Find the Relationship Among the Volumes Of a Cone

Objective

To find the relationship among the volumes of a right circular cone, a hemisphere and a right circular cylinder of equal radii and equal heights.

Materials Required

1. Cardboard
2. Geometry box
3. Hollow ball
4. Cutter
5. Marker
6. Sand or salt
7. Adhesive
8. Acrylic sheets

Prerequisite Knowledge

1. Concept of cone and its properties.
2. Concept of cylinder and its properties.
3. Concept of sphere, hemisphere and its properties.

Theory

1. For cone and its properties refer to Activity 28.
2. For concept of cylinder refer to Activity 29.

   Terms Related to Right Circular Cylinder

   - Each of the circular ends on which the cylinder rests is called its base.
   - The line segment joining the centres of two circular bases is called the axis of cylinder.
   - The radius of circular bases is called the radius of cylinder (radius of both ends will be always same).
   - The length of the axis of the cylinder is called the height of the cylinder.

3. A sphere is three dimensional figure (solid figure) which is made up of all points in the space, which lie at a constant distance, from a fixed point called the centre of

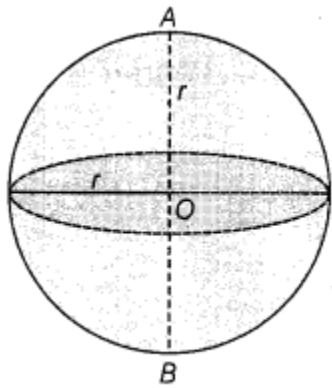the sphere and the constant distance is called its radius.

Fig. 30.1

4. A plane through the centre of a sphere divides the sphere into two equal parts, each of which is called a hemisphere.
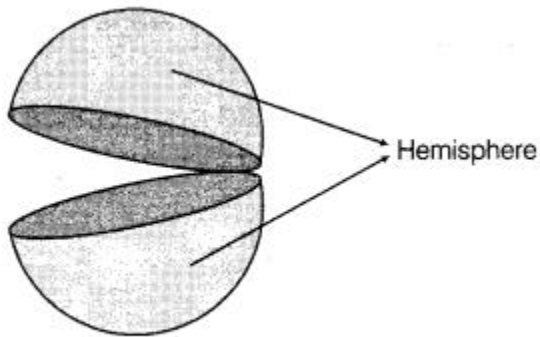

Hemisphere

Fig. 30.2

Procedure

1. Take a hollow ball having radius x units and cut it into two halves.(see Fig. 30.3)
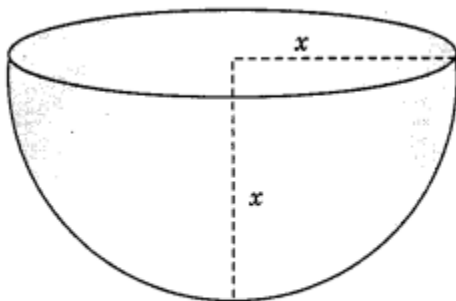

Fig. 30.3

2. Take an acrylic sheet and cut it to get right circular cone of radius x units and height x units. (see Fig. 30.4)
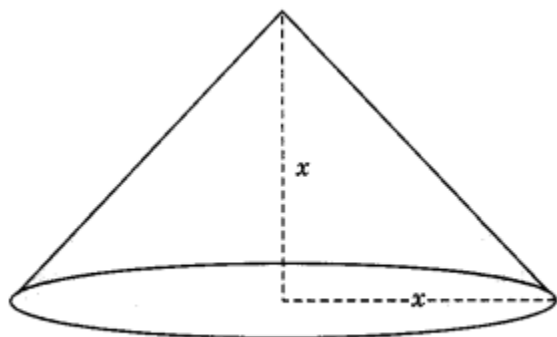

Fig. 30.4

3. Take a rectangular piece of acrylic sheet and cut it to get a right circular cylinder of radius x units and height x units. (see Fig. 30.5)
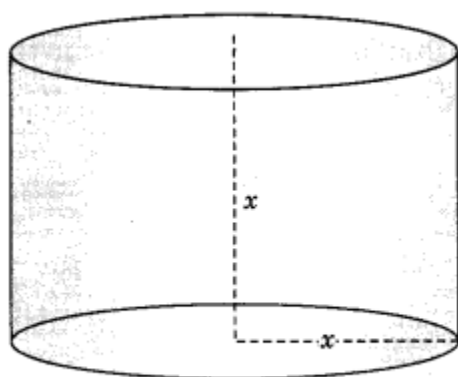

Fig. 30.5

4. Place hemisphere, right circular cone and right circular cylinder on the cardboard.
5. Let the cone be filled with sand or salt and pour it twice into the hemisphere. Then, hemisphere is fully filled with sand or salt. It means that hemisphere contains double of the amount of sand/salt as contained by cone.
6. Let the cone be filled with sand or salt and pour it thrice into cylinder. Then, cylinder is fully filled with sand or salt. It means that cylinder contains triple of the amount of sand/salt as contained by cone.

Demonstration

It is concluded from point 5 of procedure that volume of cone = $\frac{1}{2}$ volume of hemisphere.

Also, it is concluded from point 6 of procedure that volume of cone = $\frac{1}{3}$ volume of cylinder.

Hence, volume of cone : volume of hemisphere: volume of cylinder = 1 : 2 : 3

Observations

The radius of cone = The height of cone = …………

The volume of a cone = $\frac{1}{2}$ volume of …………….

The volume of a cone = $\frac{1}{3}$ volume of ……………

The volume of a cone: The volume of a hemisphere = ………….. : …………

The volume of a cone: The volume of a cylinder = ………….. : ………………

Hence, the volume of a cone: The volume of hemisphere: The volume of cylinder = ………… : ………. : …………..

Result

We have got the relationship among the volumes of a right circular cone, a hemisphere and a right circular cylinder of equal radii and equal heights.

Applications

This activity may be used in

1. Deriving the formula for volume of a cone and that of a hemisphere from the formula of volume of a cylinder.
2. Making packages of the similar material in containers of different shapes such as hemisphere, cylinder, cone, etc.

Viva-Voce

Question 1.

Do you know the formula for the volume of a right circular cone having height h and radius r?
Answer:
Yes, it is $\frac{1}{3}$ πr²h

Question 2.

What is the formula for the volume of a right circular cylinder having radius r and height h?
Answer:
πr²h

Question 3.

What is the formula for the volume of a hemisphere of radius r?
Answer:
$\frac{2}{3}$ πr³

Question 4.

"If the radius of cone is halved and height is doubled, then the volume will be doubled". Does it possible?
Answer:
No, the volume will become half.

Question 5.
Do you know the relationship among the volumes of a right circular cone, a hemisphere and a right circular cylinder of equal radii and equal heights?
Answer:
Yes, it is 1 : 2 : 3.

Question 6.
What is the volume of a sphere having radius r?
Answer:
$\frac{4}{3} \pi r^3$

Question 7.
What is the formula for finding the total surface area of a hemisphere having radius r?
Answer:
$3\pi r^2$

Suggested Activity

To find the formula for the volume of cone and that of hemisphere from the formula of the volume of the cylinder.