CHAPTER / 02

Electric Field and Potential

Topics Covered

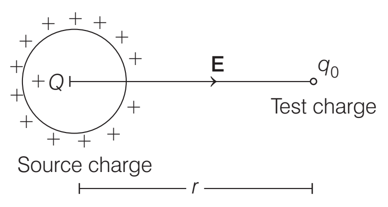
TOPIC ~01 Electric Field and Electric Dipole

Electric Field

The region surrounding a charge or distribution of charge in which its electrical effects can be observed or experienced is called the electric field of the charge or distribution of charge.

Electric Field Intensity (E)

The electric field intensity at any point due to a charge or a configuration of charges is defined as the force experienced per unit positive test charge placed at that point.



It is expressed as $\mathbf{E} = \dfrac{\mathbf{F}}{q_0}$

where, $\mathbf{E}$ = electric field intensity

and $\mathbf{F}$ = force experienced by the test charge q_0.

It is a vector quantity and its SI unit is NC^{-1} and in esu is dyne per stat-coulomb.

Important Points

(i) The charge Q which produces the electric field is called **source charge** and the charge q which experiences the effect of source charge is called **test charge.**

(ii) Test charge is a positive charge of such a small magnitude that its presence at a point does not distrub the electric field to be measured at that point.

Electric Field Intensity Due to a Point Charge

To find the electric field at the point P, we have to find the electric force on a test charge q_0 placed at point P due to source charge q.

According to Coulomb's law, the force on the test charge q_0 due to charge q is given by

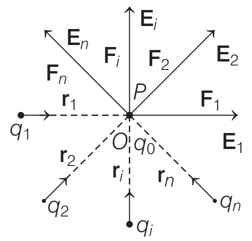
$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2}\,\hat{\mathbf{r}}$$

If $\mathbf{E}$ is the electric field at a point P, then the magnitude of the electric field at a point P is given by

$$|\mathbf{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

Electric Field Intensity Due to System of Charges

Consider n point charges $q_1, q_2, q_3, \ldots, q_n$ exert forces $\mathbf{F}_1$, $\mathbf{F}_2, \mathbf{F}_3, \ldots, \mathbf{F}_n$, respectively on a test charge q_0 placed at origin O.



According to the superposition principle of electric fields, the electric field at any point due to a group of charges is equal to the vector sum of the electric fields produced by each charge individually at that point.

If $\mathbf{E}$ is the electric field at point P due to a group of charges $\mathbf{E}_1, \mathbf{E}_2, \ldots \mathbf{E}_n$, then

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots + \mathbf{E}_n$$

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \cdot \hat{\mathbf{r}}_i$$

Electric Field Intensity Due to a Continuous Charge Distribution

Let dq be an infinitesimally small element of a continuous charge distribution.

The electric field $d\,\mathrm{E}$ at a point due to the element of charge dq,

$$d\,\mathrm{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

where, r is the distance from the element of charge dq and $\hat{\mathbf{r}}$ is the unit vector.

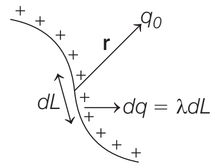
The resultant field strength due to whole of the charge distribution is obtained by integration of the above equation.

$$\mathrm{E}(\mathbf{r}) = \int d\,\mathrm{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r^2}\,\hat{\mathbf{r}}$$

There are three types of continuous charge distribution:

(i) For a Linear Charge Distribution

The charge contained per unit length of the line at any point is called **linear charge density**. It is denoted by λ.



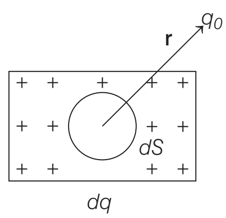
i.e. Linear charge density, $\lambda = \dfrac{dq}{dL}$

Its SI unit is Cm^{-1}. Electric field due to the linear charge distribution at the location of charge q_0 is

$$\mathrm{E}_L = \frac{1}{4\pi\varepsilon_0} \int_L \frac{\lambda}{r^2}\,dL\,\hat{\mathbf{r}}$$

(ii) For a Surface Charge Distribution

The charge contained per unit area at any point is called **surface charge density**. It is denoted by σ.



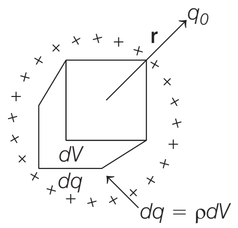
i.e. Surface charge density, $\sigma = \dfrac{dq}{dS}$

Its SI unit is Cm^{-2}. Electric field due to the surface charge distribution at the location of charge q_0 is

$$E_S = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma}{r^2}\, dS\, \hat{\mathbf{r}}$$

(iii) For a Volume Charge Distribution

The charge contained per unit volume at any point is called **volume charge density**. It is denoted by ρ.



i.e. Volume charge density, $\rho = \frac{dq}{dV}$

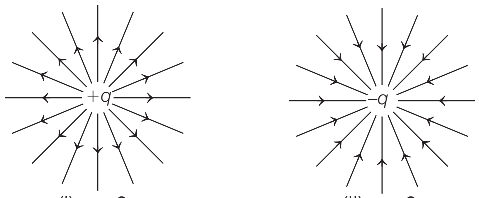
Its SI unit is coulomb per cubic metre (Cm^{-3}).

Electric field due to the volume charge distribution at the location of charge q_0 is

$$E_V = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho}{r^2}\, dV\, \hat{\mathbf{r}}$$

Electric Field Lines

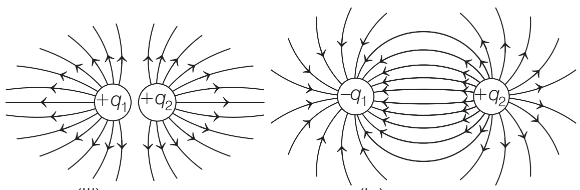
An electric field line or a line of force in general is a curve drawn in such a way that the tangent to it at each point is in the direction of the electric field at that point.



(i) $q>0$ Direction is away from positive charge

(ii) $q<0$ Direction is towards the negative charge

Electric field lines due to the positive and negative charges and their combinations are shown below in a pictorial representation.



(iii) $q_1 > q_2$ (iv) $q_1 < q_2$

Electric field lines in case of a system of two charges

Properties of Electric Field Lines

(i) The electric field lines are imaginary curves in three dimensions.

(ii) The electric field lines start from positive charge and end at negative charge.

(iii) The tangent to a field line at any point gives the direction of electric field at that point.

(iv) Two lines of forces never cross each other. If the two field lines cross each other, then there will be two tangents at the point of intersection (indicating the two field directions) which is impossible.

(v) The relative closeness of electric field lines gives a measure of strength of electric field, i.e. the field lines are

(a) close together in strong field.

(b) far apart in a weak field.

(c) parallel and equally spaced in a uniform field.

(vi) The electric field lines never pass through a conductor. Therefore, electric field strength inside a charged conductor is zero.

(vii) The electric field lines are always normal to the surface of a conductor on which the charges are in equilibrium.

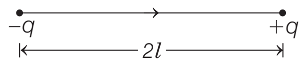
Electric Dipole

A pair of equal and opposite point charges separated by a small distance constitutes an electric dipole.

- A line passing through the two point charges (negative and positive) is called **axis of dipole** or **axial line.**
- A line perpendicular to the dipole axis at its mid-point is called **equatorial line**.

Electric Dipole Moment

The strength of an electric dipole is measured by a vector quantity known as electric dipole moment (**p**).



It is the product of the charge (q) and separation between the charges ($2l$).

$$\mathbf{p} = q \times 2\mathbf{l}$$

$$\Rightarrow \quad |\mathbf{p}| = q\,|2\mathbf{l}|$$

Its direction is from negative charge ($-q$) to positive charge ($+q$). The SI unit of electric dipole moment is C-m.

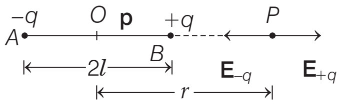
Ideal Dipole

An ideal dipole is one in which charge of each pole tends to infinity while separation between two poles tends to zero such that their product $p = 2ql$ is finite. An ideal dipole is just a point dipole having no size.

Electric Field Intensity Due to an Electric Dipole

(i) Electric Field at any Point on the Axial Line/End-on Position of a Dipole

Consider an electric dipole consisting of charges $+q$ and $-q$ separated by a distance $2l$. Let P be a point on the dipole axis at a distance r from the mid-point of dipole on the side of charge $+q$.



The electric field due to dipole at point P is

$$\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - l^2)^2}\,\hat{\mathbf{p}}$$

where, $\hat{\mathbf{p}}$ is the unit vector in the direction from $-q$ to $+q$. When $l << r$, then l can be neglected in the denominator part.

$$\mathbf{E}_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}\,\hat{\mathbf{p}}$$

$$\therefore \quad |\mathbf{E}_{\text{axial}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$$

The direction of electric field at any point on the axial line is along the direction of electric dipole moment.

(ii) Electric Field at any Point on the Equatorial Line/Broadside-on Position of a Dipole

Consider an electric dipole consisting of two point charges $+q$ and $-q$ separated by a small distance $AB = 2l$. The net electric field will be in the opposite direction of $\mathbf{p}$.

$$\mathbf{E}_{\text{equatorial}} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + l^2)^{\frac{3}{2}}}\,\hat{\mathbf{p}}$$

As $l << r$, so l can be neglected in the denominator part.

$$\mathbf{E}_{\text{equatorial}} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}\,\hat{\mathbf{p}}$$

Also, $$|\mathbf{E}_{\text{equatorial}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}$$

The direction of electric field at any point on the equatorial line of dipole will be anti-parallel to the dipole moment.

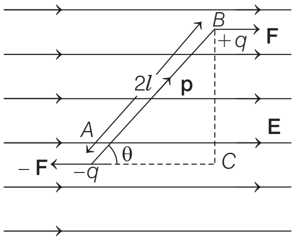
(ii) Electric Field at any Point for Far Field

Due to the positive charge of the dipole electric field at point P will be in radially outward direction and due to the negative charge it will be radially inward. Now, the net electric field (E) is

$$E_r = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p\cos\theta}{r^3} \text{ and } E_\theta = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\sin\theta}{r^3}$$

Torque on a Dipole in a Uniform Electric Field

Consider an electric dipole consisting of two charges $-q$ and $+q$ placed in a uniform external electric field of intensity $\mathbf{E}$ such that its dipole moment makes an angle θ with $\mathbf{E}$.



When an electric dipole is placed in a uniform external field of intensity E, the net force is zero. Since, the two forces are equal in magnitudes and opposite in direction and act at different points, therefore they constitute a couple. A net torque τ acts on the dipole about an axis passing through the mid-point of the dipole.

Now, τ = either force × perpendicular distance BC between parallel forces

$$= qE(2l\sin\theta) = (q \times 2l)\,E\sin\theta$$

or $\tau = pE\sin\theta$ $\quad [\because\ p = 2ql]$

In vector notation, $\tau = \mathbf{p} \times \mathbf{E}$

The SI unit of torque is N-m.

The different conditions of a dipole placed in an electric field is given by

***Case* I** If $\theta = 0°$, then $\tau = 0$, i.e. the dipole is in stable equilibrium.

***Case* II** If $\theta = 90°$, then $\tau = pE$ (maximum value), i.e. the torque acting on dipole will be maximum.

***Case* III** If $\theta = 180°$, then $\tau = 0$, i.e. the dipole is in unstable equilibrium.

Work done in Rotating an Electric Dipole in a Uniform Electric Field

∴ Work done in rotating the dipole from angle θ_1 to θ_2, w.r.t. equilibrium is given by

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

Work done in rotating the electric dipole in a uniform electric field through an angle θ from its equilibrium position, i.e. $\theta_1 = 0°$ and $\theta_2 = \theta$.

Then, $$W = pE(1 - \cos\theta)$$

PRACTICE QUESTIONS

Exam', Textbook's & Other Imp. Questions

1 MARK *Questions*

Exams' Questions

Q.1 The negative and positive charge of a dipole of moment p are placed respectively, at points $-\hat{\mathbf{i}}a$ and $+\hat{\mathbf{i}}a$. If $y >> a$, then the electric field intensity due to the dipole at the point located at $\hat{\mathbf{j}}y$, is **[2017]**

(a) $p/2\pi\varepsilon_0 y^3$ (b) $-p/2\pi\varepsilon_0 y^3$
(c) $p/4\pi\varepsilon_0 y^3$ (d) $-p/4\pi\varepsilon_0 y^3$

Sol (*d*) As, electric field at any point on the equatorial line of a dipole is given by

$$E_{\text{equa}} = -\frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2 + l^2)^{3/2}}$$

Given, $r = y$, $l = a$ and $a << y$

Hence, $E_{\text{equa}} = -\frac{1}{4\pi\varepsilon_0}\frac{p}{(y^2 + a^2)^{3/2}} \approx -\frac{1}{4\pi\varepsilon_0}\frac{p}{y^3}$

Here, negative sign shows that field is anti-parallel to the dipole moment.

