DIFFERENTIAL EQUATION

CHAPTER – 9

DIFFERENTIAL EQUATION

INTRODUCTION

An equation involving independent and dependent variables and the derivatives of the dependent variables is called a **differential equation**. There are two kinds of differential equation:

(i) ORDINARY DIFFERENTIAL EQUATION

If the dependent variables depend on one independent variable x, then the differential equation is said to be ordinary.

Example

$$\frac{dy}{dx}+\frac{dz}{dx}=y+z,$$

$$\frac{dy}{dx}+xy=\sin x,\quad \frac{d^3y}{dx^3}+2\frac{dy}{dx}+y=e^x$$

$$k\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2},\quad y=x\frac{dy}{dx}+k\sqrt{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}$$

(ii) PARTIAL DIFFERENTIAL EQUATION

If the dependent variables depend on two or more independent variables, then it is known as partial differential equation

Example

$$y2\frac{\partial z}{\partial x}+y\frac{\partial^2 z}{\partial y}=ax,\quad \frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$$

ORDER OF DIFFERNETIAL EQUATION:

The order of the differential equation is the order of the highest order derivative present in the equation.

Example

$\frac{dy}{dx}$ = 3x +2 , The order of the equation is 1

DEGREE OF DIFFERENTIAL EQUATION:

The **degree of the differential equation** is the power of the highest order derivative, where the original equation is represented in the form of a polynomial equation in derivatives such as y', y", y"', and so on.

Example

$\frac{d^2y}{dx^2}$ +y = 0 is a differential equation, so the degree of this equation here is 1.

Note

Order and degree (if defined) of a differential equation are always positive integers.

In the differential equation, all the derivatives should be expressed in the polynomial form f_1 (x, y)

$$\left[\frac{d^m y}{dx^m}\right]^{n_1} + f_2(x, y)\left[\frac{d^{m-1} y}{dx^{m-1}}\right]^{n_2} + \ldots\ldots f_k(x, y)\left[\frac{dy}{dx}\right]^{n_k} = 0$$

The above differential equation has the order m and degree n_1.

FORMATION OF DIFFERENTIAL EQUATION

Differential equation corresponding to a family of curve will have :

(i) Order exactly same as number of essential arbitrary constants in the equation of curve.

(ii) No arbitrary constant present in it.
The differential equation corresponding to a family of curve can be obtained by using the following steps:

(a) Identify the number of essential arbitrary constants in equation of curve.

Note

If arbitrary constants appear in addition, subtraction, multiplication or division, then we can club them to reduce into one new arbitrary constant.

(b) Differentiate the equation of curve till the required order.

(c) Eliminate the arbitrary constant from the equation of curve and additional equations obtained in step (ii) above.

SOLVING A DIFFERENTIAL EQUATION

Finding the dependent variable from the differential equation is called solving or integrating it. The solution or the integral of a differential equation is, therefore, a relation between dependent and independent variables (free from derivatives) such that it satisfies the given differential equation

Note

The solution of the differential equation is also called its primitive, because the differential equation can be regarded as a relation derived from it.

There can be two types of solution of a differential equation:

(i) General solution (or complete integral or complete primitive): A relation in x and y satisfying a given differential equation and involving exactly same number of arbitrary constants as order of differential equation.

(ii) Particular Solution: A solution obtained by assigning values to one or more than one arbitrary constant of general solution.

SOLUTION METHODS OF FIRST ORDER AND FIRST DEGREE DIFFERENTIAL EQUATIONS:

A differential equation of first order and first degree is of the type $\frac{dy}{dx}$ + f(x, y) = 0, which can also be written as: M dx + N dy = 0, where M and N are functions of x and y.

Example

The differential equation of the family of circles passing through the origin and having centers on the x-axis .

Solution: Let the equation of family of circle passing through origin and having center (a,0) be

$(x-a)^2 + (y-0)^2 = a^2$

or, $x^2 + a^2 - 2ax + y^2 = a^2$

or, $x^2 - 2ax + y^2 = 0$ ....(i)

On differentiating equation (i), we get

or, $2x+2y\frac{dy}{dx} - 2a=0$ or, $x + y\frac{dy}{dx} - a=0$

or, $x + y\frac{dy}{dx} - \left[\frac{x^2+y^2}{2x}\right] = 0$

or, $2x^2 + 2xy\frac{dy}{dx} - (x^2+y^2) = 0$

or, $2xy\frac{dy}{dx} = y^2 - x^2$

(i) Variables separable:

If the differential equation can be put in the form, f(x) dx = ϕ(y) dy we say that variables are separable and solution can be obtained by integrating each side separately.

A general solution of this will be $\int f(x)\,dx = \int \phi(y)\,dy + c$, where c is an arbitrary constant

Example

$\frac{dy}{dx} = ye^x$

$\Rightarrow \frac{dy}{y} = e^x dx$

Integrating on both the side

$\Rightarrow \int \frac{dy}{y} = \int e^x dx$ $\Rightarrow \ln|y| = e^x + C$

(ii) Equations reducible to the variables separable form:

If a differential equation can be reduced into a variables separable form by a proper substitution, then it is said to be "Reducible to the variables separable type".

Its general form is $\frac{dy}{dx}$ = f(ax + by + c) a, b ≠ 0.

To solve this, put ax + by + c = t.

Example

$\frac{dy}{dx} = \cos(x+y)$

Let x + y = v

$\frac{dy}{dx} = \frac{dv}{dx} - 1$

$\Rightarrow \frac{dv}{dx} - 1 = \cos v \qquad \Rightarrow \frac{dv}{dx} = \cos v + 1$

$\Rightarrow \frac{dv}{\cos v+1} = dx$

Integrating on both the side

$\Rightarrow \int \frac{dv}{\cos v+1} = \int dx$

$\Rightarrow \int \frac{1}{2} sec^2 \frac{v}{2}\, dv = \int dx$

$\Rightarrow \tan\frac{v}{2} = x + C$

$\Rightarrow \tan\left(\frac{x+y}{2}\right) = x + C$, which is the required solution.

(iii) Homogeneous differential equations:

A differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where f and g are homogeneous function of x and y, and of the same degree, is called homogeneous differential equation and can be solved easily by putting y = v.x.

Steps to Solve Homogeneous Differential Equation

To solve a homogeneous differential equation following steps are followed:-

Given differential equation of the type

$\frac{dy}{dx} = F(x,y) = g\left(\frac{y}{x}\right)$

Step 1: Substitute y = vx in the given differential equation.

Step 2: Differentiating, we get,

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Now substitute the value of and y in the given differential equation, we get

v + x$\frac{dv}{dx}$ = g(v)

Step 3: Separating the variables, we get

$\frac{dv}{g(v)-v} = \frac{dx}{x}$

Step 4: Integrating both side of equation, we have

$\int \frac{dv}{g(v)-v} = \int \frac{dx}{x}$ + C

Step 5: After integration we replace v=y/x

Example

$\frac{dy}{dx} = \frac{(x-y)}{(x+y)}$

Solution: Given, $\frac{dy}{dx} = \frac{(x-y)}{(x+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{1-\frac{y}{x}}{1+\frac{y}{x}}$

If y = vx and dy/dx = v + x dv/dx

Then,

v + x dv/dx = (1-v)/(1+v)

Subtracting v from both the sides;

x dv/dx = (1-v)/(1+v) − v

x dv/dx = [(1-v)/(1+v)] -[(v+v^2)/(1+v)]

x dv/dx = (1-2v-v^2))/(1+v)

Now we can use the separation of variables method;

(1+v)/(1-2v-v^2) dv = (1/x) dx

Integrating both the sides;

∫(1+v)/(1-2v-v^2) dv = ∫(1/x) dx

1/2 ln(1-2v-v^2) = ln(x) + C

Put C = ln(k)

-1/2 ln(1-2v-v^2) = ln(x) + ln(k)

$(1-2v-v^2)^{\frac{-1}{2}}$ = k x

or we can write;

1-2v-v^2 = $\frac{1}{k^2x^2}$

Again, putting v = y/x;

1 - 2$\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2 = \frac{1}{k^2x^2}$

Eliminating x^2 term from denominator on both the sides, we get;

$y^2 + 2xy - x^2 = -1/k^2$

Now, put -1/k^2 = c

Adding 2x^2 on both the sides;

y^2 + 2 x y + x^2 = c + 2 x^2

Now factoring the above equation, we get;

(y + x)^2 = 2x^2+c

y + x = √(2x^2) +c

Or y = ±√(2x^2+c) − x

This is the solution for the given equation.

(iv) Equations reducible to the homogeneous form

Equations of the form $\frac{dy}{dx}=\frac{ax+by+c}{Ax+By+C}$ ....(a)

can be made homogeneous (in new variables X and Y) by substituting x = X + h and y = Y + k, where h and k are constants to obtain, $\frac{dY}{dX}$ = $\frac{aX+bY+(ah+bk+c)}{AX+BY+(Ah+Bk+C)}$ . .....(b)

These constants are chosen such that ah + bk + c = 0, and Ah + Bk + C = 0. Thus we obtain the following differential equation $\frac{dY}{dX}=\frac{aX+bY}{AX+BY}$.

The differential equation can now be solved by substituting Y = v X.

(v) Linear differential equation of first order

The differential equation $\frac{dy}{dx}$ + Py = Q , is linear in y. (Where P and Q are functions of x only).

Integrating Factor (I.F.): It is an expression which when multiplied to a differential equation converts it into an exact form.

