Chapter – 7

Cubes and Cube Roots

Exercises 7.1

Question 1. Which of the following numbers are not perfect cubes?

(i) 216 (ii) 128

(iii) 1000 (iv) 100

(v) 46656

Answer:

(i) Prime factorization of 216:

$216 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^3$

Here we can see that each prime factor is appearing as many times as a perfect multiple of 3, hence, 216 is a perfect cube.

(ii) The prime factorization of 128 is:

$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Here, we can observe that factor 2 is not appearing as many times as a perfect multiple of 3.

(iii) The prime factorization of 1000 is:

$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

Here, we can observe that each prime factor is appearing as many times as a perfect multiple of 3.

Hence, 1000 is a perfect cube

(iv) The prime factorization of 100 is:

$100 = 2 \times 2 \times 5 \times 5$

Here, we can see that every prime factor is not appearing as many times as a perfect multiple of 3.

Two 2s and two 5s are remaining if we group the triplets.

Therefore, 100 is not a perfect cube

(v) The prime factorization of 46656 is:

$46656 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

Here, we can see that number of 2s and 3s is 6 each and we know that 6 is divisible by 3.

Hence, 46656 is a perfect cube

Question 2. Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube

(i) 243 (ii) 256

(iii) 72 (iv) 675

(v) 100

Answer:

(i) $243 = 3 \times 3 \times 3 \times 3 \times 3$

Here, we can find that two 3s are left which are not forming a triplet. To make 243 a cube, one more 3 is required.

Hence,

$243 \times 3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

729 is a perfect cube.

Therefore, the smallest natural number by which 243 should be multiplied to make it a perfect cube is 3

(ii) 256 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2

Here, we can see that two 2s are left which are not forming a triplet. To make 256 a cube, one more 2 is required

Hence,

256 x 2 = 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 512

Whereas 512 is a perfect cube.

Therefore, the smallest natural number by which 256 should be multiplied to make it a perfect cube is 2

(iii) 72 = 2 x 2 x 2 x 3 x 3

Here, we can see that two 3s are left which are not forming a triplet. In order to make 72 a cube, one more 3 is required

Hence,

72 x 3 = 2 x 2 x 2 x 3 x 3 x 3 = 216

We know that 216 is a perfect cube

Therefore, the smallest natural number by which 72 should be multiplied to make it a perfect cube is 3

(iv) 675 = 3 x 3 x 3 x 5 x 5

Here, we can see that two 5s are left which are not forming a triplet. To make 675 a cube, one more 5 is needed

Hence,

675 x 5 = 3 x 3 x 3 x 5 x 5 x 5 = 3375

We know that 3375 is a perfect cube.

Therefore, the smallest natural number by which 675 should be multiplied to make it a perfect cube is 5

(v) 100 = 2 x 2 x 5 x 5

Here, we can see that two 2s and two 5s are left which are not forming a triplet. To make 100 a cube, we require one more 2 and one more 5

Hence,

100 x 2 x 5 = 2 x 2 x 2 x 5 x 5 x 5 = 1000

We know that 1000 is a perfect cube.

Therefore, the smallest natural number by which 100 should be multiplied to make it a perfect cube is 2 x 5 = 10

Question 3. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube

(i) 81 (ii) 128

(iii) 135 (iv) 192

(v) 704

Answer: (i) 81 = 3 x 3 x 3 x 3

Here, one 3 is left which is not forming a triplet.

So, if we divide 81 by 3, then it will become a perfect cube

Therefore,

$\frac{81}{3} =$ 27 = 3 x 3 x 3 is a perfect cube

Hence, the smallest number by which 81 should be divided to make it a perfect cube is 3

(ii) 128 = 2 x 2 x 2 x 2 x 2 x 2 x 2

Here, one 2 is left which is not forming a triplet.

Si, if we divide 128 by 2, then it will become a perfect cube

Therefore,

$\frac{128}{2}$ = 64 = 2 x 2 x 2 x 2 x 2 x 2 is a perfect cube

Hence, the smallest number by which 128 should be divided to make it a perfect cube is 2

(iii) 135 = 3 x 3 x 3 x 5

Here, one 5 is left which is not forming a triplet.

