right)$$

Here, angle θ is from vertical towards west and is written as θ, west of vertical.

Note In the above problem if the man wants to protect himself from the rain, he should hold his umbrella in the direction of relative velocity of rain w.r.t. man *i.e.,* the umbrella should be held making an angle $\theta (= \tan^{-1} v_m / v_r)$ west of vertical.

2. Crossing the River

To cross the river over shortest distance, *i.e.* to cross the river straight, the man should swim upstream making an angle θ with **OB** such that, **OB** gives the direction of resultant velocity ($\mathbf{v}_{mR}$) of velocity of swimmer and velocity of river water as shown in figure. Let us consider



$$\mathbf{AB} = \mathbf{v}_R \quad \text{(velocity of river water)}$$

$$\mathbf{OA} = \mathbf{v}_m \quad \text{(velocity of man in still river water)}$$

$$\mathbf{OB} = \mathbf{v}_{mR} \quad \text{(relative velocity of man w.r.t. river)}$$

$$\mathbf{v}_{mR} = \sqrt{\mathbf{v}_m^2 - \mathbf{v}_R^2}$$

In ΔOAB, $\sin\theta = \dfrac{\mathbf{v}_R}{\mathbf{v}_m}$

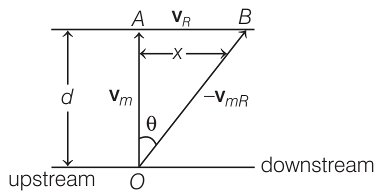
where θ is the angle made by man with shortest distance OB,

$$\tan\theta = \frac{\mathbf{v}_R}{\mathbf{v}_m} = \frac{\mathbf{v}_R}{\sqrt{\mathbf{v}_m^2 - \mathbf{v}_R^2}}$$

(a) **Time taken to cross the river** If d be the width of the river, then time taken cross to the river given by

$$t_1 = \frac{d}{\mathbf{v}_{mR}} = \frac{d}{\sqrt{\mathbf{v}_m^2 - \mathbf{v}_R^2}}$$

(b) **To cross the river in possible shortest time** The man should go along OA. Now the swimmer will be going along OB, which is the direction of resultant velocity of $\mathbf{v}_m$ and $\mathbf{v}_R$



In ΔOAB, $\qquad \tan\theta = \dfrac{AB}{OA} = \dfrac{\mathbf{v}_R}{\mathbf{v}_m}$

and $\qquad \mathbf{v}_{mR} = \sqrt{\mathbf{v}_m^2 + \mathbf{v}_R^2}$

Time of crossing the river $t = \dfrac{d}{\mathbf{v}_m}$

$$= \frac{OB}{\mathbf{v}_{mR}} = \frac{\sqrt{x^2 + d^2}}{\sqrt{\mathbf{v}_m^2 + \mathbf{v}_R^2}}$$

The boat will be reading the point B instead of point A. If $BD = x$,

if $AB = x$, then

$$\tan\theta = \frac{\mathbf{v}_R}{\mathbf{v}_m} = \frac{x}{d}$$

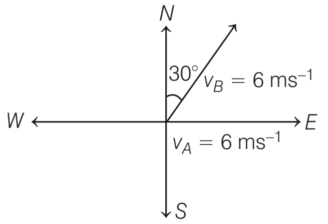
$$\Rightarrow \qquad x = \frac{d\mathbf{v}_R}{\mathbf{v}_m}$$

In this case, the man will reach the opposite bank at a distance AB downstream.

Example 19. *A man A moves due to east with velocity 6 ms^{-1} and another man B moves in N-30°E with 6 ms^{-1}. Find the velocity of B w.r.t. A.*

(a) 3 ms^{-1} (b) $\sqrt{3}$ ms^{-1}
(c) $\sqrt{6}$ ms^{-1} (d) 6 ms^{-1}

Sol. (*d*) Given, $\mathbf{v}_A = 6\,\hat{\mathbf{i}}$



$$\mathbf{v}_B = v_B \cos 60^\circ\,\hat{\mathbf{i}} + v_B \sin 60^\circ\,\hat{\mathbf{j}}$$
$$= 6\left(\frac{1}{2}\right)\hat{\mathbf{i}} + 6\left(\frac{\sqrt{3}}{2}\right)\hat{\mathbf{j}} = 3\,\hat{\mathbf{i}} + 3\sqrt{3}\,\hat{\mathbf{j}}$$

To find the velocity,

$$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = (3\,\hat{\mathbf{i}} + 3\sqrt{3}\,\hat{\mathbf{j}}) - 6\,\hat{\mathbf{i}}$$
$$= -3\,\hat{\mathbf{i}} + 3\sqrt{3}\,\hat{\mathbf{j}}$$
$$|\mathbf{v}_{BA}| = \sqrt{(-3)^2 + (3\sqrt{3})^2} = \sqrt{9+27}$$
$$= \sqrt{36} = 6\ \text{ms}^{-1}$$

Here, $\hat{\mathbf{i}}$ is –ve and $\hat{\mathbf{j}}$ is +ve. So, second quadrant is possible.

Direction, $\tan\alpha = \dfrac{\text{coefficient of } \hat{\mathbf{j}}}{\text{coefficient of } \hat{\mathbf{i}}}$

$$= \frac{3\sqrt{3}}{-3} = -\sqrt{3}$$

$\Rightarrow \qquad \alpha = -\pi/3$

Example 20. *The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight?*

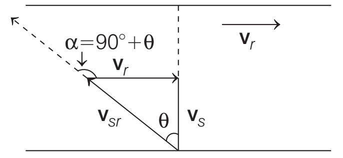
(*a*) 60°
(*b*) 120°
(*c*) 90°
(*d*) 150°

Sol. (*b*) Let the velocity of the swimmer is

$$v_s = 4\ \text{km/h}$$

and velocity of river is $v_r = 2$ km/h

Also, angle of swimmer with the flow of the river (down stream) is α as shown in the figure below.



From diagram, angle θ is

$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{2\ \text{km/h}}{4\ \text{km/h}} = \frac{1}{2}$$

$\Rightarrow \qquad \theta = 30^\circ$

Clearly, $\qquad \alpha = 90^\circ + 30^\circ = 120^\circ$

Practice Exercise

ROUND I Topically Divided Problems

Distance and Displacement

1 An aeroplane flies 400 m from North and then flies 300 m South and then flies 1200 m upwards, then net displacement is

(a) 1200 m (b) 1300 m
(c) 1400 m (d) 1500 m

2. A clerk starts from his house with a speed of 2 kmh^{-1} and reaches the office 3 min late. Next day he increases speed 1 kmh^{-1} and reaches the office 3 min earlier. Find the distance between his house and office.

(a) 1 km (b) 0.6 km
(c) 2 km (d) 3 km

3. A particle starts from rest at the point $A(1, 2, -3)$ m and reaches at the point $B(3, 4, 5)$ m. The magnitude of displacement of man is

(a) 7.1 m (b) 3.7 m
(c) 3.4 m (d) 8.5 m

4. A body starts from rest and moves with a constant acceleration. The ratio of distance covered in the nth second to the distance covered in n second is

(a) $\frac{2}{n} - \frac{1}{n^2}$
(b) $\frac{1}{n^2} - \frac{1}{n}$
(c) $\frac{2}{n^2} - \frac{1}{n}$
(d) $\frac{2}{n} + \frac{1}{n^2}$

5. A wheel of radius 1 m rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point on the wheel initially in contact with the ground is

(a) 2π (b) $\sqrt{2}\,\pi$
(c) $\sqrt{\pi^2 + 4}$ (d) π

6. A body is moving along a straight line path with constant velocity. At an instant of time the distance of time the distance travelled by it is s and its displacement is D, then

(a) $D < s$ (b) $D > s$
(c) $D = s$ (d) $D \le s$

Speed, Velocity and Acceleration

7. A particle moves with constant acceleration and v_1, v_2 and v_3 denote the average velocities in the three successive intervals t_1, t_2 and t_3 of time. Which of the following relations is correct?

(a) $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 - t_2}{t_2 + t_3}$ (b) $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 - t_2}{t_1 - t_2}$
(c) $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 - t_2}{t_2 - t_3}$ (d) $\frac{v_1 - v_2}{v_2 - v_3} = \frac{t_1 + t_2}{t_2 + t_3}$

8. A bee flies in a line from a point A to another point B in 4 s with a velocity of $|t - 2|$ ms^{-1}. The distance between A and B in metre is

(a) 2 (b) 4
(c) 6 (d) 8

9. A 2 m wide truck is moving with a uniform speed $v_0 = 8$ ms^{-1} along a straight horizontal road. A pedestrian starts to cross the road with a uniform speed v when the truck is 4 m away from him. The minimum value of v, so that he can cross the road safely is

(a) 2.62 ms^{-1} (b) 4.6 ms^{-1}
(c) 3.57 ms^{-1} (d) 1.414 ms^{-1}

10. A particle starts from rest and travels a distance s with uniform acceleration, then it travels a distance $2s$ with uniform speed, finally it travels a distance $3s$ with uniform retardation and comes to rest. If the complete motion of the particle in a straight line then the ratio of its average velocity to maximum velocity is

(a) 6/7 (b) 4/5
(c) 3/5 (d) 2/5

11. Three particles start from the origin at the same time, one with a velocity v_1 along x-axis, the second along the y-axis with a velocity v_2 and the third along $x = y$ line. The velocity of the third so that the three may always lie on the same line is

(a) $\frac{v_1 v_2}{v_1 + v_2}$ (b) $\frac{\sqrt{2}\, v_1 v_2}{v_1 + v_2}$
(c) $\frac{\sqrt{3}\, v_1 v_2}{v_1 + v_2}$ (d) zero

12. A body moves in a plane, so that the displacement along X and Y-axes is $x = 3t^2$ and $y = 4t^2$, then velocity is
(a) $15\,t^2$
(b) $20\,t^2$
(c) $5\,t^2$
(d) None of the above

13. The displacement of a body along X-axis depends on time as $\sqrt{x} = t + 1$. Then, the velocity of body
(a) increase with time (b) decrease with time
(c) independent of time (d) None of these

14. The velocity of a particle is $v = v_0 + gt + Ft^2$. Its position is $x = 0$ at $t = 0$, then its displacement after time $(t = 1)$ is **[JEE Main 2021]**
(a) $v_0 + g + F$ (b) $v_0 + \frac{g}{2} + \frac{F}{3}$
(c) $v_0 + \frac{g}{2} + F$ (d) $v_0 + 2g + 3F$

15. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$. **[NCERT Exemplar]**
(a) The displacement in time T must always take non-negative values
(b) The displacement x in time T satisfies $-v_0 T < x < v_0 T$
(c) The acceleration is always a non-negative number
(d) The motion has no turning points

16. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as
(a) t^2 (b) t
(c) $t^{1/2}$ (d) t^3

17. A particle moving with a uniform acceleration along a straight line covers distance a and b in successive intervals of p and q second. The acceleration of the particle is
(a) $\frac{pq\,(p+q)}{2\,(bp-aq)}$ (b) $\frac{2\,(aq-bp)}{pq\,(p-q)}$
(c) $\frac{bp-aq}{pq\,(p-q)}$ (d) $\frac{2\,(bp-aq)}{pq\,(p-q)}$

18. The position vector of particle changes with time according to the relation $\mathbf{r}(t) = 15t^2\,\hat{\mathbf{i}} + (4 - 20t^2)\,\hat{\mathbf{j}}$. What is the magnitude of the acceleration (in ms^{-2}) at $t = 1$? **[JEE Main 2019]**
(a) 50 (b) 100
(c) 25 (d) 40

19. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is
(a) $-2\,abv^2$ (b) $2\,bv^3$
(c) $-2\,av^3$ (d) $2\,av^2$

20. A particle starts from the origin and moves along the X-axis such that the velocity at any instant is given by $4\,t^3 - 2\,t$, where t is in second and velocity is in ms^{-1}. What is the acceleration of the particle when it is 2 m from the origin?
(a) 10 ms^{-2} (b) 12 ms^{-2}
(c) 22 ms^{-2} (d) 28 ms^{-2}

21. A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct? **[NCERT Exemplar]**
(a) $x < 0, v < 0, a > 0$
(b) $x > 0, v < 0, a < 0$
(c) $x > 0, v < 0, a > 0$
(d) $x > 0, v > 0, a < 0$

22. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be **[JEE Main 2019]**
(a) $a + \frac{b^2}{2c}$ (b) $a + \frac{b^2}{4c}$
(c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{c}$

23. The driver of a car moving with a speed of 10 ms^{-1} sees a red light ahead, applies brakes and stops after covering 10 m distance. If the same car were moving with a speed of 20 ms^{-1}, the same driver would have stopped the car after covering 30 m distance. Within what distance the car can be stopped if travelling with a velocity of 15 ms^{-1}? Assume the same reaction time and the same deceleration in each case.
(a) 18.75 m (b) 20.75 m
(c) 22.75 m (d) 25 m

24. A particle moving in a straight line with uniform acceleration is observed to be at a distance a from a fixed point initially. It is at distances b, c, d from the same point after n, $2n$, $3n$ second. The acceleration of the particle is
(a) $\frac{c-2b+a}{n^2}$ (b) $\frac{c+b+a}{9\,n^2}$
(c) $\frac{c+2\,b+a}{4\,n^2}$ (d) $\frac{c-b+a}{n^2}$

25. The retardation experienced by a moving motor boat, after its engine is cut-off, is given by $\frac{dv}{dt} = -kv^3$, where k is a constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at time t after the cut-off is
(a) v_0 (b) $\frac{v_0}{2}$
(c) $v_0\,e^{-kt}$ (d) $\frac{v_0}{\sqrt{2v_0^2\,kt + 1}}$

26. The engine of a train can impart a maximum acceleration of 1 ms^{-2} and the brakes can give a maximum retardation of 3 ms^{-2}. The least time during which a train can go from one place to the other place at a distance of 1.2 km is nearly

(a) 108 s (b) 191 s
(c) 56.6 s (d) time is fixed

27. The acceleration of a particle increasing linearly with time t is bt. The particle starts from the origin with an initial velocity v_0. The distance travelled by the particle in time t will be

(a) $v_0 t + \frac{1}{6} bt^3$ (b) $v_0 t + \frac{1}{6} bt^2$
(c) $v_0 t + \frac{1}{3} bt^3$ (d) $v_0 t + \frac{1}{3} bt^2$

Uniform and Non-uniform Motions

28. A car is moving along a straight road with uniform acceleration. It passes through two points P and Q separated by a distance with velocities 30 kmh^{-1} and 40 kmh^{-1} respectively. The velocity of car midway between P and Q is

(a) 33.3 kmh^{-1} (b) 1 kmh^{-1}
(c) $25\sqrt{2}$ kmh^{-1} (d) 35.35 kmh^{-1}

29. A particle moves from the point $(2.0\,\hat{\mathbf{i}} + 4.0\,\hat{\mathbf{j}})$ m at $t = 0$ with an initial velocity $(5.0\,\hat{\mathbf{i}} + 4.0\,\hat{\mathbf{j}})$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\,\hat{\mathbf{i}} + 4.0\,\hat{\mathbf{j}})$ ms^{-2}. What is the distance of the particle from the origin at time 2 s? **[JEE Main 2019]**

(a) 5 m (b) $20\sqrt{2}$ m (c) $10\sqrt{2}$ m (d) 15 m

30. A particle covers 4 m, 5 m, 6 m and 7 m in 3rd, 4th, 5th and 6th second respectively. The particle starts

(a) with an initial non-zero velocity and moves with uniform acceleration
(b) from rest and moves with uniform velocity
(c) with an initial velocity and moves with uniform velocity
(d) from rest and moves with uniform acceleration

31. A mosquito is moving with a velocity $\mathbf{v} = 0.5\,t^2\hat{\mathbf{i}} + 3t\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s? **[JEE Main 2021]**

(a) $\tan^{-1}\left(\frac{2}{3}\right)$, from X-axis
(b) $\tan^{-1}\left(\frac{2}{3}\right)$, from Y-axis
(c) $\tan^{-1}\left(\frac{5}{2}\right)$, from Y-axis
(d) $\tan^{-1}\left(\frac{5}{2}\right)$, from X-axis

32. A particle is moving with speed $v = b\sqrt{x}$ along positive X-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$). **[JEE Main 2019]**

(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$
(c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

33. Two balls A and B are thrown simultaneously from the top of a tower. A is thrown vertically up with a speed of 4 ms^{-1}. B is thrown vertically down with a speed of 4 ms^{-1}. The ball A and B hit the ground with speed v_A and v_B respectively, then