I.F for linear differential equation = $e^{\int P dx}$ (constant of integration will not be considered)

∴ after multiplying above equation by I.F it becomes;

$$\frac{dy}{dx}.\ e^{\int Pdx} + Py\ .e^{\int Pdx} = Q.\ e^{\int Pdx}$$

$$\Rightarrow \frac{d}{dx}(y.\ e^{\int Pdx}) = Q.\ e^{\int Pdx}$$

$$\Rightarrow\ y.\ e^{\int Pdx} = \int Q.\ e^{\int Pdx} + C\ .$$

Note

Sometimes differential equation becomes linear, if x is taken as the dependent variable, and y as independent variable. The differential equation has then the following form : $\frac{dx}{dy}+P_1 x = Q_1$.

where P_1 and Q_1 are functions of y. Then I.F. now is $e^{\int P_1\, dy}$

Example

Find the general solution of the differential equation x dy -(y + $2x^2$). dx = 0

Solution: The give differential equation is x dy - (y + $2x^2$) dx = 0. This can be simplified to represent the following linear differential equation.

$\frac{dy}{dx}-\frac{y}{x}=2x$

Comparing this with the differential equation $\frac{dy}{dx}+Py=Q$; we have the values of $P=-\frac{1}{x}$ and the value of $Q=2x$. Hence we have the integration factor as

I.F. = $e^{\int -\frac{1}{x}dx} = e^{-logx} = \frac{1}{x}$

Further, the solution of the differential equation is as follows.

$y\times\frac{1}{x}=\int 2x\times\frac{1}{x}\,dx \qquad \frac{y}{x}=2x+C$

Note

A differential equation, which has only the linear terms of the unknown or dependent variable and its derivatives, is known as a linear differential equation. It has no term with the dependent variable of index higher than 1 and do not contain any multiple of its derivatives. It cannot have nonlinear functions such as trigonometric functions, exponential function, and logarithmic functions with respect to the dependent variable. Any differential equation that contains above mentioned terms is a nonlinear differential equation.

(vi) Equations reducible to linear form

(a) By change of variable.
Often differential equation can be reduced to linear form by appropriate substitution of the non-linear term

(b) Bernoulli's equation:

(c) Equations of the form $\frac{dy}{dx}$ + Py = Q.y^n, n≠ 0 and n ≠ 1

(d) where P and Q are functions of x, is called Bernoulli's equation and can be made linear in v by dividing by y^n and putting $y^{-n+1} = v$. Now its solution can be obtained as linear differential equation in v.

Example

$2 \sin x \frac{dy}{dx} - y \cos x = xy^3 e^x$.

EXACT DIFFERENTIAL EQUATION

The differential equation $M + N\frac{dy}{dx} = 0$ .....(a)

Where M and N are functions of x and y is said to be exact if it can be derived by direct differentiation (without any subsequent multiplication, elimination etc.) of an equation of the form f(x, y) = c

Example

$y^2 dy + x dx + \frac{dx}{x} = 0$ is an exact differential equation.

Remember: (i) The necessary condition for (a) to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(ii) For finding the solution of exact differential equation, following results on exact differentials should be remembered :

(a) $xdy + y\,dx = d(xy)$

(b) $\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$

(c) $2(x\,dx + y\,dy) = d(x^2 + y^2)$

(d) $\frac{xdy - ydx}{xy} = d\left(\ell n\frac{y}{x}\right)$

(e) $\frac{xdy - ydx}{x^2 + y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$

(f) $\frac{xdy + ydx}{xy} = d(\ln xy)$

(g) $\frac{xdy + ydx}{x^2\ y^2} = d\left(-\frac{1}{xy}\right)$

Geometrical application of differential equation:

Form a differential equation from a given geometrical problem. Often following formulae are useful to remember

(i) Length of tangent $(L_T) = \left|\frac{y\sqrt{1+m^2}}{m}\right|$

(ii) Length of normal $(L_N) = \left|y\sqrt{1+m^2}\right|$

(iii) Length of sub-tangent $(L_{ST}) = \left|\frac{y}{m}\right|$

(iv) Length of subnormal $(L_{SN}) = |my|$

where y is the ordinate of the point, m is the slope of the tangent $= \left(\frac{dy}{dx}\right)$

QUESTIONS

MCQ

Q1 If $y^{\frac{1}{4}} + y^{\frac{-1}{4}} =$ 2x be a curve satisfying the differential equation $(x^2 - 1)\frac{d^2y}{dx^2} + \beta x\frac{dy}{dx}$ +αy = 0 then ordered pair $(\alpha - \beta, \alpha + \beta)$ is :

(a) (-15, -14) (b) (-13, -12)
(c) (-15, -17) (d) (-13, -15)

Q2 The solution of the differential equation "$\frac{2}{y}\frac{dy}{dx} + \sec x = \frac{\tan x}{y^2}$, where $0 \le x < \frac{\pi}{2}$ and y(0) = 1, is given by

(a) $y^2 = 1 - \frac{x}{\sec x + \tan x}$ (b) $y^2 = 1 + \frac{x}{\sec x + \tan x}$
(c) $y = 1 + \frac{x}{\sec x + \tan x}$ (d) $y = 1 - \frac{x}{\sec x + \tan x}$

Q3 Solve the following differential equation:
$\frac{dy}{dx} + 2y\tan x = \sin x$

(a) $y = \sin x + C\cos^2 x$
(b) $y = \cos x + C\sin^2 x$
(c) $y = \sin x + C\operatorname{cosec}^2 x$
(d) $y = \cos x + C\cos^2 x$

Q4 Find the order of the differential equation of the family of circles with their center at origin.

(a) 1 (b) 2
(c) 0 (d) Not defined

Q5 Solve : $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$ if y(0) =1

(a) $\tan^{-1} y + \tan^{-1}(e^x) = \frac{\pi}{2}$
(b) $\tan^{-1} y - \tan^{-1}(e^x) = \frac{\pi}{2}$
(c) $\tan^{-1} y + \tan^{-1}(e^{-x}) = \frac{\pi}{2}$
(d) $\tan^{-1} y - \tan^{-1}(e^{-x}) = \frac{\pi}{2}$

Q6 Solve the following differential equation:
$\frac{dy}{dx} = 5x^2 + 2$

(a) $10x^3 + 12x - 3y^2 + C = 0$
(b) $10x^3 - 12x - 3y^2 + C = 0$
(c) $10x^3 - 12x + 3y^2 + C = 0$
(d) $10x^3 + 12x + y^2 + C = 0$

Q7 $y + x\frac{dy}{dx} = x^2$ passes through point (3, -3)

(a) $x^3 + 3xy = 54$
(b) $x^3 - 3xy = 50$
(c) $x^3 - 3xy = 54$
(d) Not defined

Q8 Let y(x) be the solution of the differential equation $(x\log x)\frac{dy}{dx} + y = 2x\log x, (x \ge 1)$ Then the integral constant c is?

(a) e (b) 0
(c) 2 (d) -1

Q9 An integrating factor for the differential equation $(1 + y^2)dx - (\tan^{-1} y - x)dy = 0$

(a) $-e^{\tan^{-1} y}$ (b) $e^{\tan y}$
(c) $e^{\tan^{-2} y}$ (d) $e^{\tan^{-1} y}$

Q10. Solution of differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is?

(a) $x^2 - y^2 = cx^3$
(b) $x^2 + y^2 = cx^3$
(c) $-x^2 - y^2 = px^3$
(d) $x^2 + y^2 = -px^3$

Q11. Let y = y(x) be the solution of the differential equation" $\sin x\frac{dy}{dx} + y\cos x = 4x$,
$x \in (0, \pi)$. If $y\left(\frac{\pi}{6}\right) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(a) $\frac{-8}{9}\pi^2$ (b) $\frac{4}{9}\pi^2$
(c) $\frac{8}{9}\pi^2$ (d) $\frac{-4}{9}\pi^2$

Q12. What is the order and degree of each of the following differential equation: $\left(\frac{dy}{dx}\right)^4 + \left(\frac{d^2y}{dx^2}\right) = 0$

(a) 1,2 (b) 2,3
(c) 2,2 (d) 2,1

Q13. What is the order and degree (if defined) of each of the following differential equation" :
$x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$

(a) 1,2 (b) 2,3
(c) 2,2 (d) 2,1

Q14. What is the order and degree (if defined) of each of the following differential equation:
$\left(\frac{d^2s}{dt^2}\right)^2 + \left(\frac{ds}{dt}\right)^3 + 4 = 0$

(a) 1,2 (b) 2,3
(c) 2,2 (d) 2,1

Q15. "What is the order and degree (if defined) of each of the following differential equation " $\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0$

(a) 3,1 (b) 2,2
(c) 1,2 (d) 2,3

Q16. Which of the following is linear differential Equation?

(a) $\frac{dy}{dx} + \frac{y}{x} = x$ (b) $\left(\frac{dy}{dx}\right)^2 + \frac{y}{x} = x$
(c) $\frac{dy}{dx} + \frac{\sin y}{x} = x$ (d) $\frac{d^2y}{dx^2} + \frac{y^2}{x} = x$

Q17. What is the solution of the differential equation?
$(x^2 + y^2)\frac{dy}{dx} - xy = 0$

(a) $x^2 = -2y^2\log y$ (b) $x^2 = 2y^2\log y$
(c) $x^2 = 2y\log y$ (d) $x^2 = -2y\log y$

Q18. Is $y = e^x \cos bx$ the solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

(a) Yes (b) No

(c) Can't say (d) None

Q19. Form the differential equation representing the family of ellipses having centre at the origin and foci on x-axis.

(a) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$

(b) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(c) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(d) $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

Q20. *Is* $y = e^x(A\cos x + B\sin x)$ the general solution of the differential equation" $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

(a) Yes (b) No

(c) Can't say (d) None

Q21. Form the differential equation of the family of straight lines $y = mx + c$, where " m" and " c" are arbitrary constants.

