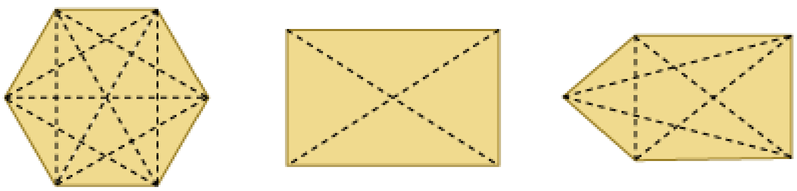
Understanding Quadrilaterals

Classification of Polygons as Convex and Concave

Let us consider some polygons such as a hexagon, a quadrilateral, and a pentagon as shown in the following figure.



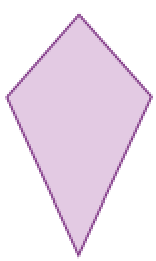
These polygons are known as convex polygons. How can we say these are convex polygons?

Let us now look at some examples.

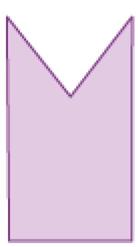
Example:

Classify the following figures as concave or convex polygons.

1.



2.



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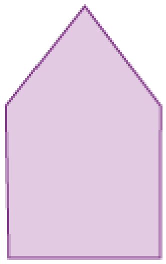

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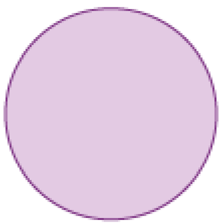
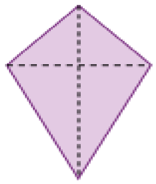


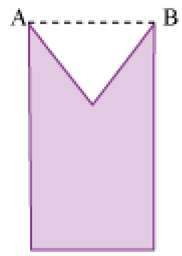
5.

Solution:

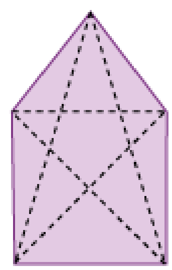
1. In this figure, we can clearly see that all the diagonals of the polygon lie inside the polygon. Therefore, it is a convex polygon.



2. In this polygon, diagonal AB lies in the exterior of the polygon. Therefore, it is a concave polygon.

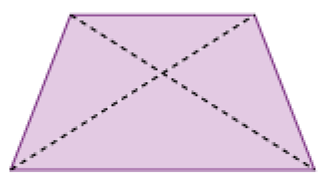


3. In this polygon, all diagonals lie in the interior of the polygon. Therefore, it is a convex polygon.



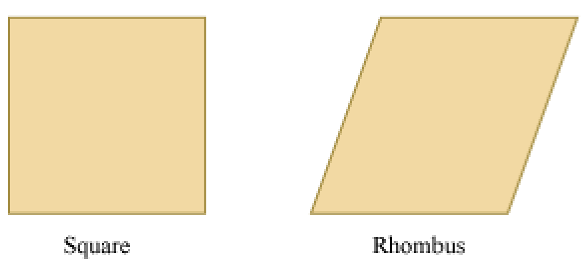
4. The given figure is a curve. It is not made up of line segments. Therefore, it is not a polygon.

5. In this polygon, all diagonals lie in the interior of the polygon. Therefore, it is a convex polygon.



Classification of Polygons as Regular and Irregular

Let us consider a square and a rhombus.



What is the difference between the two figures?

We can see that in a square, all the sides are equal and all the angles are also of equal measure. On the other hand, in a rhombus, all sides are equal; however, the measures of all angles are not equal.

We thus say that a **square is a regular polygon** and a **rhombus is an irregular polygon**.

The **regular** and **irregular polygons** can be defined as follows.

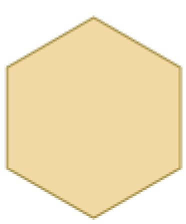
"Polygons in which all sides are of equal length and all interior angles of equal measure are known as regular polygons".

"Polygons in which all sides are not of equal length and all angles are not of equal measure are known as irregular polygons".

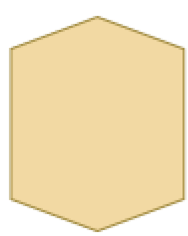
Let us see another example.

A **regular hexagon** has all sides of equal length. Moreover, all the angles are of equal measure 120°.

However, in case of an **irregular hexagon**, all the sides are not of equal length. Also, all the angles are not equal. A regular and an irregular hexagon are shown in the following figure.



Regular hexagon



Irregular hexagon

Formulas Related to Regular Polygons:

(i) The sum of the interior angles of an *n* sided polygon = $(2n-4)\times 90^\circ$
(ii) A regular polygon has all its exterior angles equal.

The sum of its exterior angles = 360°
So, the sum of each exterior angle = $360^\circ/n$

(iii) Number of sides of a regular polygon, $n = \frac{360^\circ}{\text{exterior angle}}$

Note: For a polygon, regardless of the fact whether it is regular or non-regular, at each vertex the sum of exterior and interior angle = 180°180°

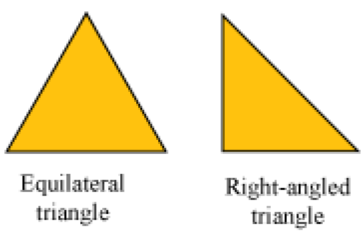
i.e Exterior angle + Interior angle = 180°180°

Let us now look at some more examples to understand this concept better.

Example 1:

Show that an equilateral triangle is a regular polygon and a right-angled triangle is an irregular polygon.

Solution:



An equilateral triangle is a regular polygon as all the sides of equilateral triangle are of equal length and all angles are of equal measure 60°.

In case of a right-angled triangle, neither all the sides are of equal length nor the measure of all angles are equal. Therefore, right-angled triangle is an example of irregular polygon.

Example 2:

Write the name of a regular polygon having

(i) 3 sides

(ii) 4 sides

Solution:

A regular polygon is a polygon in which all the sides are of equal length and all interior angles are of equal measure.

Therefore, a regular polygon having 3 sides is an equilateral triangle. A regular polygon having 4 sides is a square.

Angle Sum Property Of Polygons

Let us suppose that we have a quadrilateral and we want to find the sum of all the interior angles made by its sides.

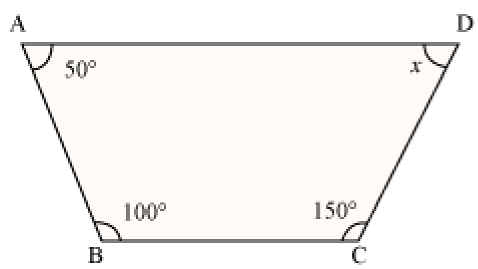
One simple way to find the sum of the angles is to find the measure of the angles and then add them. But how will we find its angles?

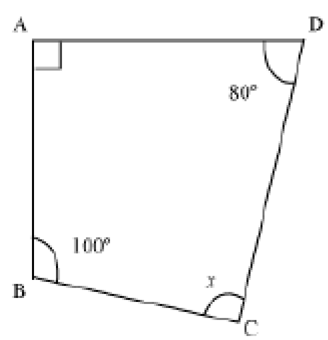
Is it possible to find the sum of all the angles of a quadrilateral without finding the measure of each angle? Is the sum of the interior angles of every quadrilateral same?

Let us solve some examples now.

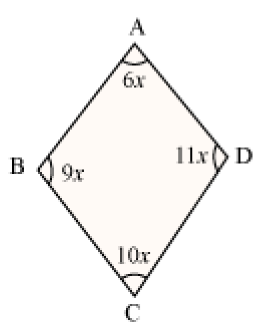
Example:

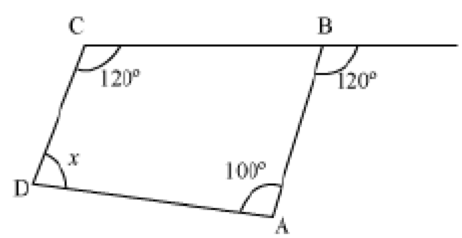
Find the value of *x* in the following figures.