(a) $v_A < v_B$ (b) $v_A > v_B$
(c) $v_A \geq v_B$ (d) $v_A = v_B$

34. A stone is allowed to fall from the top of a tower 100 m high and at the same time another stone is projected vertically upwards from the ground with a velocity of 25 ms^{-1}. The two stones will meet after

(a) 4 s (b) 0.4 s
(c) 0.04 s (d) 40 s

35. A boy released a ball from the top of a building. It will clear a window 2 m high at a distance 10 m below the top in nearly

(a) 1 s (b) 1.3 s
(c) 0.6 s (d) 0.13 s

36. A ball P is dropped vertically and another ball Q is thrown horizontally with the same velocity from the same height and at the same time. If air resistance is neglected, then

(a) ball P reaches the ground first
(b) ball Q reaches the ground first
(c) both reach the ground at the same time
(d) the respective masses of the two balls will decide the time

37. A ball A is thrown up vertically with a speed u and at the same instant another ball B is released from a height h. At time t, the speed of A relative to B is

(a) u (b) $2u$
(c) $u - gt$ (d) $\sqrt{(u^2 - gt)}$

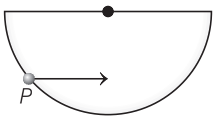
38. From a balloon rising vertically upwards at 5 m/s a stone is thrown up at 10 m/s relative to the balloon. Its velocity with respect to ground after 2 s is (assume $g = 10\,m/s^2$)

(a) 0 (b) 20 m/s
(c) 10 m/s (d) 5 m/s

39. A body thrown vertically upward with an initial velocity u reaches maximum height in 6 second. The ratio of the distances travelled by the body in the first second and seventh second is

(a) 1 : 1 (b) 11 : 1 (c) 1 : 2 (d) 1 : 11

40. A particle of mass m is initially situated at the point P inside a hemispherical surface of radius r as shown in figure. A horizontal acceleration of magnitude a_0 is suddenly produced on the particle in the horizontal direction. If gravitational acceleration is neglected, the time taken by particle to touch the sphere again is

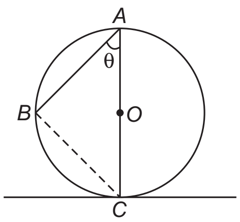


(a) $\sqrt{\dfrac{4r\sin\alpha}{a_0}}$ (b) $\sqrt{\dfrac{4r\tan\alpha}{a_0}}$
(c) $\sqrt{\dfrac{4r\cos\alpha}{a_0}}$ (d) None of these

41. A ball is thrown vertically upwards. It is observed that it remains at a height h twice with a time interval Δt, then the initial velocity of the ball is

(a) $\sqrt{8gh + g^2(\Delta t)^2}$ (b) $\sqrt{8gh + \left(\dfrac{g\Delta t}{2}\right)^2}$
(c) $1/2\sqrt{8gh + g^2(\Delta t)^2}$ (d) $\sqrt{8gh + 4g^2(\Delta t)^2}$

42. A frictionless wire AB is fixed on a sphere of radius R. A very small spherical ball slips on this wire. The time taken by this ball to slip from A to B is



(a) $\dfrac{2\sqrt{gR}}{g\cos\theta}$ (b) $2\sqrt{gR}\,\dfrac{\cos\theta}{g}$
(c) $2\sqrt{\dfrac{R}{g}}$ (d) $\dfrac{gR}{\sqrt{g\cos\theta}}$

43. A body is thrown vertically up with a velocity u. It passes three points A, B and C in its upward journey with velocities $\dfrac{u}{2}, \dfrac{u}{3}$ and $\dfrac{u}{4}$ respectively. The ratio of the separations between points A and B and between B and C, *i.e.* $\dfrac{AB}{BC}$ is

(a) 1 (b) 2 (c) $\dfrac{10}{7}$ (d) $\dfrac{20}{7}$

44. A juggler keeps on moving four balls in the air throws the balls in regular interval of time. When one ball leaves his hand (speed = 20 ms^{-1}), the position of other ball will be (Take, $g = 10\ ms^{-2}$)

(a) 10 m, 20 m, 10 m (b) 15 m, 20 m, 15 m
(c) 5 m, 15 m, 20 m (d) 5 m, 10 m, 20 m

45. A body freely falling from rest has a velocity v after it falls through distance h. The distance it has to fall down further for its velocity to become double is

(a) h (b) 2 h
(c) 3 h (d) 4 h

46. A ball is thrown vertically upwards from the top of a tower of height h with velocity v. The ball strikes the ground after

(a) $\dfrac{v}{g}\left[1+\sqrt{1+\dfrac{2gh}{v^2}}\right]$ (b) $\dfrac{v}{g}\left[1+\sqrt{1-\dfrac{2gh}{v^2}}\right]$
(c) $\dfrac{v}{g}\left(1+\dfrac{2gh}{v^2}\right)^{1/2}$ (d) $\dfrac{v}{g}\left(1-\dfrac{2gh}{v^2}\right)^{1/2}$

47. From an elevated point A, a stone is projected vertically upwards. When the stone reaches a distance h below A, its velocity is double of what was at a height above A? The greatest height attained by the stone is

(a) $\dfrac{h}{3}$ (b) $\dfrac{2h}{2}$ (c) $\dfrac{h}{2}$ (d) $\dfrac{5h}{3}$

48. A particle starting from rest falls from a certain height. Assuming that the value of acceleration due to gravity remains the same throughout motion, its displacements in three successive half second intervals are s_1, s_2, s_3. Then,

(a) $s_1 : s_2 : s_3 = 1:5:9$ (b) $s_1 : s_2 : s_3 = 1:2:3$
(c) $s_1 : s_2 : s_3 = 1:1:1$ (d) $s_1 : s_2 : s_3 = 1:3:5$

49. A body released from a great height falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies two second after the release of the second body is

(a) 9.8 m (b) 4.9 m
(c) 24.5 m (d) 19.6 m

50. A ball released from the top of a tower travels $\dfrac{11}{36}$ of the height of the tower in the last second of its journey. The height of the tower is (Take, $g = 10\ ms^{-2}$)

(a) 11 m (b) 36 m (c) 47 m (d) 180 m

51. A stone thrown vertically upwards attains a maximum height of 45 m. In what time, the velocity of stone become equal to one-half the velocity of throw? (Take, $g = 10\ ms^{-2}$)

(a) 2 s (b) 1.5 s (c) 1 s (d) 0.5 s

52. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is **[JEE Main 2014]**

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2$
(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)^2u$

53. Water drops fall from a tap on the floor 5 m below at regular intervals of time, the first drop striking the floor when the fifth drop begins to fall. The height at which the third drop will be, from ground, at the instant when first drop strikes the ground, will be (Take, $g = 10 \text{ ms}^{-2}$)
(a) 1.25 m (b) 2.15 m
(c) 2.73 m (d) 3.75 m

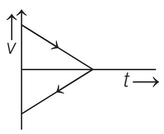
54. A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 second. If this ball is thrown downward from the top of the same tower with speed v it reaches the ground in t_2 second. In what time the ball shall reach the ground if it is allowed to falls freely under gravity from the top of the tower?
(a) $\frac{t_1 + t_2}{2}$ (b) $\frac{t_1 - t_2}{2}$
(c) $\sqrt{t_1 t_2}$ (d) $t_1 + t_2$

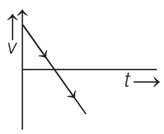
55. A ball is dropped on the floor from a height of 10 m. It rebounds to a height of 2.5 m. If the ball is in contact with the floor for 0.01 s, the average acceleration during contact is nearly
(Take, $g = 10 \text{ ms}^{-2}$)
(a) $500\sqrt{2} \text{ ms}^{-2}$ upwards
(b) 1800 ms^{-2} downwards
(c) $1500\sqrt{5} \text{ ms}^{-2}$ upwards
(d) $1500\sqrt{2} \text{ ms}^{-2}$ upwards

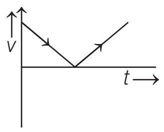
56. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to (Here, g is the acceleration due to gravity.) **[JEE Main 2020]**
(a) $t = \frac{2}{3}\sqrt{\frac{h}{g}}$ (b) $t = 1.8\sqrt{\frac{h}{g}}$
(c) $t = \sqrt{\frac{2h}{3g}}$ (d) $t = 3.4\sqrt{\frac{h}{g}}$

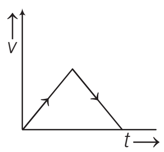
Problem Related to Graph

57. A body is thrown vertically upwards. Which one of the following graph correctly represent the velocity *versus* time? **[JEE Main 2017]**

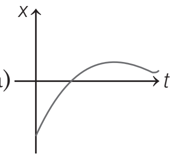
(a) 

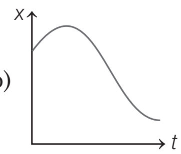
(b) 

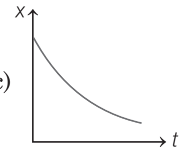
(c) 

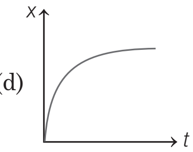
(d) 

58. Among the four graphs, there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for a suitably chosen T. Which one is it? **[NCERT Exemplar]**

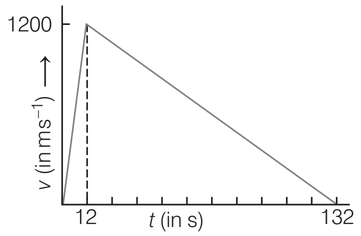
(a) 

(b) 

(c) 

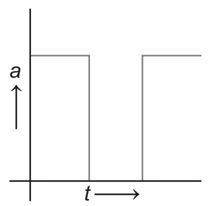
(d) 

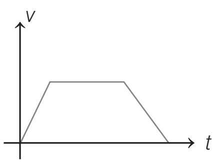
59. A rocket is fired upwards. Its engine explodes fully is 12 s. The height reached by the rocket as calculated from its velocity-time graph is

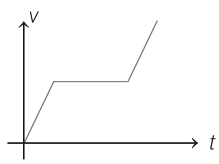


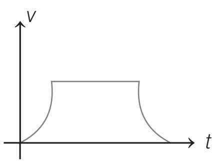
(a) 1200 × 66 m
(b) 1200 × 132 m
(c) $\frac{1200}{12}$ m
(d) 1200×12^2 m

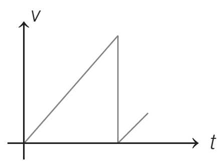
60. Figure shows the acceleration-time graphs of a particle. Which of the following represents the corresponding velocity-time graphs?



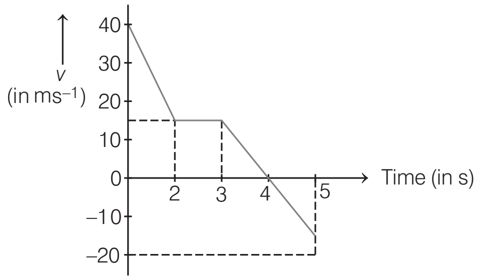
(a) 

(b) 

(c) 

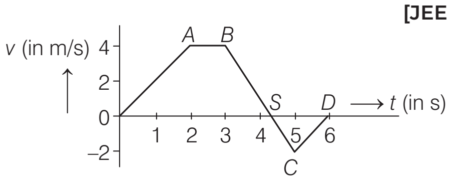
(d) 

61. In the given v-t graph, the distance travelled by the body in 5 s will be



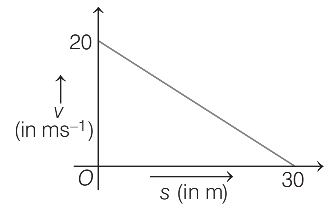
(a) 20 m (b) 40 m
(c) 80 m (d) 100 m

62. The v-t graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 s. The total distance covered by the body in 6 s is **[JEE Main 2020]**



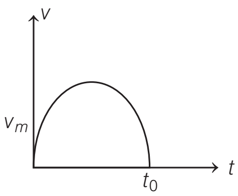
(a) $\frac{37}{3}$ m (b) 12 m
(c) 11 m (d) $\frac{49}{4}$ m

63. If the velocity v of a particle moving along a straight line decreases linearly with its displacement s from $20\ \text{ms}^{-1}$ to a value approaching zero at $s = 30$ m, then acceleration of the particle at $s = 15$ m is



(a) $\frac{2}{3}\ \text{ms}^{-2}$ (b) $-\frac{2}{3}\ \text{ms}^{-2}$
(c) $\frac{20}{3}\ \text{ms}^{-2}$ (d) $-\frac{20}{3}\ \text{ms}^{-2}$

64. The velocity of a particle moving in a straight line varies with time in such a manner that v *versus* t graph is velocity is v_m and the total time of motion is t_0

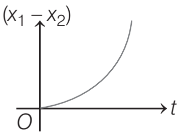


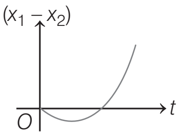
(i) Average velocity of the particle is $\frac{\pi}{4} v_m$

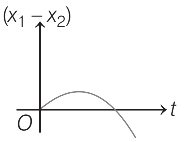
(ii) Such motion cannot be realised in practical terms

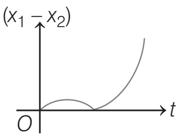
(a) Only (i) is correct
(b) Only (ii) is correct
(c) Both (i) and (ii) are correct
(d) Both (i) and (ii) are wrong

65. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant, another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time?

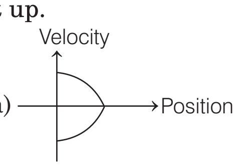
(a) 

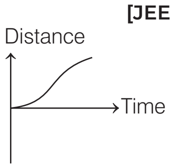
(b) 

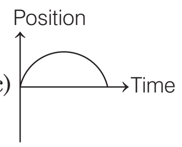
(c) 

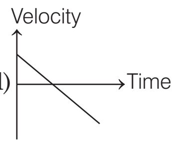
(d) 

66. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE Main 2018]**

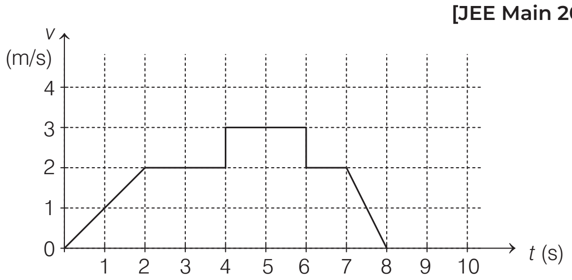
(a) 

(b) 

(c) 

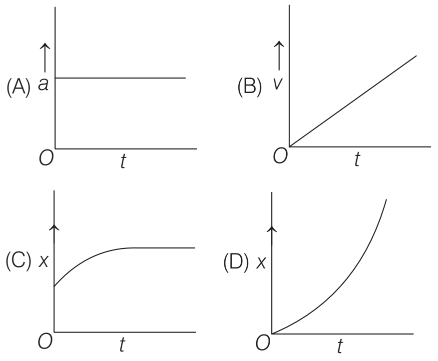
(d) 

67. A particle starts from the origin at time $t = 0$ and moves along the positive X-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5$ s? **[JEE Main 2019]**



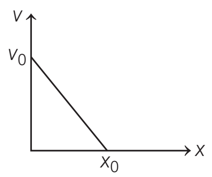
(a) 6 m (b) 3 m (c) 10 m (d) 9 m

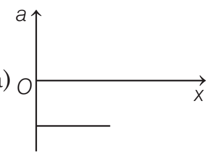
68. A particle starts from origin O from rest and moves with a uniform acceleration along the positive X-axis. Identify all figures that correctly represent the motion qualitatively.
(where, a = acceleration, v = velocity, x = displacement, t = time.) **[JEE Main 2019]**

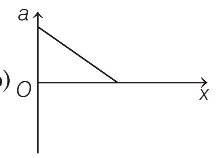


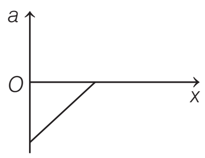
(a) (A) (b) (A), (B), (C)
(c) (B), (C) (d) (A), (B), (D)

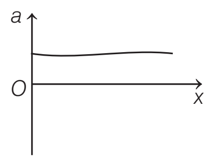
69. The velocity-displacement graph of a particle is shown in the figure. The acceleration-displacement graph of the same particle is represented by **[JEE Main 2021]**



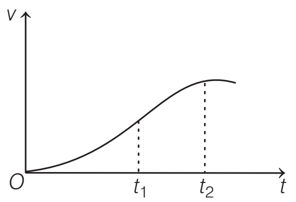
(a)


(b)


(c)


(d)


70. The velocity-time graph of a particle in one-dimensional motion is shown in figure. Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2?