(a) $\frac{d^2x}{dy^2} = 0$ (b) $\frac{d\,y}{dx} = 0$

(c) $\frac{d\,x}{dy} = 0$ (d) $\frac{d^2y}{dx^2} = 0$

Q22. Form the differential equation of the family of concentric circle s $x^2 + y^2 = a^2$, where $a > 0$ and a is a parameter.

(a) $-x + y\frac{dy}{dx} = 0$ (b) $x - y\frac{dy}{dx} = 0$

(c) $x + y\frac{dy}{dx} = 0$ (d) $x + \frac{dy}{dx} = 0$

Q23. "Form the differential equation of the family of curves" y=a sin (bx + c), Where a and c are parameters."

(a) $\frac{d^2y}{dx^2} - b^2y = 0$ (b) $\frac{-d^2y}{dx^2} + b^2y = 0$

(c) $\frac{d^2y}{dx^2} + b^2y = 0$ (d) $\frac{d^2y}{dx^2} + b\,y = 0$

Q24. "Form the differential equation of the family of curves" $y = ae^{bx}$, where a and b are arbitrary constants.

(a) $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ (b) $y\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^2$

(c) $x\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ (d) $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)$

Q25. "Form the differential equation of the family of" curves $y^2 = m(a^2 - x^2)$, where a and m are parameters."

(a) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(b) $-xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(c) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} = 0$ (d) None

Q26. Form the differential equation of the family of circles having centres on the x-axis and radius unity.

(a) $\left(y\frac{dy}{dx}\right)^2 - y^2 = 1$ (b) $\left(y\frac{dy}{dx}\right)^2 + y^2 = -1$

(c) $\left(y\frac{dy}{dx}\right)^2 + y^2 = 1$ (d) None

Q27. Find the integrating factor of $\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$

(a) $(x+1)^3$ (b) $\frac{1}{(x+1)}$

(c) $\frac{1}{(x+1)^3}$ (d) None of these

Q28. The differential equation 2ydx + (3y + 2x)dy=0" is

(a) exact and homogenous but not linear

(b) exact, homogenous and linear

(c) exact and linear but not homogenous

(d) homogenous and linear but not exact

Q29. The number of arbitrary constants in the particular solution of a differential equation of third order is:

(a) 3 (b) 2

(c) 0 (d) 1

Q30. The curve for which the slope of the tangent a any point is equal to the ratio of the abscissa to the ordinate of the point is:

(a) Ellipse (b) Parabola

(c) Circle (d) Hyperbola

SUBJECTIVE QUESTIONS

Q1. Solve the differential equation (1 + x) y dx = (y – 1) x dy

Q2. Solve the differential equation x. y $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ $(1 + x + x^2)$

Q3. Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

Q4. Solve $x^2dy + y(x + y)\,dx = 0$

Q5. Solve : $\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$

NUMERICAL TYPE QUESTIONS

Q1. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and y(0) = 1, then $y\left(\frac{\pi}{2}\right)$ is equal to ______.

Q2. If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, y(1 + xy) dx = x dy, then $f\left(-\frac{1}{2}\right)$ is equal to _______.

Q3. Let y(x) be the solution of the differential equal $(x \log x)\frac{dy}{dx} + y = 2x \log x$, $(x \geq 1)$. Then y(e) is equal to_______.

Q4. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to _____.

Q5. If the solution of $\frac{dy}{dx} = \frac{ax+3}{2y+1}$ represents a circle, then the value of a is ______.

TRUE AND FALSE

Q1. The differential equation $\frac{dy}{dx} + Py = Q$, is linear in x.

Q2. The slope of a curve at any point is the reciprocal of twice the ordinate of that point and it passes through the point (4, 3). The equation of the curve is $y^2 = x + 5$.

Q3. The solution of the differential equation $y\,dx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to 3

Q4. The value of $\lim_{x\to\infty} y(x)$ obtained from the differential equation $\frac{dy}{dx} = y - y^2$, where $y(0) = 2$ is -1.

Q5. The general solution of the differential equation $y\,dy + \sqrt{1+y^2}\,dx = 0$ represents a family of hyperbola.

ASSERTION AND REASONING

Directions (Q.No. 1 – 5) Each of these questions contains two statements , one is Assertion (A) and other is Reason (R) .Each of these questions also has four alternative choices , only one of which is the correct answer .

(a) Both A and R are individually true and R is the correct explanation of A.

(b) Both A and R are individually true and R is not the correct explanation of A.

(c) A is true but R is false

(d) A is false but R is true

Q1. Assertion(A) : The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$ is 2.

Reason (R): The degree of the differential equation which is not a polynomial in differential coefficients, cannot be defined.

Q2. Assertion(A) : The differential equation of all parabola whose axis of symmetry is parallel to x – axis is of order 3

Reason (R): The order of equation depends upon the number of unknown in equation of the curve.

Q3. Assertion(A) : The elimination of two arbitrary constants in $y = (a+b)x$ results into a differential equation of the first order $x\frac{dy}{dx} = y$

Reason (R): Elimination of n arbitrary requires in general , a differential equation of the nth order.

Q4. Assertion(A) : The solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$ is $y(1 + x^2) = c + \sin x$

Reason (R): Solution of $\frac{dy}{dx} + P(x)y = Q(x)$ is $y.\ e^{\int P dx} = \int Q.\ e^{\int P dx} + C$.

Q5. Assertion(A) : Solution of differential equation $x\,dy - y\,dx = 0$ represents : straight line passing through origin

Reason (R): Degree of $\left(\frac{dy}{dx}\right)^5 + y = 0$ is 7.

HOMEWORK

MCQ

Q1. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y^4 = 0$ are respectively

(a) 2, 2 (b) 2, 1

(c) 1, 2 (d) 3, 2

Q2. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 = y$ are respectively

(a) 2, 2 (b) 3, 2

(c) 2, 3 (d) 1, 3

Q3. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2 - y^2)\frac{dy}{dx} = g(x)\, y$, then g(x) equals

(a) $2x^2$ (b) $\frac{1}{2}x^2$

(c) $\frac{1}{2}x$ (d) 2x

Q4. If differential equation of family of curves y ln |cx| = x, where c is an arbitrary constant, is $y' = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$, for some function ϕ, then ϕ (b) is equal to :

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) – 4 (d) 4

Q5. If $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5, the value of x for y = 3 is

(a) e^5 (b) $e^6 + 1$

(c) $\frac{e^6+9}{2}$ (d) ln6

Q6. If $\frac{dy}{dx} = 1 + x + y + xy$ and y (– 1) = 0, then function y is

(a) $e^{(1-x)^2/2}$ (b) $e^{(1+x)^2/2} - 1$

(c) ln(1 + x) – 1 (d) 1 + x

Q7. If y(x) is the solution of the differential equation $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and y(0) =0, then y(– 4) is equal to :

(a) 2 (b) 0

(c) –1 (d) 1

Q8. The solution of differential equation $\frac{dy}{dx} = \frac{x(2\ell nx+1)}{\sin y + y\cos y}$ is -

(a) y sin y = x ln x + c (b) y sin y = x^2ln x + c

(c) sin y = x^2ln x + c (d) y cos y = x^2ln x + c

Q9. The solution of the differential equation $\frac{dy}{dx} - ky = 0$, y(0) = 1, approaches zero when $x\to\infty$, if

(a) k = 0 (b) k > 0

(c) k < 0 (d) k ≥ 0

Q10. Integral curve satisfying $y' = \frac{x^2+y^2}{x^2-y^2}$, y (a) = 2, has the slope of the curve at the point (1, 2), is equal to

(a) $-\frac{5}{3}$ (b) – 1

(c) 1 (d) $\frac{5}{3}$

SUBJECTIVE QUESTIONS

Q1. Form the differential equation of all circles touching the x-axis at the origin and centre on the y-axis

Q2. Solve : $e^{\frac{dy}{dx}} = x + 1$, given that when x = 0, y = 3

Q3. Solve $\frac{dy}{dx} = (4x + y + 1)^2$

Q4. Solve : y sin x $\frac{dy}{dx}$ = cos x (sin x – y^2)

Q5. Solve x dx + y dy = $\frac{xdy - ydx}{x^2+y^2}$

NUMERICAL TYPE QUESTIONS

Q1. If $\frac{dy}{dx} + y\tan x = \sin 2x$ and y(0) = 1, then y(π) is equal to ____.

Q2. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c > 0is a parameter , is of order ______.

Q3. The solution of the differential equation $\frac{dy}{dx}$ +y cot x =sin x is y sin x = k (2x –sin2x) +c then k is ______.

Q4. If (2 + sin x) $\frac{dy}{dx}$ + (y + 1) cos x = 0 and y(0) = 1, then $3y\left(\frac{\pi}{2}\right)$ is equal to _______.

Q5. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w. r. t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is _______.

TRUE AND FALSE

Q1. If the dependent variables depend on one independent variable x, then the differential equation is said to be ordinary.

Q2. If the dependent variables depend on only one independent variables, then it is known as partial differential equation

Q3. $y^2\, dy + x\, dx + \frac{dx}{x} = 0$ is an exact differential equation.

Q4. Equations of the form $\frac{dy}{dx} + Py = Q.y^n$, $n \neq 0$ and $n \neq 1$ where P and Q are functions of x, is called Homogenous differential equation.

Q5. I.F for linear differential equation = $e^{\int Pdx}$

ASSERTION AND REASONING

Directions (Q.No. 1 – 5) Each of these questions contains two statements , one is Assertion (A) and other is Reason (R) .Each of these questions also has four alternative choices , only one of which is the correct answer .