Hence, option (d) is correct. **(1)**

Q.2 Which is the unit of electric field intensity? **[2013, 2012 Instant, 2009]**

(a) N-C (b) N-m
(c) NC^{-2} (d) NC^{-1}

Sol (*d*) Since, electric field intensity,

$$E = \frac{F}{q} = \frac{\text{N}}{\text{C}} = \text{NC}^{-1}$$ **(1)**

Q.3 The unit of electric dipole moment is **[2011]**

(a) C/m (b) C-m
(c) C/m^2 (d) $C\text{-}m^2$

Sol (*b*) $\therefore$ Electric dipole moment,

$$|\mathbf{p}| = q \times |2\mathbf{l}| = \text{C} \times \text{m} = \text{C-m}$$ **(1)**

Q.4 What is the dipole moment of a dipole of charge 0.1C and length 0.1 mm in SI unit? **[2011 Instant]**

Sol Given, $q = 0.1$ C, $2l = 0.1$ mm $= 0.1 \times 10^{-3}$ m and $p = ?$

$\therefore$ Dipole moment, $p = q \times 2l = 0.1 \times 0.1 \times 10^{-3}$

$= 10^{-5}$ C-m **(1)**

Q.5 Define dipole moment. Mention its formula. **[2005]**

Sol The dipole moment ($\mathbf{p}$) of an electric dipole is defined as the product of magnitude of one of its charges (q) and distance between the charges ($2\mathbf{l}$), i.e. $\mathbf{p} = q \times 2\mathbf{l}$ **(1)**

Q.6 The unit of electric field strength is .... (Fill in the blank) **[2008 Instant]**

Sol Newton/Coulomb

Since, $E = \frac{F}{q} = \frac{\text{N}}{\text{C}}$ **(1)**

Important Questions

Q.7 To measure electric field at a point due to a positive charge, positive test charge q_0 is put at the point. The measured electric field is **[Textbook]**

(a) $\mathbf{E} > \frac{\mathbf{F}}{q_0}$ (b) $\mathbf{E} = \frac{\mathbf{F}}{q_0}$ (c) $\mathbf{E} < \frac{\mathbf{F}}{q_0}$ (d) $\mathbf{E} < \frac{\mathbf{F}}{q_0}$

Sol (*b*) The electric field intensity $\mathbf{E}$ due to a point charge q_0 at the position of test charge is given by

$$\mathbf{E} = \frac{\mathbf{F}}{q_0}$$ **(1)**

Q.8 The electric field intensity at the surface of a charged conductor is **[Textbook]**

(a) zero
(b) directed normally to the surface
(c) directed tangentially to the surface
(d) directed at an angle to the surface

Sol (*b*) Electric field intensity at the surface of a charged conductor is directed normally to the surface. **(1)**

Q.9 Six charges $+q$ each are placed at the corners of a regular hexagon of each side a. The electric field at the centre of the hexagon is **[Textbook]**

(a) zero (b) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a^2}$ (c) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{6q}{a^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\cdot\frac{3q}{a^2}$

Sol (*a*) The electric field at the centre of the hexagon is zero. Since, due to symmetry in hexagon geometry vector sum of all electric fields by each charge particle cancel out each other. **(1)**

Q.10 Electric field intensity at a point varies as r^{-3} for **[Textbook]**

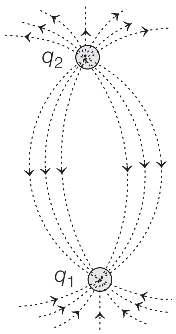
(a) a point charge
(b) an electric dipole
(c) a line charge of infinite length
(d) a plane infinite sheet of charge

Sol (*b*) Electric field intensity due to dipole at any point on the axial is given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2p}{r^3}$$

Here, electric field intensity $\mathbf{E} \propto r^{-3}$ for electric dipole. **(1)**

Q.11 At the mid-point of an electric dipole, **[Textbook]**



(a) electric field is zero
(b) electric potential is zero
(c) Both electric field and potential are zero
(d) Both electric field and potential are not zero

Sol (*b*) At the mid-point of an electric dipole, electric field is non-zero but electric potential is zero, due to presence of opposite charges. **(1)**

Q.12 When a charge of 3 C is placed in a uniform electric field, it experiences a force of 3000 N. The potential difference between two points separated by a distance of 1 cm within this field is **[Textbook]**

(a) 10 V (b) 90 V
(c) 1000 V (d) 3000 V

Sol (*a*) Given, force, $F = 3000$ N, charge, $q = 3$ C and distance, $d = 1 \text{ cm} = 1 \times 10^{-2}$ m

Electric force experience by the charge,

$$F = qE$$

$$\Rightarrow \quad F = q\frac{V}{d} \quad \left[\because E = \frac{V}{d}\right]$$

$$3000 = 3 \times \frac{V}{1 \times 10^{-2}}$$

$$\Rightarrow \quad V = 10 \text{ V}$$ **(1)**

Q.13 State whether a charge is affected by its own electric field. **[Textbook]**

Sol No, a charge is unaffected by its own electric field. **(1)**

Q.14 How does the electric field at a point due to an electric dipole depend on the distance of the point from the dipole? **[Textbook]**

Sol $E_p \propto \frac{1}{x^3}$, where x is the distance of the point from the dipole. **(1)**

Q.15 An electric field can deflect **[Textbook]**

(a) an electron (b) a proton
(c) an α-particle (d) All of these

Sol (*d*) An electric field deflects electrons, protons and α-particles. **(1)**

Q.16 If the electric field is uniform, then electric lines of force are **[Textbook]**

(a) divergent (b) convergent
(c) circular (d) parallel

Sol (*d*) Parallel

As, the electric field is uniform, so equal force is acting throughout the field, i.e. the lines of force are parallel. **(1)**

Q.17 Define electric field. **[Textbook]**

Sol Electric field due to a given charge (source charge) is defined as the space around the charge in which another charge (test charge) experiences an electrostatic force of attraction or repulsion. **(1)**

2 MARKS *Questions*

Exams' Questions

Q.18 A point charge of 3×10^{-9}C is released from rest in a uniform electric field and moves a distance of 5 cm after which its kinetic energy becomes 4.5×10^{-5} J. Calculate the magnitude of the electric field. **[2018]**

Sol Given, $q = 3 \times 10^{-9}$ C, $u = 0$, $s = 5 \text{ cm} = 5 \times 10^{-2}$ m, $\text{KE} = 4.5 \times 10^{-5}$ J and $E = ?$

As, $$E = \frac{F}{q} = \frac{\text{KE}}{q \times s}$$ **(1)**

[$\because$ change in kinetic energy = work done and work done = $F \times s$]

$$= \frac{4.5 \times 10^{-5}}{3 \times 10^{-9} \times 5 \times 10^{-2}} = 0.3 \times 10^6 \text{ N/C}$$

$\therefore$ Magnitude of electric field,

$$E = 3 \times 10^5 \text{ N/C}$$ **(1)**

Q.19 A charged oil drop is suspended in a uniform electric field 27.5×10^4 V/m, so that it neither rises nor falls. Find the charge on the drop. [Take, mass of the drop is 9×10^{-15} kg] (Neglect air resistance). **[2011]**

Sol Given, $E = 27.5 \times 10^4$ V/m, $m = 9 \times 10^{-15}$ kg,

$g = 10$ m/s^2 (say) and $q = ?$

In equilibrium, $F_e = F_g$

So, $qE = mg$ **(1)**

$\Rightarrow$ Charge, $q = \dfrac{mg}{E} = \dfrac{9 \times 10^{-15} \times 10}{27.5 \times 10^4} = \dfrac{900}{275} \times 10^{-19}$

$\Rightarrow$ $q = 3.2 \times 10^{-19}$ C **(1)**

Q.20 Find the electric field intensity in which a proton experiences a force equal to its own weight. (Take, $m_p = 1.67 \times 10^{-27}$ kg, $e^- = 1.6 \times 10^{-19}$C) **[2008]**

Sol Given, mass of proton, $m_p = 1.67 \times 10^{-27}$ kg,

charge on proton, $q_p = e^- = 1.6 \times 10^{-19}$ C,

$g = 10$ m/s^2 and $E = ?$

$\therefore$ Electric field intensity, $E = \dfrac{F}{q} = \dfrac{m_p g}{q}$ **(1)**

$= \dfrac{1.67 \times 10^{-27}}{1.6 \times 10^{-19}} \times 10$

$\Rightarrow$ $E = 1.04 \times 10^{-7}$ V/m **(1)**

Q.21 Find an expression for electric field intensity due to a point charges. **[2008]**

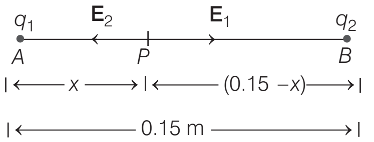
Sol **Electric field intensity due to a point charges**

Refer to text on page 11. **(2)**

Q.22 Two point charges of + 1 μC and + 4 μC are placed 0.15 m apart. Find the location of null-points between them. **[2008 Instant]**

Sol Given, $q_1 = +1\ \mu C = 1 \times 10^{-6}$ C,

$q_2 = +4\ \mu C = 4 \times 10^{-6}$ C, $r = 0.15$ m



x = location of null-point from + 1 μC charge.

At null-point, the net electric field will be zero.

i.e. $\mathbf{E}_1 - \mathbf{E}_2 = 0$

$\Rightarrow$ $\mathbf{E}_1 = \mathbf{E}_2$ **(1)**

$\Rightarrow$ $\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1}{x^2} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_2}{(0.15 - x)^2}$

$\Rightarrow$ $\dfrac{(0.15 - x)^2}{x^2} = \dfrac{4 \times 10^{-6}}{1 \times 10^{-6}}$

$\Rightarrow$ $\dfrac{0.15 - x}{x} = 2 \Rightarrow 0.15 - x = 2x$

$\Rightarrow$ $x = \dfrac{0.15}{3} = 0.05$ m

So, the null-point is at a distance 0.05 m from +1 μC charge. **(1)**

Q.23 Calculate the electric field intensity at a point 1 m away from a charge of 1C in air. **[2006]**

Sol Given, $q = 1$ C, $r = 1$ m, $k = 9 \times 10^9$ N-m^2/C^2 and $E = ?$

$\therefore$ Electric field intensity, $E = k\dfrac{q}{r^2} = 9 \times 10^9 \times \dfrac{1}{1^2}$

$\Rightarrow$ $E = 9 \times 10^9$ N/C **(2)**

Q.24 Why two electric lines of force do not intersect? **[2005]**

Sol Two electric lines of force never intersect. If they intersect, then at the point of intersection there would be two tangents. It shows two directions of electric field which is impossible. **(2)**

3 MARKS *Questions*

Important Questions

Q.25 A charged particle of mass 5 g and charge 1μC is in limiting equilibrium on a horizontal table when a horizontal electric field of 10^4 NC^{-1} is applied. Find the coefficient of friction between the table and the particle. (Take, $g = 10$ ms^{-2}) **[Textbook]**

Sol Given, $m = 5$ g $= 5 \times 10^{-3}$ kg, $q = 1\mu C = 1 \times 10^{-6}$C,

$E = 10^4$NC^{-1}, $\mu_s = ?$

$\therefore$ $\mu_s mg = Eq$ **(1)**

$\Rightarrow$ Coefficient of friction, $\mu_s = \dfrac{Eq}{mg}$

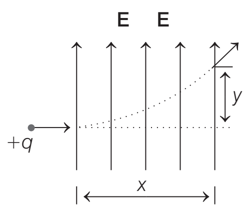
$= \dfrac{10^4 \times 1 \times 10^{-6}}{5 \times 10^{-3} \times 10} = 0.2$ **(2)**

Q.26 Define electric field lines of force. Explain it with diagram and state their properties.

Sol **Electric field lines and properties of electric field lines** Refer to text on page 12. **(1 + 1 + 1)**

Q.27 Show that motion of a charged particle when it is projected at right angles to a uniform electric field is parabolic. **[Textbook]**

Sol Let m is mass of charge particle q is charge on the particle.



E is a uniform electric field.

$u_x = u \Rightarrow u_y = 0$ **(1)**

Initially, along X-axis,

$t = \dfrac{x}{u_x} = \dfrac{x}{u}$ and $ma = qE$ $\left[\because a = \dfrac{qE}{m}\right]$ **(1)**

Along Y-axis,

$$y = u_y + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{u}\right)^2$$

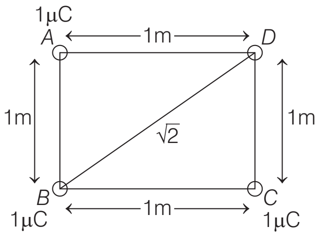
$$\Rightarrow \qquad y = \left(\frac{qE}{2mu^2}\right)x^2$$

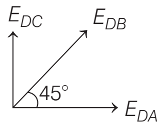
$$\Rightarrow \qquad y \propto x^2$$

This relation show the path is parabolic. **(1)**

Q.28 Three charges each equal to 1 μC are placed at the three corners of a square of side 1.0m. Find electric field at the fourth corner. **[Textbook]**

Sol





(1)

$$E_{DA} = \frac{k \times 10^{-6}}{(1)^2}\cos 45° \; ; \; E_{DB} = \frac{k \times 10^{-6}}{(\sqrt{2})^2}$$

$$E_{DC} = \frac{k \times 10^{-6}}{(1)^2}\cos 45°$$ **(1)**

$$\therefore \qquad E_D = E_{DA} + E_{DB} + E_{DC}$$

$$= 17.2 \times 10^3 \text{ NC}^{-1}$$

45° with one of the sides at the fourth corner. **(1)**

7 MARKS *Questions*

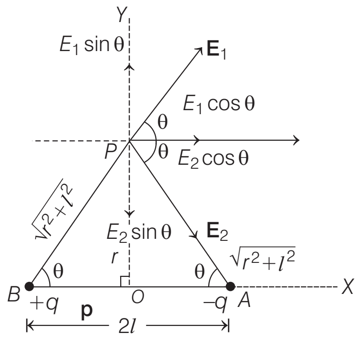
Exams' Questions

Q.29 What is an electric dipole? Derive an expression for the electric field intensity at a point on the axis of the dipole. **[2018, 2016]**

Sol **Electric dipole and electric field intensity due to an electric dipole** Refer to text on pages 12 and 13. **(2+5)**

Q.30 An electric dipole of small length lies along X-axis of a coordinate system. Derive an expression for the magnitude of electric field **E** due to the dipole at any point on its equatorial line. Mention its direction. **[2014]**

Sol Consider an electric dipole consisting of two point charges $+q$ and $-q$ separated by a small distance $AB = 2l$. Let P be a point on the equatorial line of a dipole at a distance r from the mid-point of dipole.



