Chapter - 5

Electric Current

Up till now, our discussion regarding the electrical phenomenon was concentrated only on electrostatics, i.e. charges are at rest. Now, we will discuss the situations in which the charges are in motion. The word electric current is used to explain the flow of charges in space. A large number of electrical applications are based on electric current. For example the electric equipment like bulbs, fans in houses. In study of electric current is also important in other areas of science. For example geophysists have interest in charge in atomsphere whereas biolosists study the neuro current in humans that controls muscles .

In this chapter, we define electric current, after that we will discuss its principles. For flow of electric current in a conductor, potential difference is necessary and the device which is used in it is called a cell or battery. So, in this chapter, we will also study regarding cells.

5.1 Electric Current

Net charge flowing per second from any area of cross section is called electric current. If ΔQ is the net charge flowing through the area of cross section in time Δt, then average current, is given by -

$$I_{av} = \frac{\Delta Q}{\Delta t} \quad \quad ...(5.1)$$

If the rate of flow of charge does not remain constant over interval of time, then, we define the instantaneous current I as the limit of the preceding equations (5.1) as Δt tends to zero.

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \quad \quad ...(5.2)$$

For defining the current, we must use the word net charge. Even though the current is due to the flow of charges, but all the moving charges do not produce electric current. In the absence of external field, there is no net current, because the motion of free electron is random and net effect in any particular direction is zero.

SI unit of electric current is ampere (A), which is a fundamental unit.

$$1A = \frac{1 \text{ coulomb}}{1 \text{ second}} = 1C/s$$

(From 20 May 2019 in SI system of unit 1 ampere is redefined and given in terms of a the fundamental constant *e* (the electronic charge). It can be searched at http://physics.nist.gov.

According to convention, the direction of current is given by the direction of flow of +ve charge. Hence, it is assumed that the direction of current is in the opposite direction, i.e. in the direction of flow of -ve charge. Moving charges are called charge carriers. In different cases, the current will be due to flow of different type of charges. Hence,

(i) In conductors, the current is due to the flow of free electrons.

(ii) In electrolysis (electrolytes), current flow is due to the flow of +ve and -ve ions.

(iii) In semi-conductors, the current is due to the flow of electrons and holes.

(iv) In discharge tube, the current is due to the flow of +ve ions of the gas and electrons.

The current can also flow in vacuum, e.g. in picture tube of TV, electrons flow in vacuum and hence the current flows. Although, direction is considered in the flow of electric current, yet it is a scalar quantity, because it is defined in terms of charge and time which are scalar quantities. Current does not follow the law of vector addition, which is explained in the figure 5.1. Here the junction is shown by three wires. The flow does not depend on the shape and direction of wires. Hence, the current is not a vector quantity.

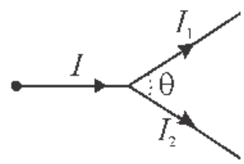


Fig 5.1 The current is represented

by $I = I_1 + I_2$, whatever be the value of θ .

In this chapter we will confine ourselves to the flow of current in conductors. Simultaneously, we will concentrate our study on steady current, which does not depend on time.

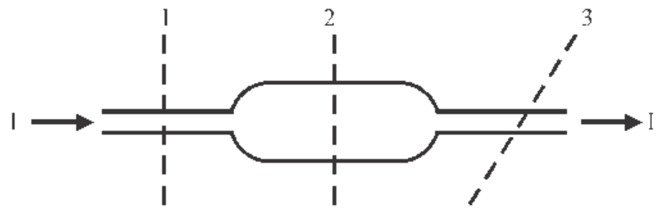


Fig 5.2 The current I flowing through all planes 1,2 and 3 is same.

In steady state, the current in any section of the conductor will remain same, whatever be the position of cross section as shown in the fig (5.2). The current I flowing through all planes 1, 2 and 3 is same. It is also the outcomes of principle of conservation of charge.

5.2 Current Density

In certain cases we are interested in studying the flow of charge from any point on the cross section of the conductor. In this situation we use current density, a vector quantity. At any point P, to define this quantity we consider a small area dS at point P. Which is perpendicular to the direction of flow of current. Fig (5.3A). If ΔI is the current passes through a small area ΔS. Then average current density

$$J_{av} = \frac{\Delta I}{\Delta S} \qquad \text{. . . 5.3 (a)}$$

and the current density at point

$$J = \lim_{\Delta S \to 0} \frac{\Delta I}{\Delta S} = \frac{dI}{dS} \qquad \text{. . . 5.3 (b)}$$

Here, J = Current density at point P. It is a vector quantity.

If current flows due to positive charges, then direction of $\vec{J}$ will be same as that of +ve charge. If the current flow is due to negative charges then direction of $\vec{J}$ will be opposite. Hence, direction of $\vec{J}$ at that point or area will be in the direction of current. If current is uniformly distributed and perpendicular to point, then

$$J = \frac{I}{S} \qquad \text{. . . 5.3(c)}$$

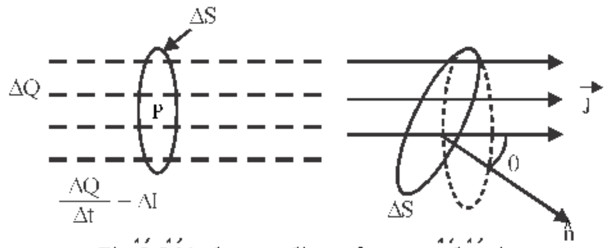


Fig 5.3 Understanding of current density

If area ΔS is not normal, we can use a unit vector n̂ which is perpendicular to it and makes an angle θ with the direction of current. Then the current density will be,

$$J = \frac{\Delta I}{\Delta S \cos\theta} \qquad \text{. . . (5.4a)}$$

$$I = J \Delta S \cos\theta$$

$$\Delta I = \vec{J} \cdot \Delta\vec{S} \qquad \text{. . . (5.4b)}$$

Unit of current density is $\frac{\text{ampere}}{m^2} = \frac{\text{A}}{m^2}$

Here, we have to note that current density is a vector quantity whereas electric current is a scalar quantity.

In a conductor of non-uniform area, current flowing will be same but current density will be different. For a finite area

$$I = \int \vec{J} \cdot d\vec{s} \qquad \text{. . . (5.4c)}$$

5.3 Flow of Electric Charge in Metallic Conductors:

In atoms of metallic conductor, attractive force between nucleus and valance electrons is very weak due to which, the valance electrons are weakly bound with nucleus and a few of them get free to move randomly within metal. These electrons are called free electrons or conduction electrons. In conductors, number of free electrons are very large. e.g. in copper where every atom release one free electron, their free electron density is nearly 8.49×10^{28} / m^3. In conductors free electrons move randomly in whole volume just as gas molecules move in gas chamber. In the absence of electric field, these electrons move randomly due to collisions of electrons with ions of conductor the direction of electron will suddenly change. As in molecular theory of gases this random motion of electrons can be related to free path λ which mean free path between two succesive collisions. The time between two succesive collisions is τ (τ is also called relaxation time this is average time between two succesive collision. Due to collisions, the directions of electron will change as zig-zag path). In metals, the electrons are in random motion at large. So within a given time interval and given area ΔS, number of electrons through transverse section is equal to the number of electrons crossing opposite to the direction. Therefore, resultant current is zero. When external electric field is applied on the conductor. A force ($\vec{F} = -e\vec{E}$) acts in opposite direction to the field. Due to this electric field, electrons drift in direction opposite to the direction of electric field. The electron gain a velocity. This type of velocity is called drift velocity.

5.4 Drift Velocity and Mobility

5.4.1 Drift Velocity

In the presence of electric field, random motion of electrons is modified such that these electrons move with average slow speed (drift) in opposite direction of electric field. This speed is known as drift speed. To represent in form of vector the drift of electron is expressed in form of drifts velocity $\vec{v}_d$. The drift velocity $\vec{v}_d$ is always opposite in direction to applied electric field $\vec{E}$.

The value of this drift speed is very less then the random average speed between the collisions of electrons. (approximately less than 10^{10} times). Figure 5.4 helps us in understanding the drift velocity. In this figure, the path followed by the electrons is shown in the presence of electric field and without electric field. The continuous lines represent the path of electrons in the absence of electric field and the dotted line represent the path of electrons in the presence of electric field.

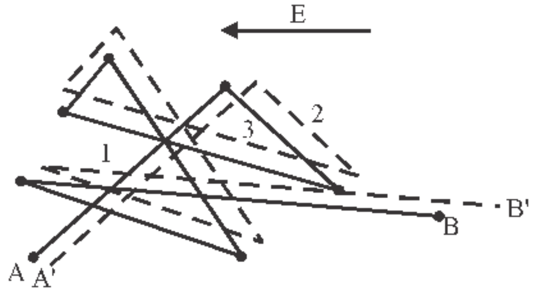


Fig 5.4: The continuous lines represent the path of electrons in the absence of electric field and the dotted line represent the path of electrons in the presence of electric field.

In the absence of electric field, electrons start from A, after some collision it reaches point B. Whereas in the presence of electric field, after same number of collisions the electron reaches B' instead of B. In this way after drift, electron reaches B' in place of B in the direction opposite to the direction of electric field. (It can be compared with still air and slow-motion wind or air). In still air every molecule will move randomly with thermal velocity and then will be no velocity in particular direction. In case of slow-motion in air every molecule has random motion as well as small velocity in the direction of air flow.

Now we will find the relation between the drift velocity and electric field. In the presence of electric field the force on every free electron of metal will be eE. Due to this the acceleration of electron will be,

$$a = \frac{eE}{m}$$

In vector representation,

$$\vec{a} = -\frac{e\vec{E}}{m}$$

For given value of $\vec{E}$, $\vec{a}$ will be constant.

Here it is important that electrons are accelerated in time interval between two successive collisions. Its reason is that due to collisions with vibration ions its drifting nature stops momentarily and its velocity becomes random in any direction.

If the velocity of n electron is $\vec{u}$ and after time t its velocity just before the next collision is $\vec{v}$, then

$$\vec{v} = \vec{u} + \vec{a}t$$

or

$$\vec{v} = \vec{u} - \frac{e\vec{E}t}{m} \quad \ldots (5.5)$$

If we take the average over the all free electrons of metal, then,

$$\langle \vec{v} \rangle = \langle \vec{u} \rangle - \frac{e\vec{E}}{m}\langle t \rangle \quad \ldots (5.6)$$

Here, the quantity $\langle \vec{u} \rangle = 0$ because for electrons average of random velocities is zero and just immediately after collision electron will not acquire any energy from electric field.