(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true and R is not the correct explanation of A.
(c) A is true but R is false
(d) A is false but R is true

Q1. **Assertion(A) :** The differential equation formed by eliminating a and b from $y = ae^x + be^{-x}$ is $\frac{d^2y}{dx^2} - y = 0$

Reason (R): y = a$e^x + b\, e^{-x}$ ....(i)

Differentiating w.r.t 'x'

$\frac{dy}{dx} = ae^x - be^{-x}$

Differentiating again w.r.t x

$\frac{d^2y}{dx^2} = ae^x + be^{-x}$ ......(ii)

Subtracting equation (i) from equation (ii)

$\frac{d^2y}{dx^2} - y = 0$

Q2. **Assertion(A) :** The degree of the differential equation given by $\frac{dy}{dx} = \frac{x^4 - y^4}{(x^2+y^2)xy}$ is 1.

Reason (R): The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction. The given differential equation has first order derivative which is free from radical and fraction with power = 1 , thus it has a degree 1.

Q3. **Assertion(A) :** The solution of differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ is $\cos\frac{y}{x} = xc$

Reason (R): $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ we can clearly see that it is homogenous equation ,substituting y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = v + tan\, v$

Separating the variables and integrating we get

$\Rightarrow \int \frac{1}{tan\, v} dv = \int \frac{1}{x} dx$

$\Rightarrow log\,(\,sin\, v) = log\, x + log\, C$

$\Rightarrow sin\, v = xC$

$\Rightarrow sin\left(\frac{y}{x}\right) = xC$

Q4. **Assertion(A) :** The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are 2 and 1

Reason (R): The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{\frac{1}{2}} = x$ is of order 1 and degree 3.

Q5. **Assertion(A) :** The integrating factor of $\frac{dy}{dx} - \frac{y}{x} = x^2$ is $\frac{1}{x}$

Reason (R): For linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$, integrating factor is equal to $e^{\int Pdx}$

SOLUTIONS

MCQ

S1. **(a)** $y^{\frac{1}{4}} + y^{\frac{-1}{4}} = 2x$

Differentiating, $\frac{1}{4y}\left(y^{\frac{1}{4}} - y^{\frac{-1}{4}}\right)\frac{dy}{dx} = 2$

$\Rightarrow \left(y^{\frac{1}{4}} - y^{\frac{-1}{4}}\right)\frac{dy}{dx} = 8y$

Now,

$y^{\frac{1}{4}} - y^{\frac{-1}{4}} = \sqrt{\left(y^{\frac{1}{4}} + y^{\frac{-1}{4}}\right)^2 - 4} = 2\sqrt{x^2 - 1}$

$\Rightarrow \sqrt{x^2 - 1}\frac{dy}{dx} = 4y$

Again Diff. $\sqrt{x^2 - 1}\frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2-1}}\frac{dy}{dx} = 4\frac{dy}{dx}$

$(x^2 - 1)\frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} = 4\sqrt{x^2 - 1}\frac{dy}{dx} = 16y$

$\alpha = -16, \beta = 1$

Thus, $(\alpha + \beta, \alpha - \beta) = (-15, -17)$

S2. **(a)** $\frac{2}{y}\frac{dy}{dx} + \sec\, x = \frac{\tan x}{y^2}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$

$\Rightarrow 2y\frac{dy}{dx} + y^2\sec\, x = \tan\, x$

$y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\sec\, x = \tan\, x$

I.F. $= e^{\int \sec x dx} = e^{\ln(\sec x + \tan x)}$

$= \sec x + \tan x$

$\Rightarrow$ t. $(\sec\, x + \tan\, x) = \int (\sec\, x + \tan\, x)\tan\, xdx$

$\Rightarrow$ t. $(\sec x + \tan x) = \int \sec\, x\tan\, xdx + \int \tan^2\, xdx$

$\Rightarrow$ $y^2(\sec x + \tan x) = \sec x + \tan x - x + c$

If $y(0) = 1$

$\Rightarrow c = 0$

$\Rightarrow y^2 = 1 - \frac{x}{\sec\, x + \tan\, x}$

S3. **(d)** Given $\frac{dy}{dx} + 2\tan\, x \cdot y = \sin\, x$

It is a linear differential equation in y,

I.F. & $= e^{\int 2\tan\, xdx} = e^{-2\log\, \cos\, x}$

$= e^{\log\,(\cos\, x)^{-2}} = (\cos\, x)^{-2} = \frac{1}{\cos^2 x}$

$y \cdot \frac{1}{\cos^2 x} = \int \sin\, x \cdot \frac{1}{\cos^2 x}dx + \mathrm{C} = \int \sec\, x.\tan\, xdx + \mathrm{C} = \sec\, x + \mathrm{C}$

$\Rightarrow y = \cos\, x + C\cos^2\, x, C$ is arbitrary constant.

S4. **(a)** For a family of circle general equation is $(x - 0)^2 + (y - 0)^2 = r^2$ ... (i)

Here (h, k) = (0, 0) and radius = r

Differentiating equation (i) w.r.t x

2x + 2y$\frac{dy}{dx} = 0$

x + y$\frac{dy}{dx} = 0$

$= \frac{dy}{dx} = -\frac{x}{y}$

Therefore , order of differential equation is 1

S5. **(a)** Given,

$(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$

$(1 + e^{2x})dy = -(1 + y^2)e^x dx$

$\frac{dy}{dx} = \frac{-(1+y^2)\cdot e^x}{1+e^{2x}}$

$\frac{dy}{(1+y)^2} = \frac{-e^x dx}{1+e^{2x}}$

Integrating both sides,

$\int \frac{dy}{(1+y)^2} = -\int \frac{e^x dx}{1+e^{2x}}$

Let $t = e^x$

$\frac{dt}{dx} = e^x$

$\frac{dt}{e^x} = dx$

∴Our equation becomes

$\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+t^2}\frac{dt}{e^x}$ $\int \frac{dy}{1+y^2} = -\int \frac{dt}{1+t^2}$ $\tan^{-1} y = -\tan^{-1} t + C$ (As $\int \frac{dx}{1+x^2} = \tan^{-1}\, x$)

Putting back the value of $t = e^x$

$\tan^{-1}\, y = -\tan^{-1}\,(e^x) + C$... (1)

Put y = 1 and x = 0 in equation (1)

$\tan^{-1}\,(1) = -\tan^{-1}\,(e^0) + C$

$\tan^{-1}\, 1 = -\tan^{-1}\, 1 + C$

$\tan^{-1}\, 1 + \tan^{-1}\, 1 = C$

$2\tan^{-1}\, 1 = C$

$2 \times \frac{\pi}{4} = C$

$2 \times \frac{\pi}{2} = C$

$C = \frac{\pi}{2}$

Putting value of C in (1)

$\tan^{-1}\, y = -\tan^{-1}\,(e^x) + C$

$\tan^{-1}\, y = -\tan^{-1}\,(e^x) + \frac{\pi}{2}$

$\tan^{-1}\, y + \tan^{-1}\,(e^x) = \frac{\pi}{2}$

is the required particular solution.

S6. **(a)** Given that, $\frac{dy}{dx} = 5x^2 + 2$

Separating the variables, we get

$dy = (5x^2 + 2)dx - (1)$

Integrating both sides of (1), we get

$\int ydy = \int 5x^2 + 2dx$

$\frac{y^2}{2} = \frac{5x^3}{3} + 2x + C_1$

$3y^2 = 10x^3 + 12x + 6C_1$

$10x^3 + 12x - 3y^2 + C = 0$ (where $6C_1 = C$)

S7. (c)

$\boldsymbol{y + x\frac{dy}{dx} = x^2}$

$\boldsymbol{\frac{dy}{dx} + \frac{1}{x}y = x}$

If $= \boldsymbol{e^{\int \frac{1}{x}dx} = e^{lnx} = x}$

So the complete solution is:

$yx = \int x^2 dx$

$xy = \frac{x^3}{3} + c$

Passes through point (3,-3)

$-9 = \frac{27}{3} + c$

$c = -\frac{54}{3}$

$\therefore xy = \frac{x^3}{3} - \frac{54}{3}$

So $3xy = x^3 - 54$

$x^3 - 3xy = 54$

S8. (c) The given equation is,

$(x\log x)\frac{dy}{dx} + y = 2x\log x$

Dividing by x log x, we get

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\log x} = 2$

I.F=$e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

y (I.F) =$\int$ Q × $I.F$

$\Rightarrow y\log x = \int 2\log x dx$

Using product rule on RHS, we get

$y\log x = 2[x(\log x - 1)] + C$

Putting x=1, we get

$0 = 2[1(0-1)] + C$

$\Rightarrow C = 2$

S9. (d) $(1+y^2)dx - (\tan^{-1} y - x)dy = 0$

$dy/dx = (1+y^2)/(\tan^{-1} y - x)$

$dx/dy = [\tan^{-1} y/1+y^2] - [x/1+y^2]$

$\frac{dy}{dx} + \frac{x}{y^2+1} = \frac{\tan^{-1} y}{y^2+1}$

This is equation of the form " dx/dy + Px=Q

So, I.F. $= e^{\int \text{Pdy}}$

$= e^{\int 1/1+y^2 \cdot dy}$

$= e^{\tan^{-1} y}$

S10. (b) It is homogeneous equation

$\frac{dy}{dx} = [x^2 + 3y^2]/2xy$

Put y = vx and $dy/dx = v + x\frac{dv}{dx}$

So we get, v +x$\frac{dv}{dx}$= $\frac{1+3v^2}{2v}$

x$\frac{dv}{dx}$ = $\frac{1+v^2}{2v}$

On integrating, we get

$x^2 + y^2 = cx^3$. (where c is a constant)

S11. (b) Given, $\sin x\frac{dy}{dx} + y\cos x = 4x$

$\frac{dy}{dx} + y\cot x = \frac{4x}{\sin x}$

I.F. $= e^{\int \cot x dx} = e^{\log_e |\sin x|} = \sin x$ as $x \in (0, \pi)$

$\& y \cdot \sin x = \int 4x dx + c$

$y\sin x = c + 2x^2$

$y\left(\frac{\pi}{6}\right) = 0$

$\Rightarrow 0 \cdot \sin\frac{\pi}{6} = c + 2\left(\frac{\pi}{6}\right)^2$. So, c = $\frac{-\pi^2}{18}$

$y\sin x = 2x^2 - \frac{\pi^2}{18}$

Put $x = \frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right).\sin\frac{\pi}{2} \& = 2\left(\frac{\pi}{2}\right)^2 - \frac{\pi^2}{18}$

$y\left(\frac{\pi}{2}\right) \& = \frac{\pi^2}{2} - \frac{\pi^2}{18} = \frac{-\pi^2 + 9\pi^2}{18}$

$y\left(\frac{\pi}{2}\right) \& = \frac{4}{9}\pi^2$

S12. (d) The order of a differential equation is the order of" the highest derivative involved in the equation.

