6 Mathematical Induction and Binomial Theorem

QUICK LOOK

Statement and Predicate

- *A* sentence which is either definitely true or definitely false is called *a* statement. "Snow is white" is a statement but "Ram is a good boy" is not a statement.
- Some sentences depend on a variable for its truth value (*i.e*, true or false). $"1+2+3+...+n=2n-1"$ is a mathematical sentence which is true for $n=1,2$ but false for $n=3,4$ etc. As the sentence is definitely true or definitely false for a particular positive integral value of *n*, the sentence is a statement and it depends on $n \in N$ for truth value. Such statements are called predicates and smybolized as $P(n)$.

Peano's Axiom (Principle of Mathematical Induction)

- A statement $P(n)$ is true for all $n \in N$ if
  $P(1)$ is true $P(m)$ is true $\Rightarrow$ $P(m+1)$ is true

Verification of Truth of P(n): PMI is a principle which can be used to verify whether a mathematical statement $P(n)$ is true for all $n \in N$ in any branch of mathematics. For this take the following steps:

- Put $n=1$ on one side of the statement and then simplify it to take the shape of the other side where $n=1$.
- Then assume $P(m)$ to be true. Use the mathematical result obtained by putting $n=m$ in the statement to establish $P(m+1)$ is true.

Use of Substitution in PMI: PMI may be used to prove whether a statement $P(n)$ is true for a particular infinite sequence of value of *n*.

- If the true of $P(n)$ is to be established for all positive even integral values of *n* then take $\phi(n)=P(2n)$ and use PMI to prove that $\phi(n)$ is true for all $n \in N$.
- If the truth of $P(n)$ is to be established for all positive odd integral values of *n* then take $\phi(n)=P(2n-1)$ and use PMI to prove that $\phi(n)$ is true for all $n \in N$.
- If the truth of $P(n)$ is to be established for all $n>k, n \in N, k \in N$ then take $\phi(n) = P(n+k)$ and use PMI to prove that $\phi(n)$ is true for all $n \in N$.
- Similarly use $\phi(n)=P(-n)$ for all negative integral values
  $\phi(n)=P\left(\frac{1}{n}\right)$ for all fractional values of the form $\frac{1}{r}, r \in N$
  $\phi(n)=P(3m)$ for all positive multiples of 3, etc.

Alternative Forms of PMI

- $P(n)$ is true for all $n \in N$ if
  $P(1)$ and $P(2)$ are true
  $P(m)$ and $P(m+1)$ are true
  $\Rightarrow$ $P(m+2)$is true
- $P(n)$ is true for all $n \in N$ if
  $P(1)$, $P(2)$, and $P(3)$ are true
  $P(m), P(m+1)$ and $P(m+2)$ are true
  $\Rightarrow$ $P(m+3)$ is true
- $P(n)$ is true for all $n \ge k, n \in N$ and k is a fixed positive integer, if $P(k)$ is true
  $P(m)$ is true $\Rightarrow$ $P(m+1)$ is true
- $P(n)$ is true for all $n \in N$ if
  $P(1), P(2), ..., P(k)$ are true
  $P(1), P(2), ..., P(m)$ are true $\Rightarrow$ $P(m+1)$ is true

Use of PMI in Statements P(*m*, *n*): Some mathematical statements are predicates of two arguments (*i.e*., truth value depending on two variables $m \in N, n \in N$). The method of establishing the truth of $P(m,n)$ for all $m \in N, n \in N$ is as follows:

- Keep *m* fixed and treat the statement $P(m,n)$ as $\phi(n)$. Establish the truth of $\phi(n)$ for all $n \in N$ by using PMI. Next keep *n* fixed and treat the statement $P(m,n)$ as $\psi(m)$. Establish the truth of $\psi(n)$ for all $n \in N$ by using PMI.

Some Formulae Based on Principle of Induction

For any natural number *n*

$$\sum n = 1+2+3+.......+n = \frac{n(n+1)}{2}$$

$$\sum n^2 = 1^2+2^2+3^2+.......+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum n^3 = 1^3+2^3+3^3+......+n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2$$

Divisibility Problems

To show that an expression is divisible by an integer.

(i) If a, p, n, r are positive integers, then first of all we write $a^{pn+r} = a^{pn}.a^r = (a^p)^n.a^r$.

(ii) If we have to show that the given expression is divisible by c.S

Then express, $a^p = [1+(a^p-1]$, if some power of (a^p-1) has c as a factor.

$a^p = [2+(a^p-2)]$, if some power of (a^p-2) has c as a factor.

$a^p = [K+(a^p-K)]$, if some power of (a^p-K) has c as a factor.

Binomial Theorem for Positive Integral Index

- $(a+x)^n = {}^nC_0a^n + {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 + ... + {}^nC_nx^n$ where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$
- $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$
- $(1+x)^n = (x+1)^n = {}^nC_0x^n + {}^nC_1x^{n-2} + ... + {}^nC_n$

 when expanded in descending powers of x.
- $(a-x)^n = {}^nC_0a^n - {}^nC_1a^{n-1}x + {}^nC_2a^{n-2}x^2 - ... + (-1)^n.{}^nC_nx^n$

Terms in Expansion: In the expansion of $(a+x)^n, n \in N$

- The number of terms $= n+1$
- $(r+1)$ th term $= t_{r+1} = {}^nC_ra^{n-r}.x^r$
- kth term from the end $\{(n+1)-(k-1)\}th$ term $= t_{(n+1)-(k-1)}$
- middle term $= t_{\frac{(n+1)+1}{2}}, i.e. t_{\frac{n}{2}+1}$ when n is even middle term $= t_{\frac{n+1}{2}}, t_{\frac{n+3}{2}}$ when n odd.

Properties of ${}^nC_r$ for Simplification

- ${}^nC_0 = 1, {}^nC_n = 1$
- ${}^nC_r = {}^nC_{n-r}$
- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
- ${}^nC_r = {}^nC_k \Rightarrow r = k$ or $r+k=n$
- $r.{}^nC_r = n.{}^{n-1}C_{r-1}$
- $\dfrac{1}{r+1}\cdot{}^nC_r = \dfrac{1}{n+1}\cdot{}^{n+1}C_{r+1}$
- The greatest among binomial coefficients

  ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n = {}^nC_{\frac{n}{2}}$ when n is even

  ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$ when n is odd.

Summation of Series Involving Binomial Coefficients: For $(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n$, the binomial coefficients are ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$.

A number of series may be formed with these coefficients figuring in the terms of a series. Standard series of the binomial coefficients are as follows:

- ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$ . . .(i)

  It is obtained by putting $x = 1$ in the binomial expansion for $(1+x)^n$.
- ${}^nC_0 - {}^nC_1 + {}^nC_2 - .... + (-1)^n.{}^nC_n = 0$ . . .(ii)

  It is obtained by putting $x = -1$ in the binomial expansion for $(1+x)^n$.
- ${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = 2^{n-1}$

  It is obtained by adding (i) and (ii).
- ${}^nC_1 + {}^nC_3 + {}^nC_5 + ... = 2^{n-1}$

  It is obtained by subtracting (ii) from (i).
- ${}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_{n-1} + \dfrac{1}{2}.{}^{2n}C_n = 2^{2n-1}$

  We have ${}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_{2n-1} + {}^{2n}C_{2n} = 2^{2n}$

$\because$ ${}^{2n}C_0 = {}^{2n}C_{2n}, {}^{2n}C_1 = {}^{2n}C_{2n-1}$, etc. combining the terms equidistant from the beginning and end we get

- $2({}^{2n}C_0 + {}^{2n}C_1 + {}^{2n}C_2 + ... + {}^{2n}C_{n-1}) + {}^{2n}C_n = 2^{2n}$
- ${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = 2^{2n}$ (as above)
- Sum of the first half of ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n$

  = sum of the last half of ${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^{n-1}$

Bino-geometric series

${}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n = (1+x)^n$.

Bino-arithmetic series

$a\,{}^nC_0 + (a+d)^nC_1 + (a+2d)^nC_2 + ... + (a+nd)^nC_n$

It is made by the sum of the products of corresponding terms of the sequences

$a, a+d, a+2d, ..., a+nd$ (AP) and ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ (sequence of binomial coefficients)

Such series can be added in two ways:

(i) by elimination of r in the multiplier of binomial coefficient from the $(r+1)th$ term of the series (using $r.{}^nC_r = n\,{}^{n-1}C_{r-1}$)

(ii) by differentiating the expansion of $x^a(1+x^d)^n$

- **Bino-harmonic series**

$$\frac{{}^nC_0}{a}+\frac{{}^nC_1}{a+d}+\frac{{}^nC_2}{a+2d}+....+\frac{{}^nC_n}{a+nd}$$

It is made by the sum of the products of corresponding terms of the sequences

$$\frac{1}{a},\frac{1}{a+d},\frac{1}{a+2d},....,\frac{1}{a+nd}\ \text{(HP)}$$

and ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ (sequence of binomial coefficients).

Such series are added in two ways :

(i) by elimination of r in the multiplier of binomial coefficients from the $(r+1)th$ term of the series

$$\left(\text{using}\frac{1}{\text{r}+1}{}^nC_r=\frac{1}{n+1}{}^{n+1}C_{r+1}\right)$$

(ii) by integrating suitable expansion.

- **Bino-binomial series**

$${}^nC_0.{}^nC_r+{}^nC_1.{}^nC_{r+1}+{}^nC_2.{}^nC_{r+2}+....+{}^nC_{n-r}.{}^nC_n$$

or ${}^mC_0.{}^nC_r+{}^mC_1.{}^nC_{r-1}+{}^mC_2.{}^nC_{r-2}+....+{}^mC_r.{}^nC_0$

Such series are added by multiplying two expansions, one involving the first factors as coefficients and the other involving the second factors as coefficients and finally equating coefficients of a suitable power of x on both sides.

Binomial Theorem for any Index

- $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+....$ to $\propto$, provided $|x|<1$.

General term $\{(r+1)th$ term$\}$ in the expansion of $(1+x)^n$ is

$$t_{r+1}=\frac{n(n-1)(n-2)....(n-r+1)}{r!}x^r$$

Some useful Binomial Expansions for Summation of Series

- $(1-x)^{-n}=1+{}^nC_1x+{}^{n+1}C_2x^2+{}^{n+2}C_3x^3+....+{}^{n+r-1}C_rx^r+....$ to $\propto$, where n is a positive integer.

$(1+x)^{-n}=1-{}^nC_1x+{}^{n+1}C_2x^2-{}^{n+2}C_3x^3+....$ to $\propto$ where $n\in N$

- $(1-x)^{-1}=1+x+x^2+x^3+x^4+....$ to $\propto$

$(1-x)^{-1}=1-x+x^2-x^3+x^4-....$ to $\propto$

- $(1-x)^{-2}=1+2x+3x^2+4x^3+....$ to $\propto$

$(1+x)^{-2}=1-2x+3x^2-4x^3+....$ to $\propto$

- $(1-x)^{-\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2$

$+\frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3+....$ to $\propto$

$(1+x)^{-\frac{p}{q}}=1-\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2-\frac{p(p+q)(p+2q)}{3!}\left(\frac{x}{q}\right)^3+....$ to $\propto$

- $(1+x)^{\frac{p}{q}}=1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p-q)}{2!}\left(\frac{x}{q}\right)^2+\frac{p(p-q)(p-2q)}{3!}\left(\frac{x}{q}\right)^3+....$ to $\propto$

$(1-x)^{\frac{p}{q}}=1-\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p-q)}{2!}\left(\frac{x}{q}\right)^2-\frac{p(p-q)(p-2q)}{3!}\left(\frac{x}{q}\right)^3+....$ to $\propto$

Number of Terms in the Expansion of $(a + b + c)^n$ and $(a + b + c + d)^n$.

$(a+b+c)^n$ can be expanded as: $(a+b+c)^n=\{(a+b)+c\}^n$

$=(a+b)^n+{}^nC_1(a+b)^{n-1}(c)^1+{}^nC_2(a+b)^{n-2}(c)^2+.....+{}^nC_nc^n$

$=(n+1)$ term $+n$ term $+$ $(n-1)$term $+...+1$term

$\therefore$ Total number of terms

$=(n+1)+(n)+(n-1)+....+1$

$=\frac{(n+1)(n+2)}{2}$.

Similarly, Number of terms in the expansion of

$(a+b+c+d)^n=\frac{(n+1)(n+2)(n+3)}{6}$.

To Determine a Particular Term in the Expansion

In the expansion of $\left(x^\alpha\pm\frac{1}{x^\beta}\right)^n$, if x^m occurs in T_{r+1}, then r is given by $n\alpha-r(\alpha+\beta)=m$

$\Rightarrow\quad r=\frac{n\alpha-m}{\alpha+\beta}$

Thus in above expansion if constant term which is independent of x, occurs in T_{r+1} then r is determined by

$n\alpha-r(\alpha+\beta)=0$

$\Rightarrow\quad r=\frac{n\alpha}{\alpha+\beta}$

Recoginising the type of the Infinite Series

- Each of the binomial, exponential and logarithmic series has infinite terms.
- Series involving n ! s in the decominators of terms are generally binomial or exponential series. But in a binomial series the number of factors in the numerators (other than the power of a fixed number) of terms goes on increasing.
- Logarithmic series do not contain n ! s in the denominators of terms. The terms contain $\frac{1}{n}s$.

