Chapter 3

Matrices

Miscellaneous Exercise

Q. 1

Let, show that $(aI + bA)^n = a^nI + nan^{-1}bA$, where I is the identity matrix of order 2 and $n \epsilon N$.

Answer:

To prove: $(aI + bA)^n = a^nI + na^{n-1}bA$

Proof: Given $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

We will be proving the above equation using mathematical induction.

Steps involved in mathematical induction $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$are-

1. Prove the equation for n = 1
2. Assume the equation to be true for n=k, where $k \epsilon N$
3. Finally prove the equation for n = k+1

I is the identity matrix of order 2,

i.e. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Let P(n): $(aI + bA)^n = a^nI + na^{n-1}bA, n \epsilon N$

For n=1,

L.H.S: $(aI + bA)^1 = aI + bA$

R.H.S: $a^1I + 1a^{1-1}bA= aI + a^0bA= aI + bA$

So, L.H.S = R.H.S

$\therefore$ P(n) is true for n=1

Now assuming P(n) to be true for n=k, where k ϵ N

P(k): $(aI + bA)^{k} = a^{k}I + ka^{k-1}bA$ …… (1)

Now proving for n=k+1, i.e. P(k+1) is also true

L.H.S = $(aI + bA)^{k+1}$

$= (aI + bA)^{k}. (aI + bA)^{1}$

$= (a^{k}I + ka^{k-1}bA). (aI + bA)$ ……from (1)

$= aI\ (a^{k}I + ka^{k-1}bA) + bA\ (a^{k}I + ka^{k-1}bA)$

$= aI(a^{k}I) + aI(ka^{k-1}bA) + bA(a^{k}I) + bA(ka^{k-1}bA)$

$= (a.\ a^{k})\ (I\times I) + kb\ (a.\ a^{k-1})\ (IA) + (ba^{k})(AI) + (bb)\ ka^{k-1}(AA)$

$= ak^{+1}\ I^{2} + ka^{1+k-1}\ bA + ba^{k}A + b2^{k}a^{k-1}A^{2}$ ($IA = AI= A$ & $I^{2} = I$)

$= a^{k}+1\ I + ka^{k}\ bA + ba^{k}A + b2^{k}a^{k-1}A^{2}$

Calculating A^{2}

$A^{2} = A.A$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0.0 + 1.0 & 0.1 + 0.1 \\ 0.0 + 0.0 & 0.1 + 0.0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore A^{2} = O$ (O is the null matrix)

Putting value of A^{2} in L.H.S

L.H.S = $a^{k+1}\ I + ka^{k}\ bA + ba^{k}A + b2^{k}a^{k-1}(O)$

$= a^{k+1}\ I + ka^{k}\ bA + ba^{k}A + 0$

$= a^{k+1}\ I + ka^{k}\ bA + ba^{k}A$

$= a^{k+1} I + (k+1) a^k bA$

Putting n=k+1 in R.H.S

R.H.S $= a^{k+1} I + (k+1) a^k bA$

∴ L.H.S = R.H.S

All conditions are proved. Hence P(k+1) is true.

∴ By mathematical induction we have proved that P(n) is true for all n ϵ N.

Thus, $(aI + bA)^n = a^nI + na^{n-1}bA$, where I is the identity matrix of order 2 and n ϵ N.

Q. 2

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$, n ∈ N.

Answer:

We will be proving the above equation by putting different values of n (i.e. n = 1, 2, 3 …. n)

For n=1,

$$A^1 = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

For n=2,

$A^2 = A.A$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.1+1.1+1.1 & 1.1+1.1+1.1 & 1.1+1.1+1.1 \\ 1.1+1.1+1.1 & 1.1+1.1+1.1 & 1.1+1.1+1.1 \\ 1.1+1.1+1.1 & 1.1+1.1+1.1 & 1.1+1.1+1.1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 3^{2-1} & 3^{2-1} & 3^{2-1} \\ 3^{2-1} & 3^{2-1} & 3^{2-1} \\ 3^{2-1} & 3^{2-1} & 3^{2-1} \end{bmatrix}$$