(1)

Resultant electric field intensity at point P is

$$E_P = E_1 + E_2$$ **(1)**

The vectors E_1 and E_2 are acting at an angle 2θ.

Here, $E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2 + l^2}$

and $E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2 + l^2}$

On resolving E_1 and E_2 into rectangular components, we observe that vectors $E_1 \sin\theta$ and $E_2 \sin\theta$ are equal in magnitude and opposite to each other and hence cancel out. **(1)**

The vectors $E_1 \cos\theta$ and $E_2 \cos\theta$ are acting along the same direction and thus add up.

$$\therefore \; E_P = E_1 \cos\theta + E_2 \cos\theta$$ **(1)**

$$= 2E_1 \cos\theta \qquad [\because E_1 = E_2]$$

$$= \frac{2}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)} \cdot \frac{l}{(r^2 + l^2)^{1/2}} \quad \left[\because \cos\theta = \frac{l}{(r^2 + l^2)^{1/2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{2ql}{(r^2 + l^2)^{3/2}}$$

$$\therefore \; E_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{(r^2 + l^2)^{3/2}} \qquad [\because |\mathbf{p}| = q \times 2\mathbf{l}]$$

The net electric field will be in the opposite direction of **p**, i.e. opposite to AB.

$$\mathbf{E}_{\text{equatorial}} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + l^2)^{\frac{3}{2}}}\hat{\mathbf{p}}$$ **(1)**

As $l << r$, so l can be neglected in denominator part.

$$\mathbf{E}_{\text{equatorial}} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}\hat{\mathbf{p}} \Rightarrow |\mathbf{E}_{\text{equatorial}}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}$$ **(1)**

The direction of electric field at any point on the equatorial line of dipole will be anti-parallel to the dipole moment. **(1)**

Q.31 Derive an expression for the electric field intensity due to an electric dipole at any point on its equatorial line and show that the direction of the field is parallel to the axis of the dipole. **[2012]**

Sol Refer to solution 30. **[4 + 3]**

Q.32 Define electric lines of force. Explain why two lines of force never cross each other. **[2012 Instant, 2009]**

Sol. **Electric field lines** Refer to text on page 13. **(2+5)**

Q.33 What is electric dipole and dipole moment? Derive an expression for the electric field strength at a point lying on its equatorial line. **[2003]**

Sol **Electric dipole** Refer to text on page 12. **(1)**

Electric dipole moment Refer to text on page 12. **(1)**

For expression of electric field strength

Refer to solution 30. **(5)**

Important Question

Q.34 What do you mean by electric field? Give Faraday's concept of electric field. Find an expression for electric field intensity due to a point charge. **[Textbook]**

Sol **Electric field** is a region of space surrounding a charge or a system of charges in which any other charge brought in experiences a force. **(1)**

Faraday's law of electric field According to Faraday, the electric force between two charged bodies is a two steps process:

(i) A charged body known as source charge creates an electric field around it. **(2)**

(ii) This field acting on any other charge present in it produces a force but it cannot exert a force on the source itself. **(2)**

Electric field intensity due to a point charge

Refer to text on page 11. **(2)**

TOPIC TEST 1

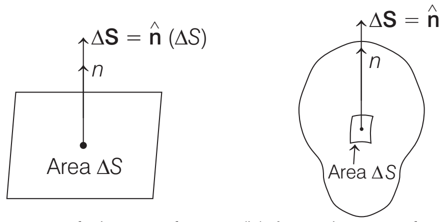
1. If a charge q is projected in a uniform electric field E. What would be its work done when it moves a distance r? [**Ans.** qEr]
2. In which manner would you determine the direction of electric field intensity, on a line of force?
3. How would you represent the electric field due to a point charge situated at infinity?
4. The vectors for gravitational field point towards the earth while the vectors for electric field of a proton point away from it. Explain.
5. State whether a charge is affected by its own field.
6. Does an electric dipole always experience a torque when placed in a uniform electric field?
7. Two point charges of $+3\,\mu C$ each are 1m apart. At what point on the line joining the charges, the electric field intensity be zero? [**Ans.** 50 cm]
8. Find electric field intensity in which an electron experiences a force equivalent to its own weight. (Take, $m_e = 9.1 \times 10^{-31}$ kg) [**Ans.** 5.6×10^{-11} N/C]
9. What do you mean by a test charge? Write a formula for electric field intensity.
10. Equal charges each of $20\,\mu C$ are placed at 0, 2, 4, 8, 16 cm on X-axis. Find the electric field experienced by the charge at $x = 2$ cm. [**Ans.** 60×10^6 NC^{-1}]

TOPIC ~02 Electric Flux and Gauss's Theorem

Area Vector

The vector associated with every area element of a closed surface is taken to be in the direction of the outward normal. Thus, the area element vector $\Delta \mathbf{S}$ at a point on a closed surface is equal to $\Delta S\hat{\mathbf{n}}$.

$$\Delta \mathbf{S} = \hat{\mathbf{n}}\,(\Delta S)$$

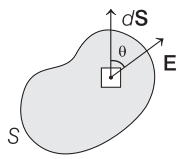


(a) Area element of plane surface (b) Area element of curved surface

where, $\Delta \mathbf{S}$ is the magnitude of the area element and $\hat{\mathbf{n}}$ is a unit vector in the direction of outward normal at the point.

Electric Flux

Electric flux linked with any surface is defined as the total number of electric lines of force that normally pass through that surface.



Electric flux $d\phi$ through a small area element $d\mathbf{S}$ due to an electric field $\mathbf{E}$ at an angle θ with $d\mathbf{S}$ is

$$d\phi = \mathbf{E}\cdot d\mathbf{S} = E\,dS\cos\theta$$

It is proportional to the number of field lines cutting the area element. Total electric flux ϕ over the whole surface S due to an electric field $\mathbf{E}$ is given by

$$\phi = \oint_S \mathbf{E}\cdot d\mathbf{S} = \oint_S E\,dS\cos\theta$$

Electric flux is a scalar quantity but it is a property of vector field. The SI unit of electric flux is N-m^{2}C^{-1} or J-mC^{-1} or V-m.

If $\oint \mathbf{E}\cdot d\mathbf{S}$ over a closed surface is negative, then the surface encloses a net negative charge.

Special cases

(i) For $0° < \theta < 90°$, ϕ is positive.

(ii) For $\theta = 90°$, ϕ is zero.

(iii) For $90° < \theta < 180°$, ϕ is negative.

Gauss's Theorem

The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i = \frac{q}{\varepsilon_0}$$

where, $q = \sum_{i=1}^{i=n} q_i$ is the algebraic sum of all the charges inside the closed surface. Hence, total electric flux over a closed surface in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface regardless of how the charges may be distributed.

Applications of Gauss's Theorem

(i) **Field due to an Infinitely Long Straight Charged Wire**

$$E(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of the electric field is radially outward for the positive line charge. For negative line charge, it will be radially inward.

(ii) **Field due to Uniformly Charged Infinite Plane Sheet**

$$2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0}$$

E is independent of r, the distance of the point from the plane charged sheet. It means that the electric field intensity is same for all points close to the plane sheet of charge. $\mathbf{E}$ at any point is directed away from the sheet in case of positive charge and directed towards the sheet in case of negative charge.

(iii) **Field due to an Uniformly Charged Thin Spherical Shell**

(a) At a point outside the shell, $(r > R)$

$$\therefore \quad E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

In vector form, $\mathbf{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\hat{\mathbf{r}}$

(b) At a point on the surface of the shell (on the surface or $r = R$) $\Rightarrow E = \frac{\sigma}{\varepsilon_0}$

(c) At a point inside the shell, $(r < R)$
Here, the charge inside the Gaussian surface shell is $q = 0$.

$$\therefore \quad E(4\pi r^2) = 0 \Rightarrow E = 0$$

PRACTICE QUESTIONS

Exam', Textbook's & Other Imp. Questions

1 MARK *Questions*

Exams' Question

Q.1 A point charge is placed at the centre of a spherical Gaussian surface. The electric flux through its surface is 1.5×10^4 N-m^2/C. If the radius of the Gaussian surface is doubled. The flux passing through its surface will be **[2013 Instant]**

(a) 1.5×10^4 N-m^2/C (b) 3×10^4 N-m^2/C
(c) 6×10^4 N-m^2/C (d) 20×10^4 N-m^2/C

Sol (*a*) 1.5×10^4 N-m^2/C

Because, flux is a number of lines crossing per unit surface area. So, it is independent of the radius of sphere. **(1)**

Important Questions

Q.2 A charge q is placed at the centre of an imaginary cubical body. Electric flux through any one of its faces is **[Textbook]**

(a) $\frac{q}{\varepsilon_0}$ (b) $\frac{6q}{\varepsilon_0}$ (c) $\frac{q}{6\varepsilon_0}$ (d) $\frac{q}{3\varepsilon_0}$

Sol (*c*) By Gauss's theorem, total electric flux linked with a closed surface is given by $\phi = \frac{q}{\varepsilon_0}$, where q is total charge enclosed by the closed surface.

$\therefore$ Total electric flux linked with cube, $\phi = \frac{q}{\varepsilon_0}$

As charge is at centre, therefore electric flux is symmetrically distributed through all six faces.

Flux linked with each face $= \frac{1}{6}\phi = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$ **(1)**

Q.3 An electric dipole is enclosed within a Gaussian surface. What is the electric flux through that surface? **[Textbook]**

Sol Since, $Q_{\text{net}} = -q + q = 0$

So, $\phi_E = \frac{Q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$ **(1)**

Q.4 Define Gaussian surface.

Sol Any hypothetical closed surface enclosing a charge is called Gaussian surface of that charge. Using Gaussian surface, we can easily find the electric field produced by certain symmetric charge configurations. **(1)**

Q.5 If $\mathbf{E} = 6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$, then calculate the electric flux through a surface of area 20 units in YZ-plane.

Sol As, the area vector $\mathbf{S}$ in the YZ-plane points along outward drawn normal. So, the direction of $\mathbf{S}$ is along positive X-axis.

$\therefore$ $\mathbf{E} = 20\,\hat{\mathbf{i}}$

Now, $\phi_E = \mathbf{E} \cdot \mathbf{S} = (6\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) \cdot 20\hat{\mathbf{i}}$

$\therefore$ $\phi_E = 120$ units **(1)**

Q.6 What is the electric flux through a closed surface enclosing an electric dipole?

Sol As, the total charge of a dipole $(+q - q = 0)$ is zero.

So, from Gauss's theorem,

$$\phi = \frac{q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$ **(1/2)**

So, the flux through a closed surface enclosing an electric dipole is zero. **(1/2)**

Q.7 The electric field intensity on the surface of a charged spherical conductor of radius r is E. Its value at a point $r/2$ from its centre is **[Textbook]**

(a) $4E$ (b) E (c) $2E$ (d) 0

Sol (*d*) Its value at a point $r/2$ from its centre is zero. **(1)**

2 MARKS *Questions*

Exams' Question

Q.8 State Gauss's law in electrostatics. **[2012, 2010]**

Sol The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i = \frac{q}{\varepsilon_0}$$ **(1)**

where, $q = \sum_{i=1}^{n} q_i$ is the algebraic sum of all the charges inside the closed surface. Hence, total electric flux over a closed surface in vacuum is $\frac{1}{\varepsilon_0}$.

Times the total charge within the surface regardless of how the charges may be distributed. **(1)**

Important Questions

Q.9 A point charge of 2 μC is at the centre of a cubical Gaussian surface 9 cm on the edge. Calculate the net electric flux through the surface.

Sol Given, $q = 2\ \mu C = 2 \times 10^{-6}$ C

and $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

From Gauss's theorem, ϕ_E

$$= \frac{q}{\varepsilon_0} = \frac{2 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= 2.26 \times 10^5\ \text{N-m}^2\text{C}^{-1}$$ **(2)**

Q.10 Calculate the electric flux through a closed surface enclosing a charge of 1C.

Sol Here, $q = 1$ C

Flux through this charge, i.e.

$$\phi = \frac{q}{\varepsilon_0} = \frac{1}{8.85 \times 10^{-12}} = 1.12 \times 10^{11}\ \text{N-m}^2\text{C}^{-1}$$ **(2)**

Q.11 If the radius of the Gaussian surface enclosing a charge is halved, how does the electric flux through the Gaussian surface change?

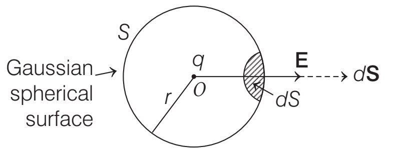
Sol Total charge enclosed by the Gaussian surface remains same even when radius is halved. Therefore, total electric flux remains constant as per Gauss' theorem. Hence, there is no change in the electric flux through the Gaussian surface. **(2)**

3 MARKS *Questions*

Exams' Question

Q.12 Deduce Coulomb's law from Gauss' law of electrostatics. **[2013, 2005]**

Sol **Coulomb's law from Gauss's law** Consider an isolated positive point charge q placed at centre of a spherical Gaussian surface S of radius r.



By symmetry, **E** has same magnitude at all the points on S. At any point on S, **E** and $d\mathbf{S}$ are directed outward and the angle between them is 0°. **(1)**

So, electric flux through small area $d\mathbf{S}$ is

$$d\phi_E = \mathbf{E} \cdot d\mathbf{S} = EdS \cos 0° = EdS$$

Net electric flux through closed surface S is

$$\phi_E = \oint_S d\phi_E = \oint_S E\ dS = E\oint_S dS$$

$$\phi_E = E \times 4\pi r^2 \quad \text{...(i)}$$

From Gauss's theorem, $\phi_E = \dfrac{q}{\varepsilon_0}$ ...(ii) **(1)**

From Eqs. (i) and (ii), we get

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

The force on a point charge q_0, if placed on surface S will be

$$F = q_0 E \Rightarrow F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2} \quad \text{...(iii)}$$

Eq. (iii) proves Coulomb's law. **(1)**

Important Questions

Q.13 A uniform electric field is given as $[\mathbf{E} = 100\hat{\mathbf{i}}$ N/ C] for $x > 0$ and $\mathbf{E} = -100\ \hat{\mathbf{i}}$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the X-axis, so that one face is at $x = +10$ cm and other is at $x = -10$ cm.