MULTIPLE CHOICE QUESTIONS

Second Principle of Mathematical Induction

1. The smallest positive integer n, for which $n! < \left(\frac{n+1}{2}\right)^n$ hold is:

a. 1 **b.** 2 **c.** 3 **d.** 4

2. Let $S(k) = 1+3+5+...+(2k-1) = 3+k^2$. Then which of the following is true:

a. Principle of mathematical induction can be used to prove the formula

b. $S(k) \Rightarrow S(k+1)$

c. $S(k) \not\Rightarrow S(k+1)$

d. $S(1)$ is correct

3. $(1+x)^n - nx - 1$ is divisible by: (where $n \in N$)

a. $2x$ **b.** x^2

c. $2x^3$ **d.** All of these

4. Sum of odd terms is A and sum of even terms is B in the expansion of $(x+a)^n$, then:

a. $AB = \frac{1}{4}(x-a)^{2n} - (x+a)^{2n}$

b. $2AB = (x+a)^{2n} - (x-a)^{2n}$

c. $4AB = (x+a)^{2n} - (x-a)^{2n}$

d. None of these

5. If the 4^{th} term in the expansion of $(px + x^{-1})^m$ is 2.5 for all $x \in R$ then:

a. $p = 5/2, m = 3$ **b.** $p = \frac{1}{2}, m = 6$

c. $p = -\frac{1}{2}, m = 6$ **d.** None of these

6. If $\frac{T_2}{T_3}$ in the expansion of $(a+b)^n$ and $\frac{T_3}{T_4}$ in the expansion of $(a+b)^{n+3}$ are equal, then $n = ?$

a. 3 **b.** 4 **c.** 5 **d.** 6

Independent Term and Middle Term

7. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$ will be :

a. $\frac{3}{2}$ **b.** $\frac{5}{4}$ **c.** $\frac{5}{2}$ **d.** $\frac{2}{3}$

8. The term independent of x in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$ is:

a. $C_0^2 + 2C_1^2 + ... + (n+1)C_n^2$

b. $(C_0 + C_1 + ... + C_n)^2$

c. $C_0^2 + C_1^2 + ... + C_n^2$

d. None of these

9. The coefficient of x^n in the expansion of $(1 + x)(1 - x)^n$ is:

a. $(-1)^{n-1} n$ **b.** $(-1)^n (1-n)$

c. $(-1)^{n-1}(n-1)^2$ **d.** $(n-1)$

10. 6^{th} term in expansion of $\left(2x^2 - \frac{1}{3x^2}\right)^{10}$ is:

a. $\frac{4580}{17}$ **b.** $-\frac{896}{27}$

c. $\frac{5580}{17}$ **d.** None of these

11. If the coefficients of r^{th} term and $(r+4)^{th}$ term are equal in the expansion of $(1+x)^{20}$, then the value of r will be:

a. 7 **b.** 8 **c.** 9 **d.** 10

12. If coefficient of $(2r+3)^{th}$ and $(r-1)^{th}$ terms in the expansion of $(1+x)^{15}$ are equal, then value of r is:

a. 5 **b.** 6

c. 4 **d.** 3

13. If the $(r+1)^{th}$ term in the expansion of $\left(\sqrt[3]{\frac{a}{\sqrt{b}}} + \sqrt{\frac{b}{\sqrt[3]{a}}}\right)^{21}$ has the same power of a and b, then the value of r is:

a. 9 **b.** 10

c. 8 **d.** 6

14. The first 3 terms in the expansion of $(1+ax)^n$ $(n \neq 0)$ are 1, $6x$ and $16x^2$. Then the value of a and n are respectively:

a. 2 and 9 **b.** 3 and 2

c. 2/3 and 9 **d.** 3/2 and 6

15. Coefficient of x in the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ is:

a. $9a^2$ **b.** $10a^3$ **c.** $10a^2$ **d.** $10a$

16. If the expansion of $\left(y^2 + \frac{c}{y}\right)^5$, the coefficient of y will be:

a. $20c$ **b.** $10c$

c. $10c^3$ **d.** $20c^2$

17. In the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, the coefficient of x^4 is:

a. $\frac{405}{256}$ b. $\frac{504}{259}$

c. $\frac{450}{263}$ d. None of these

18. The coefficient of x^7 in the expansion of $\left(\frac{x^2}{2}-\frac{2}{x}\right)^8$ is:

a. – 56 b. 56

c. – 14 d. 14

19. The coefficient of x^{32} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is

a. ${}^{15}C_5$ b. ${}^{15}C_6$

c. ${}^{15}C_4$ d. ${}^{15}C_7$

20. If in the expansion of $(1+x)^m(1-x)^n$, the coefficient of x and x^2 are 3 and – 6 respectively, then m is:

a. 6 b. 9

c. 12 d. 24

21. If coefficients of 2^{nd}, 3^{rd} and 4^{th} terms in the binomial expansion of $(1+x)^n$ are in A.P., then n^2-9n is equal to:

a. – 7 b. 7

c. 14 d. – 14

22. The coefficient of x^{39} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$ is:

a. – 455 b. – 105

c. 105 d. 455

23. If the coefficients of second, third and fourth term in the expansion of $(1+x)^{2n}$ are in A.P., then $2n^2-9n+7$ is equal to:

a. – 1 b. 0

c. 1 d. 3/2

24. If the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are the same, then the value of a is:

a. $-\frac{7}{9}$ b. $-\frac{9}{7}$ c. $\frac{7}{9}$ d. $\frac{9}{7}$

Number of Terms in the Expansion of $(a+b+c)^n$ and $(a+b+c+d)^n$

25. If the number of terms in the expansion of $(x-2y+3z)^n$ is 45, then $n=?$

a. 7 b. 8 c. 9 d. 5

26. The middle term in the expansion of $(1+x)^{2n}$ is:

a. $\frac{1.3.5......(2n-1)}{n!}x^{2n+1}$ b. $\frac{2.4.6......2n}{n!}x^{2n+1}$

c. $\frac{1.3.5......(2n-1)}{n!}x^n$ d. $\frac{1.3.5......(2n-1)}{n!}x^n.2^n$

27. The term independent of x in the expansion of $\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$ is:

a. 7/12 b. 7/18

c. – 7/12 d. – 7/16

Greatest Term and Greatest Coefficient

28. The largest term in the expansion of $(3+2x)^{50}$, where $x=\frac{1}{5}$ is:

a. 5^{th} b. 8^{th} c. 7^{th} d. 6^{th}

Properties of Binomial Coefficients

29. If $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$. Then $\frac{t_n}{S_n}$ is equal to:

a. $\frac{2n-1}{2}$ b. $\frac{1}{2}n-1$

c. $n-1$ d. $\frac{n}{2}$

30. In the expansion of $(1+x)^5$, the sum of the coefficient of the terms is:

a. 80 b. 16

c. 32 d. 64

31. If the sum of coefficient in the expansion of $(\alpha^2x^2-2\alpha x+1)^{51}$ vanishes, then the value of α is:

a. 2 b. –1

c. 1 d. –2

32. ${}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9=?$

a. 2^9 b. 2^{10}

c. $2^{10}-1$ d. None of these

33. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^2$, then $C_0^2+C_1^2+C_2^2+C_3^2+...+C_n^2=?$

a. $\frac{n!}{n!n!}$ b. $\frac{(2n)!}{n!n!}$

c. $\frac{(2n)!}{n!}$ d. None of these

34. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$, then $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + ... + \frac{nC_n}{C_{n-1}} = ?$

a. $\frac{n(n-1)}{2}$ **b.** $\frac{n(n+2)}{2}$

c. $\frac{n(n+1)}{2}$ **d.** $\frac{(n-1)(n-2)}{2}$

35. $2C_0 + \frac{2^2}{2}C_1 + \frac{2^3}{3}C_2 + ... + \frac{2^{11}}{11}C_{10}$

a. $\frac{3^{11}-1}{11}$ **b.** $\frac{2^{11}-1}{11}$

c. $\frac{11^3-1}{11}$ **d.** $\frac{11^2-1}{11}$

36. The sum to $(n+1)$ terms of the following series $\frac{C_0}{2} - \frac{C_1}{3} + \frac{C_2}{4} - \frac{C_3}{5} + ...$ is:

a. $\frac{1}{n+1}$ **b.** $\frac{1}{n+2}$

c. $\frac{1}{n(n+1)}$ **d.** None of these

37. The value of $\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + ...$ is equal to:

a. $\frac{2^n - 1}{n+1}$ **b.** $n.2^n$

c. $\frac{2^n}{n}$ **d.** $\frac{2^n + 1}{n+1}$

38. The sum of all the coefficients in the binomial expansion of $(x^2 + x - 3)^{319}$ is:

a. 1 **b.** 2

c. – 1 **d.** 0

39. If n is an integer greater than 1, then $a - {}^nC_1(a-1) + {}^nC_2(a-2) + ... + (-1)^n(a-n) = ?$

a. a **b.** 0

c. a^2 **d.** 2^n

40. If the sum of the coefficients in the expansion of $(\alpha^2x^2 - 2\alpha x + 1)^{51}$ vanishes, then the value of α is:

a. 2 **b.** –1

c. 1 **d.** – 2

Use of Differentiation and Integration

41. $C_1 + 2C_2 + 3C_3 + ...{}^nC_n = ?$

a. 2^n **b.** $n.\ 2^n$

c. $n.\ 2^{n-1}$ **d.** $n.\ 2^{n+1}$

42. $\frac{C_0}{1} + \frac{C_1}{2} + \frac{C_2}{3} + ... + \frac{C_n}{n+1} = ?$

a. $\frac{2^n}{n+1}$ **b.** $\frac{2^n - 1}{n+1}$

c. $\frac{2^{n+1} - 1}{n+1}$ **d.** None of these

Binomial theorem for any index

43. Let $R = (5\sqrt{5} + 11)^{2n+1}$ and $f = R - [R]$ where [.] denotes the greatest integer function. The value of R.f is:

a. 4^{2n+1} **b.** 4^{2n}

c. 4^{2n-1} **d.** 4^{-2n}

44. The coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is:

a. – 83 **b.** – 82 **c.** – 81 **d.** 0

45. Find the coefficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6$?

a. 0 **b.** 60 **c.** –60 **d.** 25

46. If $(1+ax)^n = 1 + 8x + 24x^2 + ...$ then the value of a and n is:

a. 2,4 **b.** 2,3

c. 1,2 **d.** 1,2

47. Coefficient of x^r in the expansion of $(1-2x)^{-1/2}$?

a. $\frac{(2r)!}{(r!)^2}$ **b.** $\frac{(2r)!}{2^r.(r!)^2}$

c. $\frac{(2r)!}{(r!)^2.2^{2r}}$ **d.** $\frac{(2r)!}{2^r.(r+1)!(r-1)!}$

48. If x is so small that its two and higher power can be neglected and $(1-2x)^{-1/2}(1-4x)^{-5/2} = 1 + kx$ then $k = ?$

a. 1 **b.** –2

c. 10 **d.** 11

49. If a_1, a_2, a_3, a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$, then $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = ?$

a. $\frac{a_2}{a_2 + a_3}$ **b.** $\frac{1}{2}\frac{a_2}{a_2 + a_3}$

c. $\frac{2a_2}{a_2 + a_3}$ **d.** $\frac{2a_3}{a_2 + a_3}$

50. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is:

a. 32 **b.** 33

c. 34 **d.** 35

51. The fourth term in the expansion of $(1–2x)^{3/2}$ will be:

a. $-\frac{3}{4}x^4$ **b.** $\frac{x^3}{2}$

c. $-\frac{x^3}{2}$ **d.** $\frac{3}{4}x^4$

52. The expansion of $\frac{1}{(4-3x)^{1/2}}$ binomial theorem will be valid, if:

a. $x<1$ **b.** $|x|<1$

c. $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$ **d.** None of these

53. $\frac{1}{\sqrt[3]{6-3x}}=?$

a. $6^{1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+...\right]$ **b.** $6^{-1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+...\right]$

c. $6^{1/3}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-...\right]$ **d.** $6^{-1/3}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-...\right]$

54. $\left(\frac{a}{a+x}\right)^{\frac{1}{2}}+\left(\frac{a}{a-x}\right)^{\frac{1}{2}}=?$

a. $2+\frac{3x^2}{4a^2}+....$ **b.** $1+\frac{3x^2}{8a^2}+....$

c. $2+\frac{x}{a}+\frac{3x^2}{4a^2}+....$ **d.** $2-\frac{x}{a}+\frac{3x^2}{4a^2}+......$

55. $(r+1)^{th}$ term in the expansion of $(1–x)^{–4}$ will be:

a. $\frac{x^r}{r!}$ **b.** $\frac{(r+1)(r+2)(r+3)}{6}x^r$

c. $\frac{(r+2)(r+3)}{2}x^r$ **d.** None of these

56. $\frac{1}{(2+x)^4}=?$

a. $\frac{1}{2}\left(1-2x+\frac{5}{2}x^2-....\right)$ **b.** $\frac{1}{16}\left(1-2x+\frac{5}{2}x^2-....\right)$

c. $\frac{1}{16}\left(1+2x+\frac{5}{2}x^2+....\right)$ **d.** $\frac{1}{2}\left(1+2x+\frac{5}{2}x^2+....\right)$

57. If $|x|>1$, then $(1+x)^{-2}$ =?

a. $1-2x+3x^2-....$ **b.** $1+2x+3x^2+....$

c. $1-\frac{2}{x}+\frac{3}{x^2}-....$ **d.** $\frac{1}{x^2}-\frac{2}{x^3}+\frac{3}{x^4}-...$

58. The approximate value of $(7.995)^{1/3}$correct to four decimal places is:

a.1.9995 **b.**1.9996

c.1.9990 **d.**1.9991

59. $1+\frac{1}{3}x+\frac{1.4}{3.6}x^2+\frac{1.4.7}{3.6.9}x^3+....$ is equal to:

a. x **b.** $(1+x)^{1/3}$

c. $(1-x)^{1/3}$ **d.** $(1-x)^{-1/3}$

60. If $(r+1)^{th}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r is:

a. 5 **b.** 6

c. 4 **d.** 7

61. The coefficient of x^n in the expansion of $(1+x+x^2+...)^{-n}$ is:

a. 1 **b.** $(-1)^n$

c. n **d.** $n+1$

NCERT EXEMPLAR PROBLEMS

More than One Answer

62. The value of $C_0^2+3C_1^2+5C_2^2+...$ to $(n+1)$ terms, (given that $C_r\equiv{}^nC_r$) is:

a. ${}^{2n-1}C_{n-1}$ **b.** $(2n+1).{}^{2n-1}C_n$

c. $2(n+1).{}^{2n-1}C_n$ **d.** ${}^{2n-1}C_n+(2n+1).{}^{2n-1}C_{n-1}$

63. The number of distinct terms in the expansion of $(x+2y-3z+5w-7u)^n$ is:

a. $n+1$

b. ${}^{n+4}C_4$

c. ${}^{n+4}C_n$

d. $\frac{(n+1)(n+2)(n+3)(n+4)}{24}$

64. In the expansion of $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}$?

a. the number of rational terms = 4

b. the number of irrational terms =19

c. the middle term is irrational

d. the number of irrational terms = 17

65. Let $a_n=\frac{(1000)^n}{n!}$ for $n\in N$. Then a_n is greatest, when:

a. n = 998 **b.** n = 999

c. n = 1000 **d.** n = 1001

66. The expression $\{x+\sqrt{(x^3-1)}\}^5+\{x-\sqrt{(x^3-1)}\}^5$ is a polynomial of degree:

a. ${}^9C_2$ **b.** ${}^7C_6$

c. 7 **d.** ${}^8C_1$

67. In the expansion of $(x+y+z)^{25}$?

a. every term is of form ${}^{25}C_r.{}^rC_k.x^{25-r}.y^{r-k}.z^k$

b. the coefficient of $x^8y^9z^9$ is 0

c. the number of terms is 351

d. none of the above

68. The coefficient of the middle term in the expansion of $(1+x)^{2n}$ is:

a. ${}^{2n}C_n$ **b.** $\frac{1\cdot 3\cdot 5...(2n-1)}{n!}2^n$

c. $2\cdot 6...(4n-2)$ **d.** none of the above

69. The last digit of $3^{3^{4n}}+1, n\in N$, is:

a. ${}^4C_3$ **b.** ${}^8C_7$

c. 8 **d.** 4

70. In the expansion of $(2-2x+x^2)^9$?

a. number of distinct terms is 10

b. coefficient of x^4 is 97

c. sum of coefficient is 1

d. number of distinct terms is 55

71. If the term independent of x in the expansion of $(\sqrt{x}-\lambda/\lambda^2)^{10}$ is 405, then value of λ is:

a. –3 **b.** 9

c. –9 **d.** 3

Assertion and Reason

Note: Read the Assertion (A) and Reason (R) carefully to mark the correct option out of the options given below:

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of the assertion.

c. If assertion is true but reason is false.

d. If the assertion and reason both are false.

e. If assertion is false but reason is true.