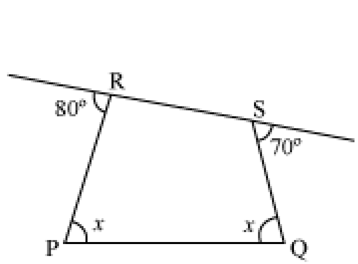
(a) **(b)**

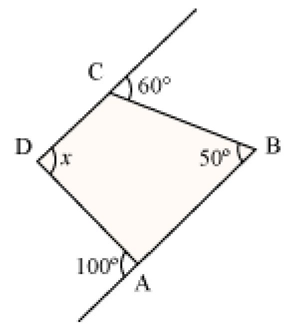




(c)

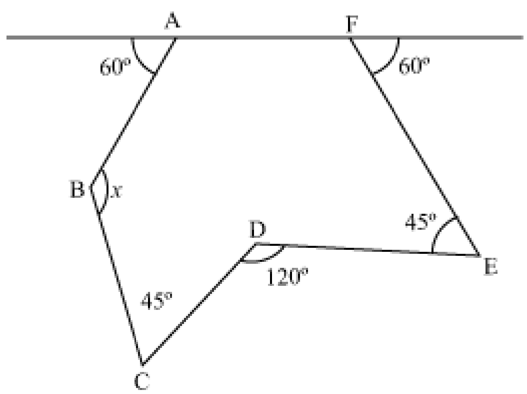
(d)





(e)

(f)



(g)

Solution:

(a) The sum of all the interior angles of a quadrilateral is 360°.

Therefore, from the figure,

100° + 150° + x + 50° = 360°

$\Rightarrow 300° + x = 360°$

$\Rightarrow x = 360° - 300°$

$\Rightarrow x = 60°$

(b) The sum of all the interior angles of a quadrilateral is 360°.

Therefore,

$90° + 80° + x + 100° = 360°$

$\Rightarrow 270° + x = 360°$

$\Rightarrow x = 360° - 270°$

$\Rightarrow x = 90°$

(c) The sum of all the interior angles of a quadrilateral is 360°. Therefore, from the figure,

$9x + 6x + 11x + 10x = 360°$

$\Rightarrow 36x = 360°$

On dividing both sides by 36, we obtain $x = 10°$

Thus, the angles of the quadrilateral are 90°, 60°, 110°, and 100°.

(d) The sum of the angles which forms a linear pair is 180°.

$\therefore \angle ABC + 120° = 180°$

$\Rightarrow \angle ABC = 180° - 120° = 60°$

Also, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 100° + 60° + 120° = 360°$

$\Rightarrow x + 280° = 360°$

$\Rightarrow x = 360° - 280°$

$\Rightarrow x = 80°$

(e) The sum of the adjacent angles on a straight line is 180°.

$\therefore \angle PSR + 80° = 180°$

$\Rightarrow \angle PSR = 180° - 80°$

$\Rightarrow \angle PSR = 100°$

Also, $\angle SRQ + 70° = 180°$

$\Rightarrow \angle SRQ = 180° - 70°$

$\Rightarrow \angle SRQ = 110°$

The sum of all the interior angles of a quadrilateral is 360°. Therefore,

$100° + 110° + x + x = 360°$

$\Rightarrow 210° + 2x = 360°$

$\Rightarrow 2x = 360° - 210°$

$\Rightarrow 2x = 150°$

$\Rightarrow x = 75°$

(f) The sum of the angles which forms a linear pair is 180°.

$\therefore 100° + \angle DAB = 180°$

$\Rightarrow \angle DAB = 180° - 100°$

$\Rightarrow \angle DAB = 80°$

Similarly, $\angle DCB + 60° = 180°$

$\Rightarrow \angle DCB = 180° - 60°$

$\Rightarrow \angle DCB = 120°$

Now, the sum of all the interior angles of a quadrilateral is 360°. Therefore,

$x + 80° + 50° + 120° = 360°$

$\Rightarrow x + 250° = 360°$

$\Rightarrow x = 360° - 250°$

$\Rightarrow x = 110°$

(g) ∠BAF = 180° − 60° = 120°

Similarly, ∠AFE = 120°

∠CDE = 360° − 120° = 240°

The polygon ABCDEF is a hexagon.

∴ Sum of all the interior angles of a hexagon = 180° × (6 − 2)

= 180° × 4

= 720°

$\therefore x = 720° - (120° + 120° + 45° + 240° + 45°)$

$\Rightarrow x = 720° - 570°$

$\Rightarrow x = 150°$

Exterior Angle Sum Property of Polygons

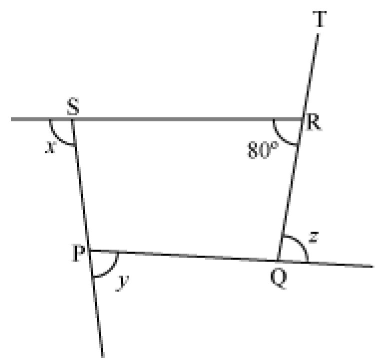
Let us consider a quadrilateral. We know that the sum of all the interior angles of a quadrilateral is 360°.

But if we want to find the sum of all the exterior angles of a quadrilateral, then how will we proceed?

Let us look at some examples now.

Example 1:

Find the value of ($x + y + z$) from the following figure.



Solution:

In the given figure, we can see that ∠QRS and ∠SRT form linear pair of angles.

Therefore, their sum should be 180°.

Thus, we obtain

80° + ∠SRT = 180°

⇒ ∠SRT = 180° – 80°

= 100°

The sum of the exterior angles of a quadrilateral is 360°.

∴ $x + y + z + 100° = 360°$

⇒ $x + y + z = 360° – 100°$

⇒ $x + y + z = 260°$

Example 2:

Find the number of sides of a regular polygon in which each exterior angle has a measure of 40°.

Solution:

The measure of all the exterior angles of a polygon is 360°. It is given that the measure of each exterior angle is 40° and the given polygon is a regular polygon, therefore all the exterior angles are same.

Therefore, number of exterior angles = $\frac{360^\circ}{40^\circ}$ = 9

Thus, the polygon has 9 sides.

Example 3:

Find the measure of each exterior angle of a regular polygon having 12 sides.

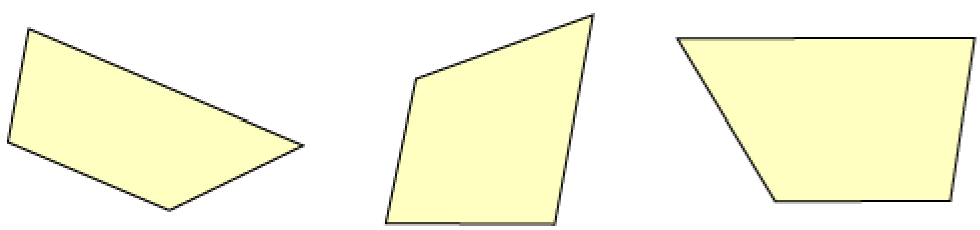
Solution:

The measure of all the exterior angles of a polygon is 360°. It is given that the polygon has 12 sides. Since it is a regular polygon, all its exterior angles are equal.

∴ Measure of each exterior angle = $\frac{360^\circ}{12}$ = 30°

Trapeziums, Kites And Parallelograms

Let us look at the following quadrilaterals.