So, the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and degree of differential equations is the power of the highest derivative in a differential equation.

So, the power at this order is 1. So the answer is 2,1.

S13. (c) The order of a differential equation is the order of the highest derivative involved in the equation.

So, the order comes out to be 2 as we have $\frac{d^2y}{dx^2}$ and the degree is the highest power to which a derivative is raised.

So the power at this order is 2.

So the answer is 2,2.

S14. (c) The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 2 as we have $\frac{d^2s}{dt^2}$ and the degree is the highest power to which a derivative is raised.

So the power at this order is 2. So the answer is 2, 2.

S15. (a) The order of a differential equation is the order of the highest derivative involved in the equation.

So the order comes out to be 3 as we have $\frac{d^3y}{dx^3}$ and the degree is the highest power to which a derivative is raised.

So the power at this order is 1.

So the answer is " 3,1"."

S16. (a) A differential equation, which has only the linear terms of the unknown or dependent variable and its derivatives, is known as a linear differential equation. It has no term with the dependent variable of index higher than 1 and do not contain any multiple of its derivatives. It cannot have nonlinear functions such as trigonometric functions, exponential function, and logarithmic functions with respect to the dependent variable. Any differential equation that contains above mentioned terms is a nonlinear differential equation. Hence option (a) is correct.

S17. **(b)** Given $x^2 = 2y^2\log\ y$ On differentiating both sides with respect to x, we get

$2x = 2(2y)\log\ y\left(\frac{dy}{dx}\right) + 2y^2\left(\frac{1}{y}\right)\frac{dy}{dx}$

$x = (2y)\log\ y\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right)$

$x = \left(\frac{dy}{dx}\right)((2y)\log\ y + y)$

Multiply both sides with y

$xy = (2y^2\log\ y + y^2)\frac{dy}{dx}$

We know, $x^2 = 2y^2\log y$. So replace $2y^2$logy with x^2 in the above equation.

$xy = (x^2 + y^2)\frac{dy}{dx}$

$(x^2 + y^2)\frac{dy}{dx} - xy = 0$

$\therefore x^2 = 2y^2\log y$ is the solution of $(x^2 + y^2)\frac{dy}{dx} - xy = 0$

S18. **(b)** Given $y = e^x\cos bx$ On differentiating both sides with respect to x, we get

$\frac{dy}{dx} = e^x\cos\ bx + e^x(-b\sin\ bx)$

On differentiating both sides with respect to x, we get

$\frac{d^2y}{dx^2} = e^x\cos\ bx + e^x(-b\sin\ bx) + e^x(-b^2\cos\ bx) + e^x(-b\sin\ bx)$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$= e^x\cos bx + e^x(-b\sin bx) + e^x(-b^2\cos bx) + e^x(-b\sin bx) - 2e^x\cos\ bx - 2e^x(-b\sin\ bx) + 2e^x\cos\ bx$

$= e^x\cos\ bx - e^x(b^2\cos\ bx)$

This is not a solution

$\therefore \mathrm{y} = \mathrm{e^x}\cos \mathrm{bx}$ is not the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

S19. **(c)** The equation of the family of ellipses having a center at the origin and foci on the x-axis is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ....(i)

It is a two-parameter family of curves.

Differentiating (i) twice with respect to x, we get

$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$

and $\frac{2}{a^2} + \frac{2}{b^2}\left(\frac{dy}{dx}\right)^2 + \frac{2y}{b^2}\frac{d^2y}{dx^2} = 0$

On simplifying we get

$\Rightarrow \frac{x}{a^2} + \frac{y}{b^2}\frac{dy}{dx} = 0$........(ii) and,

$\frac{1}{a^2} + \frac{1}{b^2}\left(\frac{dy}{dx}\right)^2 + \frac{y}{b^2}\frac{d^2y}{dx^2} = 0$ ......(iii)

Multiplying (iii) by x

$\frac{x}{a^2} + \frac{x}{b^2}\left(\frac{dy}{dx}\right)^2 + \frac{xy}{b^2}\frac{d^2y}{dx^2} = 0$

and subtracting this from (ii), we get

$\frac{1}{b^2}\left\{y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 - xy\frac{d^2y}{dx^2}\right\} = 0$

$\Rightarrow xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

This is the required differential equation.

S20. **(a)** Given $y = e^x(A\cos\ x + B\sin\ x)$

On differentiating both sides with respect to x, we get

$\frac{dy}{dx} = e^x(A\cos\ x + B\sin\ x) + e^x(-A\sin\ x + B\cos\ x)$

On differentiating both sides with respect to x, we get

$\frac{d^2y}{dx^2} = e^x(A\cos\ x + B\sin\ x) + e^x(-A\sin\ x + B\cos\ x) + e^x(-A\sin x + B\cos x)$
$+e^x(-A\cos\ x - B\sin\ x)$

Now let's see what is the value of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$= e^x(\mathrm{A}\cos x + \mathrm{B}\sin x) + e^x(-\mathrm{A}\sin x + \mathrm{B}\cos x) + e^x(-A\sin x + B\cos x)\& + e^x(-A\cos\ x - B\sin\ x) - 2e^x(A\cos\ x + B\sin\ x) - 2e^x(-A\sin\ x + B\cos\ x) + 2e^x(A\cos\ x + B\sin\ x)$

= 0

Therefore, y= $e^x(A\cos\ x + B\sin\ x)$ is the solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

S21. **(d)** The equation of a straight line is represented as, $y = mx + c$

Differentiating the above equation with respect to x, $\frac{dy}{dx} = m$

Differentiating the above equation with respect to x, $\frac{d^2y}{dx^2} = 0$

This is the differential equation of the family of straight lines y = mx + c, where m and c are arbitrary constants

S22. **(c)** Now, in the general equation of of the family of concentric circles $x^2+y^2=a^2$,

where $a>0$, 'a' represents the radius of the circle and is an arbitrary constant.

The given equation represents a family of concentric circles centred at the origin.

$x^2+y^2=a^2$ Differentiating the above equation with respect to x on both sides, we have," $2x+2y\frac{dy}{dx}=0$ (As a >0, derivative of a with respect to x is 0.)

$x+y\frac{dy}{dx}=0$

S23. **(c)** Equation of the family of curves, $y=\text{a}\sin(bx+c)$,

Where a and c are parameters.

Differentiating the above equation with respect to x on both sides, we have,

$y=a\sin(bx+c)\ldots(1)$

$\frac{dy}{dx}=ab\cos\,(bx+c)$

$\frac{\text{d}^2\text{y}}{\text{dx}^2}=-\text{ab}^2\sin\,(\text{bx}+\text{c})$

$\frac{d^2y}{dx^2}=-b^2y$

$\frac{d^2y}{dx^2}+b^2y=0$

This is the required differential equation.

S24. **(a)** Equation of the family of curves, y=ae $^{\text{bx}}$,where a and " b" are arbitrary constants .

Differentiating the above equation with respect to x on both sides, we have,

$y=ae^{bx}\quad\ldots(1)$

$\frac{dy}{dx}=abe^{bx}\quad\ldots(2)$

$\frac{d^2y}{dx^2}=ab^2e^{bx}$

$y\frac{d^2y}{dx^2}=ab^2e^{bx}(ae^{bx})$

$y\frac{d^2y}{dx^2}=(abe^{bx})^2$

$y\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

This is the required differential equation.

S25. **(a)** Equation of the family of curves,

$y^2=m(a^2-x^2)$,"where a and m are parameters.

Differentiating the above equation with respect to x on both sides, we have,

$2y\frac{dy}{dx}=m(-2x)$

$y\frac{dy}{dx}=-mx$

$m=-\frac{y}{x}\frac{dy}{dx}\ldots(1)$

Differentiating the above equation with respect to x on both sides,

$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-m\ldots(2)$

From equations (1) and (2),

$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

This is the required differential equation.

S26. **(c)** Equation of the family of circles having centers on the x-axis and radius unity can be represented by $(x-a)^2+(y)^2=1$, where a is an arbitrary constants.

$(x-a)^2+y^2=1\ldots(1)$

Differentiating the above equation with respect to x on both sides, we have ,

$2(x-a)+2(y)\frac{dy}{dx}=0$

$x-a+y\frac{dy}{dx}=0$

$a=x+y\frac{dy}{dx}$

Substituting the value of a in equation (1)

$\left(x-x-y\frac{dy}{dx}\right)^2+y^2=1$

$\left(y\frac{dy}{dx}\right)^2+y^2=1$

This is the required differential equation.