72. Let $S_1=\sum_{j=1}^{10} j(j-1)({}^{10}C_j), S_2=\sum_{j=1}^{10} j({}^{10}C_j)$ and

$S_3=\sum_{j=1}^{10}(j^2)({}^{10}C_j)$

Assertion: $S_3 = 55\times 2^9$

Reason: $S_1 = 90\times 2^8$ and $S_2 = 10\times 2^8$

73. Let $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$.

Assertion: $5C_0^2+7C_1^2+9C_2^2+...+(5+2n)C_n^2=(5+n)\frac{(2n)!}{n!n!}$

Reason: $C_0^2+C_1^2+...+C_n^2={}^{2n}C_n$

74. Let $(1+t)^n=C_0+C_1t+C_2t^2+...+C_nt^n$

Assertion: $\frac{C_0}{1.2}+\frac{C_1}{2.3}+\frac{C_2}{3.4}+...+\frac{C_n}{(n+1)(n+2)}=\frac{1}{n+1}\left[\frac{2^{n+2}}{n+2}-1\right]$

Reason: $\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-...+(-1)^n\frac{C_n}{n+2}=0$

75. Let $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$.

Assertion: $S=C_0+(C_0+C_1)+(C_0+C_1+C_2)+...$
$+(C_0+C_1+...+C_{n-1})=n(2^{n-1})$

Reason: $\sum_{j=1}^{n}\sum_{i<j}(C_i+C_j)=(n+1)2^n$.

76. Let $(1+x)^{3n}=C_0+C_1x+C_2x^2+...+C_{3n}x^{3n}$, and $\omega\neq 1$ be a cube root of unity

Assertion: $C_0+C_1\omega+C_2\omega^2+C_3+C_4\omega+C_5\omega^2+...=(-1)^n$

Reason: Cube roots unity form a triangle of area $\sqrt{3}$ square units.

77. Let $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$

Assertion: $\sum_{r=0}^{n}C_r\sin(rx)\cos(n-r)x=2^n\sin(nx)$

Reason: $\sum_{r=0}^{n}C_r=2^n$

78. **Assertion:** The coefficient of the term of independent of x in the expansion of $\left(x+\frac{9}{x}+6\right)^n$ is $\frac{3^n(2n)!}{n!n!}$.

Reason: The coefficient of x^r in the expansion of $(1+x)^n$ is $\binom{n}{r}$.

79. **Assertion:** For any positive integers m, n (with $n\geq m$),

$\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+....+\binom{m}{n}=\binom{n+1}{m}$

Reason: Coefficient of x^r in the expansion of $(1+x)^n$ is $\binom{n}{r}$.

80. **Assertion:** If n is an odd prime, then greatest integer contained in $\left(2+\sqrt{5}\right)^n-2^{n+1}$ is divisible by 20 n.

Reason: If p is a prime and $1\leq r\leq p-1$, then $\binom{p}{r}$ is divisible by p.

81. **Assertion:** Greatest term in the expansion of $(\sqrt{3}+\sqrt{2})^{50}$ is $\binom{50}{22}3^{14}2^{11}$

Reason: Greatest term in the expansion $(1+x)^n$, $x>0$ of is the r^{th} term if $\frac{(n+1)x}{x+1}$ is not an integer and $r=\left[\frac{(n+1)x}{x+1}\right]$, where [y] denotes the greatest integer $\le y$.

Comprehension Based

Paragraph –I

Consider $(1+x+x^2)^{2n}=\sum_{r=0}^{4n} a_r x^r$, where $a_0, a_1, a_2 ..., a_{4n}$ are real number and n is a positive integer.

82. The value of $\sum_{r=0}^{n-1} a_{2r}$ is:

a. $\frac{9^n-2a_{2n}-1}{4}$

b. $\frac{9^n+2a_{2n}+1}{4}$

c. $\frac{9^n-2a_{2n}+1}{4}$

d. $\frac{9^n+2a_{2n}-1}{4}$

83. The value of $\sum_{r=1}^{n} a_{2r-1}$ is:

a. $\left(\frac{9^n-1}{2}\right)$ **b.** $\left(\frac{3^{2n}-1}{4}\right)$

c. $\left(\frac{3^{2n}+1}{4}\right)$ **d.** $\left(\frac{9^n+1}{2}\right)$

84. The value of a_2 is:

a. ${}^{4n+1}C_2$

b. ${}^{3n+1}C_2$

c. ${}^{2n+1}C_2$

d. ${}^{n+1}C_2$

85. The value of a_{4n-1} is:

a. $2n$

b. $2n^2+4n$

c. $2n+3$

d. $2n^2+3n$

86. The correct statement is:

a. $a_r = a_{n-r}$, $0\le r\le n$

b. $a_{r-r} = a_{n-r}$, $0\le r\le n$

c. $a_r = a_{2n}$, $0\le r\le 2n$

d. $a_r = a_{4n-r}$, $0\le r\le 4n$

Paragraph –II

If m, n, r are positive integers and if $r<m$, $r<n$ then ${}^mC_r + {}^mC_{r-1}\cdot{}^mC_{r-2}\cdot{}^nC_2+...+{}^nC_r$ = Coefficient of x^r in $(1+x)^m(1+x)^n$ = Coefficient of x^r in $(1+x)^{m+n}$ $={}^{m+n}C_r$

87. The value of ${}^nC_0\cdot{}^nC_n+{}^nC_1\cdot{}^nC_{n-1}+...+{}^nC_n\cdot{}^nC_0$ is:

a. ${}^{2n}C_{n-1}$ **b.** ${}^{2n}C_n$

c. ${}^{2n}C_{n+1}$ **d.** ${}^{2n}C_2$

88. The value of r for which ${}^{30}C_r\cdot{}^{20}C_0+{}^{30}C_{r-1}\cdot{}^{20}C_1+...+{}^{30}C_0\cdot{}^{20}C_r$ is maximum, is:

a. 10 **b.** 15

c. 20 **d.** 25

89. The value of $r(0\le r\le 30)$for which ${}^{20}C_r\cdot{}^{10}C_0+{}^{20}C_{r-1}\cdot{}^{10}C_1+...+{}^{20}C_0\cdot{}^{10}C_r$ is minimum, is:

a. 0 **b.** 1

c. 5 **d.** 15

90. If $S_n={}^nC_0\cdot{}^nC_1+{}^nC_1\cdot{}^nC_2+...+{}^nC_{n-1}\cdot{}^nC_n$ and if $\frac{S_{n+1}}{S_n}=\frac{15}{4}$, then n equals:

a. 2, 4 **b.** 4, 6

c. 6, 8 **d.** 8, 10

91. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$ and n is odd, then the value of $C_0^2-C_1^2=C_2^2-C_3^2+...+(-1)^nC_n^2$ is:

a. 0 **b.** ${}^{2n}C_n$

c. $(-1)^n\,{}^{2n}C_{n-1}$ **d.** $3\,{}^{2n}C_{n-2}$

Match the Column

92. Observe the following columns:

Column I	Column II
(A) If λ be the number of terms in the expansion of $(1+5x+10x^2+10x^3+5x^4+x^5)^{20}$ and if unit's place and ten's place digits in 3^λ are O and T, then	**1.** $O+T=3$
(B) If λ be the number of terms in the expansion of $\left(x^2+1+\frac{1}{x^2}\right)$ and if unit's place and ten's place digits in 7^λ are O and T, then	**2.** $O+T=7$

(C) If λ be then number of terms in the expansion of $(1+x)^{101}$ $(1+x^2-x)^{100}$ and if unit's place and ten's place digits in 9^{λ} are O and T, then	**3.** $O+T=9$
	4. $T-O=7$
	5. $O-T=7$

a. A→1, B→2→5, C→3-4 **b.** A→1, B→3-5, C→2-4
c. A→2, B→4-5, C→3-1 **d.** A→2, B→1-4, C→3-5

93. Observe the following columns:

Column I	Column II
(A) If last digit of the number 9^{9^9} is λ and last digit of $2^{\lambda^{100}}$ is μ then	**1.** $\lambda+\mu=9$
(B) If last digit of the number 2^{9999} is λ and last digit of $3^{\lambda\lambda\lambda}$ is μ then	**2.** $\lambda+\mu=11$
(C) Let $a=\dfrac{72!}{(36!)^2}-1$ is divisible by $10\lambda+\mu$ then	**3.** $\lambda-\mu=7$
	4. $\lambda-\mu=4$
	5. $\lambda^{\mu}+\mu^{\lambda}=9$

a. A→2-3, B→1-3-5, C→4
b. A→1-3, B→2-3-4, C→5
c. A→1-3, B→2-3-4, C→5
d. A→4-3, B→1-3-5, C→2

94. Observe the following columns:

Column I	Column II
(A) If n be the degree of the polynomial $\sqrt{(3x^2+1)}\{(x+\sqrt{3x^2+1}))^7$ $-(x-\sqrt{(3x^2+1)}\}$, then n divisible by	**1.** 2
(B) In the expansion of $(x+a)^n$ there is only one middle term for x = 3, a = 2 and seventh term is numerically greatest term, then n divisible by	**2.** 4
(C) The sum of the binomial coefficients in the expansion of $(x^{-3/4}+nx^{5/4})^m$, where m is positive integer lies between 200 and 400 and the term independent of x equals 448. Then n^5 divisible by.	**3.** 8
	4. 16
	5. 32

a. A→1,2,3; B→1,2,3,4; C→1,2,3,4,5
b. A→3,2,1; B→4,2,3,1; C→1,2,3,4,5
c. A→1,2,3; B→3,2,1,4; C→5,2,3,4,1
d. A→1,2,3; B→1,2,3,4; C→1,3,2,5,4

Integer

95. The coefficient of x^{50} in the polynomials after parenthesis have been removed and like terms have been collected in the expansion

$(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}$ $+...+x^{1000}$ is $\dfrac{\lambda!}{\mu!\nu!}$, then the value of $\lambda+2\mu$ $+3\nu$ must be $(\nu>\mu)$?

96. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$ and $\sum_{r=0}^{51}\dfrac{C_r^2}{(r+1)}$ $=\dfrac{\lambda!}{\left\{\left(\frac{\lambda+1}{2}\right)!\right\}^2}$ then the number of zeros in $\lambda!$must be:

97. If $(1+x)^n=C_0+C_1x+C_2x^2+...+C_nx^n$ and $\sum_{r=0}^{n}(r+1)^2C_r$ $=2^{n-2}f(n)$, if roots of the equation $f(x)$ = 0 are α andβ, then $\alpha^4+\beta^4$ must be:

98. If $\sum_{r=0}^{n}{}^nC_r\sin(rx)\cos\{(n-r)x\}=f(n)\sin(nx)$, then the value of $f(13)$ must be:

99. Let $a_r=r^2\cdot\dfrac{^{100}C_r}{^{100}C_{r-1}}$ and λ be the coefficient of x^{99} in $\prod_{i=1}^{100}(x-a_i)$, then the value of $-\dfrac{\lambda}{50}$ must be:

100. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then, n is equal to:

ANSWER

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
b	c	b	c	b	c	b	c	b	b
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
c	a	a	c	b	c	a	c	c	c
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
d	a	b	d	b	d	b	c,d	d	c
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
c	a	b	c	a	d	a	c	b	c
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
c	c	a	c	b	a	b	d	c	b
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
b	d	b	a	b	b	d	a	d	a
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
b	c	d	c	c	b	d	a	d	b
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
b	c	a	b	c	c	d	a	d	a
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**	**88.**	**89.**	**90.**
d	c	b	c	a	d	b	d	a	a
91.	**92.**	**93.**	**94.**	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
a	a	a	a	3954	0024	257	4096	3333	6

SOLUTION

Multiple Choice Questions

1. **(b)** Let $P(n)$: $n! < \left(\frac{n+1}{2}\right)^n$

Step (i): For $n = 2$

$\Rightarrow \quad 2! < \left(\frac{2+1}{2}\right)^2$

$\Rightarrow \quad 2 < \frac{9}{4}$

$\Rightarrow \quad 2 < 2.25$

which is true. Therefore, $P(2)$ is true.

Step (ii): Assume that $P(k)$ is true, then $p(k)$: $k! < \left(\frac{k+1}{2}\right)^k$

Step (iii): For $n = k + 1$,

$P(k+1) : (k+1)! < \left(\frac{k+2}{2}\right)^{k+1}$

$\Rightarrow \quad k! < \left(\frac{k+1}{2}\right)^k$

$\Rightarrow \quad (k+1)k! < \frac{(k+1)^{k+1}}{2^k}$

$\Rightarrow \quad (k+1)! < \frac{(k+1)^{k+1}}{2^k}$ . . . (i)

and $\frac{(k+1)^{k+1}}{2^k} < \left(\frac{k+2}{2}\right)^{k+1}$ . . . (ii)

$\Rightarrow \quad \left(\frac{k+2}{k+1}\right)^{k+1} > 2 \Rightarrow \left[1+\frac{1}{k+1}\right]^{k+1} > 2$

$\Rightarrow \quad 1+(k+1)\frac{1}{k+1}+{}^{k+1}C_2\left(\frac{1}{k+1}\right)^2+..... > 2$

$\Rightarrow \quad 1+1+{}^{k+1}C_2\left(\frac{1}{k+1}\right)^2+.... > 2$

Which is true, hence (ii) is true.