Simultaneously, t is the time of travel between two successive collisions. Hence $\langle t \rangle$ is the average of time intervals between two successive collisions of all the electrons and it is equal to τ .

In this situation, if $\langle \vec{v} \rangle = v_d$. Here $\vec{v}_d$ is the average drift velocity,

Then, $\vec{v}_d = \dfrac{-e\vec{E}}{m}\tau$ . . . (5.7)

And average drift speed

$$v_d = \frac{eE}{m}\tau \quad \text{. . . (5.8)}$$

In general, if the charge is q which is moving in electric field. Then drift velocity of this charge will be

$$v_d = \frac{q\tau}{m}E$$

If the charge is positive then $\vec{v}_d$ will be in the direction of $\vec{E}$, if the charge is negative then $\vec{v}_d$ will be in the direction opposite of $\vec{E}$.

5.4.2 Mobility

In the equation for drift velocity, $v_d = (eE/m)\ \tau$, in the right-hand term eE/m , e and m are constants and the relaxation time (τ) is the characteristic property of material of the conductor. Therefore, the drift velocity is constant for the material of the conductor. This constant is called the mobility of the conductor (μ), and it is represented by the following equations,

$$\vec{v}_d = -\mu\vec{E} \quad \text{... (5.9A)}$$

$$v_d = \mu E \quad \text{... (5.9B)}$$

From these relations it is clear that for any conductor $v_d \propto E$, hence drift velocity (v_d) is proportional to applied electric field (E).

$$\mu = \frac{e\tau}{m} = \frac{v_d}{E} \quad \text{. . . (5.10)}$$

(In general for any charge q, $\mu = \dfrac{q\tau}{m}$)

Hence, according to equation (5.10), mobility can be defined as drift velocity per unit electric field. It is a positive quantity having the unit

$$\frac{m/s}{V/m} = m^2 s^{-1} V^{-1}$$

For two given metals, when the electric field is same,

$$\frac{\mu_1}{\mu_2} = \frac{v_{d1}}{v_{d2}}$$

From this relation, it is clear that mobility of electron in a conductor is more if the drift velocity is more. We shall study the mobility in semiconductors in chapter 16 of this book.

5.4.3 Relation between drift velocity and electric current

After understanding the concept of drift velocity, now we can use it to find the electric current in a conductor. Let n be the free electrons per unit volume (free electron density) inside a metallic conductor. A is the area of cross section of this conductor. If electric field is applied across a conductor, the free electrons inside the conductor will move with drift velocity, opposite to the direction of electric field. All the free electrons in the conductor can be assumed to be moving with the same drift velocity v_d . Now, let us think of a small element ΔL of this conductor as shown in the figure (5.5). The number of charge carriers in this small element will be $nA\Delta L$ and the net charge on these charge carriers will be $(nA\Delta L)e$.

Therefore, $\Delta Q = (nA\Delta L)e$

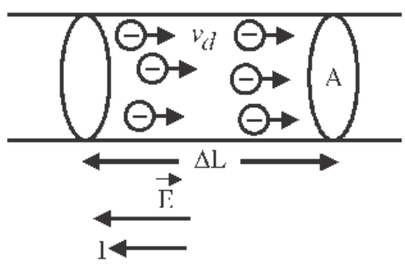


Fig 5.5 Drift velocity in a conductor

Because, all the charge carriers (free electrons) in the wire are moving with the same drift velocity v_d . Then, the time taken by these charge carriers in crossing this element of conductor is,

$$\Delta t = \frac{\Delta L}{v_d}$$

By the definition, the current is the free charge passing through any area of cross section of the conductor per second.

$$I = \frac{\Delta Q}{\Delta t} = \frac{(nA\Delta L)e}{\Delta L / v_d}$$

or $$I = nAev_d \qquad \ldots (5.11)$$

Equation (5.11) represents the relation between the current and drift velocity. By equation (5.9B), $v_d = \mu E$, Therefore Eq. (5.11) can be written as,

$$I = nAe\mu E \qquad \ldots (5.12)$$

The equation (5.12) shows the relation between the current and mobility of the charge carriers. From equation (5.11) for a constant electric current,

$$nAev_d = \text{constant.}$$

Here, e is constant and for a given metallic conductor, n is also constant. Hence for a given conductor, for constant current,

$$Av_d = \text{constant}$$

or $$A_1 v_{d1} = A_2 v_{d2} = \text{constant} \qquad \ldots (5.13)$$

Thus, us for a given conducting wire of non-uniform cross section area, the drift velocity (v_d) of electron will be more where the area of cross section is small and the drift velocity (v_d) of electron will be small where the area of cross section is more.

5.4.4 Relation between drift velocity and potential difference

According to section 5.4.3, electric field inside the conductor of length l will be,

$$E = \frac{V}{l} \qquad \ldots (5.14)$$

So, the drift velocity of electron according to the equation (5.7) will be,

$$v_d = \frac{eE}{m}\tau$$

By putting the value of E from the Eq. (5.14),

$$v_d = \frac{eV}{ml}\tau = \frac{e\tau}{ml}V \qquad \ldots (5.15)$$

From the equation (5.15) it is clear that the drift velocity (v_d) of the free electron in a conductor is proportional to potential difference (V).

Hence, $v_d \propto V$

The drift velocity (v_d) of the free electron in a conductor does not depend on the length of the conductor.

Example 5.1 : The dependence of charge Q crossing a surface in time t is given as,

$$Q = 4t^3 + 5t + 6$$

Then calculate the instantaneous current from the surface at $t = 1\ sec$.

Solution : Instantanous current,

$$I = \frac{dQ}{dt} = \frac{d}{dt}(4t^3 + 5t + 6) = (12t^2 + 5)\,\text{A}$$

At $t = 1\ sec$,

$$I = 12(1)^2 + 5 = 17A$$

Example 5.2 : Estimate the average drift speed of electrons in a copper wire of cross-sectional area $1.0 \times 10^{-7}\,\text{m}^2$ carrying a current of 1.5 A. Assuming that each copper atom contributes roughly one conduction electron. The density of copper is 9.0×10^3 kg/m^3 and its atomic mass is 63.5 a.m.u.

Solution : The formula for drift speed is given by,

$$v_d = \frac{I}{neA}$$

Given, $I = 1.5$ A

$e = 1.6 \times 10^{\ 19}$ C.

$A = 1.0 \times 10^{\ 7}\,\text{m}^2$

$n =$ the number of electrons per unit volume in copper

$= N_A = 6.0 \times 10^{23}$ atoms per mole

Given that atomic mass of copper is 63.5 amu and each copper atom contributes roughly one conduction electron to the current flow.

Number of atoms in 63.5 amu copper = ($N_A = 6.0 \times 10^{23}$ Avogadro number).

number of atoms in 1 gram copper

$$= \frac{6.02 \times 10^{23}}{63.5}$$

number of free electrons per unit volume

$$n = \frac{6.02 \times 10^{23}}{63.5 \times 10^{-3}} \times 9 \times 10^{3} = 8.5 \times 10^{28}\, m^{-3}$$

Hence, drift speed will be,

$$v_d = \frac{1.5}{8.5 \times 10^{28} \times 1.6 \times 10^{\ 19} \times 1.0 \times 10^{\ 7}}$$

$$= 1.10 \times 10^{\ 3}\, \text{m/s}$$

Note : If this speed is compared with the speed of electric field (electromagnetic wave, i.e. 3×10^8 m/sec, then it was found that drift velocity is very less than the velocity of e.m. wave. Simultaneously the drift velocity is also very much less than the thermal velocity (~10^6 m/s) of electrons. The time taken by an electron in travelling the length of 1m with drift velocity $10^{\ 3}$ m/s in a conductor is about 15 minutes. Now the question arises how an electric bulb lights up instantaneously after the circuit is switched on even though the bulb is few meters from the electric switch. Here one simple example can simplify our problem. If one end of a very long pipe is connected to tap and the other end to the tank, then water will take some time to reach the other end. But, if the pipe is already filled with water, then water will not take much time to reach the tank.

Example 5.3 : The electron revolves in an orbit of radius $5.3\times10^{\ 11}$ m of hydrogen atom with speed of 2.2×10^6 m/s. Calculate the average electric current.

Solution : Given,

Radius of orbit, $r = 5.3 \times 10^{\ 11}$ m

Speed of electron, $v = 2.2 \times 10^6$ m/s

Charge on electron, $e = 1.6 \times 10^{-19}$ C

Therefore, the time period of orbital motion of electron

$$T = \frac{2\pi r}{v} = \frac{2\times3.14\times5.3\times10^{-11}}{2.2\times10^{6}}$$

$= 15.13 \times 10^{\ 17}$ sec

So, the average current produced due to orbital motion of electron is,

$$I = \frac{q}{T} = \frac{e}{T} = \frac{1.6\times10^{\ 19}}{15.13\times10^{-17}}$$

$= 1.06 \times 10^{-3}$ A $= 1.06$ mA

5.5 Ohm's Law:

In the year 1828, German scientist G.S. Ohm, after performing so many experiments, enunciated a law regarding the flow of current in a conductor. To honour him, the law is named after him.

According to this law, when the physical conditions of the conductor remains same (length, temperature, nature of material of the conductor and area of cross section), then the current flowing through the conductor will be proportional to the potential drop across its ends.

$$V \propto I$$

$$V = RI \qquad \ldots (5.16)$$

Where, the constant of proportionality is called the resistance of the conductor.

$$R = \frac{V}{I} \qquad \ldots (5.17)$$

The S.I. unit of resistance is ohm and is represented by the Greek symbol Ω. If on applying a potential difference of one volt across the ends of a conductor a current of 1 ampere flows through the conductor, then resistance of the conductor is said to be one ohm (Ω).

By knowing the values of electric current (I) for the potential difference (V) applied across the terminals of the conducting wire a graph can be plotted between V and I. This graph will be a straight line as shown in the fig 5.6.

