Agniveer Vayu Science (Group X) - 13 Oct 2023 Shift 1 - Memory Based Paper

Physics

Question 1

Kepler's second law regarding the areal velocity of a planet is an application of the law of conservation of

Options:

A. Angular momentum

- B. Energy
- C. Linear momentum
- D. None of these

Answer: A

Solution:

CONCEPT:

Johannes Kepler proposed laws of planetary motion.

• Kepler's Second Law: The line that joins any planet to the sun sweeps equal areas in equal intervals of time.

This law comes from the observations that planets appear to **move slower** when they are **farther from the sun** than when they are **nearer**, they appear to **move faster**. It is also termed as the **Law of Equal Areas**.



The planet P moves around the sun in an elliptical orbit. The shaded area is the area ΔA swept out in a small interval of time Δt .

i.e. $\frac{dA}{dt} = constant$

• Law of conservation of Angular Momentum: The law of conservation of angular momentum states that when no external torque acts on an object, no change of angular momentum will occur.

EXPLANATION:

As we know from Kepler's second law

$$\frac{dA}{dt} = constant$$
, and

If take angular momentum 'L' and mass 'm' then it can be related by areal velocity is constant,

$$rac{\mathrm{d}\mathbf{A}}{\mathrm{d}\mathbf{t}} = rac{\mathbf{L}}{2\ \mathbf{m}} = \mathbf{constant},$$

So this law can be derived from, the **law of conservation of angular momentum**. So option 1 follows.

So option 1 is correct.

Question 2

The ratio of the dimension of Planck's constantand that of the moment of inertia is thedimension of

Options:

A. Frequency

- B. Velocity
- C. Acceleration

D. Time

Answer: A

Solution:

Concept:

Planck's Constant:

The Planck's constant relates the energy of a photon with its frequency.

The energy of a photon is given as

E = hv - (1)

h is Planck's constant, v is frequency.

Moment of Inertia:

- Moment of inertia is the sum of the products of the mass of each particle in the body with the square of its distance from the axis of rotation.
- It is defined for an axis.
- The moment of inertial of a particle m at distance r from the axis is

$I = mR^2$

Explanation:

The Planck constant is given as from the first equation.

$$h = \frac{E}{\nu}$$

So, the dimensions of h

 $rac{Dimension \ of \ Energy}{Dimension \ of \ Frequency} = rac{ML^2T^{-2}}{T^{-1}} = ML^{-2}T^{-1}$

The dimension of Moment of Inertia

 $I = ML^2$

The ratio of h and I

$$R = rac{ML^2T^{-1}}{ML^2} = T^{-1}$$

 T^{-1} is the Dimension of Frequency which is equal to the **reciprocal of the time period**.

So, frequency is the correct answer.

Question 3

A body executing simple harmonic motion has an amplitude of 0.01 m and a frequency of 50 Hertz. The ratio of the magnitude of maximum acceleration and maximum velocity of the body is

Options:

Α. 25π

Β. 50π

C. 100π

D. 200π

Answer: C

Solution:

The correct answer is option 3) i.e. 100π

CONCEPT:

- **Simple harmonic motion(SHM)**: It is a type of oscillatorymotionin which the restoring force is directly proportional to the displacement of the body from its mean position.
 - This means that thenet forcewhich in turn is theaccelerationisproportionalto the displacement of the object and acts in theopposite direction of the displacement.
 - The displacement in an SHM for a system starting at equilibrium (x = 0) is given by the equation:

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x = A sin \omega t
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Where x is the displacement, A is the amplitude, w is the angular frequency and t is the time taken.

The velocity of a particle in SHM, $v = \frac{dx}{dt} = A\omega cos\omega t$

Acceleration of a particle in SHM, a = $\frac{d(dx/dt)}{dt} = -A\omega^2 sin\omega t$

CALCULATION:

Given that:

Frequency, $f = 50 \text{ Hz} \Rightarrow \omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s}$

Amplitude, A = 0.01 m

Velocity, $v = a\omega cos\omega t$

Maximum velocity occurs when $\cos \omega t = 1$.

Therefore, **maximum velocity**, $\mathbf{v}_{max} = A\omega = 0.01 \times 100\pi = 1\pi \text{ m/s}$

Acceleration, $a = -A\omega^2 \sin\omega t$

Maximum acceleration occurs when $\sin\omega t = 1$.

Therefore, maximum acceleration, $a_{max} = A\omega^2 = 0.01 \times (100\pi)^2 = 100\pi^2 \text{ m/s}^2$

Ratio = $\frac{a_{max}}{v_{max}} = \frac{100\pi^2}{1\pi} = 100\pi$

Question 4

A conductor carries a current of 20A and is at right-angles to a magnetic field having a flux density of 0.9T. If the length of the conductor in the field is 30 cm, calculate the force acting on the conductor.