(i) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$

(ii) $v(t_2) = v(t_1) + a(t_2 - t_1)$

(iii) $v_{av} = \left[\frac{x(t_2) - x(t_1)}{(t_2 - t_1)}\right]$

(iv) $a_{av} = \frac{[v(t_2) - v(t_1)]}{(t_2 - t_1)}$

(v) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \frac{1}{2}a_{av}(t_2 - t_1)^2$

(vi) $x(t_2) - x(t_1)$ = Area under v-t curve bounded by the t-axis and the dotted line shown

(a) (iii) and (vi)
(b) (iii), (iv) and (vi)
(c) (ii), (iii) and (iv)
(d) (iv) and (vi)

Relative Motion

71. An express train is moving with a velocity v_1 and its driver finds another train is moving on the same track in the same direction with velocity v_2. To avoid collision, driver applies a retardation a on the train. The minimum time of avoiding collision will be

(a) $t = \frac{v_1 - v_2}{a}$ (b) $t = \frac{v_1^2 - v_2^2}{2}$
(c) Neither (a) nor (b) (d) Both (a) and (b)

72. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be **[NCERT Exemplar]**

(a) $(t_1 + t_2)/2$ (b) $t_1 t_2/(t_2 - t_1)$
(c) $t_1 t_2/(t_2 + t_1)$ (d) $t_1 - t_2$

73. Two cars move in the same direction along parallel roads. One of them is a 100 m long travelling with a velocity of 7.5 ms^{-1}. How long will it take for the first car to overtake the second car?

(a) 24 s (b) 40 s
(c) 60 s (d) 80 s

74. A 210 m long train is moving due North at a speed of 25 m/s. A small bird is flying due South, a little above the train with speed 5 m/s. The time taken by the bird to cross the train is

(a) 6 s (b) 7 s
(c) 9 s (d) 10 s

75. Rain drops fall vertically at a speed of 20 ms^{-1}. At what angle do they fall on the wind screen of a car moving with a velocity of 15 ms^{-1}, if the wind screen velocity inclined at an angle of 23° to the vertical?

$$\left[\cot^{-1}\left(\frac{4}{3}\right) \approx 36°\right]$$

(a) 60° (b) 30° (c) 45° (d) 90°

76. Two trains travelling on the same track are approaching each other with equal speeds of $40\ ms^{-1}$. The drivers of the trains begin to decelerate simultaneously when they are just 2 km apart. If the decelerations are both uniform and equal, then the value of deceleration to barely avoid collision should be

(a) $0.8\ ms^{-2}$ (b) $2.1\ ms^{-2}$
(c) $11.0\ ms^{-2}$ (d) $13.2\ ms^{-2}$

77. A passenger train of length 60 m travels at a speed of 80 km/h. Another freight train of length 120 m travels at a speed of 30 km/h. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction and (ii) in the opposite direction is **[JEE Main 2019]**

(a) $\frac{3}{2}$ (b) $\frac{25}{11}$
(c) $\frac{11}{5}$ (d) $\frac{5}{2}$

78. The distance between two particles moving towards each other is decreasing at the rate of 6 m/s. If these particles travel with same speed and in the same direction, then the separation increase at the rate of 4 m/s. The particles have speed as

(a) 5 m/s; 1 m/s (b) 4 m/s; 1 m/s
(c) 4 m/s; 2 m/s (d) 5 m/s; 2 m/s

79. A police jeep is chasing with velocity of 45 km/h a thief in another jeep moving with velocity of 153 km/h. Police fires a bullet with muzzle velocity of 180 m/s towards thief's car. The velocity with which the bullet will strike the car of the thief is

(a) 150 m/s (b) 27 m/s
(c) 450 m/s (d) 250 m/s

80. A train is moving towards east and a car is along north, both with same speed. The observed direction of a car to the passenger in the train is

(a) east-north direction
(b) west-north direction
(c) south-east direction
(d) None of the above

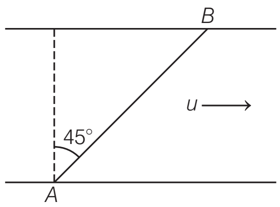
81. A steam boat goes across a lake and comes back (i) on a quiet day when the water is still and (ii) on a rough day when there is a uniform current so as to help the journey onwards and to impede the journey back. If the speed of the launch on both days was same, the time required for complete journey on the rough day, as compared to the quiet day will be

(a) more (b) less
(c) same (d) None of these

82. Trains A and B are running on parallel tracks in the opposite directions with speeds of 36 km/h and 72 km/h, respectively. A person is walking in train A in the opposite direction to its motion with a speed of 1.8 km/h. Speed (in ms^{-1}) of this person as observed from train B will be close to (Take, the distance between the tracks as negligible) **[JEE Main 2020]**

(a) 28.5 (b) 30.5 (c) 29.5 (d) 31.5

83. A man wants to reach point B on the opposite bank of a river flowing at a speed as shown in figure. What minimum speed relative to water should the man have, so that he can reach point B?



(a) $u\sqrt{2}$ (b) $u/\sqrt{2}$
(c) $2u$ (d) $u/2$

84. A boat crosses a river from part A to part B which are just on opposite side. The speed of the water is v_w and that of boat is v_b relative to still water. Assume $v_b = 2v_w$. What is the time taken by the boat? If it has to cross the river directly on the AB line.

(a) $\frac{2D}{v_b\sqrt{3}}$ (b) $\frac{\sqrt{3}D}{2v_b}$
(c) $\frac{D}{v_b\sqrt{2}}$ (d) $\frac{D\sqrt{2}}{v_b}$

85. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops are coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1+\beta)v$, this angle changes to 45°. The value of β is close to **[JEE Main 2020]**

(a) 0.50 (b) 0.41
(c) 0.37 (d) 0.73

86. In a car race on a straight path, car A takes a time t less than car B at the finish and passes finishing point with a speed v more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively, then v is equal to **[JEE Main 2019]**

(a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}\ t$
(c) $\sqrt{a_1a_2}\ t$ (d) $\frac{a_1+a_2}{2}t$

ROUND II Mixed Bag

Only One Correct Option

1. A particle moving in a straight line covers half the distance with speed of 3 m/s. The other half of the distance is covered in two equal time intervals and with speeds of 4.5 m/s and 7.5 m/s, respectively. The average speed of the particle during this motion is
(a) 4.0 m/s (b) 5.0 m/s
(c) 5.5 m/s (d) 4.8 m/s

2. In a race for 100 m distance, the first and the second runners have a gap of one metre at the mid way stage. Assuming the first runner goes steady, by what percentage should the second runner increases his speed just to win the race.
(a) 2% (b) 4%
(c) more than 4% (d) less than 4%

3. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is **[JEE Main 2021]**
(a) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$ (b) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$
(c) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$ (d) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$

4. A particle is moving with a velocity $\mathbf{v} = k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where k is a constant. The general equation for its path is
(a) $y = x^2$ + constant
(b) $y^2 = x$ + constant
(c) $xy =$ constant
(d) $y^2 = x^2$ + constant

5. An automobile travelling with a speed of 60 kmh^{-1} can brake to stop with a distance of 20 m. If the car is going twice as fast, *i.e.* 120 kms^{-1}, the stopping distance will be
(a) 20 m (b) 40 m
(c) 60 m (d) 80 m

6. A point initially at rest moves along x-axis. Its acceleration varies with time as $a = (6t + 5)\ \text{m/s}^2$. If it starts from origin, the distance covered in 2 s is
(a) 20 m (b) 18 m
(c) 16 m (d) 25 m

7. The motion of a body is given by the equation $\frac{dv(t)}{dt} = 6.0 - 3v(t)$, where $v(t)$ is speed in ms^{-1} and t in second. If body was at rest at $t = 0$ and the acceleration is half the initial value, then find its speed.
(a) 2.0 ms^{-1} (b) 3.0 ms^{-1}
(c) 1.0 ms^{-1} (d) 6.0 ms^{-1}

8. An elevator ascends with an upward acceleration of 2.0 ms^{-2}. At the instant its upward speed is 2.5 ms^{-1}, a loose bolt is dropped from the ceiling of the elevator 3.0 m from the floor. If $g = 10\ \text{ms}^{-2}$, then find the distance covered by the bolt during the free fall relative to the elevator shaft.
(a) 0.11 m (b) 0.75 m (c) 1.38 m (d) 2.52 m

9. A particle is moving with a uniform acceleration along a straight line AB. Its speed at A and B are 2 ms^{-1} and 14 ms^{-1} respectively. Then
(a) its speed at mid-point of AB is 20 ms^{-1}
(b) its speed at a point P such that $AP : PB = 1:5$ is 4 ms^{-1}
(c) the time to go from A to mid-point of AB is double of that to go from mid-point of B
(d) None of the above

10. A bullet emerges from a barrel of length 1.2 m with a speed of 640 ms^{-1}. Assuming constant acceleration, the approximate time that it spends in the barrel after the gun is fired is
(a) 4 ms (b) 40 ms (c) 400 ms (d) 1 s

11. A train accelerated uniformly from rest attains a maximum speed of 40 ms^{-1} in 20 s. It travels at this speed for 20 s and is brought to rest with uniform retardation in 40 s. The average velocity during this period is
(a) (80/3) ms^{-1} (b) 30 ms^{-1}
(c) 25 ms^{-1} (d) 40 ms^{-1}

12. A car starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed in 15 s, then
(a) $S = \frac{1}{4}ft^2$ (b) $S = \frac{1}{72}ft^2$
(c) $S = \frac{1}{6}ft^2$ (d) $S = ft$

13. A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distances of 1 m each, will then be
(a) all being equal to $\sqrt{2/9}$ s
(b) in the ratio of the square roots of the integers 1, 2, 3, ...
(c) in the ratio of the difference in the square roots of the integers is $(\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3})$
(d) in the ratio of the reciprocal of the square roots of the integers is $\left(\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}\right)$

14. A man throws balls with the same speed vertically upwards one after the other at an interval of 2 s. What should be the speed of the throw so that more than two balls are in the sky at any time? (Take, $g = 9.8 \text{ m/s}^2$)
(a) At least 0.8 m/s
(b) Any speed less than 19.6 m/s
(c) Only with speed 19.6 m/s
(d) More than 19.6 m/s

15. From the top of a tower of height 50 m, a ball is thrown vertically upwards with a certain velocity. It hits the ground 10 s after it is thrown up. How much time does it take to cover a distance AB where A and B are two points 20 m and 40 m below the edge of the tower? (Take, $g = 10 \text{ ms}^{-2}$)
(a) 2.0 s (b) 1.0 s
(c) 0.5 s (d) 0.4 s

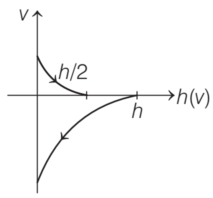
16. From the top of a tower, a stone is thrown up and reaches the ground in time $t_1 = 9$ s. A second stone is thrown down with the same speed and reaches the ground in time $t_2 = 4$ s. A third stone is released from rest and reaches the ground in time t_3, which is equal to
(a) 6.5 s (b) 6.0 s
(c) $\frac{5}{36}$ s (d) 65 s

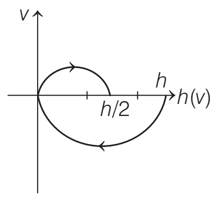
17. A ball which is at rest, is dropped from a height h metre. As it bounces off the floor, its speed is 80% of what it was just before touching the ground. The ball will then rise nearly to a height
(a) 0.94 h (b) 0.80 h
(c) 0.75 h (d) 0.64 h

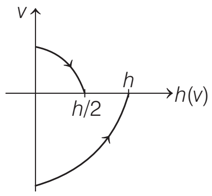
18. A parachutist after alling out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2}. He reaches the ground with a speed of 3 ms^{-1}. At what height, did he fallen out?
(a) 111 m (b) 293 m
(c) 182 m (d) 91 m

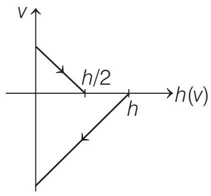
19. A tennis ball is released from a height h and after freely falling on a wooden floor, it rebounds and reaches height $\frac{h}{2}$. The velocity *versus* height of the ball during its motion may be represented graphically by

(Graphs are drawn schematically and on not to scale) **[JEE Main 2020]**

(a) 

(b) 

(c) 

(d) 

20. A motorboat covers a given distance in 6 h moving downstream on a river. It covers the same distance in 10 h moving upstream. The time it takes to cover the same distance in still water is
(a) 9 h (b) 7.5 h
(c) 6.5 h (d) 8 h

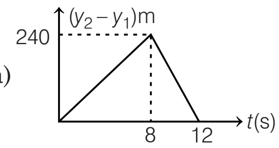
21. Two trains are moving with equal speed in opposite directions along two parallel railway tracks. If the wind is blowing with speed u along the track so that the relative velocities of the trains w.r.t. the wind are in the ratio 1 : 2, then the speed of each train must be
(a) $3u$ (b) $2u$
(c) $5u$ (d) $4u$

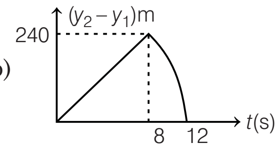
22. A boat crosses a river of width 1 km by shortest path in 15 min. If the speed of boat in still water is 5 kmh^{-1}, then what is the speed of the river?
(a) 5 kmh^{-1}
(b) 12 kmh^{-1}
(c) 3 kmh^{-1}
(d) 4 kmh^{-1}

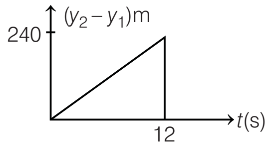
23. Two buses of equal 5 m lengths are moving with the same velocity in the same direction on a highway. The first bus is 40 m ahead of the second bus. The driver of the second bus thinks to overtake the first bus and gives an acceleration of 1 ms^{-2} to the bus. After what time the second bus just passes the first bus?
(a) 5 s (b) 10 s
(c) 15 s (d) 20 s

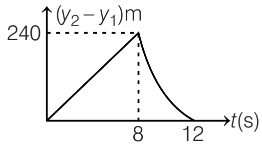
24. Two cars A and B are travelling in the same direction with velocities v_A and v_B $(v_A > v_B)$. When the car A is at a distance s behind car B, the driver of the car A applies the brakes producing a uniform retardation a, there will be no collision when
(a) $s < \frac{(v_A - v_B)^2}{2a}$ (b) $s = \frac{(v_A - v_B)^2}{2a}$
(c) $s \geq \frac{(v_A - v_B)^2}{2a}$ (d) $s \leq \frac{(v_A - v_B)^2}{2a}$

25. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\,\text{m/s}^2$) **[JEE Main 2015]**

(a) 

(b) 

(c) 

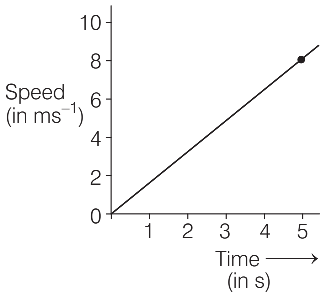
(d) 

26. Ship A is sailing towards north-east with velocity $\mathbf{v} = 30\hat{\mathbf{i}} + 50\hat{\mathbf{j}}$ km/h, where $\hat{\mathbf{i}}$ points east and $\hat{\mathbf{j}}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/h. A will be at minimum distance from B in **[JEE Main 2019]**

(a) 4.2 h
(b) 2.6 h
(c) 3.2 h
(d) 2.2 h

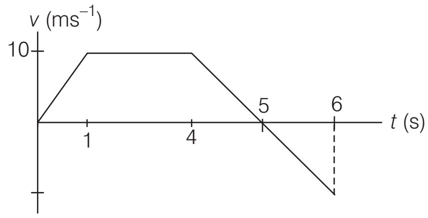
Numerical Value Questions

27. The speed *versus* time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval $t = 0$ s to $t = 5$ s will be ....



28. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is ....

29. A particle moving on straight line whose velocity-time graph is shown in the figure. The average speed from $t = 0$ to $t = 6$ s is $v = \dfrac{15}{n}$ ms^{-1}, the value of n is ....



30. A particle is moving on a straight line whose velocity as function of time is $(t - 2)$ ms^{-1}. The distance travelled by particle (in m) in 4 s is ....

31. A particle starts from rest to move along X-axis. The acceleration of the particle is $a = (t - x)$ ms^{-2}. During motion, maximum acceleration of the particle is $a_0 = 2$ ms^{-2}.The velocity (in ms^{-1}) of the particle at $t = \dfrac{\pi}{3}$ s is ....

32. A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is x m/s. The value of x to the nearest integer is .... **[JEE Main 2021]**

33. A particle is moving along the X-axis with its coordinate with the t given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the Y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is ....