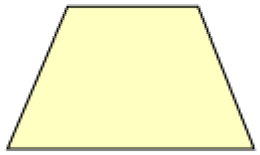
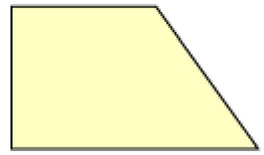
What similarity do we find between the three figures?

We can observe that in all these figures, a pair of opposite sides is parallel. These quadrilaterals are known as **trapeziums.**

> **"Quadrilaterals which have exactly one pair of parallel sides are known as trapeziums".**

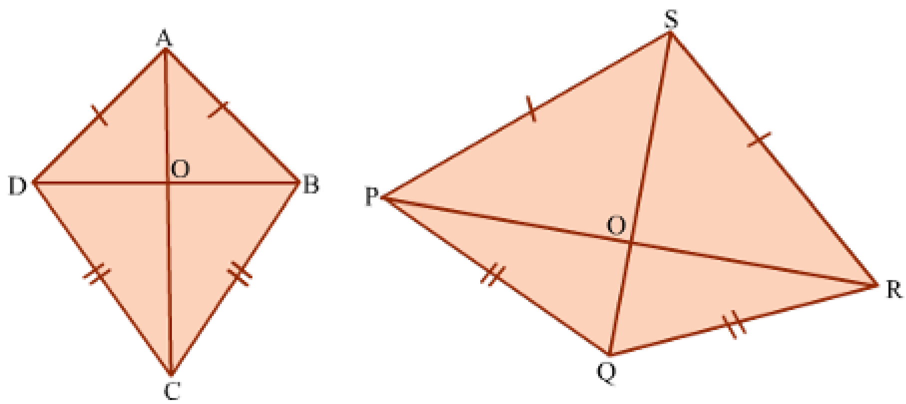
Note: If the non parallel sides of a trapezium are of equal lengths, then it is known as an ***isosceles trapezium.***

An isosceles trapezium and a non isosceles trapezium are shown in the following figure.

Isosceles Trapezium Non isosceles Trapezium

Now, let us look at the following quadrilaterals.



What is common in the two figures?

We can easily see that two pairs of adjacent sides are equal in both the figures.

Such quadrilaterals in which two distinct pairs of adjacent sides are equal in length are known as kites.

Therefore, we can say that

"**Kites are the quadrilaterals having exactly two distinct pairs of sides of equal length".**

In a kite, diagonal joining the points of intersection of equal sides bisect the other diagonal at right angle.

Therefore, in kite ABCD, we have $l(DO) = l(BO)$.

Also, $m\angle AOB = m\angle BOC = m\angle COD = m\angle DOA = 90°$.

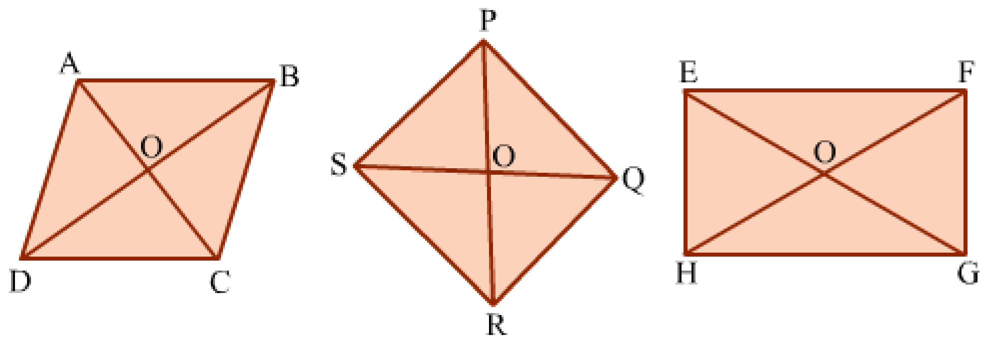
Similarly, in kite PQRS, we have $l(PO) = l(RO)$.

And, $m\angle POQ = m\angle QOR = m\angle ROS = m\angle SOP = 90°$.

Also, in a kite, one of the diagonals bisects the apex angles.

Therefore, in kite ABCD, we have diagonal AC bisects the apex angles ∠A and ∠C.

Now, let us have a look at the following quadrilaterals.



What do we find common in all the above given quadrilaterals?

Observe that in all these quadrilaterals, the opposite sides are parallel. Such quadrilaterals are known as **parallelograms.**

"A quadrilateral in which opposite sides are parallel is known as a parallelogram".

Diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, we have *l*(AO) = *l*(CO) and *l*(DO) = *l*(BO).

Similarly, in parallelogram PQRS, we have *l*(PO) = *l*(RO) and *l*(QO) = *l*(SO).

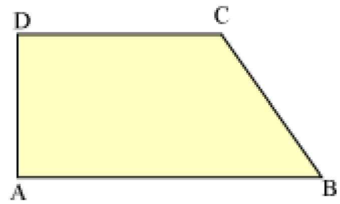
And, in parallelogram EFGH, we have *l*(EO) = *l*(GO) and *l*(HO) = *l*(FO).

Let us go through some examples now.

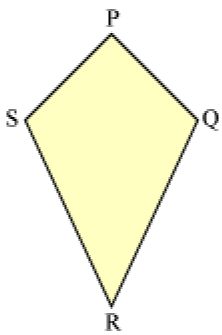
Example 1:

Classify the following figures as kite, parallelogram, or trapezium.

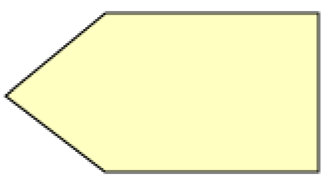
A.



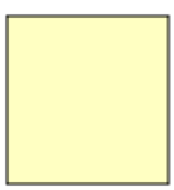
B.



C.



D.



Solution:

A. The given quadrilateral ABCD is a trapezium, since it has only one pair of parallel sides i.e., AB is parallel to CD.

B. The given quadrilateral PQRS is a kite, since it has two distinct pairs of sides of equal length which are

(1) PS and PQ, and

(2) SR and RQ

C. The given figure is not a quadrilateral as it has five sides.

D. The given figure is a parallelogram, since its two pairs of opposite sides are parallel.

Example 2:

State whether the following statements are correct or incorrect. If incorrect, then give reasons.

A. In a kite, opposite sides are equal in length.

B. A trapezium has a pair of parallel lines.

C. A kite is a parallelogram.

Solution:

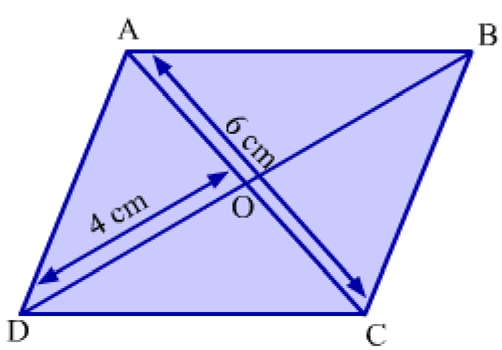
A. The given statement is incorrect since in a kite, opposite sides are not of equal lengths.

B. The given statement is correct.

C. The given statement is incorrect, since in a parallelogram the opposite sides are equal which is not true in a kite.

Example 3:

Observe the figure of given parallelogram.



Find the following components of this parallelogram.