Options:

- A. 2.6 N
- B. 5.4 N
- C. 54 N
- D. 27 N

Answer: B

Solution:

Concept:

The force experienced by a current-carrying conductor lying in a magnetic field is with an angle θ is given by

$\mathbf{F} = \mathbf{BIL} \, \mathbf{sin} \, \boldsymbol{\theta}$

Where,

- B = Magnetic flux density of the field
- I = Current
- L = Length of the conductor perpendicular to the magnetic field
- Here angle θ represents the direction of the field and the current.
- Therefore, the force on the current-carrying conductor in a magnetic field depends upon

- The magnetic flux density of the field (B)
- The strength of the current (I)
- The length of the conductor perpendicular to the magnetic field (l)
- The directions of the field and the current

Calculation:

Given that, current (I) = 20 A Magnetic flux density (B) = 0.9T Length of conductor (I) = 30cm = 0.3 m Angle between conductor and magnetic field (θ) = 90° The force on the conductor = Bil sin θ = 0.9× 20 × 0.3 × sin 90° = 5.4 N

Question 5

The induced emf in a coil is given as:

Options:

A. $e = L \frac{d\phi}{dt}$ B. $e = L \frac{di}{dt}$ C. $e = -L \frac{di}{dt}$ D. $e = L \frac{d^2i}{dt^2}$

Answer: C

Solution:

Concept:

Emf induced in a coil or inductor is given by

$$e = -Lrac{di}{dt}$$

Where,

L = Inductance

di = final value of current - initialvalue of current

Question 6

The efficiency of Carnot heat engine is 75% and the amount of heat absorbed is 300 Joule, then the work done is?

Options:

A. 225 J

B. 300 J

C. 220 J

D. None of these

Answer: A

Solution:

CONCEPT:

- Carnot engine: Itis atheoretical thermodynamic cycleproposed by Leonard Carnot.
 - It gives an idea of the maximum possible efficiency that a heatengineduring the conversion process of heat into work and conversely, working between two reservoirs, can possess.

Pressure-Volume diagram:



The efficiency of the Carnot cycle:

Efficiency = work done by the engine (W)/heat supplied(Q)

where, W =work done in joules, Q = amount of heat absorbed.

Efficiency(η) = 1 - T₂/T₁

Where, T_1 = input temperature (absolute) or source temperature, and T_2 = output temperature (absolute) or sink temperature.

CALCULATION:

Given that, $\eta = 75\%$, Q = 300 J

So from the above discussion its clear that,

 $\eta = W/Q$

75/100 = W/300

W = 225 J

The correct option is 225 J.

Question 7

A ball, initially at rest, is dropped from a height *h* above the floor bounces again and again vertically. If the coefficient of restitution between the ball and the floor is 0.5, the total distance travelled by the ball before it comes to rest is

Options:

A. 8h/3

B. 5h/3

C. 3h

D. 2h

Answer: B

Solution:

Concept:

Projectile motion is the motion of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a projectile, and its path is called its trajectory.

Calculation:

 $v = \sqrt{2gh} \text{ and } v = e\sqrt{2gh}$ $0 = (ev)^2 - 2gh_1$ $h_1 = \frac{e^2 \times 2gh}{2g} = e^2h$ Similarly, $h_2 = e^4h$ $H = h + 2h_1 + 2h_2 + \dots \infty$ $= h + 2(e^2h + e^4h + \dots \infty)$ $= h + 2e^2h(\frac{1}{1 - e^2})$ $= h \times (\frac{1 + e^2}{1 - e^2})$ The event of vertication has

The coefficient of restitution between the ball and the floor is 0.5.

e = 0.5

H = 5h/3

The correct answer is option (2).

Question 8

The ratio of the amplitude of the magnetic field to the amplitude of the electric field for electromagnetic wave propagation in a vacuum is equal to

Options:

A. Unity

- B. Speed of light in vacuum
- C. Reciprocal of the speed of light in vacuum

D. The ratio of magnetic permeability to electrical susceptibility in a vacuum.

Answer: C

Solution:

Concept:

In electromagnetic waves, the ratio of amplitudes of the electric field and the magnetic field is equal to the velocity of the electromagnetic waves in free space.

$$rac{E_0}{B_0}=c$$
 $rac{B_0}{E_0}=rac{1}{c}$

Where:

 $E_0 = Electric field$

 $B_0 =$ Magnetic field

c = velocity

Derivation:

The intrinsic impedance of the wave is defined as the ratio of the electric field and the magnetic field phasor (complex amplitude), i.e.

 $rac{|E|}{|H|}=\eta=\sqrt{rac{\mu}{\epsilon}}$

Since $B = \mu H$

$$H=rac{B}{\mu}$$
 $rac{|E|}{|B|}\mu=\sqrt{rac{\mu}{\epsilon}}$

$$rac{|E|}{|B|} = \sqrt{rac{1}{\mu\epsilon}}$$

 $c=rac{1}{\sqrt{\mu_0\epsilon_o}},\;rac{|E|}{|B|}=c$

Question 9

The magnitude of electric flux through the given square surface is:



Options:

A. 20 N $m^{2}C^{-1}$

B. 200N m²C⁻¹

C. $0.1N \text{ m}^2\text{C}^{-1}$

D. $1N m^2 C^{-1}$

Answer: D

Solution:

The correct answer is option 4) i.e.1N m²C⁻¹

CONCEPT:

• Electric flux: It is the number of electric field lines passing through a given surface area.

