Quadratic Equations

Exercise – 4.1

Question 1:

Examine whether the following equations are quadratic equations or not :

1. $x + \frac{1}{x} = 2$, X ≠ 0
2. (x – 2)(x + 3) = 0
3. $2x^2 - \sqrt{5}x + 2 = 0$
4. $\frac{1}{x+1} - \frac{1}{x-1} = 3$ (x ≠ ±1)
5. (2x + 1)(2x – 1)-(4x + 3)(x – 5)
6. $\frac{x-1}{x+1} - \frac{x+1}{x-1} = \frac{2}{3}$ (x ≠ ±1)
7. $(2x + 3)^2 - (3x + 2)^2 = 13$

Question 1(1):

Solution :

$$x + \frac{1}{x} = 2$$

$\therefore x^2 + 1 = 2x$

$\therefore x^2 - 2x + 1 = 0$

Comparing $x^2 - 2x + 1 = 0$ with the standard quadratic equation

$ax^2 + bx + c = 0$, we get, $a = 1$, $b = -2$, $c = 1$

Here $a = 1 \neq 0$ and $a, b, c \in R$.

Here degree of the polynomial is 2.

$\therefore x^2 - 2x + 1 = 0$ i.e., $x + \frac{1}{x} = 2$ is a quadratic equation.

Question 1(2):

Solution :

$(x - 2)(x + 3) = 0$
$\therefore x^2 + 3x - 2x - 6 = 0$
$\therefore x^2 + x - 6 = 0$
Comparing $x^2 + x - 6 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$, we get, $a = 1$, $b = 1$, $c = -6$
It is clear that $a = 1 \neq 0$ and $a, b, c \in R$.
Here degree of the polynomial $x^2 + x - 6$ is 2.
$\therefore x^2 + x - 6 = 0$ i.e., $(x - 2)(x + 3) = 0$ is a quadratic equation.

Question 1(3):

Solution :

Comparing $2x^2 - \sqrt{5}x + 2 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$,
we get, $a = 2$, $b = -\sqrt{5}$, $c = 2$.
Here $a = 2 \neq 0$ and $a, b, c \in R$.
Here degree of the polynomial $2x^2 - \sqrt{5}x + 2$ is 2.
$\therefore 2x^2 - \sqrt{5}x + 2 = 0$ is a quadratic equation.

Question 1(4):

Solution :

$$\frac{1}{x+1} - \frac{1}{x-1} = 3 (x \neq \pm 1)$$
$$\Rightarrow \frac{x - 1 - (x + 1)}{(x + 1)(x - 1)} = 3$$
$$\Rightarrow \frac{x - 1 - x - 1}{x^2 - 1} = 3$$
$$\Rightarrow -2 = 3x^2 - 3$$
$$\Rightarrow 3x^2 - 1 = 0$$
Comparing $3x^2 - 1 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$,
we get $a = 3$, $b = 0$, $c = -1$
It is clear that $a = 3 \neq 0$, $a, b, c \in R$.
Here degree of the polynomial $3x^2 - 1$ is 2.
$\therefore 3x^2 - 1 = 0$ i.e., $\frac{1}{x+1} - \frac{1}{x-1} = 3$ is a quadratic equation.

Question 1(5):

Solution :

For $(2x + 1)(2x – 1) = (4x + 3)(x – 5)$

$\Rightarrow 4x^2 – 2x + 2x – 1 = 4x^2 – 20x + 3x – 15$

$\Rightarrow -1 = -17x – 15$

$\Rightarrow 17x + 14 = 0$

Comparing $17x + 14 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$,

We get, $a = 0$

But for any quadratic equation $a \neq 0$.

$\Rightarrow 17x + 14 = 0$ i.e., the given equation is not a quadratic equation.

Question 1(6):

Solution :

$\frac{x-1}{x+1} - \frac{x+1}{x-1} = \frac{2}{3} (x \neq \pm 1)$

$\Rightarrow \frac{x^2 - 2x + 1 - x^2 - 2x - 1}{x^2 - 1} = \frac{2}{3}$

$\Rightarrow \frac{-4x}{x^2 - 1} = \frac{2}{3}$

$\Rightarrow -6x = x^2 - 1$

$\Rightarrow x^2 + 6x - 1 = 0$

Comparing $x^2 + 6x - 1 = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$, we get,

$a = 1, b = 6, c = -1$

Here $a = 1 \neq 0$ and $a, b, c \in R$.

Here degree of the polynomial $x^2 + 6x - 1$ is 2.

$\therefore x^2 + 6x - 1 = 0$ i.e., the given equation is a quadratic equation.

Question 1(7):

Solution :

$(2x+3)^2 – (3x+2)^2=13$

$\therefore 4x^2 + 12x + 9 – (9x^2 + 12x + 4) = 13$

$\therefore -5x^2 + 5 = 13$

$\therefore 5x^2 + 8 = 0$

Comparing $5x^2+8=0$ with the standard quadratic equation $ax^2+bx+c=0$, we get, $a = 5, b = 0, c = 8$

Here $a = 5 \neq 0$ and $a, b, c \in R$.

Here degree of the polynomial $5x^2 + 8$ is 2.

$\therefore 5x^2 + 8 = 0$ i.e., the given equation is a quadratic equation.

Question 2:

Verify whether the given value of x is a solution of the quadratic equation or not :

1. $X^2 - 3x + 2 = 0$, x = 2
2. $X^2 + x - 2 = 0$, x = 2
3. $\frac{1}{3x+1} - \frac{1}{2x-1} + \frac{3}{4} = 0$, x = 1[x ≠ $\frac{1}{2}$, $-\frac{1}{3}$]
4. (3x – 8)(2x + 5) = 0, x = $-\frac{5}{2}$

Question 2(1):

Solution :

Let $p(x) = x^2 - 3x + 2$.
Substituting x = 2 in p(x).
$p(2) = (2)^2 - 3(2) + 2$
= 4 – 6 + 2
= 0
∴ x = 2 is the zero of the polynomial $x^2 - 3x + 2$.
∴ One of the solutions of p(x) is x = 2.

Question 2(2):

Solution :

Let $p(x) = x^2 + x - 2$
Substituting x = 2 in p(x)
$P(2) = (2)^2 + 2 - 2$
= 4 + 2 – 2
= 4 ≠ 0
∴ x = 2 is not the zero of the polynomial $x^2 + x - 2$.
∴ x = 2 is not a solution of $x^2 + x - 2 = 0$.

Question 2(3):

Solution :

$$\frac{1}{3x+1} - \frac{1}{2x-1} + \frac{3}{4} = 0$$
$$\frac{1}{3x+1} - \frac{1}{2x-1} + \frac{3}{4} = 0$$
$\Rightarrow 4(2x-1) - 4(3x+1) + 3(3x+1)(2x-1) = 0$
$\therefore 8x - 4 - 12x - 4 + 3(6x^2 - 3x + 2x - 1) = 0$
$\therefore -4x - 8 + 18x^2 - 3x - 3 = 0$
$\therefore 18x^2 - 7x - 11 = 0$
Let $p(x) = 18x^2 - 7x - 11$
Substituting x = 1 in the p(x),
$p(1) = 18(1)^2 - 7(1) - 11$
= 18 – 7 – 11
= 0
∴ x=1 is the zero of the polynomial $18x^2 - 7x - 11$
∴ One of the solutions of $\frac{1}{3x+1} - \frac{1}{2x-1} + \frac{3}{4} = 0$ is x = 1.

Question 2(4):

Solution :

$(3x-8)(2x+5)=0$

$\therefore 6x^2 + 15x - 16x - 40 = 0$

$\therefore 6x^2 - x - 40 = 0$

Let $p(x) = 6x^2 - x - 40 = 0$

Substituting $x = -\frac{5}{2}$ in $p(x)$,

$6\left(-\frac{5}{2}\right)^2 - \left(-\frac{5}{2}\right) - 40 = \frac{150}{4} + \frac{5}{2} - 40 = \frac{150+10-160}{4} = 0$

$x = -\frac{5}{2}$ is the zero of the polynomial $6x^2 - x - 40$.

$\therefore$ One of the solutions of $(3x-8)(2x+5) = 0$ is $x = -\frac{5}{2}$.

Question 3:

1. If $x = 1$ is a root of $ax^2 + bx + c = 0$, $a \neq 0$, $a, b, c \in R$, prove that $a + b + c = 0$.
2. If $x = -1$ is a root of $x^2 - px + q = 0$, $p, q \in R$, prove that $p + q + 1 = 0$.
3. Find k, if one of the roots of $x^2 - kx + 6 = 0$ is 3.
4. Find k, if one of the roots of $x^2 + 3(k+2)x - 9 = 0$ is -3.

Question 3(1):

Solution :

$x = 1$ is given as the solution of $ax^2 + bx + c = 0$

$\therefore p(1) = 0$

$\therefore a(1)^2 + b(1) + c = 0$

$\therefore a + b + c = 0$

If $x = 1$ is the solution of $ax^2 + bx + c = 0$, then $a + b + c = 0$.

Question 3(2):
Solution :

$x = -1$ is given as the solution of $x^2 - px + q = 0$.
$\therefore p(-1) = 0$.
$\therefore (-1)^2 - p(-1) + q = 0$
$\therefore 1 + p + q = 0$
$\therefore p + q + 1 = 0$
Thus, if $x = -1$ is a root of the equation $x^2 - px + q = 0$, then $p + q + 1 = 0$.