A. ***l*****(BD)**

B. *l*(AO)

Solution:

A. Diagonals of a parallelogram bisect each other.

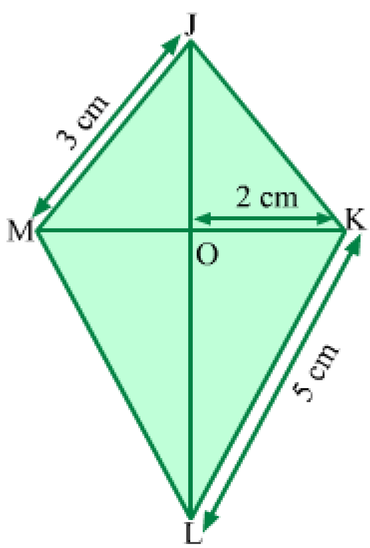
∴ $l(BD) = 2 \times l(DO) = (2 \times 4)$ cm = 8 cm

B. Diagonals of a parallelogram bisect each other.

∴ $l(AO) = \frac{1}{2} \times l(AC) = \left(\frac{1}{2} \times 6\right) \text{cm} = 3\text{cm}$

Example 4:

Observe the figure of given kite.



Find the following components of this kite.

A. *l*(JK)

B. *l*(LM)

C. *m*∠LOM

D. *l*(MO)

Solution:

In kite JKLM, (KL and LM) is a pair of longer sides, while (JK and MJ) is a pair of shorter sides.

We know that, kite is a quadrilateral having exactly two distinct pairs of sides of equal length.

Therefore, l(KL) = l(LM) and l(JK) = l(MJ)

Also, in kite JKLM, JL bisects MK at right angle.

Now, the required components of kite JKLM will be follows:

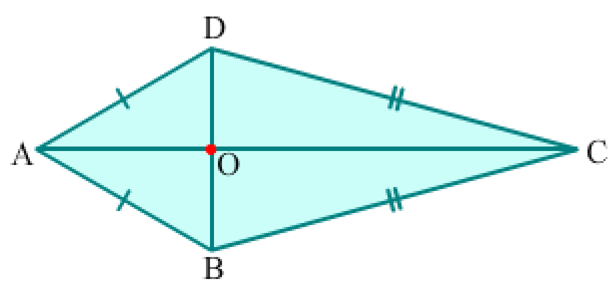
A. l(JK) = l(MJ) = 3 cm

B. l(LM) = l(KL) = 5 cm

C. $m\angle$LOM = 90°

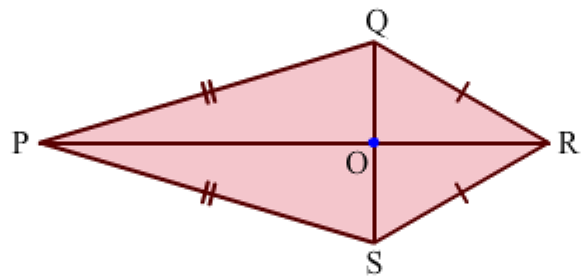
D. l(MO) = l(KO) = 2 cm

Example 5:
In the figure given below, ABCD is a kite with AB = 5 cm and BC = 10 cm. Find the perimeter of the kite ABCD.



Solution:
Since ABCD is a kite, we have
AD = AB = 5 cm
DC = BC = 10 cm
Therefore, perimeter of kite ABCD = AB + BC + CD + DA
= (5 + 10 + 10 + 5) cm
= 30 cm

Example 6:
In the figure given below, PQRS is a kite with PQ: QR = 2: 1 and the perimeter of kite PQRS is 36 cm. Find all the sides of the kite PQRS.



Solution:
Sides PQ and QR of the kite PQRS are in the ratio 2: 1. Let PQ be 2*x* and QR be *x*.

Also, we have in the kite PQRS,
PQ = PS = 2*x*
RS = QR = *x*

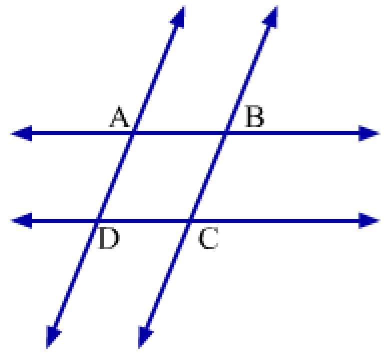
Perimeter of the kite PQRS = PQ + QR + RS + SP
= (2*x* + *x* + 2*x* + *x*) cm
= 6*x* cm
It is given that perimeter of the kite 36 cm.

$$6x = 36$$
$$\Rightarrow x = \frac{36}{6}$$
$$\Rightarrow x = 6\,\text{cm}$$

Thus, we get
$$PQ = 2x = 2 \times 6 \text{ cm} = 12 \text{ cm}$$
$$QR = x = 6 \text{ cm}$$
$$RS = QR = 6 \text{ cm}$$
$$SP = PQ = 12 \text{ cm}$$

Properties of The Sides of a Parallelogram

Consider the given pairs of parallel lines.



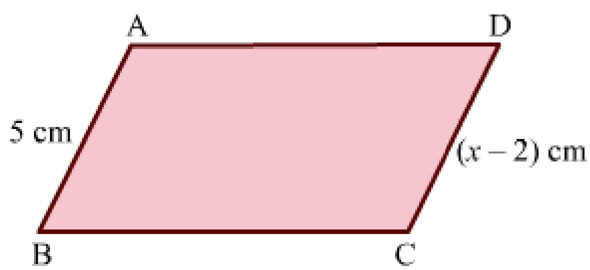
A closed figure ABCD is formed by the intersection of the two pairs of parallel lines. This figure is a parallelogram. A property of parallelograms defines the relation between the sides of a parallelogram as follows:

Opposite sides of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples related to the same.

Opposite Sides of a Parallelogram Are Equal

Consider the given parallelogram ABCD.



We can find the value of x by using the property of parallelograms which states that:

Opposite sides of a parallelogram are equal.

Thus, in the given figure, we have AB = DC and AD = BC.

Since AB = DC, we have:

$x - 2 = 5$

$\Rightarrow x = 7$

Concept Builder

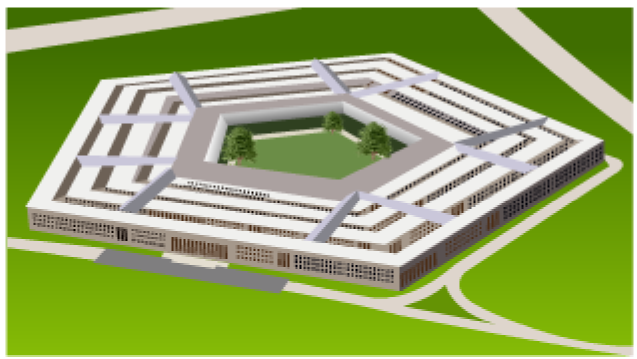
- A quadrilateral is a polygon having four sides.
- The sum of the interior angles of a quadrilateral is 360°.

Know More

- A pentadecagon is a fifteen-sided polygon. The sum of its interior angles is 2340°.
- An icosagon is a twenty-sided polygon. The sum of its interior angles is 3240°.

Did You Know?

The headquarters of the US Department of Defense is called 'the Pentagon'. It is one of the world's largest office buildings. It is virtually a city in itself.



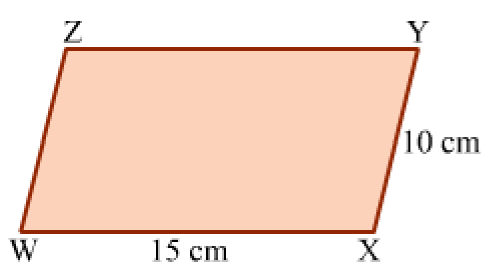
Solved Examples

Easy

Example 1:

What is the perimeter of the given parallelogram WXYZ if WX = 15 cm and XY = 10 cm?

Solution:



We know that the opposite sides of a parallelogram are equal.