From (i) and (ii), $(k+1)! < \frac{(k+1)^{k+1}}{2^k} < \left(\frac{k+2}{2}\right)^{k+1}$

$\Rightarrow \quad (k+1)! < \left(\frac{k+2}{2}\right)^{k+1}$

Hence $P(k+1)$ is true. Hence by the principle of mathematical induction $P(n)$ is true for all $n \in N$

By check option

(a) For $n = 1$, $1! < \left(\frac{1+1}{2}\right)^1 \Rightarrow 1 < 1$ which is wrong

(b) For $n = 2$, $2! < \left(\frac{3}{2}\right)^2 \Rightarrow 2 < \frac{9}{4}$ which is correct

(c) For $n = 3$, $3! < \left(\frac{3+1}{2}\right)^3 \Rightarrow 6 < 8$ which is correct

(d) For $n = 4$, $4! < \left(\frac{4+1}{2}\right)^4 \Rightarrow 24 < \left(\frac{5}{2}\right)^4$

$\Rightarrow$ 24 < 39.0625 which is correct. But smallest positive integer n is 2.

2. **(c)** We have $S(k) = 1+3+5+......+(2k-1) = 3+k^2$

$S(1) \Rightarrow 1 = 4$, Which is not true and $S(2) \Rightarrow 3 = 7$.

Which is not true?

Hence induction cannot be applied and $S(k) \not\Rightarrow S(k+1)$

3. **(b)** $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+...$

$\Rightarrow \quad (1+x)^n - nx - 1 = x^2\left[\frac{n(n-1)}{2!}+\frac{n(n-1)(n-3)}{3!}x+.....\right]$

From above it is clear that $(1+x)^n - nx - 1$ is divisible by x^2.

Short Trick: $(1+x)^n - nx - 1$.

Put $n = 2$ and $x = 3$

Then $4^2 - 2.3 - 1 = 9$

Is not divisible by 6, 54 but divisible by 9.

Which is given by option (c) = $x^2 = 9$.

4. **(c)** $(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 + ... + {}^nC_nx^{n-n}.a^n =$

$(x^n + {}^nC_2x^{n-2}a^2 + ..) + ({}^nC_1x^{n-1}a^1 + {}^nC_3x^{n-3}a^3 + ...)$

$= A + B$ . . .(*i*)

Similarly, $(x-a)^n = A - B$ . . . (*ii*)

From (*i*) and (*ii*), we get $4AB = (x+a)^{2n} - (x-a)^{2n}$

Short Trick: Put $n = 1$ in $(x+a)^n$.

Then, $x + a = A + B$. Comparing both sides $A = x, B = a$.

Option (c) L.H.S. $4AB = 4xa$, R.H.S. $(x+a)^2 - (x-a)^2 = 4ax$. *i.e.,* L.H.S. = R.H.S

5. **(b)** We have $T_4 = \frac{5}{2}$

$\Rightarrow T_{3+1} = \frac{5}{2}$

$\Rightarrow {}^mC_3\,(px)^{m-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$

$\Rightarrow {}^mC_3\,p^{m-3}x^{m-6} = \frac{5}{2}$ . . .(*i*)

Clearly, R.H.S. of the above equality is independent of *x*

$\therefore\ m - 6 = 0,\ m = 6$

Putting $m = 6$ in (*i*)

We get ${}^6C_3p^3 = \frac{5}{2} \Rightarrow p = \frac{1}{2}$.

Hence $p = 1/2, m = 6$.

6. **(c)** $\frac{T_2}{T_3} = \frac{2}{n-2+1}.\frac{b}{a} = \frac{2}{n-1}\left(\frac{b}{a}\right)$

and $\frac{T_3}{T_4} = \frac{3}{n+3-3+1}.\left(\frac{b}{a}\right) = \frac{3}{n+1}\left(\frac{b}{a}\right)$

$\because\ \frac{T_2}{T_3} = \frac{T_3}{T_4}$ (given)

$\therefore\ \frac{2}{n-1}\left(\frac{b}{a}\right) = \frac{3}{n+1}\left(\frac{b}{a}\right)$

$\Rightarrow 2n + 2 = 3n - 3 \Rightarrow n = 5$

7. **(b)** $(10-r)\left(\frac{1}{2}\right) + r(-2) = 0 \Rightarrow r = 2$

$\therefore\ T_3 = {}^{10}C_2\left(\frac{1}{3}\right)^{8/2}\left(\frac{3}{2}\right)^2 = \frac{5}{4}$

8. **(c)** We know that, $(1+x)^n = {}^nC_0 + {}^nC_1x^1 + {}^nC_2x^2 + ... + {}^nC_nx^n$

$\left(1+\frac{1}{x}\right)^n = {}^nC_0 + {}^nC_1\frac{1}{x^1} + {}^nC_2\frac{1}{x^2} + ... + {}^nC_n\frac{1}{x^n}$

Obviously, the term independent of *x* will be

${}^nC_0.{}^nC_0 + {}^nC_1{}^nC_1 + ... + {}^nC_n.{}^nC_n = C_0^2 + C_1^2 + ... + C_n^2$

Put $n = 1$ in the expansion of $(1+x)^1\left(1+\frac{1}{x}\right)^1$

$= 1 + x + \frac{1}{x} + 1 = 2 + x + \frac{1}{x}$ . . . (*i*)

We want coefficient of x^0. Comparing to equation (*i*). Then, we get 2 *i.e.,* independent of *x*.

Option (c): $C_0^2 + C_1^2 + ...C_n^2$;

Put $n = 1$; Then ${}^1C_0^2 + {}^1C_1^2 = 1 + 1 = 2$.

9. **(b)** Coefficient of x^n in $(1+x)(1-x)^n$ = Coefficient of x^n in $(1-x)^n$ + coefficient of x^{n-1} in $(1-x)^n$ = Coefficient of x^n in $[{}^nC_n(-x)^n + x.{}^nC_{n-1}(-x)^{n-1}]$ = $(-1)^n\ {}^nC_n + (-1)^{n-1}.{}^nC_1$ = $(-1)^n + (-1)^n.(-n) = (-1)^n[1-n]$.

10. **(b)** Applying $T_{r+1} = {}^nC_rx^{n-r}a^r$ for $(x+a)^n$

Hence $T_6 = {}^{10}C_5(2x^2)^5\left(-\frac{1}{3x^2}\right)^5$

$= -\frac{10!}{5!5!}32 \times \frac{1}{243} = -\frac{896}{27}$

11. **(c)** ${}^{20}C_{r-1} = {}^{20}C_{r+3}$

$\Rightarrow 20 - r + 1 = r + 3 \Rightarrow r = 9$.

12. **(a)** ${}^{15}C_{2r+2} = {}^{15}C_{r-2}$

But ${}^{15}C_{2r+2} = {}^{15}C_{15-(2r+2)} = {}^{15}C_{13-2r}$

$\Rightarrow {}^{15}C_{13-2r} = {}^{15}C_{r-2}$

$\Rightarrow r = 5$.

13. **(a)** We have $T_{r+1} = {}^{21}C_r\left(\sqrt[3]{\frac{a}{\sqrt{b}}}\right)^{21-r}\left(\sqrt{\frac{b}{\sqrt[3]{a}}}\right)^r$

$= {}^{21}C_ra^{7-(r/2)}b^{(2/3)r-(7/2)}$

Since the powers of *a* and *b* are the same, therefore

$7 - \frac{r}{2} = \frac{2}{3}r - \frac{7}{2} \Rightarrow r = 9$

14. **(c)** $T_1 = {}^nC_0 = 1$ . . . (i)

$T_2 = {}^nC_1ax = 6x$ . . . (ii)

$T_3 = {}^nC_2(ax)^2 = 16x^2$ . . . (iii)

From (ii), $\frac{n!}{(n-1)!}a = 6 \Rightarrow na = 6$ . . . (iv)

From (iii), $\frac{n(n-1)}{2}a^2 = 16$ . . . (v)

Only (c) is satisfying equation (iv) and (v).

15. **(b)** In the expansion of $\left(x^2+\frac{a}{x}\right)^5$ the general term is

$$T_{r+1}={}^5C_r(x^2)^{5-r}\left(\frac{a}{x}\right)^r={}^5C_ra^rx^{10-3r}$$

Here, exponent of x is $10-3r=1\Rightarrow r=3$

$\therefore\quad T_{2+1}={}^5C_3a^3x=10a^3.x$

Hence coefficient of x is $10a^3$.

16. **(c)** $2(5-r)+(-1)\,r=1$

$\Rightarrow\quad 10-2r-r=1\Rightarrow r=3$

Thus coefficient of y is ${}^5C_3c^3=\frac{5\times4}{2\times1}c^3=10c^3$.

17. **(a)** In the expansion of $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$, the general term is

$$T_{r+1}={}^{10}C_r\left(\frac{x}{2}\right)^{10-r}.\left(-\frac{3}{x^2}\right)^r$$

$$={}^{10}C_r(-1)^r.\frac{3^r}{2^{10-r}}x^{10-r-2r}$$

Here, the exponent of x is $10-3r=4\Rightarrow r=2$

$\therefore\quad T_{2+1}={}^{10}C_2\left(\frac{x}{2}\right)^8\left(-\frac{3}{x^2}\right)^2$

$$=\frac{10.9}{1.2}.\frac{1}{2^8}.3^2.x^4=\frac{405}{256}x^4$$

$\therefore\quad$ The required coefficient $=\frac{405}{256}$.

18. **(c)** $(8-r)(2)+r(-1)=7\Rightarrow 16-2r-r=7$

$\Rightarrow\quad r=3$

Thus coefficient of x^7 is

$$={}^8C_3\left(\frac{1}{2}\right)^5(-2)^3=-\frac{8\times7\times6}{3\times2\times1}\times\frac{1}{4}=-14.$$

19. **(c)** Let T_{r+1} term containing x^{32}.

Therefore ${}^{15}C_rx^{4r}\left(\frac{-1}{x^3}\right)^{15-r}$

$\Rightarrow\quad x^{4r}x^{-45+3r}=x^{32}\Rightarrow 7r=77\Rightarrow r=11$.

Hence coefficient of x^{32} is ${}^{15}C_{11}$ or ${}^{15}C_4$

20. **(c)** $(1+x)^m(1-x)^n$

$$=\left(1+mx+\frac{m(m-1)x^2}{2!}+...\right)\left(1-nx+\frac{n(n-1)}{2!}x^2-...\right)$$

$$=1+(m-n)x+\left[\frac{n^2-n}{2}-mn+\frac{(m^2-m)}{2}\right]x^2+...$$

Given, $m-n=3$ or $n=m-3$

Hence $\frac{n^2-n}{2}-mn+\frac{m^2-m}{2}=-6$

$\Rightarrow\quad \frac{(m-3)(m-4)}{2}-m(m-3)+\frac{m^2-m}{2}=-6$

$\Rightarrow\quad m^2-7m+12-2m^2+6m+m^2-m+12=0$

$\Rightarrow\quad -2m+24=0\Rightarrow m=12$

21. **(d)** Coefficients of 2nd, 3rd and 4th terms are respectively ${}^nC_1, {}^nC_2$ and ${}^nC_3$ are in A.P.

$\Rightarrow\quad 2.{}^nC_2={}^nC_1+{}^nC_3$

$\Rightarrow\quad \frac{2n!}{2!(n-2)!}=\frac{n!}{(n-1)!}+\frac{n!}{3!(n-3)!}$

On solving, $n^2-9n+14=0$

$\Rightarrow\quad n^2-9n=-14$.

22. **(a)** $T_{r+1}={}^{15}C_r(x^4)^{15-r}(-1/x^3)^r=(-1)^r\,{}^{15}C_r(x)^{60-7r}$

For coefficient of x^{39}, $60-7r=39\Rightarrow r=3$

$\therefore\quad T_4={}^{15}C_3(x^4)^{12}(-1/x^3)^3$

$=-455\,x^{39}$

Hence the required coefficient is – 455.

23. **(b)** $T_2={}^{2n}C_1\;x$, $T_3={}^{2n}C_2\;x^2$

$T_4={}^{2n}C_3\;x^3$

Coefficient of T_2, T_3, T_4 are in A.P.

$\Rightarrow\quad 2.{}^{2n}C_2={}^{2n}C_1+{}^{2n}C_3$

$\Rightarrow\quad 2\frac{2n!}{2!(2n-2)!}=\frac{2n!}{(2n-1)!}+\frac{2n!}{3!(2n-3)!}$

$\Rightarrow\quad \frac{2.2n(2n-1)}{2}=2n+\frac{2n(2n-1)(2n-2)}{6}$

$\Rightarrow\quad n(2n-1)=n+\frac{(n)(2n-1)(2n-2)}{6}$

$\Rightarrow\quad 6(2n^2-n)=6n+4n^3-6n^2+2n$

$\Rightarrow\quad 6n(2n-1)=2n(2n^2-3n+4)$

$\Rightarrow\quad 6n-3=2n^2-3n+4$

$\Rightarrow\quad 0=2n^2-9n+7$

$\Rightarrow\quad 2n^2-9n+7=0$.

24. **(d)** $T_{r+1}={}^9C_r(3)^{9-r}(ax)^r={}^9C_r(3)^{9-r}a^rx^r$

$\therefore\quad$ Coefficient of $x^r={}^9C_r3^{9-r}a^r$

Hence, coefficient of $x^2={}^9C_23^{9-2}a^2$ and coefficient of $x^3={}^9C_33^{9-3}a^3$

So, we must have ${}^9C_23^7a^2={}^9C_33^6a^3$

$\Rightarrow\quad \frac{9.8}{1.2}.3=\frac{9.8.7}{1.2.3}.a\Rightarrow a=\frac{9}{7}.$

25. **(b)** Given, total number of terms

$= \frac{(n+1)(n+2)}{2} = 45$

$\Rightarrow \quad (n+1)(n+2) = 90 \Rightarrow n = 8$.

26. **(d)** Since $2n$ is even, so middle term $T_{\frac{2n}{2}+1} = T_{n+1}$

$\Rightarrow \quad T_{n+1} = {}^{2n}C_n x^n = \frac{(2n)!}{n!.n!} x^n$

$= \frac{1.3.5........(2n-1)}{n!} .2^n x^n$.