For n=3,

$A^3 = A^2. A$

$$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3.1+3.1+3.1 & 3.1+3.1+3.1 & 3.1+3.1+3.1 \\ 3.1+3.1+3.1 & 3.1+3.1+3.1 & 3.1+3.1+3.1 \\ 3.1+3.1+3.1 & 3.1+3.1+3.1 & 3.1+3.1+3.1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \end{bmatrix} = \begin{bmatrix} 3^{3-1} & 3^{3-1} & 3^{3-1} \\ 3^{3-1} & 3^{3-1} & 3^{3-1} \\ 3^{3-1} & 3^{3-1} & 3^{3-1} \end{bmatrix}$$

r n=4,

$A^4 = A^3. A$

$$= \begin{bmatrix} 9 & 9 & 9 \\ 9 & 9 & 9 \\ 9 & 9 & 9 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9.1+9.1+9.1 & 9.1+9.1+9.1 & 9.1+9.1+9.1 \\ 9.1+9.1+9.1 & 9.1+9.1+9.1 & 9.1+9.1+9.1 \\ 9.1+9.1+9.1 & 9.1+9.1+9.1 & 9.1+9.1+9.1 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 27 & 27 \\ 27 & 27 & 27 \\ 27 & 27 & 27 \end{bmatrix} = \begin{bmatrix} 3^{4-1} & 3^{4-1} & 3^{4-1} \\ 3^{4-1} & 3^{4-1} & 3^{4-1} \\ 3^{4-1} & 3^{4-1} & 3^{4-1} \end{bmatrix}$$

d so on for other values of n.

If we notice each result, then we will see that it is of same type that we are trying to prove.

So we can generalize the above results for all n ϵ N

$$\therefore A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix} \text{ (n = 1, 2, 3, … n)}$$

Hence Proved

Q. 3

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that An = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer.

Answer:

We will be proving the above equation by putting different values of n. Because n is a positive integer so it will take values which are greater than 0 i.e. n = 1, 2, 3 …n

For n=1,

L.H.S = $A^n = A^1 = A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

R.H.S = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

$$= \begin{bmatrix} 1+2.1 & -4.1 \\ 1 & 1-2.1 \end{bmatrix}$$

$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

∴ L.H.S = R.H.S

For n=2,

L.H.S = A^2

= A.A

$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 3.3 + (-4).1 & 3.(-4) + (-4).(-1) \\ 1.3 + (-1).1 & 1.(-4) + (-1).(-1) \end{bmatrix}$

$= \begin{bmatrix} 9 - 4 & -12 + 4 \\ 3 - 1 & -4 + 1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

R.H.S. $= \begin{bmatrix} 1 + 2n & -4n \\ n & 1 - 2n \end{bmatrix}$

$= \begin{bmatrix} 1 + 2.2 & -4.2 \\ 2 & 1 - 2.2 \end{bmatrix}$

$= \begin{bmatrix} 1 + 4 & -8 \\ 2 & 1 - 4 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

∴ L.H.S = R.H.S

For n = 3,

L.H.S = A^3

= A^2. A

$= \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$= \begin{bmatrix} 5.3 + (-8).1 & 5.(-4) + (-8).(-1) \\ 2.3 + (-3).1 & 2.(-4) + (-3).(-1) \end{bmatrix}$

$= \begin{bmatrix} 15 - 8 & -20 + 8 \\ 6 - 3 & -8 + 3 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

R.H.S. = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

= $\begin{bmatrix} 1+2.3 & -4.3 \\ 3 & 1-2.3 \end{bmatrix}$

= $\begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

∴ L.H.S = R.H.S

For n = 4,

L.H.S = A^4

= A^3. A

= $\begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

= $\begin{bmatrix} 7.3+(-12).1 & 7.(-4)+(-12).(-1) \\ 3.3+(-5).1 & 3.(-4)+(-5).(-1) \end{bmatrix}$

= $\begin{bmatrix} 21-12 & -28+12 \\ 9-5 & -12+5 \end{bmatrix}$ = $\begin{bmatrix} 9 & -16 \\ 4 & -7 \end{bmatrix}$

R.H.S = $\begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

= $\begin{bmatrix} 1+2.4 & -4.4 \\ 4 & 1-2.4 \end{bmatrix}$

= $\begin{bmatrix} 9 & 16 \\ 4 & -7 \end{bmatrix}$

∴ L.H.S = R.H.S

And so on for other values of n.

If we notice each result, then we will see that it is of same type that we are trying to prove.

So we can generalize the above results for all positive integer values of n.

$\therefore A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ (n = 1, 2, 3 … n)

Hence Proved

Q. 4

If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Answer:

To prove: AB – BA is a skew symmetric matrix.

Symmetric matrix: A symmetric matrix is a square matrix that is equal to its transpose. In simple words, matrix A is symmetric if

A = A’

where A’ is the transpose of matrix A.