S27. **(c)** Given $\frac{dy}{dx}-\frac{3y}{x+1}=(x+1)^4$

Now integrating factor = $e^{\int\frac{-3}{x+1}dx}=\frac{1}{(x+1)^3}$

S28. **(b)** $\because\ 2ydx+(2x+3y)dy=0$

It is homogeneous. The given equation can be written as $\frac{dy}{dx}=\frac{-2y}{2x+3y}$. It is a linear form.

$Mdx+Ndy=0$

$2ydx+(3y+2x)dy=0$

hence, $M=2y$ and $N=2x+3y$

$\frac{\partial M}{\partial y}=\frac{\partial(2y)}{\partial y}=2$ and $\frac{\partial N}{\partial x}=\frac{\partial(2x+3y)}{\partial x}=2$

As $\frac{\partial M}{\partial y}=\frac{\partial N}{\partial y}$

so, it is an exact equation.

S29. **(c)** The solution free from arbitrary constants i.e., the solution obtained from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation."

S30. **(d)** This is true for hyperbola.

SUBJECTIVE QUESTIONS

S1. The equation can be written as –

$\left(\frac{1+x}{x}\right)dx=\left(\frac{y-1}{y}\right)\ dy$

$\Rightarrow \int\left(\frac{1}{x}+1\right) dx = \int\left(1-\frac{1}{y}\right) dy$

$\Rightarrow \ln x + x = y - \ln y + c$

$\Rightarrow \ln y + \ln x = y - x + c$

$\Rightarrow x.\ y = c\ e^{y-x}$

S2. Differential equation can be rewritten as x .y $\frac{dy}{dx} = (1 + y^2)\left(1+\frac{x}{1+x^2}\right)$

$\Rightarrow \frac{y}{1+y^2} dy = \left(\frac{1}{x}+\frac{1}{1+x^2}\right) dx$

Integrating, we get

$\Rightarrow \frac{1}{2} \ln(1 + y^2) = \ln x + \tan^{-1}x + \ln c$

$\Rightarrow \sqrt{1+y^2} = cxe^{\tan^{-1}x}$

S3. $\frac{dy}{dx} = \sin (x + y)$

Putting $x + y = t$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$\therefore \frac{dt}{dx} - 1 = \sin t$

$\Rightarrow \frac{dt}{dx} = 1 + \sin t$

$\Rightarrow \frac{dt}{1+\sin t} = dx$

Integrating both sides,

$\int\frac{dt}{1+\sin t} = \int dx$

$\Rightarrow \int\frac{1-\sin t}{\cos^2 t} dt = x + c$

$\Rightarrow \int(\sec^2 t - \sec t\ \tan t)\ dt = x + c$

$\Rightarrow \tan t - \sec t = x + c$

$\Rightarrow -\frac{1-\sin t}{\cos t} = x + c$

$\Rightarrow \sin t - 1 = x \cos t + c \cos t$ substituting the value of t

$\Rightarrow \sin (x + y) = x \cos (x + y) + c \cos (x + y) + 1$

S4. The given differential equation can be rewritten as

$\frac{dy}{dx} = \frac{-y(x+y)}{x^2}$ or $\frac{dy}{dx} = -\frac{y}{x} - \frac{y^2}{x^2}$

putting y = v x

$\Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$

Given equation transforms to $v + x\frac{dv}{dx} = -v - v^2$

$\Rightarrow \int\frac{dv}{v^2+2v} = -\int\frac{dx}{x}$

$\Rightarrow \frac{1}{2}\int\left[\frac{1}{v}-\frac{1}{v+2}\right] dv = -\int\frac{dx}{x}$

$\Rightarrow \ln |v| - \ln |v + 2| = -2\ln|x| + \ln c$ c > 0

$\Rightarrow \left|\frac{vx^2}{v+2}\right| = c$

$\Rightarrow \left|\frac{x^2y}{2x+y}\right| = c\ ;\ c > 0$

S5. Dividing both sides by y^2

$\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{xy} = \frac{1}{x^2}$ ..... (a)

Putting $\frac{1}{y} = t$

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore$ differential equation (a) becomes,

$-\frac{dt}{dx} - \frac{t}{x} = \frac{1}{x^2}$

$\frac{dt}{dx} + \frac{t}{x} = -\frac{1}{x^2}$ which is linear differential equation in $\frac{dt}{dx}$ IF $= e^{\int\frac{1}{x}dx} = e^{\ln x} = x$

$\therefore$ General solution is $-$ t. x = $\int -\frac{1}{x^2}.x\ dx + c$

$\Rightarrow t\ .x = -\ln x + c$

$\Rightarrow \frac{x}{y} = -\ln x + c$

NUMERICAL TYPE QUESTIONS

S1. $\left(\frac{1}{3}\right)$ $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$

$(2 + \sin x)\frac{dy}{dx} + y\cos x + \cos x = 0$

$(2 + \sin x)\frac{dy}{dx} + y\cos x = -\cos x$

Now, Divide by $(2 + \sin x)$

$\frac{dy}{dx} + \left(\frac{\cos x}{2 + \sin x}\right) y = \frac{-\cos x}{(2 + \sin x)}$

$P = \frac{\cos x}{2 + \sin x}\ Q = \frac{-\cos x}{2 + \sin x}$

$I.F = e^{\int p.dx} \Rightarrow e^{\int\frac{\cos x}{2+\sin x}} \Rightarrow e^{\log(2+\sin x)}$

$I.F \Rightarrow 2 + \sin x$

$y(I.F) = \int Q.\text{IF}dx + c$

$$y(2+\text{sinx}) = \int -\frac{\text{cosx}}{2+\text{sinx}}.2+\text{sindx}$$

$(2+\text{sinx})y = -\text{sinx}+c$

if $y(0) = 1$

$\Rightarrow (2+\sin 0)(1) = -\sin(0)+c$

$2 = c\ y$

$= \frac{2-\text{sinx}}{2+\text{sinx}}$

$\Rightarrow \frac{2-1}{2+1} = \frac{1}{3}$

$y(\pi/2) = \frac{1}{3}$

S2. **($\frac{4}{5}$)** $y(1+xy)dx = xdy$

$$\Rightarrow \frac{ydx - xdy}{y^2} - xdx$$

$\Rightarrow d\left(\frac{x}{y}\right) = -xdx$

Integrating both sides, we get

$$\frac{x}{y} = \frac{-x^2}{2} + c \qquad ...(i)$$

Since the curve passes through (1, -1)

$$-1 = \frac{-1}{2} + c \Rightarrow c = \frac{-1}{2}$$

$$\therefore \frac{x}{y} = \frac{-x^2}{2} - \frac{1}{2} \dots [from(i)]$$

$$\Rightarrow y = \frac{-2x}{x^2+1}$$

$$i.e., f(x) = \frac{-2x}{x^2+1}$$

$\therefore f\left(-\frac{1}{2}\right) = \frac{4}{5}$

S3. **(2)** $(\text{xlogx})\frac{\text{dy}}{\text{dx}} + \text{y} = 2\text{xlogx}$

$\rightarrow$ dividing by x.log x, we get

$$\rightarrow \frac{dy}{dx} + \frac{\text{y}}{\text{xlogx}} = 2$$

Multiplying by IF, we get

$\rightarrow d(\text{IF} \times y) = 2\text{logxdx}$

$\rightarrow \text{IF} = e^{\int \frac{1}{\text{xlogx}}\text{dx}} = e^{\log(\log x)} = \text{logx}$

Multiplying by IF, we get

$\rightarrow d(\text{IF} \times y) = 2\text{logxdx}$

By integrating, we get

$\rightarrow \text{ylogx} = \int 2\text{logxdx}$

By using Product Rule on RHS, we get

$\rightarrow \text{ylogx} = 2[\text{logx} \int 1 - \int((\int 1)\frac{\text{d}}{\text{dx}}(\text{logx}))\text{dx}$

$\rightarrow \text{ylogx} = 2[\text{x}(\text{logx} - 1)] + \text{C}$

Put $\text{x} = 1$, we get $0 = 2[1(0-1)] + \text{C}$

$\text{C} = 2$

$\rightarrow \text{ylogx} = 2[\text{x}(\text{logx} - 1)] + 2$

at $\text{x} = \text{e}$

$\text{y} = 2$

S4. **(7)** $\frac{dy}{dx} = y + 3$ and $y(0) = 2$

then $y(\ln 2) = (?)$ $\therefore \frac{dy}{dx} = y + 3$

$\therefore \frac{1}{y+3}\text{dy} = \text{dx}$ $\int \frac{1}{y+3}\text{dy} = \int \text{dx}$

$\therefore \ln(y+3) = x + c$

$\rightarrow$ Initial condition $y(0) = 2$

$\therefore \ln(2+3) = 0 + c$

$\therefore c = \ln 5$

$\rightarrow \ln(y+3) = x + \ln 5$

$\therefore \ln(y+3) - \ln 5 = x$

$\therefore \ln\left(\frac{y+3}{5}\right) = x$

$\therefore \frac{y+3}{5} = e^x$

$\therefore y = 5e^x - 3$

for y (ln2) $y = 5e^{\ln 2} - 3$

$= 5 \times 2 - 3$

$= 10 - 3$

$= 7$

S5. **(-2)** $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$\int(2y+f)\,dy = \int(ax+3)\,dx$