27. **(b)** n =9, $\alpha = 2$, $\beta = 1$.

Then $r = \frac{9(2)}{1+2} = 6$.

Hence, $T_7 = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6$

$= \frac{9\times8\times7}{3\times2\times1}.\frac{1}{2^3.3^3} = \frac{7}{18}$

28. **(c,d)** $(3+2x)^{50} = 3^{50}\left[1+\frac{2x}{3}\right]^{50}$,

Now greatest term in $\left(1+\frac{2x}{3}\right)^{50}$

$r = \left|\frac{x(n+1)}{1+x}\right| = \left|\frac{\frac{2x}{3}(50+1)}{\frac{2x}{3}+1}\right| = \frac{\frac{2.\frac{1}{5}}{3}(51)}{\frac{2}{15}+1} = 6$ (an integer)

$\therefore$ T_r and $T_{[r]+1}$= T_6 and $T_{[6]+1}$= T_6 and T_7 are numerically greatest terms

29. **(d)** Take $n = 2m$,

Then, $S_n = \frac{1}{{}^{2m}C_0} + \frac{1}{{}^{2m}C_1} + ... + \frac{1}{{}^{2m}C_{2m}}$

$= 2\left[\frac{1}{{}^{2m}C_0} + \frac{1}{{}^{2m}C_1} + ... + \frac{1}{{}^{2m}C_{m-1}}\right] + \frac{1}{{}^{2m}C_m}$

$t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r} = \sum_{r=0}^{2m} \frac{r}{{}^{2m}C_r} = \frac{1}{{}^{2m}C_1} + \frac{2}{{}^{2m}C_2} + ... + \frac{2m}{{}^{2m}C_{2m}}$

$t_n = \left(\frac{1}{{}^{2m}C_1} + \frac{2m-1}{{}^{2m}C_{2m-1}}\right) + \left(\frac{2}{{}^{2m}C_2} + \frac{2m-2}{{}^{2m}C_{2m-2}}\right) +$

$...\left(\frac{m-1}{{}^{2m}C_{m-1}} + \frac{m+1}{{}^{2m}C_{m+1}}\right) + \frac{m}{{}^{2m}C_m} + \frac{2m}{{}^{2m}C_{2m}}$

$= 2m\left[\frac{1}{{}^{2m}C_1} + \frac{1}{{}^{2m}C_2} + ...\right] + \frac{m}{{}^{2m}C_m} + 2m$

$= 2m\left[\frac{1}{{}^{2m}C_0} + \frac{1}{{}^{2m}C_1} + ... + \frac{1}{{}^{2m}C_{m-1}}\right] + \frac{m}{{}^{2m}C_m}$

$= m\left[S_n - \frac{1}{{}^{2m}C_m}\right] + \frac{m}{{}^{2m}C_m} = mSn$

$t_n = mS_n \Rightarrow \frac{t_n}{S_n} = m = \frac{n}{2}$

30. **(c)** Putting $x = 1$ in $(1+x)^5$, the required sum of coefficient

$= (1+1)^5 = 2^5 = 32$

31. **(c)** The sum of coefficient of polynomial $(\alpha^2x^2 - 2\alpha x + 1)^{51}$ is obtained by putting $x = 1$ in $(\alpha^2x^2 - 2\alpha x + 1)^{51}$.

Therefore by hypothesis $(\alpha^2 - 2\alpha + 1)^{51} = 0$

$\Rightarrow \quad \alpha = 1$

32. **(a)** We know that

$2^{n-1} = {}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}^nC_5 + ...$

So, ${}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + ... + {}^{10}C_9 = 2^{10-1} = 2^9$

33. **(b)** $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n \quad ...(i)$

and $\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\left(\frac{1}{x}\right)^2 + ... + C_n\left(\frac{1}{x}\right)^n \quad ...(ii)$

If we multiply (i) and (ii), we get $C_0^2 + C_1^2 + C_2^2 + ... + C_n^2$ is the term independent of x and hence it is equal to the term independent of x in the product $(1+x)^n\left(1+\frac{1}{x}\right)^n$ or in $\frac{1}{x^n}(1+x)^{2n}$ or term containing x^n in $(1+x)^{2n}$. Clearly the coefficient of x^n in $(1+x)^{2n}$ is T_{n+1} and equal to ${}^{2n}C_n = \frac{(2n)!}{n!\,n!}$

Solving conversely.

Put $n = 1, n = 2,.....$ then we get $S_1 = {}^1C_0^2 + {}^1C_1^2 = 2$,

$S_2 = {}^2C_0^2 + {}^2C_1^2 + {}^2C_2^2 = 1^2 + 2^2 + 1^2 = 6$

Now check the options

(a) Does not hold given condition,

(b) (i) Put $n = 1$, then $\frac{2!}{1!1!} = 2$

(ii) Put $n = 2$, then $\frac{4!}{2!2!} = \frac{4\times3\times2\times1}{2\times1\times2\times1} = 6$

Students should remember this question as an identity.

34. **(c)** $\frac{C_1}{C_0}+2.\frac{C_2}{C_1}+3.\frac{C_3}{C_2}+.....+n.\frac{C_n}{C_{n-1}}$

$=\frac{n}{1}+2\frac{n(n-1)/1.2}{n}+3\frac{n(n-1)(n-2)/3.2.1}{n(n-1)/1.2}+....+n.\frac{1}{n}$

$=n+(n-1)+(n-2)....+1=\sum n=\frac{n(n+1)}{2}$

Put $n=1,2,3$ ....., then $S_1=\frac{{}^1C_1}{{}^1C_0}=1$,

$S_2=\frac{{}^2C_1}{{}^2C_0}+2\frac{{}^2C_2}{{}^2C_1}=\frac{2}{1}+2.\frac{1}{2}=2+1=3$

By option, (put n=1,2......) (a) and (b) does not hold condition, but (c) $\frac{n(n+1)}{2}$, put n =1, 2......

$S_1=1, S_2=3$ which is correct.

35. **(a)** We have $(1+x)^{10}=C_0+C_1x+C_2x^2+...+C_{10}x^{10}$

Integrating both sides from 0 to 2, we get

$\frac{3^{11}-1}{11}=2C_0+\frac{2^2}{2}C_1+\frac{2^3}{3}C_2+....+\frac{2^{11}}{11}C_{10}$.

36. **(d)** $(1-x)^n=C_0-C_1x+C_2x^2-C_3x^3+.....$

$\Rightarrow$ $x(1-x)^n=C_0x-C_1x^2+C_2x^3-C_3x^4+.....$

$\Rightarrow$ $\int_0^1 x(1-x)^n dx=\int_0^1 (C_0x-C_1x^2+C_2x^3....)dx$ . . . (i)

The integral on the LHS

$=\int_1^0 (1-t)t^n(-dt)$, by putting $1-x=t$

$=\int_0^1 (t^n-t^{n+1})\,dt=\frac{1}{n+1}-\frac{1}{n+2}$

Whereas the integral on the RHS of (i)

$=\left[\frac{C_0x^2}{2}-\frac{C_1x^3}{3}+\frac{C_2x^4}{4}-....\right]=\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-....$

$\therefore$ $\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-....$ to $(n+1)$ terms

$=\frac{1}{n+1}-\frac{1}{n+2}=\frac{1}{(n+1)(n+2)}$.

37. **(a)** We know that $\frac{(1+x)^n-(1-x)^n}{2}=C_1x+C_3x^3+C_5x^5+...$

Integrating from $x=0$ to $x=1$, we get

$\frac{1}{2}\int_0^1 \{(1+x)^n-(1-x)^n\}\,dx$

$=\int_0^1 (C_1x+C_3x^3+C_5x^5+....)dx$

$\Rightarrow$ $\frac{1}{2}\left\{\frac{(1+x)^{n+1}}{n+1}+\frac{(1-x)^{n+1}}{n+1}\right\}_0^1=\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+....$

or $\frac{C_1}{2}+\frac{C_3}{4}+\frac{C_5}{6}+....=\frac{1}{2}\left\{\frac{2^{n+1}-1}{n+1}+\frac{0-1}{n+1}\right\}$

$=\frac{1}{2}\left(\frac{2^{n+1}-2}{n+1}\right)=\frac{2^n-1}{n+1}$

38. **(c)** Putting x = 1 in $(x^2+x-3)^{319}$

We get the sum of coefficient = $(1+1-3)^{319}=-1$

39. **(b)** L.H.S. $=a[C_0-C_1+C_2-C_3+...(-1)^n.C_n]$

$+[C_1-2C_2+3C_3-....+(-1)^{n-1}n.C_n]$

$=a.0+0=0$

40. **(c)** The sum of the coefficients of the polynomial $(\alpha^2x^2-2\alpha x+1)^{51}$ is obtained by putting $x=1$ in $(\alpha^2x^2-2\alpha x+1)^{51}$.

Therefore by hypothesis $(\alpha^2-2\alpha+1)^{51}=0\Rightarrow\alpha=1$

41. **(c)** We know that,

$(1+x)^n=C_0+C_1x+C_2x^2+.....+C_nx^n$ . . . (i)

Differentiating both sides w.r.t. x,

We get $n(1+x)^{n-1}=0+C_1+2.C_2x+3C_3x^2+...+nC_nx^{n-1}$

Putting $x=1$,

We get, $n.2^{n-1}=C_1+2C_2+3C_3+......+nC_n$.

42. **(c)** Consider the expansion

$(1+x)^n=C_0+C_1x+C_2x^2+....+C_nx^n$ . . .(i)

Integrating both sides of (i) within limits 0 to 1.

We get $\int_0^1 (1+x)^n dx=\int_0^1 C_0+\int_0^1 C_1x+\int_0^1 C_2x^2+...+\int_0^1 C_nx^n dx$

$\left[\frac{(1+x)^{n+1}}{n+1}\right]_0^1=C_0[x]_0^1+C_1\left[\frac{x^2}{2}\right]_0^1+...C_n\left[\frac{x^{n+1}}{n+1}\right]_0^1$

$\frac{2^{n+1}}{n+1}-\frac{1}{n+1}=C_0[1]+C_1\frac{1}{2}+C_2\frac{1}{3}+...C_n.\frac{1}{n+1}$

$C_0+\frac{C_1}{2}+\frac{C_2}{3}+...\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$.

43. **(a)** Since $f=R-[R]$, $R=f+[R]$

$[5\sqrt{5}+11]^{2n+1}=f+[R]$, where $[R]$ is integer

Now let $f'=[5\sqrt{5}-11]^{2n+1}, 0<f'<1$

$f+[R]-f'=[5\sqrt{5}+11]^{2n+1}-[5\sqrt{5}-11]^{2n+1}$

$=2[{}^{2n+1}C_1(5\sqrt{5})^{2n}(11)^1+{}^{2n+1}C_3(5\sqrt{5})^{2n-2}(11)^3+...]$

$=2.$(Integer) $=2K$ $(K\in N)$ = Even integer

Hence $f - f' =$ even integer – [R], but $-1 < f - f' < 1$.

Therefore, $f - f' = 0$

$\therefore \quad f = f'$

Hence R.f = $R.f^1 = (5\sqrt{5}+1)^{2n+1}(5\sqrt{5}-11)^{2n+1} = 4^{2n+1}$.

44. (c) Coefficient of x^5 in the expansion of $(x^2 - x - 2)^5$ is

$$\sum \frac{5!}{n_1!.n_2!n_3!}(1)^{n_1}(-1)^{n_2}(-2)^{n_3}.$$

Where $n_1 + n_2 + n_3 = 5$ and $n_2 + 2n_3 = 5$. The possible value of n_1, n_2 and n_3 are shown in margin

n_1	n_2	n_3
1	3	1
2	1	2
0	5	0

$\therefore$ The coefficient of x^5

$$= \frac{5!}{1!3!1!}(1)^1(-1)^3(-2)^1 + \frac{5!}{2!1!2!}(1)^2(-1)^1(-2)^2$$

$$+ \frac{5!}{0!5!0!}(1)^0(-1)^5(-2)^0$$

$$= 40 - 120 - 1 = -81$$

45. (b) In this case, $a^3b^4c^5 = (ab)^x(bc)^y(ca)^z = a^{x+z}.b^{x+y}.c^{y+z}$

$z + x = 3,\ x + y = 4, y + z = 5;\ 2(x + y + z) = 12;$

$x + y + z = 6$.

Then $x = 1, y = 3, z = 2$

Therefore the coefficient of $a^3b^4c^5$ in the expansion of $(bc + ca + ab)^6 = \frac{6!}{1!3!2!} = 60$.

46. (a) We know that $(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + ...$

$$(1+ax)^n = 1 + \frac{n(ax)}{1!} + \frac{n(n-1)(ax)^2}{2!} + ...$$

$$\Rightarrow \quad 1 + 8x + 24x^2 + ... = 1 + \frac{n(ax)}{1!} + \frac{n(n-1)(ax)^2}{2!} + ...$$

Comparing coefficients of both sides we get, $na = 8$,

and $\frac{n(n-1)a^2}{2!} = 24$ on solving, $a = 2$, $b = 4$.

47. (b) Coefficient of

$$x^r = \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)....\left(-\frac{1}{2}-r+1\right)}{r!}(-2)^r$$

$$= \frac{1.3.5...(2r-1).(-1)^r.(-1)^r.2^r}{2^r\, r!}$$

$$= \frac{1.3.5...(2r-1)}{r!} = \frac{(2r)!}{r!r!2^r}$$

48. (d) $(1-2x)^{-1/2}(1-4x)^{-5/2} = 1 + kx$

$$\left[1 + \frac{(-1/2)(-2x)}{1!} + \frac{(-1/2)(-3/2)(-2x)^2}{2!} + ...\right]$$

$$\left[1 + \frac{(-5/2)(-4x)}{1!} + \frac{(-5/2)(-7/2)(-4x)^2}{2!} + ...\right] = 1 + kx$$

Higher power can be neglected.

Then $\left[1 + \frac{x}{1!}\right]\left[1 + \frac{10x}{1!}\right] = 1 + kx$; $1 + 10x + x = 1 + kx$

$k = 11$

49. (c) Let a_1, a_2, a_3, a_4 be respectively the coefficients of $(r+1)^{th}, (r+2)^{th}, (r+3)^{th}, (r+4)^{th}$ terms in the expansion of $(1+x)^n$.

Then $a_1 = {}^nC_r, a_2 = {}^nC_{r+1}, a_3 = {}^nC_{r+2}, a_4 = {}^nC_{r+3}$.