Skew Symmetric matrix: A skew symmetric matrix is a square matrix that is equal to minus of its transpose. In simple words, matrix A is skew symmetric if

A = -A’

Given: A and B are symmetric matrices i.e.

A = A’ … (1)

B = B’ … (2)

Now calculating the transpose of AB – BA,

(AB – BA)’ = (AB)’ – (BA)’

(By property of transpose i.e. (A – B)’ = A’ – B’)

= B’A’ – A’B’

(By property of transpose i.e. (AB)’ = B’A’)

= BA – AB

= - (AB – BA)

Or we can say that: (AB – BA) = - (AB – BA)'

Clearly it satisfies the condition of skew symmetric matrix.

Hence AB – BA is a skew symmetric matrix.

Q. 5

Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Answer:

Case 1: When A is a symmetric matrix i.e.

A = A'……. (1)

where A' is the transpose of A

To prove: B'AB is also a symmetric matrix.

Calculating the transpose of B'AB

(B'AB)'= B'A'(B')' (By property of transpose i.e. (AB)' = B'A')

= B'A'B (By property of transpose i.e. (A')' = A)

= B'AB (from (1))

It satisfies the condition of symmetric matrix as matrix B'AB is equal to its transpose.

Hence B'AB is a symmetric matrix when A is symmetric.

Case 2: When A is a skew symmetric matrix i.e.

A = -A'……. (2)

where A' is the transpose of A.

To prove: B'AB is also a skew symmetric matrix.

Calculating the transpose of B'AB

(B'AB)'= B'A'(B')' (By property of transpose i.e. (AB)' = B'A')

= B'A'B (By property of transpose i.e. (A')' = A)

= B'(-A) B (from (2))

= - (B'AB)

It satisfies the condition of skew symmetric matrix as matrix (B'AB) is equal to its transpose.

Hence (B'AB) is a skew symmetric matrix when A is skew symmetric.

∴ Both results are proved.

Q. 6

Find the values of x, y, z if the matrix satisfy A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ the equation A'A =I.

Answer:

Given A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

Transpose of a matrix: If A be an m × n matrix, then the matrix obtained by interchanging the rows and columns of A is called the transpose of A. It is denoted by A' or AT.

∴ Transpose of A = A' = $\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

Given equation

A'A =I

$$= \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

=

$$\begin{bmatrix} 0.0 + 2y.2y + z.z & 0.x + 2y.y + z.(-z) & 0.x + 2y(-y) + z.z \\ x.0 + y.2y + (-z).z & x.x + y.y + (-z).(-z) & x.x + y.(-y) + (-z).z \\ x.0 + (-y).(2y) + z.z & x.x + (-y).y + z.(-z) & x.x + (-y).(-y) + z.z \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As these matrices are equal to each other that means each element of matrix on L.H.S is equal each element of matrix on R.H.S.

∴ On comparing elements on both sides we get

$4y^2 + z^2 = 1$ …… (1)

$2y^2 – z^2 = 0$ …… (2)

$x^2 + y^2 + z^2 = 1$ …… (3)

$– y^2 – z^2 = 0$ …… (4)

From equation (4) we get,

$x^2 = y^2 + z^2$ …… (5)

Substituting this value in equation (3) we get,

$2y^2 + 2z^2 = 1$ …… (6)

Subtracting equation (2) and (6) we get,

$3z^2 = 1$

$z^2 = 1/3$

$z = 1/3$

Substituting value of z in equation (2) we get,

$2y^2 = 1/3$

$y^2 = 1/6$

y = �1/6

Substituting values of y and z in equation (5) we get,

$x^2 = \frac{1}{6} + \frac{1}{6}$

$x^2 = 1/2$

$x = 1/2$

Hence values of x, y, z is 1/2, 1/6, 1/3 respectively.

Q. 7

For what values of x: $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$ = O?

Answer:

Multiplying matrices on the left hand side

L.H.S = $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$

$= \begin{bmatrix} 1.1 + 2.2 + 1.1 & 1.2 + 2.0 + 2.0 & 1.0 + 2.1 + 1.2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$

$= \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$

= [6.0 + 2.2 + 4.x]

= [4 + 4x]

R.H.S = O (O is the null matrix)

= 0

∴ L.H.S = R.H.S, so we get,

[4+4x] = O

4 + 4x = 0

4x = -4

x = -1

Hence value of x is equal to -1.

Q. 8

If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$.