$2y^2 + fy = ax^2 + 3x + c$

$\Rightarrow ax^2 - 2y^2 + 3x - fy + c = 0$

In the equation of circle

Co -efficient of $'x^2'$ = C- efficient of $'y^2'$

a = -2

TRUE AND FALSE

S1. **(False)** The differential equation $\frac{\text{dy}}{\text{dx}}$ + P y = Q , is linear in y

S2. **(True)** According to the question, we have

$$\frac{dy}{dx} = \frac{1}{2y}$$

$\Rightarrow 2\text{ydy} = \text{dx}$

On integrating, we get

$\text{y}^2 = \text{x} + \text{C}$

This passes through $(4,\ 3)$

Therefore, $9 = 4 + \text{C} \Rightarrow \text{C} = 5$

Hence, the equation of the curve is $\text{y}^2 = \text{x} + 5.$

S3. **(True)** Given, $\text{ydx} - (\text{x} + 2\text{y}^2)\text{dy} = 0$

$\Rightarrow \text{ydx} - \text{xdy} = 2\text{y}^2\text{dy}$

$\Rightarrow \frac{\text{ydx} - \text{xdy}}{\text{y}^2} = 2\text{dy}$

$\Rightarrow \text{d}\left(\frac{\text{x}}{\text{y}}\right) = 2\text{dy}$

Integrating we get,

$$\Rightarrow \left(\frac{x}{y}\right) = 2y + c \Rightarrow 2y^2 + cy = x = f(y)$$

$$\text{Given } f(-1) = 1 \Rightarrow 2(-1)^2 - c = 1 \Rightarrow c = 1$$

$$\Rightarrow f(y) = 2y^2 + y$$

$$\therefore f(1) = 2(1)^2 + 1 = 3$$

S4. **(False)** $\frac{dy}{dx} = y - y^2$ y (0) = 2

$$\frac{dy}{y-y^2} = dx$$

$$\frac{-dy}{y^2-y} = dx$$

$$\frac{-dy}{y^2\left(\frac{1}{2}\right)^2 - 2\times\frac{1}{2}y - \frac{1}{4}} = dx$$

$$\frac{-dy}{y^2\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = -dx$$

$$\frac{1}{2\times\frac{1}{2}} ln\left|\frac{y-\frac{1}{2}-\frac{1}{2}}{y-\frac{1}{2}+\frac{1}{2}}\right| = -x + c$$

$$\ln\left(\frac{y-1}{y}\right) = x + c$$

Given $y(0) = 2$

$$\ln\left(\frac{2-1}{2}\right) = C$$

$$C = \ln\left(\frac{1}{2}\right)$$

$$\ln\left(\frac{y-1}{y}\right) = -x + \ln\left(\frac{1}{2}\right)$$

$$\ln\left(\frac{y}{y-1}\right) = x + \ln(2)$$

$$\left(\frac{y}{y-1}\right) = e^{x+\ln(2)}$$

$$\left(\frac{y-1}{y}\right) = \frac{1}{e^{x+\ln(2)}}$$

$$1 - \frac{1}{y} = e^{-(x+\ln(2))}$$

$$\frac{1}{y} = 1 - e^{-(x+\ln(2))}$$

$$y = \frac{1}{1-e^{-(x+\ln(2))}}$$

$$\lim_{x\to\infty}\frac{1}{1-e^{-\infty}} = \frac{1}{1-0} = 1$$

$$y\frac{dy}{dx} + x = 0$$

S5. **(False)** $\therefore \int y\,dy + \int x\,dx$

$$\therefore \frac{y^2}{2} + \frac{x^2}{2} = c$$

$\therefore x^2 + y^2 = 2c$ which is a circle.

ASSERTION AND REASONING

S1. **(d)** The given differential equation is $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$

This equation is not a polynomial in differential coefficients. So , its degree is not defined. Thus ,A is false but R is true.

S2. **(a)** Let the general equation of the given parabola is $x = ay^2 + by + c$, where a , b and c are constants. Hence , the order of the required differential equation is 3.Thus , both A and R are true and R is the correct explanation of A.

S3. **(b)** We have , $y = (a+b)x$

$$\Rightarrow x\frac{dy}{dx} = y$$

It is true

∴ Both A and R are true and R is not the correct explanation of A.

S4. **(a)**

$$(1+x^2)\frac{dy}{dx} + 2xy = \cos x$$

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\cos x}{1+x^2}$$

$$\frac{dy}{dx} + Py = Q$$

$$P = \frac{2x}{1+x^2}, Q = \frac{\cos x}{1+x^2}$$

Integrating factor = $e^{\int Pdx}$

I.F = $e^{\int\frac{2x}{1+x^2}dx} = 1 + x^2$

So , the solution is

$\Rightarrow y \times (1+x^2) = \int\frac{\cos x}{1+x^2}\times(1+x^2)dx$ + C

$\Rightarrow y(1+x^2) = \sin x + C$

Thus , both A and R are true and R is the correct explanation of A.

We have,

$$xdy - ydx = 0$$

S5. **(c)** $\Rightarrow \frac{1}{y}dy - \frac{1}{x}dx = 0$

$$\Rightarrow \log y - \log x = \log C \;[On\ integrating]$$

$$\Rightarrow \frac{y}{x} = C \Rightarrow y = Cx$$

Clearly, it represents a straight line passing through the origin.

Degree of $\left(\frac{dy}{dx}\right)^5 + y = 0$ is 5.

Thus A is correct but R is false.

HOMEWORK

MCQ

S1. **(a)** The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+y^4=0$ are respectively, Then order = 2 and degree = 2

S2. **(b)** The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2+\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^4=y$ are respectively, Then order = 3 , degree = 2

S3. **(d)** $(x^2-y^2)y'=g(x)y$

$x^2+(y-a)^2=a^2$

Differentiating the equation with respect to x

$2sx+2(y-a)y'=0$

$a=\frac{x+yy'}{y'}.....(1)$

Put 'a' in original equation, we get

$x^2+\left(y-\left(\frac{x+yy'}{y'}\right)\right)^2=\left(y'\frac{x+yy'}{y'}\right)^2$

$y'^2x^2+(yy'-(x+yy'^2))^2=(x+yy')^2$

By solving this equation

$(x^2-y^2)y'=2xy$

$g(x)=2x$

S4. **(b)** $\frac{dy}{dx}\ln|cx|+\frac{cy}{cx}=1$

$\frac{dy}{dx}\frac{x}{y}=1-\frac{y}{x}$

$\frac{dy}{dx}=\frac{y}{x}-\frac{y^2}{x^2}$

For $\phi(2)$, we get $-\frac{1}{4}$

S5. **(c)** Given that, $\frac{dy}{dx}=e^{-2y}\Rightarrow\frac{dy}{e^{-2y}}=dx$

$\Rightarrow\int e^{2y}=\int dx\Rightarrow\frac{e^{2y}}{2}=x+C....(i)$

When x=5 and y=0, then substituting these values in Eq. (i), we get

$\frac{e^0}{2}=5+C$

$\Rightarrow\frac{1}{2}=5+C\Rightarrow\frac{1}{2}-5=\frac{9}{2}$

Eq. = (i) becomes $e^{2y}=2x-9$

When, $y=3, e^6=2x-9\Rightarrow 2x=e^6+9$

$x=\frac{(e^6)+9}{2}$

S6. **(b)** $\frac{dy}{dx}=1+x+y+xy$

$\Rightarrow\frac{dy}{dx}=(1+x)+y(1+x)$

$\Rightarrow\frac{dy}{dx}=(1+x)(1+y)$

$\int 1+\frac{dy}{y}=\int(1+x)dx$

$\log(1+y)=x+\frac{x^2}{2}+C.....(1)$

$\because y(-1)=0$

Therefore,

$\log(1+0)=-1+\frac{1}{2}+C$

$\Rightarrow C=\frac{1}{2}$

Substituting the value of C in equation (1), we get

$\log(1+y)=x+\frac{x^2}{2}+\frac{1}{2}$

$\log(1+y)=\frac{(x+1)^2}{2}$

$y=e^{\frac{(x+1)^2}{2}}-1$

S7. **(b)** $(x+2)\frac{dy}{dx}=x^2+4x-9$

$\Rightarrow\int dy=\int\frac{((x+2)^2-13)dx}{(x+2)}$

$\Rightarrow y=\left(\frac{x^2}{2}+2x\right)-13\,ln\,|x+2|+c$

As

$y(0)=0\Rightarrow 0=-13\,ln\,2+c\Rightarrow c=13\,ln\,2$

So, $y=\frac{x^2}{2}+2x+13\,ln\left(\frac{2}{|x+2|}\right)$

$\Rightarrow y(-4)=8-8+13\,ln\left(\frac{2}{2}\right)=0$

S8. **(b)** Given equation is

$\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$

$\Rightarrow(\sin y+y\cos y)dy=x(2\log x+1)dx$

On integrating both sides, we get

$\int\sin y dy+y\sin y-\int\sin y dy=x^2\log x-\int x^2\cdot x\frac{1}{x}dx+\int x dx+C$

$\Rightarrow y\sin y=x^2\log x+C$

S9. **(c)** $\frac{dy}{dx}-ky=0,\frac{dy}{y}=kdx$

In $y=kx+c$

At $x = 0, y = 1 \therefore c = 0$
Now, In $y = kx$
$y = e^{kx}$
$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{kx} = 0$
$\therefore k < 0$

