

### 9.3 Coordinate transformations (astronomical)

#### Time in astronomy

Julian day number <sup>a</sup> $JD = D - 32075 + 1461 * (Y + 4800 + (M - 14)/12)/4$ $+ 367 * (M - 2 - (M - 14)/12 * 12)/12$ $- 3 * ((Y + 4900 + (M - 14)/12)/100)/4$ <span style="float: right;">(9.1)</span>	<i>JD</i> Julian day number <i>D</i> day of month number <i>Y</i> calendar year, e.g., 1963 <i>M</i> calendar month (Jan=1) <i>*</i> integer multiply  / integer divide <i>MJD</i> modified Julian day number
Modified Julian day number $MJD = JD - 2400000.5$ <span style="float: right;">(9.2)</span>	
Day of week $W = (JD + 1) \mod 7$ <span style="float: right;">(9.3)</span>	<i>W</i> day of week (0=Sunday, 1=Monday, ... )
Local civil time $LCT = UTC + TZC + DSC$ <span style="float: right;">(9.4)</span>	<i>LCT</i> local civil time <i>UTC</i> coordinated universal time <i>TZC</i> time zone correction <i>DSC</i> daylight saving correction
Julian centuries $T = \frac{JD - 2451545.5}{36525}$ <span style="float: right;">(9.5)</span>	<i>T</i> Julian centuries between 12 <sup>h</sup> UTC 1 Jan 2000 and 0 <sup>h</sup> UTC <i>D/M/Y</i>
Greenwich sidereal time $GMST = 6^h 41^m 50^s.54841$ $+ 8640184^s.812866T$ $+ 0^s.093104T^2$ $- 0^s.0000062T^3$ <span style="float: right;">(9.6)</span>	<i>GMST</i> Greenwich mean sidereal time at 0 <sup>h</sup> UTC <i>D/M/Y</i> (for later times use 1s=1.002738 sidereal seconds)
Local sidereal time $LST = GMST + \frac{\lambda^\circ}{15^\circ}$ <span style="float: right;">(9.7)</span>	<i>LST</i> local sidereal time <i>λ°</i> geographic longitude, degrees east of Greenwich

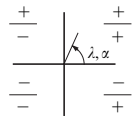
<sup>a</sup>For the Julian day starting at noon on the calendar day in question. The routine is designed around integer arithmetic with “truncation towards zero” (so that  $-5/3 = -1$ ) and is valid for dates from the onset of the Gregorian calendar, 15 October 1582. *JD* represents the number of days since Greenwich mean noon 1 Jan 4713 BC. For reference, noon, 1 Jan 2000 = *JD*2451545 and was a Saturday (*W* = 6).

#### Horizon coordinates<sup>a</sup>

Hour angle $H = LST - \alpha$ <span style="float: right;">(9.8)</span>	<i>LST</i> local sidereal time <i>H</i> (local) hour angle
Equatorial to horizon $\sin a = \sin \delta \sin \phi + \cos \delta \cos \phi \cos H$ <span style="float: right;">(9.9)</span> $\tan A \equiv \frac{-\cos \delta \sin H}{\sin \delta \cos \phi - \sin \phi \cos \delta \cos H}$ <span style="float: right;">(9.10)</span>	<i>α</i> right ascension <i>δ</i> declination <i>a</i> altitude <i>A</i> azimuth (E from N) <i>φ</i> observer's latitude
Horizon to equatorial $\sin \delta = \sin a \sin \phi + \cos a \cos \phi \cos A$ <span style="float: right;">(9.11)</span> $\tan H \equiv \frac{-\cos a \sin A}{\sin a \cos \phi - \sin \phi \cos a \cos A}$ <span style="float: right;">(9.12)</span>	

<sup>a</sup>Conversions between horizon or alt-azimuth coordinates, (*a*, *A*), and celestial equatorial coordinates, (*δ*, *α*). There are a number of conventions for defining azimuth. For example, it is sometimes taken as the angle west from south rather than east from north. The quadrants for *A* and *H* can be obtained from the signs of the numerators and denominators in Equations (9.10) and (9.12) (see diagram).