(i) What is the net outward flux through each flat face?
(ii) What is the flux through the side of cylinder?
(iii) What is the net outward flux through the cylinder?

Sol (i) On the left face, the outward flux is

$$\phi_L = \mathbf{E} \cdot \Delta\mathbf{S} = -100\hat{\mathbf{i}} \cdot \Delta\mathbf{S} = 100\Delta S$$

Since, $\hat{\mathbf{i}} \cdot \Delta\mathbf{S} = -\Delta S \ = 100 \times \pi(0.05)^2$

$$= 0.785\ \text{N-m}^2\text{C}^{-1}$$

Similarly, on the right face, **E** and $\Delta\mathbf{S}$ are parallel and therefore

$$\phi_R = \mathbf{E} \cdot \Delta\mathbf{S}$$

$$= 0.785\ \text{N-m}^2\text{C}^{-1}$$ **(1)**

(ii) For any point on the side of the cylinder, $\mathbf{E} \perp \Delta\mathbf{S}$ and hence $\mathbf{E} \cdot \Delta\mathbf{S} = 0$ [$\because \cos 90° = 0$]

$\therefore$ Flux out of the side of the cylinder $= 0$. **(1)**

(iii) Net outward flux through the cylinder,

$$\phi = 0.785 + 0.785 + 0 = 1.57\ \text{N-m}^2\text{C}^{-1}$$ **(1)**

Q.14 It has been experimentally observed that the electric field in a large region of the earth's atmosphere is directed vertically down. At an altitude of 300 m, the electric field is 60 Vm^{-1}. At an altitude of 200 m, the field is 100 Vm^{-1}. Calculate the net amount of charge contained in the cube of 100 m edge located between 200 m and 300 m altitude.

Sol Let E_1 be electric field at 200 m

and $E_1 = 100$ V/m (given), E_2 be electric field at 300 m and $E_2 = 60$ V/m (given)

As wire of 100 m edge is situated in electric field with its area normal to electric field, so flux through the cube at height of 200 m is

$$\phi_1 = \mathbf{E}_1 \cdot \mathbf{A} = E_1 A \cos 0°$$

$= E_1 A = 100 \times (100 \times 100)$
$= 10 \times 10^5$ V-m **(1)**

Flux through the cube at height of 300 m is

$\phi_2 = \mathbf{E}_2 \cdot \mathbf{A} = E_2 A \cos 180°$
$= -[60 \times (100 \times 100)]$
$= -6 \times 10^5$ V-m **(1/2)**

$\therefore$ Net flux through the cube is

$\phi = \phi_1 + \phi_2$

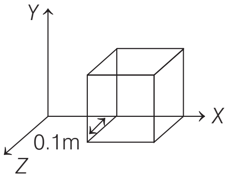
$\Rightarrow \quad \phi = 10 \times 10^5 - 6 \times 10^5 \Rightarrow \phi = 4 \times 10^5$ V-m **(1/2)**

By Gauss's theorem,

$\phi = \frac{q}{\varepsilon_0}$ (where, q is net amount of charge)

$q = \phi\, \varepsilon_0 = (4 \times 10^5)(8.85 \times 10^{-12})$
$= 3.54 \times 10^{-6}$ C **(1)**

Q.15 The electric field components due to a charge inside the cube of side 0.1 m are as shown below:



$E_x = \alpha x$

where, $\alpha = 500$ N/C-m, $E_Y = 0$, $E_Z = 0$

Calculate (a) the flux through the cube and (b) the charge inside the cube.

Sol ***Step* I** Since, the electric field has only x-component.

$\therefore$ $\phi_E = \mathbf{E} \cdot \Delta \mathbf{S} = 0$ for each of the four faces of cube perpendicular to Y-axis and Z-axis. So, electric flux is only non-zero for left and right faces perpendicular to X-axis. **(1)**

***Step* II** Electric field at the left face situated at $x = a$ is given by, $E_L = \alpha\, a$

$\therefore \quad \phi_L = \mathbf{E}_L \cdot \Delta \mathbf{S} = E_L \Delta S \cos 180°$
$= \alpha\, a \cdot a^2 \times -1 = -\alpha\, a^3$ **(1/2)**

Electric field at the right face,

$x = a + a = 2a$ is

$E_R = \alpha (2a)$

$\Rightarrow \quad \phi_R = \mathbf{E}_R \cdot \Delta \mathbf{S}$
$= \alpha (2a) a^2 \cos 0° = 2\, \alpha\, a^3$ **(1/2)**

***Step* III** (a) $\therefore$ Net flux through the cube

$= \phi_L + \phi_R$
$= -\alpha a^3 + 2\, \alpha a^3$
$= \alpha a^3 = 500 \times (0.1)^3$
$= 0.5$ N-m^{2}C^{-1} **(1/2)**

(b) By Gauss's law,

$q = \varepsilon_0 \phi$
$= 8.85 \times 10^{-12} \times 0.5$
$= 4.425 \times 10^{-12}$ C **(1/2)**

7 MARKS *Questions*

Exams' Questions

Q.16 State Gauss's law of electrostatics and write down its mathematical form with identification of the symbols. Derive Coulomb's law from Gauss's law. **[2005 Instant]**

Or

State Gauss's law of electrostatics. Using it, derive Coulomb's law. **[Textbook]**

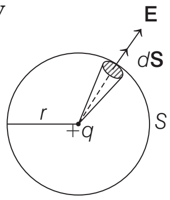
Sol **Gauss's law** Refer to text on page 19. **(2 + 1)**

Coulomb's law from Gauss' law Refer to solution 12. **(4)**

Q.17 State and prove Gauss' theorem in electrostatics. Apply it to find the electric field intensity due to a charged plane lamina in its neighbourhood. **[2004, 2002]**

Sol **Gauss' law** Refer to text on page 19. **(1)**

Proof of Gauss' law for spherically symmetric surface Consider an isolated positive point charge q. Let S be the spherical Gaussian surface of radius r centred on q.



Let an area element $d\mathbf{S}$ on surface S.

$\therefore$ Electric flux through an area element $d\mathbf{S}$ on the surface S is

$$d\phi = \mathbf{E} \cdot d\mathbf{S} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\, \hat{\mathbf{r}}\, (dS\, \hat{\mathbf{n}})$$

As the unit vectors $\hat{\mathbf{r}}$ and $\hat{\mathbf{n}}$ both are in the same direction, so $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = 1.1 \cos 0° = 1$

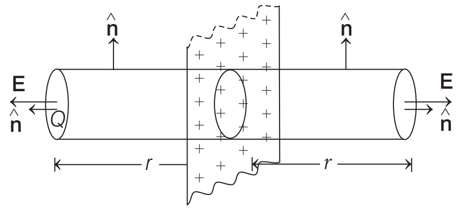
$$\Rightarrow \quad d\phi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\, dS}{r^2}$$

Total electric flux through the spherical surface is

$$\phi_E = \oint_S d\phi = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \oint_S dS$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow \phi_E = \frac{q}{\varepsilon_0}$$ **(3)**

Electric field intensity due to a charged plane lamina Let σ be the surface charge density of the sheet. From symmetry, **E** on either side of the sheet must be perpendicular to the plane of the sheet having same magnitude at all points equidistant from the sheet.



We take a cylinder of cross-sectional area A and length $2r$ as the Gaussian surface. On the curved surface of the cylinder, $\mathbf{E}$ and $\hat{\mathbf{n}}$ are perpendicular to each other. Therefore, flux through curved surface is zero. Flux through the flat surfaces

$$= EA + EA = 2EA$$

where, A = area of each flat surface.

$\therefore$ Total electric flux over the entire surface of cylinder, $\phi_E = 2EA$

Total charge enclosed by the cylinder, $q = \sigma A$

According to Gauss's law,

$$\phi_E = \frac{q}{\varepsilon_0} \Rightarrow 2EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

E is independent of r, the distance of the point from the plane charged sheet. It means that the electric field intensity is same for all points close to the plane sheet of charge. $\mathbf{E}$ at any point is directed away from the sheet in case of positive charge and directed towards the sheet in case of negative charge. **(3)**

Important Questions

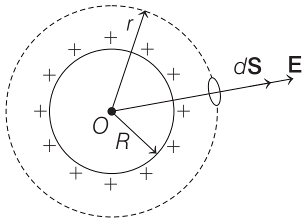
Q.18 Define electric flux. Give its SI unit.
Using Gauss' law, deduce an expression for the electric field due to a uniformly charged spherical conducting shell of radius R at a point

(i) outside and (ii) inside the shell

Sol **Electric flux** Refer to text on page 19. **(2)**

Electric field due to a uniformly charged spherical conducting shell Let σ be the uniform surface charge density of a thin spherical shell of radius R. The Gaussian surface will be a spherical surface centred at the centre of shell. **(1)**

(i) At a point outside the shell, $(r > R)$
Since, $\mathbf{E}$ and $d\mathbf{S}$ are in the same direction,



Gaussian surface for field outside the shell

$$\therefore \quad \phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

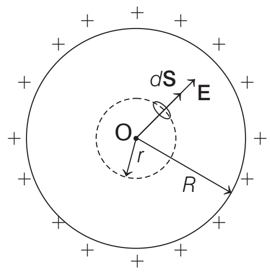
$$\Rightarrow \quad \phi_E = E\oint_S dS = \frac{q}{\varepsilon_0}$$

or $\quad E(4\pi r^2) = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

Since, $\quad q = \sigma \times 4\pi R^2 \Rightarrow E = \dfrac{\sigma R^2}{\varepsilon_0 r^2}$

Vectorially, $\mathbf{E} = \dfrac{\sigma R^2}{\varepsilon_0 r^2}\hat{\mathbf{r}}$ **(2)**

(ii) At a point inside the shell, $(r < R)$



Gaussian surface for field inside the shell

Here, the charge inside the Gaussian surface shell is given by, $q = 0$

$\Rightarrow \quad E(4\pi r^2) = 0$

$\Rightarrow \quad E = 0$ **(2)**

TOPIC TEST 2

1. If charge of 12 nC is placed at the centre of a cube. Calculate the electric flux linked with each face of cube. [**Ans.** $2.26 \times 10^2 \text{N-m}^2\text{C}^{-1}$]
2. Is it possible that a surface placed in an electric field may have zero electric flux? If so, under what circumstances?
3. What solid angle will be subtended by a surface area dS at a point distant r from it? [**Ans.** $\Omega = \dfrac{dS}{r^2}$ steradian]
4. What will be the net flux through a closed surface due to a charge lying outside the closed surface?
5. What do you mean by flux density?
6. What should be the value of electric field due to a uniformly charged plane lamina? [**Ans.** $\dfrac{\sigma}{2\varepsilon_0}$]
7. A charge of 5.31×10^{-7}C is placed at the centre of an imaginary cube. Find the electric flux through one of its faces. [**Ans.** $10^4 \text{ N-m}^2/\text{C}$]
8. What do you mean by term area vector?
9. State Gauss' theorem in electrostatics. Using this theorem, prove that no electric field exists inside a hollow charged conducting sphere.

TOPIC ~03 Electric Potential and Potential Difference

The electric field around a charge can be described into two ways:

(i) By electric field (**E**)

(ii) By electrostatic potential (V)

Electric Potential

The electric potential at any point in an electric field is defined as the amount of work done in bringing a unit positive test charge from infinity to that point along any arbitrary path.

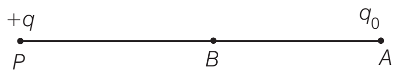
i.e. $$\text{Electric potential} = \frac{\text{Work done}}{\text{Charge}} \Rightarrow V = \frac{W}{q_0}$$

- The SI unit of electric potential is volt (V). So, 1 V = 1 J/C = 1 N-m/C.
- Its dimensional formula is $[ML^2T^{-3}A^{-1}]$.
- Electric potential is a scalar quantity and it can be positive or negative.

Potential Difference

The potential difference between any two points in an electric field is defined as the amount of work done in moving a unit positive charge from one point to the other against the electric field.

Consider a point charge $+q$ is placed at a point P. Let A and B be the two points in the electric field as shown in the figure.



When a test charge q_0 is moved from A to B, then a certain amount of work is done in moving against repulsive force exerted by charge $+q$.

$$\therefore \text{Potential difference, } V = V_B - V_A = \frac{W_{AB}}{q_0}$$

Its SI unit and dimensional formula both are same as that of electric potential.

Zero Reference Potential

In electrostatics, a point at infinity with respect to source charge is considered to be at zero potential. While for electrical circuits, the potential at earth is chosen to be at zero potential, because the potential of earth remains constant even, if it gains or loses electrons.

Positive Potential, Negative Potential and Zero Potential

A conductor is said to be at **positive potential**, if it is connected to earth, electrons flow from earth to the conductor.

A conductor is said to be at **negative potential**, if it is connected to earth, electrons flow from conductor to the earth.

A conductor is said to be at **zero potential**, if there is no transfer of electrons between the conductor and earth.