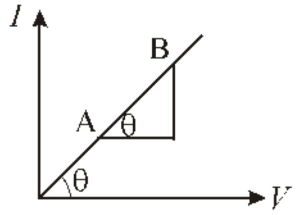


Fig: 5.6 Graph between V and I according to Ohm's Law

Slope of this straight line curve will be equal to the reciprocal of resistance of the wire.

slope of the curve =

$$(\tan q) = \frac{I}{V} = \frac{1}{R} \qquad \ldots (5.18)$$

5.5.1 Deduction of Ohm's Law

In section 5.4.3 of this chapter regarding free electron model, we have seen expression for electron current related to drift speed. According to equation (5.11), we have -

$$I = nAev_d$$

Thus, the current density will be,

$$J = \frac{I}{A} = nev_d \qquad \ldots (5.19)$$

But we have seen in equation (5.8)

$$v_d = \frac{e\tau}{m}E$$

Thus, $J = \frac{ne^2\tau}{m}E \qquad \ldots (5.20)$

In this equation, the terms of right side $\frac{ne^2\tau}{m}$, m and e are constant and terms n and τ are the characteristics of the conductor. For anisotropic homogeneous conductor, the quantity $\frac{ne^2\tau}{m}$, can be considered as a constant. It is called as conductivity and represented by the symbol σ.

Hence the conductivity

$$\sigma = \frac{ne^2\tau}{m} \qquad \ldots (5.21)$$

Now equation (5.20) can be written as,

$$J = \sigma E$$

Since $\vec{J}$ and $\vec{E}$ are vector quantities, therefore in vector notation the above equation can be rewritten as,

$$\vec{J} = \sigma \vec{E} \qquad \ldots (5.22)$$

According to these equations, the conductivity (σ) of any conductor does not depend on electric field (E). Whereas the current density ($\vec{J}$) in the conductor is proportional to the electric field ($\vec{E}$). Equation (5.22) is called Ohm's Law in microscopic form. For so many conductors it is applicable for long range of electric field. Now we see the formal definition of ohm's law which is equivalent to equation (5.16)

Now let us consider a conductor of length l and area of cross section A connected to a battery of potential difference V as shown in the figure 5.7. The field produced inside the conductor is $E = \frac{V}{\ell}$ and current density is $J = I/A$. Now put these values of E and J in equation (5.22), we get,

$$\frac{I}{A} = \sigma\frac{V}{\ell}$$

or $$V = \frac{1}{\sigma}\frac{\ell}{A}I$$

$$= \left(\frac{\rho\ell}{A}\right)I \qquad \ldots (5.23)$$

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \qquad \ldots (5.24)$$

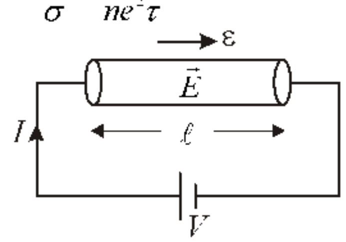


Fig: 5.7 Current flow in a conductor

ρ is called the resistivity of the material of the conductor. For any given material of length l and area of cross section A are constant and the resistivity (ρ) is the characteristic property of the material of the conductor.

Thus, the quantity $\frac{\rho \ell}{A}$ is a constant known as the resistance of the conductor (R).

$$V = IR$$

where $$R = \frac{\rho \ell}{A} \qquad \ldots (5.25)$$

We see that the equation (5.24) is same as that of Ohm's Law. Equation, $V = IR$ is sometimes also called macroscopic form of Ohm's Law. The macroscopic quantities V, I and R can be measured directly with the help of meters. When we are interested to know the fundamental electrical properties, then quantities E, σ and J are useful.

5.5.2 Resistivity

After performing so many experiments it was found that resistance of isotropic and homogenous conductor is directly proportional to length l and inversely proportional to area of cross section A of the conductor. i.e.

$$R \propto \frac{\ell}{A}$$

or $$R = \frac{\rho \ell}{A}$$

Here, the constant of proportionality ρ is called the resistivity of the material of conductor. Here, it is essential to note the difference between resistivity and resistance. The resistivity is a property of the material of the substance whereas the resistance of the material is the property related to the material and the geometric parameters such as length and area of cross section. Two wires of same material can have different resistance but their resistivity will be same. Similarly, two wires of different material will have different resistivity but may have same resistance. This we have already seen in the derivation of Ohm's law. There we have also seen the formula for resistivity as, in equation (5.24).

$$\rho = \frac{m}{ne^2 \tau} \qquad \ldots (5.26)$$

Because the quantities n and τ are the characteristics of the substance, therefore resistivity depends on the nature of the substance as well as the temperature of the substance. Resistivity of the material is also known as specific resistance. From equation (5.25),

$$\rho = \frac{RA}{\ell} \qquad \ldots (5.27)$$

Unit of resistivity is $\frac{\Omega m^2}{m} = \Omega \text{m}$ and the dimensions are $M^1 L^3 T^{-3} A^{-2}$.

If the conductor is in the form of a cylinder of radius r, then $A = \pi r^2$. Then,

$$\rho = \frac{RA}{\ell} = \frac{R(\pi r^2)}{\ell}$$

In the above equation, if we take, $A = 1 \text{m}^2$ and $\ell = 1 \text{m}$, then, $\rho = R$.

Thus, the resistivity of a material will be numerically equal to the resistance of a sample whose length is unity and area of cross section is also unity.

Reciprocal of resistivity of any substance is known as conductivity.

The unit of conductivity is ohm ^{1} meter ^{1} and the dimensions of conductivity are $M^{\ 1} L^{\ 3} T^3 A^2$.

The table 5.1 shows the resistivity of some common materials. It is clear from the table that the resistivity of a conductor lies between $10^{\ 8} \Omega \times m$ to $10^{\ 6} \Omega \times m$. At the other end are insulators like ceramic, rubber and plastic having resistivity of the order of $10^{16} \Omega \times m$. In between the two are semiconductors like Germanium and Silicon, which behave as an insulators at 0 K. On increasing the temperature the resistivity of semiconductors decreases.

Table 5.1 Resistivity of some common materials

Material	Resistivity ρ at 0^0 (In $\Omega \times m$ units)
Conductor	
Silver	1.6×10^{-8}
Copper	1.7×10^{-8}
Aluminium	2.7×10^{-8}
Tungsten	5.6×10^{-8}
Iron	10×10^{-8}
Platinum	11×10^{-8}
Mangnin	48×10^{-8}
Mercury	98×10^{-8}
Nichrome	100×10^{-8}
Semiconductor	
Carbon (Graphite)	3.5×10^{-5}
Germanium	0.60
Silicon	2300
Insulators	
Pure water	2.5×10^{5}
Glass	$10^{10} - 10^{14}$
Hard rubber	10^{14}
Dry wood	10^{8} to 10^{14}

5.6 Electric Resistance

The property of conductor which creates obstacle in the flow of electric current is called electric resistance. We know that in every conductor there are free electrons. On applying potential across its ends, electrons flow from one end to the other by doing random motion. This causes the flow of electric current in the conductor. During the motion electrons keep on colliding with ions and atoms. In this way obstacle is created in the flow of electric current and it is called electric resistance. Resistance of a specimen of a substance not only depends on the nature of the substance but also on the length and area of cross section of the wire. In addition to this the resistance of any conductor also depends on the faces of the conductor across which the potential difference is applied. The resistance also depends on the temperature of the conductor. The resistances which are prepared for a particular value are called resisters.

5.6.1 Ohmic and Non Ohmic Resistance

The conductors (or devices) obeying Ohm's law, i.e. for which, the graph between potential difference V and the current I is a straight line and passing from origin are called Ohmic conductors (or devices). In such devices the current flow does not depend on the polarity of the applied potential difference.

There are several devices or substances used in electric circuits which do not obey ohm's law. i.e. the graph between potential difference V and the current I is not a straight line but a curve. These types of devices or substances are known as non-ohmic.

In addition to this there are certain substances in which the flow of electric current depends on the polarity of the applied voltage. The examples of such devices are vacuum tube, semi-conductor diode, liquid electrolyte, transistor etc.

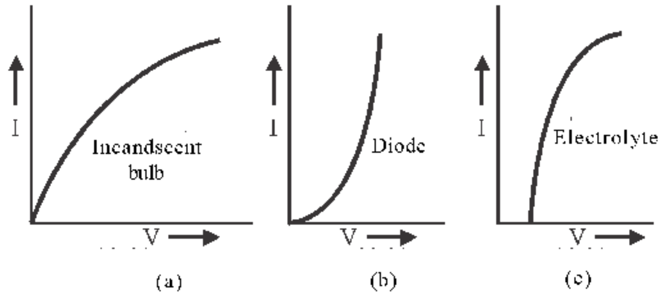


Fig 5.8 Non-ohmic behaviour

Figure (5.8A) is a *V-I* curve for a torch bulb. It is clear from the graph that Ohm's law is not obeyed. The reason is that when current flow increases in the filament of the bulb, the temperature of the filament increases. The resistance of the filament will increase. Therefore, the ratio of V and I will not remain constant. Graph (5.8B) is drawn for semi-conductor diode device and graph (5.8C) is drawn for a liquid electrolyte. From the graphs it is evident that these devices do not obey Ohm's law.

5.7 Carbon Resistance and colour codes for Carbon Resistance

Commercially produced resistances for domestic use or laboratory use are of two major types: wire wound resistors and carbon resisters. Wire wound resisters are made by winding the wires of an alloy, viz. magainin, constantan, nichrome etc. The temperature coefficient of resistance of these alloys is very small due to this the conductivity is least affected by temperature. Resisters of

(b)

very high value cannot be made using these materials because it will need very long length of wire which is inconvenient. On the other hand, the resisters of very high value are made up of Carbon resisters, which are inexpensive and compact, therefore they are extensively used in electronic circuits. Carbon resisters are moulded into the cylindrical shape by using a binding agent and wire leads (for connecting the carbon resister in any electric circuit) are attached to the ends.

The value of carbon resister in indicated by four coloured bands. Every coloured band (strip) has a special coded meaning. Every colour has a special colour code. Key to these colour codes are is given in the table 5.2.

Table 5.2 Colour code for resistances

Colour	digit	multiple	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Golden		10^{-1}	5
Silver		10^{-2}	10
No colour			20

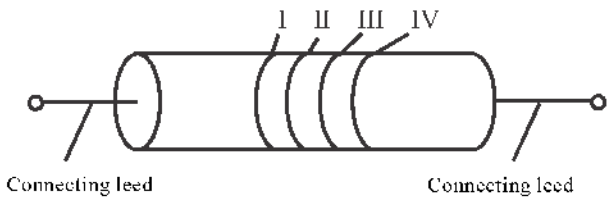


Fig 5.9 Carbon resistance colour code

Identification of the value of a carbon resister:

(i) Hold the Carbon resister such that the tolerance ring (Silver or Golden Colour) is on the right side.