∴ WX = ZY = 15 cm and XY = WZ = 10 cm

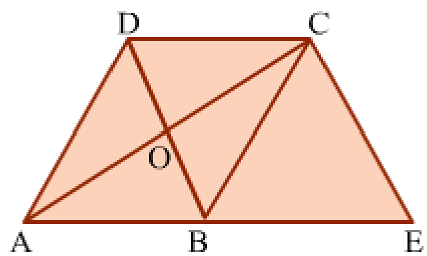
Perimeter of parallelogram WXYZ = WX + XY + YZ + ZW
= (15 + 10 + 15 + 10) cm

= 50 cm

Medium

Example 1:

In the given figure, ABCD is a parallelogram and B is the midpoint of AE. If DB = CE, then prove that BECD is also a parallelogram.



Solution:

We know that the opposite sides of a parallelogram are equal.

∴ AB = DC … (1)

It is given that B is the midpoint of AE.

∴ AB = BE … (2)

From equations 1 and 2, we get:

DC = BE

Also, it is given that DB = CE.

Now, in quadrilateral BECD, the opposite sides are equal (i.e., DC = BE and DB = CE). Therefore, it is a parallelogram.

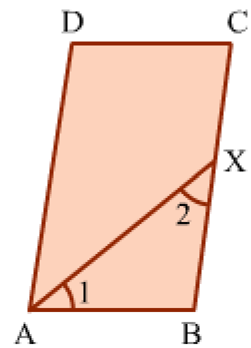
Hard

Example 1:

In a parallelogram ABCD, the bisector of ∠BAD also bisects side BC. Prove that the length of side AD is twice the length of side AB.

Solution:

The parallelogram ABCD according to the given specifications is shown below.



Here, AX is the bisector of ∠BAD.

$$\therefore \angle 1 = \frac{1}{2}\angle BAD \qquad ...(1)$$

Since ABCD is a parallelogram, AD||BC and AB is the transversal between these lines.

∴ ∠BAD + ∠CBA = 180° ... (2)

In ΔABX, by the angle sum property of triangles, we have:

∠1 + ∠2 + ∠ABX = 180°

$$\Rightarrow \frac{1}{2}\angle BAD + \angle 2 + 180° - \angle BAD = 180° \qquad \text{(Using equations 1 and 2)}$$

$$\Rightarrow \angle 2 + \frac{1}{2}\angle BAD = 0$$

$$\Rightarrow \angle 2 = \frac{1}{2}\angle BAD$$

$$\Rightarrow \angle 2 = \angle 1$$

We know that the sides opposite equal angles are also equal.

∴ AB = BX ... (3)

Since ABCD is a parallelogram, AD = BC.

Now, BC = BX + XC

⇒ AD = BX + XC

⇒ AD = 2BX(∵ AX bisects BC)

⇒ ∴ AD = 2AB (Using equation 3)

Thus, in parallelogram ABCD, the length of side AD is twice the length of side AB.

Properties of The Angles of a Parallelogram

Opposite Angles of a Parallelogram

Look at the postage stamp shown below.

Observe how the stamp is shaped like a parallelogram. What can you say about its opposite angles? Is there any relation between them? Are they equal?

A property of parallelograms relates the opposite angles of a parallelogram as follows:

Opposite angles of a parallelogram are equal.

In this lesson, we will study the above-stated property and its converse. We will also solve some examples based on the same.

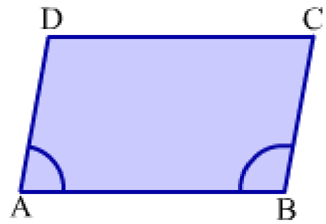
Whiz Kid

The sum of the measures of all the exterior angles of a quadrilateral (i.e., one at each vertex) is equal to the sum of the measures of all the interior angles of the quadrilateral, i.e., 360°.

Concept Builder

Adjacent angles in a parallelogram are supplementary.

In parallelogram ABCD, AD||BC and AB is the transversal intersecting these lines.



Therefore, ∠A and ∠B are **interior angles**

on the same side of the transversal and, hence, **supplementary**
.

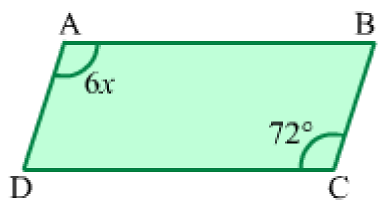
Similarly, we can say that ∠B and ∠C, ∠C and ∠D, and ∠D and ∠A are supplementary angles.

Solved Examples

Easy

Example 1:

Find the value of *x* if ABCD is a parallelogram.



Solution:

We know that the opposite angles of a parallelogram are equal.

∴ ∠A = ∠C

⇒ $6x = 72°$

∴ $x = 12°$

Example 2:
Find the measure of all the angles of a parallelogram whose adjacent angles are in the ratio 1:2.

Solution:

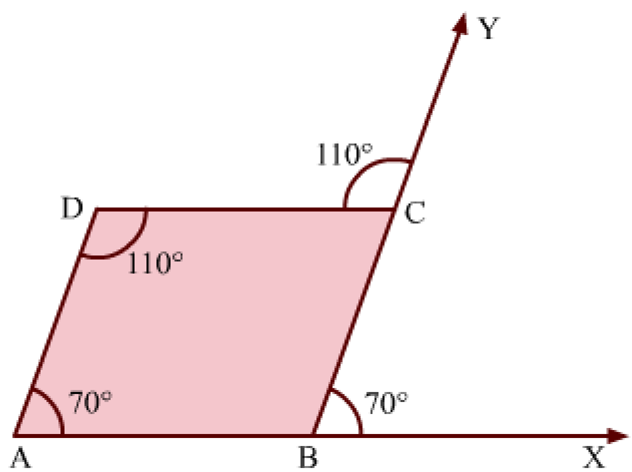
In a parallelogram ABCD, let $\angle A = x^\circ$ and $\angle B = 2x^\circ$.
In a parallelogram, the adjacent angles are supplementary.
$\angle A + \angle B = 180^\circ$
$\Rightarrow x^\circ + 2x^\circ = 180^\circ$
$\Rightarrow 3x^\circ = 180$
$\Rightarrow x^\circ = \frac{180^\circ}{3}$
$\Rightarrow x^\circ = 60^\circ$
Thus, we get
$\angle A = x^\circ = 60^\circ$
$\angle B = 2x^\circ = 2 \times 60^\circ = 120^\circ$

In a parallelogram, the opposite angles are equal.
Thus, we get
$\angle A = \angle C = 60^\circ$ and $\angle B = \angle D = 120^\circ$

Medium

Example 1:

Is the shown quadrilateral ABCD a parallelogram?



Solution:

In the given figure, ∠CBX and ∠CBA form a linear pair.

∴ ∠CBX + ∠CBA = 180°

⇒ ∠CBA = 180° − ∠CBX

⇒ ∠CBA = 180° − 70°

⇒ ∠CBA = 110°

⇒ ∠CBA = ∠CDA

Similarly, ∠DCY and ∠BCD form a linear pair.

∴ ∠DCY + ∠BCD = 180°

⇒ ∠BCD = 180° − ∠DCY

⇒ ∠BCD = 180° − 110°

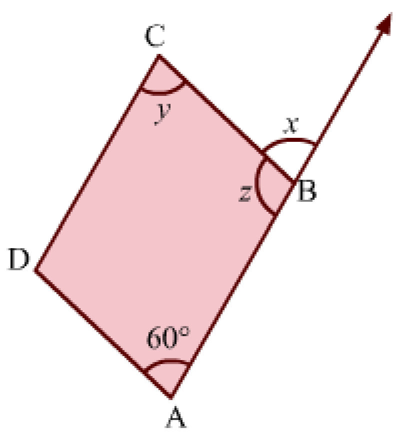
⇒ ∠BCD = 70°

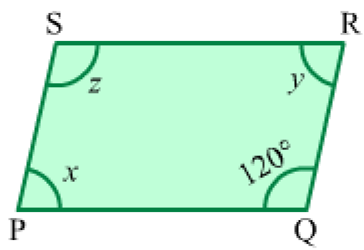
⇒ ∠BCD = ∠BAD

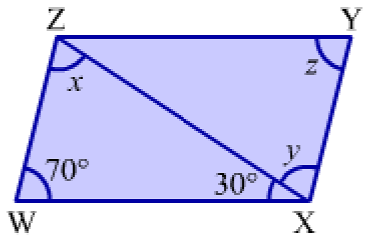
Thus, quadrilateral ABCD has two pairs of equal opposite angles. Hence, it is a parallelogram.