Units of Electric Potential (V)

In SI system, $V = \frac{W}{q} = \frac{\text{joule}}{\text{coulomb}} = \text{volt}$

In CGS system (esu), $V = \frac{W}{q} = \frac{\text{erg}}{\text{stat-coulomb}} = \text{stat-volt}$

In CGS system (emu), $V = \frac{W}{q} = \frac{\text{erg}}{\text{ab-coulomb}} = \text{ab-volt}$

Relation between volt, stat-volt and ab-volt

$$1\text{ volt} = \frac{1\text{ joule}}{1\text{ coulomb}} = \frac{10^7\text{ erg}}{3\times 10^9\text{ stat-coulomb}}$$

$$= \frac{1}{300}\text{ stat-volt}$$

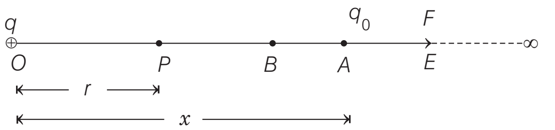
Again, $$1\text{ volt} = \frac{1\text{ joule}}{1\text{ coulomb}} = \frac{10^7\text{ erg}}{\frac{1}{10}\text{ ab-coulomb}}$$

$$= 10^8\text{ ab-volt}$$

Hence, stat-volt = 300 volt = 3×10^{10} ab-volt.

Electric Potential due to a Point Charge

Consider a positive point charge q is placed at point O. We wish to find its potential V at point P at a distance r from it.



Hence, total work done in moving the charge q_0 from infinity to the point P will be

$$W = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r}$$

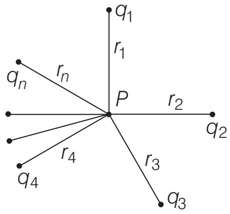
Thus, the electric potential at point P will be

$$V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{rq_0} \Rightarrow V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Clearly, $V \propto \frac{1}{r}$, so the electric potential due to a point charge is spherically symmetric.

Potential Due to a System of Charges

Let there be a number of point charges $q_1, q_2, q_3, \ldots\ldots\ldots\ldots, q_n$ placed at distances $r_1, r_2, r_3, \ldots\ldots\ldots, r_n$ respectively, from the point P, where electric potential is to be calculated.



Using superposition principle, we obtain resultant potential at P due to total charge configuration by the algebraic sum of the potentials due to individual charges.

$$V = V_1 + V_2 + V_3 + \ldots\ldots + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_3} + \ldots\ldots\ldots + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_n}{r_n}$$

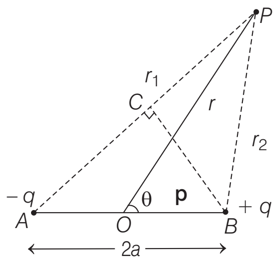
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\ldots + \frac{q_n}{r_n}\right)$$

$$\Rightarrow V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

Electric Potential Due to a Electric Dipole

Consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a distance $2a$.

Let O be the centre of dipole and P be a point at a distance r from the centre of dipole, the direction OP makes an angle θ with the dipole moment $\mathbf{p}$.



$\therefore$ Then, electric potential,

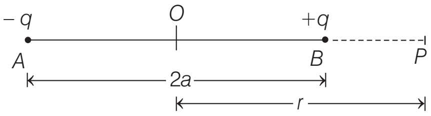
$$V = \frac{q}{4\pi\varepsilon_0} \cdot \frac{2a\cos\theta}{r^2}$$

$$\Rightarrow \quad V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad [\because p = 2qa]$$

In vector form, $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p} \cdot \mathbf{r}}{r^3}$

Special Cases

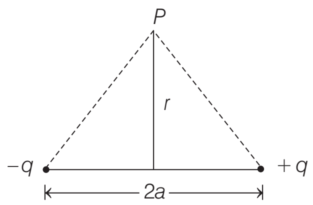
(i) When the point P lies on the axial line, then $\theta = 0°$ or $180°$.



$\therefore \quad \cos\theta = 1$ or -1

So, $\quad V = \pm \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2}$

(ii) When the point P lies on the equatorial line, then



$\theta = 90°$

$\Rightarrow \quad \cos\theta = 0$

So, $\quad V = 0$

Relation between Electric Field Intensity and Potential Gradient

$$\because \quad E = -\frac{dV}{dr} \qquad \ldots(i)$$

Here, $\frac{dV}{dr}$ is the rate of change of potential with distance and it is known as potential gradient.

It is clear from Eq. (i) that the electric field at any point is equal to the negative potential gradient.

Note Potential inside the charged spherical conductor always remain constant and that is

$$V_{\text{inside}} = V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

where, R = radius of conductor.

PRACTICE QUESTIONS

Exam', Textbook's & Other Imp. Questions

1 MARK *Questions*

Exams' Questions

Q.1 A test charge q_0 is brought from infinity along the perpendicular bisector of an electric dipole. The work done on q_0 by the electric field of the dipole is **[2019]**

(a) zero (b) negative
(c) positive (d) proportional to q_0

Sol (*a*) At a line along the perpendicular bisector of dipole (i.e. along equatorial line), the electric potential is zero, so work done in carrying the charge along this line is zero ($W = Vq$, if $V = 0$, then $W = 0$). **(1)**

Q.2 The electric potential at a point at a distance of 2 m from a point charge of 0.1 μC is 450 V. The electric field at this point will be …… N/C. (Fill in the blank) **[2018]**

Sol Given, $V = 450$ V, $r = 2$ m and $q = 0.1\,\mu$C

Since, electric field, $|E| = \frac{dV}{dr} = \frac{450}{2} = 225$ V/m

$= 225$ N/C **(1)**

Q.3 A force of 2.5 N is experienced by a point charge of 5×10^{-6} C at a point in an electric field. The potential gradient at that point in SI unit is **[2016]**

(a) 5×10^5 N/C (b) 5×10^{-5} V/m
(c) -5×10^5 N/C (d) -5×10^{-5} V/m

Sol (*a*) Given, force, $F = 2.5$ N and charge, $q = 5\times10^{-6}$C

∵ Potential gradient, can be given as

$$\frac{dV}{dr} = E = \frac{F}{q}$$

$$\Rightarrow \quad \frac{dV}{dr} = \frac{2.5}{5\times10^{-6}} = 5\times10^5 \text{ N/C}$$ **(1)**

Q.4 The electric potential due to an electric dipole at an axial point, distant r from the dipole, is related to r as

(a) r (b) r^{-1} (c) r^2 (d) r^{-2} **[2013]**

Sol (*d*) As, potential at axial point,

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} \Rightarrow V \propto \frac{1}{r^2}$$ **(1)**

Q.5 The electric potential at a point in an electric field has the unit of **[2012]**

(a) N-m^2/C (b) N-m/C (c) N-C/m (d) C-m/N

Or

What is the SI unit of electric potential? **[2005]**

Sol (*b*) N-m/C is the SI unit of electric potential. **(1)**

Q.6 There exists a potential difference of 5 V between two points in an electric field. Work done in moving a charge of 7 C from one point to the other is **[2010]**

(a) 5/7 J (b) 7/5 J (c) 35 J (d) $\frac{1}{35}$ J

Sol (*c*) Given, potential difference, $\Delta V = 5$ V,

$q = 7$ C, $W = ?$

$$\therefore \quad \Delta V = \frac{W}{q}$$ **(1/2)**

$\Rightarrow$ Work done, $W = \Delta V q = 5\times 7$

$\therefore \quad W = 35$ J **(1/2)**

Q.7 What is the relation between electric field intensity and electric potential at a point? **[2007 Instant]**

Sol Electric field intensity at a point in the field of an isolated point charge is given by

$$|\mathbf{E}| = \frac{kQ}{r^2}$$

Electric potential at a point in the field of an isolated point charge is given by

$$V = \frac{kQ}{r} \Rightarrow \frac{E}{V} = \frac{1}{r}$$ **(1)**

Q.8 How is electric field intensity at a point related to the potential gradient there? **[2007, 2001]**

Sol Both are related as, $E = -\frac{dV}{dr}$ **(1)**

Q.9 What happens to the potential when one moves towards a negative charge? **[2006 Instant]**

Sol It increases. **(1)**

Q.10 Give an example, where electric field exists but no potential. **[2006 Instant]**

Sol At a point on the perpendicular bisector of an electric dipole. **(1)**

Important Questions

Q.11 The potential of a charged spherical conductor of radius r is 10 V. The potential at a point $\frac{r}{2}$ from its centre is **[Textbook]**

(a) 20 V (b) 0 (c) 10 V (d) 40 V

Sol (c) Potential inside the charged spherical conductor always remains constant. So, the potential at a point $r/2$ from its centre is 10 V. **(1)**

Q.12 A hollow spherical conductor of radius 2 m carries a charge of 1 nC. The electric potential at the centre of the spherical conductor is **[Textbook]**

(a) 0 (b) 4.5 V (c) 1.5 V (d) 2.25 V

Sol (b) Given, radius of hollow spherical conductor,

$$r = 2 \text{ m}$$

Charge, $q = 1 \text{ nC} = 1 \times 10^{-9}$ C

The electric potential at the centre of the spherical conductor is given by

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

$$= \frac{9 \times 10^9 \times 1 \times 10^{-9}}{2} \quad \left[\text{here, } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\right]$$

$\therefore \quad V = 4.5$ V **(1)**

Q.13 The electric potential V as a function of distance x (in metre) is given by

$$V = (2x^2 + 5x - 9) \text{ V}$$

The value of electric field at $x = 1$ m would be **[Textbook]**

(a) 9 V/m (b) – 9 V/m (c) –7 V/m (d) –2 V/m

Sol (b) $\therefore E = -\frac{dV}{dx}$

Given, electric potential, $V = (2x^2 + 5x - 9)$ V

The value of electric field at $x = 1$ m is

$$E = -\frac{d}{dx}[2x^2 + 5x - 9] = -[4x + 5] = -9 \text{ V/m}$$ **(1)**

Q.14 Charges of $\frac{10}{3} \times 10^{-9}$ C are placed at each of the four corners of a square of sides 8 cm each. The potential at the intersection of its two diagonals is **[Textbook]**

(a) $900\sqrt{2}$ V (b) 900 V (c) $150\sqrt{2}$ V (d) $1500\sqrt{2}$ V

Sol (d) Given, side length of square 8 cm

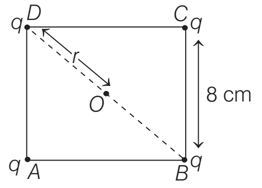
Therefore, the length of the diagonal of square

$$= \sqrt{(8)^2 + (8)^2} = \sqrt{128} = 8\sqrt{2} \text{ cm}$$

The distance of the intersection of its two diagonals from a charge is

$$r = \frac{8\sqrt{2}}{2} = 4\sqrt{2} \text{ cm}$$

Therefore, potential,

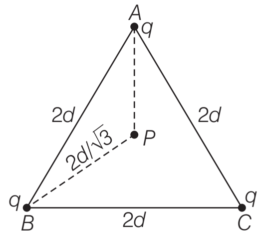


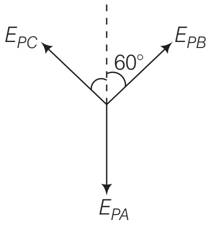
$$V = 4 \times \frac{kq}{r} = \frac{4 \times 9 \times 10^9 \times \frac{10}{3} \times 10^{-9}}{4\sqrt{2} \times 10^{-2}}$$

$\therefore \quad V = 1500\sqrt{2}$ V **(1)**

Q.15 What is the electric field at the centre of an equilateral triangle, if three identical charges each of charge q are fixed at the three corners of the triangle? **[Textbook]**

Sol





$$\therefore \quad E_{PC} = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left(\frac{2d}{\sqrt{3}}\right)^2} = E_0$$

$\therefore \quad E_{PB} = E_0$ and $E_{PA} = E_0$

Also, $\quad E' = E_0 \cos 60° \times 2 = E_0$

$\Rightarrow \quad E = E_0 - E_0 = 0$ **(1)**

Q.16 Name of the physical quantity whose SI unit is JC^{-1}. Is it a scalar or a vector quantity? **[Textbook]**

Sol Electrostatic potential. It is a scalar quantity. **(1)**

Q.17 A charge of 15 C is sent through an electric lamp when the difference of potential is 120 V. How much of energy is expended? **[Textbook]**

Sol $\therefore W = \text{Potential} \times \text{Charge} = 120 \times 15 = 1800$ J **(1)**

Q.18 What is the electric potential at any point on the dipole equator?

Sol Electric potential at any point on the equatorial line of a dipole is zero. **(1)**

Q.19 The electric potential at a point distant r from an electric dipole is proportional to **[Textbook]**

(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) r

Sol (b) Electric potential due to a dipole is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^2} \Rightarrow V \propto \frac{1}{r^2}$$ **(1)**

2 MARKS *Questions*

Exams' Questions

Q.20 Calculate the dimensional formula of electric potential. **[2019]**

Sol As we know that, electric potential, $V = \frac{W}{q}$

$\Rightarrow \quad V = \frac{W}{IT}$ $[\because q = IT]$ **(1)**

$\Rightarrow \quad V = \frac{[ML^2T^{-2}]}{[AT]} \Rightarrow V = [ML^2T^{-3}A^{-1}]$ **(1)**

Q.21 A region in an electric field is specified by the following potential $V(x) = \frac{a}{x} + b$, where V is in volt and x is in metre. Find out the units of a and b. **[2019]**

Sol Given, $V(x) = \frac{a}{x} + b$

Units of both the terms in summation of given equation should be same. **(1/2)**

Hence, unit of b should be volt (V) and unit of $\frac{a}{x}$ should be volt (V). **(1/2)**

Therefore, $\frac{a}{x} = \text{V}$ **(1/2)**

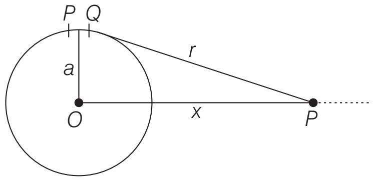
Here unit of x is metre, hence unit of a should be volt-metre. **(1/2)**

Q.22 An electric charge Q is uniformly distributed around a thin metallic circular coil of single turn and of radius a. Find an expression for the electric potential at a point on its axis at a distance x from its centre. **[2018]**

Sol Total charge on the coil $= Q$

Charge per unit length $= \frac{Q}{2\pi a}$

[given, radius of circular coil $= a$]



Charge on small element, $PQ = dq$ **(1)**

Electric potential at P due to $PQ = dV$

$\therefore \quad dV = k\frac{dq}{r} = k\frac{dq}{\sqrt{x^2 + a^2}}$

For total electric potential at P due to ring integrate both sides with proper limit,

$$\int_0^V dV = \frac{k}{\sqrt{x^2 + a^2}}\int_0^Q dq \Rightarrow V = \frac{kQ}{\sqrt{x^2 + a^2}}$$ **(1)**

Q.23 A region in an electric field is given by the potential function $V(x) = ax^2 + b$, where V is expressed in volt and x in metre. Find out the units of a and b. **[2013 Instant]**

Sol Given, $V(x) = ax^2 + b$

where, x is in metre and V is in volt.