Theelectric flux associated with a surface kept in a uniform electric field is given by:



 $\phi = E.A = EAcos\theta$

Where E is the uniform electric field, A is the area of the surface and θ is the angle between E and area vector of A.

CALCULATION:

Given that:

Electric field, E = 200 N/C

Side of the square surface = 10 cm = 0.1 m

Area of the square surface, $A = 0.1^2 = 0.01 \text{ m}^2$



The angle between E and $A, \theta = 60^{\circ}$

Electric flux, $\phi = EA\cos\theta = 200 \times 0.01 \times \cos 60^\circ = 1 \text{ M m}^2 \text{C}^{-1}$

Question 10

For the destructive interference of the two similar waves, the phase difference between the waves must be:

Options:

A. 0

B. $\frac{\pi}{2}$

C. π

D. None of these

Answer: C

Solution:

CONCEPT:

The principle of superposition of waves:

• Whentwo or more waves come together at some point in space then the resultant disturbance wave is the vector sum of disturbance of the individual waves.



Constructive interference Destructive interference

- The principle of superposition is basic to the phenomenon of interference.
- Let us consider two harmonic travelling waves on a stretched string, both with the same ω(angular frequency) and k (wave number) and, therefore, the same wavelength λ. Their wave speed will be identical. Let us further assume that their amplitudes are equal and they are both travelling in the positive direction of the x-axis. The waves only differ in their initial phase.
- The two waves are described by the functions:

 \Rightarrow y₁= a.sin(kx - ω t)

 \Rightarrow y₂= a.sin(kx - ω t + ϕ)

• By the principle of superposition, the net displacement is given by,

 $\Rightarrow y=2a.\,cosrac{\phi}{2}sin\left(kx-\omega t+rac{\phi}{2}
ight)$

• The resultant wave is also a harmonic travelling wave in the positive direction of the x-axis, with the same frequency and wavelength. However, its initial phase angle is $\frac{\phi}{2}$.

Interference:

- Whentwo waves superimpose then the resultant amplitude of the wave at that point is the vector sum of amplitudes of each individual wave. This phenomenonis called the interference of waves.
- There are two types of
 - 1. Constructive Interference:
 - 2. Destructive Interference:

S. No	Constructive Interference	Destructive Interference
	If two waves superimpose with	If two waves superimpose with
1.	amplitude of the resultant is equal to the sum of the amplitudes of individual waves resulting in the maximum intensity of light, this is	the amplitude of the resultant is equal to the difference in amplitude of individual waves, resulting in the minimum intensity of light, this is

EXPLANATION:

- If two waves superimpose with each other in the opposite phase, the amplitude of the resultant is equal to the difference in amplitude of individual waves, resulting in the minimum intensity of light, this is known as destructive interference.
- Hence, option 3 is correct.

Question 11

The torque experienced by a magnetic dipole, having dipole moment M, when placed in a uniform magnetic field of intensity B is:

Options:

A. MB

B. **M.B**

C. $\frac{|\mathbf{M}|}{|\mathbf{B}|}$

D. M× B

Answer: D

Solution:

Concept:

A rectangular coil always acts as a**magnetic dipole** of dipole moment M.

The dipole moment of the coil (M) = N I AN is the number of turns

I is current in the coiland

A is the area of the rectangular coil



The torque of the magnetic field on any magnetic dipole moment is given by;

$\mathbf{T} = \mathbf{M} \times \mathbf{B} = \mathbf{M}\mathbf{B}\,\mathbf{Sin}\boldsymbol{\theta}$

 $M \times B = cross \text{ product of Area vector A and magnetic field vector B}$

 θ = the angle between area vector A and magnetic field B.

SinceM= N I A, the torque on the rectangular coil (T) will be:

 $M \times B = MB \sin\theta = N I A B \sin\theta$

Observations:

- According to the above formula of the torque on a rectangular coil in a magnetic field, torque will be large when the number of turns is large.
- If the magnetic field is perpendicular to the plane of the coil then it will be along the area vector. So the angle between the area vector and the magnetic field will be zero and finally, the torque will be zero.
- If the area will be small then torques will also be small.

Question 12

Consider a force acts on a 1 kg block so that its position is given as a function of time as $x = 3t^2 + 5$. What will be the work done by this force in first 5 seconds?

Options:

A. 900 J

B. 500 J

C. 800 J

D. 450 J

Answer: D

Solution:

Concept:

• In classical physics, the **kinetic energy** of an object is defined as the energy carried by an object due to virtue of its motion and it is directly proportional to the mass of an object and square of its velocity at any instance

i.e., $K.E = \frac{1}{2}mv^2$

whereas work done by the system is directly proportional to its mass and the rate at which it is accelerated

i. e., W = F. x

- The relation between the kinetic energy of an object and work done is called Work-Energy Theorem.
- It states that Work is done if there is any change in Kinetic Energy.
- Energy is measured in the same unit as work because energy is defined via work (i.e. the capacity of a physical system to do work),

thus the SI unit for energy is the same as the unit of work which is given as Joule (J).