Question 3(3):

Solution :

One of the roots of $x^2 - kx + 6 = 0$ is given as 3.
$\therefore p(3) = 0$.
$\therefore (3)^2 - k(3) + 6 = 0$
$\therefore 9 - 3k + 6 = 0$
$\therefore 15 - 3k = 0$
$\therefore 3k = 15$
$\therefore k = 5$

Question 3(4):

Solution :

One of the roots of $x^2 + 3(k+2)x - 9 = 0$ is given as 3
$\therefore p(-3) = 0$,
$\therefore (-3)^2 + 3(k + 2)(-3) - 9 = 0$
$\therefore 9 - 9k - 18 - 9 = 0$
$\therefore -9k - 18 = 0$
$\therefore 9k = -18$
$\therefore k = -2$

Question 4:

Solve the following equations using the method of factorization :

1. $27x^2 - 48 = o$
2. $(x - 7)^2 - 16 = 0$
3. $6x^2 + 13x + 6 = 0$
4. $15x^2 - 16x + 1 = O$
5. $\sqrt{5}x^2 - 4x - \sqrt{5} = 0$
6. $x + \frac{1}{x} = 2\frac{1}{6}$

Question 4(1):

Solution :

Here $27x^2 - 48 = 0$

$\therefore\ 9x^2 - 16 = 0 \quad (\because \text{ Dividing by } 3)$

$\therefore\ (3x)^2 - (4)^2 = 0 \quad (\because\ a^2 - b^2 = (a-b)(a+b))$

$\therefore\ (3x + 4)(3x - 4) = 0$

$\therefore\ 3x + 4 = 0 \quad \text{or} \quad 3x - 4 = 0$

$\therefore\ x = -\frac{4}{3} \text{ or } x = \frac{4}{3}$

$\therefore$ Roots of the given equation are $-\frac{4}{3}$ and $\frac{4}{3}$.

Question 4(2):

Solution :

Here $(x - 7)^2 - 16 = 0$

$\therefore (x - 7)^2 - (4)^2 = 0$

$\therefore (x - 7 - 4)(x - 7 + 4) = 0$

$\therefore (x - 11)(x - 3) = 0$

$\therefore x - 11 = 0$ or $x - 3 = 0$

$\therefore x = 11$ or $x = 3$

$\therefore$ Roots of the given equation are 3 and 11.

Question 4(3):

Solution :

$6x^2 + 13x + 6 = 0$

$\therefore\ 6x^2 + 9x + 4x + 6 = 0 \quad [\because 36 = 9 \times 4]$

$\therefore\ 3x(2x + 3) + 2(2x + 3) = 0$

$\therefore\ (2x + 3)(3x + 2) = 0$

$\therefore\ 2x + 3 = 0 \text{ or } 3x + 2 = 0$

$\therefore\ x = -\frac{3}{2} \text{ or } x = -\frac{2}{3}$

$\therefore$ Roots of the given equation are $-\frac{3}{2}$ and $-\frac{2}{3}$.

Question 4(4):

Solution :

Here $15x^2 - 16x + 1 = 0$

$\therefore\ 15x^2 - 15x - x + 1 = 0 \quad [\because 15 = 15 \times 1]$

$\therefore\ 15x(x - 1) - 1(x - 1) = 0$

$\therefore\ (15x - 1)(x - 1) = 0$

$\therefore\ 15x - 1 = 0 \text{ or } x - 1 = 0$

$\therefore\ x = \frac{1}{15} \text{ or } x = 1$

The roots of the given equation are $\frac{1}{15}$ and 1.

Question 4(5):

Solution :

$$\sqrt{5}x^2 - 4x - \sqrt{5} = 0$$
$$\therefore \sqrt{5}x^2 - 5x + x - \sqrt{5} = 0 \qquad [\because \sqrt{5} \times \sqrt{5} = 5]$$
$$\therefore \sqrt{5}x\left(x - \sqrt{5}\right) + 1\left(x - \sqrt{5}\right) = 0$$
$$\therefore x - \sqrt{5} = 0 \text{ or } \sqrt{5}x + 1 = 0$$
$$\therefore x - \sqrt{5} \text{ or } x = -\frac{1}{\sqrt{5}}$$

The roots of the given equation are $\sqrt{5}$ and $-\frac{1}{\sqrt{5}}$.

Question 4(6):

Solution :

$$x + \frac{1}{x} = 2\frac{1}{6}$$
$$\therefore x + \frac{1}{x} = \frac{13}{6}$$
$$\therefore \frac{x^2 + 1}{x} = \frac{13}{6}$$
$$\therefore 6x^2 + 6 = 13x$$
$$\therefore 6x^2 - 13x + 6 = 0$$
$$\therefore 6x^2 - 9x - 4x + 6 = 0 \qquad [\because 36 = 9 \times 4]$$
$$\therefore 3x(2x - 3) - 2(2x - 3) = 0$$
$$\therefore (2x - 3)(3x - 2) = 0$$
$$\therefore 2x - 3 = 0 \text{ or } 3x - 2 = 0$$
$$\therefore x = \frac{3}{2} \text{ or } x = \frac{2}{3}$$

The roots of the given equation are $\frac{3}{2}$ and $\frac{2}{3}$.

Exercise – 4.2

Question 1:

Find the discriminant of the following quadratic equations and discuss the nature of the roots :

1. $6x^2 - 13x + 6 = 0$
2. $\sqrt{6}x^2 - 5x + \sqrt{6} = 0$
3. $24x^2 - 17x + 3 = 0$
4. $X^2 + 2x + 4 = 0$
5. $X^2 + x + 1 = 0$
6. $X^2 - 3\sqrt{3}x - 30 = 0$

Question 1(1):

Solution :

From equation $6x^2 - 13x + 6 = 0$, we get,

$a = 6$, $b = -13$, $c = 6$

$\therefore D = b^2 - 4ac = (-13)^2 - 4(6)(6)$

$= 169 - 144$

$= 25 > 0$

Here, D > 0 and it is a perfect square.

Also, a, b, c ∈ Q, so the two roots of the quadratic equation are rational and distinct.

Question 1(2):

Solution :

From equation $\sqrt{6}x^2 - 5x + \sqrt{6} = 0$, we get,

$a = \sqrt{6},\ b = -5,\ c = \sqrt{6}$

$\therefore D = b^2 - 4ac = (-5)^2 - 4(\sqrt{6})(\sqrt{6})$

$= 25 - 24$

$= 1 > 0$

Here, D > 0 and it is a perfect square.

Also a, b, c ∈ Q, so the two roots of the quadratic equation are real.

Question 1(3):

Solution :

From the equation $24x^2 - 17x + 3 = 0$ we get,

a = 24, b = -17, c = 3

$\therefore D = b^2 - 4ac$

$= (-17)^2 - 4(24)(3)$

$= 289 - 288$

$= 1 > 0$

Here, D > 0 and it is a perfect square.

Also a, b, c ∈ Q, so the two roots of the quadratic equation are rational and distinct.

Question 1(4):

Solution :

From the equation $x^2 + 2x + 4 = 0$, we get,

a = 1, b = 2, c = 4

$\therefore D = b^2 - 4ac$

$= (2)^2 - 4(1)(4)$

$= 4 - 16$

$= -12$

$\therefore D < 0$

Here D < 0, so the quadratic equation has no real roots.

Question 1(5):

Solution :

From the equation $x^2 + x + 1 = 0$, we get,

a = 1, b = 1, c = 1

$\therefore D = b^2 - 4ac = (1)^2 - 4(1)(1)$

$= 1 - 4$

$= -3 < 0$

$\therefore D < 0$

Here $D < 0$, so the quadratic equation has no real roots.

Question 1(6):

Solution :

From the equation $x^2 - 3\sqrt{3}x - 30 = 0$ we get,

$a = 1,\ b = -3\sqrt{3},\ c = -30$

$\therefore D = b^2 - 4ac = (-3\sqrt{3})^2 - 4(1)(-30)$

$= 27 + 120$

$= 147 > 0$

Here, $D > 0$ and it is not a perfect square.

$\therefore$ The given quadratic equation has two real distinct roots.

Question 2:

If $a, b, c \in R$, $a > 0$, $c < 0$, then prove that the roots of $ax^2 + bx + c = 0$ are real and distinct.

Solution :

Here, a, b, and $c \in R$ and $a > 0$, $c < 0$

So, $ac < 0$

$\therefore 4ac < 0$

$\therefore -4ac > 0$

$\therefore b^2 - 4ac > 0 \quad (\because b^2 \geq 0)$

$\therefore D > 0$

As $D > 0$ and a, b and $c \in R$, the roots of the quadratic equation are real and distinct.

Question 3:

Find k, if the roots of $x^2 - (3k - 2)x + 2k = 0$ are equal and real.

If the roots of the quadratic equation $(k + 1)x^2 - 2(k - 1)x + 1 = O$ are real and equal, find the value of k.

Question 3(1):

Solution :

From the equation $x^2 - (3k - 2)x + 2k = 0$
we get, $a = 1$, $b = -(3k - 2)$, $c = 2k$
We are given that the roots of the given equation are real and equal.
$\therefore D = 0$
$\therefore b^2 - 4ac = 0$
$\therefore [-(3k-2)]^2 - 4(1)(2k) = 0$
$\therefore 9k^2 - 12k + 4 - 8k = 0$
$\therefore 9k^2 - 20k + 4 = 0$
$\therefore 9k^2 - 18k - 2k + 4 = 0$
$\therefore 9k(k-2) - 2(k-2) = 0$
$\therefore (k-2)(9k-2) = 0$
$\therefore k - 2 = 0$ or $9k - 2 = 0$
$\therefore k = 2$ or $k = \frac{2}{9}$
$\therefore 2$ or $\frac{2}{9}$ are the values of k.