Answers

Round I

1. (a)	**2.** (b)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (c)	**10.** (c)
11. (b)	**12.** (d)	**13.** (a)	**14.** (b)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (c)
21. (a)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (d)	**26.** (c)	**27.** (a)	**28.** (d)	**29.** (b)	**30.** (a)
31. (b)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (d)	**39.** (b)	**40.** (c)
41. (c)	**42.** (c)	**43.** (d)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (d)	**49.** (c)	**50.** (d)
51. (b)	**52.** (c)	**53.** (d)	**54.** (c)	**55.** (d)	**56.** (d)	**57.** (b)	**58.** (b)	**59.** (a)	**60.** (b)
61. (c)	**62.** (a)	**63.** (d)	**64.** (c)	**65.** (b)	**66.** (b)	**67.** (d)	**68.** (d)	**69.** (c)	**70.** (b)
71. (a)	**72.** (c)	**73.** (a)	**74.** (b)	**75.** (a)	**76.** (a)	**77.** (c)	**78.** (a)	**79.** (a)	**80.** (b)
81. (a)	**82.** (c)	**83.** (b)	**84.** (a)	**85.** (d)	**86.** (c)				

Round II

1. (a)	**2.** (c)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (c)	**10.** (a)
11. (c)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (b)	**19.** (c)	**20.** (b)
21. (a)	**22.** (c)	**23.** (b)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** 20	**28.** 3	**29.** 2	**30.** 4
31. 1	**32.** 5	**33.** 580							

Solutions

Round I

1. Displacement along North = 400 − 300 = 100 m

Upward displacement = 1200 m

$\therefore$ Net displacement $= \sqrt{(100)^2 + (1200)^2}$

$= 1204.15 \text{ m} \approx 1200 \text{ m}$

2. If distance between his house and office is s.

$$\therefore \quad 2 = \frac{s}{t + \frac{3}{60}} \quad \text{(for 1st event)} \qquad \ldots\text{(i)}$$

$$\text{and} \quad 2 + 1 = \frac{s}{\left(t - \frac{3}{60}\right)} \quad \text{(for 2nd event)} \qquad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$s = 0.6$ km and $t = 0.25$ h

3. Here, initial position vector of particle is

$$\mathbf{r}_i = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

and final position vector of particle is

$$\mathbf{r}_f = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$$

$\therefore$ The displacement of particle is

$$\mathbf{s} = \mathbf{r}_f - \mathbf{r}_i$$
$$= (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$
$$= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$$

$\therefore$ Magnitude of displacement, $|\mathbf{s}| = \sqrt{2^2 + 2^2 + 8^2}$

$= \sqrt{72}$ m $= 8.5$ m

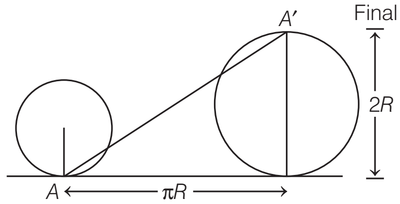
4. Here, $S_n = \frac{1}{2} a n^2$

$S_{n\text{th}}$ = distance travelled in n second − distance travelled in $(n-1)$ second

$$= \left(\frac{2n-1}{2}\right) a$$

$$\therefore \quad \frac{S_{n\text{th}}}{S_n} = \frac{2n-1}{n^2} = \frac{2}{n} - \frac{1}{n^2}$$

5. Horizontal distance covered by the wheel in half revolution πR.



So, the displacement of the point which was initially in contact with ground

$$= AA' = \sqrt{(\pi R)^2 + (2R)^2}$$
$$= R\sqrt{\pi^2 + 4} = \sqrt{\pi^2 + 4} \qquad (\because R = 1 \text{ m})$$

6. A body is moving on straight line with constant velocity. Between A and B, the straight path length is the shortest distance. This is the distance travelled. The particle starts at A and reaches B along the straight line. Therefore displacement is also AB, *i.e.* $D = s$.

7. As, $v_1 = \dfrac{u + v_1'}{2} = \dfrac{u + u + at_1}{2} = u + \dfrac{1}{2}at_1$

$$v_2 = \frac{v_1' + v_2'}{2} = \frac{(u + at_1) + u + a\,(t_1 + t_2)}{2}$$

$$= u + at_1 + \frac{1}{2}at_2$$

$$v_3 = \frac{v_2' + v_3'}{2} = \frac{(u + at_1 + at_2) + u + a\,(t_1 + t_2 + t_3)}{2}$$

$$= u + at_1 + at_2 + \frac{1}{2}at_3$$

Then, $v_1 - v_2 = -\dfrac{1}{2}a\,(t_1 + t_2)$

$$v_2 - v_3 = -\frac{1}{2}a\,(t_2 + t_3)$$

$\therefore$ $\dfrac{v_1 - v_2}{v_2 - v_3} = \dfrac{t_1 + t_2}{t_2 + t_3}$

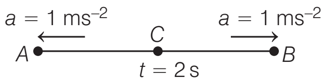
8. Here, $v = |t - 2|\ \text{ms}^{-1}$

$v = t - 2$, when $t > 2\,\text{s}$

$v = 2 - t$, when $t < 2\,\text{s}$

$\therefore$ $a = \dfrac{dv}{dt} = 1\ \text{ms}^{-2}$ when $t > 2\,\text{s}$

$a = -1\ \text{ms}^{-2}$ when $t < 2\,\text{s}$



In the direction of motion from A to C, bee decelerates but for C to B, bee accelerates.

Let $AC = s_1, BC = s_2$

$u_A = 2\ \text{ms}^{-1}, t = 0$

$u_C = 0$ at $t = 4\,\text{s}$

$\therefore$ $s_1 = \left(\dfrac{u_A + u_C}{2}\right)t_1$

$$s_2 = \left(\frac{u_C + u_B}{2}\right)t_2$$

$\therefore$ $s = s_1 + s_2 = \left(\dfrac{2+0}{2}\right)2 + \left(\dfrac{0+2}{2}\right)2 = 4\ \text{m}$

9. Let the man starts crossing the road at an angle θ with the roadside. For safe crossing, the condition is that the man must cross the road by the time truck describes the distance $(4 + 2\cot\theta)$,

So, $\dfrac{4 + 2\cot\theta}{8} = \dfrac{2\sin\theta}{v}$

or $v = \dfrac{8}{2\sin\theta + \cos\theta}$

For minimum v, $\dfrac{dv}{d\theta} = 0$

or $\dfrac{-8\,(2\cos\theta - \sin\theta)}{(2\sin\theta + \cos\theta)^2} = 0$

or $2\cos\theta - \sin\theta = 0$

or $\tan\theta = 2$

So, $\sin\theta = \dfrac{2}{\sqrt{5}}$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$\therefore$ $v_{\min} = \dfrac{8}{2\left(\dfrac{2}{\sqrt{5}}\right) + \dfrac{1}{\sqrt{5}}}$

$$= \frac{8}{\sqrt{5}} = 3.57\ \text{ms}^{-1}$$

10. When a particle is moving with uniform acceleration, let v be the velocity of particle at a distance s,

then average velocity $= \dfrac{0 + v}{2} = \dfrac{v}{2}$

Time taken, $t_1 = \dfrac{s}{(v/2)} = \dfrac{2\,s}{v}$

When particle moves with uniform velocity, time taken,

$$t_2 = \frac{2\,s}{v}$$

When particle moves with uniform acceleration, time taken,

$$t_3 = \frac{3\,s}{(0 + v)/2} = \frac{6\,s}{v}$$

Total time $= t_1 + t_2 + t_3$

$$= \frac{2\,s}{v} + \frac{2\,s}{v} + \frac{6\,s}{v} = \frac{10\,s}{v}$$

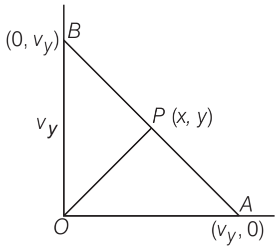
$\therefore$ $v_{av} = \dfrac{s + 2\,s + 3s}{10s/v} = \dfrac{6\,v}{10}$

or $\dfrac{v_{av}}{v} = \dfrac{6}{10} = \dfrac{3}{5}$

11. Let time interval be chosen as 1 s

$$\frac{PA}{PB} = \frac{OA}{OB} = \frac{v_x}{v_y}$$

So, $P\,(x, y)$ divides AB in the ratio of $v_x : v_y$.



Using section formula,

$$x = \frac{v_x \times 0 + v_y \times v_x}{v_x + v_y} = \frac{v_x v_y}{v_x + v_y}$$

$$y = \frac{v_x v_y + v_y \times 0}{v_x + v_y} = \frac{v_x v_y}{v_x + v_y}$$

$$v = \sqrt{x^2 + y^2} = \sqrt{2}\,\frac{v_x v_y}{v_x + v_y}$$

Now, replace v_x by v_1 and v_y by v_2.

$$v = \frac{\sqrt{2}\,v_1 v_2}{v_1 + v_2}$$

12. Given, $x = 3t^2 \Rightarrow v_x = \frac{dx}{dt} = 3\,(2t) = 6t$

and $y = 4t^2 \Rightarrow v_y = \frac{dy}{dt} = 4\,(2t) = 8t$

$\therefore$ Velocity, $\mathbf{v} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} = 6t\,\hat{\mathbf{i}} + 8t\,\hat{\mathbf{j}}$

$$|\mathbf{v}| = \sqrt{(6t)^2 + (8t)^2} = 10\,t$$

13. $\sqrt{x} = t + 1$

Squaring both sides, we get

$$x = (t+1)^2 = t^2 + 1 + 2\,t$$

Differentiating it w.r.t. t, we get

$$\frac{dx}{dt} = 2t + 2$$

Velocity, $v = \frac{dx}{dt} = 2t + 2$

So, velocity increases with time.

14. Given, $v = v_0 + gt + Ft^2$

$\Rightarrow \frac{dx}{dt} = v_0 + gt + Ft^2 \qquad \left[\because v = \frac{dx}{dt}\right]$

$\Rightarrow dx = (v_0 + gt + Ft^2)\,dt$

So, $\int_0^x dx = \int_0^1 (v_0 + gt + Ft^2)\,dt$

$\Rightarrow x = v_0 + \frac{g}{2} + \frac{F}{3}$

Since, vertical component of velocity is zero.

15. The maximum distance covered in time $T = v_0\,T$.

Therefore, for the object having one dimensional motion, the displacement x in time T satisfies $-v_0\,T < x < v_0 T$.

16. Given, $v = \alpha\sqrt{x}$ or $\frac{dx}{dt} = \alpha\sqrt{x} \qquad \left[\because v = \frac{dx}{dt}\right]$

or $\frac{dx}{\sqrt{x}} = \alpha\,dt$

On integrating, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\,dt$$

[$\because$ at $t = 0, x = 0$ and let at any time t, particle be at x]

$\Rightarrow \left[\frac{x^{1/2}}{1/2}\right]_0^x = \alpha\,t$ or $x^{1/2} = \frac{\alpha}{2}\,t$

or $x = \frac{\alpha^2}{4} \times t^2$ or $x \propto t^2$

17. According to problem, when $s = a$, $t = p$

$\because \quad s = ut + \frac{1}{2}ft^2$ (here, f = acceleration)

$\therefore \quad a = up + \frac{fp^2}{2} \qquad \ldots\text{(i)}$

For $s = b, t = q$

$b = uq + \frac{fq^2}{2} \qquad \ldots\text{(ii)}$

After solving Eqs. (i) and (ii), we get

$$f = \frac{2\,(aq - bp)}{pq\,(p - q)}$$

18. Position vector of particle is given as

$$\mathbf{r} = 15\,t^2\,\hat{\mathbf{i}} + (4 - 20\,t^2)\hat{\mathbf{j}}$$

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}\,[15\,t^2\,\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}]$$
$$= 30\,t\,\hat{\mathbf{i}} - 40\,t\,\hat{\mathbf{j}}$$

Acceleration of particle is

$$\mathbf{a} = \frac{d}{dt}\,(\mathbf{v}) = \frac{d}{dt}\,(30\,t\hat{\mathbf{i}} - 40\,t\hat{\mathbf{j}}) = 30\hat{\mathbf{i}} - 40\hat{\mathbf{j}}$$

So, magnitude of acceleration at $t = 1$ s is

$$|\,\mathbf{a}\,|_{t = 1\text{s}} = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 40^2} = 50\text{ ms}^{-2}$$

19. Given, $t = ax^2 + bx$

Differentiating it w.r.t. t, we get

$$\frac{dt}{dt} = 2\,ax\,\frac{dx}{dt} + b\,\frac{dx}{dt}$$
$$v = \frac{dx}{dt} = \frac{1}{(2ax + b)}$$

Again, differentiating w.r.t. t, we get

$$\frac{d^2x}{dt^2} = \frac{-2\,a}{(2\,ax + b)^2} \cdot \frac{dx}{dt}$$

$\therefore \quad f = \frac{d^2x}{dt^2} = \frac{-1}{(2\,ax + b)^2} \cdot \frac{2\,a}{(2\,ax + b)}$

or $\quad f = \frac{-2\,a}{(2\,ax + b)^3} \Rightarrow f = -2\,av^3$

20. $\frac{dx}{dt} = 4\,t^3 - 2\,t$

or $dx = 4t^3\,dt - 2\,t\,dt$

Integrating, $x = \frac{4t^4}{4} - \frac{2t^2}{2} = t^4 - t^2$

When $x = 2$,

$\therefore \quad t^4 - t^2 - 2 = 0$

$$t^2 = \frac{-(-1) \pm \sqrt{1 + 8}}{2}$$

or $t^2 = \frac{1 \pm 3}{2} = 2 \qquad$ (Ignoring –ve sign)

Again, $\frac{d^2x}{dt^2} = 12\,t^2 - 2$

When $t^2 = 2$, acceleration $= 12 \times 2 - 2 = 22\text{ ms}^{-2}$

21. As lift is coming from 8th to 4th floor, the value of x becomes negative, *i.e.* $x < 0$. Velocity is downwards (*i.e.* negative). So, $v < 0$. Before reaching 4th floor lift is retarded, *i.e.* acceleration is upwards. Hence, $a > 0$.

22. Position of particle is, $x(t) = at + bt^2 - ct^3$

So, its velocity is, $v = \frac{dx}{dt} = a + 2bt - 3\,ct^2$

and acceleration is, $a = \frac{dv}{dt} = 2b - 6\,ct$

Acceleration is zero, then $2b - 6\,ct = 0$

$\Rightarrow \quad t = \frac{2b}{6\,c} = \frac{b}{3\,c}$

Substituting this t in expression of velocity, we get

$$v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$$

$$= a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}$$

23. If t_0 is the reaction time, then the distance covered during decelerated motion is $10 - 10\,t_0$.

Now, in the first case,

$$10^2 = 2a\,(10 - 10\,t_0) \quad \ldots\text{(i)}$$

Similarly, in the second case,

$$20^2 = 2a\,(30 - 20\,t_0) \quad \ldots\text{(ii)}$$

Again, in the third case,

$$15^2 = 2a\,(x - 5\,t_0) \quad \ldots\text{(iii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{20^2}{10^2} = \frac{30 - 20\,t_0}{10 - 10\,t_0}$$

or $\quad 40 - 40\,t_0 = 30 - 20\,t_0$

or $\quad 20\,t_0 = 10$

or $\quad t_0 = \frac{1}{2}\,\text{s}$

Dividing Eq. (iii) by Eq. (i), we get

$$\frac{225}{100} = \frac{x - 15\,t_0}{10 - 10\,t_0}$$

or $$\frac{9}{4} = \frac{x - 15 \times \frac{1}{2}}{10 - 10 \times \frac{1}{2}}$$

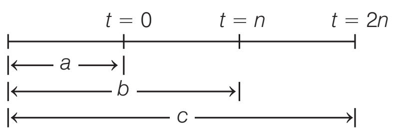
$$45 = 4x - 30$$

or $\quad 4x = 75$

or $\quad x = \frac{75}{4}\,\text{m} = 18.75\,\text{m}$

24. As, $b - a = un + \frac{1}{2}An^2$

$\therefore \quad 2b - 2a = 2\,un + An^2 \quad \ldots\text{(i)}$



Again, $\quad c - a = u\,(2\,n) + \frac{1}{2}A\,(2\,n)^2 \quad \ldots\text{(ii)}$