So, if we divide 135 by 5, then it will become a perfect cube

Therefore,

$\frac{135}{5}$= 27 = 3 x 3 x 3 is a perfect cube

Hence, the smallest number by which 135 should be divided to make it a perfect cube is 5

(iv) 192 = 2 x 2 x 2 x 2 x 2 x 2 x 3

Here, one 3 is left which is not forming a triplet.

So, if we divide 192 by 3, then it will become a perfect cube

Therefore,

$\frac{192}{3}$ = 64 = 2 x 2 x 2 x 2 x 2 x 2 is a perfect cube

Hence, the smallest number by which 192 should be divided to make it a perfect cube is 3

(v) 704 = 2 x 2 x 2 x 2 x 2 x 2 x 11

Here, one 11 is left which is not forming a triplet.

So, if we divide 704 by 11, then it will become a perfect cube.

Therefore,

$\frac{107}{11}$ = 64 = 2 x 2 x 2 x 2 x 2 x 2 is a perfect cube

Hence, the smallest number by which 704 should be divided to make it a perfect cube is 11

Question 4. Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?

Answer: Dimension of cuboid given = 5 x 5 x 2 cm

Volume of 1 cuboid = 5 x 5 x 2 = 50 cm^3

Let the volume of the cube to be formed from given cuboids = x cm^3

Now we know that this value x will be a perfect cube Number of cuboids x Volume of 1 cuboid = Volume of cube Thus we can see that we have to multiply volume of 1 cuboid such that the resulting number is a perfect cube. Thus, 50 x (Some number) = Perfect cube Thus, 2 × 5 × 5 × n is a perfect cube Now, to make this a perfect cube, minimum value of 'n' should be 2 × 2 × 5 (such that it has 3 factors of 2 and 5 each) i.e. n = 20

Exercises 7.2

Question 1. Find the cube root of each of the following numbers by prime factorization method

(i) 64 (ii) 512

(iii) 10648 (iv) 27000

(v) 15625 (vi) 13824

(vii) 110592 (viii)46656

(ix) 175616 (x) 91125

Answer: For finding the cube root of a number. We need to first write the prime factors of the number. Then group the number in pair of 3's. And then multiply the groups.

For example: 125 = 5 x 5 x 5It will be one group of 5 only. So the cube root of 125 will be 5. (i) Prime factorization of 64 is as follows:

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

Hence,

$\sqrt[3]{64} = 2 * 2$

$= 4$

(ii) Prime factorization of 512 is as follows:

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

Hence,

$\sqrt[3]{512} = 2 * 2 * 2$

= 8

(iii) Prime factorization of 10648 is as follows:

$\underline{2 \times 2 \times 2} \times \underline{11 \times}\ \underline{11 \times 11}$

Hence,

$\sqrt[3]{10648} = 2 * 11$

$= 22$

(iv) Prime factorization of 27000 is as follows:

$= \underline{2 \times 2 \times 2} \times \underline{3 \times 3}\ \underline{\times 3} \times \underline{5 \times 5 \times 5}$

Hence,

$\sqrt[3]{27000} = 2 * 3 * 5$

$= 30$

(v) Prime factorization of 15625 is as follows:

$= \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

Hence,

$\sqrt[3]{15625} = 5 * 5$

= 25

(vi) Prime factorization of 13824 is as follows:

$= \underline{2 \times 2 \times 2} \times\ \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Hence,

$\sqrt[3]{13824} = 2 * 2 * 2 * 3$

$= 24$

(vii) Prime factorization of 110592 is as follows:

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Hence,

$\sqrt[3]{110592} = 2 * 2 * 2 * 2 * 3$

$= 48$

(viii) Prime factorization of 46656 is as follows:

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

Hence,

$\sqrt[3]{46656} = 2 * 2 * 3 * 3$

$= 36$

(ix) Prime factorization of 175616 is as follows:

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

Hence,

$\sqrt[3]{175616} = 2 * 2 * 2 * 7$

$= 56$

(x) Prime factorization of 91125 is as follows:

$= \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Hence,

$\sqrt[3]{91125} = 3 * 3 * 5$

$= 45$

Question 2. State true or false

(i) Cube of any odd number is even

(ii) A perfect cube does not end with two zeros

(iii) If square of a number ends with 5, then its cube ends with 25

(iv) There is no perfect cube which ends with 8

(v) The cube of a two-digit number may be a three-digit number

(vi) The cube of a two-digit number may have seven or more digits

(vii) The cube of a single digit number may be a single digit number

Answer: (i) False.