(ii) The first two coloured rings indicate the first two significant digits of the value of carbon resistance as per colour code. The colour of the third ring indicates the decimal multiplier and the fourth ring indicates the tolerance percentage. Sometimes the fourth ring is absent, and it indicates that the tolerance is 20%. It indicates $\pm$% in the value of resistor.

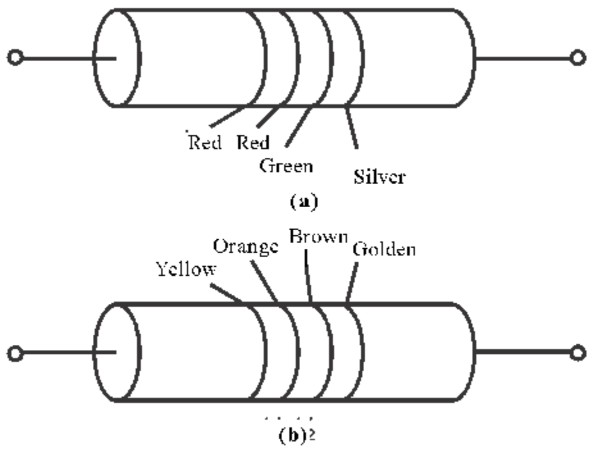


Fig 5.10 Carbon resistance

For carbon resistance shown in the Fig 5.10 (a) and (b),

the value of resistance (a) is $22 \times 10^5\,\Omega \pm 10\%$ and that of (b) is $(43 \times 10^1\ \Omega) \pm 5\%$ respectively.

Example 5.4 : A wire of length l and cross sectional area of A has a resistance R. Find the percentage change in the resistance, when it is stretched to double of its length.

Solution : Given, the length of the wire is l and its cross sectional area is A, then the resistance of wire will be,

$$R = \rho \frac{\ell}{A}$$

On stretching the wire to double the length, its cross sectional area decreases, because the mass and volume of the substance remains constant.

$$A\ell d = A'(2\ell)d$$

$$\Rightarrow \quad A' = \frac{A}{2}$$

Hence the new resistance of the substance,

$$R' = \frac{\rho(2\ell)}{A'} = \frac{\rho(2\ell)}{A/2} = 4\frac{\rho\ell}{A} = 4R$$

Therefore, percentage change in resistance,

$$= \frac{R'-R}{R}\times 100\%$$

$$= \frac{4R-R}{R}\times 100\% = 300\%$$

Example 5.5 : The value of a carbon resistance is $62\times 10^3\,\Omega$. The percentage tolerance is 5%. Write down the colour code in sequence.

Solution : Given,

$$R = (62\times 10^3\,\Omega)\pm 5\%$$

According to the colour code, the colour of strips on the carbon resistance are Blue, Red, Orange and Golden.

Example 5.6 : If the length of two conductors,

$X = 4\Omega$ and $Y = 48\times 10^{-8}\,\Omega\times m$ reduced to half, then write down the corresponding value of X and Y.

Solution : In the question, the X is resistance of the wire and Y is resistivity of the material of wire. Therefore, by changing the length the resistance will change whereas the resistivity remains unaltered.

Therefore $X' = 2\Omega$ and Y remains same, i.e. $Y' = 48\times 10^{-8}\,\Omega m$.

Example 5.7 : A potential difference of 0.9V is applied across the ends of a tungsten wire of length 1.5 m and cross-sectional area $0.60\times 10^{-6}\,m^2$. Find the current flowing through the wire. Specific resistance (r) of Tungsten is $5.6\times 10^{-8}\,\Omega\times m$.

Solution : Given,

$$\ell = 1.5\,m$$

$$A = 0.60\times 10^{-6}\,m^2$$

$$r = 5.6´\ 10^{-8} W´\ m$$

Therefore, resistance of the wire,

$$R = \rho\frac{\ell}{A}$$

$$R = \frac{5.6\times 10^{-8}\times 1.5}{0.60\times 10^{-6}}\Omega$$

$$R = 0.14\,\Omega$$

Thus, the current through the wire,

$$I = \frac{V}{R} = \frac{0.90}{0.14} = 6.43\,A$$

5.8 Effect of temperature on Resistance and Resistivity

The resistivity of various materials changes with temperature in different manner according to their nature.

(A) For conductors

Previously we have seen that the resistivity of a conductor is given as,

$$\rho = \frac{m}{ne^2\tau}$$

Where, m is the mass and e is the charge of an electron and n is the free electron number density of the conductor. All these quantities are constant. On increasing temperature, the amplitude of vibration in the conductor and the frequency of collision of free electrons increase due to which the relaxation time t decreases. Thus, the resistivity of the conductor increases and thereby the conductivity decreases. If ρ_0 and ρ_t are the resistivity of a conductor at 0^0C and t^0C, then a close relation between these quantities is,

$$\rho_t = \rho_0(1+\alpha t) \qquad \ldots (5.28)$$

Where, α is a constant known as temperature coefficient of resistivity. α depends on the nature of material. The value of some common materials is given in the table (5.3). For some substances is positive, whereas for others it is negative.

Table 5.3 Temperature coefficient of Resistivity

Material	Temperature coefficient of Resistivity $(^{\circ}C)^{-1}$
A. Conductor	
(A) Metals	
Silver	4.1×10^{-3}
Copper	3.9×10^{-3}
Aluminium	4.3×10^{-3}
Tungsten	4.5×10^{-3}
Iron	6.5×10^{-3}
Platinum	3.9×10^{-3}
Mercury	0.9×10^{-3}
(b) ***Alloys***	
Nichrome	0.4×10^{-3}
Mangnin	0.002×10^{-3}
Constantan	0.001×10^{-3}
B. ***Semiconductor***	
Carbon	− 0.0005
Germanium	− 0.05
Silicon	− 0.07

According to the above equation (5.28), the temperature coefficient of resistivity will be given by,

$$\alpha = \frac{\rho_t - \rho_0}{\rho_0 t} = \frac{\Delta\rho}{\rho_0 t} \qquad \ldots (5.29)$$

The dimensions of α in the equation (5.29) will be θ^{-1}. Thus, for a unit change in temperature (Δt=1), the ratio of change in resistivity to the resistivity at 0^{0}C,

i.e. $\left(\frac{\Delta\rho}{\rho_0}\right)$ is equal to the temperature coefficient of resistivity. The unit of temperature coefficient of resistivity is $^{0}C^{-1}$.

Similarly, the dependence of resistance on temperature can be written as,

$$R_t = R_0\left(1+\alpha t\right) \qquad \ldots (5.30)$$

If the change in temperature of a conductor is Δt, then, equivalent value of resistivity and resistance will be given by,

$$\rho_{t_2} = \rho_{t_1}(1+\alpha\Delta t) \qquad \ldots (5.31)$$

$$R_{t_2} = R_{t_1}(1+\alpha\Delta t) \qquad \ldots (5.32)$$

Here, t_1 and t_2 are the initial and final temperature of the conductor.

Here $\Delta t = t_2 - t_1$

If a graph is plotted between the resistivity and temperature for metals, then it will be more or less a straight line. But at low temperature it is curved, i.e deviation from the straight line.

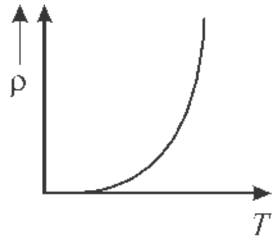


Fig 5.11 Resistivity of a conductor at low temperature

Some materials like Nichrome, which is an alloy (Nickel 80 % and Chromium 20%) exhibits very weak dependence of resistivity on temperature. Such materials are widely used in wire wound standard resisters. Figure (5.12) shows a graph between ρ_T and T.

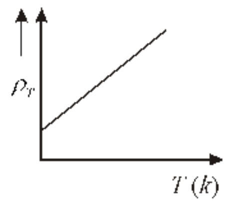


Fig 5.12 Resistivity of a conductor at high temperature

(B) For insulators

The resistivity of insulating material decreases exponentially on increasing temperature and increases on decreasing temperature. At absolute zero temperature, the resistivity becomes very high and tends to infinite value. Thus, the conductivity of insulators at absolute zero temperature becomes zero. The variation of resistivity of insulator with temperature is given in the following relation,

$$\rho = \rho_0 e^{E_g/2K_BT} \qquad \ldots (5.33)$$

Where,

K_B = Boltzmann constant

T = Temperature of the material in kelvin

E_g = Energy gap between valance band and conduction band.

(C) For semi-conductors:

On increasing the temperature of semi-conductors, the bonds of semi-conductors break up rapidly. Then the number of electron-hole pairs increase exponentially. Thus, the resistivity of semi-conductors decreases exponentially as temperature increases. The temperature coefficient of resistivity in this case will be *–ve*. Figure (5.13) shows a graph between r and T.

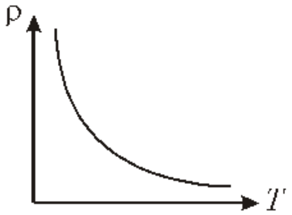


Fig 5.13 Conductivity of semi-conductors

Example 5.8 : The resistance of a platinum wire of a platinum resistance thermometer at the ice point is 5 Ω and at steam point is 5.23 Ω. When the thermometer is inserted in hot bath, the resistance of platinum wire is 5.795 Ω. Calculate the temperature of the hot bath.

Solution : Given,

$$R_0 = 5\Omega, \qquad R_{100} = 5.23\,\Omega$$

and $\quad R_t = 5.795\,\Omega$

We know the formula for the resistance of a wire at temperature t,

$$R_t = R_0(1+\alpha t)$$

Therefore, the resistance $R_{100} = R_0[1+\alpha 100]$ is,

$$R_{100} - R_0 = R_0\alpha \times 100 \qquad \text{. . . (i)}$$

$$R_t - R_0 = R_0\alpha t \qquad \text{. . . (ii)}$$

Diving eq (2) by eqn (1),

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

$$t = \frac{5.795 - 5}{5.23 - 5} \times 100$$

$$t = \frac{0.795}{0.23} \times 100 = 345.65°C$$

Example 5.9 : A platinum resistance thermometer, a device used to measure the change in temperature, has a resistance of 50 Ω at a temperature of 20°C. When this thermometer is placed inside a container filled with silver at its melting point the resistance increases to 80 Ω. Assuming that there is linear change in the resistance of platinum in this temperature range, calculate the melting point of silver. Given $\alpha = 3.8 \times 10^{-3}\ °C^{-1}$ for silver.