Subtracting Eq. (i) from Eq. (ii), we get

$$c - a - 2b + 2a = An^2$$

$$A = \frac{c - 2\,b + a}{n^2}$$

25. As, $\frac{dv}{dt} = -kv^3$

$\Rightarrow \quad \int_{v_0}^{v} \frac{dv}{v^3} = -k\int_0^t dt$

$\Rightarrow \quad -\frac{1}{k}\int_{v_0}^{v} v^{-3}\,dv = t$

or $\quad -\frac{1}{k}\left|\frac{v^{-3+1}}{-3+1}\right|_{v_0}^{v} = t$

or $\quad \frac{1}{2k}\left[\frac{1}{v^2} - \frac{1}{v_0^2}\right] = t$

or $\quad \frac{1}{v^2} - \frac{1}{v_0^2} = 2\,kt$

or $\quad \frac{1}{v^2} = \frac{1}{v_0^2} + 2\,kt$

or $\quad \frac{1}{v^2} = \frac{1 + 2v_0^2kt}{v_0^2}$

or $\quad v = \frac{v_0}{\sqrt{2v_0^2kt + 1}}$

26. As, $1 = \frac{v}{t_1}$ and $3 = \frac{v}{t_2}$

$\therefore \quad 1200 = \frac{1}{2}(t_1 + t_2)\,v, \quad \ldots\text{(i)}$

$$1200 = \frac{1}{2}\left(v + \frac{v}{3}\right)v = \frac{1}{2}\,\frac{4\,v^2}{3} = \frac{2\,v^2}{3}$$

or $\quad v^2 = 1800$

From Eq. (i), we get

$\therefore \quad 1200 = \frac{1}{2}\,(t_1 + t_2) \times \sqrt{1800}$

$$(t_1 + t_2) = \frac{2400}{\sqrt{1800}}\,\text{s} = \frac{2400}{42.43}\,\text{s} = 56.6\,\text{s}$$

27. Given, $\frac{dv}{dt} = bt$ or $dv = bt\,dt$

$\Rightarrow \quad \int_{v_0}^{v} dv = \int_0^t bt\,dt$

or $\quad v - v_0 = \frac{bt^2}{2}$

or $\quad v = v_0 + \frac{bt^2}{2}$

or $\quad dx = v_0 dt + \frac{bt^2}{2}\,dt$

$$\int_0^x dx = \int_0^t v_0 dt + \frac{b}{2}\int_0^t t^2\,dt$$

or $\quad x = v_0 t + \frac{1}{2}\,\frac{bt^3}{3}$

$$= v_0 t + \frac{bt^3}{6}$$

28. $40^2 - 30^2 = 2\,as$, and $v^2 - 30^2 = 2a\,\frac{s}{2}$

or $\quad 2\,(v^2 - 30^2) = 2\,as$

Comparing, we get

$$2\,(v^2 - 900) = 1600 - 900 = 700$$

or $\quad v^2 = 900 + 350 = 1250$

or $\quad v = 35.35\,\text{kmh}^{-1}$

29. Given, initial position of particle, $\mathbf{r}_0 = (2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\,\text{m}$,

Initial velocity of particle, $\mathbf{u} = (5\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\,\text{m/s}$

Acceleration of particle, $\mathbf{a} = (4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}})\,\text{m/s}^2$

According to second equation of motion,
position of particle at time t is, $\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$
At $t = 2$s, position of particle,
$$\mathbf{r} = (2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) + (5\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \times 2 + \frac{1}{2}(4\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}) \times 4$$
or $\mathbf{r} = (2 + 10 + 8)\,\hat{\mathbf{i}} + (4 + 8 + 8)\,\hat{\mathbf{j}}$
$\Rightarrow$ $\mathbf{r} = 20\,\hat{\mathbf{i}} + 20\,\hat{\mathbf{j}}$
$\therefore$ Distance of particle from origin, $|\mathbf{r}| = 20\sqrt{2}$ m

30. $4 = u + \frac{a}{2}(2 \times 3 - 1)$

or $\qquad 4 = u + \frac{5a}{2}$

$\qquad 5 = u + \frac{a}{2}(2 \times 4 - 1)$

or $\qquad 5 = u + \frac{7a}{2}$

Subtracting, $\quad 1 = \frac{7a}{2} - \frac{5a}{2} = \frac{2a}{2} = a$

Again, $\qquad 4 = u + \frac{5}{2}$

or $\qquad u = 4 - \frac{5}{2} = 1.5\ \text{ms}^{-1}$

So, the initial velocity is non-zero and acceleration is uniform.

31. $\mathbf{v} = 0.5t^2\hat{\mathbf{i}} + 3t\hat{\mathbf{j}} + 9\hat{\mathbf{k}}$

$\therefore \qquad \mathbf{a} = \frac{d\mathbf{v}}{dt} = t\,\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

At $t = 2$ s, $\mathbf{a} = 2\,\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

$\therefore \qquad \tan\theta = \frac{2}{3}$

or $\qquad \theta = \tan^{-1}\left(\frac{2}{3}\right)$, from Y-axis

32. Given, speed, $v = b\sqrt{x}$
Now, differentiating it with respect to time, we get
$$\frac{dv}{dt} = \frac{d}{dt}\,b\sqrt{x}$$
Now, acceleration,
$$\Rightarrow \qquad a = \frac{b}{2\sqrt{x}} \cdot \frac{dx}{dt} \qquad \left[\because \frac{dv}{dt} = a\right]$$
$$\Rightarrow \qquad a = \frac{b}{2\sqrt{x}} \cdot v = \frac{b}{2\sqrt{x}} \cdot b\sqrt{x} = \frac{b^2}{2}$$
As acceleration is constant, we use
$$v = u + at \qquad \ldots\text{(i)}$$
Now, it is given that $x = 0$ at $t = 0$.
So, initial speed of particle is
$$u = b\sqrt{x}\Big|_{x=0} = b \times 0 = 0$$
Hence, when time $t = \tau$, speed of the particle using Eq. (i) is
$$v = u + at = 0 + \frac{b^2}{2} \cdot \tau = \frac{b^2}{2} \cdot \tau$$

33. When A returns to the level of top of tower, its downward velocity is 4 ms^{-1}. This velocity is the same as that of B. So, both A and B hit the ground with the same velocity.

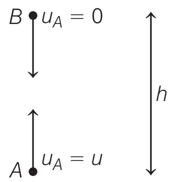
34. As, $x = \frac{1}{2}gt^2$ and $100 - x = 25x - \frac{1}{2}gt^2$,

Adding $\qquad 25t = 100$
or $\qquad t = 4$ s

35. $$\Delta t = \sqrt{\frac{2 \times 12}{10}} - \sqrt{\frac{2 \times 10}{10}} \qquad \left(\because t = \sqrt{\frac{2H}{g}}\right)$$
$$= 1.549\ \text{s} - 1.414\ \text{s} = 0.135\ \text{s} \simeq 0.13\ \text{s}$$

36. Vertical component of velocities of both the balls are same and equal to zero, so $t = \sqrt{\frac{2h}{g}}$ which is same for both the balls.

37. At time t



Velocity of A, $v_A = u - gt$ (upward)
Velocity of B, $v_B = gt$ (downward)
If we assume that height h is smaller than or equal to maximum height reached by A, then at every instant v_A and v_B are in opposite direction.

$\therefore \qquad v_{AB} = v_A + v_B$
$\qquad = u - gt + gt$
(Speeds in opposite directions get added)
$\qquad = u$

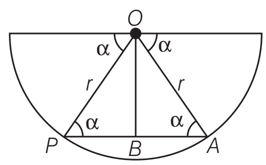
38. Initial velocity of balloon with respect to ground
$$v = 10 + 5 = 15\ \text{m/s (upward)}$$
After 2 s its velocity $v = u - gt$
$$= 15 - 10 \times 2 = -5\ \text{m/s}$$
$$= 5\ \text{m/s (downward)}$$

39. Time of ascent $= \frac{u}{g} = 6$ s $\Rightarrow u = 60$ m/s
Distance in first second,
$$h_{\text{first}} = 60 - \frac{g}{2}(2 \times 1 - 1) = 55\ \text{m}$$
Distance in seventh second will be equal to the distance in first second of vertical downward motion.
$$h_{\text{seventh}} = \frac{g}{2}(2 \times 1 - 1) = 5\ \text{m}$$
$\Rightarrow \qquad h_{\text{first}} / h_{\text{seventh}} = 11 : 1$

40. Let the particle touches the sphere at the point A.
Let $PA = l$

$\therefore \qquad PB = \frac{l}{2}$

In ΔOPB, $\cos\alpha = \frac{PB}{r}$



$\therefore \qquad PB = r\cos\alpha$

or $\qquad \frac{l}{2} = r\cos\alpha$

$\qquad l = 2r\cos\alpha$

But $l = \frac{1}{2} a_0 t^2$

$\therefore \quad t = \sqrt{\left(\frac{2l}{a_0}\right)} = \sqrt{\left(\frac{2 \times 2r \cos \alpha}{a_0}\right)} = \sqrt{\left(\frac{4r \cos \alpha}{a_0}\right)}$

41. Let the ball be at height h at time t and $(t + \Delta t)$, then

$$h = ut - \frac{1}{2} gt^2 \quad \ldots(i)$$

and $$h = u(t + \Delta t) - \frac{1}{2} g (t + \Delta t)^2 \quad \ldots(ii)$$

Equating Eqs. (i) and (ii), we get

$$t = \frac{2u - g\Delta t}{2g}$$

Substituting Eq. (ii) in Eq. (i), we get,

$$h = \frac{4u^2 - g^2 (\Delta t)^2}{8g}$$

$$\Rightarrow \quad u = \frac{1}{2}\sqrt{8gh + g^2 (\Delta t)^2}$$

42. Acceleration of body along AB is $g \cos \theta$

Distance travelled in time t sec $= AB = \frac{1}{2}(g \cos \theta) t^2$

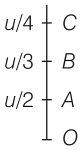
From ΔABC, $\quad AB = 2R \cos \theta$

$$2R \cos \theta = \frac{1}{2} g \cos \theta \, t^2$$

$$t^2 = \frac{4R}{g}$$

$$t = 2\sqrt{\frac{R}{g}}$$

43. Here, $\quad OA = \dfrac{u^2 - \dfrac{u^2}{4}}{2g} = \dfrac{u^2}{2g} \cdot \dfrac{3}{4}$



$$OB = \frac{u^2 - \dfrac{u^2}{9}}{2g} = \frac{u^2}{2g} \cdot \frac{8}{9}$$

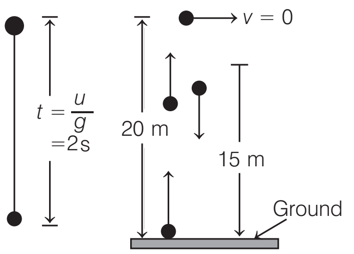
and $$OC = \frac{u^2 - \dfrac{u^2}{16}}{2g} = \frac{u^2}{2g} \cdot \frac{15}{16}$$

$$\therefore \quad AB = OB - OA = \frac{u^2}{2g}\left\{\frac{8}{9} - \frac{3}{4}\right\} = \frac{4^2}{2g} \cdot \frac{5}{36}$$

$$BC = OC - OB = \frac{u^2}{2g}\left\{\frac{15}{16} - \frac{8}{9}\right\} = \frac{u^2}{2g} \cdot \frac{7}{144}$$

$$\therefore \quad \frac{AB}{BC} = \frac{5}{36} \times \frac{144}{7} = \frac{20}{7}$$

44.



Position of balls

$$h_1 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times 1^2 = 5 \text{ m}$$

$$h_2 = \frac{1}{2} gt^2 = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$$

From ground, 5 m, 20 m, 15 m (shown in figure)

45. Now, retain only the positive sign.

$$(2v)^2 - v^2 = 2gh'$$

or $\quad 4v^2 - v^2 = 2gh'$ or $3v^2 = 2gh'$

or $\quad 3 \times 2gh = 2gh'$ or $h' = 3h$

46. As, $\quad h = -vt + \frac{1}{2} gt^2$

or $\quad gt^2 - 2vt - 2h = 0$

$$\Rightarrow \quad t = \frac{-(-2v) \pm \sqrt{4v^2 + 8gh}}{2g} = \frac{2v \pm \sqrt{v^2 + 2gh}}{2g}$$

$$= \frac{v}{g} \pm \frac{(v^2 + 2gh)^{1/2}}{g} = \frac{v}{g}\left[1 \pm \sqrt{1 + \frac{2gh}{v^2}}\right]$$

$$= \frac{v}{g}\left[1 + \sqrt{1 + \frac{2gh}{v^2}}\right] \quad \text{[taking +ve sign]}$$

47. Let u be the velocity with which the stone is projected vertically upwards.

Given that, $\quad v_{-h} = 2v_h$

$$(v_{-h})^2 = 4v_h^2$$

$$\therefore \quad u^2 - 2g(-h) = 4(u^2 - 2gh)$$

$$\therefore \quad u^2 = \frac{10gh}{3}$$

Now, $\quad h_{\max} = \dfrac{u^2}{2g} = \dfrac{5h}{3}$

48. As, $s' \propto t^2$

Now, $\quad s_1' : s_2' : s_3' = \frac{1}{4} : 1 : \frac{9}{4}$ or $1 : 4 : 9$

For successive intervals,

$$s_1 : s_2 : s_3 = 1 : (4 - 1) : (9 - 4)$$

or $\quad s_1 : s_2 : s_3 = 1 : 3 : 5$

49. As, $\quad \Delta x = \frac{1}{2} gt^2 - \frac{1}{2} g (t - 1)^2$

$$= \frac{1}{2} g [t^2 - (t - 1)^2] = \frac{1}{2} g (2t - 1)$$

$$= \frac{1}{2} \times 9.8 \times 5 \text{ m} = 24.5 \text{ m}$$

50. Clearly, $\dfrac{11h}{36} = \dfrac{9.8}{2}(2n - 1)$

or $\quad \dfrac{11}{36} \times \dfrac{1}{2} \times 9.8 n^2 = \dfrac{9.8}{2}(2n - 1)$

or $\quad 2n - 1 = \dfrac{11}{36} n^2$

or $\quad 11n^2 = 72n - 36$

or $\quad 11n^2 - 72n + 36 = 0$

or $\quad 11n^2 - 66n - 6n + 36 = 0$

or $\quad 11n(n - 6) - 6(n - 6) = 0 \Rightarrow n = 6$

(Rejecting fractional value)

$$h = \frac{1}{2} \times 10 \times 6 \times 6 = 180 \text{ m}$$

51. Let us solve the problem in terms of relative initial velocity, relative acceleration and relative displacement of the coin with respect to floor of the lift.

Given, $u = 0\ \text{ms}^{-1}, a = 9.8\ \text{ms}^{-2}, s = 4.9\ \text{m}, t = ?$

As, $$4.9 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

or $$4.9\, t^2 = 4.9$$

or $$t = 1\ \text{s}$$

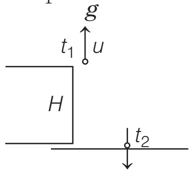
$$15 = 30 - 10\, t$$

or $$10\, t = 15$$

or $$t = 1.5\ \text{s}$$

52. Time taken to reach the maximum height

$$t_1 = \frac{u}{g}$$



If t_2 is the time taken to hit the ground

i.e. $$-H = ut_2 - \frac{1}{2} gt_2^2$$

But $$t_2 = nt_1 \quad \text{[given]}$$

So, $$-H = u\frac{nu}{g} - \frac{1}{2} g \frac{n^2u^2}{g^2}$$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g}$$

$$H = \frac{1}{2}\frac{n^2u^2}{g} - \frac{nu^2}{g} = \frac{n^2u^2 - 2nu^2}{2g}$$

$$2gH = n^2u^2 - 2nu^2$$

$$2gH = nu^2\,(n-2)$$

53. By the time 5th water drop starts falling, the first water drop reaches the ground.

As $u = 0$, $$h = \frac{1}{2} gt^2$$

$$= \frac{1}{2} \times 10 \times t^2$$

or $$5 = \frac{1}{2} \times 10 \times t^2$$

or $$t = 1\ \text{s}$$

Hence, the interval of each water drop $= \dfrac{1\ \text{s}}{4} = 0.25\ \text{s}$

When the 5th drop starts its journey towards ground, the third drop travels in air for

$$t_1 = 0.25 + 0.25 = 0.5\ \text{s}$$

$\therefore$ Height (distance) covered by 3rd drop in air is

$$h_1 = \frac{1}{2} gt_1^2 = \frac{1}{2} \times 10 \times (0.5)^2$$

$$= 5 \times 0.25 = 1.25\ \text{m}$$

So, third water drop will be at a height of

$$= 5 - 1.25 = 3.75\ \text{m}$$

54. As, $h = -vt_1 + \frac{1}{2} gt_1^2$

or $$\frac{h}{t_1} = -v + \frac{1}{2} gt_1 \quad \text{...(i)}$$

and $$h = vt_2 + \frac{1}{2} gt_2^2$$

or $$\frac{h}{t_2} = v + \frac{1}{2} gt_2 \quad \text{...(ii)}$$

$\therefore$ $$\frac{h}{t_1} + \frac{h}{t_2} = \frac{1}{2} g\,(t_1 + t_2)$$

or $$h = \frac{1}{2} gt_1t_2$$

For fall under gravity from the top of the tower,

$$h = \frac{1}{2} gt^2$$

$\therefore$ $$\frac{1}{2} gt_1t_2 = \frac{1}{2} gt^2$$

$\Rightarrow$ $$t = \sqrt{t_1\, t_2}$$

55. Average acceleration $= \dfrac{\Delta v}{\Delta t}$

$$= \frac{\sqrt{2\, gh'} - (-\sqrt{2\, gh})}{\Delta t}$$

$$= \frac{\sqrt{2\, gh'} + \sqrt{2\, gh}}{\Delta t}$$

$$= \frac{\sqrt{2 \times 10 \times 2.5} + \sqrt{2 \times 10 \times 10}}{0.01}\ \text{ms}^{-2}$$

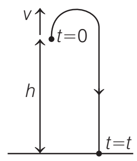
$$= \frac{\sqrt{50} + \sqrt{200}}{0.01}\ \text{ms}^{-2}$$

$$= \frac{5\sqrt{2} + 10\sqrt{2}}{0.01}\ \text{ms}^{-2}$$

$$= \frac{15\sqrt{2}}{0.01} = 1500\sqrt{2}\ \text{ms}^{-2}$$

The upward velocity has been taken as positive. Since, average acceleration is positive, therefore its direction is vertically upward.