 $Work = K. E = \frac{1}{2}mv^2$

Calculation:

Given that,

Displacement, $x = 3t^2 + 5$

t = 5 sec

we know velocity of particle can be given as

$$v=rac{dx}{dt}=6t+0\Rightarrow v=6t$$

Now at t = 5 sec

$$v = 6 \times 5 = 30 \text{ m/s}$$

Now according to relation of work and energy

Work = Kinetic energy

 $\dot{v} W = rac{1}{2}mv^2 = rac{1}{2} imes 1 imes 30^2 = rac{900}{2} = 450 \; J$

Hence work done by the force in first 5 sec will be 450 Joules

Mathematics

Question 13

If the equation of parabola is $y^2 = -12x$, then which of the following is true ?

Options:

- A. Equation of axis is: y = 0
- B. Equation of directrix is: x 3 = 0
- C. Equation of latus rectum is: x + 3 = 0

D. All of the above

Answer: D

Solution:

Concept:

For a parabola of the form $y^2 = -4ax$ where a > 0 we have:

- Focus is given by: (- a, 0)
- Vertex is given by: (0, 0)
- Equation of directrix is given by: x a = 0
- Equation of axis is given by: y = 0
- Equation of latus rectum is given by: x + a = 0
- Length of latus rectum is given by: 4a

Calculation:

Given:

Equation of parabola is $:y^2 = -12x$

We can re-write the equation of the given parabola as:

$$\Rightarrow y^2 = -4 \cdot (3) \cdot x$$

Now by comparing the above equation with $y^2 = -4ax$ where a > 0 we get: a = 3.

So, equation of directrix is: x - 3 = 0

Similarly, equation of axis is: y = 0 and equation of latus rectum is:

x + 3 = 0.

Hence, option D is true.

Question 14

The equation of circle passing through (4, 5) and having the centre at (2, 2), is

Options:

A. $x^{2} + y^{2} + 4x + 4y - 5 = 0$ B. $x^{2} + y^{2} - 4x - 4y - 5 = 0$ C. $x^{2} + y^{2} - 4x = 13$ D. $x^{2} + y^{2} - 4x - 4y + 5 = 0$

Answer: B

Solution:

Concept:

The equation of a circle with centre (h,k) and radius R is,

0

 $(x - h)^2 + (y - k)^2 = R^2$

Calculation:

The radius of the circlepassing through (4, 5) and the centre at (2, 2),

$$is\sqrt{(4-2)^2+(5-2)^2}=\sqrt{13}$$

 \Rightarrow The equation of the circle is,

$$(x - 2)^{2+} (y - 2)^{2} = 13$$

$$\Rightarrow x^{2+} y^{2-} 4x - 4y + 8 - 13 =$$

$$\Rightarrow x^{2+} y^{2-} 4x - 4y - 5 = 0$$

 \therefore The correct answer is option (2).

Question 15

If the sum of the matrices $\begin{bmatrix} x \\ x \\ y \end{bmatrix}$, $\begin{bmatrix} y \\ y \\ z \end{bmatrix}$ and $\begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix}$ is the matrix $\begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$ then

what is the value of y?

Options:

A. -5

B. 0

C. 5

D. 10

Answer: B

Solution:

Concept:

Matrices are added or subtracted only if they have the same number of rows and columns.

Calculation:

Matrices havesame number of rows and columns, they can be added.

$$\begin{bmatrix} x \\ x \\ y \end{bmatrix} + \begin{bmatrix} y \\ y \\ z \end{bmatrix} + \begin{bmatrix} z \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} x + y + z \\ x + y \\ y + z \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 5 \end{bmatrix}$$

From above, we can write

x + y + z = 10 ----(i) x + y = 5 ----(ii)

and y + z = 5 ----(iii)

Form (i) and (ii), we get 5 + z = 10 $\Rightarrow z = 5$ Now, from (iii), we get y + 5 = 5 $\Rightarrow y = 0$ \therefore The value of y = 0.

Question 16

The general solution of $\cot\theta + \tan\theta = 2$ is

Options:

A. $\theta = n\pi + (-1)^n \frac{\pi}{8}$ B. $\theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}$ C. $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$ D. $\theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{8}$

Answer: C

Solution:

<u>Calculation:</u>

Given $\cot \theta + \tan \theta = 2$ $\frac{1}{\tan \theta} + \tan \theta = 2$ $\tan^2 \theta - 2 \tan \theta + 1 = 0$ $(\tan \theta - 1)^2 = 0$ $\tan \theta = 1$ $\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4} \dots = \frac{n\pi}{2} + (-1)^n \frac{\pi}{4}, (n \in N)$

Question 17

If
$$i^2 = -1$$
 then the value of $\sum_{n=1}^{200} i^n$ is

Options:

A. 50

B. -50

C. 0

D. 100

Answer: C

Solution:

CONCEPT:

Let x and y be any two real numbers, then any number of the form x + iy is called a complex number, where $i = \sqrt{-1}$ or $i^2 = -1$.