Answer:

To prove: $A^2 - 5A + 7I = 0$

Given: A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

L.H.S: $A^2 - 5A + 7I$

R.H.S = 0

I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Calculating value of A^2:

$A^2 = A.A$

= $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3.3 + 1.(-10 & 3.1 + 1.2 \\ (-1).3 + 2.(-1) & (-1).+2.2 \end{bmatrix}$

$= \begin{bmatrix} 9 - 1 & 3 + 2 \\ -3 - 2 & -1 + 4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

Substituting value in L.H.S we get,

$= A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 - 15 + 7 & 5 - 5 + 0 \\ -5 - (-5) + 0 & 3 - 10 + 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

= 0 = R.H.S

L.H.S = R.H.S

Hence $A^2 - 5A + 7I = 0$ is proved.

Q. 10

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Products
I	10,000, 2,000, 18, 000
II	6,000, 20,000, 8,000

(a) If unit sale prices of x, y and z are Rs. 2.50, Rs. 1.50 and Rs. 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs. 2.00, Rs. 1.00 and 50 paise respectively. Find the gross profit.

Answer:

(a) Given the unit sale prices of x, y and z as Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively.

Unit sale prices can be represented in form of matrix as: $\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

Calculating total revenue in market I:

Number of products in the form of matrix: $\begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix}$

So the total revenue is given by:

$= \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

$= [10000.(2.50) + 2000.(1.50) + 18000.(1.00)]$

$= [25000 + 3000 + 18000]$

= [46000]

∴ Total revenue in market is Rs 46000.

Calculating total revenue in market II:

Number of products in the form of matrix: $\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix}$

So the total revenue is given by:

$= \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

= [6000. (2.50) + 20000. (1.50) + 8000. (1.00)]

= [15000 + 30000 + 8000]

= [53000]

∴ Total revenue in market is Rs. 53000.

(b) Given the unit cost prices of x, y and z as Rs. 2.00, Rs. 1.00 and 50 paisa respectively.

Calculating gross profit in market I:

Unit cost prices can be represented in form of matrix as: $\begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

So the total cost of products in market I is given by:

$= \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

$= [20000 + 2000 + 9000\ ]$

= [31000]

Since the total revenue in market I is Rs. 46000, the gross profit in this market is given by:

(Rs. 46000 – Rs. 31000)

= Rs. 15000.

Calculating gross profit in market II:

The total cost of products in market II is given by:

$= \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

= [6000. (2.00) + 20000. (1.00) + 8000. (0.50)]

= [12000 + 20000 + 4000]

= [36000]

Since the total revenue in market II is Rs. 53000, the gross profit in this market is given by:

(Rs. 53000 – Rs. 36000)

= Rs. 17000.

Q. 11

Find the matrix X so that X = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Answer:

Given X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

From above equation it can be observed that matrix on R.H.S is a 2×3 matrix and that on the L.H.S is also a 2×3 matrix. Therefore, X must be a 2×2 matrix.

Let X $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

So the equation is given by:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} a.1 + b.4 & a.2 + b.5 & a.3 + b.6 \\ c.1 + d.4 & c.2 + d.5 & c.3 + d.6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} a + 4b & 2a + 5b & 3a + 6b \\ c + 4d & 2c + 5d & 3c + 6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Now equating the corresponding elements of both the matrices we get,

a + 4b = -7, 2a + 5b = -8, 3a + 6b = -9

c + 4d = 2, 2c + 5d = 4, 3c + 6d = 6

Now, a + 4b = -7 h ⇒ a = -4b -7

∴ 2a + 5b = -8 ⇒ 2. (-4b -7) + 5b = -8

$\Rightarrow$ -8b -14 + 5b = -8

$\Rightarrow$ -3b = 6

$\Rightarrow$ b = -2

$\therefore$ a = -4b -7 $\Rightarrow$ a = -4. (-2) -7

$\Rightarrow$ a = 1

Now, c + 4d = 2 $\Rightarrow$ c = -4d + 2

$\therefore$ 2c + 5d = 4 $\Rightarrow$ 2. (-4d + 2) + 5d = 4

$\Rightarrow$ -8d + 4 + 5d = 4

$\Rightarrow$ -3d = 0

$\Rightarrow$ d = 0

$\therefore$ c = -4d + 2 $\Rightarrow$ c = -4.0 + 2

$\Rightarrow$ c = 2

Thus, a = 1, b = -2, c = 2, d = 0.

Hence X becomes $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Q. 12

If A and B are square matrices of the same order such that AB = BA, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n$ = AnBn for all n ϵ N.