Question 3(2):

Solution :

From the equation $(k + 1)x^2 - 2(k - 1)x + 1 = 0$, we get, $a = k + 1$, $b = -2(k - 1)$, $c = 1$
Now, the roots of the given quadratic equation are real and equal.
$\therefore D = 0$
$\therefore b^2 - 4ac = 0$
$\therefore 4(k - 1)^2 - 4(k + 1)(1) = 0$
$\therefore 4k^2 - 12k = 0$
$\therefore k^2 - 3k = 0$ [Taking 4 common]
$\therefore k(k - 3) = 0$
$\therefore k = 0$ or $k = 3$
$\therefore$ 0 or 3 are the values of k.

Question 4:

If the roots of $ax^2 + 2bx + c = 0$, $a \neq 0$, $a, b, c \in R$ are real and equal, then prove that $a : c = b : c$.

Solution :

Comparing the equation $ax^2 + 2bx + c = 0$ with $Ax^2 + Bx + C = 0$, we get,
$A = a$, $B = 2b$, $C = c$
We are given the roots of the given quadratic equation are real and equal.
$\therefore D = 0$
$\therefore B^2 - 4AC = 0$
$\therefore (2b)^2 - 4(a)(c) = 0$
$\therefore 4b^2 - 4ac = 0$
$\therefore ac = b^2$
$\therefore \frac{a}{b} = \frac{b}{c}$
$\therefore a : b = b : c$
So if the roots of equation $ax^2 + 2bx + c = 0$ are real and equal, then $a : b = b : c$

Question 5:

Solve the following equations using the general formula :

1. $x^2 + 10x + 6 = 0$
2. $x^2 + 5x – 1 = 0$
3. $x^2 – 3x – 2 = 0$
4. $x^2 – 3\sqrt{6}x + 12 = 0$
5. $3x^2 + 5\sqrt{5}x + 2 = 0$
6. $\frac{x2-1}{x2+1} = \frac{4}{5}$

Question 5(1):

Solution :

For the equation $x^2 + 10x + 6 = 0$, we get,

$a = 1, b = 10, c = 6$

$\therefore D = b^2 - 4ac = (10)2 - 4(1)(6)$

$= 100 - 24$

$= 76$

Here, $D > 0$, so the equation has real and distinct roots.

Then,

$\alpha = \frac{-b + \sqrt{D}}{2a}$ $\quad \beta = \frac{-b - \sqrt{D}}{2a}$

$= \frac{-10 + \sqrt{76}}{2(1)}$ $\quad = \frac{-10 - \sqrt{76}}{2(1)}$

$= \frac{-10 + 2\sqrt{19}}{2}$ $\quad = \frac{-10 - 2\sqrt{19}}{2}$

$= -5 + \sqrt{19}$ $\quad = -5 - \sqrt{19}$

$\therefore$ Roots of the given equation are $-5 + \sqrt{19}$ and $-5 - \sqrt{19}$.

Question 5(2):

Solution :

For the equation, $x^2 + 5x - 1 = 0$ with $ax^2 + bx + c = 0$, we get,

$a = 1, b = 5, c = -1$

$\therefore D = b^2 - 4ac = (5)2 - 4(1)(-1)$

$= 25 + 4$

$= 29$

Here, $D > 0$, so the equation has real and distinct roots.

Then,

$\alpha = \frac{-b + \sqrt{D}}{2a}$ $\quad \beta = \frac{-b - \sqrt{D}}{2a}$

$= \frac{-5 + \sqrt{29}}{2(1)}$ $\quad = \frac{-5 - \sqrt{29}}{2(1)}$

$= \frac{-5 + \sqrt{29}}{2}$ $\quad = \frac{-5 - \sqrt{29}}{2}$

$\therefore$ Roots of the given equation are $\frac{-5 - \sqrt{29}}{2}$ and $\frac{-5 - \sqrt{29}}{2}$

Question 5(3):

Solution :

For the equation $x^2 - 3x - 2 = 0$, we get,

$a = 1,\ b = -3,\ c = 0$

$\therefore D = b^2 - 4ac = (-3)2 - 4(1)(-2)$

$= 9 + 8$

$= 17$

Here, $D > 0$, so the equation has real and distinct roots.

Then,

$\alpha = \dfrac{-b + \sqrt{D}}{2a}$	$\beta = \dfrac{-b - \sqrt{D}}{2a}$
$= \dfrac{3 + \sqrt{17}}{2(1)}$	$= \dfrac{3 - \sqrt{17}}{2(1)}$
$= \dfrac{3 + \sqrt{17}}{2}$	$= \dfrac{3 - \sqrt{17}}{2}$

$\therefore$ Roots of the given equation are $\dfrac{3 + \sqrt{17}}{2}$ and $\dfrac{3 - \sqrt{17}}{2}$

Question 5(4):

Solution :

For the equation $x^2 - 3\sqrt{6}x + 12 = 0$, we get,

$a = 1,\ b = -3\ ,\ c = 12$

$D = b^2 - 4ac = (-3\sqrt{6})^2 - 4(1)(12)$

$= 54 - 48$

$= 6$

Here, $D > 0$, so the equation has real and distinct roots.

Then,

$\alpha = \dfrac{-b + \sqrt{D}}{2a}$	$\beta = \dfrac{-b - \sqrt{D}}{2a}$
$= \dfrac{-\left(-3\sqrt{6}\right) + \sqrt{6}}{2(1)}$	$= \dfrac{-\left(-3\sqrt{6}\right) - \sqrt{6}}{2(1)}$
$= \dfrac{3\sqrt{6} + \sqrt{6}}{2}$	$= \dfrac{3\sqrt{6} - \sqrt{6}}{2}$
$= \dfrac{4\sqrt{6}}{2}$	$= \dfrac{2\sqrt{6}}{2}$
$= 2\sqrt{6}$	$= \sqrt{6}$

$\therefore$ Roots of the given equation are $2\sqrt{6}$ and $\sqrt{6}$

Question 5(5):

Solution :

For the equation $3x^2 + 5\sqrt{2}x + 2 = 0$, we get,

$a = 3,\ b = 5\sqrt{2},\ c = 2$

$\therefore D = b^2 - 4ac = (5\sqrt{2})^2 - 4(3)(2)$

$= 50 - 24$

$= 26$

Here, $D > 0$, so the equation has real and distinct roots.

Now,

$\alpha = \dfrac{-b + \sqrt{D}}{2a}$	$\beta = \dfrac{-b - \sqrt{D}}{2a}$
$= \dfrac{-5\sqrt{2} + \sqrt{26}}{2(3)}$	$= \dfrac{-5\sqrt{2} - \sqrt{26}}{2(3)}$
$= \dfrac{-5\sqrt{2} + \sqrt{26}}{6}$	$= \dfrac{-5\sqrt{2} - \sqrt{26}}{6}$

$\therefore$ Roots of the given equation are $\dfrac{-5\sqrt{2} + \sqrt{26}}{6}$ and $\dfrac{-5\sqrt{2} - \sqrt{26}}{6}$

Question 5(6):

Solution :

$\dfrac{x^2 - 1}{x^2 + 1} = \dfrac{4}{5}$

$\therefore 5x^2 - 5 = 4x^2 + 4$

$\therefore x^2 - 9 = 0$

For equation $x^2 - 9 = 0$, we get, $a = 1,\ b = 0,\ c = -9$

$D = b^2 - 4ac = (0)^2 - 4(1)(-9)$

$= 0 + 36$

$= 36$

As $D > 0$, the equation has real and distinct roots.

Then,

$\alpha = \dfrac{-b + \sqrt{D}}{2a}$	$\beta = \dfrac{-b - \sqrt{D}}{2a}$
$= \dfrac{-(0) + \sqrt{36}}{2(1)}$	$= \dfrac{-(0) - \sqrt{36}}{2(1)}$
$= \dfrac{6}{2}$	$= -\dfrac{6}{2}$
$=3$	$= -3$

$\therefore$ Roots of the given equation are -3 and 3.

Exercise – 4.3

Question 1:

Find two numbers whose sum is 27 and the product is 182.

Solution :

Let the first number be x.

$\therefore$ Second number $= \frac{182}{x}$. ($\because$ product of numbers = 182)

Next,

Sum of these two numbers is given as 27.

$\therefore x + \frac{182}{x} = 27$

$\therefore x^2+182 = 27x$

$\therefore x^2 - 27x + 182 = 0$

$\therefore x^2 - 14x - 13x + 182 = 0$

$\therefore x(x - 14) - 13(x - 14) = 0$

$\therefore (x - 14)(x - 13) = 0$

$\therefore x - 14 = 0$ or $x - 13 = 0$

$\therefore x = 14$ or $x = 13$

Thus, if $x = 14$, then other number is $\frac{182}{14} = 13$.

If $x = 13$, then other number is $\frac{182}{13} = 14$.

Thus, the required two numbers are 13 and 14.

Question 2:

Find two consecutive natural numbers, sum of whose squares is 365.

Solution :

Let the two consecutive natural numbers be x and x + 1.

Sum of their squares is given as 365.

$\therefore (x)^2 + (x + 1)^2 = 365$

$\therefore x^2 + x^2 + 2x + 1 = 365$

$\therefore 2x^2 + 2x - 362 = 0$

$\therefore x^2 + x - 182 = 0$ [$\because$ Taking 2 common]

$\therefore x^2 + 14x - 13x - 182 = 0$

$\therefore x(x + 14) - 13(x + 14) = 0$

$\therefore (x + 14)(x - 13) = 0$

$\therefore x = -14$ or $x = 13$

x cannot be negative as it is a natural number.