Here, unit of b = unit of V

$\Rightarrow \quad$ unit of b = volt or JC^{-1}

There is a summation in equation, hence

unit of b = unit of ax^2

$\Rightarrow \quad a\,(\text{metre})^2 = JC^{-1}$

$\Rightarrow \quad a = JC^{-1}\,m^{-2}$

So, unit of $b = JC^{-1}$ and $a = JC^{-1}\,m^{-2}$ **(2)**

Q.24 In SI unit, V at a point in an electric field is given by $V = -\frac{6}{x} + 2$. Find the value of $\mathbf{E}$ in the field at the point (2, 0, 0). **[2012]**

Sol Given, $\quad V = -\frac{6}{x} + 2$

$\therefore \quad |\mathbf{E}| = \left|-\frac{dV}{dx}\right| = \left|-\left(\frac{-6}{x^2} + 0\right)\right| \Rightarrow E = \frac{6}{x^2}$ **(1)**

Given point is (2, 0, 0), i.e. $x = 2$ units.

So, $\quad E = \frac{6}{2^2} = 1.5$ units **(1)**

Q.25 A region in electric field is specified by the potential function $V(x) = 4x^2$ **[2011]**

Sol Since, $E = -\frac{dV}{dx}$ **(1/2)**

Given, potential function,

$V(x) = 4x^2$ **(1/2)**

$\Rightarrow \quad E = -\frac{dV}{dx} = -\frac{d}{dx}(4x^2) = -8x$

$\therefore$ Electric field at $x = 2$,

$E = -16\,\text{V/m}$ **(1)**

Important Questions

Q.26 There are two conducting spheres of radii r_1 and r_2. Find (i) ratio of charges on them, if both of them are at the same potential (ii) ratio of potential, if charge on them are equal. **[Textbook]**

Sol Given, radii are r_1 and r_2

(i) $V_1 = V_2 \Rightarrow \frac{kq_1}{r_1} = \frac{kq_2}{r_2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1}{r_2}$ **(1)**

(ii) $q_1 = q_2 = q$

$\therefore \quad V_1 = \frac{kq}{r_1}, V_2 = \frac{kq}{r_2} \Rightarrow \frac{V_1}{V_2} = \frac{r_2}{r_1}$ **(1)**

Q.27 If 100 J of work is done to move 4 C charge from a place, where potential is 10 volt to another place, where potential is V volt, then find the value of V.

Sol Given, $q_0 = 4\,\text{C}$,

$$V_A = -10\text{ V},\ V_B = V \text{ volt},\ W_{AB} = 100\text{ J}$$

So, $$V_B - V_A = \frac{W_{AB}}{q_0} \quad \textbf{(1)}$$

$$\Rightarrow \quad V - (-10) = \frac{100}{4} = 25$$

$$\therefore \quad V = 15 \text{ volt} \quad \textbf{(1)}$$

Q.28 A bird perches on a bare highpower line but nothing happens to it, while a man standing on the ground suffers a shock on touching the same line. Why? **[Textbook]**

Sol Current will be able to flow, if there is some potential difference. Bird have the potential of their body equal to the potential of the wire. Hence, no current will flow, whereas man is touching both high potential wire and zero potential earth, hence current will flow. **(2)**

Q.29 The electric potential at 0.1 m from a point charge is 50 V. What is the magnitude of the charge?

Sol Given, $V = 50\text{ V},\ r = 0.1\text{ m}$

Now, $$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r} \quad \textbf{(1)}$$

$$\Rightarrow \quad q = 4\pi\varepsilon_0 Vr = \frac{1}{9\times10^9}\times 50\times 0.1 = \frac{5}{9}\times10^{-9}\text{ C}$$

$$\therefore \quad q = 0.55\text{ nC} \quad \textbf{(1)}$$

3 MARKS *Questions*

Exams' Question

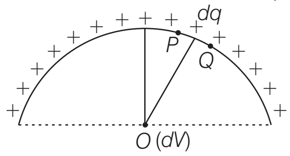
Q.30 An electric charge Q is uniformly distributed around a semi-circle of radius a. Calculate the electric potential at the centre of this semi-circle. **[2018]**

Sol Q is total charge distributed over given semi-circle.

$\therefore$ Charge per unit length $= \dfrac{Q}{\pi R}$ **(1)**

Electric potential at O due to small element PQ is dV

and $dV = k\,dq$ $\quad \left(\because dq = \dfrac{Q}{\pi}\times dl\right)$ **(1)**



Now, integrate both sides with proper limit, we get

$$\int_0^V dV = \frac{kQ}{\pi^2}\int_0^{\pi} dl \Rightarrow V = \frac{kQ}{\pi^2}\times\pi \Rightarrow V = kQ \quad \textbf{(1)}$$

Important Questions

Q.31 10^{-9} C of charge is distributed over two concentric hollow spheres of radii 1 cm and 2 cm such that surface densities of charges are equal. Find the potential at their common centre. **[Textbook]**

Sol Given, $q_1 + q_2 = 10^{-9}\text{C}$,

$$r_1 = 1\text{ cm and } r_2 = 2\text{ cm}$$

$$\because \quad \sigma_1 = \sigma_2$$

$$\frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2} \Rightarrow \frac{q_1}{q_2} = \frac{r_1^2}{r_2^2} = \frac{1}{4} \Rightarrow 4q_1 = q_2 \quad \textbf{(1)}$$

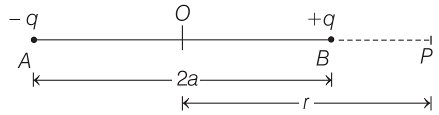
$$\therefore \quad V_B - V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} - \frac{q_2}{r_2}\right) \quad \textbf{(1)}$$

Substituting the values, we get

$$V = 540\text{ V} \quad \textbf{(1)}$$

Q.32 Derive an expression for the potential at a point along the axial line of a short electric dipole.

Sol **Electric potential at an axial point of a dipole**
Let us consider an electric dipole consisting of charges $+q$ and $-q$ separated by a distance $2a$.



(1/2)

Let P be a point on the axis of a dipole at a distance r from the centre of dipole.

Electric potential at point P due to charge $-q$ is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{AP}$$

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r+a} \quad \textbf{(1/2)}$$

Electric potential at point P due to charge $+q$ is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{BP}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a} \quad \textbf{(1/2)}$$

Hence, electric potential at point P due to the dipole is given by

$$V = V_1 + V_2$$

$$\Rightarrow \quad V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right] = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a}{r^2 - a^2}$$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2 - a^2} \quad [\because q\times 2a = p] \quad \textbf{(1)}$$

For short dipole $a << r$, a can be neglected in denominator.

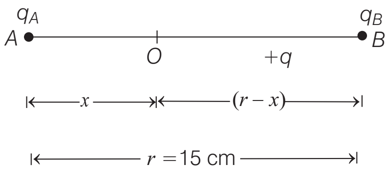
$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2} \quad \textbf{(1/2)}$$

Q.33 Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electrical potential zero? (Take, the potential at infinity to be zero)

Sol Given, $q_A = 3 \times 10^{-8}$ C,

$$q_B = -2 \times 10^{-8} \text{C and } r = 15 \text{ cm} = 0.15 \text{ m}$$

Let O be the point, where the electric potential is zero due to the two charges.



(1)

Suppose that the distance $AO = x$.

Then, $BO = r - x = 0.15 - x$

Electric potential at point O due to q_A,

$$V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{AO} = 9 \times 10^9 \times \frac{3 \times 10^{-8}}{x} = \frac{270}{x}$$ **(1)**

Electric potential at point O due to q_B,

$$V_B = \frac{1}{4\pi\varepsilon_0} \frac{q_B}{BO} = 9 \times 10^9 \times \frac{(-2 \times 10^{-8})}{0.15 - x} = -\frac{180}{0.15 - x}$$

Since, the electric potential at point O is zero, we have $V_A + V_B = 0$

or $\frac{270}{x} + \left(-\frac{180}{0.15 - x}\right) = 0$ or $\frac{270}{x} = \frac{180}{0.15 - x}$

or $x = 0.09$ m $= 9$ cm (from charge 3×10^{-8} C) **(1)**

7 MARKS *Questions*

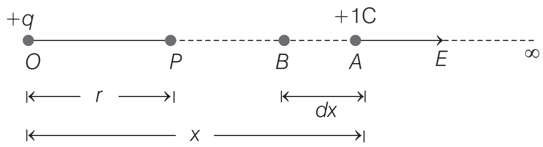
Exams Question

Q.34 (i) Derive an expression for the potential at a distance r from a point charge $+q$.

(ii) What is the electric potential due to a dipole in the broadside-on position? **[2007]**

Sol (i) **Potential due to a point charge**

Let P be the point at a distance r from the origin O at which the electric potential due to charge $+q$ is required. **(1)**



The electric potential at a point P is the amount of work done in carrying a unit positive charge from ∞ to P. As work done is independent of the path, we choose a convenient path along the radial direction from infinity to the point P without acceleration.

Let A be an intermediate point on this path, where $OA = x$. The electrostatic force on unit positive charge at A is

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}, \text{ (along } OA) \quad \text{...(i)}$$

Small work done in moving the charge through a distance dx from A to B,

$$dW = \mathbf{F}\, d\mathbf{x} = Fdx \cos 180° = -Fdx \quad [\because \cos 180° = -1]$$

$$dW = -Fdx \quad \text{...(ii)}$$

Total work done in moving unit positive charge from ∞ to the point P is

$$W = \int_\infty^r -Fdx = \int_\infty^r -\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{x^2}\, dx$$

$$= -\frac{q}{4\pi\varepsilon_0} \int_\infty^r x^{-2} dx = -\frac{q}{4\pi\varepsilon_0} \left[\frac{-1}{x}\right]_\infty^r$$

$$\left[\because \int x^{-2}\, dx = -\frac{1}{x}\right]$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\Rightarrow W = \frac{q}{4\pi\varepsilon_0 r} \quad \text{...(iii)}$$

From the definition of electric potential, this work is equal to the potential at point P,

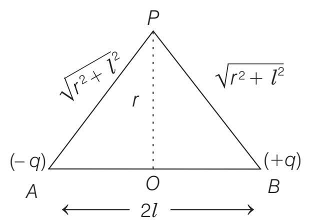
$$V = \frac{q}{4\pi\varepsilon_0 r} \quad \text{...(iv)}$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential. **(4)**

(ii) **Electric potential due to a dipole in the broadside-on position**

Consider a point P on the perpendicular bisector of the dipole at a distance r from mid-point O of the dipole. The potential at point P is

$$V_P = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{\sqrt{r^2 + l^2}} + \frac{-q}{\sqrt{r^2 + l^2}}\right] = 0$$ **(1)**



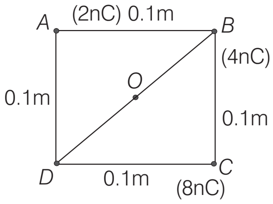
Electric potential at any point on the perpendicular bisector of an electric dipole is zero. **(1)**

Important Questions

Q.35 Derive an expression for electric potential due to a point charge. $ABCD$ is a square measuring 0.1 m on each side. Positive charges of 2 nC, 4 nC and 8 nC are placed at points A, B and C, respectively. Calculate the work done to transfer 1 C of charge from point D to centre of square.