CALCULATION:

$$\sum_{n=1}^{200} i^n = i + i^2 + i^3 + i^4 + \dots i^{200}$$

= i+ (-1) + (-i) + 1 + i²⁰⁰

As we know that, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$

As we can see that, $i + i^2 + i^3 + i^4 = 0$

 \therefore Sum of every 4 consecutive terms in the above sequence will result to 0

$$\therefore \sum_{n=1}^{200} i^n = 0$$

Question 18

What is
$$\int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x} \right) dx$$
 equal to ?

Where c is the constant of integration

Options:

- A. 2 cosec 2x + c
- B. $-2 \cot 2x + c$
- C. 2 sec 2x + c
- D. -2 tan 2x + c

Answer: A

Solution:

Concept:

 $\int \sec^2 x dx = \tan x + c \int \csc^2 x dx = -\cot x + c$

<u>Calculation:</u>

Let $I = \int \left(\frac{1}{\cos^2 x} - \frac{1}{\sin^2 x}\right) dx$ $= \int \sec^2 x dx - \int \csc^2 x dx$ $= \tan x - (-\cot x) + c$ $= \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + c = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} + c = \frac{1}{\sin x \cos x} + c$ $= \frac{2}{2\sin x \cos x} + c = \frac{2}{\sin 2x} + c (\because 2 \sin x \cos x = \sin 2x)$ $= 2 \operatorname{cosec} 2x + c$

Question 19

If
$$\mathbf{x} = \mathbf{A} \cos 4\mathbf{t} + \mathbf{B} \sin 4\mathbf{t}$$
, then $\frac{d^2x}{dt^2}$ is equal to -

Options:

A. -16 x

B. 16 x

C. x

D. -x

Answer: A

Solution:

CONCEPT:

•
$$rac{d}{dx} \{f(x) \pm g(x)\} = rac{d\{f(x)\}}{dx} \pm rac{d\{g(x)\}}{dx}$$

•
$$\frac{u(\sin x)}{dx} = \cos x$$

•
$$\frac{d(\cos x)}{dx} = -\sin x$$

- Chain Rule:
- Let y = f(v) be a differentiable function of v and v = g(x) be a differentiable function of x. Then $\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$

CALCULATION:

Given: $x = A \cos 4t + B \sin 4t$

Let's differentiate the given function with respect to t

As we know that, $\frac{d}{dx} \{ f(x) \pm g(x) \} = \frac{d\{f(x)\}}{dx} \pm \frac{d\{g(x)\}}{dx}$ $\Rightarrow \frac{dx}{dt} = \frac{d(A\cos 4t)}{dt} + \frac{d(B\sin 4t)}{dt}$ As we know that, $\frac{d(\sin x)}{dx} = \cos x$ and $\frac{d(\cos x)}{dx} = -\sin x$ $\Rightarrow \frac{dx}{dt} = 4A \cdot (-\sin 4t) + 4B \cdot (\cos 4t)$

Again by differentiating the above equation with respect to t we get

$$egin{array}{lll} \Rightarrow rac{d^2x}{dt^2} = & -16A\cdot\cos4t - 16B\cdot\sin4t \ \Rightarrow rac{d^2x}{dt^2} = & -16\cdot\left[A\cos4t + B\sin4t
ight] = & -16 \end{array}$$

Hence, option A is the correct answer.

 $\cdot x$

Question 20

$$\int_0^1 rac{1}{\sqrt{4-x^2}} \mathrm{d}x =$$

Options:

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{4}$
- C. $\frac{\pi}{3}$
- D. $\frac{\pi}{2}$

Answer: A

Solution:

<u>Concept:</u>

 $\int rac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(rac{x}{a}) + c$

Calculation:

Let, $I = \int_0^1 \frac{1}{\sqrt{4-x^2}} dx$ = $\int_0^1 \frac{1}{\sqrt{2^2-x^2}} dx$ = $[\sin^{-1}(\frac{x}{2})]_0^1$ (:: $\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}(\frac{x}{a}) + c)$ = $\sin^{-1}(\frac{1}{2}) - \sin^{-1}(0)$ = $\frac{\pi}{6}$

Hence, option (1) is correct.

Question 21

What is $\lim_{x \to 0} rac{x}{\sqrt{1 - \cos 4x}}$ equal to ?