$\therefore x = 13$

Other natural number = x + 1 = 13 + 1 = 14

Thus, the two consecutive natural numbers are 13 and 14.

Question 3:

The sum of ages of two friends is 20 years. Four years ago the product of their ages was 48. Show that these statements can not be true.

Solution :

Let the present age of the first friend = x years.

Now,

Sum of ages of both friends is given 20

$\therefore$ Age of second friend = (20 – x) years.

Before four years,

Age of first friend = $(x – 4)$ years

Age of second friend = $(20 – x) – 4 = (16 – x)$ years.

Now, four years ago the product of the numbers showing their ages was 48.

$\therefore (x – 4)(16 – x) = 48$

$\therefore 16x – x^2 – 64 + 4x = 48$

$\therefore x^2 – 20x + 112 = 0$

Comparing the equation $x^2 – 20x + 112 = 0$ with $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$, $c = 112$

$\therefore D = b^2 – 4ac=(-20)^2 – 4(1)(112)$

$= 400 – 488 = -48 < 0$

As $D < 0$, so the roots are not real.

But age can be represented only by a positive number.

$\therefore$ The given statement cannot be true.

Question 4:

A rectangular garden is designed such that the length of the garden is twice its breadth and the area of the garden is 800 m^2. Find the length of the garden.

Solution :

Let the breadth of the rectangular garden = x metres.

Given, the length of the garden is twice the breadth.

i.e. length = 2(breadth) = 2x metres

Area of a rectangular garden is given as 800 m^2

Now,

Area = length × breadth

$\therefore 800 = (2x)\times x$

$\therefore x^2 = 400$

$\therefore x = 20$ ($\because x = -20$ is not possible.)

Thus, length of the garden = 2x = 2 × 20m = 40 metres.

Question 5:

Perimeter of a rectangular garden is 360 m and its area is 8000 m^2. Find the length of the garden and also find its breadth. (The length is greater than the breadth)

Solution :

Let the length of the rectangular garden = x metres

Also, Area of the garden is 8000 m^2

$\therefore$ Breadth of the garden = $\frac{8000}{x}$ m ($\because$ area = length × breadth)

Here, perimeter of a rectangular garden is given as 360 m.

$\therefore$ 2(length + breadth) = 360

$\therefore 2\left(x + \frac{8000}{x}\right) = 360$

$\therefore x + \frac{8000}{x} = 180$

$\therefore x^2 - 180x + 8000 = 0$

$\therefore x^2 - 100x - 80x + 8000 = 0$

$\therefore x(x - 100) - 80(x - 100) = 0$

$\therefore (x - 100)(x - 80) = 0$

$\therefore x - 100 = 0$ or $x - 80 = 0$

$\therefore x = 100$ or $x = 80$

Now,

For $x = 100$, breadth = $\frac{8000}{100} = 80$

For $x = 80$, breadth = $\frac{8000}{80} = 100$

But, according to given, length is greater than the breadth.

$\therefore$ Length of the rectangular garden is 100 metres and its breadth is 80 metres.

Question 6:

If a cyclist travels at a speed 2 km/hr more than his usual speed, he reaches the destination 2 hours earlier. If the destination is 35 km away, what is the usual speed of the cyclist ?

Solution :

Let the usual speed of the cyclist be x km/hr.

$\therefore$ Time taken to travel 35 km = $t_1 = \frac{35}{x}$ hours $\left(\because \text{Time} = \frac{\text{dist.}}{\text{speed}}\right)$ ... ... (1)

If the cyclist increases his speed by 2 km/hr, then

New speed =(x + 2) km/hr.

$\therefore$ Time taken to travel the same distance with new speed = $t_2 = \frac{35}{x+2}$ hours ... ... (2)

But, the cyclist reaches the destination 2 hours earlier with his new speed.

$t_1 - t_2 = 2$

$\therefore \frac{35}{x} - \frac{35}{x+2} = 2$ [$\because$ From the equations (1) and (2)]

$\therefore 35(x+2) - 35x = 2x^2(x+2)$ [Multiplying both the sides by x(x+2)]

$\therefore 35x + 70 - 35x = 2x^2 + 4x$

$\therefore 2x^2 + 4x - 70 = 0$

$\therefore x^2 + 2x - 35 = 0$

$\therefore x^2 + 7x - 5x - 35 = 0$

$\therefore x(x + 7) - 5(x + 7) = 0$

$\therefore (x + 7)(x - 5) = 0$

$\therefore x + 7 = 0$ or $x - 5 = 0$

$\therefore x = -7$ or $x = 5$

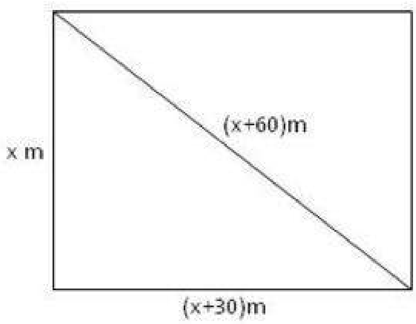
$\therefore x = 5$ ($\because$ speed cannot be negative.)

Thus, the usual speed of the cyclist is 5 km/hr.

Question 7:

The diagonal of a rectangular ground is 60 meters more than the breadth of the ground. If the length of the ground is 30 meters more than the breadth, find the area of the ground.

Solution :



Let the breadth of the rectangular ground be x metres.

Length of the ground is 30 m more than the breadth, so length = (x + 30) m.

Now, the length of the diagonal of the rectangular ground is 60 m more than its breadth,

Length of the diagonal = (x + 60) m.

For any rectangle,

$(\text{breadth})^2 + (\text{length})^2 = (\text{diagonal})^2$ ($\because$ Pythagoras' thm)

$\therefore x^2 + (x + 30)^2 = (x + 60)^2$

$\therefore x^2 + x^2 + 60x + 900 = x^2 + 120x + 3600$

$\therefore x^2 - 60x - 2700 = 0$

$\therefore x^2 - 90x + 30x - 2700 = 0$

$\therefore x(x - 90) + 30(x - 90) = 0$

$\therefore (x - 90)(x + 30) = 0$

$\therefore x - 90 = 0$ or $x + 30 = 0$

$\therefore x = 90$ or $x = -30$

$\therefore x = 90$ ($\because$ breadth cannot be negative.)

$\therefore$ Breadth of the rectangle = x = 90 m

$\therefore$ Length of the rectangle

= (x + 30) m

= (90 + 30) m

= 120 m

Now, area of the rectangular ground

= length × breadth

= (120)×(90)

= $10800\ m^2$

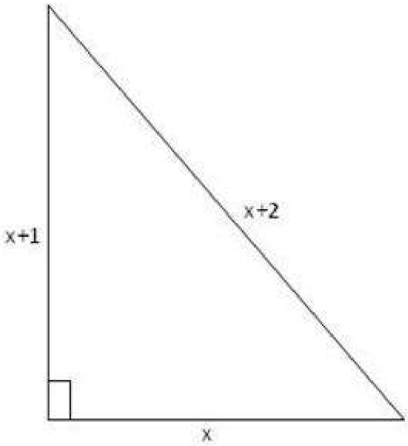
$\therefore$ Area of the ground is $10,800 m^2$.

Question 8:

The sides of a right angled triangle are consecutive positive integers. Find the area of the triangle.

Solution :

Let the consecutive positive integers of the sides of a right angled triangle be x, x + 1 and x + 2.



The size of largest measure x + 2 is taken as the hypotenuse.
By Pythagoras theorem,
$(x)^2 + (x+1)^2 = (x+2)^2$
$\therefore x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$
$\therefore x^2 - 2x - 3 = 0$
$\therefore x^2 - 3x + x - 3 = 0$
$\therefore x(x-3) + 1(x-3) = 0$
$\therefore (x-3)(x+1) = 0$
$\therefore x - 3 = 0$ or $x + 1 = 0$
$\therefore x - 3 = 0$ or $x + 1 = 0$
$\therefore x = 3$ or $x = -1$
$\therefore x = 3$ ($\because$ x is a positive integer.)
Now, the area of right angled triangle
$= \frac{1}{2}$base $\times$ altitude
$= \frac{1}{2}x(x+1)$
$= \frac{1}{2}(3)(3+1)$
$= \frac{1}{2}(12)$
$= 6$
Thus, the area of the triangle is 6 square units.

Exercise – 4

Question 1:

Solve the following quadratic equations using factorization :

1. $x^2 - 12 = 0$
2. $x^2 - 7x - 60 = 0$
3. $x^2 - 15x + 56 = 0$
4. $\frac{2x+3}{2x-3} + \frac{2x-3}{2x+3} = \frac{17}{4}$, $x \neq \pm\frac{3}{2}$
5. $\frac{1}{x+5} + \frac{3}{4(3x+1)} = \frac{1}{x+2}$, $x \neq -5$, $x \neq -2$, $x \neq -\frac{1}{3}$

Question 1(1):

Solution :

For $x^2 - 12 = 0$

$\therefore (x)^2 - (2\sqrt{3})^2 = 0$

$\therefore (x + 2\sqrt{3})(x - 2\sqrt{3}) = 0$

$\therefore x + 2\sqrt{3} = 0$ or $x - 2\sqrt{3} = 0$

$\therefore x = -2\sqrt{3}$ or $x = 2\sqrt{3}$

$\therefore$ Roots of the given equation are $-2\sqrt{3}$ and $2\sqrt{3}$.