Solution : For limited temperature range, the resistance at $t_2°C$ is given by,

$$R_{t_2} = R_{t_1}(1 + \alpha\Delta t)$$

Where,

Δt = initial temperature, t_2°C = final temperature

and Δt = increase in temperature

Given $\quad \alpha = 3.8 \times 10^{-3}\ °C^{-1}$,

$$R_t = 80\Omega, \quad R_0 = 50\Omega$$

$$\Delta t = \frac{R_t - R_0}{\alpha R_0} = \frac{80 - 50}{3.8 \times 10^{-3} \times 50}$$

$$= \frac{3 \times 10^3}{19}$$

or $\quad t_2 - t_1 = 157.9°C$

or $\quad t_2 = t_1 + 157.9°C$

or $\quad t_2 = 20°C + 157.9°C = 177.9°c$

∴ melting point of silver = t_2 = 177.9°C

5.8.1 Super conductivity

It was found in some metals or composite metals that at particular very low temperature their resistivity decreases abnormally or rapidly and becomes zero. Such substances are called Super conductors and this property is called Superconductivity. The temperature at which this phenomenon occurs is called critical temperature.

The phenomenon of Superconductivity was first observed by the physicist Heike Kamerlingh Onnes in the year 1911 by cooling mercury to 4.2 K. In the state of superconductivity, magnetic field inside the conductor will also be zero. This effect is called Meissner Effect.

The phenomenon of superconductivity is exhibited at very low temperature (10 K to 0.1 K). Although now some materials have been found to exhibit the property of superconductivity at higher temperature of the order of 90K. Now a days scientists are putting great efforts to search superconductors at normal temperature so to get rid of the energy lossproblem during transmission.

Uses of Superconductors: To construct magnets known as Superconducting magnets which can create high magnetic fields (of the order of 10 tesla), small and very efficient transformers, motor electric generators, energy transmission and super computers etc.

5.9 Series and Parallel Combination of Resistances:

According to the requirement of certain electric current in any electric circuit, we must have definite resistance in that circuit. Suppose this resistance is not available with us. Then we use the combination of the resistances available with us so that the resistance of desired value is obtained. There are two ways in which the combination is carried out.

5.9.1 Series Combination

Two or more than two resistances are said to be connected in series if same current is passed through all the resistances. In series combination, second end of every resistance is connected to first end of the next resistance.

In fig. (5.14),three resistance R_1, R_2 and R_3 are connected between two ends. A & B in series and a current I is flowing in this. The potential difference at resistance are V_1, V_2 and V_3 respectively than according to ohm's law $V_1 = TR_1$, $V_2 = IR_2$, $V_3 = IR_3$

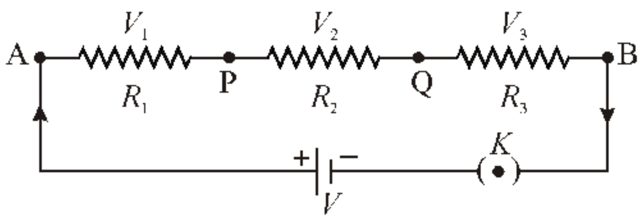


Fig 5.14 Series Combination of resistances

If the potential difference between points A and B is V, then,

$$V = V_1 + V_2 + V_3$$

or $$V = IR_1 + IR_2 + IR_3$$

or $$V = I(R_1 + R_2 + R_3) \quad \ldots (5.34)$$

If the equivalent resistance of the series combination is R_{eq}, Then,

$$V = IR_{eq} \quad \ldots (5.34A)$$

By combining equation (5.34) and (5.34A)

$$IR_{eq} = I(R_1 + R_2 + R_3)$$

or $$R_{eq} = R_1 + R_2 + R_3 \quad \ldots (5.35)$$

i.e. the equivalent resistance will be the sum of all individual resisters. If there are n resisters then,

$$R_{eq} = R_1 + R_2 + \ldots\ldots\ldots + R_n$$

If all the resistances are identical and equal to R, then,

$$R_{eq} = R + R + \ldots\ldots.\ up\ to\ n$$

or $$R_{eq} = nR \quad \ldots (5.36)$$

Important points for series combination of resistances :

(a) In all the resistances the current flowing will be same.

(b) Resultant potential difference will be sum of potential difference across each resistance.

(c) Equivalent resistance will be greater than any of the resistance connected in series.

5.9.2 Parallel Combination:

Two or more than two resistances are said to be connected in parallel if same potential difference exists across each of the resisters. In Parallel combination, one end of all resistances is connected at one point and other end of all resistances are connected to another point. In figure 5.15 three resistances of value R_1, R_2 and R_3 connected in parallel.

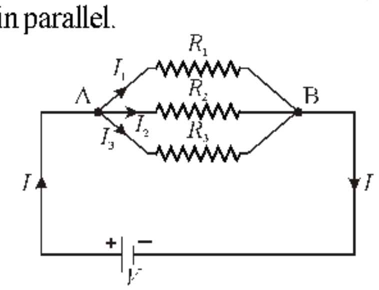


Fig 5.15 Parallel Combination of resistances.

The parallel combination of resistances have two end points A and B. One end of each resistance is connected to A and the other end of each resistance is connected to B. Points A and B of the parallel combination is connected to a battery of e.m.f. V volt.

In this combination, the potential difference across each resistance will be same and equal to V. In this circuit main current, I at point A is divided among the resistances such that the potential difference across each resistance is same and equal to potential difference V of the battery. If I_1, I_2 and I_3 are the currents through the resistances R_1, R_2 and R_3, then total current I is given by,

$$I = I_1 + I_2 + I_3 \qquad \ldots(5.37)$$

According to Ohm's law,

$$V = I_1 R_1 = I_2 R_2 = I_3 R_3$$

Hence, $I_1 = \dfrac{V}{R_1}$, $I_2 = \dfrac{V}{R_2}$, $I_3 = \dfrac{V}{R_3}$

Put these values in Eq (5.37),

$$I = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad \ldots(5.38)$$

If equivalant resistance of combination is R_{eq}

thus, $I = \dfrac{V}{R_{eq}}$

Compare this equation with Eq (5.38)

$$\frac{V}{R_{eq}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

or $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \ldots(5.39)$$

In this way, the reciprocal of equivalent resistance will be sum of reciprocal of individual resistances.

If n resistances are connected in parallel, then,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots.. + \frac{1}{R_n} \qquad \ldots(5.40)$$

If n identical resistances are connected in parallel, then,

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \ldots.. + \frac{1}{R} \ \ldots\ldots \ up\ to\ n$$

If all resistances connected in parallel are equal, then,

$$R_{eq} = \frac{R}{n} \qquad \ldots(5.41)$$

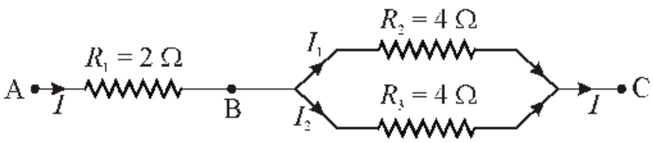
Important points for parallel combination of resistances :

(a) Potential difference across each resistance will be same.

(b) If R_1, R_2 and R_3 are values of resistances connected in series and if $R_1 > R_2 > R_3 > \ldots.. > R_n$. The current flowing in these resistances will be I_1, I_2 and I_3, where $I_1 < I_2 < I_3 < \ldots.. < I_n$

(c) In parallel combination, equivalent resistance will be lesser than the least.

Example 5.10 : In the given electric circuit. Calculate the equivalent resistance between terminals A and C.



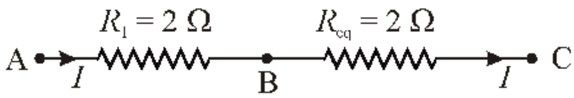
Solution : In the given figure, R_2 and R_3 are in parallel across B and C, therefore this parallel combination can be replaced by,

$$\frac{1}{R_{eq_{23}}} = \frac{1}{R_2} + \frac{1}{R_3}$$

$$R_{eq_{23}} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{eq_{23}} = \frac{4 \times 4}{4 + 4} = \frac{16}{8} = 2\Omega$$

Now, the simplified circuit can be redrawn as,

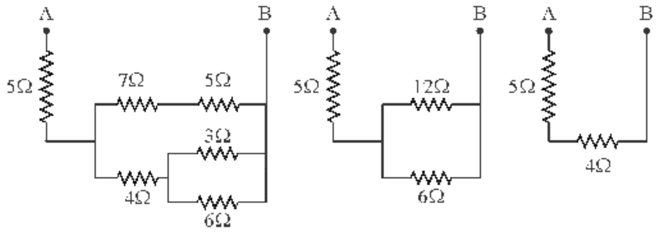


In this simplified circuit, R_1 and Req_{23} are in series.

$$R_{eq_{123}} = R_1 + R_{eq_{23}} = 2\Omega + 2\Omega$$

$$R_{eq_{23}} = 4\Omega$$

Example 5.11: Calculate the equivalent resistance of the given circuit between terminals A and B.



Solution : In such type of problems, it is useful to start with the smallest identifiable series or parallel sub-combination of the given circuit.

Step I: The equivalent resistance of 3 Ω and 6 Ω resisters connected in parallel is,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or $$R_{eq} = \frac{3 \times 6}{3+6} = 2\Omega$$

Step II : Thus, these two resisters are equivalent to 2 Ω. When this 2 Ω resister is connected in series to 4 Ω resistance we obtain 6 Ω resistance.

Step III : This 6Ω is now connected in parallel to 12Ω (a series combination of 7Ω and 5Ω). Combining there resistances in parallel we get,

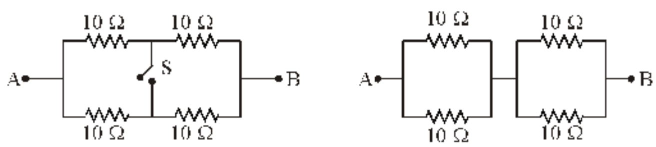
$$R_{eq}^{1} = \frac{6 \times 12}{6+12} = 4\Omega$$

Step IV : Now, this 4Ω is connected in series to 5Ω and the total resistance 9Ω is obtained.