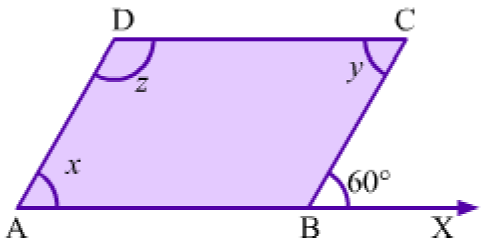
Example 2:

Find the values of *x*, *y* and *z* in the following parallelograms.

1. 

2. 

3. 

4. 

Solution:

1. We know that the opposite angles of a parallelogram are equal.

   So, ∠BCD = ∠DAB

   ∴ $y = 60°$

   We also know that the adjacent angles of a parallelogram are supplementary.

   So, ∠CBA + ∠DAB = 180°

   ⇒ $z + 60° = 180°$

   ⇒ $z = 180° - 60°$

   ⇒ ∴ $z = 120°$

   Now, x and z form a linear pair of angles; so, their sum is 180°.

   So, $x + z = 180°$

   ⇒ $x + 120° = 180°$

   ⇒ ∴ $x = 180° - 120° = 60°$

2. We know that the opposite angles of a parallelogram are equal.

   So, ∠PSR = ∠PQR

$\therefore z = 120^\circ$

$\angle QPS$ and $\angle PQR$ are adjacent angles.

So, $\angle QPS + \angle PQR = 180^\circ$

 $x + 120^\circ = 180^\circ$

 $x = 180^\circ - 120^\circ$

  $x = 60^\circ$

$\angle QRS$ and $\angle QPS$ are opposite angles.

So, $\angle QRS = \angle QPS$

 $y = x$

 $\therefore y = 60^\circ$

3. We know that the opposite angles of a parallelogram are equal.

   So, $\angle XYZ = \angle XWZ$

    $z = 70^\circ$

   $\angle XYZ$ and $\angle WXY$ are adjacent angles.

    $\angle XYZ + \angle WXY = 180^\circ$

    $z + 30^\circ + y = 180^\circ$

    $70^\circ + 30^\circ + y = 180^\circ$

    $100^\circ + y = 180^\circ$

    $y = 180^\circ - 100^\circ$

    $\therefore y = 80^\circ$

   Now, XY||WZ; so, $\angle WZX$ and $\angle YXZ$ are alternate interior angles.

   So, $\angle WZX = \angle YXZ$

⇒ $x = y$

⇒∴ $x = 80°$

4. It is given that ∠CBX = 60°.

   ∠CBA and ∠CBX form a linear pair.

   So, ∠CBA + ∠CBX = 180°

   ⇒ ∠CBA + 60° = 180°

   ⇒ ∠CBA = 180° − 60°

   ⇒ ∴ ∠CBA = 120°

   ∠CDA and ∠CBA are opposite angles.

   So, ∠CDA = ∠CBA

   ⇒ z = ∠CBA

   ⇒ ⇒ $z = 120°$

   ∠BCD and ∠CBA are adjacent angles.

   So, ∠BCD + ∠CBA = 180°

   ⇒ $y + 120° = 180°$

   ⇒ $y = 180° − 120°$

   ⇒ ∴ $y = 60°$

   ∠BAD and ∠BCD are opposite angles.

   So, ∠BAD = ∠BCD

   ⇒ $x = y$

   ⇒ ⇒ $x = 60°$

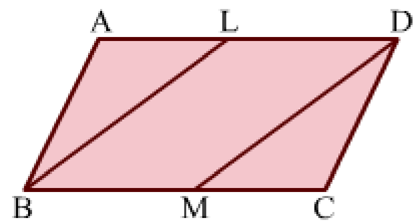
   Hard

Example 1:

Show that the bisectors of opposite angles of a parallelogram are parallel to each other.

Solution:

Let ABCD be a parallelogram. Let BL and DM be the bisectors of ∠ABC and ∠ADC respectively.



Since BL and DM are the bisectors of ∠ABC and ∠ADC respectively, we have:

$\angle LBM = \frac{\angle ABC}{2}$ ...(1)

$\angle LDM = \frac{\angle ADC}{2}$ ...(2)

We know that the opposite angles of a parallelogram are equal.

∴ ∠ABC = ∠ADC

On dividing both sides of the above equation by 2, we obtain:

$\frac{\angle ABC}{2} = \frac{\angle ADC}{2}$

Using equations 1 and 2, we obtain:

∠LBM = ∠LDM

Now, LD and BM are parallel.

So, ∠DLB + ∠LBM = 180° (Interior angles on the same side of a transversal)

 ∠DLB = 180° − ∠LBM

Similarly, ∠DMB = 180° − ∠LDM

∴ ∠DLB = ∠DMB (∵ ∠LBM = ∠LDM)

In quadrilateral LDMB, the opposite angles ∠DLB and ∠DMB are equal. Hence, it is a parallelogram.

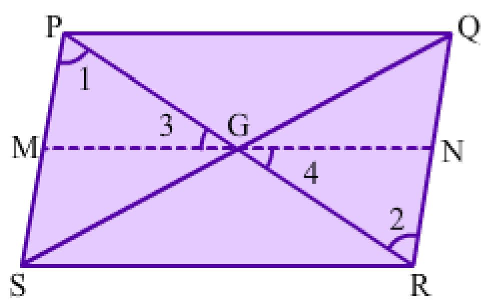
⇒ BL||DM

We know that BL and DM are the bisectors of opposite angles of parallelogram ABCD. Thus, the bisectors of opposite angles of a parallelogram are parallel.

Properties of The Diagonals of a Parallelogram

Relation between the Diagonals of a Parallelogram

Consider the following parallelogram PQRS.



In the figure, GM = GN, but can we prove this?

In order to prove GM = GN, we need to show that ΔGMP is congruent to ΔGNR.

In ΔGMP and ΔGNR, we have two sets of equal angles as follows:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

Now, to apply the ASA congruence rule, we need to show that GP and GR are equal.

A property of parallelograms helps us establish this equality and it can be stated as follows:

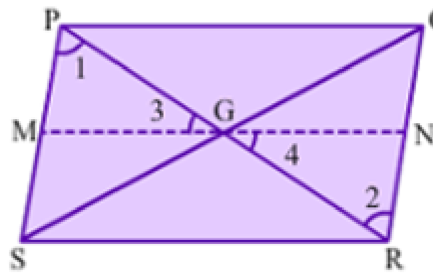
The diagonals of a parallelogram bisect each other.

In this lesson, we will study the above-stated property and solve some problems based on it.

Using the Property

Let us use the property of the diagonals of a parallelogram to solve the problem discussed at the beginning.

Let us once again consider parallelogram PQRS.



We have to prove that GM = GN.

Since diagonals PR and QS bisect each other, we obtain:

GP = GR and GS = GQ… (1)

In ΔGMP and ΔGNR, we have:

∠3 = ∠4(Vertically opposite angles)

∠1 = ∠2(Alternate interior angles; since PS||QR and PR is the transversal)

GP = GR(Using 1)

Thus, by the ASA congruence rule, we obtain:

ΔGMP ≅ ΔGNR

⇒ GM = GN(By CPCT)

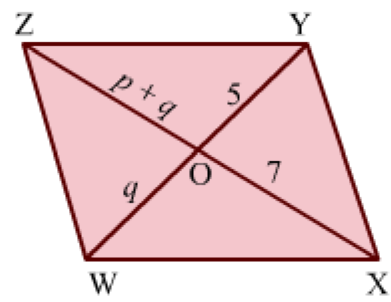
Similarly, we can use the property of the diagonals of a parallelogram to solve other problems.