Question 1(2):

Solution :

For $x^2 - 7x - 60 = 0$

$\therefore x^2 - 5x - 12x - 60 = 0$

$\therefore x(x + 5) - 12(x + 5) = 0$

$\therefore (x + 5)(x - 12) = 0$

$\therefore x + 5 = 0$ or $x - 12 = 0$

$\therefore x = 5$ or $x = 12$

$\therefore$ Roots of the given equation are 5 and 12.

Question 1(3):

Solution :

$x^2 - 15x + 56 = 0$

$\therefore x^2 - 7x - 8x + 56 = 0$

$\therefore x(x - 7) - 8(x - 7) = 0$

$\therefore (x - 7)(x - 8) = 0$

$\therefore x - 7 = 0$ or $x - 8 = 0$

$\therefore x = 7$ or $x = 8$

$\therefore$ Roots of the given equation are 7 and 8.

Question 1(4):

Solution :

$$\frac{2x+3}{2x-3} + \frac{2x-3}{2x+3} = \frac{17}{4}$$

Taking $(2x - 3)(2x+3)$ as the l.c.m for L.H.S.

$$\therefore \frac{(2x+3)^2 + (2x-3)^2}{(2x-3)(2x+3)} = \frac{17}{4}$$

$$\therefore (2x+3)^2 + (2x-3)^2 = \frac{17}{4}(2x-3)(2x+3)$$

$\therefore 4[4x^2 + 12x + 9 + 4x^2 - 12x + 9] = 17(4x^2 - 9)$

$\therefore 4(8x^2 + 18) = 68x^2 - 153$

$\therefore 32x^2 + 72 = 68x^2 - 153$

$\therefore 36x^2 - 225 = 0$

$\therefore 4x^2 - 25 = 0$ $\quad$ ($\because$ Dividing by 9)

$\therefore (2x)^2 - (5)^2 = 0$

$\therefore (2x + 5)(2x - 5) = 0$

$\therefore 2x + 5 = 0$ or $2x - 5 = 0$

$$\therefore x = -\frac{5}{2} \text{ or } x = \frac{5}{2}$$

The roots of the given equation are $-\frac{5}{2}$ and $\frac{5}{2}$.

Question 1(5):

Solution :

$$\frac{1}{x+5}+\frac{3}{4(3x+1)}=\frac{1}{x+2}$$

Take $4(x+5)(3x+1)$ as L.C.M. of the L.H.S.

$$\frac{4(3x+1)+3(x+5)}{4(3x+1)(x+5)}=\frac{1}{x+2}$$

$\therefore 4(3x+1)(x+2)+3(x+5)(x+2)=4(3x+1)(x+5)$

$\therefore 4(3x^2+7x+2)+3(x^2+7x+16x+5)$

$\therefore 12x^2+28x+8+3x^2+21x+30=12x^2+64x+20$

$\therefore 3x^2-15x+18=0$

$\therefore x^2-5x+6=0$

$\therefore x^2-2x-3x+6=0$

$\therefore x(x-2)-3(x-2)=0$

$\therefore (x-2)(x-3)=0$

$\therefore x-2=0$ or $x-3=0$

$\therefore x=2$ or $x=3$

$\therefore$ Roots of the given equation are 2 and 3.

Question 2:

Find the roots of the following equations by the method of perfect square :

1. $x^2 - 24x - 16 = 0$
2. $3x^2 + 7x - 20 = 0$
3. $X^2 - 10x + 25 = 0$
4. $X^2 + (x + 5)^2 = 625$
5. $(x + 2)(x + 3) = 240$

Question 2(1):

Solution :

For $x^2-24x-16=0$, we need to find the third term to make it a perfect square.

Third trem $=\frac{(M.T.)^2}{4\times F.T.}=\frac{576x^2}{4\times x^2}=144$

Then,

$x^2-24x+144-160=0$

$\therefore (x-12)^2-(4\sqrt{10})^2=0$

$\therefore (x-12+4\sqrt{10})(x-12-4\sqrt{10})$

$\therefore x-12+4\sqrt{10}=0$ or $x-12-4\sqrt{10}=0$

$\therefore x=12-4\sqrt{10}$ or $x=12+4\sqrt{10}$

$\therefore$ Roots of the given equation are $12+4\sqrt{10}$ and $12-4\sqrt{10}$.

Question 2(2):

Solution :

For $3x^2 + 7x - 20 = 0$, multiply it by 12.

$\therefore\ 36x^2 + 84x - 240 = 0$

$\therefore\ 36x^2 + 84x + 49 - 289 = 0 \qquad \left(\because L.T. = \dfrac{(M.T.)^2}{4 \times F.T.}\right)$

$\therefore\ (6x+7)^2 - (17)^2 = 0$

$\therefore\ (6x + 7 + 17) - (6x + 7 - 17)^2 = 0$

$\therefore\ 6x + 24 = 0$ or $6x - 10 = 0$

$\therefore\ x = -\dfrac{24}{6}$ or $x = \dfrac{10}{6}$

$\therefore\ x = -4$ or $x = \dfrac{5}{3}$

Thus, the roots of the given equation are -4 and $\dfrac{5}{3}$.

Question 2(3):

Solution :

For $x^2 - 10x + 25 = 0$

$(x)^2 - 2(x)(5) + (5)^2 = 0$

$\therefore (x - 5)^2 = 0$

$\therefore (x - 5)(x - 5) = 0$

$\therefore x - 5 = 0$ or $x - 5 = 0$

$\therefore x = 5$ or $x = 5$

$\therefore$ Root of the given equation is 5.

Question 2(4):

Solution :

For $x^2 + (x + 5)^2 = 625$ we find the third term.

$\therefore\ 2x^2 + 10x - 600 = 0$

$\therefore\ x^2 + 5x - 300 = 0$

$\therefore\ x^2 + 5x - 300 = 0$

$\therefore\ x^2 + 5x + \dfrac{25}{4} - \dfrac{25}{4} - 300 = 0 \qquad \left(\because \text{Third trem} = \dfrac{(M.T.)^2}{4 \times F.T.} = \dfrac{(5x)^2}{4 \times x^2} = \dfrac{25}{4}\right)$

$\therefore\ \left(x + \dfrac{5}{2}\right)^2 - \left(\dfrac{25}{4} + 300\right) = 0$

$\therefore\ \left(x + \dfrac{5}{2}\right)^2 - \left(\dfrac{25 + 1200}{4}\right) = 0$

$\therefore\ \left(x + \dfrac{5}{2}\right)^2 - \left(\dfrac{1225}{4}\right) = 0$

$\therefore\ \left(x + \dfrac{5}{2}\right)^2 - \left(\dfrac{35}{2}\right)^2 = 0$

$\therefore\ \left(x + \dfrac{5}{2} + \dfrac{35}{2}\right)^2 - \left(x + \dfrac{5}{2} - \dfrac{35}{2}\right) = 0$

$\therefore\ (x + 200)(x - 15) = 0$

$\therefore\ x + 20 = 0$ or $x - 15 = 0$

$\therefore\ x = -20$ or $x = 15$

Thus, the roots of the given equation are -20 and 15.

Question 2(5):

Solution :

For $(x+2)(x+3) = 240$ we write it in standard form.

$\therefore x^2 + 5x + 6 = 240$

$\therefore x^2 + 5x - 234 = 0$

$$\therefore x^2 + 5x + \frac{25}{4} - \frac{25}{4} - 234 = 0 \qquad \left(\because \text{Third trem} = \frac{(M.T.)^2}{4 \times F.T.} = \frac{(5x)^2}{4 \times x^2} = \frac{25}{4} \right)$$

$$\therefore \left(x + \frac{5}{2}\right)^2 - \left(\frac{25}{4} + 234\right) = 0$$

$$\therefore \left(x + \frac{5}{2}\right)^2 - \left(\frac{25 + 936}{4}\right) = 0$$

$$\therefore \left(x + \frac{5}{2}\right)^2 - \left(\frac{961}{4}\right) = 0$$

$$\therefore \left(x + \frac{5}{2}\right)^2 - \left(\frac{31}{4}\right)^2 = 0$$

$$\therefore \left(x + \frac{5}{2} + \frac{31}{4}\right) - \left(x + \frac{5}{2} - \frac{31}{4}\right) = 0$$

$\therefore (x + 18)(x - 13) = 0$

$\therefore x + 18 = 0$ or $x - 13 = 0$

$\therefore x = 18$ or $x = 13$

Thus, the roots of the given equation are -18 and 13.

Question 3:

Divide 20 into two parts such that the sum of the square of the parts is 218.

Solution :

Let the two parts be x and 20 – x.

It is given that the sum of their squares is 218.

$\therefore x^2 + (20 - x)^2 = 21$

$\therefore x^2 + 400 - 40x + x^2 = 218$

$\therefore 2x^2 - 40x + 182 = 0$

$\therefore x^2 - 20x + 91 = 0$

$\therefore x^2 - 7x - 13x + 91 = 0$

$\therefore x(x - 7) - 13(x - 7) = 0$

$\therefore (x - 7)(x - 13) = 0$

$\therefore x - 7 = 0$ or $x - 13 = 0$

$\therefore x = 7$ or $x = 13$

If one part is 7, then the second part is (20- x) i.e., 20-7=13

If one part is 13, then the second part is (20 – x) i.e., 20 – 13 = 7.

Hence two parts of 20 are 7 and 13 respectively.

Question 4:

A car takes 1 hour less to cover a distance of 200 km if its speed is increased by 10 km/hr, than its usual speed. What is the usual speed of the car ?

Solution :

Let the usual speed of a car is x km/hr.