Example 5.12 : Find the equivalent resistance of the electric circuit between terminals A and B in the following two cases.

(a) when switch S is open

(b) when switch S is closed



Solution :

(a) When switch S is open, then the resistances in the upper branch of the circuit will be in series combination for which the equivalent resistance will be 20Ω. Similarly the resistances in the lower branch of the circuit will also be in series combination for which the equivalent resistance will also be 20Ω.

These two resistances (20Ω each) are connected in parallel. Hence the equivalent resistance will be,

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{eq} = \frac{20 \times 20}{20+20} = 10\Omega$$

(b) When switch S is closed, then the resistances in the two branches can be simplified as shown in the figure. The left circuit will be the parallel combination for which the equivalent resistance will be 5Ω. Similarly the resistances in the right circuit will also be in parallel combination for which equivalent resistance will be 5Ω.

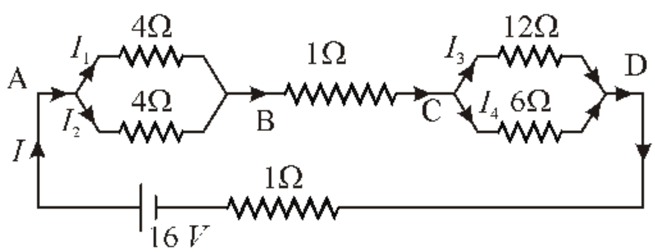
Now left and right circuit are connected in series combination. And equivalent series resistance will be 5Ω+5Ω= 10Ω.

Example 5.13 : A battery of e.m.f. 16 V and internal resistance 1Ω is connected to the network of resistances shown in the figure. Calculate,

(a) Equivalent resistance of network between points A and D.

(b) Current in each resistance.

(c) Potential drop V_{AB}, V_{BC} and V_{CD}.



Solution : Case (a) :

Step I : Two 4Ω resistances are connected in parallel between the terminals A and B. Hence the equivalent resistance R_{AB} will be,

$$R_{AB} = \frac{4 \times 4}{4 + 4} = 2\Omega$$

Step II : Given $R_{BC} = 1\Omega$.

Step III : Two resistances 12Ω and 6Ω are connected between the terminals C and D. Hence the equivalent resistance R_{CD} will be,

$$R_{CD} = \frac{12 \times 6}{12 + 6} = 4\Omega$$

Step IV : Now the resistances R_{AB}, R_{BC} and R_{CD} are all connected in series between the points A and D. Therefore,

$$R = R_{AB} + R_{BC} + R_{CD}$$

$$= 2\Omega + 1\Omega + 4\Omega$$

$$= 7\Omega$$

Case (b): Total current in the circuit,

$$I = \frac{Total\ e.m.f.}{total\ resistance}$$

$$= \frac{\varepsilon}{R + r} = \frac{16V}{(7+1)\Omega} = 2A$$

Step I : Potential difference between the points A and B will be $V_{AB} = I_1 R_1 - I_2 R_2$

$$4 \times I_1 = 4 \times I_2$$

Hence, $I_1 = I_2$ But $I_1 + I_2 = 2A$,

or $2I_1 = 2A$

$I_1 = 1A$ $I = 2A$

Step II : Current in the wire between B and C will be $2A$.

Step III : Totalcurrent in the wires between C and D will be $2A$.

i.e. $I_3 + I_4 = 2A$ ...(1)

Now, $V_{CD} = I_3 R_3 = I_4 R_4$

$I_3 \times 12 = I_4 \times 6$

or $I_4 = 2I_3$ ...(2)

On Solving equation (1) and (2), we get,

$$I_3 = \left(\frac{2}{3}\right)A \quad \text{and} \quad I_4 = \left(\frac{4}{3}\right)A$$

Case (c): Potential difference between A & B=

$V_{AB} = I \times R_{AB} = 2 \times 2 = 4\ V$

Similarly,

$V_{BC} = I \times R_{BC} = 2 \times 1 = 2V$

$V_{CD} = I \times R_{CD} = 2 \times 4 = 8V$

$V_r = I \times r = 2 \times 1 = 2V$

Total e.m.f. in the circuit E,

$E - V_{AB} - V_{BC} - V_{CD} - V_r$

$E - 4 - 2 + 8 - 2 - 16\ V$

5.10 : Cell, Electro Motive Force, Terminal Voltage and Internal Resistance

Electric Cell is a device which maintains the steady electric current in an electric circuit or it is a simple device which converts chemical energy into electrical energy.

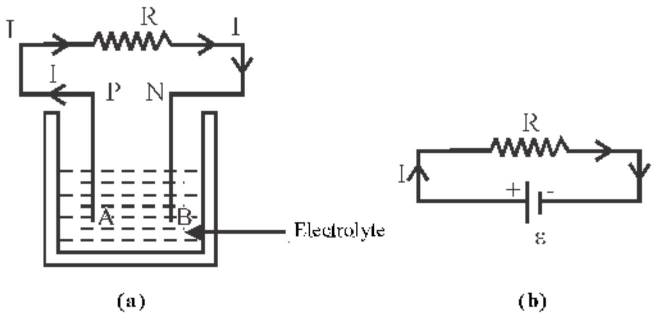


Fig 5.16 (a) & (b) Electric cell

Basically, an electric cell has two electrodes as shown in the fig (5.16), which are immersed in an electrolyte. These electrodes are shown as +*ve* electrode (P) and -*ve* electrode (N). These electrodes, immersed in electrolyte exchange charges with the electrolyte. Due to this reason, +ve electrode P develops a +ve potential V_+ ($V_+ > 0$) with respect to its adjacent electrolyte marked A. Similarly, the -*ve* electrode N develops a -*ve* potential V_- ($V_- < 0$) with respect to its adjacent electrolyte marked B. When no current flows through the cell, the electrolyte has the same potential difference throughout, so that the potential difference between the two electrodes P and N is,

$$V_+ - (-V_-) = V_+ + V_-$$

When cell is in open circuit i.e. no current is drawn from the cell, then the potential difference across its electrodes is known as Electro Motive Force (EMF) and is represented as ε .

It is to be remembered that ε is actually a potential difference and not a force. The term e.m.f. is however used due to historical reasons and this name was given at a time when this phenomenon was not understood properly. When current is flowing through external resistance R from the cell, then electric current will flow in the electrolyte from -ve electrode to +ve electrode.

The electrolyte through which a current flows has a finite resistance r, called internal resistance of the cell, i.e. internal resistance is the hinderance produced by the electrolyte of the cell in the flow of electric current.

Internal resistance of an ideal cell is zero, but practically all cells have a finite internal resistance.

If current is flowing in an external resistance due to a cell, i.e. cell is in closed circuit, then potential difference across two electrodes will be known as terminal voltage. This terminal voltage is represented by V and $V < \varepsilon$. In a closed circuit, the value of $(\varepsilon - V)$ will be equal to voltage drop across the internal resistance of the cell.

$$\varepsilon - V = Ir \qquad \ldots (5.42)$$

here, I is the electric current from the cell.

Hence, terminal voltage,

$$V = \varepsilon - Ir \qquad \ldots (5.43)$$

But, from Ohm's law,

$$V = IR \qquad \ldots (5.44)$$

Combining Eq 5.43 and Eq 5.44, we get

$$I(R + r) = \varepsilon$$

$$I = \frac{\varepsilon}{R + r} \qquad \ldots (5.45)$$

$$r = \frac{\varepsilon - V}{I} = \frac{\varepsilon - V}{V / R}$$

$$r = \left(\frac{\varepsilon - V}{V}\right) R \qquad \ldots (5.46)$$

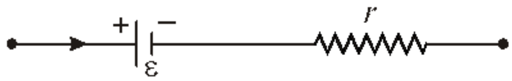


Fig 5.17 Charging of a cell

Note : If a cell is being charged as shown in the figure 5.17, then, the current enters from the +ve electrode. Therefore, the terminal voltage in this case will be,

$$V = \varepsilon + Ir$$

$$V > \varepsilon$$

5.11 Combination of Cells :

In general, cells can be connected in two manners:

(a) Series combination

(b) Parallel combination

5.11.1 Series Combination of Cells :

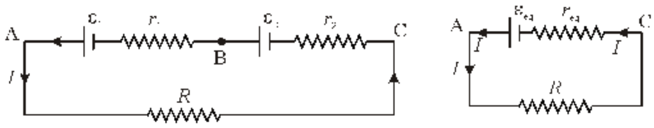


Fig 5.18 Series combination of cells

An arrangement of two or more than two cells, in which cells are connected such that the negative terminal of one cell is connected to the positive terminal of the next cell is called series combination. The end points of the combination are of opposite polarities across which an external resistance is connected.

Suppose two cells having emf ε_1 & ε_2 and internal resistance r_1 & r_2 are connected in series as shown in the figure 5.18. End points of the combination are connected to external resistance R. We want to find the equivalent emf and equivalent internal resistance and current in the circuit.

According to ohm's law, the potential difference across the resistance R.

$$V = IR = V_A - V_C$$

Potential difference between the end points A and B will be,

$$V_{AC} = (V_A - V_B) + (V_B - V_C)$$

$$= (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$$

$$= (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2) \quad \ldots (5.47)$$

If ε_{eq} and r_{eq} are the equivalent emf and equivalent internal resistance of the combination then,

$$V_{AC} = \varepsilon_{eq} - I\, r_{eq} \quad \ldots (5.48)$$

By comparing Eq 5.47 & 5.48

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 \quad \ldots (5.49)$$

$$r_{eq} = r_1 + r_2 \quad \ldots (5.50)$$

The value of terminal voltage =

potential drop across the external resistance R

Therefore, using equation 5.44 and the Ohm's law,

$$V_A - V_C = IR = \varepsilon_1 - Ir_1 + \varepsilon_2 - Ir_2$$

or $$I(R + r_1 + r_2) = \varepsilon_1 + \varepsilon_2$$

or $$I = \frac{\varepsilon_1 + \varepsilon_2}{R + r_1 + r_2} = \frac{\varepsilon_{eq}}{R + r_{eq}} \quad \ldots (5.51)$$

From the above expression it is clear that in series combination,

(i) Net emf of cells will be the sum of the emf of cells.