Options:

A. $\frac{1}{2\sqrt{2}}$

B.
$$-\frac{1}{2\sqrt{2}}$$

C.
$$\sqrt{2}$$

D. Limit does not exist

Answer: D

Solution:

Concept:

L's Hospital Rule-

 $\mathop{\mathrm{Iflim}}_{x
ightarrow a}f(x)=\lim_{x
ightarrow a}g(x)=0$ then,

 $\lim_{x
ightarrow a}rac{f(x)}{g(x)}=\lim_{x
ightarrow a}rac{f'(x)}{g'(x)}$

Condition of limit to exist:

Let y = f(x) be a function. Then for a function, we say,

 $\lim_{x \to a} f(x)$ exists i.e $\lim_{x \to a} f(x) = l$,

Where l is a finite value if

 $\lim_{\mathrm{x}\to\mathrm{a}^-}\mathrm{f}(\mathrm{x})=\lim_{\mathrm{x}\to\mathrm{a}^+}\mathrm{f}(\mathrm{x})$

Formula used:

 $1 - \cos 2x = 2\sin^2 x$

Calculation:

Given,
$$\lim_{x \to 0} \frac{x}{\sqrt{1 - \cos 4x}}$$
$$\Rightarrow \lim_{x \to 0} \frac{x}{\sqrt{2 \sin^2 2x}}$$
$$\Rightarrow \lim_{x \to 0} \frac{x}{\sqrt{2} |\sin 2x|}$$

Left-hand limit:

$$\Rightarrow \lim_{x
ightarrow 0^-} f(x) = \lim_{h
ightarrow 0} rac{0-h}{\sqrt{2} |sin \ 2(0-h)|}$$

$$\Rightarrow \lim_{h \to 0} \frac{-h}{\sqrt{2} \sin 2h} \times \frac{2}{2}$$
$$\Rightarrow \lim_{h \to 0} \frac{-2h}{\sqrt{2} \sin 2h} = \frac{-1}{2\sqrt{2}}$$

Right-hand limit:

$$\Rightarrow \lim_{x \to 0^+} f(x) = \lim_{h \to 0} \frac{0+h}{\sqrt{2}|\sin 2(0+h)|}$$

$$\Rightarrow \lim_{h \to 0} \frac{h}{\sqrt{2}\sin 2h} \times \frac{2}{2}$$

$$\Rightarrow \lim_{h \to 0} \frac{2h}{\sqrt{2}\sin 2h} = \frac{1}{2\sqrt{2}}$$

We can see that, $LHL \neq RHL$

 \therefore The limit does not exist.

Question 22

Find the value of $\vec{a}\times\vec{a}$

Options:

A. 1

B. 0

C. $|\vec{a}|$

D. $|\vec{a}|^2$

Answer: B

Solution:

Concept:

Dot product of two vectors is defined as:

 $ec{\mathbf{A}}. \, ec{\mathbf{B}} = |\mathbf{A}| imes |\mathbf{B}| imes \cos heta$

Cross/Vector product of two vectors is defined as:

 $ec{\mathbf{A}} imes ec{\mathbf{B}} = |\mathbf{A}| imes |\mathbf{B}| imes \sin \theta imes \hat{\mathbf{n}}$

where θ is the angle between \vec{A} and \vec{B}

Calculation:

To Find: Value of $\vec{a}\times\vec{a}$

Hereangle between them is 0°

 $ec{\mathbf{a}} imes ec{\mathbf{a}} = |\mathbf{a}| imes |\mathbf{a}| imes \sin \mathbf{0} imes \hat{\mathbf{n}} = \mathbf{0}$

Question 23

If ${}^{n}C_{10} = {}^{n}C_{15}$, then ${}^{27}C_{n}$ equals

Options:

A. 702

B. 351

C. 729

D. 243

Answer: B

Solution:

Given:

 ${}^{n}C_{10} = {}^{n}C_{15} - \dots - (1)$

Concept: ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

Calculation:

We can write, ${}^{n}C_{10} = \frac{n!}{10!(n-10)!}$ -----(2) ${}^{n}C_{15} = \frac{n!}{15!(n-15)!}$ -----(3) By equation (1) comparing equations (2) and (3), we get $\frac{n!}{10!(n-10)!} = \frac{n!}{15!(n-15)!}$

 $\frac{1}{10!(n-10)!} = \frac{1}{15!(n-15)!}$ -----(4)

Comparing both sides of equation (4), we get

10 = n - 15, and

 $\Rightarrow 15 = n - 10$

 \Rightarrow n = 25

Now, ${}^{27}C_n$ for n = 25, will be

 ${}^{27}C_{25} = \frac{27!}{25!(27-25)!} = \frac{27 \times 26 \times 25!}{25!(2)!}$ $\Rightarrow {}^{27}C_{n} = \frac{27 \times 26}{2} = 27 \times 13 = 351$ $\therefore {}^{27}C_{n} \text{ or } {}^{27}C_{25} \text{ equals to } 351.$

Question 24

If $y = \frac{(\sin x - \cos x)}{\sin 2x}$, find $\frac{dy}{dx}$

Options:

A. $\frac{1}{2}(\text{secx. cotx} + \text{cosecx. tanx})$ B. $\frac{1}{2}(\text{secx. cotx} - \text{cosecx. tanx})$

C. $\frac{1}{2}(\text{secx. tanx} + \text{cosecx. cotx})$

D. $\frac{1}{2}$ (secx. tanx – cosecx. cotx)

Answer: C

Solution:

Concept used:

Trigonometry formula

 $\sin 2x = 2\sin x \cos x$

Calculation:

$$y = \frac{(\sin x - \cos x)}{\sin 2x}$$

$$y = \frac{(\sin x - \cos x)}{2 \cdot \sin x \cdot \cos x}$$

$$\Rightarrow y = \frac{\sin x}{2 \cdot \sin x \cdot \cos x} - \frac{\cos x}{2 \cdot \sin x \cdot \cos x}$$

$$\Rightarrow y = \left(\frac{1}{2 \cdot \cos x}\right) - \left(\frac{1}{2 \cdot \sin x}\right)$$

$$\Rightarrow y = \frac{1}{2} (\operatorname{secx-cosecx})$$
Differentiate both sides, we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \left[\frac{d(\text{secx})}{dx} - \frac{d(\text{cosecx})}{dx} \right]$$
$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} (\text{secx. tanx} + \text{cosecx. cotx})$$

English

Question 25

Select the most appropriate synonym for the given word.

Merciful

Options:

- A. Brutal
- B. Stubborn
- C. Kind
- D. Grateful

Answer: C

Solution:

The correct answer is **option 3 i.e. Kind.**

The sentence talks about how the king was known for his kindness when it came to forgiving first-time wrongdoers.

- Merciful: To be forgiving and compassionate towards someone.(किसी के प्रति क्षमाशील और दयावान होना)
- Kind: To be generous, helpful, and caring about other people.(उदार, मददगार और दूसरे लोगों की देखभाल करने वाला होना)

Thus, it is clear that 'merciful' and 'kind' are synonyms.

Let's look at the meanings of the other words:

- Brutal: Violent in a savage way (एक क्रूर तरीके से हिंसक)
- Stubborn: Showing extreme determination to not change what one believes in (जिस पर कोई विश्वास करता है उसे न बदलने के लिए अत्यधिक दृढ़ संकल्प दिखा रहा है)
- Grateful: Showing appreciation for something that has happened. (जो कुछ हुआ है उसके लिए प्रशंसा दिखाना।)

Question 26

In the following question, out of the four alternatives, select the word opposite in meaning to the word given.

Lethargic

Options:

A. Torpid

B. Dull

C. Energetic

D. Narcotic

Answer: C

Solution:

Correct Answer: Energetic

'Lethargic' means 'slow-moving and inactive or sluggish'.

On the other hand, 'energetic' means 'full of energy and activity'.

Hence, they have opposite meanings. Torpid means inactive, dull means lifeless and narcotic is related to drowsiness.

Question 27

Choose the word that can substitute the given group of words.

One who looks at the dark side of life

Options:

- A. Libertine
- B. Pessimist
- C. Anarchist
- D. Extrovert
- Answer: B

Solution:

The correct answer is 'Option 2' i.e. 'Pessimist'.

🔗 <u>Key Points</u>

- "Pessimist" is a person who habitually or systematically expects the worst outcome or has a negative outlook on life or a specific situation.(निराशावादी)
- Explanation of **other options**:
 - **Libertine:** A libertine is a person who leads an immoral or unrestrained lifestyle, often disregarding social norms or moral principles.
 - Anarchist: An anarchist is a person who believes in the absence of government or authority, advocating for a society based on voluntary cooperation and absence of hierarchy.
 - **Extrovert:** An extrovert is a person who is outgoing, sociable, and energized by being around others.

Therefore, the correct answer is: 'Pessimist'.

Question 28

Select the wrongly spelt word.

Options:

- A. Celebrate
- B. Celebrant
- C. Celebrity
- D. Celeberation

Answer: D

Solution:

The correct answer is **option 4**) i.e.celeberation.

- The correct spelling will be'celebration'.
- All other words are correctly spelt.
 - **celebrate:**acknowledge a significant or happy day or eventwith a social gathering or enjoyable activity.
 - **celebrant:**a person who celebrates something.
 - celebrity: a famous person, especially in entertainment or sport.

Question 29

What is the noun form of 'Restrict'?

Options:

- A. Restricted
- B. Restricting
- C. Restriction
- D. Restrictive

Answer: C

Solution:

The correct answer is "Restriction".

🔗 <u>Key Points</u>

- The word "Restrict" is a verb. According to the rules of English grammar, when we want to turn a verb into a noun, we often use specific suffixes. A frequently used suffix to convert verbs to nouns is "-ion".
- Hence, when we apply this rule to the verb "Restrict", we add the suffix "-ion" to get "Restriction", which is a noun. Additionally, it's worth noting that many other English verbs follow this pattern, converting to nouns through the addition of "-ion".
- The noun "Restriction" is used to denote something that limits or controls what people can do or what is possible. It represents a limit or control on someone or something to prevent them from behaving in a particular way. It falls in line logically with the meaning of "Restrict', which involves limiting or controlling something.

Hence, the correct answer is "option 3".

Question 30

In the given sentence, identify the segment that contains a grammatical error.

There is a few water in the bowl on the table.