$\therefore$Time taken to cover 200km = t_1

Let the usual speed of the car be x km/hr.

$\therefore$ Time taken to cover 200km $= t_1$

$$t_1 = \frac{200}{x}\text{hours} \qquad \left(\because t = \frac{d}{s}\right) \qquad \ldots (1)$$

Next, if the speed of the car is increased by 10 km/hr then the new speed is $(x + 10)$ km/hr.

$\therefore$ Time taken to cover 200 km by new speed $= t_2$

$$t_2 = \frac{200}{x}\text{hours} \qquad \left(\because t = \frac{d}{s}\right) \qquad \ldots (2)$$

But, the car takes 1 hour less to cover a distance of 200 km by new speed.

$\therefore t_1 - t_2 = 1$

$\therefore \dfrac{200}{x} - \dfrac{200}{x+10} = 1 \qquad [\because \text{By eq. (1) and (2)}]$

$\therefore 200(x + 10) - 200x = x(x + 10) \quad [\because \text{Mult. both sides by } x(x + 10)]$

$\therefore 200x + 2000 - 200x = x^2 + 10x$

$\therefore x^2 + 10x - 2000 = 0$

$\therefore x^2 - 40x + 50x - 2000 = 0$

$\therefore x(x - 40) + 50(x - 40) = 0$

$\therefore (x - 40)(x + 50) = 0$

$\therefore x - 40 = 0$ or $x + 50 = 0$

$\therefore x = 40$ or $x = -50$

$\therefore x = 40 \qquad (\because \text{Speed cannot be negative.})$

Hence the usual speed of the car is 40 km/hr.

Question 5:

When there is a decrease of 5 km/hr in the usual uniform speed of a goods train, due to track repair work going on it takes 4 hours more than the usual time for travelling the distance of 400 km. Find the usual speed of the train.

Solution :

Let the usual speed of goods train be x km/hr.

$\therefore$ Time taken to cover 400 km by original speed = t_1

$$t_1 = \frac{400}{x} \text{hours} \qquad \left(\because t = \frac{d}{s}\right)$$

If the speed of a goods train is decreased by 5 km/hr then the new speed is $(x - 5)$ km/hr.

Time taken to cover 400 km by new speed = t_2

$$t_2 = \frac{400}{x-5} \text{hours} \qquad \left(\because t = \frac{d}{s}\right)$$

But, the train takes 4 hours more to cover the same distance by a new speed.

$\therefore t_2 = t_1 + 4$

$\therefore \dfrac{400}{x-5} = \dfrac{400}{x} + 4$

$400x = 400(x - 5) + 4x(x - 5) \qquad [\because \text{Mult. both sides by } x(x - 5)]$

$\therefore 400x = 400x - 2000 + 4x^2 - 20x$

$\therefore 4x^2 - 20x - 2000 = 0$

$\therefore x^2 - 5x - 500 = 0$

$\therefore (x - 25)(x + 20) = 0$

$\therefore x - 25 = 0$ or $x + 20 = 0$

$\therefore x = 25$ or $x = -20$

But, speed of train cannot be negative.

$\therefore x = -20$

$\therefore x = 25$

Hence the usual speed of the goods train is 25 km/hr.

Question 6:

A river flows at a speed of I km/hr. A boat takes 15 hours to travel 112 km downstream and coming back the same distance upstream. Find the speed of the boat in still water. (Speed of the river flow is less than the speed of the boat in still water)

Solution :

Let the speed of boat in still water be x km/hr.

Speed of the river is given as 1 km/hr.

$\therefore$ Speed of the boat downstream = (x + 1) km/hr.

$\therefore$ Time taken for going 112 km downstream = t_1

$$t_1 = \frac{112}{x+1} \text{hours} \qquad \left(\because t = \frac{d}{s}\right) \qquad \dots (1)$$

Next,

Speed of the boat upstream is (x − 1) km/hr.

$\therefore$ Time taken for going 112 km upstream = t_2

$$t_2 = \frac{112}{x-1} \text{hours} \qquad \left(\because t = \frac{d}{s}\right) \qquad \dots (2)$$

Total time of journey is given as 15 hour.

$\therefore t_1 + t_2 = 15 \qquad [\because \text{By eq.}(1) \ \& \ (2)]$

$\therefore \dfrac{112}{x+1} + \dfrac{112}{x-1} = 15$

$\therefore \ 112(x-1) + 112(x+1) = 15(x+1)(x-1)$

$\therefore \ 112x - 112 + 112x + 112 = 15(x^2 - 1)$

$\therefore 224x = 15x^2 - 15$

$\therefore 15x^2 - 224x - 15 = 0$

$\therefore 15x(x-15) + 1(x-15) = 0$

$\therefore (x-15)(15x+1) = 0$

$\therefore x - 15 = 0$ or $15x + 1 = 0$

$\therefore x = 15$ or $x = -\dfrac{1}{15}$

$\therefore x = 15 \qquad (\because \text{speed cannot be negative.})$

Hence the speed of boat in still water is 15 km/hr.

Question 7:

Find a number greater than 1 such that the sum of the number and its reciprocal is $\frac{4}{15}$

Solution :

Let the non-zero number be x.

$\therefore$ Its reciprocal is $\frac{1}{x}$.

It is given the sum of the number and its reciprocal is $2\frac{4}{15}=\frac{34}{15}$.

$\therefore x + \frac{1}{x} = \frac{34}{15}$.

$\therefore 15x^2 + 15 = 34x$ (Multiplying both the sides by 15x)

$\therefore 15x^2 - 34x + 15 = 0$

$\therefore 15x^2 - 9x - 25x + 15 = 0$

$\therefore 3x(5x-3) - 5(5x-3) = 0$

$\therefore (5x-3)(3x-5) = 0$

$\therefore 5x - 3 = 0$ or $3x - 5 = 0$

$\therefore x = \frac{3}{5}$ or $x = \frac{5}{3}$

$x = 0.6$ or $x = 1.6$

But the required number is greater than 1, so $x = \frac{3}{5}$ is not possible.

$\therefore x = \frac{5}{3} > 1$ is the required number.

Thus, the required number greater than 1 is $\frac{5}{3}$.

Question 8:

The difference of the speed of a faster car and a slower car is 20 km/hr. If the slower car takes 1 hour more than the faster car to travel a distance of 400 km, find speed of both the cars.

Solution :

Let the speed of the slower car be x km/hr.

$\therefore$ Speed of the faster car = (x + 20) km/hr.

Time taken to cover 400 km by the slower car = t_1

$t_1 = \frac{400}{x}$ hours $\left(\because t = \frac{d}{s}\right)$

Time taken to cover 400 km by the faster car = t_2

$t_2 = \frac{400}{x+20}$ hours $\left(\because t = \frac{d}{s}\right)$

According to the given conditions,

$t_1 - t_2 = 1$

$\therefore \frac{400}{x} - \frac{400}{x+20} = 1$

Multiplying both the sides by x(x+20),

$400(x + 20) - 400x = x(x+20)$

$\therefore 400x + 8000 - 400x = x^2 + 20$

$\therefore x^2 + 20x - 8000 = 0$

$\therefore (x + 100)(x - 80) = 0$

$\therefore x + 100 = 0$ or $x - 80 = 0$

$\therefore x = -100$ or $x = 80$

But, speed of a car cannot be negative. $\therefore x = -100$ is not possible.

$\therefore x = 80$

Speed of the slower car = x = 80km/hr

$\therefore$ Speed of the faster car = (x + 20) = 80 + 200 = 100 km/hr

Question 9:

Product of the ages of Virat 7 years ago and 7 years later is 480. Find his present age.

Solution :

Let the present age of Virat be x years.
Before 7 years,
Virat's age = (x – 7) years
After 7 years,
Virat's age = (x + 7) years
Product of the numbers showing these ages is 480.
∴ $(x - 7)(x + 7) = 480$
∴ $x^2 - 49 = 480$
∴ $x^2 - 529 = 0$
∴ $(x)^2 - (23)^2 = 0$
∴ $x - 23 = 0$ or $x + 23 = 0$
∴ $x = 23$ or $x = -23$
∴ $x = 23$ (∵ Age cannot be negative.)
Hence the present age of Virat is 23 years.

Question 10:

If the age of Sachin 8 year ago is multiplied by his age two years later, the result is 1200. Find the age of Sachin at present.

Solution :

Let the present age of Sachin be x years.
Before 8 years,
Sachin's age = (x – 8) years
After 2 years,
Sachin's age = (x + 2) years.
Product of the numbers showing these ages is 1200.
∴ $(x - 8)(x + 2) = 1200$
∴ $x^2 - 6x - 16 = 1200$
∴ $x^2 - 6x - 1216 = 0$
∴ $x^2 - 38x + 32x - 1216 = 0$
∴ $x(x - 38)+32(x - 38) = 0$
∴ $(x - 38)(x + 32) = 0$
∴ $x - 38 = 0$ or $x + 32 = 0$
∴ $x = 38$ or $x = -32$
∴ $x = 38$ (∵ Age cannot be negative.)
Hence Sachin's present age is 38 years.

Question 11:

Sunita's age at present is 2 years less than 6 times the age of her daughter Anita. The product of their ages 5 years later will be 330. What was the age of Sunita when her daughter Anita was born ?

Solution :

Let the present age of Anita be x years

It is given that Sunita's age at present is 2 years less than 6 times the age of her daughter Anita.

$\therefore$ Present age of Sunita = (6x – 2) years

After five years,

Age of Anita = (x + 5) years

Age of Sunita = [(6x – 2) + 5] years = (6x + 3) years

Also, the product of the numbers showing these ages is 330.