Sol **Electric potential due to a point charge** Refer to solution 34. **(4)**

According to the principle of superposition of potentials, the potential at D is



$$V_D = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{AD}+\frac{q_B}{BD}+\frac{q_C}{CD}\right]$$

Here, $AD = CD = 0.1\text{m}$ **(1)**

But $BD = \sqrt{2} \times \text{side}$ ($\because BD$ is diagonal of the square)

$= \sqrt{2} \times 0.1\text{m}$

$$\therefore \quad V_D = 9\times10^9\left[\frac{2\times10^{-9}}{0.1}+\frac{4\times10^{-9}}{0.1\sqrt{2}}+\frac{8\times10^{-9}}{0.1}\right]$$

$= 9\,[20 + 20\sqrt{2} + 80] = 1154.56\text{ V}$ **(1)**

Potential at centre O of square $ABCD$,

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{2\times10^{-9}}{0.1\sqrt{2}/2}+\frac{4\times10^{-9}}{0.1\sqrt{2}/2}+\frac{8\times10^{-9}}{0.1\sqrt{2}/2}\right]$$

$= 9\,[20\sqrt{2} + 40\sqrt{2} + 80\sqrt{2}]$

$= 9 \times 140 \times 2 = 1781.91\text{ V}$

Potential difference,

$$\Delta V = V_O - V_D = 1781.91 - 1154.56 = 627.35\text{ V}$$

$\therefore$ Work done $= \Delta V q = 627.35 \times 1 = 627.35\text{ J}$ **(1)**

Q.36 State the principle of superposition of potentials. Find an expression for the potential at a point due to an electric dipole on its axis, if P is electric dipole moment. **[Textbook]**

Sol **Principle of superposition of potentials** state that potential at a point due to a group of charges is equal to scalar sum of potentials due to all individual charges constituting a group. The potential due to each charge is calculated as if other charges were not present. **(2)**

Electric potential at an axial point of a dipole Refer to solution 32. **(5)**

TOPIC TEST 3

1. State the significance of negative sign in relation between work done and potential difference.
2. What do you mean by line integral of vector field?
3. Name the physical quantity whose SI unit is JC^{-1}. Is it scalar or vector? [**Ans.** potential, scalar]
4. Define electric potential difference between two points.
5. A region is specified by the potential function
$$V = 2x^2 + 3y^3 - 5z^2$$
Calculate the electric field strength at a point $(2,4,5)$ in this region. [**Ans.** $(-8\hat{\mathbf{i}} - 144\hat{\mathbf{j}} + 50\hat{\mathbf{k}})$]
6. Will any work be done in moving a charge +8 between two points on the surface of an isolated charged metallic sphere?
7. A charge of 2 C moves between two plates maintained at a potential difference of 1 V. What is the energy acquired by the charge? [**Ans.** 2 J]
8. How the electric field at a point is related to the potential gradient? $\left[\textbf{Ans. } E = -\dfrac{dV}{dx}\right]$
9. Two point charges $+q$ and $-q$ are separated by a distance d. Except at infinity, where can the electric potential due to these two point charges be zero?
10. Calculate the potential at point P due to a charge 4×10^{-7} C located 9 cm away. [**Ans.** 4×10^4]
11. Define electric potential and derive its dimensional formula.
12. If the electric potential is constant in a region, then what will be the value of electric field in that region?
13. A spherical oil drop of radius 10^{-4} cm has on it at a certain time, a charge of 40 electrons. Calculate the energy that would be required to place an additional electron on the drop. (Take, charge on electron $= 1.6\times10^{-19}$ C) [**Ans.** 9.216×10^{-21} J]
14. Two point charges, each of 3×10^{-9} C located at the two vertices of an equilateral triangle of side 20 cm. How much work must be done to bring a charge of 10^{-9} C upto the third corner of the triangle from infinity? [**Ans.** 2.7×10^7 J]
15. What is the potential gradient (in Vm^{-1}) at a distance of 10^{-12} m from the centre of the platinum nucleus? Atomic number of platinum is 78 and the radius of platinum nucleus may be taken as 5×10^{-15} m. [**Ans.** 1.1×10^{17}]

TOPIC ~04 Electrostatic Potential and Electrostatic Potential Energy

Electrostatic Potential Energy

The electric potential energy of a group of point charges is defined as the work done in assembling the group from the situation when they were at infinite distance from each other (i.e. their initial electrical potential energy is zero as there is no interaction between them).

Electron Volt (eV)

An electron volt (1 eV) is defined as the potential energy gained or lost by an electron in moving through a potential difference of 1 V.

Consider an electron is moved through a potential difference of 1 V, then the change in potential energy will be 1 eV.

i.e. $1\text{ eV} = \Delta U = q\Delta V = e\Delta V = (1.6\times 10^{-19}\text{ C})\,(1\text{V})$

So, $1\text{ eV} = 1.6\times 10^{-19}\text{ J}$

As electron volt is very small unit, so we generally use mega electron volt (MeV).

i.e. $1\text{ MeV} = 10^6\text{ eV} = 1.6\times 10^{-13}\text{ J}$

Electrostatic Potential Energy of a Point Charge

As the potential V at a point P is the amount of work done in bringing a positive charge q from infinity to the point P. So, the work done in bringing a charge q from infinity to the point P will be

$$W = qV$$

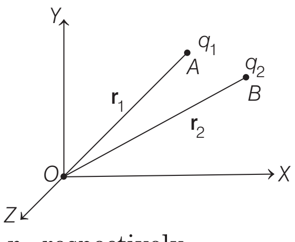
This work is stored as the potential energy U of the charge q.

i.e. $$U = W = qV \Rightarrow V = \frac{U}{q}$$

So, the electric potential at a given point in an external electric field is the potential energy of a unit positive charge at that point.

Electrostatic Potential Energy of a System of Two Point Charges

Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations. Consider two point charges q_1 and q_2 lying at points A and B, whose locations are $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively.



$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1\,q_2}{r_{AB}}$$

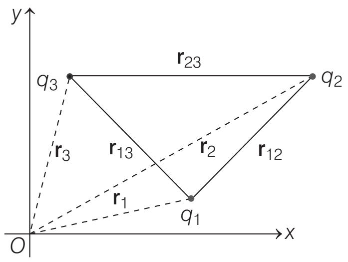
Electrostatic potential energy is a scalar quantity.

In the given formula, the values of q_1 and q_2 must be put with proper signs. If $q_1, q_2 > 0$, then potential energy is positive. It means that two charges are of same sign, i.e. they repel each other.

If $q_1 > 0, q_2 < 0$, then potential energy is negative. It means that two charges are of opposite sign, i.e. they attract each other.

Electrostatic Potential Energy of a System of Three Point Charges

Consider a system of three point charges q_1, q_2 and q_3 having position vectors $\mathbf{r}_1, \mathbf{r}_2$ and $\mathbf{r}_3$ respectively, as from origin.



The potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{\mathbf{r}_{12}} + \frac{q_1q_3}{\mathbf{r}_{13}} + \frac{q_2q_3}{\mathbf{r}_{23}}\right)$$

This result can also be expressed in summation form as follow:

$$U = \left[\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{3}\sum_{\substack{i=1\\ j\neq i}}^{3}\frac{q_iq_j}{\mathbf{r}_{ij}}\right]$$

Due to the conservative nature of electrostatic force, the value of U is independent of the manner in which the configuration is assembled.

Electric Potential Energy of an Electric Dipole

When an electric dipole is placed in a uniform electric field in a direction parallel to the field ($\mathbf{E}$), the electric potential energy of dipole will be, $U_0 = -pE$

In this position, the dipole is in stable equilibrium and it has minimum potential energy. If the dipole is rotated through an angle θ, electric potential energy is given by

$$U = U_0 + pE(1-\cos\theta) = -pE + pE(1-\cos\theta)$$

$$= pE\,[1-1-\cos\theta]$$

$$U = -pE\cos\theta$$

$$\therefore \quad U = -\mathbf{p}\cdot\mathbf{E}$$

Equipotential Lines and Surfaces

In order to show the variation of potential in a region, we generally draw equipotential lines and surfaces graphically in an electric field.

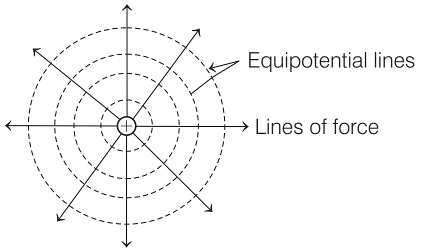
Equipotential Lines

An equipotential line is a curve drawn in an electric field such that all the points on the curve are at the same potential.

Characteristics of Equipotential Lines

The characteristics of equipotential lines are given as below:

(i) The potential difference between any two points of an equipotential line is zero. Hence, no electrical work is done, if a charge is displaced from one point to another on the same equipotential line.



(ii) Equipotential lines and the lines of force always intersect at right angles to each other.

(iii) Two different equipotential lines cannot intersect. If they intersect at the point of intersection, there will be two values of potential which is not possible.

Equipotential Surfaces

Any surface, which has same electrostatic potential at every point, is called an equipotential surface.

The equipotential surfaces can be drawn through any region, in which there is a electric field. If all the points at same potential in the electric field are joined, then an equipotential surface is obtained.

The shape of equipotential surface due to

(i) line charge is cylindrical.

(ii) point charge is spherical.

Characteristics of Equipotential Surfaces

The characteristics of equipotential surfaces are given below:

(i) Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point, which is not possible.

(ii) Equipotential surfaces are closely spaced in the region of strong electric field and *vice-versa*.

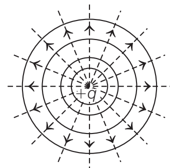
(iii) For any charge configuration, equipotential surface through a point is normal to the electric field at that point and directed from one equipotential surface at higher potential to the equipotential surface at lower potential.

(iv) No work is required to move a test charge on an equipotential surface.

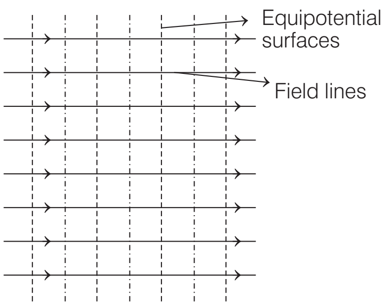
(v) For an uniform electric field E, let along X-axis, the equipotential surfaces are normal to the X-axis, i.e. planes parallel to the YZ-plane.

Equipotential Surfaces in Different Cases

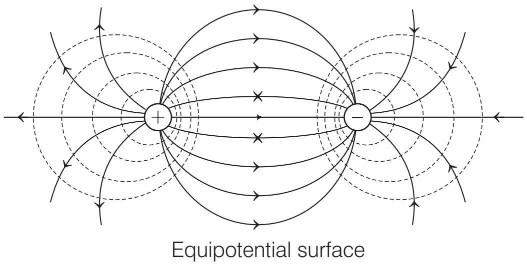
***Case* I** The equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution is a family of concentric spheres as shown below in the figure.



***Case* II** The equipotential surfaces for a uniform electric field is shown below in figure by dotted lines.



***Case* III** The equipotential surfaces for an electric dipole is shown below in figure by dotted lines.



PRACTICE QUESTIONS

Exam', Textbook's & Other Imp. Questions

1 MARK *Questions*

Exams' Questions

Q.1 Two points in an electric field have a potential difference of 5V. What is the amount of work done in moving a charge of 3 C from one point to the other? (Write the answer only) **[2019]**

Sol Given, potential difference, $\Delta V = 5\text{V}$

Charge, $Q = 3\text{C}$

As we know that, work done,

$$W = Q \times \Delta V = 3 \times 5 = 15 \text{ J}$$ **(1)**

Q.2 A positive charge Q' is moved around another positive charge Q on a circular path. If the radius of the circular path is r, then work done on the charge Q' in making one complete revolution is

(a) $Q / 4\pi\varepsilon_0 r$ (b) $QQ'/4\pi\varepsilon_0 r$ **[2013]**

(c) zero (d) $Q'/4\pi\varepsilon_0 r$

Or

A charge Q is the centre of circular path of radius R. Another charge q is taken around the circumference of circular path once. The work done will be **[2013 Instant]**

(a) $\frac{Qqr}{2\varepsilon_0 R^2}$ (b) $Qq\frac{r}{R^2}$ (c) zero (d) $\frac{Qqr^2}{R}$

Sol (*c*) Zero, since the circumference of a circular path around a charge Q is an equipotential surface. So, the work done will be zero. **(1)**

Q.3 A student said that a possible unit for electric field is J $\text{C}^{-1}\text{m}^{-1}$. Is this correct? **[2018]**

(Write 'Yes' or 'No')

Sol Yes;

$\text{JC}^{-1}\text{m}^{-1}$ is unit of electric field because JC^{-1} is unit of electric potential (i.e. volt) and Vm^{-1} is unit of electric field. Hence, $\text{JC}^{-1}\text{m}^{-1}$ is unit of electric field. **(1)**

Q.4 As isolated point charge is placed in a homogeneous, isotropic medium. The equipotential surfaces due to the charge are **[2008 Instant]**

(a) cylindrical (b) spherical

(c) elliptical (d) hyperboloid

Sol (*b*) Spherical **(1)**

Q.5 How much work is done when a charge of 1 μC is moved through 1 cm on a charged conducting sphere? **[2008]**

Sol Zero work is done when a charge of 1μC is moved through 1 cm on a charged conducting sphere. **(1)**

Q.6 How the electric potential and electric potential energy are related? **[2006]**

Sol Electric potential = $\frac{\text{Electric potential energy}}{\text{Charge}}$

i.e. $V = \frac{U}{q} \Rightarrow U = qV$ **(1)**

Q.7 The unit of which physical quantity is electron volt? **[2006]**

Sol An electron volt is the unit of energy. **(1)**

Q.8 Define one electron volt. **[2006 Instant, 2005]**

Sol The energy gained by an electron when it moves through a potential difference of one volt is one electron volt. **(1)**

Q.9 Why is it that two equipotential lines do not intersect? **[2005]**

Or

Why do equipotential lines not intersect each other? **[2001]**

Sol If the two equipotential lines intersect, then at the point of intersection, there would be two potentials which is impossible. So, two equipotential lines never intersect. **(1)**

Important Questions

Q.10 The electric lines of force due to an isolated negative charge is **[Textbook]**

(a) divergent (b) convergent

(c) circular (d) parallel

Sol (*b*) Lines of force originate from a positive charge are converge on a negative charge. **(1)**

Q.11 An electric line of force in XY-plane is given by equation $x^2 + y^2 = 4$. A positively charged particle is set free at $x = 2$, $y = 0$ in the XY-plane. The particle will **[Textbook]**

(a) not move at all

(b) move initially in a straight line

(c) move in a circular path

(d) move first in a straight line then in a circular path

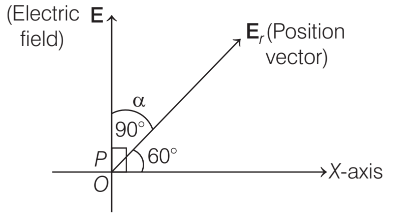
Sol (*c*) The electric line of force in XY-plane is given by equation $x^2 + y^2 = 4$.

Its a equation of circle, so the positively charged particle will move in a circular path. **(1)**

Q.12 An electric dipole is placed at the origin parallel to X-axis. The angle made by the electric field with the X-axis at a point whose position vector makes an angle 60° with the X-axis is **[Textbook]**

(a) 60° (b) 30° (c) 0° (d) 90°

Sol (*d*) An electric dipole is placed at the origin parallel to X-axis.