Solved Examples

Easy

Example 1:

If the shown quadrilateral WXYZ is a parallelogram, then find the values of *p* and *q*.



Solution:

We know that the diagonals of a parallelogram bisect each other.

∴ WO = OY

⇒ $q = 5$

Similarly, XO = OZ

∴ $p + q = 7$

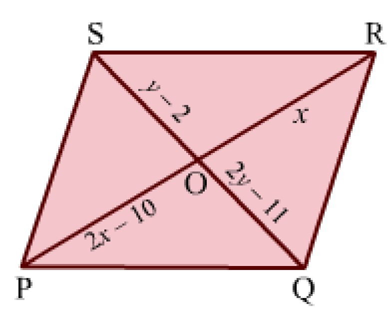
⇒ $p = 7 - q$

⇒ $p = 7 - 5$

⇒ ∴ $p = 2$

Example 2:

In the given parallelogram PQRS, find the lengths of the diagonals PR and QS.



Solution:

We know that the diagonals of a parallelogram bisect each other.

$\therefore$ PO = OR

$\Rightarrow 2x - 10 = x$

$\Rightarrow 2x - x = 10$

$\Rightarrow \therefore x = 10$

Similarly, QO = OS

$\Rightarrow y - 2 = 2y - 11$

$\Rightarrow 2y - y = -2 + 11$

$\Rightarrow \therefore y = 9$

Now, PR = PO + OR

$= 2x - 10 + x$

$= 3x - 10$

$= 3 \times 10 - 10$

$= 30 - 10$

$= 20$

Similarly, QS = QO + OS

$= y - 2 + 2y - 11$

$= 3y - 13$

$= 3 \times 9 - 13$

$= 27 - 13$

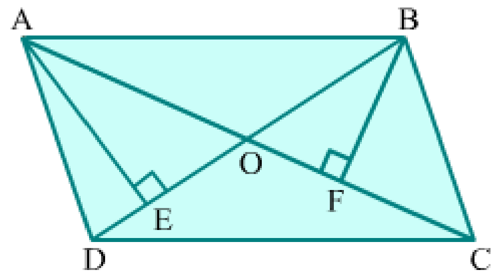
$= 14$

Thus, the lengths of the diagonals PR and QS are 20 units and 14 units respectively.

Medium

Example 1:

ABCD is a parallelogram with diagonals AC and BD of lengths 10 cm and 8 cm respectively. If the perpendiculars on DO and OC are 5 cm each, then find the sum of the areas of ΔAOD and ΔBOC.



Solution:

We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, we have:

AO = OC = $\frac{AC}{2}$ and BO = OD = $\frac{BD}{2}$

⇒ AO = OC = $\frac{10}{2}$ cm = 5 cm and BO = OD = $\frac{8}{2}$ cm = 4 cm

Now, area of ΔAOD= $\frac{1}{2}$ × Base × Height

= $\frac{1}{2}$ × OD × AE

= $\frac{1}{2}$ × 4 × 5 cm^2

= 10 cm^2

Similarly, area of ΔBOC= $\frac{1}{2}$ × OC × BF

= $\frac{1}{2}$ × 5 × 5 cm^2

$= 12.5\ cm^2$

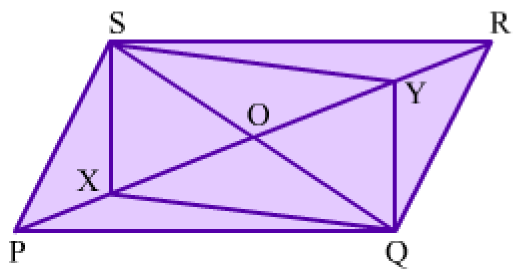
Therefore, sum of the areas of ΔAOD and ΔBOC = $(10 + 12.5)\ cm^2 = 22.5\ cm^2$

Hard

Example 1:

In parallelogram PQRS, X and Y are points on PR such that PX = YR. Prove that:

1. **XQYS is a parallelogram**
2. **ΔSXP ≅ ΔQYR**



Solution:

1. We know that the diagonals of a parallelogram bisect each other.

   ∴ OS = OQ … (1)

   And OP = OR… (2)

   Also, PX = YR… (3) [Given]

   On subtracting equation 3 from equation 2, we obtain:

   OP − PX = OR − YR

   ⇒ OX = OY … (4)

   In quadrilateral XQYS, XY and QS are the diagonals.

   We know from equations 1 and 4 that the diagonals bisect each other.

   Thus, XQYS is a parallelogram.

2. In ΔSXP and ΔQYR, we have:

PS = QR (Opposite sides of parallelogram PQRS)

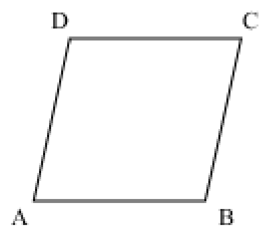
SX = QY (Opposite sides of parallelogram XQYS)

PX = YR (Given)

∴ ΔSXP ≅ ΔQYR (By the SSS congruence rule)

Properties Of Rhombuses

Let us consider a parallelogram ABCD.



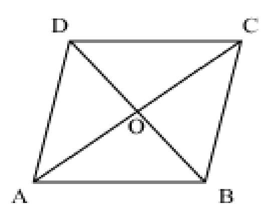
The above parallelogram has been drawn such that all its sides are of equal length. Therefore, can we give any special name to the parallelogram ABCD?

Yes, ABCD is called a **rhombus.**

"***A rhombus is a parallelogram in which all sides are of equal length".***

Being a parallelogram, ***the diagonals of a rhombus bisect each other.***

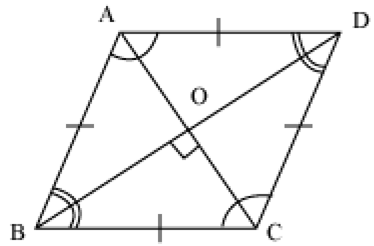
Therefore, OA = OC and OB = OD.



Can we say anything else about the diagonals of a rhombus?

So, we can conclude that:
"A quadrilateral is a rhombus, if its diagonals bisect each other at right angles."

We can thus summarize the properties of a rhombus as follows:

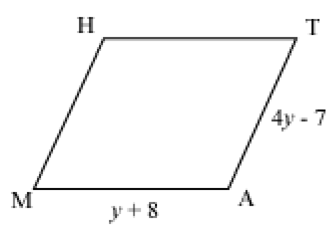


All sides are equal.
Opposite sides are parallel.
Opposite angles are equal.
Diagonals are perpendicular bisectors of each other.
The two diagonals divide the rhombus into four congruent right angled triangles.
Diagonals bisect the angles.

Let us now look at some examples.

Example 1:

In a rhombus MATH, MA = y + 8 and AT = 4y − 7. Find the length of MA. Also, find the perimeter of the rhombus.