$\therefore (x + 5)(6x + 3) = 330$

$\therefore 6x^2 + 33x + 15 = 330$

$\therefore 6x^2 + 33x - 315 = 0$

$\therefore 2x^2 + 11x - 105 = 0$ ($\because$ Dividing by 3)

$\therefore 2x^2 + 21x - 10x - 105 = 0$

$\therefore x(2x + 21) - 5(2x + 21) = 0$

$\therefore (2x + 21)(x - 5) = 0$

$\therefore 2x + 21 = 0$ or $x - 5 = 0$

or $x = 5$

$\therefore x = 5$ ($\because$Age cannot be negative.)

Thus, the present age of Anita (x) is 5 years.

$\therefore$Present age of Sunita

$= 6x - 2$

$= 6(5) - 2$

$= 28$ years

Now, Anita was born before five years, so the age of Sunita at that time was = 28 – 5 = 23 years.

Question 12:

The formula of the sum of first n natural numbers is $S = \frac{n(n+1)}{2}$. If the sum of first n natural number is 325, find n.

Solution :

It is given the sum of first natural numbers is 325.

For $S = \frac{n(n+1)}{2}$,

Substituting S = 325 we get,

$325 = \frac{n(n+1)}{2}$

$650 = n^2 + n$

$\therefore n^2 + n - 650 = 0$

$\therefore n^2 - 25n + 26n - 650 = 0$

$\therefore n(n-25) + 26(n-25) = 0$

$\therefore (n-25)(n+26) = 0$

$\therefore n - 25 = 0$ or $n + 26 = 0$

$\therefore n = 25$ or $n = -26$

$\therefore n = 25$ ($\because$ n is a natural number.)

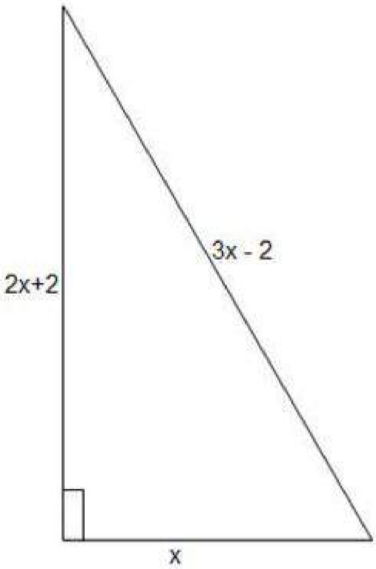
Thus, n = 25 is obtained.

Question 13:

Hypotenuse of a right angled triangle is 2 less than 3 times its shortest side. If the remaining side is 2 more than twice the shortest side, find the area of the triangle.

Solution :

Let the length of the shortest side of the right angled triangle be x.
Here hypotenuse is given is 2 less than 3 times its shortest side.
∴ Length of the hypotenuse is $(3x - 2)$.



Then, length of the remaining side is 2 more than twice the shortest side = $(2x + 2)$
By Pythagoras theorem,
$(\text{Length of hypotenuse})^2$
$= (\text{Length of a side})^2 + (\text{Length of other side})^2$
$\therefore (3x - 2)^2 = (x)^2 + (2x + 2)^2$
$\therefore 9x^2 - 12x + 4 = x^2 + 4x^2 + 8x + 4$
$\therefore 4x^2 - 20x = 0$
$\therefore x^2 - 5x = 0$ ($\because$ Dividing by 4)
$\therefore x(x - 5) = 0$
$\therefore x = 0$ or $x = 5$
$\therefore x = 5$ ($\because$ Length of a side cannot be zero.)
The length of the shortest side of a right angled triangle is 5 units.
Length of the other side
$= 2x + 2$
$= 2(5)+2$
$= 10 + 2$
$=12$
Now, area of a right angled triangle
$= \frac{1}{2}\text{base} \times \text{altitude}$
$= \frac{1}{2}(5)\times(12)$
$=30$ square units

Question 14:

The sum of the squares of two consecutive odd positive integers is 290. Find the numbers.

Solution :

Let the first odd positive integer be x.

∴ Second consecutive odd positive integer = $x + 2$.

According to the given conditions, sum of the square of these two numbers is 290.

∴ $x^2 + (x + 2)^2 = 290$

∴ $x^2 + x^2 + 4x + 4 = 290$

∴ $2x^2 + 4x – 286 = 0$

∴ $x^2 + 2x – 143 = 0$ (∵ Dividing by 2)

∴ $x^2 + 13x – 11x – 143 = 0$

∴ $x(x + 13) – 11(x + 13) = 0$

∴ $(x + 13)(x – 11) = 0$

∴ $x + 13 = 0$ or $x – 11 = 0$

∴ $x = – 13$ or $x = 11$

But $x = -13$ is not possible.

∴ $x = 11$ (∵ x is a positive integer.)

∴ First odd positive integer = $x = 11$

Second consecutive odd positive integer = $x + 2$

$= 11 + 2 = 13$

∴ Thus the two consecutive odd positive numbers are 11 and 13.

Question 15:

The product of two consecutive even natural numbers is 224. Find the numbers.

Solution :

Let the two consecutive even natural numbers be x and $x + 2$.

Product of these two numbers is 224.

∴ $x(x + 2) = 224$

∴ $x^2 + 2x – 224 = 0$

∴ $x^2 + 16x – 14x – 224 = 0$

∴ $x(x + 16) – 14(x + 16) = 0$

∴ $(x + 16)(x – 14) = 0$

∴ $x + 16 = 0$ or $x – 14 = 0$

∴ $x = -16$ or $x – 14 = 0$

∴ $x = -16$ or $x = 14$

∴ $x = 14$ (∵ x is a natural number.)

∴ First even natural number $x = 14$

second even natural number = $x + 2 = 14 + 2 = 16$

∴ The required consecutive even natural numbers are 14 and 16.

Question 16:

The product of digits of a two-digit number is 8 and the sum of the squares of the digits is 20. If the number is less than 25. Find the number.

Solution :

$\therefore$ The digit at units place becomes $\frac{8}{x}$.

$\therefore$ Original number $= 10x + \frac{8}{x}$ ... ... (1)

The sum of the squares of the digit is 20.

$\therefore$ (Digit at tens place)^2+(Digit at units place)^2=20

$\therefore (x)^2+\left(\frac{8}{x}\right)^2=20$

$\therefore x^2+\frac{64}{x^2}=20$

$\therefore x^4 + 64 = 20x^2 = 0$

$\therefore x^4 - 20x^2 + 64 = 0$

$\therefore x^4 - 16x^2 - 4x^2 + 64 = 0$

$\therefore x^2(x^2-16)-4(x^2-16)=0$

$\therefore (x^2-16)(x^2-4)=0$

$\therefore x^2-16=0$ or $x^2-4=0$

$\therefore x^2 = 16$ or $x^2 = 4$

$\therefore x = 4$ or $x = 2$ ($\because x$ is a digit, so it is positive.)

Substituting $x = 4$ in the equation (1),

Original number $= 10(4) + \frac{8}{4} = 40 + 2 = 42$

Next, substituting $x = 2$ in the equation (1),

Original number $= 10(2) + \frac{8}{2} = 20 + 4 = 24$

But, the required number is less than 25, so the required two-digit number is 24.

Question 17:

If price of sugar decreases by ₹ 5, one can buy 1 kg more sugar in ₹ 150, what is the price of the sugar ?

Solution :

Let the price of 1 kg sugar be Rs x.

$\therefore$ Amount of sugar obtained at this price $= \frac{150}{x}$ kg ... (1)

If the price of sugar is decreased by Rs. 5,

New price of sugar = Rs. $(x-5)$per kg.

$\therefore$ The amount of sugar obtained at this new rate

$=\frac{150}{x-5}$ kg ... (2)

One can by 1 kg more sugar in Rs 150

$\therefore$ Amount of sugar by new rate − Amount of sugar by old rate = 1

$\therefore 150x-150(x-5)=x(x-5)$

[$\because$ Multiplying both the sides by x(x-5)]

$\therefore x^2-5x-750=0$

$\therefore x^2-30x+25x-750=0$

$\therefore x(x-30)+25(x-30)=0$

$\therefore (x-30)(x+25)=0$

$\therefore x-30=0$ or $x+25=0$

$\therefore x=30$ or $x=-25$

$\therefore x = 30$ ($\because$ The rate of sugar cannot be negative.)

Thus, the original price of sugar is Rs. 30/kg.

Question 18:

If the price of petrol is increased by ₹ 5 per litre. One gets 2 litres less petrol spending ₹ 1320. What is the increased price of the petrol ?

Solution :

Let the original price of one litre petrol be Rs. x.

$\therefore$ For Rs. 1320, $\frac{1320}{x}$ litres petrol is obtained.

If the price of petrol is increased by Rs. 5 per litre,

New price of petrol = Rs. (x + 5) per litre.

One gets 2 litres less petrol spending Rs. 1320.

$\therefore \frac{1320}{x} - \frac{1320}{x+5} = 2$

$\therefore 1320(x+5) - 1320x = 2x(x+5)$ [$\because$ Mult. both the sides by x(x+5)]

$\therefore 1320x + 6600x - 1320x - 2x^2 + 10x = 0$

$\therefore 2x^2 + 10x - 6600 = 0$

$\therefore x^2 + 5x - 3300 = 0$

$\therefore x^2 + 60x - 55x - 3300 = 0$

$\therefore x(x+60)(x-55) = 0$

$\therefore x + 60 = 0$ or $x - 55 = 0$

$\therefore x = -60$ or $x = 55$

$\therefore x = 55$ ($\because$ Price of petrol cannot be negative.)