The angle made by the electric field with X-axis is,

$$\theta = \alpha + 60° \quad \text{[from figure]}$$

$$\theta = 90°$$ **(1)**

Q.13 Proton has a mass of 1840 times that of an electron. If a proton is accelerated from rest by a potential difference of 1 V, its kinetic energy is **[Textbook]**

(a) 1840 eV (b) 1 eV (c) 1 meV (d) 0

Sol (*b*) Charge on proton = e

Hence, when proton is accelerated through a potential difference 1V. Its KE = 1 eV **(1)**

Q.14 A cloud is at a potential of 8×10^6 V relative to the ground. A charge of 40 C is transferred in a lightning stroke between the cloud and the ground. The energy dissipated is **[Textbook]**

(a) 5×10^{-6} J (b) 3.2×10^{-7} J (c) 3.8×10^8 J (d) 3.2×10^8 J

Sol (*d*) Given, potential difference, $\Delta V = 8 \times 10^6$ V

Charge, $q = 40$ C

The energy dissipated is, $E = \Delta V \times q = 8 \times 10^6 \times 40$

$= 3.2 \times 10^8$ J **(1)**

Q.15 What is the work done in moving a charge between two points on an equipotential surface?

Sol Electric field is always perpendicular to an equipotential surface and as a result, work done in moving a charge between two points on an equipotential surface is zero. **(1)**

Q.16 A charge of $+1$C is placed at the centre of a spherical shell of radius 10 cm. What will be the work done in moving a charge of $+1\ \mu$C on its surface through a distance of 5 cm?

Sol Since, the surface of spherical shell will be an equipotential surface due to charge of $+1$ C. So, the work done on this surface will be zero. **(1)**

Q.17 The kinetic energy of an electron accelerated through a potential difference of 100 V is

(a) 1.6×10^{21} J (b) 1.6×10^{-29} J
(c) 1.6×10^{-17} J (d) 1.6×10^{-34} J

Sol (*c*) Given, $V = 100$ V and $q = e = 1.6 \times 10^{-19}$ J

As, $W = qV = 1.6 \times 10^{-19} \times 100$

So, kinetic energy, KE $= W = 1.6 \times 10^{-17}$ J **(1)**

2 MARKS *Questions*

Exams' Questions

Q.18 Define equipotential surface. Mention the angle between the equipotential surface and the lines of force on this surface. **[2019]**

Sol Any surface which has same electrostatic potential at every point is called an equipotential surface. Electric field lines are always perpendicular (90°) to an equipotential surface. **(2)**

Q.19 Can two equipotential surfaces never intersect? Explain your answer. **[2006 Instant]**

Sol Two equipotential surfaces never intersect each other. If they intersect, then there will be two normals at the point of intersection which gives two directions of electric field which is impossible. **(2)**

Q.20 What will be the amount of work done when a charge of 10μ C is moved between two points on an equipotential surface? Explain. **[2006 Instant]**

Sol The surface is equipotential.

So, ΔV = change in potential or potential difference = 0

Also, $q = 10\ \mu\text{C} = 10 \times 10^{-6}\text{C}$, **(1)**

$W = ?$

$\therefore \quad \Delta V = \dfrac{W}{q}$

$\Rightarrow$ Work done, $W = \Delta V q = 0 \times 10 \times 10^{-6} = 0$

So, the work done on equipotential surface is zero. **(1)**

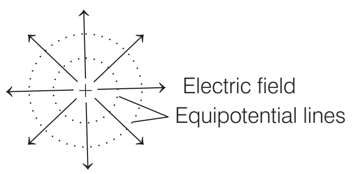
Important Questions

Q.21 State the properties of equipotential lines. **[Textbook]**

Sol **Characteristics of equipotential lines** Refer to text on page 33. **(2)**

Q.22 Draw in the same graph, the curves showing the variation of electric field and potential due to a point charge with distance. **[Textbook]**

Sol



(2)

Q.23 A positive charge $+q$ is located at a point. What is the work done, if a test charge q_0 is carried around this charge along a circle of radius r about this point?

Sol Since, the circumference of a circular path around a charge $+q$ is equipotential surface. So, the work done will be zero. **(2)**

Q.24 What is the difference between potential energy of a charge at a certain point and the potential at that point?

Sol Potential energy at any point in an electrostatic field is amount of work done in moving the charge from infinity to that point, on the other hand, the potential is the work done in moving a unit charge from infinity to that point. **(2)**

3 MARKS *Questions*

Exams' Questions

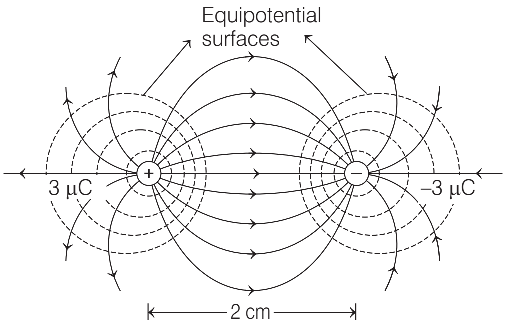
Q.25 Two point charges 3 μC and – 3 μC are placed respectively, at points A and B, 2 cm apart.

(i) Mention the location of equipotential surface of the system.

(ii) Write the value of potential on it.

(iii) Mention the direction of electric field at every point on this surface. **[2013 Instant]**

Sol (i) Location of equipotential surface is shown as given below:



(1)

(ii) The potential on the system will be zero. **(1)**

(iii) The direction of electric field is indicated by arrows on the field lines in above figure. **(1)**

Important Questions

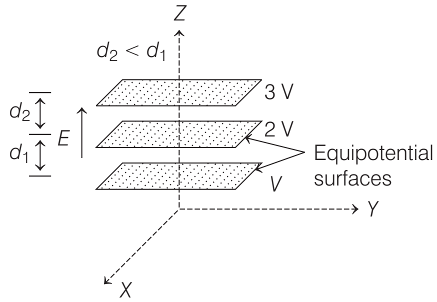
Q.26 Define equipotential lines. Why do equipotential lines not intersect each other? **[Textbook]**

Sol **Equipotential lines** Refer to text on page 33. **(1)**

Refer to solution 09. **(2)**

Q.27 Draw three equipotential surfaces corresponding to a field that uniformly increases in magnitude but remains constant along z-direction. How are these surfaces different from that of a constant electric field along z-direction?

Sol The figure is shown as given below:



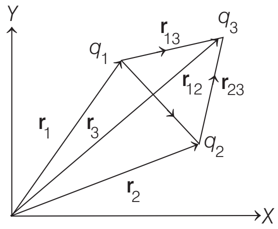
(1)

In case of constant electric field along z-direction, the perpendicular distance between equipotential surfaces remain same, whereas for field of increasing magnitude, equipotential surfaces get closer as we go away from the origin. **(1)**

In both cases, surfaces will be planes parallel to XY-plane. **(1)**

Q.28 Find out the expression for the potential energy of a system of three charges q_1, q_2 and q_3 located respectively, at r_1, r_2 and r_3 with respect to the common origin O.

Sol Let three point charges q_1, q_2 and q_3 have position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$.



Potential energy of the charges q_1 and q_2,

$$U_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_{12}|}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\mathbf{r}_3 - \mathbf{r}_1|} \quad \textbf{(1)}$$

Similarly, $U_{23} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|}$

$\Rightarrow$ $U_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|}$ **(1)**

$\therefore$ Net potential energy of the system,

$$U = U_{12} + U_{23} + U_{13}$$

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{|\mathbf{r}_2 - \mathbf{r}_2|} + \frac{q_2 q_3}{|\mathbf{r}_3 - \mathbf{r}_2|} + \frac{q_1 q_3}{|\mathbf{r}_3 - \mathbf{r}_1|}\right] \quad \textbf{(1)}$$

7 MARKS *Questions*

Important Questions

Q.29 What do you mean by equipotential line and equipotential surface. State the properties of equipotential lines and equipotential surface. Show that electric lines of force and equipotential lines are mutually perpendicular to each other. **[Textbook]**

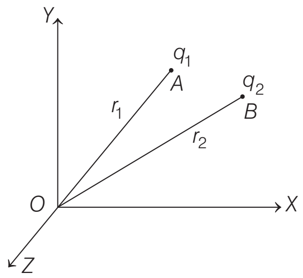
Sol **Equipotential lines and surfaces** Refer to text on pages 33. **(1+4+2)**

Q.30 Define electrostatic potential energy. Derive an expression for it due to a system of two charges.

Sol **Electrostatic potential energy** Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations. **(1)**

Electrostatic Potential Energy of a System of Two Point Charges

Consider two point charges q_1 and q_2 lying at points A and B, whose locations are r_1 and r_2, respectively.



To find the electric potential energy of this two charge system, we must mentally build the system, starting with both charges infinitely far away and at rest. First, the charge q_1 is brought from infinity to the point r_1.

There is no external field against which work needs to be done, so work done in bringing q_1 from infinity to r_1 is zero.

V is potential that has been set up by q_1 at the point B, where q_2 is to be placed.

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{AB}}$$

where, r_{AB} is the distance between points A and B.

By definition, work done in carrying charge q_2 from ∞ to B.

$$W = \text{Potential} \times \text{Charge}$$

$$W = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{AB}} \cdot q_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{AB}}$$

This work is stored in the system of two point charges q_1 and q_2 in the form of electrostatic potential energy U of the system.

$$\text{Thus,} \qquad U = W = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{AB}}$$

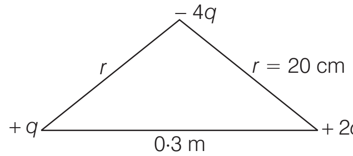
Electrostatic potential energy is a scalar quantity. In the above formula, the values of q_1 and q_2 must be put with proper signs.

If $q_1 q_2 > 0$, then potential energy is positive. It means that two charges are of same sign, i.e. they repel each other.

If $q_1 > 0, q_2 < 0$, then potential energy is negative. It means that two charges are of opposite sign, i.e. they attract each other. **(6)**

TOPIC TEST 4

1. How would you prove that the surface of a good conductor is always equipotential?
2. Give the relationship between electron volt and erg? [**Ans.** $1\text{eV} = 1.6 \times 10^{-12}$ erg]
3. "Potential energy of a system of charges is a unique property of the system". Comment.
4. Give two properties of equipotential surface.
5. Define equipotential surface.
6. Draw an equipotential surface in a uniform electric field.
7. Show that electric field is always directed perpendicular to an equipotential surface.
8. Two point charges + 10 μC and – 10 μC are separated by a distance of 40 cm in air. Draw an equipotential surface for the system.
9. Three point charges are arranged at the three vertices of a triangle as shown in the figure. Determine the electrostatic potential energy of the system, if $q = 3 \times 10^{-7}$C.



[**Ans.** 0.0432 J]

Chapter Test

1 MARK *Questions*

1 The unit of electric dipole moment is
(a) C/m
(b) C-m
(c) C/m^2
(d) $C\text{-}m^2$

2 An electric dipole is kept in a non-uniform electric field. It experiences
(a) a force and a torque
(b) a force but not a torque
(c) a torque but not a force
(d) Neither a force nor a torque

3 What is the angle between the directions of electric field strength and dipole moment at any point in (i) end-on position and (ii) broadside-on position? **[Textbook]**

4 In a non-uniform electric field, a dipole experiences a torque and a net force. True or False?

5 What are the dimensions of (i) dielectric constant and (ii) permittivity?

6 What do you mean by a test charge? **[Textbook]**

2 MARKS *Questions*

7 Define an electric field intensity. Is it a scalar or vector? What is its SI unit? **[Textbook]**

8 Define an electric lines of force and state their important properties.

9 Why do two electric lines of force not intersect each other?

10 What is an electric dipole? What is an electric dipole moment? Define an ideal dipole. **[Textbook]**

11 What is SI unit of electric dipole moment?

12 With the help of a diagram, show the lines of force due to an isolated (i) positive charge and (ii) negative charge.

13 An electrostatic field line cannot be discontinuous. Why?

14 Define electrostatic potential energy. Derive an expression for it due to a system of two charges. **[Textbook]**

15 A region in electric field is specified by the potential function $V(x) = 4x^3$.

Determine the electric field at the point $x = 1$ located in the region.

16 What is the relation between electric field intensity and potential gradient? **[Textbook]**

3 MARKS *Questions*

17 Find the electric field intensity in which an electron experiences a force equivalent to its own weight. (Take, $m_e = 9.1 \times 10^{-31}$ kg) **[Textbook]**

18 There small spheres each carrying a charge q are placed on the circumference of a circle of radius r to form an equilateral triangle. Find the electric field at the centre of the circle.

19 Find electric field intensity due to an electric dipole of moment 16.9×10^{-7} C-m at a distance of 12 cm from its centre on its equator, if the length of the dipole is 10 cm. **[Textbook]**

20 Use Gauss' law and derive an expression for the electric field between two uniformly charged parallel sheets with surface charge densities σ and – σ, respectively.

7 MARKS *Questions*

21 What do you mean by a Gaussian surface? What is the Gaussian surface for a line charge? Apply Gauss' theorem to find an expression for electric field near a line charge of linear charge density λ.

22 What do you mean by equipotential line? State the properties of equipotential lines. Show that electric lines of force and equipotential lines are mutually perpendicular to each other. **[Textbook]**

HINTS and ANSWERS

1. (b) **2.** (a)

4. True

5. (i) $M^0L^0T^0$ (ii) $M^{-1}L^{-3}T^4A^2$

15. Hint $E = -\dfrac{dV}{dx}$ [**Ans.** – 12 V/m]

17. Hint $E = \dfrac{mg}{q}$ [**Ans.** 5.6×10^{-11} NC^{-1}]

18. Hint $E = kq/r$ [**Ans.** Zero]

19. Hint $E = \dfrac{p}{4\pi\varepsilon_0 x^3}$ [**Ans.** 6.9×10^6 N/C]