CHAPTER 16

Capacitance

CAPACITOR & CAPACITANCE

A capacitor consists of two conductors carrying charges of equal magnitude and opposite sign. The capacitance C of any capacitor is the ratio of the charge Q on either conductor to the potential difference V between them $C = \frac{Q}{V}$

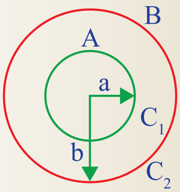
The capacitance depends only on the geometry of the conductors.

Capacitance of an Isolated Spherical Conductor

$C = 4\pi \in_0 \in_r R$ in a medium $C = 4\pi \in_0 R$ in air.

This sphere is at infinite distance from all the conductors.

Spherical Capacitor :



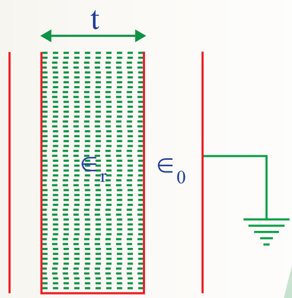
It consists of two concentric spherical shells as shown in figure. Here capacitance of region between the two shells is C_1 and that outside the shell is C_2. We have $C_1 = \frac{4\pi \in_0 ab}{b-a}$ and $C_2 = 4\pi\in_0 b$.

Parallel Plate Capacitor

(i) Uniform Di-Electric Medium : If two parallel plates each of area A & separated by a distance d are charged with equal & opposite charge Q, then the system is called a **parallel plate capacitor** & its capacitance is given by, $C = \frac{\in_0 \in_r A}{d}$ in a medium, $C = \frac{\in_0 A}{d}$ with air as medium.

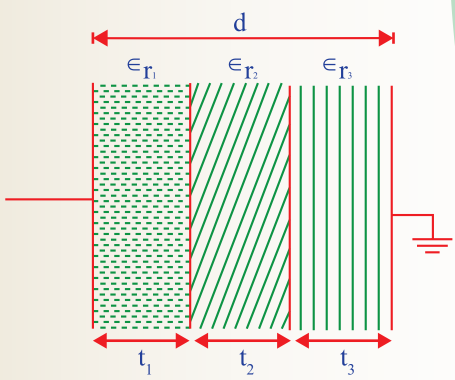
This result is only valid when the electric field between plates of capacitor is constant.

(ii) Medium Partly Air : $C = \dfrac{\epsilon_0 A}{d - \left(t - \dfrac{t}{\epsilon_r}\right)}$



When a di-electric slab of thickness t & relative permitivity ϵ_r is introduced between the plates of an air capacitor, then the distance between the plates is effectively reduced by $\left(t - \dfrac{t}{\epsilon_r}\right)$ irrespective of the position of the di-electric slab.

(iii) Composite Medium :

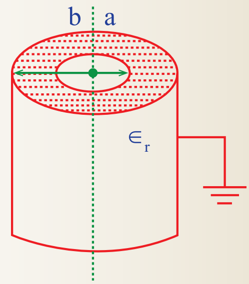


$$C = \frac{\epsilon_0 A}{\dfrac{t_1}{\epsilon_{r1}} + \dfrac{t_2}{\epsilon_{r2}} + \dfrac{t_3}{\epsilon_{r3}}}$$

Cylindrical Capacitor :

It consists of two co-axial cylinders of radii a & b, the outer conductor is earthed.The di-electric constant of the medium filled in the space between the cylinders is ϵ_r.

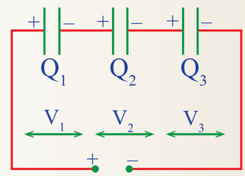
The capacitance per unit length is $C = \dfrac{2\pi \epsilon_0 \epsilon_r}{\ell n\left(\dfrac{b}{a}\right)}$



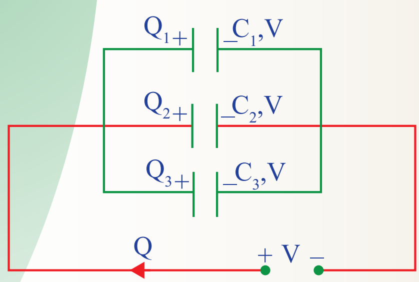
Combination of Capacitors :

(i) Capacitors in Series : In this arrangement all the capacitors when uncharged get the same charge Q but the potential difference across each will differ (if the capacitance are unequal).

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$



(ii) Capacitors in Parallel : When one plate of each capacitor is connected to the positive terminal of the battery & the other plate of each capacitor is connected to the negative terminals of the battery, then the capacitors are said to be parallel connection. The capacitors have the same potential difference V, but the charge on each one is different (if the capacitors are unequal). $C_{eq.} = C_1 + C_2 + C_3 + ... + C_n$.



Energy Stored in a Charged Capacitor

Capacitance C, Charge Q & potential difference V; then energy stored is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

This energy is stored in the electrostatic field set up in the di-electric medium between the conducting plates of the capacitor.

Heat Produced in Switching in Capacitive Circuit

Due to charge flow always some amount of heat is produced when a switch is closed in a circuit which can be obtained by energy conservation as

Heat = Work done by battery – Energy absorbed by capacitor

Work done by battery to charge a capacitor

$$W = CV^2 = QV = Q^2/C$$

Sharing of Charges: When two charged conductors of capacitance C_1 & C_2 at potential V_1 & V_2 respectively are connected by a conducting wire, the charge flows from higher potential conductor to lower potential conductor, until the potential of the two condensers become equal. The common potential (V) after sharing of charges;

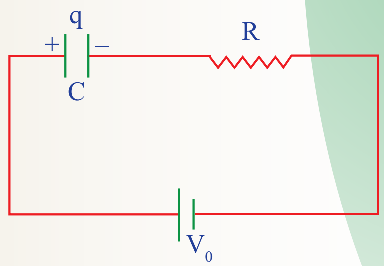
$$V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Charges after sharing $q_1 = C_1V$ & $q_2 = C_2V$. In this process energy is lost in the connecting wire as heat.

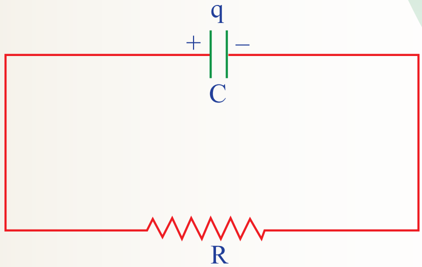
This loss of energy is $U_{\text{initial}} - U_{\text{final}} = \dfrac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2$

Attractive force between capacitor plate: $F = \left(\dfrac{\sigma}{2\epsilon_0}\right)(\sigma A) = \dfrac{Q^2}{2\epsilon_0 A}$

Charging of a capacitor: $q = q_0\,(1 - e^{-t/RC})$ where $q_0 = CV_0$



Discharging of a capacitor : $q = q_0e^{-t/RC}$



KEY TIPS

- The energy of a charged conductor resides outside the conductor in its electric field, where as in a condenser it is stored within the condenser in its electric field.
- The energy of an uncharged condenser = 0.
- The capacitance of a capacitor depends only on its size & geometry & the dielectric between the conducting surface.
- The two adjacent conductors carrying same charge can be at different potential bacause the conductors may have different sizes and means different capacitance.
- On filling the space between the plates of a parallel plate air capacitor with a dielectric, capacity of the capacitor is increased because the same amount of charge can be stored at a reduced potential.
- The potential of a grounded object is taken to be zero because capacitance of the earth is very large.

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