(ii) If n cells of equal emf and internal resistance are connected in series, then

$$\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \ldots\ldots \varepsilon_n = \varepsilon$$

and $$r_1 = r_2 = r_3 = \ldots\ldots = r_n = r \text{ है तो}$$

$$I = \frac{n\varepsilon}{R + nr} = \frac{\varepsilon}{r + R/n} \quad \ldots (5.52)$$

$$\varepsilon_{eq} = n\varepsilon \quad \ldots (5.53)$$

$$r_{eq} = nr \quad \ldots (5.54)$$

(iii) If the polarity of either of the cells in the combination is reversed, the value of equivalent emf will be $\varepsilon_1 - \varepsilon_2$ or $\varepsilon_2 - \varepsilon_1$,

But $r_{eq} = r_1 + r_2 + r_3 + \ldots r_n$. will remain same.

5.11.2 Parallel Combination of Cells :

An arrangement of two or more cells are connected in such a way that all the terminals of same polarity are joined together is known as the parallel combination of cells.

In figure 5.19 a parallel combination of two cells is shown. Let the emf of two cells are ε_1 and ε_2 and their respective internal resistances r_1 and r_2. External resistance R is connected to the end terminal A and B as shown in the figure.

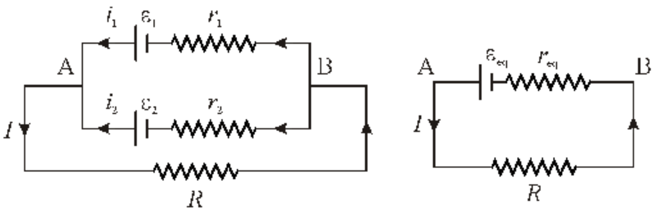


Fig 5.19: Parallel combination of cells.

If I_1 and I_2 are the current through each cell and I is the total current in the external circuit.

$$I = I_1 + I_2 \quad \ldots (5.55)$$

If terminal voltage between the terminals A and B for two cells is V_1 and V_2 each equal to V, then

$$V = \varepsilon_1 - I_1 r_1 \quad \ldots (5.56)$$

$$V = \varepsilon_2 - I_2 r_2 \quad \ldots (5.57)$$

Solving above equations,

$$I_1 = \frac{\varepsilon_1 - V}{r_1}$$

and $$I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Putting these values of I_1 and I_2 in eq. (5.55)

$$I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$I = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$I = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2}\right) - V\left(\frac{r_1 + r_2}{r_1 r_2}\right) \quad \ldots (5.58)$$

On solving

$$V = \left(\frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \quad \ldots (5.59)$$

If ε_{eq} and r_{eq} are the equivalent emf and internal resistance respectively of the cell combination, then terminal voltage will be,

$$V = \varepsilon_{eq} - I\, r_{eq} \quad \ldots (5.60)$$

After comparing equation (5.59) and (5.60),

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad \ldots (5.61)$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \ldots (5.62)$$

And the value of current through the external resistance is,

$$I = \frac{\varepsilon_{eq}}{R + r_{eq}} \quad \ldots (5.63)$$

In the above equation, the values of ε_{eq} and r_{eq} are given by eqs (5.61) and (5.62).

It is clear from the parallel combination of cells, that,

(i) If $\varepsilon_1 = \varepsilon_2 = \varepsilon$ and $r_1 = r_2 = r$. i.e. two cells of same emf and same internal resistance then $\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{2}$.

(ii) n cells of same emf and same internal resistance are connected in parallel then, $\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{n}$ and current in the external resistance will be,

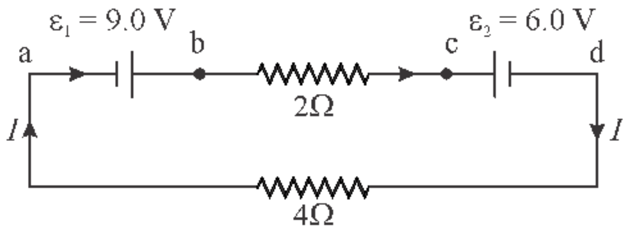
$$I = \frac{\varepsilon}{R + r/n} \quad \ldots (5.64)$$

(iii) Equations (5.61) and (5.62) can also be written as follows,

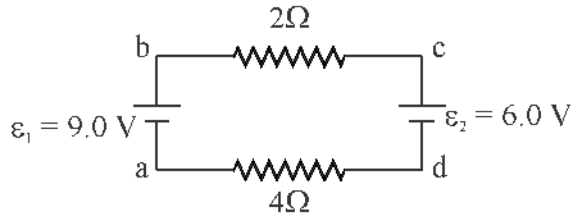
$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad \ldots (5.65)$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \quad \ldots (5.66)$$

Example 5.14 : The circuit shown in the figure consists of two ideal batteries connected in series through two resistances. Find the value of current in the circuit (a).



(a)



(b)

Solution : The simplified equivalent circuit of the given circuit (a) is redrawn in figure (b). Let I be the current flowing through the circuit, then the terminal voltage across the ends a and d will be,

$$\varepsilon_{aq} = \varepsilon_1 - \varepsilon_2 \qquad (\because \varepsilon_1 < \varepsilon_2)$$

$$\varepsilon_{eq} = 9.0 - 6.0 = 3.0V$$

Effective resistance of the circuit,

$$R = 2\Omega + 4\Omega = 6\Omega$$

Hence Electric current in the circuit,

$$I = \frac{\varepsilon_{eq}}{R} = \frac{3.0V}{6\Omega} = 0.5A$$

5.12 Electric Energy

Total work done (or the energy supplied) by the source of emf (i.e. cell) in maintaining an electric current in a circuit for a given time is called electric energy consumed in the circuit. If I current is flowing in a resistance R for the time t, then charge flowing in time t will be

$q = I \times t$

If V potential difference is applied across the ends of a wire then, by the definition of potential difference, work done by electric source in taking a charge q from one end to the other end of wire will be,

$$W = qV = VIt \qquad \ldots (5.67)$$

But according to Ohm's law, $V = IR$

hence, $$W = I^2Rt \qquad \ldots (5.68)$$

or $$W = \frac{V^2}{R}t \qquad \ldots (5.69)$$

SI unit of electric energy is joule.

Here, 1 joule = 1 watt × second

Commercial unit of electric energy is kilo watt hour (kWh). This unit is also called Board of Trade Unit (B.O.T.U.)

Therefore,

1 electric Unit = 1kWh

=1kW×1h

=1000W×3600 second

= 1 kWh = 3.6 × 10^6 watt second or joule

Important Note :

Using joule's law we can find equivalence between work and heat.

W = JH

$$H = \frac{W}{J} = \frac{VIt}{J} = \frac{I^2Rt}{J} = \frac{V^2}{R}\frac{t}{J}$$

Here, H= heat generated and J= mechanical equivalent of heat.

The value of $J = 4.2 \frac{\text{J}}{\text{cal}}$

5.13 Electric Power

In any electric circuit, work done per second by electric source to the flow of electric current or loss of energy per second is known as power of electric circuit.

It is generally represented by the symbol P.

In electric circuit, loss of energy in time t for current flow i.e. work done is W.

Then power,

$$P = \frac{W}{t} \qquad \ldots (5.70)$$

We have studied in the earlier section that,

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

Therefore, $$P = VI = I^2R = \frac{V^2}{R} \qquad \ldots (5.71)$$

SI unit of electric power is watt.

Here, 1 watt = $\frac{\text{1 joule}}{\text{second}}$

In practice electric power is measured in kilo watt or megawatt.

1 kilowatt = 10^3 watt

1 megawatt = 10^6 watt

There is another unit used to measure electric power known as Horse Power (HP).

1 HP = 746 watt

The equations $P = VI$ and $P = \frac{V^2}{R}$ have important role in electric power loss in electric power transmission. The electric power is transmitted from power station to homes / factories through the cable even thousands of miles away from the power station. We want such an arrangement in which the transmission power loss would be minimum. If P power is transmitted to a device of V volt through a cable of resistance R_C, then, power loss in connecting wire will be,

$$P_C = \frac{P^2R_C}{V^2}$$

This power loss is inversely proportional to V^2. Due to this reason, to reduce electric power loss in electric power circuit, current is passed at high voltage. Due to this high voltage danger, the transmission power lines are away from residential areas. Near homes, this high voltage is again converted to low voltage by means of electric transformers.

Example 5.15 : A Bulb of 220 V and 110 Watt is connected with a source of 110 Volt, calculate the value of the power consumed by the bulb.

Solution :

According to the question,

V = 220 V, P = 100 W

So, the resistance of the bulb,

$$R = \frac{V^2}{P} = \frac{220 \times 220}{100}$$

or $R = 484\Omega$

New voltage of the source, $V' = 110V$

Therefore, the power consumed by the bulb, when bulb is connected to new voltage of the source, $V' = 110V$,

$$P^1 = \frac{(V^1)^2}{R} = \frac{(110)^2}{484} = 25W$$

Important Points

1. The flow of charge per unit time is known as electric current.
2. When an electric field is applied across a conductor, then the average velocity by which free electrons of that conductor move is called its drift velocity.

   $$\text{Drift velocity} = \vec{v}_d = \frac{-e\vec{E}}{m}\tau$$

3. Mobility is numerically equal to drift velocity per unit electric field. Hence mobility (μ) is given by :

   $$\mu = \frac{|v_d|}{E}$$

   The S.I.unit of mobility is $\frac{m^2}{V\,s}$

4. The relation between electric current and drift velocity is $I = neAv_d$. The relation between the current density $J = nev_d$, drift velocity and potential difference is $v_d = \frac{eV}{ml}\tau$
5. According to ohm's law, when physical conditions of the conductor remains same, the electric current flowing through the conductor is proportional to the applied potential difference across its ends. i.e. $V \propto I$ or $V = RI$, where R is the resistance of the conductor. The unit of R is ohm $\Omega = VA^{-1}$.
6. The resistance of conductor is $R = \frac{m\ell}{ne^2A\tau}$. On increasing the temperature, relaxation time decreases, hence R will increase.
7. The resistance $R \propto \ell$ and $R \propto \frac{1}{A}$, hence $R = \rho\frac{\ell}{A}$, where ρ is the resistivity or specific resistance of the conductor. The resistivity ρ of the conductor depends on the material of the conductor and temperature,

whereas it does not depend on the length and area of cross section.