Now, $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}} + \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$

$$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} + \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} = \frac{{}^nC_r}{\frac{n+1}{r+1}\,{}^nC_r} + \frac{{}^nC_{r+2}}{\frac{n+1}{r+3}\,{}^nC_{r+2}}$$

$$= \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2(r+2)}{n+1}$$

$$= 2.\frac{{}^nC_{r+1}}{{}^{n+1}C_{r+2}} = 2.\frac{{}^nC_{r+1}}{{}^nC_{r+1} + {}^nC_{r+2}} = \frac{2a_2}{a_2 + a_3}$$

50. (b) $T_{r+1} = {}^{256}C_r.3^{\frac{256-r}{2}}.5^{\frac{r}{8}}$

First term = ${}^{256}C_0\, 3^{128}5^0 =$ integer and after eight terms, *i.e.*, 9th term = ${}^{256}C_8 3^{124}.5^1 =$ integer

Continuing like this, we get an A.P., $1^{st}, 9^{th}.......257^{th}$;

$T_n = a + (n-1)d$

$\Rightarrow \quad 257 = 1 + (n-1)8$

$\Rightarrow \quad n = 33$

51. (b) Expansion of $(1-2x)^{3/2}$

$$= 1 + \frac{3}{2}(-2x) + \frac{3}{2}.\frac{1}{2}.\frac{1}{2}(-2x)^2 + \frac{3}{2}.\frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{6}(-2x)^3 + ...$$

Hence 4^{th} term is $\frac{x^3}{2}$

52. (d) The given expression can be written as

$4^{-1/2}\left(1 - \frac{3}{4}x\right)^{-1/2}$ and it is valid only when

$\left|\frac{3}{4}x\right| < 1 \Rightarrow -\frac{4}{3} < x < \frac{4}{3}$.

53. **(b)** $\frac{1}{(6-3x)^{1/3}} = (6-3x)^{-1/3}$

$= 6^{-1/3}\left[1-\frac{x}{2}\right]^{-1/3}$

$= 6^{-1/3}\left[1+\left(-\frac{1}{3}\right)\left(-\frac{x}{2}\right)x+\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2.1}\left(-\frac{x}{2}\right)^2+....\right]$

$= 6^{-1/3}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+....\right]$

54. **(a)** $\left(\frac{a+x}{a}\right)^{-1/2}+\left(\frac{a-x}{a}\right)^{-1/2}$

$=\left(1+\frac{x}{a}\right)^{-1/2}+\left(1-\frac{x}{a}\right)^{-1/2}$

$=\left[1+\left(-\frac{1}{2}\right)\left(\frac{x}{a}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2.1}\left(\frac{x}{a}\right)^2+....\right]$

$+\left[1+\left(-\frac{1}{2}\right)\left(-\frac{x}{a}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2.1}\left(-\frac{x}{a}\right)^2+....\right]$

$=2+\frac{3x^2}{4a^2}+...$

Here odd terms cancel each other.

55. **(b)** $(1-x)^{-4}=\left[\frac{1.2.3}{6}x^0+\frac{2.3.4}{6}x+\frac{3.4.5}{6}x^2+\frac{4.5.6}{6}x^3+...\right.$

$\left.+\frac{(r+1)(r+2)(r+3)}{6}x^r+...\right]$

Therefore $T_{r+1}=\frac{(r+1)(r+2)(r+3)}{6}x^r$.

56. **(b)** $(x+2)^{-4}=2^{-4}\left[1+\frac{x}{2}\right]^{-4}$

$=\frac{1}{16}\left[1-2x+\frac{5}{2}x^2-....\right]$

57. **(d)** Given that $|x|>1$.

So given expression can be written as

$x^{-2}\left(1+\frac{1}{x}\right)^{-2}$

$=x^{-2}\left[1-\frac{2}{x}+\frac{3}{x^2}-\frac{4}{x^3}+....\right]$

$=\left[\frac{1}{x^2}-\frac{2}{x^3}+\frac{3}{x^4}-\frac{4}{x^5}+....\right]$

58. **(a)** $(7.995)^{1/3}=(8-0.005)^{1/3}=(8)^{1/3}\left[1-\frac{0.005}{8}\right]^{1/3}$

$=2\left[1-\frac{1}{3}\times\frac{0.005}{8}+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2+1}\left(\frac{0.005}{8}\right)^2+.....\right]$

$=2\left[1-\frac{0.005}{24}-\frac{\frac{1}{3}\times\frac{1}{3}}{1}\times\frac{(0.005)^2}{8}+.....\right]$

$=2(1-0.000208)=2\times0.999792=1.9995$

59. **(d)** Let $(1+y)^n=1+\frac{1}{3}x+\frac{1.4}{3.6}x^2+\frac{1.4.7}{3.6.9}x^3+....$

$=1+ny+\frac{n(n-1)}{2!}y^2+.....$

Comparing the terms, we get

$ny=\frac{1}{3}x, \frac{n(n-1)}{2!}y^2=\frac{1.4}{3.6}x^2$

Solving, $n=-\frac{1}{3}, y=-x$.

Hence given series $=(1-x)^{-1/3}$

60. **(a)** We have, $T_{r+1}=\frac{\frac{7}{2}\left(\frac{7}{2}-1\right)\left(\frac{7}{2}-2\right)...\left(\frac{7}{2}-r+1\right)x^r}{r!}$

This will be the first negative term when $\frac{7}{2}-r+1<0$ *i.e.* $r>\frac{9}{2}$ Hence $r=5$.

61. **(b)** We have, $(1+x+x^2+...)^{-n}=[(1-x)^{-1}]^{-n}=(1-x)^n$

$={}^nC_0-{}^nC_1x+{}^nC_2x^2+...+(-1)^n\ {}^nC_n.x^n$

Coefficient of x^n is $(-1)^n\ {}^nC_n=(-1)^n$.

NCERT Exemplar Problems

More than One Answer

62. **(c)** $C_0^2+3C_1^2+5C_2^2+...+(2n+1)C_n^2$

$\Rightarrow$ $(C_0^2+C_1^2+C_2^2+...+C_n^2)+2(C_1^2+2C_2^2+...+nC_n^2)$

$={}^{2n}C_n+2(C_1^2+2C_2^2+...+nC_n^2)$ . . .(*i*)

$\because$ $(1+x)^n=C_0+C_1x+C_2x^2+C_3x^3+...+C_nx^n$

Differentiating both sides *w.r.t* x,

We get $n(1+x)^{n-1}=C_1+2C_2x+3C_3x^2+...+nC_nx^{n-1}$ . . .(*ii*)

and $(x+1)^n=C_0x^n+C_1x^{n-1}+C_2x^{n-2}+...+C_n$ . . .(*iii*)

Multiplying Equation (*ii*) and (*iii*), we get

$n(1+x)^{2n-1}=(C_1+2C_2x+3C_3x^2+...+nC_nx^{n-1})$

$\times(C_0x^n+C_1x^{n-1}+C_2x^{n-2}+...+C_n)$

Comparing the coefficients of x^{n-1} in both sides, then

$C_1^2+2C_2^2+3C_3^2+...+nC_n^2=n^{2n-1}C_{n-1}$ . . .(*iv*)

From Equation (*i*) and (*iv*),

We get $C_0^2+3C_1^2=5C_2^2+...+(2n+1)C_n^2$

$={}^{2n}C_n=2n.{}^{2n-1}C_{n-1}$

$=\frac{2n}{n}.{}^{2n-1}C_{n-1}+2n.{}^{2n-1}C_{n-1}$

$=2(n+1).{}^{2n-1}C_{n-1}=2(n+1)^{2n-1}C_n$

$={}^{2n-1}C_{n-1}+(2n+1)^{2n-1}C_{n-1}={}^{2n-1}C_n+(2n+1).{}^{2n-1}C_{n-1}$

63. **(d)** Number of distinct terms

$={}^{n+4}C_4=\frac{(n+4)(n+3)(n+2)(n+1)}{1\cdot2\cdot3\cdot4}$

$=\frac{(n+1)(n+2)(n+3)(n+4)}{24}$

$\because$ ${}^{n+4}C_4={}^{n+4}C_{n+4-4}={}^{n+4}C_n$

64. **(c)** $\left(\sqrt[3]{4}+\frac{1}{\sqrt[4]{6}}\right)^{20}=\left(4^{1/3}+6^{-1/4}\right)^{20}$

$=\left(2^{2/3}+6^{-1/4}\right)^{20}$

$T_{r+1}={}^{20}C_r(2^{2/3})^{20-r}(6^{-1/4})^r$

$={}^{20}C_r2^{(160-11r)/12}.3^{-r/4}$

$\therefore$ For $r=8,20;T_{r+1}$ is rational.

$\therefore$ Only two terms are rational.

So, $21-2=19$ terms are irrational.

65 **(c)** $\frac{a_{n+1}}{a_n}+\frac{(1000)^{n+1}}{(n+1)!}.\frac{n!}{(1000)^n}=\frac{1000}{(n+1)}\geq1$

For $n=1,2,3,...,999$

$\Rightarrow$ $a_{n+1}\geq a_n$ $a_{1000}=\frac{(1000)^{1000}}{1000!}=\frac{(1000)^{999}}{999!}=a_{999}$

Hence, a_{999} and a_{1000} are equal and are the greatest.

66 **(b)** Let $P=\{x+\sqrt{(x^3-1)}\}^5+\{x-\sqrt{(x^3-1)}\}^5$ . . .(*i*)

Let $\sqrt{(x^3-1)}=\lambda$

$\therefore$ $\lambda^2=x^3-1$ . . .(*ii*)

From Eq. (*i*) $P=(x+\lambda)^5+(x-\lambda)^5$

$=2\{x^5+{}^5C_2x^3\lambda^2+{}^5C_4x\lambda^4\}$

$=2(x^5+10x^3(x^3-1)+5x(x^3-1)^2\}$

$=2\{5x^7+10x^6+x^5-10x^4-10x^3+5x\}$

Hence, P is a polynomial of degree 7.

67. **(d)** $(x+y+z)^{25}=[x+y+z]^{25}$

$={}^{25}C_0x^{25}+{}^{25}C_1x^{24}(y+z)^1+...+{}^{25}C_rx^{25-r}(y+z)^r+...$

$=...+{}^{25}C_rx^{25-r}(...+{}^rC_ky^{r-k}z^k+...)+...$

Hence, every term is of the form ${}^{25}C_rx^{25-r}\ {}^rC_ky^{r-k}z^k$

$\therefore$ $25-r+r-k+k=25\neq8+9+9$

Hence, coefficient of $x^8y^9z^9$ is 0 and number of terms

$={}^{25+r}C_2=\frac{27\times26}{2}=27\times13=351$

68. **(a)** Total term $=(2n+1)$

Middle term $=\frac{(2n+1)+1}{2}=(n+1)^{th}\ T_{n+1}={}^{2n}C_n\,x^n$

Coefficient of $x^n={}^{2n}C_n=\frac{2n!}{n!n!}$

$=\frac{2^n\,n!\{1\cdot3\cdot5\cdot...\cdot(2n-1)\}}{n!n!}$

$=\left\{\frac{1\cdot3\cdot5\cdot...\cdot(2n-1)}{n!}\right\}2^n$

69 **(d)** $3^{4n}=81^n=(1+80)^n=1+80\lambda,\lambda\in N$

$\therefore$ $3^{3^{4n}}=3^{1+80\lambda}=3\cdot3^{80\lambda}=3\cdot(9)^{40\lambda}=3(10-1)^{40\lambda}$

$=3(1+10\mu)=3+30\mu$

$\therefore$ Last digit of $3^{34n}+1$ is 4.

70. **(b)** Number of distinct terms $={}^{9+3-1}C_{3-1}={}^{11}C_2=55$

Sum of coefficients $=(2-2+1)^9=1^9=1$

and $(2-2x+x^2)^9=\sum\frac{9!}{\alpha!\beta!\gamma!}(2)^\alpha(-2x)^\beta(x^2)^\gamma$

Here, $\beta+2\gamma=4$, $\alpha+\beta+\gamma=9$

α	β	γ
5	4	0
6	2	1
7	0	2

$\therefore$ Coefficient of x^4 is

$=\frac{9!}{5!4!0!}\cdot2^5\cdot(-2)^4+\frac{9!}{6!2!1!}(2)^6(-2)^2+\frac{9!}{7!0!2!}(2)^7(-2)^0$

$\Rightarrow$ $2^9(126+126+9)=133632$

71. **(b)** $T_{r+1}={}^{10}C_r(\sqrt{x})^{10-r}.\left(-\frac{\lambda}{x^2}\right)^r={}^{10}C_r\cdot x^{5-\frac{r}{2}-2r}\cdot(-\lambda)^r$

$\therefore$ Put $5-\frac{r}{2}-2r=0$

$\Rightarrow$ $r=2$

Then, $T_{2+1}={}^{10}C_2\cdot(-\lambda)^2=45\lambda^2=405$ (given)

$\therefore$ $\lambda^2=9\Rightarrow\lambda=\pm3$

Assertion and Reason

72. **(c)** We have $(1+x)^{10}=\sum_{j=0}^{10}({}^{10}C_j)x^j$

Differentiating both are sides with respect to x, we get

$$10(1+x)^9=\sum_{j=1}^{10}j({}^{10}C_j)x^{j-1} \quad ...(i)$$

Again differentiating both the sides with respect to x,

We get $(10)(9)(1+x)^8=\sum_{j=1}^{10}j(j-1)({}^{10}C_j)x^{j-2}$

Putting $x=1$ in (i) and (ii),

We get $S_2=\sum_{j=1}^{10}j({}^{10}C_j)=10(2^9)$

and $S_1=\sum_{j=1}^{10}j(j-1)({}^{10}C_j)=(10)(9)(2^8)=(90)(2^8)$

Adding the above two equation, we get

$$S_3=\sum_{j=1}^{10}[j+j(j-1)]({}^{10}C_j)=(10)(2^8)(2+9)$$

$\Rightarrow$ $S_3=(55)\,(2^9)$

Thus, Assertion is true but Reason is false.