56. Let t be the time taken by the packet to reach the ground. As, the helicopter rises from rest in upward direction, its final velocity is



$$v = \sqrt{0^2 + 2gh} \Rightarrow v = \sqrt{2gh}$$

From second equation of motion,

$$s = ut + \frac{1}{2} at^2$$

Here, $$s = -h$$

$$u \text{ or } v = \sqrt{2gh}$$

$\Rightarrow$ $$a = g$$

Substituting all these values in above equation, we get

$$-h = \sqrt{2gh}\ t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \qquad \frac{1}{2}gt^2 - \sqrt{2gh}\ t - h = 0$$

This is a quadratic equation in t.

$$\therefore \qquad t = \frac{\sqrt{2gh} \pm \sqrt{(\sqrt{2gh})^2 - 4 \times \frac{g}{2}(-h)}}{2 \times \frac{g}{2}}$$

$$= \frac{\sqrt{2gh} \pm \sqrt{2gh + 2gh}}{g}$$

$$= \frac{\sqrt{2gh}}{g}(1 + \sqrt{2})$$

$$= \sqrt{\frac{2h}{g}}\ (1 + \sqrt{2})$$

$$= (2 + \sqrt{2})\sqrt{\frac{h}{g}} = 3.4\sqrt{\frac{h}{g}}$$

57. Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

58. In graph (b), for one value of displacement, there are two timings. As a result of it, for one time, the average velocity is positive and for other time is equivalent negative. Due to it, the average velocity for the two timings (equal to time period) can vanish.

59. Height reached $= \frac{1}{2} \times 132 \times 1200$ m $= 66 \times 1200$ m

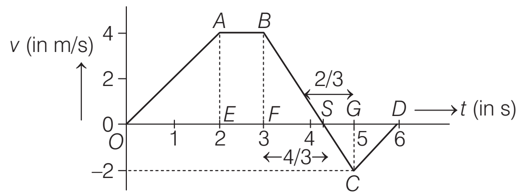
60. Since acceleration is constant, therefore there is a uniform increase in velocity. So, the v-t graph is a straight line slopping upward to the right. When acceleration becomes zero, velocity is constant. So, v-t graph is a straight line parallel to the time-axis.

61. Area between v-t graph and time axis gives the distance.

$$\therefore \qquad \Delta = \frac{1}{2} \times 2 \times 20 + 15 \times 3 + 2 \times \frac{1}{2} \times 15 \times 1$$

$$= 80 \text{ m}$$

62. The given v-t graph is shown below



Distance covered by the body in 6s = Sum of magnitudes of area of each part in v-t curve

s = Area of ΔAOE + Area of rectangle $ABFE$ + Area of ΔBSF + Area of ΔSCG + Area of ΔGCD

$$= \frac{1}{2}(OE \times AE) + (EF \times AE) + \frac{1}{2}(SF \times BF) + \frac{1}{2}(SG \times GC) + \frac{1}{2}(GD \times GC)$$

$$= \frac{1}{2}(2 \times 4) + 1 \times 4 + \frac{1}{2} \times 4 \times \left(\frac{4}{3}\right) + \frac{1}{2} \times 2 \times \left(\frac{2}{3}\right) + \frac{1}{2} \times 2 \times 1$$

$$= 4 + 4 + \frac{8}{3} + \frac{2}{3} + 1 = \frac{37}{3} \text{ m}$$

63. Slope of line $= -\frac{2}{3}$

Equation of line is $(v - 20) = -\frac{2}{3}(s - 0)$

$$\Rightarrow \qquad v = 20 - \frac{2}{3}s \qquad \ldots\text{(i)}$$

Velocity at $s = 15$ m, *i.e.*

$$v = \left.\frac{ds}{dt}\right|_{s=15\text{ m}} = 20 - \frac{2}{3}(15) = 10 \text{ ms}^{-1}$$

Differentiate Eq. (i) with respect to time,

$$\text{acceleration} = \frac{dv}{dt} = -\frac{2}{3}\frac{ds}{dt}$$

$$\therefore \qquad a = \left.\frac{dv}{dt}\right|_{s=15\text{ m}} = -\frac{2}{3}\left.\frac{ds}{dt}\right|_{s=15\text{ m}} = -\frac{20}{3}\text{ ms}^{-2}$$

64. The displacement of the particle is determined by the area bounded by the curve, this area is $s = \frac{\pi}{4} v_m t_0$.

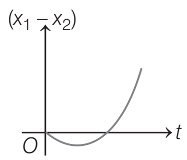
The average velocity is $< v > = \frac{s}{t_0} = \frac{\pi}{4} v_m$

Such motion cannot be realised in practical terms since at the initial and final moments, the acceleration (which is slope of v-t graph) is infinitely large. Hence, both (i) and (ii) are correct.

65. Here, $\quad x_2 = vt \quad$ and $\quad x_1 = \frac{at^2}{2}$

$$\therefore \qquad x_1 - x_2 = -\left(vt - \frac{at^2}{2}\right)$$

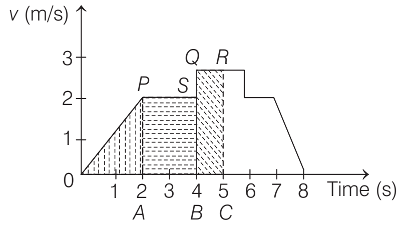
From the above expression, it is clear that at $t = 0$, $x_1 - x_2 = 0$, further for increasing values, the graph is as follows



Hence, option (b) is true.

66. In option (a), (c) and (d), velocity of the body becomes zero after sometime or travelling some distance at least once. Hence, these graphs represent the same motion. In option (b), distance increases with time and becomes constant, hence velocity is not zero after starting motion, hence it represents different motion.

67. To get exact position at $t = 5$ s, we need to calculate area of the shaded part in the curve as shown below



$\therefore$ Displacement of particle = Area of OPA + Area of $PABSP$ + Area of $QBCRQ$

$$= \left(\frac{1}{2} \times 2 \times 2\right) + (2 \times 2) + (3 \times 1)$$

$$= 2 + 4 + 3 = 9 \text{ m}$$

68. Since, the particle starts from rest, this means, initial velocity, $u = 0$.

Also, it moves with uniform acceleration along positive X-axis. This means, its acceleration (a) is constant.

$\therefore$ Given, a-t graph in (A) is correct.

As we know, for velocity-time graph,
slope = acceleration.

Since, the given v-t graph in (B) represents that its slope is constant and non-zero.

$\therefore$ Graph in (B) is also correct.

Also, the displacement of such a particle w.r.t. time is given by

$$x = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 \Rightarrow x \propto t^2$$

So, x *versus* t graph would be a parabola with starting from origin.

This is correctly represented in displacement-time graph given in (D).

69. Given line have positive intercept but negative slope so its equation can be written as

$$v = -mx + v_0 \quad \ldots\text{(i)}$$

$$\left[\text{where, } m = \tan\theta = \frac{v_0}{x_0}\right]$$

By differentiating w.r.t. time, we get

$$\frac{dv}{dt} = -m\frac{dx}{dt} = -mv$$

Now substituting the value of v from Eq. (i), we get

$$\frac{dv}{dt} = -m(-mx + v_0) = m^2x - mv_0$$

$$\therefore \quad a = m^2x - mv_0$$

The graph between a and x should have positive slope but negative intercept on a-axis. So, graph (c) is correct.

70. The slope of the given graph over the time interval t_1 to t_2 is not constant and is not uniform. It means acceleration is not constant and is not uniform, therefore relations (i), (ii) and (v) are not correct which is for uniform accelerated motion, but relations (iii), (iv) and (vi) are correct, because these relations are true for both uniform or non-uniform accelerated motion.

71. As the trains are moving in the same direction, so the initial relative speed $(v_1 - v_2)$ and by applying retardation, final relative speed becomes zero.

From $v = u - at$, we have

$$\Rightarrow \quad 0 = (v_1 - v_2) - at$$

$$\Rightarrow \quad t = \left(\frac{v_1 - v_2}{a}\right)$$

72. Velocity of girl, $v_g = \dfrac{L}{t_1}$

Velocity of escalator, $v_e = \dfrac{L}{t_2}$

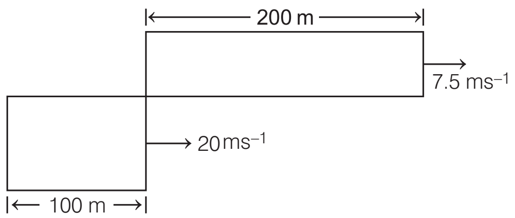
Effective velocity of girl on escalator $= v_g + v_e$

$$= \frac{L}{t_1} + \frac{L}{t_2}$$

If t is the time taken, then $\dfrac{L}{t} = \dfrac{L}{t_1} + \dfrac{L}{t_2}$

or $$t = \frac{t_1 t_2}{t_1 + t_2}$$

73. From the figure, the relative displacement is



$$s_{\text{rel}} = (200 + 100) \text{ m} = 300 \text{ m}$$

$$v_{\text{rel}} = v_1 - v_2 = (20 - 7.5) \text{ ms}^{-1} = 12.5 \text{ ms}^{-1}$$

$$\therefore \quad t = \frac{s_{\text{rel}}}{v_{\text{rel}}} = \frac{300}{12.5} = 24 \text{ s}$$

74. Relative velocity of bird w.r.t. train $= 25 + 5 = 30$ m/s

Time taken by the bird to cross the train

$$t = \frac{210}{30} = 7 \text{ s}$$

75. Let the required angle is θ.

$$\tan(90° - \theta) = \frac{20}{15}$$

$$\therefore \quad \cot\theta = \frac{20}{15} = \frac{4}{3}$$

$$\Rightarrow \quad \theta = 37°$$

$\therefore$ Required angle $= \theta + 23° = 37° + 23° = 60°$

76. Let us calculate relative deceleration by considering relative velocity.

Using, $\quad v^2 - u^2 = 2as$

$$0^2 - 80^2 = 2 \times a \times 2000$$

or $$a = -\frac{80 \times 80}{4000} = -\frac{64}{40} \text{ ms}^{-2} = -1.6 \text{ ms}^{-2}$$

Deceleration of each train is $\frac{1.6}{2}\text{ms}^{-2}$, *i.e.* 0.8 ms^{-2}.

77. When trains are moving in same direction, then relative speed $= |v_1 - v_2|$ and in opposite direction, relative speed $= |v_1 + v_2|$.

Hence, ratio of time when trains move in same direction with time when trains move in opposite direction is

$$\frac{t_1}{t_2} = \frac{\left(\frac{l_1 + l_2}{|v_1 - v_2|}\right)}{\left(\frac{l_1 + l_2}{|v_1 + v_2|}\right)} = \frac{|v_1 + v_2|}{|v_1 - v_2|}$$

where, $l_1 + l_2 =$ sum of lengths of trains which is same as distance covered by trains to cross each other

So, $$\frac{t_1}{t_2} = \frac{80+30}{80-30} = \frac{110}{50} = \frac{11}{5}$$

78. When two particles moves towards each other, then

$$v_1 + v_2 = 6 \qquad \text{...(i)}$$

When these particles moves in the same direction, then

$$v_1 - v_2 = 4 \qquad \text{...(ii)}$$

By solving Eqs. (i) and (ii), we get $v_1 = 5$ and $v_2 = 1$ m/s.

79. Effective speed of the bullet

= Speed of bullet + Speed of police jeep

= 180 m/s + 45 km/h

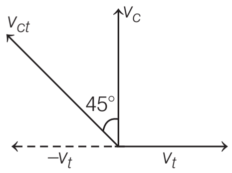
= (180 + 12.5) m/s = 192.5 m/s

Speed of thief's jeep = 153 km/h = 42.5 m/s

Velocity of bullet w.r.t. thief's car

= 192.5 − 42.5 = 150 m/s

80. $\mathbf{v}_{ct} = \mathbf{v}_c - \mathbf{v}_t$



$$\mathbf{v}_{ct} = \mathbf{v}_c + (-\mathbf{v}_t)$$

Velocity of car w.r.t. train (v_{ct}) is towards west-north.

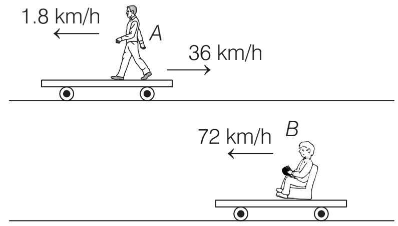
81. Here, $t_1 = \frac{x}{v} + \frac{x}{v} = \frac{2x}{v}$

$$t_2 = \frac{x}{v+\omega} + \frac{x}{v-\omega} = \frac{2xv}{v^2 - \omega^2}$$

or $$t_2 = \frac{2xv}{v^2\left(1 - \frac{\omega^2}{v^2}\right)} = \frac{2x}{v\left(1 - \frac{\omega^2}{v^2}\right)}$$

or $$t_2 = \frac{t_1}{1 - \frac{\omega^2}{v^2}} \quad \text{or} \quad t_2 > t_1$$

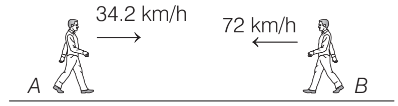
82. Condition given in question is as shown,



Speed of person on train A with respect to ground

$= 36 - 1.8 = 34.2$ km/h in direction of A

So, the situation becomes as shown

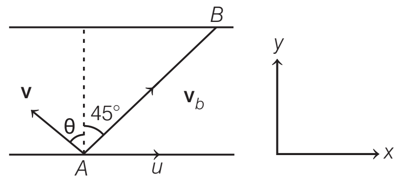


Hence, speed of person on train A as observed by an observer on train B is

$$v_{AB} = 34.2 + 72 = 106.2 \text{ km/h}$$

$$= 106.2 \times \frac{5}{18}\frac{\text{m}}{\text{s}} = 29.5 \text{ ms}^{-1}$$

83. Let v be the speed of boatman in still water.



Resultant of v and u should be along AB. Components of $\mathbf{v}_b$ (absolute velocity of boatman) along x and y directions are

$$v_x = u - v\sin\theta$$

and $$v_y = v\cos\theta$$

Further, $$\tan 45° = \frac{v_y}{v_x}$$

or $$1 = \frac{v\cos\theta}{u - v\sin\theta}$$

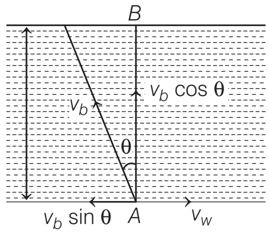
$$v = \frac{u}{\sin\theta + \cos\theta} = \frac{u}{\sqrt{2}\sin(\theta + 45°)}$$

v is minimum, at

$$\theta + 45° = 90° \quad \text{or} \quad \theta = 45°$$

and $$v_{\min} = \frac{u}{\sqrt{2}}$$

84. From figure,



$$v_b \sin\theta = v_\omega$$

$$\sin\theta = \frac{v_\omega}{v_b} = \frac{1}{2}$$

$\Rightarrow \qquad \theta = 30°$

Time taken to cross the river,

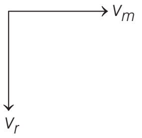
$$t = \frac{D}{v_b \cos\theta} = \frac{D}{v_b \cos 30°} = \frac{2D}{v_b\sqrt{3}}$$

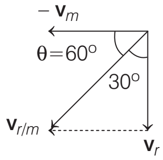
85. Given, rain drop is falling vertically downwards (w.r.t. ground).

