ACTIVITY 2

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation.

Material Required

A piece of plywood, 8 pieces of wire, plywood, some nails, white paper, gum etc.

Method of Construction

Take a piece of plywood and paste a white paper on it. Fix all the 8 wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in figure given below.

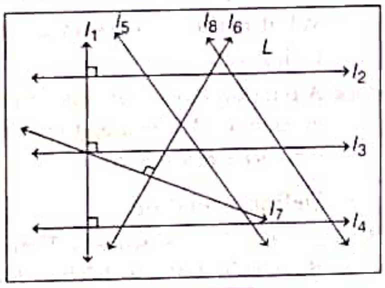

Figure 2.1

Demonstration

1. Let the wires represent the lines $l_1, l_2, \dots l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 [see figure].
3. l_6 is perpendicular to l_7.
4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.
5. $(l_2, l_3), (l_3, l_4), (l_5, l_8) \in R$

Observation

1. In fig., every line is parallel to itself. So the relation $R = \{(l, m) : l \parallel m\}$ .... reflexive relation (is/is not)
2. In fig., observe that $l_2 \parallel l_3$. Is $l_3 \dots\dots l_2$? (II/II)
   So, $(l_2, l_3) \in R \Rightarrow (l_3, l_2) \dots . R (\notin / \in)$
   Similarly, $l_3 \parallel l_4$. Is $l_4 \dots . l_3$? (II/II)
   So, $(l_3, l_4) \in R \Rightarrow (l_4, l_3) \dots . R (\notin / \in)$
   and $(l_5, l_8) \in R \Rightarrow (l_8, l_5) \dots . R (\notin / \in)$
   $\therefore$ The relation R .... symmetric relation (is/is not)
3. In fig., observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is $l_2 \dots . l_4$? (II/II)
   So, $(l_2, l_3) \in R$ and $(l_3, l_4) \in R \Rightarrow (l_2, l_4) \dots . R (\notin / \in)$
   Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is $l_3 \dots . l_2$? (II/II)
   So, $(l_3, l_4) \in R$,
   $(l_4, l_2) \in R \Rightarrow (l_3, l_2) \dots . R (\notin / \in)$
   Thus, the relation R... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

Application

From this activity we can easily understand start the concept of an equivalence relation.

Note

1. This activity can be repeated by taking fixing some more wires in different positions.
2. In this case, the relation is an equivalence relation.

VIVA-VOCE

1 Define a symmetric relation.

Ans A relation R on a set A is said to be a symmetric relation iff
$$(a, b) \in R \Rightarrow (b, a) \in R \; \forall \; a, b \in A$$

2 Define a transitive relation.

Ans Let A be any set A relation R on A is said to be transitive relation iff.
$$(a, b) \in R \text{ and } (b, c) \in R$$
$$\Rightarrow (a, c) \in R \; \forall \; a, b, c \in A$$

3 Define an equivalence relation.

Ans A relation R on a set A is said to be an equivalence relation on A iff it is reflexive, symmetric and transitive.

4 For the set $A = \{1, 2, 3\}$ define a relation R on the set A as follow: $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$

Write the ordered pairs to be added to R to make the smallest equivalence relation.

Ans (3, 1)

5 Write the smallest equivalence relation on the set $A = \{1, 2, 3\}$

Ans $\{(1, 1), (2, 2), (3, 3)\}$

6 If R is a symmetric relation on a set A then write a relation between R and R^{-1}

Ans $R = R^{-1}$

7 Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. write the equivalence class of [0].

Ans Clearly 2-0 and 4-0 are divisible by 2. Therefore, $(2, 0) \in R$ and $(4, 0) \in R$.
$\therefore \; [0] = [2, 4]$

8 Is it true, the inverse of an equivalence relation is an equivalence relation.

Ans Yes.