Options:

A. There is

B. in the bowl

C. on the table

D. a few water

Answer: D

Solution:

The erroneous part of the sentence is:a few water

🔗 <u>Key Points</u>

- Replace 'few' with 'little' to make the sentence correct.
- We use few and a few with countable nouns in the plural form.
 Example Few dogs are chasing a goat.
- We use little and a little with uncountable nouns.

- **Example -** We had **little** time to prepare before we had to go.
- In the given sentence Water is an uncountable noun. So, we must use 'little' in place of 'few'.

Therefore, the correct answer is **option 4** which contains an error.

Corrected sentence:*There is a little water in the bowl on the table.*

Question 31

Select the option that expresses the given sentence in indirect speech. She said, "I wrote a book."

Options:

- A. She said that he has written a book.
- B. She said that he had been written a book.
- C. She said that she had written a book.
- D. She said that she has written a book.

Answer: C

Solution:

The correct answer is "She said that she had written a book."

🔗 <u>Key Points</u>

- When we convert direct speech into indirect speech, we need to follow some certain rules.
 - One of the rules involves verb tense changes.
 - In this case, the tense in the direct speech is simple past ("I wrote").
 - When converting to indirect speech, we typically shift this to past perfect ("she had written").
- The correct sentence is: "She said that she had written a book."
 - It maintains the correct subject 'she' (referencing 'I' in the original sentence) and uses 'had written', which is the past perfect tense appropriate for this shift in direct to indirect speech.

Therefore, the correct answer is "Option 3".

誟 Additional Information

- Let's consider other options:
 - She said that he has written a book.
 - This option changes the primary subject from 'I' to 'he' and uses 'has written', which is present perfect tense.

- It is not consistent with the tense shift rules for converting direct to indirect speech, and it changes the subject incorrectly.
- She said that he had been written a book.
 - This option also incorrectly changes 'I' to 'he', but it uses 'had been written', which is past perfect passive.
 - This changes the meaning, implying that someone else wrote the book for her.
- She said that she has written a book.
 - This option keeps the correct subject 'she', but the verb tense 'has written' is present perfect, which is not the appropriate tense shift for converting this sentence.

Question 32

Select the option that expresses the given sentence in passive voice. I am eating mangoes.

Options:

- A. Mangoes is being eaten by me.
- B. Mangoes are being eaten by me.
- C. Mangoes have been eaten by me.
- D. Mangoes are eaten by me.

Answer: B

Solution:

The correct answer is "Mangoes are being eaten by me."

🔗 <u>Key Points</u>

- In the English language, presenting a sentence in active or passive voice changes the focus of attention in that sentence.
- In active voice sentences, the subject performs the action (verb), but in passive voice sentences, the subject receives the action.
- The original given sentence, "I am eating mangoes." is in active voice, where the subject 'I' is performing the action (eating) on the object 'mangoes'.
- When transforming this into passive voice, the verb tense needs to be maintained.
- In this sentence, the tense is present continuous (am eating).
- "Mangoes are being eaten by me." is the correct passive voice equivalent of the original sentence.
 - This sentence maintains the present continuous tense "are being eaten" instead of "am eating", and the subject and the object positions have been interchanged.
- So, the answer is: "Mangoes are being eaten by me."

誟 Additional Information

- Let's examine the options:
 - Mangoes is being eaten by me.
 - In this option, the verb agrees incorrectly with the subject. "Mangoes" is plural, so the verb should be "are", not "is".
 - Mangoes have been eaten by me.
 - This option changes the tense of the verb to present perfect ("have been eaten"), which is not an accurate conversion of the original present continuous tense ("am eating").
 - Mangoes are eaten by me.
 - This option uses simple present tense ("are eaten"), which again, is not an accurate conversion of the original present continuous tense ("am eating").

Question 33

In the following sentences, choose the correct verb which agrees with the subject.

Many a soldier_____ killed in the war.

Options:

A. have

B. was

C. did

D. are

Answer: B

Solution:

The correct answer is **'was'**.



- "Many a + singular noun" is a common phrase, and it always takes a singular verb.
- The phrase "many a soldier" is followed by a singular verb because "a soldier" is singular.
- Correct Sentence: "Many a soldierwas killed in the war."

Therefore. the correct answer is'Option 2'.

Additional Information

- **Option 1:** 'have' is incorrect because "many a man" requires a singular verb.
- **Option 3:** 'did' is incorrect as it doesn't fit here. The verb 'did' is used for simple past tense, not present perfect.
- **Option 4:** 'are' is incorrect as it is a plural verb form and doesn't complement the singular subject "many a man".

Question 34

The idiom 'A snake in the grass' means:

Options:

- A. a hidden enemy
- B. a recognizable danger
- C. not a reliable person
- D. unexpected happening

Answer: A

Solution:

The correct answer is a hidden enemy.

🔗 <u>Key Points</u>

- This phrase means 'a secretly faithless friend'.
 For example -Don't trust her, she may seem honest but she's a snake in the grass.
- So the correct answer is "a hidden enemy".
- Hence, **Option 1** is the correct answer.