73. **(a)** $C_0^2+C_1^2+...+C_n^2=C_0C_n+C_1C_{n-1}+C_nC_0$

= number of ways of choosing n persons out of m men and n women $={}^{2n}C_n$

$\therefore$ Reason is true

Let $S=5C_0^2+7C_1^2+9C_2^2+...+(5+2n)C_n^2 \quad ...(i)$

Using $C_r=C_{n-r}$, we can rewrite (i) as

$$S=(5+2n)C_0^2+(3+2n)C_1^2+...+5C_n^2 \quad ...(ii)$$

Adding (i) and (ii),

We get $2S=(10+2n)(C_0^2+C_1^2+...+C_n^2)$

$\Rightarrow$ $S=(5+n)({}^{2n}C_n)=(5+n)\dfrac{(2n)!}{n!n!}$

74. **(b)** From $(1+t)^n=C_0+C_1t+C_2t^2+...+C_nt^n \quad ...(i)$

$$\int_0^x(1+t)^n=\int_0^x[C_0+C_1t+C_2t^2+...+C_nt^n]dt$$

$\Rightarrow$ $\dfrac{1}{n+1}[(1+x)^{n+1}-1]=\dfrac{C_0}{1}x+\dfrac{C_2}{2}x^2+...+\dfrac{C_n}{n+1}x^{n+1} \quad ...(ii)$

Multiplying (i) by t and integrating, we get

$$\int_0^x t(1+t)^n\,dt=\int_0^x[C_0t+C_1t_2+...+C_nt^{n+1}]dt$$

$\Rightarrow$ $\dfrac{x(1+x)^{n+1}}{n+1}-\dfrac{(1+x)^{n+2}}{(n+1)(n+2)}$

$$=\frac{C_0}{2}x^2+\frac{C_1}{3}x^3+\frac{C_2}{4}x^4+...+\frac{C_n}{n+2}x^{n+2} \quad ...(iii)$$

Putting $x=-1$ in (iii), we obtain that the Reason is true.

Putting $x=1$ in (ii) and (iii) and subtracting, we obtain that Assertion is also true.

75. **(c)** We can write

$$S=nC_0+(n-1)C_1+(n-2)C_2+...+1C_{n-1}+0C_n \quad ...(i)$$

Using $C_r=C_{n-r}$, we can rewrite (i) as

$$S=0C_0+{}^1C_1+2C_2+...+(n-1)C_{n-1}+nC_n \quad ...(ii)$$

Adding (i) and (ii)

We obtain $2S=n[C_0+C_1+C_2+...+C_n]=n(2^n)$

$\Rightarrow$ $S=n(2^{n-1})$ In the expression $\sum_{j=1}^{n}\sum_{i<j}(C_i+C_j)$

Each $C_i(0\le i\le n)$ occurs exactly n times.

Thus $\sum_{j=1}^{n}\sum_{i<j}(C_i+C_j)=n\sum_{k=0}^{n}C_k=n(2^n)$

76. **(c)** $C_0+C_1\omega+C_2\omega^2+C_3+C_4\omega+C_5\omega^2+...$

$$=\sum_{k=0}^{3n}C_k\omega^k=(1+\omega)^{3n}=(-\omega^2)^{3n}=(-1)^{3n}\omega^{6n}$$

$=(-1)^n(1)=(-1)^n$.

Reason is false as area of triangle formed by cube roots of unity is $\sqrt{3}/4$ square units.

77. **(d)** Let $S=\sum_{r=0}^{n}C_r\sin(rx)\cos(n-r)x] \quad ...(i)$

Using $C_r=C_{n-r}$, we can write

$$S=\sum_{r=0}^{n}C_{n-r}\sin(rx)\cos[(n-r)x]$$

$$\sum_{r=0}^{n}C_r\sin[(n-r)x]\cos(rx) \quad ...(ii)$$

Adding (i) and (ii), We get $2S=\sum_{r=0}^{n}C_r\ \{\sin(rx)\cos[(n-r)x]\ +\sin[(n-r)]x\cos(rx)\}$

$$=\sum_{r=0}^{n}C_r\sin(nx)=\sin(nx)\sum_{r=0}^{n}C_r=2^n\sin(nx)$$

$\Rightarrow$ $S=2^{n-1}\sin(nx)$

78. **(a)** $\left(x+\dfrac{9}{x}+6\right)^n=\left(\dfrac{x^2+6x+9}{x}\right)^n=\dfrac{(3+x)^{2n}}{x^n}=\dfrac{3^{2n}}{x^n}\left(1+\dfrac{x}{3}\right)^{2n}$

$\therefore$ Coefficient of the term independent of $x\left(x+\dfrac{9}{x}+6\right)^n$ in

$$=3^{2n}\left[\text{Coefficient of } x^n \text{in the expansion of}\left(1+\frac{x}{3}\right)^{2n}\right]$$

$$=3^{2n}\binom{2n}{n}\left(\frac{1}{3}\right)^n=\frac{3^n(2n)!}{n!n!} \quad \text{[using Reason]}$$

79. **(d)** LHS of $\binom{n}{m}+\binom{n-1}{m}+\binom{n-2}{m}+...+\binom{m}{n}=\binom{n+1}{m}$ . . .(i)

= coefficient of x^m in $\{(1+x)^m+(1+x)^{m+1}+...+(1+x)^n\}$

[using Reason]

= coefficient of x^m in $(1+x)\frac{\{(1+x)^{n-m+1}-1\}}{1+x-1}$

= coefficient of x^{m+1} in $\left[(1+x)^{n+1}-(1+x)^m\right]=\binom{n+1}{m+1}$

80. **(a)** We have $\binom{p}{r}=\frac{p!}{r!(p-r)!}$

$\Rightarrow \quad r!(p-r)!\binom{p}{r}=p!$

As $p\,|\,p!$, we get $p\,|\,p!(p-r)\binom{p}{r}$

But for $1 \le r \le p-1$, neither $r!$ nor $(p-r)!$ is divisible by p.

$\therefore \quad p\left|\binom{p}{r}\right.$

We have $\sqrt{5}-2=\frac{1}{\sqrt{5}+2}$

$\Rightarrow \quad 0<\sqrt{5}-2<1$

$\Rightarrow \quad 0<f=(\sqrt{5}-2)<1.$

Let $(2+\sqrt{5})^n=N+F$ Where $0<F<1$.

Now, $N+F-f-2^{n+1}=\left(\sqrt{5}+2\right)^n-\left(\sqrt{5}-2\right)^n$

$=2\left[\binom{n}{1}5^{(n-1)}(2)+\binom{n}{3}5^{(n-3)/2}(2^3)+...+\binom{n}{n-2}(5)2^{n-1}\right]$ . . .(i)

[$\because$ n is odd]

Since n is an odd prime, each of $\binom{n}{1},\binom{n}{3},...\binom{n}{n-2}$ is

divisible by n.

Thus RHS of (i) is divisible by 20 n

Also, $F–f$ is an integer. Since $0 < F < 1$

and $0 < f < 1$,

W get $–1 < F –f < 1$.

As $F-f$ is an integer,

We get $F-f=0$ or $F=f$.

$\therefore$ integral part of $(2+\sqrt{5})^n-2^{n+1}$ is $N-2^{n+1}$ which is divisible by 20 n.

81. **(d)** That Reason is false can be seen from theory.

We can write $(\sqrt{3}+\sqrt{2})^{50}=2^{25}\left(1+\sqrt{\frac{3}{2}}\right)^{50}$.

Let $l=\frac{(50+1)\sqrt{3/2}}{1+\sqrt{3/2}}=\frac{51\sqrt{3}(\sqrt{3}-\sqrt{2})}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)}=51\left(3-\sqrt{6}\right)$

$=51\,(0.5505)\simeq 28.07 \;\Rightarrow [k]=28.$

Thus, the greatest term in the expansion of $\left(\sqrt{3}+\sqrt{2}\right)^{50}$ is the 29th term and it equals $2^{25}\binom{50}{28}\left(\sqrt{\frac{3}{2}}\right)^{28}=\binom{50}{22}3^{14}2^{11}$

Comprehension Based

For 82 to 86

Comprehension Based Multiple Choice Questions

$\because \quad (1+x+x^2)^{2n}=\sum_{r=0}^{4n}a_rx^r$ . . .(i)

Replacing x by $\frac{1}{x}$ in Eq. (i),

Then $\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^{2n}=\sum_{r=0}^{4n}a_r\left(\frac{1}{x}\right)^r$

or $\quad (1+x+x^2)^{2n}=\sum_{r=0}^{4n}a_rx^{4n-r}$ . . .(ii)

From Equation (i) and (ii), We get $\sum_{r=0}^{4n}a_rx^r=\sum_{r=0}^{4n}a_rx^{4r-r}$

Comparing coefficient of on both sides, then

We get $\quad a_r=a_{4n-r}$ . . .(iii)

82. **(c)** Put $x=1$

and $\quad x=-1$ in Eq. (i), then

$9^n=a_0+a_1+a_2+a_3+a_4+...+a_{2n}+...+a_{4n}$ and

$1=a_0-a_1+a_2-a_3+a_4-...+a_{2n}+...+a_{4n}$

Adding and subtracting, then

We get $\frac{9^n+1}{2}=a_0+a_2+a_4+...+a_{2n}+...+a_{4n-2}+a_{4n}$..... .($iv$)

and $\frac{9^n-1}{2}=a_1+a_3+a_s+...+a_{2n-1}+...a_{4n-1}$ . . .(v)

Now, $a_r=a_{4n-r}$

Put $r=0,2,4,6,...,a_{2n-2,}a_{2n}$

$\therefore \quad a_0=a_{4n}$

$a_2=a_{4n-2}$

$a_4=a_{4n-4}$

………..

$a_{2n-2} = a_{2n+2}$

$\therefore \quad a_0 + a_2 + a_4 + ... + a_{2n-2}$

$= a_{2n+2} + ... + a_{4n-4} + a_{4n-2} + a_{4n}$

Now, from equation. (*iv*)

We get $\dfrac{9^n + 1}{2} = 2(a_0 + a_2 + a_4 + ... + a_{2n-2}) + a_{2n}$

$\Rightarrow \quad \dfrac{9^n + 1 - 2a_{2n}}{4} = a_0 + a_2 + a_4 + ... + a_{2n-2}$

$\therefore \quad \sum_{r=0}^{n-1} a_{2r} = \left(\dfrac{9^n + 1 - 2a_{2n}}{4}\right)$

83. **(b)** $a_r = a_{4n-r}$ Put $r = 1, 3, 5, 7, ..., 2n-3, 2n-1$

$a_1 = a_{4n-1}$

$a_3 = a_{4n-3}$

$a_5 = a_{4n-5}$

....

$a_{2n-3} = a_{2n+3}$

$a_{2n-1} = a_{2n+1}$

$\therefore \quad a_1 + a_3 + a_5 + ... + a_{2n-1} = a_{2n+1} + a_{2n+3} + ... + a_{4n-3} + a_{4n-1}$

Now, from equation (*v*)

We get $\dfrac{9^n + 1}{2} = 2(a_1 + a_3 + a_5 + ... + a_{2n-1})$

$\therefore \quad \sum_{r=1}^{n} a_{2r-1} = \left(\dfrac{9^n - 1}{4}\right)$

84. **(c)** a_2= coefficient of x^2 in$(1 + x + x^2)^{2n}$ = coefficient of x^2 in $\{1 + {}^{2n}C_1(x + x^2) + {}^{2n}C_2(x + x^2)^2 + ...\}$

$= {}^{2n}C_1 + {}^{2n}C_2 = {}^{2n+1}C_2$

85. **(a)** $a_{4n-1} = a_1 (\because a_r = a_{4n-r})$

=coefficient of x in $(1 + x + x^2)^{2n}$

=coefficient of x in $= {}^{2n}C_1 = 2n$

86. **(d)** From Eq. (*iii*), $a_r = a_{4n-r}, 0 \le r \le 4n$

87. **(b)** Required sum = coefficient of x^n in $(1+x)^n (1+x)^n$

$=$coefficient of x^n in $(1+x)^{2n} = {}^{2n}C_n$

88. **(d)** ${}^{30}C_r \cdot {}^{20}C_0 + {}^{30}C_{r-1} \cdot {}^{20}C_1 \cdot + ... + {}^{30}C_0 \cdot {}^{20}C_r$

= coefficient of x^r in $(1+x)^{30}(1+x)^{20}$

= coefficient of x^r in $(1+x)^{50} = {}^{50}C_r$

$\because \quad {}^{50}C_r$ is maximum

$\therefore \quad r = \dfrac{50}{2} = 25$

89. **(a)** ${}^{20}C_r \cdot {}^{10}C_0 + {}^{20}C_{r-1} \cdot {}^{10}C_1 + ... + {}^{20}C_0 \cdot {}^{10}C_r$

$=$coefficient of x^r in $(1+x)^{20}(1+x)^{10}$

= coefficient of x^r in $(1+x)^{30} = {}^{30}C_r$

$\because \quad {}^{30}C_r$ is maximum

$\therefore \quad r = 0$

90. **(a)** $S_n = {}^nC_0 \cdot {}^nC_1 + {}^nC_1 \cdot {}^nC_2 + ... + {}^nC_{n-1} \cdot {}^nC_n$

$= {}^nC_0 \cdot {}^nC_{n-1} + {}^nC_1 \cdot {}^nC_{n-2} + ... + {}^nC_{n-1} \cdot {}^nC_0 \; (\because {}^nC_r = {}^nC_{n-r})$

$=$coefficient of x^{n-1} in $(1+x)^n(1+x)^n$

$=$coefficient of x^{n-1} in $(1+x)^{2n} = {}^{2n}C_{n-1}$

$\because \quad \dfrac{S_{n+1}}{S_n} = \dfrac{15}{4}$

$\Rightarrow \quad \dfrac{{}^{2n+2}C_n}{{}^{2n}C_{n-1}} = \dfrac{15}{4}$

$\Rightarrow \quad \left(\dfrac{2n+2}{n}\right) . \dfrac{{}^{2n+1}C_{n-1}}{{}^{2n}C_{n-1}} = \dfrac{15}{4}$

$\Rightarrow \quad \left(\dfrac{2n+2}{n}\right) . \dfrac{\dfrac{(2n+1)!}{(n-1)!(n+2)!}}{\dfrac{2n!}{(n-1)!(n+1)!}} = \dfrac{15}{4}$

$\Rightarrow \quad \left(\dfrac{2n+2}{n}\right) . \dfrac{(2n+1)}{(n+2)} = \dfrac{15}{4}$