Answer:

To prove: $AB^n = B^nA$

Given A and B are square matrices of same order such that AB = BA.

We have to prove it using mathematical induction.

Steps involved in mathematical induction are-

1. Prove the equation for n=1

2. Assume the equation to be true for n=k, where k ϵ N

3. Finally prove the equation for n=k+1

Let P(n): $AB^n = B^nA$

For n=1,

L.H.S: $AB^n = AB^1 = AB$

R.H.S: $B^nA = B^1A = BA = AB$

So, L.H.S = R.H.S

∴ P(n) is true for n=1.

Now assuming P(n) to be true for n=k, where k ϵ N

P(k): $AB^k = B^kA$ …… (1)

Now proving for n=k+1, i.e. P(k+1) is also true

L.H.S = ABn

$= AB^{k+1}$

$= (AB^k). B$

$= (B^kA). B$ …… from (1)

$= B^k (A.B)$

$= B^k(BA)$ (∵AB = BA)

$= B^{k+1A}$

R.H.S = BnA

$= B^{k+1}A$

∴ L.H.S = R.H.S

All conditions are proved. Hence P(k+1) is true.

∴ By mathematical induction we have proved that $AB^n = B^nA$.

Now, to prove: $(AB)^n = A^nB^n$ for all n ϵ N

For n=1,

L.H.S = $(AB)^n = (AB)^1 = AB$

R.H.S = $A^nB^n = A^1B^1 = AB$

∴ L.H.S = R.H.S

∴ It is true for n=1Assuming it to be true for n=k then,

$(AB)^k = A^kB^k$ …… (2)

Now proving for n=k+1,

L.H.S = (AB)n

$= (AB)^{k+1}$

$= (AB)^k(AB^{)1}$

$= (A^kB^k)AB$

$= A^k (B^k. A) B$

$= A^k (A. B^k) B$ $(AB^n = B^nA)$

$= (A^kA)(B^kB)$

$= A^{k+1}B^{k+1}$

R.H.S = AnBn

= Ak+1Bk+1

∴ L.H.S = R.H.S

All conditions are proved. Hence P(k+1) is true.

∴ By mathematical induction we have proved that $(AB)^n = A^nB^n$ for all n ϵ N

Hence proved.

Q. 13

If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

A. $1+\alpha^2 + \beta\gamma = 0$

B. $1 - \alpha^2 + \beta\gamma = 0$

C. $1 - \alpha^2 - \beta\gamma$

D. $1 + \alpha^2 - \beta\gamma = 0$

Answer:

Given A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Calculating A^2:

$A^2 = A.A$

$= \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$= \begin{bmatrix} \alpha.\alpha + \beta.Y & \alpha.\beta + \beta.(-\alpha) \\ \gamma.\alpha + (-\alpha).\gamma & \gamma.\beta + (-\alpha).(-\alpha) \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta.\gamma & \alpha.\beta - \alpha.\beta \\ \gamma.\alpha - \gamma.\alpha & \gamma.\beta + \alpha^2 \end{bmatrix}$

$= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix}$

And given that $A^2 = I$

Then $\begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Comparing corresponding elements, we get,

$= \alpha^2 + \beta\gamma =$

$= 1 - \alpha^2 - \beta\gamma = 0$

Q. 14

If the matrix A is both symmetric and skew symmetric, then

A. A is a diagonal matrix

B. A is a zero matrix

C. A is a square matrix

D. None of these

Answer:

Given A is both symmetric and skew symmetric matrix then, A = A’ and also A = -A’

⇒ A’ = -A’

⇒ 2A’ = 0

⇒ A’ = O

Clearly it is observed that transpose of A is a null matrix or zero matrix then matrix A must also be a zero matrix.

Hence A is a zero matrix.

Q. 15

If A is square matrix such that $A^2 = A$, then $(I + A)^3 – 7A$ is equal to

A. A

B. I – A

C. I

D. 3A

Answer:

Given that $A^2 = A$

Calculating value of $(I + A)^3 – 7 A$:

$\Rightarrow I^3 + A^3 + 3I^2A + 3IA^2 – 7A$

$\Rightarrow I + A^2. A + 3A + 3A^2 – 7A$ (In = I and I.A = A)

$\Rightarrow I + A.A + 3A + 3A – 7A$ ($A^2 = A$)

$\Rightarrow I + A^2 + 3A + 3A – 7A$

$\Rightarrow I + A - A$

$\Rightarrow I$

Hence $(I + A)^3 – 7 A = I$.