$\therefore$ Old price of petrol x = Rs.55/litre

Also, increased price of petrol = x + 5 = 55 + 5 = Rs. 60/litre

Question 19:

A vendor gets a profit in percentage equal to the cost price of a flower pot when he sells it for ₹ 96. Find the cost of the flower pot and the percentage of profit.

Solution :

Let the cost price of a flowerpot be Rs. x

It is given that the profit in percentage is equal to the cost price.

$\therefore$ Profit = x%

If the cost price is Rs. 100, then profit = Rs. x

$\therefore$ When cost price is Rs. x, then

$\text{Profit} = \text{Rs.}\left(x \times \frac{x}{100}\right) = \text{Rs.}\frac{x^2}{100}$

Now, Cost price + Profit = Selling price

$\therefore x + \frac{x^2}{100} = 96$

$\therefore 100x + x^2 = 9600$

$\therefore x^2 + 100x - 9600 = 0$

$\therefore (x + 160)(x - 60) = 0$

$\therefore x + 1600 = 0$ or $x - 60 = 0$

$\therefore x = -160$ or $x = 60$

Here x is the cost price of flowerpot, so it cannot be negative.

$\therefore x = 60$

$\therefore$ The cost price of flowerpot is Rs. 60.

Hence, profit obtained on selling the flowerpot = x% = 60%.

Question 20:

While selling a pen for ₹ 24 the loss in percentage is equal to its cost price. Find the cost price of pen. The cost price of pen is less than ₹ 50.

Solution :

Let the cost price of pen be Rs x. $(x < 50)$
It is given that the loss in percentage is equal to the cost price.
$\therefore$ Loss = $x\%$
If the cost is Rs.100, then the loss = Rs.x
$\therefore$ When the cost price Rs. x, then

$$\text{Loss} = \text{Rs.}\left(x \times \frac{x}{100}\right) = \text{Rs.}\frac{x^2}{100}$$

Now, loss = cost price – selling price

$$\therefore \frac{x^2}{100} = x - 24$$

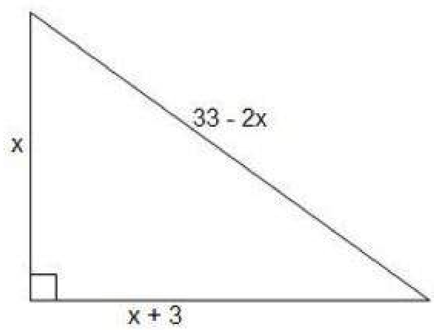
$\therefore x^2 = 100(x-24)$
$\therefore x^2 - 100x + 2400 = 0$
$\therefore (x-60)(x-40)=0$
$\therefore x - 60 = 0$ or $x - 40 = 0$
$\therefore x = 60$ or $x = 40$
But, the cost price of a pen is less than Rs. 50,
so the cost price of the pen is Rs. 40.

Question 21:

The difference of lengths of sides forming right angle in right angled triangle is 3 cm. If the perimeter of the triangle is 36 cm. Find the area of the triangle.

Solution :

Let the length of the smallest side of a right angled triangle be x cm.
∴ Measure of the larger side making a right angle with it is (x+3) cm.
Perimeter of a triangle is 36cm.
36 = x + (x + 3)+ hypotenuse
∴ Measure of the hypotenuse = 36 – x – (x + 3)
= (33 – 2x) cm
According to Pythagoras' theorem



(Length of hypotenuse)^2
= (Length of a side)^2 + (Length of other side)^2
∴ $(33 - 2x)^2 = x^2 + (x + 3)^2$
∴ $1089 - 132x + 4x^2 = x^2 + x^2 + 6x + 9$
∴ $2x^2 - 1138x + 1080 = 0$

$\therefore x^2 - 69x + 540 = 0$ [$\because$ Taking 2 common]

$\therefore (x - 60)(x - 9) = 0$

$\therefore x - 60 = 0$ or $x - 9 = 0$

$\therefore x = 60$ or $x = 9$

But, it is given that perimeter of the triangle is 36 cm, so side of the triangle cannot be 60 cm.

$\therefore x = 9$

Measure of the smallest side of this triangle = 9 cm.

Measure of other side = x + 3 = 9 + 3 = 12 cm

Now, area of a right angled triangle

$= \frac{1}{2}\text{base} \times \text{altitude}$

$= \frac{1}{2}(x + 3)\times(x)$

$= \frac{1}{2}(12)\times 9$

$= 6 \times 9 = 54$

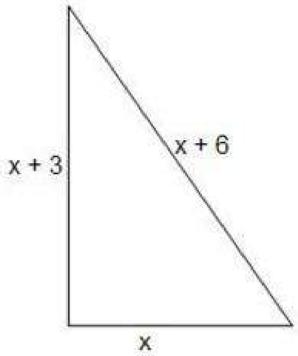
Thus, the area of a triangle is 54 cm^2

Question 22:

The sides of a right angled triangle are x, x + 3, x + 6, x being a positive integer. Find the perimeter of the triangle.

Solution :

The largest side in a right angled triangle is the hypotenuse, so measure of hypotenuse is (x + 6) cm.



$\therefore$ The measures of the remaining sides making a right angle in a right angled triangle are respectively x cm and (x + 3)cm.

$$\text{Area} = \frac{1}{2}\text{base} \times \text{altitude}$$

$\therefore 54 = \frac{1}{2}(x)(x + 3)$

$\therefore 108 = x^2 + 3x$

$\therefore x^2 + 3x - 108 = 0$

$\therefore x^2 + 12x - 9x - 108 = 0$

$\therefore x(x + 12) - 9(x + 12) = 0$

$\therefore (x + 12)(x - 9) = 0$

$\therefore x + 12 = 0$ or $x - 9 = 0$

$\therefore x = -12$ or $x = 9$

$\therefore x = 9$ ($\because$ side cannot be negative.)

Thus, measure of the smallest side of the triangle = x = 9 cm.

Measure of the other side making a right angle = x + 3 = 9 + 3 = 12 cm

Measure of the hypotenuse = x + 6 = 9 + 6 =15 cm.

$\therefore$ Perimeter of the triangle = 9 + 12 + 15 = 36 cm

Question 23:

Select a proper option (a), (b), (c) or (d) from given options :

Question 23(1):

……..is a solution of quadratic equation $x^2 - 3x + 2 = 0$

Solution :

b. 1

$x^2 - 3x + 2 = 0$

$\therefore x^2 - x - 2x + 2 = 0$

$\therefore x(x - 1) - 2(x - 1) = 0$

$\therefore (x - 1)(x - 2) = 0$

$\therefore x = 1$ or $x = 2$

$\therefore x = 1$ is the solution of the given equation.

Question 23(2):

Discriminant D = ……… for the quadratic equation $5x^2 - 6x + 1 = 0$

Solution :

a. 16

Comparing equation $5x^2 - 6x + 1 = 0$ with

$ax^2 + bx + c = 0$, we get a = – 5 , b = – 6, c = 1

Then Discriminant,

$D = b^2 = 4ac$

$= (-6)^2 - 4(5)(1)$

$= 36 - 20$

$= 16$

Question 23(3):

If $x = 2$ is a root of the equation $x^2 – 4x + a = 0$, then a = ……

Solution :

d. 4
2 is a root of a equation
$x^2 – 4x + a = 0$
$\therefore (2)^2 – 4(2) + a = 0$
$\therefore 4 – 8 + a = 0$
$\therefore a – 4 = 0$
$\therefore a = 4$

Question 23(4):

A quadratic equation has two equal roots, if …….

Solution :

c. D = 0
When D= 0, the quadratic equation has equal roots.

Question 23(5):

The quadratic equation ……. has 3 as one of its roots.

Solution :

a. $x^2 – x – 6 = 0$
Consider the first equation,
$p(x) = x^2 – x – 6$,
$p(3) = (3)^2 – 3 – 6$
$= 9 – 3 – 6$
$= 0$
$\therefore$ One solution of the equation $x^2 – x – 6 = 0$ is 3.
[Solution can be found by trial and error method, i.e. substituting x = 3 in all polynomials if it isn't true for the first equation.]

Question 23(6):

If 4 is a root of quadratic equation $x^2 + ax – 8 = 0$, then a = …….

Solution :

c. – 2
4 is a root of a quadratic equation $x^2 + ax – 8 = 0$.
$\therefore (4)^2 + a(4) – 8 = 0$
$\therefore 16 + 4a – 8 = 0$
$\therefore 4a + 8 = 0$
$\therefore a = -2$

Question 23(7):

If one of the roots of $kx^2 - 7x + 3 = 0$ is 3, then k = …….

Solution :

d. 2

3 is a root of a quadratic equation $kx^2 - 7x + 3 = 0$

$\therefore k(3)^2 - 7(3) + 3 = 0$

$\therefore 9k - 21 + 3 = 0$

$\therefore 9k - 18 = 0$

$\therefore 9k = 18$

$\therefore k = 2$

Question 23(8):

The discriminant of $x^2 - 3x - k = 0$ is 1. A value of x is …….

Solution :

b. – 2

Comparing the equation $x^2 - 3x - k = 0$ with

$ax^2 + bx + c = 0$, we get a = 1, b = -3, c = -k

Here, Discriminant = 1

$\therefore b^2 - 4ac = 1$

$\therefore (-3)^2 - 4(1)(-k) = 1$

$\therefore 9 + 4k = 11$

$\therefore 4k = -8$

$\therefore k = -2$