$\Rightarrow \quad 4(4n^2 + 6n + 2) = 15n^2 + 30n$

$\Rightarrow \quad n^2 - 6n + 8 = 0$

$\Rightarrow \quad (n-2)(n-4) = 0$

$\therefore \quad n = 2, 4$

91. **(a)** $C_0^2 - C_1^2 + C_2^2 + C_3^2 + ... + (-1)^n C_n^2$

$= C_0C_0 - C_1C_1 + C_2C_2 - C_3C_3 + ... + (-1)^n C_nC_n$

$= C_0C_n - C_1C_{n-1} + C_2C_{n-2} - C_3C_{n-3} + ... + (-1)^n C_nC_0$

$(\because C_r = C_{n-r})$

$=$coefficient of x^n in $(1+x)^n(1-x)^n$

$=$coefficient of x^n in $(1-x^2)^n = (-1)^{n/2} \cdot {}^nC_{n/2} = 0$

$(\because n$ is odd)

Match the Column

92. **(a)** **(A)** $(1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5)^{20} = \{(1+x)^5\}^{20}$

$= (1+x)^{100}$

$\therefore \quad \lambda = 100 + 1 = 101$

Then $3^\lambda = 3^{101} = 3 \cdot 3^{100} = 3(9)^{50} = 3(10-1)^{50}$

$= 3\{(10)^{50} - {}^{50}C_1(10)^{49} + - {}^{50}C_2(10)^{48} - ... - {}^{50}C_{49}(10) + 1\}$

$= 3\{100\mu + 1\} = 300\mu + 3(\mu \text{ is} + \text{ve integer})$

$\therefore$ Last two digits 03

$\therefore$ $O = 3, T = 0 \Rightarrow O + T = 3(P)$

(B) $\left(x^2 + 1 + \frac{1}{x^2}\right)^{100}$

$\therefore$ $\lambda = 2 \times 100 + 1 = 201$

Then $7^\lambda = 7^{201} = 7 \cdot 7^{200} = 7 \cdot (7^2)^{100} = 7 \cdot (49)^{100}$

$= 7(50 - 1)^{100}$

$= 7\{(50)^{100} - {}^{100}C_1(50)^{90} + {}^{100}C_2(50)^{98} - ... - {}^{100}C_{99}(50) + 1\}$

$= 7\{100\mu + 1\} = 700\mu + 7(\because \mu \text{ is} + \text{ve integer})$

$\therefore$ Last two digits 07

$\therefore$ $O = 7, T = 0$

$\Rightarrow$ $O + T = 7$

and $O - T = 7(Q, T)$

(C) $(1+x)^{101}(1 + x^2 - x)^{100}$

$= (1+x)\{(1+x)(1+x^2-x)\}^{100}$

$= (1+x)(1+x^3)^{100}$

$= (1+x)\{1 + {}^{100}C_1x^3 + {}^{100}C_2x^6 + {}^{100}C_3x^9 + ... + {}^{100}C_{100}x^{303}$

$= 1 + {}^{100}C_1x^3 + {}^{100}C_2x^6 + {}^{100}C_3x^9 + ... +$

${}^{100}C_{100}x^{303} + x + {}^{100}C_1x^4 + {}^{100}C_2x^7 + {}^{100}C_3x^{10} + ... + {}^{100}C_{100}x^{304}$

$\therefore$ $\lambda = 1 + 100 + 101 = 202$

$\Rightarrow$ $9^\lambda = 9^{202} = (10-1)^{202}$

$= (10)^{202} - {}^{202}C_1(10)^{201} + {}^{202}C_2(10)^{200} - ... - {}^{202}C_{201}(10) + {}^{202}C_{202}$

$= 100\mu - 2020 + 1(\mu \text{ is} + \text{ve integer})$

$= 100(\mu - 21) + 81$

$= 100v + 81\ ^{\bullet}(v \text{ is + ve integer})$

$\therefore$ Last tow digit 81

$\therefore$ $O = 1$ and $T = 8$

$\Rightarrow$ $O + T = 9$ and $T - O = 7(R, S)$

93. **(a) (A)** Every even power of 9 can be represented in the form $9^{2r} = 81^r = \underbrace{81 \cdot 81 \cdot 81 \cdot ... \cdot 81}_{r \text{ times}}$

It's last digit is 1, every odd power of 9 can be written as $9^{2r+1} = 9 \cdot 81^r$, therefore its last digit is 9. In particular $9^{(9^9)}$ is an odd power of 9 and consequently the last digit of 9^{9^9} is 9

$\therefore$ $\lambda = 9$

Now, $2^{\lambda^{100}} = 2^{9^{100}}$ the

$\because$ $9^{100} = (2 \cdot 4 + 1)^{100} = 4n + 1(\text{say})$

$\therefore$ $2^{9^{100}} = 2^{4n+1} = 2 \cdot (2^4)^n = 2 \cdot (16)^n$

The digit at units place in $(16)^n = 6$

$\therefore$ The digit at units place in $(16)^n \cdot 2 = 2$

$\therefore$ $\mu = 2$

$\Rightarrow$ $\lambda + \mu = 11, \lambda - \mu = 7(Q, R)$

(B) $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16$

We say that units digit in $2^12^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, ...$ are

$\underset{(1)}{2},\ \underset{(2)}{4},\ \underset{(3)}{8},\ \underset{(0)}{6},\ \underset{(1)}{2},\ \underset{(2)}{4},\ \underset{(3)}{8},\ \underset{(0)}{6}, \ldots$(period4)

```
   249
 4)999(
   996
     3
```

$\therefore$ Last digit of the number 2^{999} is 8

$\therefore$ $\lambda = 8$

Also, $3^{\lambda\lambda\lambda} = 3^{888} = (3^2)^{444} = (10-1)^{444} = 10n + 1$

$\therefore$ $\mu = 1$

$\Rightarrow$ $\lambda + \mu = 9, \lambda - \mu = 7, \lambda^\mu + \mu^\lambda = 8^1 + 1^8 = 9\ (P, R, T)$

(C) $a = \frac{72!}{(36!)^2} - 1$

$= \frac{(72 \cdot 71 \cdot 70 \cdot ... \cdot 37)36!}{(36!)^2} - 1$

$= \frac{72 \cdot 71 \cdot 70 \cdot ... \cdot 37}{36!} - 1$

$= \frac{(1-73)(2-73)(3-73)...(36-73)}{36!} - 1$

$= \frac{(1 \cdot 2 \cdot 3 \cdot ... \cdot 36) - 73(m)}{36!} - 1\ (m \text{ is an integer})$

$= 1 - \frac{73m}{36!} - 1 = -\frac{73m}{36!}$

Which is divisible by $73 = 70 + 3$

$\therefore$ $\lambda = 7, \mu = 3 \begin{pmatrix} \because a = \frac{72!}{(36!)^2} - 1 \\ = {}^{72}C_{36} - 1 \\ = \text{even} - 1 \\ = \text{odd} \end{pmatrix}$

Then $\lambda - \mu = 4(S)\ 00$

94. **(a) (A)** Given expression $= \sqrt{(3x^2+1)} \cdot 2\{{}^7C_1x^6(\sqrt{3x^2+1})$

$+ {}^7C_3x^4(\sqrt{3x^2+1})^3 + {}^7C_5\ x^2\ (\sqrt{3x^2+1}))^5$

$+ {}^7C_7(\sqrt{(3x^2+1)})^7\}$

$= (3x^2+1) \cdot 2\{7x^6 + 35x^4(3x^2+1) + 21x^2(3x^2+1)^2 + (3x^2+1)^3\}$

$\therefore \quad n = 8($

(B) $\dfrac{T_7}{T_6} \geq 1 \Rightarrow \dfrac{{}^nC_6 x^{n-6} a^6}{{}^nC_5 x^{n-5} a^5} \geq 1$

$\Rightarrow \quad \left(\dfrac{n-6+1}{6}\right) \cdot \left(\dfrac{2}{3}\right) \geq 1$

$\Rightarrow \quad 2n - 10 \geq 18$

$\Rightarrow \quad n \geq 14$

Similarly, $\dfrac{T_7}{T_8} \geq 1$

$\Rightarrow \quad n \geq \dfrac{33}{2} \Rightarrow n = 16$

$\because$ There is only one middle term, n must be even. $n = 16$ gives the greatest term.

(C) $200 < 2^m < 400$

$\Rightarrow \quad m = 8 (m \in I)$

$\therefore \quad T_{r+1} = {}^8C_r (x^{-3/4})^{8-r} \cdot (nx^{5/4})^r = 448x^0$

$\therefore \quad x^{-6+\frac{3r}{4}+\frac{5r}{4}} = x^0$

$\Rightarrow \quad -6 + 2r = 0$

$\Rightarrow \quad r = 3$

$\Rightarrow \quad {}^8C_3 \cdot n^3 = 448$

$\Rightarrow \quad n^3 = 8 \Rightarrow n = 2$

$\therefore \quad n^5 = 32$

Integer

95. **(3954)** Using the formula for the sum of a geometric progression, we find

$(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + x^{1000}$

$$= \frac{(1+x)^{1000}\left[1 - \left(\dfrac{x}{1+x}\right)^{1001}\right]}{\left[1 - \left(\dfrac{x}{1+x}\right)\right]}$$

$$= \frac{(1+x)^{1000} - \dfrac{x^{1001}}{(1+x)}}{\dfrac{x+1-x}{(1+x)}}$$

$= (1+x)^{1001} - x^{1001}$

Hence, the coefficient of

$x^{50} = {}^{1001}C_{50} = \dfrac{1001!}{50!\,951!}$

$\therefore \quad \lambda = 1001, \mu = 50$

and $\quad \text{v} = 951$

$\therefore \quad \lambda + 2\mu + 3v = 1001 + 100 + 2853 = 3954$

96. **(0024)** $(1+x)^{51} = C_0 + C_1 x + C_2 x^2 + \ldots + C_{51} x^{51}$

$= \sum_{r=0}^{51} C_r x^r$

Then, $\int_0^x (1+x)^{51}\, dx = \sum_{r=0}^{51} \int_0^x C_r x^r\, dx$

$\Rightarrow \quad \dfrac{(1+x)^{52} - 1}{52} = \sum_{r=0}^{51} \left(\dfrac{C_r x^{r+1}}{r+1}\right) \qquad \ldots(i)$

and $\quad (x+1)^{51} = \sum_{r=0}^{51} C_r x^{51+r} \qquad \ldots(ii)$

Multiply equation (i) and equation (ii), then

$\dfrac{(1+x)^{103} - (1+x)^{51}}{52} = \left(\sum_{r=0}^{51} \dfrac{C_r x^{r+1}}{r+1}\right)\left(\sum_{r=0}^{51} C_r x^{51-r}\right)$

Now, comparing the coefficient of x^{52}, then

We get $\dfrac{1}{52}({}^{103}C_{52} - 0) = \sum_{r=0}^{51} \dfrac{C_r^2}{(r+1)}$

$\Rightarrow \quad \dfrac{103!}{52 \cdot 51!52!}$ or $\dfrac{103!}{(52!)^2}$

97. **(257)** $(1+x)^n = \sum_{r=0}^{n} C_r x^r$

$\therefore \quad x(1+x)^n = \sum_{r=0}^{n} C_r x^{r+1}$

Differentiating both sides *w.r.t.*

Then $(1+x)^{n-1}(1+x+nx) = \sum_{r=0}^{n} (r+1) C_r x^r$

Again, multiplying both sides by x,

Then $(1+x)^{n-1}(x + x^2 + nx^2) = \sum_{r=0}^{n} (r+1) \cdot C_r \cdot x^{r+1}$

Again, differentiating both sides *w.r.t x,*

Then $(1+x)^{n-2}\{(1+x)(1+2x+2nx) + (n-1)(x+x^2+nx^2)\}$

$= \sum_{r=0}^{n} (r+1)^2 C_r x^r$

Putting $x = 1$ on both sides, then

We get $= \sum_{r=0}^{n} (r+1)^2 C_r = 2^{n-2}(n^2 + 5n + 4)$

$\therefore \quad f(n) = n^2 + 5n + 4$

Then, $f(x) = x^2 + 5x + 4 = (x+1)(x+4)$

$\therefore \quad \alpha = -1\ \beta = -4$

Then, $\alpha^4 + \beta^4 = (-1)^4 + (-4)^2 = 1 + 256 = 257$

98. **(4096)** $P=\sum_{r=0}^{n}{}^{n}C_r\sin(rx)\cos\{(n-r)x\}$ ...(*i*)

Replacing *r* by $n-r$,

Then $P=\sum_{r=0}^{n}{}^{n}C_{n-r}\sin\{(n-r)x\}\cos[\{n-(n-r)\}x]$

$=\sum_{r=0}^{n}{}^{n}C_r\sin\{(n-r)x\}\cos rx$ ...(*ii*)

Adding equation (*i*) and (*ii*),

Then $2P=\sum_{r=0}^{n}{}^{n}C_r\sin(nx)=\sin(nx)\cdot 2^n$ or $P=2^{n-1}\cdot\sin(nx)$

$\therefore\quad f(n)=2^{n-1}$

Then, $f(13)=2^{12}=4096$

99. **(3333)** $a_r=r^2\cdot\dfrac{(100-r+1)}{r}=r\cdot(101-r)$

$\because\quad \prod_{i=1}^{100}(x-a_i)=(x-a_1)(x-a_2)(x-a_3)...(x-a_{100})=x^{100}$

$-(a_1+a_2+a_3+...+a_{100})x^{99}+...$

$\lambda=-(a_1+a_2+a_3+\ldots+a_{100})=-\sum_{r=1}^{100}a_r=-\sum_{r=1}^{100}r\cdot(100-r)$

$=-100\sum_{r=1}^{100}r+\sum_{r=1}^{100}r^2=-505000+338350$

$=-166650-\dfrac{\lambda}{50}=\dfrac{166650}{50}=3333$

100. **(6)** Let the three consecutive terms in $(1+x)^{n+5}$ be t_r, t_{r+1}, t_{r+2}.

Having coefficients ${}^{n+5}C_{r-1}, {}^{n+5}C_r, {}^{n+5}C_{r+1}$

Given, ${}^{n+5}C_{r-1}:{}^{n+5}C_r:{}^{n+5}C_{r+1}=5:10:14$

$\therefore\quad \dfrac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}}=\dfrac{10}{5}$ and $\dfrac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r}=\dfrac{14}{10}$

$\Rightarrow\quad \dfrac{n+5-(r-1)}{r}=2$

and $\dfrac{n-r+5}{r+1}=\dfrac{7}{5}$

$\Rightarrow\quad n-r+6=2r$

and $5n-5r+25=7r+7$

$\Rightarrow\quad n+6=3r$ and $5n+18=12r$

$\therefore\quad \dfrac{n+6}{3}=\dfrac{5n+18}{12}$

$\Rightarrow\quad 4n+24=5n+18\Rightarrow n=6$

* * *