DPP No: 41

Maximum Time 50 Min

MATHS

TARGET JEE-MAIN

SYLLABUS : COMPLEX NUMBER

1. Number of complex numbers z such that |z| = 1 and $|z/\bar{z}+\bar{z}/z|=1$ is (arg(z) $\in$ [0, 2π])

2. The number of solutions of the system of equations Re (z^2) = 0, |z| = 2 is

3. If k > 0, |z| = |w| = k and $\alpha = \dfrac{z-\bar{w}}{k^2+z\bar{w}}$, then find Re($\alpha$).

4. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then arg z_1 – arg z_2 is equal to:

(A) $-\frac{\pi}{2}$ (B) 0 (C) $-\pi$ (D) $\frac{\pi}{2}$.

5. Let A = {z $\in$ C:1 $\le$ | z – (1 + i) | $\le$ 2} and B = {z $\in$ A :| z – (1– i) |=1}. Then, B :

(1) is an empty set (2) contains exactly two elements

(3) contains exactly three elements (4) is an infinite set

6. Let S = $\left\{z\in\mathbb{C}:|z-3|\le \text{and}\, z(4+3i)+\bar{z}(4-3i)\le 24\right\}$. If α + iβ is the point in S which is closest to 4i, then 25(α + β) is equal to _____.

(A) 20 (B) 40 (C) 60 (D) 80

7. If $w = \dfrac{z}{z-\frac{1}{3}i}$ and |w| = 1, then z lies on :

(A) a parabola (B) a straight line (C) a circle (D) an ellipse.

8. The points representing complex number z for which |z – 3| = |z – 5| lie on the locus given by

(A) circle (B) ellipse (C) straight line (D) none of these

9. The complex number z = x + iy which satisfy the equation $\left|\dfrac{z-5i}{z+5i}\right| = 1$ lie on :

(A) the x-axis (B) the straight line y = 5

(C) a circle passing through the origin (D) the y-axis

10. The inequality |z – 4| < | z – 2| represents :

(A) Re(z) > 0 (B) Re(z) < 0 (C) Re (z) > 2 (D) Re(z) > 3

11. Let a circle C in complex plane pass although the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to :

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$ (C) $\tan^{-1}(3) - \pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

12. If z_1, z_2, z_3 are vertices of an equilateral triangle inscribed in the circle $|z| = 2$ and if $z_1 = 1 + i\sqrt{3}$, then

(A) $z_2 = -2$, $z_3 = 1 + i\sqrt{3}$ (B) $z_2 = 2$, $z_3 = 1 - i\sqrt{3}$

(C) $z_2 = -2$, $z_3 = 1 - i\sqrt{3}$ (D) $z_2 = 1 - i\sqrt{3}$, $z_3 = -1 - i\sqrt{3}$

13. Let z_1 and z_2 be two complex numbers such that $\bar{z}_1 = i\bar{z}_2$ and $\arg\left(\frac{z_1}{\bar{z}_2}\right) = \pi$. Then

(A) arg $z_2 = \frac{\pi}{4}$ (B) arg $z_2 = -\frac{3\pi}{4}$ (C) arg $z_1 = \frac{\pi}{4}$ (D) arg $z_1 = -\frac{3\pi}{4}$

14. If $x = a + b + c$, $y = a\alpha + b\beta + c$ and $z = a\beta + b\alpha + c$, where α and β are imaginary cube roots of unity, then xyz =

(A) $2(a^3 + b^3 + c^3)$ (B) $2(a^3 - b^3 - c^3)$ (C) $a^3 + b^3 + c^3 - 3abc$ (D) $a^3 - b^3 - c^3$

15. If $x^2 + x + 1 = 0$, then the numerical value of

$\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right)^2 + \left(x^4+\frac{1}{x^4}\right)^2 + ... + \left(x^{27}+\frac{1}{x^{27}}\right)^2$ is equal to

(A) 54 (B) 36 (C) 27 (D) 18

16. Let z_1 and z_2 be two non real complex cube roots of unity and $|z - z_1|^2 + |z - z_2|^2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is

17. If 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to-

18. If $\omega\ (\neq 1)$ be a cube root of unity and $(1 + \omega^4)^n = (1 + \omega^2)^n$ then find the least positive integral value of n

19. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ =

(A) – 1 (B) 1 (C) 2 (D) –2

20. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(A) 1 (B) 2 (C) ∞ (D) 0

21. If $\omega(\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals

(A) (0, 1) (B) (1, 1) (C) (1, 0) (D) (–1, 1)

22. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that :

(A) $\beta \in (0, 1)$ (B) $\beta \in (-1, 0)$ (C) $|\beta| = 1$ (D) $\beta \in (1, \infty)$

23. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals :

(A) $-\theta$ (B) $\frac{\pi}{2}-\theta$ (C) θ (D) $\pi-\theta$

24. Let: $A = \left\{z \in C : \left|\frac{z+1}{z-1}\right| < 1\right\}$ and $B = \left\{z \in C : \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}\right\}$ Then $A \cap B$ is :

(A) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) an empty set

(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

25. A complex number z is said to be unimodular if |z| = 1. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(A) straight line parallel to x-axis (B) straight line parallel to y-axis

(C) circle of radius 2 (D) circle of radius $\sqrt{2}$

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ANSWER KEY OF DPP NO. : 41

1.	(8)	2.	(4)	3.	(0)	4.	(B)	5.	(D)	6.	(D)	7.	(B)
8.	(C)	9.	(A)	10.	(D)	11.	(B)	12.	(C)	13.	(C)	14.	(C)
15.	(A)	16.	(3)	17.	(0)	18.	(3)	19.	(B)	20.	(A)		
21.	(B)	22.	(D)	23.	(C)	24.	(B)	25.	(C)				