When we will calculate the cube of an odd number, we will get an odd number as the result because the unit place digit of an odd number is odd and we are multiplying three odd numbers.

Hence, the product will be again an odd number

For example, the cube of 5 (i.e., an odd number) is 125, which is again an odd number

(ii) True.

Perfect cube will end with a certain number of zeroes that are always a perfect multiple of 3

For example, the cube of 10 is 1000 and there are 3 zeroes at the end of it. The cube of 100 is 1000000 and there are 6 zeroes at the end of it and so on.

(iii) False.

It is not every time compulsory that if the square of a number ends with 5, then its cube will end with 25

For example, the square of 25 is 625 and 625 has its unit digit as 5. The cube of 25 is 15625. However, the square of 35 is 1225 and also has its unit place digit as 5 but the cube of 35 is 42875 which does not end with 25

(iv) False.

There are several cubes that ends with 8. The cubes of all the numbers with their unit place digit as 2 will end with 8

The cube of 2 is 8 and the cube of 22 is 10648

(v) False.

The smallest two-digit natural number is 10, and the cube of 10 is 1000 which has 4 digits in it

(vi) False.

The largest two-digit natural number is 99, and the cube of 99 is 970299 which has 6 digits in it. Hence, the cube of any two-digit number cannot have 7 or more digits in it

(vii)True

As we know that the cubes of 1 and 2 are 1 and 8 respectively.

Question 3. You are told that 1,331 is a perfect cube. Can you guess without factorization what is its cube root? Similarly, guess the cube roots of 4913, 12167, 32768

Answer: at the very first we will create groups of three digits starting from the rightmost digit of the number as $\overline{1}\ \overline{331}$

Now, there are 2 groups, having 1 and 331, in it

Taking 331,

Its digit at the unit place is 1.

We know,

If the digit 1 is at the end of a perfect cube number, then its cube root will have its unit place digit as 1 only.

Hence, the unit place digit of the required cube root can be taken as 1

Taking the next group i.e., 1, the cube of 1 exactly matches with the number of the second group. Therefore, the tens digit of our cube root will be taken as the unit place of the smaller number whose cube is close to the number of the second group i.e., 1 itself. 1 will be taken as tens place of the cube root of 1331.

Hence, $\sqrt[3]{1331} = 11$

The cube root of 4913 has to be calculated.

We will make groups of three digits starting from the right side digit of 4913, as $\overline{4}\,\overline{913}$. Hence, the groups are 4 and 913.

Taking the group 913, the unit digit of the number 913 is 3.

We know that if the digit 3 is at the end of a perfect cube number, then its cube root will have its unit place digit as 7 only.

Hence, the unit place digit of the required cube root is taken as 7.

Now,

Considering the other group i.e., 4,

We know that,

$1^3 = 1$ and $2^3 = 8$

And,

$1 < 4 < 8$

Therefore, 1 will be taken at the tens place of the required cube root

Hence, $\sqrt[3]{4913} = 17$

Now,

The cube root of 12167 has to be calculated.

Again we will create groups of three digits starting from the right side digit of the number 12167, as $\overline{12}\,\overline{167}$. The groups are 12 and 167

Analyzing the group 167,

167 ends with 7 We know that if the digit 7 is at the end of a perfect cube number, then its cube root will have its unit place digit as 3 only. Therefore, the unit place digit of the required cube root can be considered as 3.

Now,

Analyzing the other group i.e., 12,

We know that,

$2^3 = 8$ and $3^3 = 27$

And,

$8 < 12 < 27$

2 is smaller between 2 and 3.

Therefore, 2 will be taken at the tens place of the required cube root

Hence, $\sqrt[3]{12176} = 23$

The cube root of 32768 is to be calculated.

We will create groups of three digits starting from the right side digit of the number 32768, as $\overline{32}\,\overline{768}$

Analyzing, the group 768,

768 ends with 8 We know that if the digit 8 is at the end of a perfect cube number, then its cube root will have its unit place digit as 2 only.

Hence, the unit place digit of the required cube root will be taken as 2

Taking the other group i.e., 32,

We know that, $3^3 = 27$ and $4^3 = 64$

And,

$27 < 32 < 64$

3 is smaller between 3 and 4.

Therefore, 3 will be taken at the tens place of the required cube root.

Thus, $\sqrt[3]{32768} = 32$