Solution:

All the sides of a rhombus are equal. Therefore,

MA = AT = TH = HM

Now, MA = AT

$\Rightarrow y + 8 = 4y - 7$

$\Rightarrow y - 4y = -7 - 8$

$\Rightarrow -3y = -15$

Dividing both sides by 3, we obtain

$y = 5$ units

Now, MA = $y + 8$

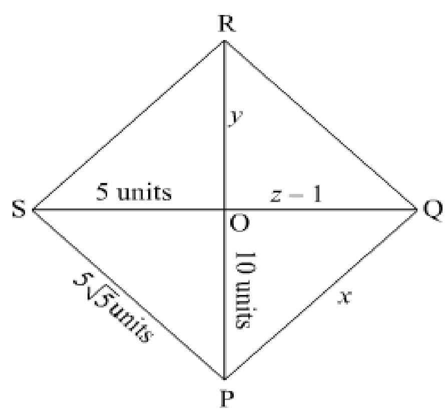
⇒ MA = 5 + 8 (Since $y = 5$ units)

⇒ MA = 13 units

∴ Perimeter of rhombus = MA + AT + TH + HM = 4MA
= 4 × 13
= 52 units

Example 2:

Find the values of *x*, *y*, and *z* from the given figure where PQRS is a rhombus.



Solution:

Since PQRS is a rhombus, all sides are equal.

∴ PQ = PS

⇒ $x = 5\sqrt{5}$ units

The diagonals of a rhombus bisect each other.

∴ OP = OR

⇒ $y = 10$ units

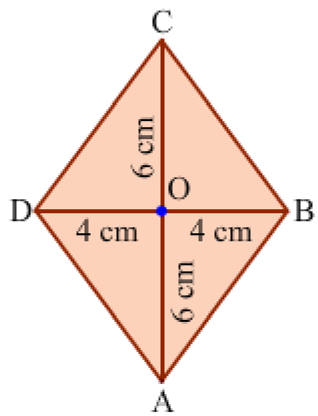
Also, OS = OQ

⇒ z − 1 = 5

⇒ z = 5 + 1

⇒ z = 6 units

Example 3:
The diagonals of a rhombus are 12 cm and 8 cm. Find the area of the rhombus.
Solution:
We have a rhombus *ABCD* with diagonals are 12 cm and 8 cm.
It is known that the diagonals of a rhombus bisect each other at right angles.
So we get OB = OD = 4 cm and OA = OC = 6 cm.



Now, area of right triangle BOC = $\frac{1}{2} \times OB \times OC = \frac{1}{2} \times 4 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$

Diagonals of a rhombus divides it into four congruent right triangles.
∴ Area of the rhombus = 4 × Area of ΔBOC = 4 × 12 cm^2 = 48 cm^2

Properties Of Rectangles

Let us suppose that we draw a parallelogram with 90° as the measure of each angle. Then we will obtain the following figure.

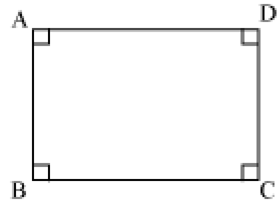


Figure ABCD is called a **rectangle.**

Thus, we can define a rectangle as follows:

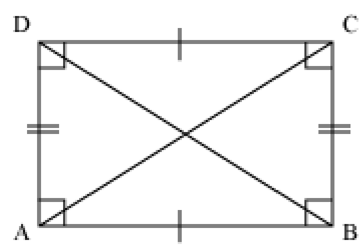
"***A rectangle is a parallelogram in which each interior angle is a right angle".***

Being a parallelogram, **a rectangle has opposite sides of equal length that are parallel to each other and its diagonals bisect each other.**

Is there any other property of the diagonals of a rectangle?

Now, we can write:
"A parallelogram is a rectangle, if its diagonals are equal."

Thus, we can summarize the properties of rectangles as follows:

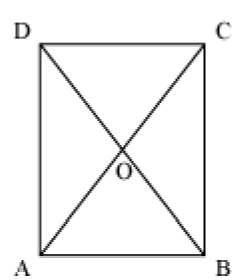


Opposite sides are equal.
Opposite sides are parallel.
Each angle is 90°.
Diagonals are equal and bisect each other.

Let us look at some examples now.

Example 1:

In the given rectangle ABCD, find the value of *x*, if OB = *x* − 2 and OC = 2*x* − 10.



Solution:

Since ABCD is a rectangle, its diagonals are equal and they bisect each other.

i.e., BD = AC

Also, AO = OC and OB = OD

Now, BD = AC

$\Rightarrow$ BO + OD = AO + OC

$\Rightarrow$ 2OB = 2OC

$\Rightarrow$ OB = OC

$\Rightarrow x - 2 = 2x - 10$

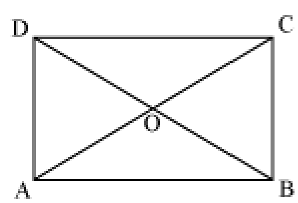
$\Rightarrow 2x - x = -2 + 10$

$\Rightarrow x = 8$

Thus, the value of x is 8.

Example 2:

In a rectangle ABCD, find the length of diagonal BD, if CO = 20 cm.



Solution:

We know that the diagonals of a rectangle bisect each other, therefore

OA = OC

It is given that OC = 20 cm

$\therefore$ OA = 20 cm

From figure,

AC = AO + OC

AC = 20 + 20

AC = 40 cm

In a rectangle, both the diagonals are of equal length.

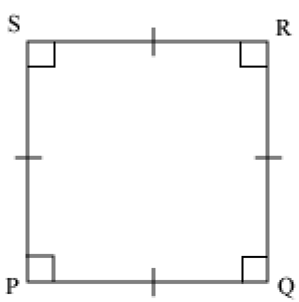
∴ AC = BD

⇒ BD = 40 cm

Thus, the length of diagonal BD is 40 cm.

Properties Of Squares

If we draw a rectangle PQRS with all sides of equal measure, then we will obtain the following figure.



The quadrilateral PQRS so formed is called a **square.**

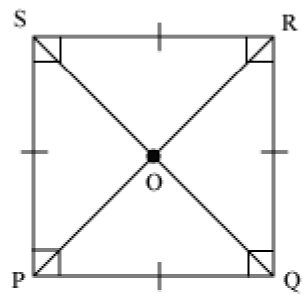
A square is a special case of parallelogram in which all the sides are equal in length and the measure of each angle is 90°.

Being a parallelogram, the diagonals of a square bisect each other, but there are some properties of diagonals of a square, which are exclusive for the square.

We have studied that the diagonals of a square are equal and act as perpendicular bisectors. Its converse is also true. If the diagonals of a quadrilateral are equal and act as perpendicular bisectors, then the quadrilateral is a square.

Now, we can state this result as follows:
"A quadrilateral is a square, if its diagonals are equal and bisect each other at right angles."

We can thus summarize the properties of squares as follows:



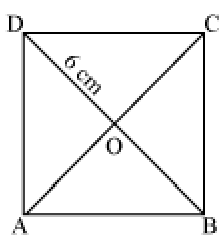
All sides are equal.
The measure of each angle is 90°.
The opposite sides are parallel.
Diagonals are of equal lengths and perpendicular bisectors of each other.

A **square** can also be thought of a **rectangle** in which adjacent sides are equal or a **rhombus** in which each angle is 90º.

Let us look at an example now.

Example:

In the given figure, find the length of the sides of the square ABCD.



Solution:

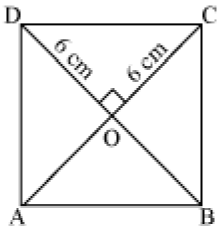
We know that the diagonals of a square are equal.

Therefore, BD = AC

Also, the diagonals bisect each other.

∴ OD = OC

Also, the diagonals are perpendicular to each other.



Now, applying Pythagoras Theorem in ΔDOC, we obtain

$$OD^2 + OC^2 = DC^2$$
$$6^2 + 6^2 = DC^2$$
$$DC^2 = 36 + 36$$
$$DC^2 = 72$$
$$DC = \sqrt{72}$$
$$DC = 6\sqrt{2}\text{cm}$$

Thus, the length of each side of square ABCD is $6\sqrt{2}$ cm.