S10. (a) Given that, $\frac{dy}{dx} = \frac{x^2+y^2}{x^2-y^2}$, y(1) = 2

Has slope at the point (1,2)

$$\frac{dy}{dx} = \frac{1^2+2^2}{1^2+2^2}$$

$$= -\frac{5}{3}$$

SUBJECTIVE QUESTIONS

S1. Such family of circle is given by $x^2 + (y - a)^2 = a^2 \therefore x^2 + y^2 - 2ay = 0$ ...(i)

differentiating, $2x + 2y\frac{dy}{dx} = 2a\frac{dy}{dx}$

or $x + y\frac{dy}{dx} = a\frac{dy}{dx}$

substituting the value of a in equation (i)

$\Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy$

(order is 1 again and degree 1)

S2. $e^{\frac{dy}{dx}} = x + 1 \Rightarrow \frac{dy}{dx} = \lambda n(x + 1)$

$\int dy = \int \ell n(x+1)\,dx$

$\Rightarrow y = (x + 1)\ln(x + 1) - x + c$

when x = 0, y = 3 gives c = 3

Hence the solution is $y = (x + 1)\ln(x + 1) - x + 3$

S3. Putting $4x + y + 1 = t$

$4 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 4$

Given equation becomes

$\frac{dt}{dx} - 4 = t^2 \Rightarrow \frac{dt}{t^2+4} = dx$ (Variables are separated)

Integrating both sides,

$\int \frac{dt}{4+t^2} = \int dx \Rightarrow \frac{1}{2}\tan^{-1}\frac{t}{2} = x + c \Rightarrow \frac{1}{2}\tan^{-1}\left(\frac{4x+y+1}{2}\right) = x + c$

S4. The given differential equation can be reduced to linear form by change of variable by a suitable substitution

Substituting $y^2 = z$

$2y\frac{dy}{dx} = \frac{dz}{dx}$

differential equation becomes

$\frac{\sin x}{2}\frac{dz}{dx} + \cos x.z = \sin x \cos x$

$\frac{dz}{dx} + 2\cot x \,.\, z = 2\cos x$ which is linear in $\frac{dz}{dx}$

$IF = e^{\int 2\cot x\,dx} = e^{2\ell n\sin x} = \sin^2 x$

$\therefore$General solution is $z.\sin^2 x = \int 2\cos x.\ \sin^2 x.\ dx + c$

$\Rightarrow y^2 \sin^2 x = \frac{2}{3}\sin^3 x + c$

S5. The differential equation can be written as

$\frac{1}{2}d(x^2 + y^2) = d\{\tan^{-1}(y / x)\}$

Integrating on both sides, we get $\frac{1}{2}(x^2 + y^2) = \tan^{-1}(y / x) + c$

NUMERICAL TYPE QUESTIONS

S1. (-5) $\frac{dy}{dx} + y\tan x = \sin 2x$ It is a linear equation first we find integrating factor

$\frac{dy}{dx} + y\tan x = \sin 2x$ It is a linear equation

$I.F. = e^{\int \tan x dx} = \sec x$ ....(since integration of tanx =ln(secx)) ysecx

$= \int \sin 2x \sec x dx + C$ ....(simplify sin2x and integrate both sides)

$\Rightarrow y\sec x = -2\cos x + C$

$\Rightarrow 1 = -2 + C$ or $C = 3$

$y(-1) = 2 + 3$ $\Rightarrow y = -5$

S2. (1) Given equation is $y^2 = 2c(x + \sqrt{c})$ ...(1)

$\therefore\ 2yy_1 = 2c$

$\therefore\ yy_1 = c$

Now putting $c = yy_1$ in (1), we get

$y^2 = 2.yy_1(x + \sqrt{yy_1})$

$\Rightarrow (y^2 - 2xyy_1)^2 = 4(yy_1)^3$

$\Rightarrow (y^2 - 2xyy_1)^2 = 4y^3y_1^3$

$\Rightarrow$ order 1, degree 3

S3. **($\frac{1}{4}$)** The solution of the differential equation $\frac{dy}{dx}$ +y cot x =sin x is y sin x = k (2x –sin2x) +c then k is

$\frac{dy}{dx}$ +y cot x =sin x

$\Rightarrow I.F = e^{\int \cot x\, dx} = e^{\ln \sin x} = \sin x$

$\Rightarrow y.\sin x = \int sin^2 x\, dx$

$\Rightarrow y\,.\sin x = \int \frac{1-\cos 2x}{2} dx$

$\Rightarrow y.\sin x = \frac{x}{2} - \frac{\sin 2x}{4} + C$

$\Rightarrow y.\sin x = \frac{2x - \sin 2x}{4} + C$

On comparing with y sin x = k (2x –sin2x) +c

$\therefore$ k = $\frac{1}{4}$

S4. **(1)** If (2 + sin x) $\frac{dy}{dx}$ + (y + 1) cos x = 0 and y(0) = 1, then 3y$\left(\frac{\pi}{2}\right)$ is equal to

$\Rightarrow (2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$

$\Rightarrow (2 + \sin x)\frac{dy}{dx} = -(y+1)\cos x$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x} dx$

Integrating on both the sides

$\Rightarrow \int \frac{dy}{y+1} = -\int \frac{\cos x}{2+\sin x} dx$

Let 2 + sin x = t

Cos x dx = dt

$\Rightarrow \ln(y+1) = -\int \frac{dt}{t}$

$\Rightarrow \ln(y+1) = -\ln t + \ln c$

$\Rightarrow \ln(y+1) = -\ln(2+\sin x) + \ln c$

$\Rightarrow \ln(y+1) + \ln(2+\sin x) = \ln c$

$\Rightarrow \ln(y+1)(2+\sin x) = \ln c$

$\Rightarrow (y+1)(2+\sin x) = c$

Given , y(0) = 1

$\Rightarrow 2(\,2 + \sin 0) = c$

$\Rightarrow c = 4$

$\Rightarrow (y+1)(2+\sin x) = 4$

$\Rightarrow y = \frac{4}{2+\sin x} - 1$

Now, $3y\left(\frac{\pi}{2}\right) = 3\left(\frac{4}{2+\sin\frac{\pi}{2}} - 1\right) = 3\left(\frac{4}{2+1} - 1\right) = 3 \times \frac{1}{3} = 1$

S5. **(3500)** $\int_{2000}^{P} dP = \int_0^{25}(100 - 12\sqrt{x})\, dx$

$(P - 2000) = 25 \times 100 - \frac{12\times 2}{3}(25)^{\frac{3}{2}}$

$P = 3500.$

TRUE AND FALSE

S1. **(True)** If the dependent variables depend on one independent variable x, then the differential equation is said to be ordinary.

Example $\frac{dz}{dx}$ + x y = sin x,

S2. **(False)** If the dependent variables depend on two or more independent variables, then it is known as partial differential equation

for Example $y^2\frac{\partial z}{\partial x} + y\frac{\partial^2 z}{\partial y} = ax$, $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

S3. **(True)** $y^2\, dy + x\, dx + \frac{dx}{x} = 0$

M = x + 1/x , N = y^2

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow 0 = 0$

$\therefore y^2\, dy + x\, dx + \frac{dx}{x} = 0$ is an exact differential equation.

S4. **(False)** Equations of the form

$\frac{dy}{dx} + Py = Q.y^n$, n≠ 0 and n ≠ 1

where P and Q are functions of x, is called Bernoulli's equation and can be made linear in v by dividing by y^n and putting y^{-n+1} = v.

S5. **(True)** I.F for linear differential equation = $e^{\int P dx}$

ASSERTION AND REASONING

S1. **(a)** The differential equation formed by eliminating a and b from $y = ae^x + be^{-x}$ is $\frac{d^2y}{dx^2} - y = 0$

y = $ae^x + b\,e^{-x}$ ....(i) Differentiating w.r.t 'x'

$\frac{dy}{dx} = ae^x - be^{-x}$

Differentiating again w.r.t x

$\frac{d^2y}{dx^2} = ae^x + be^{-x}$ ......(ii)

Subtracting equation (i) from equation (ii)

$\frac{d^2y}{dx^2} - y = 0$
Thus both A and R are true and R is correct explanation of A.

S2. **(a)** Assertion(A) : The degree of the differential equation given by
$\frac{dy}{dx} = \frac{x^4-y^4}{(x^2+y^2)xy}$ is 1.
Reason (R): The degree of a differential equation is the degree of the highest order derivative when differential coefficients are free from radicals and fraction. The given differential equation has first order derivative which is free from radical and fraction with power = 1 , thus it has a degree 1.
Thus both A and R are true and R is correct explanation of A.

S3. **(d)** $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$, it is homogenous equation
Let y = vx
$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
$\Rightarrow v + x\frac{dv}{dx} = v + tan\, v$
Separating the variables and integrating we get
$\Rightarrow \int \frac{1}{tan\, v} dv = \int \frac{1}{x} dx$
$\Rightarrow log\,(\,sin\, v) = log\, x + log\, C$
$\Rightarrow sin\, v = xC$
$\Rightarrow sin\, \left(\frac{y}{x}\right) = xC$
Thus Assertion is not correct but Reason is correct.

S4. **(b)** Assertion(A) : The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx} + 5}$ are 2 and 1
$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + 5$
∴ Order = 2 and degree = 1
∴ A is correct
Reason (R): The differential equation $\left(\frac{dx}{dy}\right)^3 + 2y^{\frac{1}{2}} = x$ is of order 1 and degree 3.
R is correct.
Thus both A and R are correct but R is not correct explanation of A.

S5. **(a)** The integrating factor of $\frac{dy}{dx} - \frac{y}{x} = x^2$
For linear differential equation
$\frac{dy}{dx} + P(x)y = Q(x)$, integrating factor is equal to $e^{\int P dx}$
$\Rightarrow I.F. = e^{\int \frac{-1}{x} dx} = e^{-\log x} = \frac{1}{x}$
Thus both A and R are true and R is correct explanation of A.