## Ecliptic coordinates<sup>a</sup>

Obliquity of the ecliptic	$\varepsilon = 23^{\circ}26'21''.45 - 46''.815 T - 0''.0006 T^2 + 0''.00181 T^3$	(9.13)	$\varepsilon$ mean ecliptic obliquity $T$ Julian centuries since J2000.0 <sup>b</sup>
Equatorial to ecliptic	$\sin \beta = \sin \delta \cos \varepsilon - \cos \delta \sin \varepsilon \sin \alpha$	(9.14)	$\alpha$ right ascension
	$\tan \lambda \equiv \frac{\sin \alpha \cos \varepsilon + \tan \delta \sin \varepsilon}{\cos \alpha}$	(9.15)	$\delta$ declination $\lambda$ ecliptic longitude $\beta$ ecliptic latitude
Ecliptic to equatorial	$\sin \delta = \sin \beta \cos \varepsilon + \cos \beta \sin \varepsilon \sin \lambda$	(9.16)	
	$\tan \alpha \equiv \frac{\sin \lambda \cos \varepsilon - \tan \beta \sin \varepsilon}{\cos \lambda}$	(9.17)	

<sup>a</sup>Conversions between ecliptic,  $(\beta, \lambda)$ , and celestial equatorial,  $(\delta, \alpha)$ , coordinates.  $\beta$  is positive above the ecliptic and  $\lambda$  increases eastwards. The quadrants for  $\lambda$  and  $\alpha$  can be obtained from the signs of the numerators and denominators in Equations (9.15) and (9.17) (see diagram).

<sup>b</sup>See Equation (9.5).

## Galactic coordinates<sup>a</sup>

Galactic frame	$\alpha_g = 192^{\circ}15'$	(9.18)	$\alpha_g$ right ascension of north galactic pole
	$\delta_g = 27^{\circ}24'$	(9.19)	$\delta_g$ declination of north galactic pole
	$l_g = 33^{\circ}$	(9.20)	
Equatorial to galactic	$\sin b = \cos \delta \cos \delta_g \cos(\alpha - \alpha_g) + \sin \delta \sin \delta_g$	(9.21)	$l_g$ ascending node of galactic plane on equator
	$\tan(l - l_g) \equiv \frac{\tan \delta \cos \delta_g - \cos(\alpha - \alpha_g) \sin \delta_g}{\sin(\alpha - \alpha_g)}$	(9.22)	
Galactic to equatorial	$\sin \delta = \cos b \cos \delta_g \sin(l - l_g) + \sin b \sin \delta_g$	(9.23)	$\delta$ declination
	$\tan(\alpha - \alpha_g) \equiv \frac{\cos(l - l_g)}{\tan b \cos \delta_g - \sin \delta_g \sin(l - l_g)}$	(9.24)	$\alpha$ right ascension $b$ galactic latitude $l$ galactic longitude

<sup>a</sup>Conversions between galactic,  $(b, l)$ , and celestial equatorial,  $(\delta, \alpha)$ , coordinates. The galactic frame is defined at epoch B1950.0. The quadrants of  $l$  and  $\alpha$  can be obtained from the signs of the numerators and denominators in Equations (9.22) and (9.24).

## Precession of equinoxes<sup>a</sup>

In right ascension	$\alpha \simeq \alpha_0 + (3^s.075 + 1^s.336 \sin \alpha_0 \tan \delta_0) N$	(9.25)	$\alpha$ right ascension of date $\alpha_0$ right ascension at J2000.0 $N$ number of years since J2000.0
In declination	$\delta \simeq \delta_0 + (20''.043 \cos \alpha_0) N$	(9.26)	$\delta$ declination of date $\delta_0$ declination at J2000.0

<sup>a</sup>Right ascension in hours, minutes, and seconds; declination in degrees, arcminutes, and arcseconds. These equations are valid for several hundred years each side of J2000.0.