So, velocity of rain $\mathbf{v}_r = v_r(-\hat{\mathbf{j}})$, velocity of man $\mathbf{v}_m = v\hat{\mathbf{i}}$.

Velocity of rain w.r.t to man $\mathbf{v}_{r/m} = \mathbf{v}_r - \mathbf{v}_m$

Drawing velocity vectors



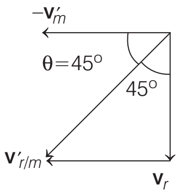


From diagram,

$$\tan 30° = \frac{v_m}{v_r} \Rightarrow \frac{1}{\sqrt{3}} = \frac{v_m}{v_r} \quad \text{or} \quad v_r = \sqrt{3}v$$

Now, man increases velocity of car to $(1+\beta)v$ in same direction, then $\mathbf{v}'_m = (1+\beta)\,v\hat{\mathbf{i}}$

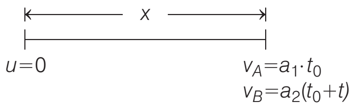
Drawing velocity vectors



$$\tan 45° = \frac{v'_m}{v_r} \Rightarrow 1 = \frac{(1+\beta)v}{\sqrt{3}v}$$

$\Rightarrow \qquad 1 + \beta = \sqrt{3} \;$ or $\beta = \sqrt{3} - 1 = 0.73$

86. Let car B takes time $(t_0 + t)$ and car A takes time t_0 to finish the race.



Given, $\qquad v_A - v_B = v = (a_1 - a_2)t_0 - a_2 t \qquad$ ...(i)

$$s_B = s_A = \frac{1}{2}a_1 t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$$

or $\qquad \sqrt{a_1}\; t_0 = \sqrt{a_2}(t_0 + t)$

or $\qquad \sqrt{a_1}t_0 = \sqrt{a_2}t_0 + \sqrt{a_2}t$

or $\qquad (\sqrt{a_1} - \sqrt{a_2})\, t_0 = \sqrt{a_2}\; t$

or $\qquad t_0 = \dfrac{\sqrt{a_2}.\, t}{(\sqrt{a_1} - \sqrt{a_2})} \qquad$ ...(ii)

Substituting the value of t_0 from Eq. (ii) into Eq. (i), we get

$$v = (a_1 - a_2)\frac{\sqrt{a_2}\; t}{\sqrt{a_1} - \sqrt{a_2}} - a_2 t$$

$$= (\sqrt{a_1} - \sqrt{a_2})\,(\sqrt{a_1} + \sqrt{a_2}) \cdot \frac{\sqrt{a_2}t}{(\sqrt{a_1} - \sqrt{a_2})} - a_2 t$$

or $\qquad v = (\sqrt{a_1} + \sqrt{a_2}) \cdot \sqrt{a_2}\; t - a_2 t$

$\qquad = \sqrt{a_1 a_2} \cdot t + a_2 t - a_2 t$

or $\qquad v = \sqrt{a_1 \cdot a_2}\; t$

Round II

1. If t_1 and $2\,t_2$ are the time taken by particle to cover first and second half distance, respectively

$$t_1 = \frac{x/2}{3} = \frac{x}{6}$$

Clearly, $\qquad x_1 = 4.5\, t_2$

and $\qquad x_2 = 7.5\, t_2$

So, $\qquad x_1 + x_2 = \dfrac{x}{2} \Rightarrow 4.5\, t_2 + 7.5\, t_2 = \dfrac{x}{2}$

$$t_2 = \frac{x}{24}$$

$$\text{Total time } t = t_1 + 2\,t_2 = \frac{x}{6} + \frac{x}{12} = \frac{x}{4}$$

So, $\qquad$ average speed = 4 m/s

2. Let v_1 and v_2 be the initial speeds of first and second runners respectively. Let t be time taken by them when the first runner has completed 50 m. During this time, the second runner has covered a distance $= 50 - 1 = 49$ m.

So, $\qquad t = \dfrac{50}{v_1} = \dfrac{49}{v_2} \qquad$ ...(i)

Suppose, the second runner increases his speed to v_3 so that he covers the remaining distance (= 51 m) in time t.

So $\qquad t = \dfrac{51}{v_3} = \dfrac{49}{v_2}$

or $\qquad v_3 = \dfrac{51}{49}\, v_2$

or $\qquad v_3 = \left(1 + \dfrac{2}{49}\right) v_2$

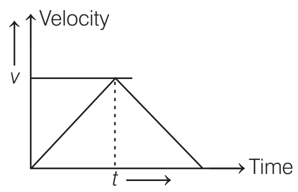
or $\qquad \dfrac{v_3}{v_2} - 1 = \dfrac{2}{49}$

or $\qquad \dfrac{v_3 - v_2}{v_2} = \dfrac{2}{49}$

or $\qquad \%\text{ increase} = \dfrac{2}{49} \times 100 = 4.1\%$

3. $v_1 = \alpha t_1$, $v_2 = \beta t_2$ and $t_1 + t_2 = t$

$\therefore \qquad v = \dfrac{\alpha\beta t}{(\alpha + \beta)}$



Distance = Area of v-t graph

$$= \frac{1}{2} \times v \times t = \frac{1}{2} \frac{\alpha\beta t^2}{\alpha + \beta}$$

4. Given, velocity of a particle is $\mathbf{v} = k(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}})$ ...(i)

Suppose, it's position is given as $\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$

$$\therefore \quad \mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}) = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{dx}{dt} = ky \quad \text{...(iii)}$$

and $$\frac{dy}{dt} = kx \quad \text{...(iv)}$$

Dividing Eq. (iii) and Eq. (iv), we get

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{y}{x}$$

$$\Rightarrow \quad x\frac{dx}{dt} = y\frac{dy}{dt}$$

or $$xdx = ydy$$

Integrating both sides, we get

$$\int xdx = \int ydy$$

or $$\frac{x^2}{2} + \frac{c_1}{2} = \frac{y^2}{2} + \frac{c_2}{2}$$

where, c_1 and c_2 are the constants of integration.

$$\Rightarrow \quad x^2 + c = y^2 \quad [\text{here, } c \text{ (constant)} = c_1 - c_2]$$

or $$y^2 = x^2 + \text{constant}$$

5. Let a be the retardation in both the cases. Using the relation, $v^2 = u^2 + 2\,as$, when automobile is stopped, $v = 0$.

So, $$0 = u^2 - 2\,as$$

or $$s \propto u^2$$

$$\therefore \quad s_2 = 4\,s_1 = 4 \times 20 = 80 \text{ m}$$

6. Given, acceleration $a = (6\,t + 5)$ m/s^2

$$\Rightarrow \quad a = \frac{dv}{dt} = (6\,t + 5),$$

$$dv = (6\,t + 5)\,dt$$

Integrating it, we have $\int dv = \int (6t + 5)\,dt$

$$v = 3\,t^2 + 5\,t + c$$

where c is constant of integration

When $t = 0$, $v = 0$ so $c = 0$

$$\therefore \quad v = 3\,t^2 + 5\,t$$

$$\Rightarrow \quad ds = (3\,t^2 + 5\,t)\,dt \qquad \left(\text{as } v = \frac{ds}{dt}\right)$$

Integrating it within the condition of motion is as t changes from 0 to 2 s, S changes from 0 to S, we have

$$\int_0^S ds = \int_0^2 (3\,t^2 + 5\,t)\,dt$$

$$\therefore \quad S = \left[t^3 + \frac{5}{2}t^2\right]_0^2 = 8 + 10 = 18 \text{ m}$$

7. Given, $$\frac{dv(t)}{dt} = 6 - 3\,v(t) \quad \text{...(i)}$$

or $$\frac{dv(t)}{6 - 3\,v(t)} = dt$$

Integrating it, we have

$$\left[-\frac{1}{3}\log(6 - 3\,v(t))\right] = t + K \quad \text{...(ii)}$$

At $$t = 0,\ v(t) = 0$$

$$\therefore \quad K = -\frac{1}{3}\log 6$$

Putting this value in Eq. (ii), we have

$$-\frac{1}{3}\log(6 - 3\,v(t)) = t - \frac{1}{3}\log 6$$

or $$\log\left(\frac{6 - 3\,v(t)}{6}\right) = -3\,t$$

or $$\frac{6 - 3\,v(t)}{6} = e^{-3\,t}$$

or $$1 - \frac{v(t)}{2} = e^{-3\,t}$$

or $$v(t) = 2\,(1 - e^{-3\,t})$$

when $$t = 0,\ v(t) = 2\,(1 - e^{-3\,t})$$

Initially, $v = 0$, From Eq. (i) acceleration,

$$a_0 = \frac{dv}{dt} = 6 - 3 \times 0 = 6 \text{ ms}^{-2}$$

When $a = \dfrac{a_0}{2} = \dfrac{6}{2} = 3 \text{ ms}^{-2}$ then from Eq. (i);

$$3 = 6 - 3\,v$$

or $$3\,v = 6 - 3 = 3$$

or $$v = 1 \text{ ms}^{-1}$$

8. Velocity of bolt relative to elevator = 2.5 – 2.5 = 0

Acceleration of bolt relative to elevator,

$$a = 10 - (-2) = 12 \text{ ms}^{-2} \quad (\because g = 10 \text{ m/s}^2)$$

Using the relation, $s = ut + \dfrac{1}{2}at^2$

we have, $$3.0 = 0 \times t + \frac{1}{2} \times 12 \times t^2$$

or $$t = \frac{1}{\sqrt{2}} \text{ s} = 0.707 \text{ s} = 0.7 \text{ s}$$

$$\text{Displacement} = -\,2.5 \times 0.71 + \frac{1}{2} \times 10 \times (0.71)^2$$

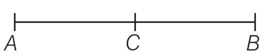
$$= -\,1.775 + 2.521 = 0.746 = 0.75 \text{ m}$$

$$\text{Distance covered} = 2 \times \frac{u^2}{2\,g} + \text{displacement}$$

$$= 2 \times \frac{(2.5)^2}{2 \times 10} + 0.75$$

$$= 0.63 + 0.75 = 1.38 \text{ m}$$

9. Here, $u = 2 \text{ ms}^{-1}$, $v = 14 \text{ ms}^{-1}$



Distance between A and $B = s$

Then acceleration, $a = \dfrac{v^2 - u^2}{2\,s} = \dfrac{14^2 - 2^2}{2\,s} = \dfrac{194}{2\,s} = \dfrac{97}{s}$

The speed at mid-point C,

$$v^2 = u^2 + 2a\frac{s}{2}$$
$$= 2^2 + 2\times\frac{97}{s}\times\frac{s}{2} = 101$$
$$v = \sqrt{101} \simeq 10 \text{ ms}^{-1}$$

As per question, $AP = \frac{1}{6}[AB] = \frac{1}{6}s$

When, $s = \frac{s}{6}$,

then, $v^2 = 2^2 + 2\times\frac{97}{s}\times\frac{s}{6}$

or $v = 3 + 2\times\frac{97}{3} = 36.3 \text{ ms}^{-1}$

$\therefore$ $v = \sqrt{36.3} \approx 6 \text{ ms}^{-1}$

Since velocity at mid-point C is 10 ms^{-1}.

$\therefore$ Taking motion from A to C, we have

$$10 = 2 + a\times t_1$$

or $t_1 = \frac{10-2}{a} = \frac{8}{a}$

Taking motion from C to B, we have

$$14 = 10 + 1\times t_2$$

or $t_2 = \frac{14-10}{a} = \frac{4}{a}$

$\therefore$ $\frac{t_1}{t_2} = 2$ or $t_2 = \frac{t_1}{2}$

10. Here, $u = 0, v = 640 \text{ ms}^{-1}, s = 1.2 \text{ m}, a = ?$ and $t = ?$

As, $v^2 = u^2 + 2as$

or $a = \frac{v^2 - u^2}{2s} = \frac{(640)^2}{2\times 1.2} = \frac{(640)^2}{2.4}\text{ms}^{-1}$

Also, $t = \frac{v-u}{a} = \frac{640-0}{\frac{(640)^2}{2.4}}$

$$= \frac{2.4}{640} = 3.75\times 10^{-3} \text{ s} \simeq 4 \text{ ms}$$

11. The acceleration of train in 20 s is given by

$$a = \frac{40-0}{20} = 2 \text{ ms}^{-2}$$

[from the formula $v = u + at$ (here, $u = 0$)].

Now the distance travelled is given by equation of motion,

$$v^2 = u^2 + 2as$$

So, $s_1 = \left(\frac{v^2-u^2}{2}\right)\Big/ a = \frac{40\times 40 - 0}{2\times 2} = 400 \text{ m}$

Now distance travelled with constant speed of 40 ms^{-1} in $t = 20$ s is

$$s_2 = 40\times 20 = 800 \text{ m}$$

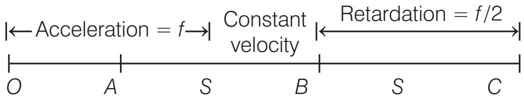
Again the distance covered in 3rd case is given by

$$s_3 = \frac{40\times 40 - 0}{2\times 1} = 800 \text{ m}$$

Therefore, average speed of the train is given by

$$v_{\text{av}} = \frac{400+800+800}{20+20+40} = \frac{2000}{80} = 25 \text{ ms}^{-1}$$

12.



Taking motion of car from 0 to A,

Here, $u = 0, a = f, s = S, v = ?$

As $v^2 = u^2 + 2as$

So $v^2 = 0 + 2\times f\times S$

or $v = \sqrt{2fS}$

The velocity of car at A = velocity of car at $B = (2fS)^{1/2}$

As magnitude of retardation of the car from B to C is half of that of acceleration from O to A when velocity changes by v, so distance $BC = 2S$

Distance, $AB = 15S - (S + 2S) = 12S$

As distance AB is covered with constant velocity in time t

So, $12S = vt = (2fS)^{1/2}\times t$

or $144S^2 = 2fSt^2$

or $S = \frac{1}{72}ft^2$

13. As, $h = ut + \frac{1}{2}gt^2$

$\Rightarrow$ $1 = 0\times t_1 + \frac{1}{2}gt_1^2$

$\Rightarrow$ $t_1 = \sqrt{2/g}$

Velocity after travelling 1m distance

$$v^2 = u^2 + 2gh$$

$\Rightarrow$ $v^2 = (0)^2 + 2g\times 1$

$\Rightarrow$ $v = \sqrt{2}\ g$

For second 1 m distance

$$1 = \sqrt{2g}\times t_2 + \frac{1}{2}gt_2^2$$

$\Rightarrow$ $gt_2^2 + 2\sqrt{2}\ gt_2 - 2 = 0$

$$t_2 = \frac{-2\sqrt{2\ g} \pm \sqrt{8\ g + 8\ g}}{2\ g}$$
$$= \frac{-\sqrt{2}\pm 2}{\sqrt{g}}$$

Taking +ve sign, $t_2 = (2-\sqrt{2}\ t)/\sqrt{g}$

$\therefore \frac{t_1}{t_2} = \frac{\sqrt{2/g}}{(2-\sqrt{2})/\sqrt{g}} = \frac{1}{\sqrt{2}-1}$ or $\frac{t_2}{t_1} = (\sqrt{2}-\sqrt{1})$ and so on.

14. Interval of all ball throw = 2 s

If we want that minimum three (more than two) ball remain in air then, time of flight of first ball must be greater than 4 s.

$$t > 4 \text{ s}$$
$$\frac{2u}{g} > 4 \text{ s}$$

$\Rightarrow$ $u > 19.6$ m/s

For $u = 19.6$, then first ball will just about to strike the ground (in air).

Second ball will be highest point (in air).

Third ball will beat point of projection or at ground (not in air).