8. The relation between electric field E and resistivity, $E = \rho J$ and the relation between current density and conductivity, $J = \sigma E$ are the microscopic forms of Ohm's law.

9. Many substances and devices do not obey ohm's law, such devises are called non-ohmic devises. For these devises the graph between V and I is no longer a straight line.

10. Dependence of resistance on temperature is given as $R_t = R_0(1+\alpha t)$. On increasing the temperature of the conductor, its resistivity as well as resistance increases whereas for semiconductors and insulators on increasing the temperature their resistivity as well as resistance decreases.

11. The equivalent resistance in series combination is $R_{eq} = R_1 + R_2 + ... + R_n$ and the equivalent resistance in parallel combination is $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$

12. When the cell is in open circuit, then the potential difference across its electrodes is called its e.m.f.

13. When the cell is in closed circuit, i.e. the current is flowing through the cell and the external circuit then the potential difference across its electrodes is called the terminal voltage.

14. The relation between the terminal voltage (at the time of discharging of the cell) and e.m.f. will be $\varepsilon = V + Ir$.

15. The relation between the terminal voltage (at the time of charging of the cell) and e.m.f. will be V=E+ir. i.e. $V > E$.

Questions for Practice

Multiple Choice Questions

1. The product of resistivity and conductivity of a conductor depends on -

(a) Area of cross section

(b) Temperature

(c) Length

(d) None of the above

2. Two similar wires of same size of resistivity ρ_1 and ρ_2 are connected in series. Equivalent resistivity of the combination will be -

(a) $\sqrt{\rho_1\rho_2}$ (b) $2(\rho_1 + \rho_2)$

(c) $\frac{\rho_1 + \rho_2}{2}$ (d) $\rho_1 + \rho_2$

3. A conducting resistance is connected to a battery. The temperature of the conductor decreases due to cooling. The current flowing through the resistance will -

(a) increase (b) decrease

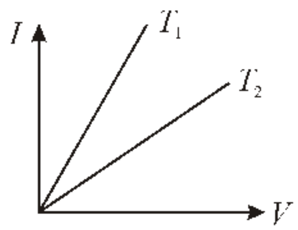
(c) remain constant (d) become zero

4. A cell of emf 2.1 volt gives a current of 0.2 A. This current passes through a 10 Ω resistance. Internal resistance of the cell will be.

(a) 0.2 Ω (b) 0.5 Ω

(c) 0.8 Ω (d) 1.0 Ω

5. The voltage current graph of a conductor at two different temperature are shown in the Figure. If the resistances corresponding to these temperatures are R_1 and R_2, then which of the following statement s is true.



(a) $T_1 = T_2$ (b) $T_1 > T_2$)

(c) $T_1 < T_2$ (d) None of the above

6. Electric power is transmitted from on city to another city through copper wire, situated 150 km apart. The voltage drop per km is 8 V and the resistance per km is 0.5 Ω, then the power loss in the transmission line will be-

(a) 19.2 W (b) 19.2 kW

(c) 19.2 J (d) 12.2 kW

7. There are 5 resistances each of R Ω. First three resistances are connected in parallel, after that remaining two are connected in series, then the equivalent resistance of the combination will be -

(a) $\frac{3}{7}R\Omega$ (b) $\frac{7}{3}R\Omega$

(c) $\frac{7}{8}R\Omega$ (d) $\frac{8}{7}R\Omega$

8. From which of the following relations between the drift velocity v_d and electric field E, obeys ohm's law.

(a) $v_d \alpha E^2$ (b) $v_d \alpha E$

(c) $v_d \alpha E^{1/2}$ (d) $v_d =$ constant

9. In a carbon resistance there are 4 rings in order blue, yellow red and silver. The resistance of the resister will be -

(a) $64\times10^2\Omega$

(b) $(64\times10^2 \pm 10\%)\Omega$

(c) $642\times10^4\ \Omega$

(d) $(26\times10^3 \pm 5\%)\Omega$

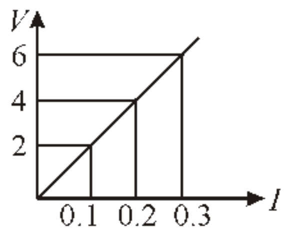
10. When a wire connected to a battery gets heated, then which of the following quantity will not change -

(a) Drift velocity (b) Resistivity

(c) Resistance (d) No. of free electrons

Very Short Answer Questions

1. From the following graph between V & I, calculate the resistance of the resister.



2. Write down the S.I. unit of current density.

3. Write down the relation between conductivity and current density of a conductor.

4. Give two examples of non-ohmic devices.

5. Give the dependence of resistivity on temperature of a conductor.

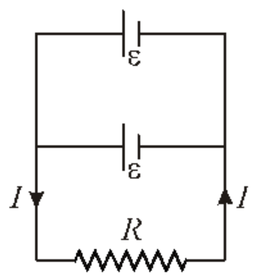
6. Write the names of two materials whose resistivity decreases on increasing the temperature.

7. Write down the value of current flowing through a bulb of 40 W 220 V.

Short Answer Questions

1. How much charge will be there when a current is flowing through a conductor.

2. In the given figure, the resistivity of some conductor are ρ_1 and $\rho_2 \Omega\times m$. What is the ratio of ρ_1, ρ_2



3. In the given figure, there are two similar cells whose emfs are same and internal resistances are negligible. These cells are connected in parallel. What is the value of current flowing through the resistance R ?

4. Explain the difference between terminal voltage and emf of a cell.

5. Define drift velocity.

6. A resistance wire of 8R is bent in the form of a circle, then what is its equivalent resistance across the ends of diameter.

7. When the shape of a conductor is deformed then what is the effect on its resistance and resistivity?

8. Can the terminal voltage of a cell be greater than the emf of a cell?

Essay Type Questions

1. Define drift velocity. On the basis of drift velocity derive Ohm's law $\vec{j} = \sigma\vec{E}$?.

2. Derive the relation between the drift velocity and electric field. What is mobility? Explain dependence of drift velocity and mobility.

3. Derive relation between resistance and resistivity of any conductor. Explain temperature dependence on the resistance of a material. Explain in reference to a conductor, insulator and semi-conductor.

4. There are two cells of emf ε_1 and v and internal resistance r_1 and r_2 respectively connected in parallel. Then find out the equivalent emf and equivalent internal resistance of this combination. If external resistance R in connected to this combination, then find out the value of electric current flowing through R.

Answer Key (Multiple Choice Questions)

1. (d) 2. (c) 3. (a) 4. (b)

5. (c) 6. (b) 7. (b) 8. (b) 9. (b) 10. (d)

Answer Key (Very Short Answer Questions)

1. 20 Ω

2. A/m^2

3. $\vec{j} = \sigma\vec{E}$

4. diode, electrolytes

5. $\rho = \rho_0(1+\alpha\Delta t)$

6. Germanium and Silicon

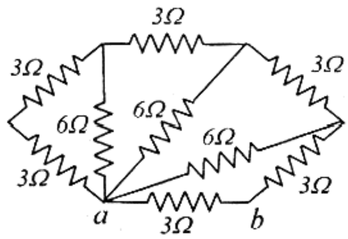
7. 0.18 A

Numerical Questions

1. A potential of 120 V is applied across the ends of a cylindrical copper rod of length 1 cm and radius 2.0 mm. Find the value of the current through the rod. (The resistivity of copper is $1.7\times10^{-8}\,\Omega m$.)

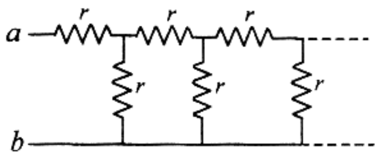
Ans. $\left[6.85\times10^{5}\text{ A}\right]$

2. Find the equivalent resistance between a and b in the given circuit diagram.



[Ans. 2 Ω]

3. Find the equivalent resistance between points of an infinite ladder network circuit as shown in the figure.



Ans. $\left[\left(\frac{1+\sqrt{5}}{2}\right)r\right]$

4. (a) Three resisters 1Ω, 2Ω and 3Ω are connected in series. What is the total resistance of the combination?

(b) If this series combination is connected to a battery of e.m.f. 12 V and negligible internal resistance, obtain the potential drop across each resistance.

[Ans. 6 Ω, 2V, 4 V and 6V]

5. At room temperature (27°C), the resistance of a heating element is $100\,\Omega$. What is the temperature of the element if the resistance is found to be $117\,\Omega$, given that the temperature coefficient of the material of resistance is $1.70\times10^{-4}\,{}^{\circ}C^{-1}$.

[Ans. 1027°C]

6. A negligibly small current passes through a wire of length 15 m and uniform cross section $6.0\times10^{\ 7}\,m^2$ and its resistance is measured to be $5.0\,\Omega$. What is the resistivity of the material at the temperature of the experiment?

[Ans. $2.0\times10^{\ 7}\,\Omega m$]

7. A Copper wire of cross section area 1.0 mm^2 is carrying a current of 0.5 A. If the density of free electrons is $8.5\times10^{22}\,cm^{-3}$. Calculate the drift velocity of free electrons.

[Ans. 3.7×10^{-5}m/s]

8. Find the temperature at which the resistance of a material is doubled that of the resistance at (0°C). The temperature coefficient of the material of resistance is $4.0\times10^{-3}\,{}^{\circ}C^{-1}$

[Ans. 250°C]

9. The storage battery of a car has an e.m.f. of 12 V. If the internal resistance of the battery is $0.4\,\Omega$. What is the maximum current that can be drawn from the battery?

[Ans. 30 A]

10. A coil of resistance $4.2\,\Omega$ is immersed in water. A current of 2A passes through it for a duration of 10 minutes. How many calories of heat will be produced in the coil? [J = 4.2 J / cal]

[Ans. 2400 cal]

11. A cylindrical tube of length l has inner and outer radii a and b. The resistivity of the material is ρ, then calculate the resistance between the two end of the cylindrical tube.

Ans. $\left[\dfrac{\rho\ell}{\pi(a^2-b^2)}\right]$

12. In a house 4 bulbs of 100 W and 4 bulbs of 40 W glow every day for 4 hours and 6 hours respectively. Two fans of 60 W are also used 8 hours every day. Calculate the electrical energy consumed in a month of 30 days. Also calculate the electricity bill for the month at the rate of Rs. 5 per unit.

[(Ans. Electricy consumed: 105.6 units, Bill amount Rs. 528)]