HARMONIC PROGRESSION - I

1. Find the fourth term in the following series : $2, 2\frac{1}{2}, 3\frac{1}{3}, ...$

2. Find the fourth term in the following series : $2, 2\frac{1}{2}, 3, ...$

3. If the p^{th}, q^{th}, r^{th} terms of a H.P. be a, b, c respectively, prove that $(q - r)bc + (r - p)ca + (p - q)ab = 0$.

4. If the m^{th} term of a H.P. be equal to n, and the n^{th} term be equal to m, prove that the $(m + n)^{th}$ term is equal to $\frac{mn}{m+n}$.

5. If a, b, c be in H.P., prove that

(A) $\frac{1}{b-a} + \frac{1}{b-c} = \frac{2}{b}$ (B) $\frac{b+a}{b-a} + \frac{b+c}{b-c} = 2$ (C) $\left(\frac{1}{a} + \frac{1}{b} - \frac{1}{c}\right)\left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right) = \frac{4}{ac} - \frac{3}{b^2}$

6. If $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$, then prove that a, b, c are in H.P. unless b = a + c.

7. If a, b, c, d are in H.P., then show that ab + bc + cd = 3ad.

8. (A) Solve the equation $6x^3 - 11x^2 + 6x - 1 = 0$ if its roots are in harmonic progression.

(B) If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then find c and all the roots.

9. If a, b, c are in G.P. and a – b, c – a and b – c are in H.P. then prove that a + 4b + c is equal to 0.

10. (A) If $\frac{1}{a(b+c)}, \frac{1}{b(c+a)}, \frac{1}{c(a+b)}$ be in H.P. then a, b, c are also in H.P.

(B) If b + c, c + a, a + b are in H.P. then prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

(C) If a, b, c be in H.P. prove that $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P.

11. Let $a_1, a_2, ..., a_{10}$ be in A.P. and $h_1, h_2, ..., h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$ then a_4h_7 is

(A) 2 (B) 3 (C) 5 (D) 6

12. If x > 1, y > 1, z > 1 are in G.P., then $\frac{1}{1+\ell nx}, \frac{1}{1+\ell ny}, \frac{1}{1+\ell nz}$ are in

(A) A.P. (B) H.P. (C) G.P. (D) None of these

13. If a, b, c, d are in H. P., then ab + bc + cd is equal to

(A) 3ad (B) (a + b) (c + d) (C) 3ac (D) (a + c)(b + d)

HARMONIC PROGRESSION - II

1. The value of n for which $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ is the harmonic mean of a and b, is equal to

(A) –1 (B) 0 (C) 1/2 (D) 1

2. The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (4+\sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

(A) 2 (B) 4 (C) 6 (D) 8

3. If 2(y – a) is the H.M. between y – x and y – z, then x – a, y – a, z – a are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

4. If a, b, c are in H.P., then $a^2(b-c)^2$, $\frac{b^2}{4}(c-a)^2$, $c^2(a-b)^2$ are in

(A) H.P. (B) G.P. (C) A.P. (D) All of the above

5. If m is a root of the equation $(1 - ab)x^2 - (a^2 + b^2)x - (1 + ab) = 0$ and m harmonic means are inserted between a and b, then the difference between the last and the first of the means equals

(A) b – a (B) ab(b – a) (C) a(b – a) (D) ab(a – b)

6. If positive number a, b, c are in A.P. and a^2, b^2, c^2 are in H.P., then

(A) a = b = c (B) 2b = a + c (C) $b^2 = \sqrt{\frac{ac}{8}}$ (D) none of these

7. If the pth term of an H.P. is qr and the qth term in rp, then the rth term of the H.P. is

(A) pqr (B) 1 (C) pq (D) pqr^2

8. If x, y, z are in A.P., a, b, c are in H.P. and ax, by, cz are in G.P., then $\frac{x}{z}+\frac{z}{x}$ is equal to

(A) $\frac{a}{c}-\frac{c}{a}$ (B) $\frac{a}{c}+\frac{c}{a}$ (C) $\frac{b}{a}+\frac{a}{b}$ (D) $\frac{b}{c}-\frac{c}{b}$

9. If the first two terms of an H.P. are 2/5 and 12/13 respectively, then the largest term is

(A) 5th term (B) 6th term (C) 10th term (D) none of these.

10. If $H_1, H_2, \ldots H_n$ be n H.M.s between a and b, then $\frac{H_1+a}{H_1-a}+\frac{H_n+b}{H_n-b}$ is equal to

(A) n (B) 2n (C) 3n (D) 4n

11. If three numbers are in HP then the numbers obtained by subtracting half of the middle number from each of them are in

(A) AP (B) GP (C) HP (D) none of these

12. If HM : GM = 4 : 5 for two positive numbers then the ratio of the numbers is

(A) 4 : 1 (B) 3 : 2 (C) 3 : 4 (D) 2 :

13. Insert two harmonic means between 5 and 11.

14. If 12 and $9\frac{3}{5}$ are the geometric and harmonic means, respectively between two numbers, find them.

15. If between any two quantities there be inserted two arithmetic means A_1, A_2; two geometric means G_1, G_2; and two harmonic means H_1, H_2; show that $G_1G_2 : H_1H_2 = A_1 + A_2 : H_1 + H_2$.

16. (A) If A be the A.M. and H the H.M. between two numbers a and b, then $\frac{a-A}{a-H}\times\frac{b-A}{b-H}=\frac{A}{H}$.

(B) If 9 arithmetic and harmonic means be inserted between 2 and 3, prove that A + 6/H = 5 where A is any of the A.M.'s and H the corresponding H.M.

17. Find $\sum_{i=1}^{100}\frac{1}{Hi}$. If $H_1, H_2 \ldots H_{100}$ are HMs between 1 and 1/100

Answers

RACE # 13

HARMONIC PROGRESSION - I

1. 5 **2.** $3\frac{1}{2}$ **8.** (a) $1, \frac{1}{2}, \frac{1}{3}$ (b) $9, 3, -\frac{3}{2}, -\frac{3}{5}$ **11.** (D) **12.** (B) **13.** (A)

HARMONIC PROGRESSION - II

1. (A) **2.** (B) **3.** (B) **4.** (D) **5.** (B) **6.** (A) **7.** (C) **8.** (B) **9.** (D) **10.** (B)

11. (B) **12.** (A) **13.** $6\frac{1}{9}, 7\frac{6}{7}$ **14.** 6, 24 **17.** 5050