15. Let the body be projected upwards with velocity u from top of tower. Taking vertical downward motion of boy from top of tower to ground, we have

$$u = -u, a = g = 10 \text{ ms}^{-2}, s = 50 \text{ m}, t = 10 \text{ s}$$

As $$s = ut + \frac{1}{2}at^2$$

So, $$50 = -u \times 10 + \frac{1}{2} \times 10 \times 10^2$$

On solving, $u = 45 \text{ ms}^{-1}$

If t_1 and t_2 are the time taken by the ball to reach points A and B respectively, then

$$20 = 45\, t_1 + \frac{1}{2} \times 10 \times t_1^2$$

and $$40 = -45\, t_2 + \frac{1}{2} \times 10 \times t_2^2$$

On solving, we get, $t_1 = 9.4 \text{ s}$ and $t_2 = 9.8 \text{ s}$

Time taken to cover the distance AB

$$= (t_2 - t_1) = 9.8 - 9.4 = 0.4 \text{ s}$$

16. Let u be the initial upward velocity of the ball from top of the tower and h be the height of the tower.

Taking the downward motion of the first stone from A to the ground, we have

$$h = -ut_1 + \frac{1}{2}gt_1^2 \quad \text{...(i)}$$

Taking the downward motion of the second stone from top of the tower to the ground, we have

$$h = ut_2 + \frac{1}{2}gt_1^2 \quad \text{...(ii)}$$

Multiplying Eq. (i) t_2 and Eq. (ii) by t_1 and adding, we get

$$h\,(t_1 + t_2) = \frac{1}{2}gt_1\,t_2\,(t_1 + t_2)$$

So, $$h = \frac{1}{2}gt_1\,t_2 \quad \text{...(iii)}$$

For falls under gravity from the top of the tower

$$h = \frac{1}{2}gt_3^2 \quad \text{...(iv)}$$

From Eqs. (iii) and (iv),

$$t_3^2 = t_1 t_2$$

or $$t_3 = \sqrt{t_1\,t_2} = \sqrt{9 \times 4} = 6 \text{ s}$$

17. Clearly, $$v = \sqrt{2\,gh} \quad \text{...(i)}$$

After rebounce, $v^2 = u^2 - 2gh$

$\Rightarrow$ $$u^2 = v^2 + 2gh$$

and $$u^2 = 2gh' \quad \text{...(ii)}$$

$\therefore$ $$\frac{v^2}{u^2} = \frac{2gh}{2gh'}$$

$\Rightarrow$ $$h' = h \times \frac{u^2}{v^2} = h \times \left(\frac{80}{100}\right)^2$$

$$= 0.64 \text{ h}$$

18. First 50 m fall is under the effect of gravity only. The velocity acquired, $u = \sqrt{2\,gh} = \sqrt{2 \times 9.8 \times 50} \text{ ms}^{-1}$.

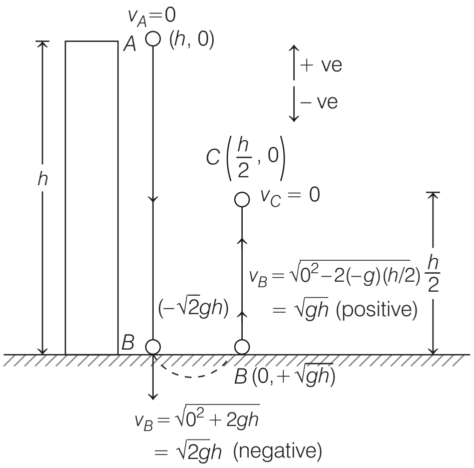
Taking onward motion of parachutist with retardation 2 ms^{-2}, we have, $u = 10\sqrt{9.8} \text{ ms}^{-1}$.

$$d = -2 \text{ ms}^{-2}, v = 3 \text{ ms}^{-1}$$

$$s = \frac{v^2 - u^2}{2\,a} = \frac{(3)^2 - (2 \times 9.8 \times 50)}{2 \times (-2)} = 243 \text{ m}$$

$\therefore$ Total height $= 50 + 243 = 293 \text{ m}$

19.



Using third equation of motion, the relation between v and h is given by,

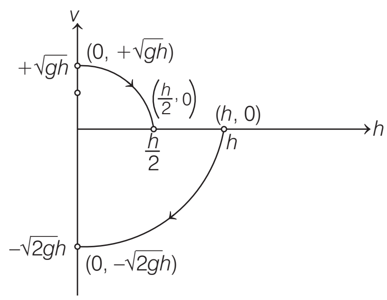
$$v^2 = u^2 + 2gh = 0^2 + 2gh = 2gh$$

$\Rightarrow$ $$v^2 \propto h$$

It is the equation of a parabola. So, the shape of v *versus* h graph will be parabolic.

Now, as we have some co-ordinate points from diagram.

On plotting them and tracing with a parabolic locus, we will get the following graph



Hence, correct option is (c).

20. If v_w be the velocity of water and v_b be the velocity of motorboat in still water.

The distance covered by motorboat in moving downstream in 6 h is given by

$$x = (v_b + v_w) \times 6 \quad \text{...(i)}$$

Same distance covered by motorboat in moving upstream in 10 h is

$$x = (v_b - v_w) \times 10 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$(v_b + v_w) \times 6 = (v_b - v_w) \times 10$$

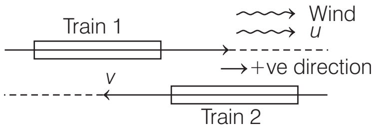
$$\Rightarrow \quad v_w = \frac{v_b}{4}$$

$$\therefore \quad x = (v_b + v_w) \times 6 = 7.5\, v_b$$

Time taken by the motorboat to cover the same distance in still water is

$$t = \frac{x}{v_b} = \frac{7.5\, v_b}{v_b} = 7.5 \text{ h}$$

21. Let the speed of trains be v.



$$v_{t_1 w} = v_{t_1} - v_w = v - u$$

$$v_{t_2 w} = v_{t_2} - v_w = (-v) - u = -v - u$$

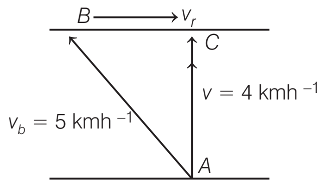
Given, $|\, v - u \,| : |\, -v - u \,| = 1 : 2$

$$\therefore \quad \frac{v - u}{v + u} = \frac{1}{2}$$

or $\quad 2v - 2u = v + u$

or $\quad v = 3u$

22. Let v_r be the velocity of river.



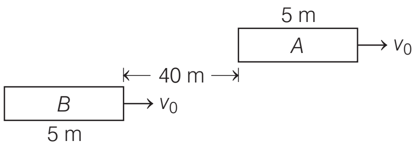
Velocity of boat when crosses river along the shortest path

$$v = \frac{1 \text{ km}}{15 \text{ min}} = \frac{1000}{15 \times 60} \times \frac{18}{5} \text{ kmh}^{-1} = 4 \text{ kmh}^{-1}$$

Therefore, speed of river $v_r = \sqrt{5^2 - 4^2} = 3 \text{ kmh}^{-1}$

During motion, boat directs itself parallel to AB, moves along AC due to the resultant velocity.

23. The initial velocity of the both buses $= v_0$



$$\therefore \quad u_{\text{rel}} = \mathbf{u}_{BA} = v_0 - v_0 = 0$$

$$\because \quad s_{\text{rel}} = u_{\text{rel}} t + \frac{1}{2} a_{\text{rel}} t^2$$

$$5 + 40 + 5 = 0 + \frac{1}{2} (1 - 0)\, t^2$$

$$\therefore \quad t^2 = 100$$

$$\Rightarrow \quad t = 10 \text{ s}$$

24. For no collision, the speed of car A should be reduced to v_B before the cars meet, *i.e.*, final relative velocity of car A with respect to car B is zero, *i.e.* $v_r = 0$

Here, initial relative velocity, $u_r = v_A - v_B$

Relative acceleration, $a_r = -a - 0 = -a$

Let relative displacement $= s_r$

The equation,

$$v_r^2 = u_r^2 + 2\, a_r s_r$$

$$(0)^2 = (v_A - v_B)^2 - 2\, a s_r$$

$$s_r = \frac{(v_A - v_B)^2}{2\, a}$$

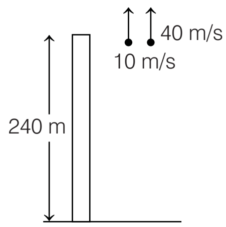
For no collision, $s_r \le s$

i.e. $\quad \dfrac{(v_A - v_B)^2}{2\, a} \le s$

25. Consider the stones thrown up simultaneously as shown in the diagram below.

Considering motion of the second particle with respect to the first we have relative acceleration

$|\, \mathbf{a}_{21} | = |\, \mathbf{a}_2 - \mathbf{a}_1 | = g - g = 0$



Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time given by

$$-240 = 10\, t - \frac{1}{2} \times 10 \times t^2$$

or $\quad t^2 - 2t - 48 = 0$

or $\quad t^2 - 8t + 6t - 48 = 0$

or $\quad t = 8, -6$ (not possible)

Thus, distance covered by second particle with respect to first particle in 8 s is

$$s_{12} = (v_{21})\, t$$

$$= (40 - 10)\, (8 \text{ s})$$

$$= 30 \times 8 = 240 \text{ m}$$

Similarly, time taken by second particle to strike the ground is given by

$$-240 = 40t - \frac{1}{2} \times 10 \times t^2$$

or $\quad -240 = 40t - 5\, t^2$

or $\quad 5\, t^2 - 40t - 240 = 0$

or $\quad t^2 - 8t - 48 = 0$

$\quad t^2 - 12t + 4t - 48 = 0$

or $\quad t\,(t - 12) + 4\,(t - 12) = 0$

or $\quad t = 12, -4$ (not possible)

Thus, after 8 s, magnitude of relative velocity will increase upto 12 s when second particle strikes the ground.

So, option (b) is correct.

26. Considering the initial position of ship A as origin, so the velocity and position of ship will be

$$\mathbf{v}_A = (30\,\hat{\mathbf{i}} + 50\,\hat{\mathbf{j}})$$

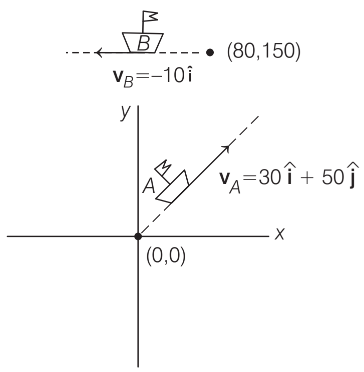
and $$\mathbf{r}_A = (0\,\hat{\mathbf{i}} + 0\,\hat{\mathbf{j}})$$

Now, as given in the question, velocity and position of ship B will be,

$$\mathbf{v}_B = -10\,\hat{\mathbf{i}}$$

and $$\mathbf{r}_B = (80\,\hat{\mathbf{i}} + 150\,\hat{\mathbf{j}})$$

Hence, the given situation can be represented graphically as



After time t, coordinates of ships A and B are $(80-10t, 150)$ and $(30t, 50t)$.

So, distance between A and B after time t is

$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$d = \sqrt{(80-10t-30t)^2 + (150-50t)^2}$$

$$\Rightarrow \quad d^2 = (80-40t)^2 + (150-50t)^2$$

Distance is minimum when $\frac{d}{dt}(d^2) = 0$

After differentiating, we get

$$\Rightarrow \quad \frac{d}{dt}\,[(80-40t)^2 + (150-50t)^2] = 0$$

$$\Rightarrow \quad 2(80-40t)(-40) + 2(150-50t)\,(-50) = 0$$

$$\Rightarrow \quad -3200 + 1600t - 7500 + 2500t = 0$$

$$\Rightarrow \quad 4100t = 10700$$

$$\Rightarrow \quad t = \frac{10700}{4100} = 2.6\ \text{h}$$

27. Distance travelled = Area under speed-time graph

Distance travelled (from $t = 0$ s to $t = 5$ s)

$$= \text{Area of } \Delta\, OAB$$

$$= \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times 5 \times 8 = 20\ \text{m}$$

28. Given, displacement (x) and time (t) relation,

$$x^2 = at^2 + 2bt + c \qquad \text{...(i)}$$

On differentiating Eq. (i) w.r.t.t, we get

$$\Rightarrow \quad 2x\frac{dx}{dt} = 2at + 2b$$

or $$xv = at + b \qquad \text{...(ii)}$$

Differentiating again w.r.t. t, we have

$$x \cdot \frac{dv}{dt} + v \cdot \frac{dx}{dt} = a \qquad \left(\because \frac{dx}{dt} = v \text{ and } \frac{dv}{dt} = A\right)$$

$$\Rightarrow \quad xA + v^2 = a$$

$$\Rightarrow \quad xA = a - v^2$$

$$\Rightarrow \quad xA = a - \left(\frac{at+b}{x}\right)^2$$

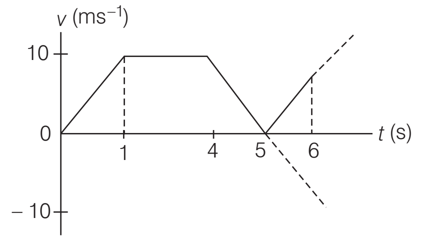
$$\left[\text{from Eq. (ii), } v = \frac{at+b}{x}\right]$$

$$\Rightarrow \quad A = \frac{ac - b^2}{x^3}$$

$$\Rightarrow \quad A \propto x^{-3}$$

Thus, $n = 3$

29. The corresponding velocity time graph is



Distance travelled = area of velocity time graph

$$= \frac{1}{2} \times 10\ (3 + 5) + \frac{1}{2} \times 10 \times 1$$

$$= 40 + 5 = 45\ \text{m}$$

$$\therefore \quad \langle v \rangle = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{45}{6} = \frac{15}{2}\ \text{ms}^{-1}$$

$$\therefore \quad n = 2$$

30. $\because \; v = t - 2$

or $$0 = t - 2$$

$$\therefore \quad t = 2\ \text{s}$$

$$\therefore \quad |v| = t - 2 \text{ for } t > 2\ \text{s}$$

and $$|v| = 2 - t \text{ for } 0 < t \le 2\ \text{s}$$

$$\therefore \quad s = \int_0^4 |v|\,dt = \int_0^2 (2-t)\,dt + \int_2^4 (t-2)\,dt$$

$$= \left[2t - \frac{t^2}{2}\right]_0^2 + \left[\frac{t^2}{2} - 2t\right]_2^4$$

$$= (4 - 2) + 2 = 4\ \text{m}$$

31. $\because \; a = t - x$

or $$\frac{da}{dt} = 1 - \frac{dx}{dt}$$

or $$\frac{da}{dt} = 1 - v$$

or $$\frac{d^2a}{dt^2} = -\frac{dv}{dt} = -a$$

The solution of this differential equation is

$$a = a_0 \sin(\omega t + \phi)$$

where, $\omega = 1$ unit

At $\quad t = 0, n = 0 \quad \Rightarrow \quad a = 0$

$\therefore \quad 0 = a_0 \sin(\omega \times 0 + \phi)$

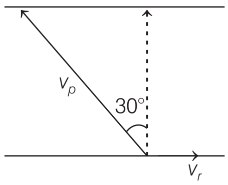
$\therefore \quad \phi = 0$

$\therefore \quad a = a_0 \sin t$

or $$\frac{dv}{dt} = a_0 \sin t \quad \text{or} \quad \int_0^v dv = a_0 \int_0^t \sin t \, dt$$

or $$v = a_0[-\cos t]_0^t$$
$$= -a_0[\cos t - 1]$$
$$v = a_0(1 - \cos t)$$
$$= 2\left(1 - \cos\frac{\pi}{3}\right)$$
$$= \left(2 \times \frac{1}{2}\right) = 1 \text{ ms}^{-1}$$

32. $v_p = 10$ m/s



$\Rightarrow \quad 10 \sin 30° = v_r$

$\Rightarrow \quad v_r = 5$ m/s

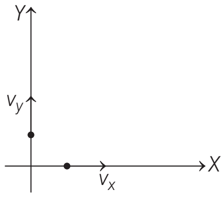
$\therefore \quad x = 5$

33. Let particle A moves along X-axis such that

$$x = 10 + 8t - 3t^2$$

and let particle B moves over Y-axis such that

$$y = 5 - 8t^3$$



Velocities of particles A and B are

$$v_A = v_x = \frac{dx}{dt} = \frac{d}{dt}(10 + 8t - 3t^2)$$
$$= 8 - 6t$$

or $\quad v_A$ at $t = 1$ s,

$$v_A = 8 - 6 \times 1 = 2 \text{ ms}^{-1}$$

$\Rightarrow \quad \mathbf{v}_A = +2\,\hat{\mathbf{i}}$ ms^{-1}

Velocity of particle B,

$$v_B = v_y = \frac{d}{dt}(5 - 8t^3) = -8 \times 3t^2$$

At $\quad t = 1$ s, $v_B = -8 \times 3 \times 1^2 = -24$ ms^{-1}

$\Rightarrow \quad \mathbf{v}_B = -24\,\hat{\mathbf{j}}$ ms^{-1}

Speed of particle B w.r.t. particle A,

$\mathbf{v}_{BA}$ = velocity of particle B w.r.t. A

$$= \mathbf{v}_B - \mathbf{v}_A$$
$$= -24\hat{\mathbf{j}} - (2\,\hat{\mathbf{i}})$$
$$= -2\hat{\mathbf{i}} - 24\hat{\mathbf{j}}$$

$\Rightarrow$ Magnitude of velocity,

$$|\mathrm{v}_{BA}| = \sqrt{(2)^2 + (24)^2}$$

But given $|\mathbf{v}_{BA}| = \sqrt{v}$

So, $$\sqrt{v} = \sqrt{24^2 + 2^2}$$

or $$v = 580 \text{ ms}^{-1}$$