METHOD OF DIFFERENTIATION

(Function which are differentiable)

Syllabus in IIT JEE : Derivative of a function, Derivative of the sum, difference, product and quotient of two functions, chain rule, derivative of polynomial, rational, trigonometric, inverse trigonometric, exponential and logarithmic functions. Derivative of implicit functions, derivatives up to order two, L'Hospital rule of evaluation of limits of functions.

1. DERIVATIVE BY FIRST PRINCIPLE :

Let $y = f(x)$; $y + \Delta y = f(x + \Delta x)$

$$\therefore \quad \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{(average rate of change of function)}$$

$$\therefore \quad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad \text{....(1)}$$

(1) denotes the instantaneous rate of change of function.

Finding the value of the limit given by (1) in respect of variety of functions is called finding the derivative by first principle / by delta method / by ab-initio / by fundamental definition of calculus.

✎ **Note that** if $y = f(x)$ then the symbols

$$\frac{dy}{dx} = Dy = f'(x) = y_1 \text{ or } y' \text{ have the same meaning.}$$

However a dot, denotes the time derivative.

e.g. $\dot{S} = \frac{dS}{dt}$; $\dot{\theta} = \frac{d\theta}{dt}$ etc.

2. DERIVATIVE OF STANDARD FUNCTIONS:

(i) $y = x^n \quad \Rightarrow \quad Dy = n\,x^{n-1}$, $n \in R$, $x > 0$

Examples : $y = (ax + b)^n$; $y = \frac{x+2}{x+1}$

(ii) $D(a^x) = a^x \ln a$, $a > 0$

$D(e^x) = e^x$; $D(\ln x) = \frac{1}{x}$; $D(\log_a x) = \frac{1}{x}\log_a e$

Examples : $e^{\sqrt{x}}$; xe^x ; $x^2 \ln x$; π^x

(iii) $D(\sin x) = \cos x$; $D(\cos x) = -\sin x$; $D(\tan x) = \sec^2 x$; $D(\cot x) - \text{cosec}^2 x$;
$D(\sec x) = \sec x \tan x$; $D(\text{cosec } x) = -\text{cosec } x \cot x$

✎ **Note that** the derivative value of all co-trigonometric functions begin with a – ve sign.

Examples : $\sin\sqrt{x}$; $\sqrt{\sin x}$; $\sin x^2$; $\sin^2 x$; $\tan^2(ax + b)$; $\sqrt{\cos\sqrt{x}}$; $x \sin x$;
$e^{\sqrt{\cos x}}$; ln(sinx)

(iv) $D(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$; $D(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$; $D(\tan^{-1}x) = \frac{1}{1+x^2}$

$D(\cot^{-1}x) = -\frac{1}{1+x^2}$; $D(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}}$; $D(\text{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}}$

D $\left(\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)\right) =$ ____

[Hint: If $x > 0$, $y = \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \Rightarrow y' = 0$

if $x < 0$, $y = \tan^{-1}x + \cot^{-1}x - \pi = -\frac{-\pi}{2} \Rightarrow y' = 0$]

(ii) **Supplementry theorems / results :**

(a) Product rule : $D\left(f(x)\cdot g(x)\right) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$ (prove by 1^{st} principle)

(b) Quotient rule : $y = \frac{f(x)}{g(x)}$ (prove by first principle)

$D\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{g^2(x)}$, to be remembered as

$D\left(\frac{N^r}{D^r}\right) = \frac{D^r \frac{d}{dx}(N^r) - N^r \frac{d}{dx}(D^r)}{(D^r)^2}$; Note: If $y = \frac{1}{f(x)}$ then $D(y) = -\frac{f'(x)}{f^2(x)}$

Note: If f (x) is derivable at x = a and f (a) = 0 and g (x) is continuous at x = a then f (x) g (x) will be derivable at x = a.

(c) Chain Rule : (Prove it by first principle)

If y = f (u) and u = g (x) then $\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$. It can be extended for any number of chains. In general if $y = (E)^C$ where E = function of x and C = constant then

$\frac{dy}{dx} = C(E)^{C-1} \cdot \frac{d}{dx}(E)$

(A) LOGARITHMIC DIFFERENTIATION:

To find the derivative of :

(i) a function which is the product or quotient of a number of functions **OR**

(ii) a function of the form $[f(x)]^{g(x)}$ where f & g are both derivable, it will be found convinient to take the logarithm of the function first & then differentiate **OR**

express $y = \left(f(x)\right)^{g(x)} = e^{g(x).\ln\left(f(x)\right)}$ and then differentiate.

(B) PARAMETRIC DIFFERENTIATION :

In some situation curves are represented by the equations e.g. x = sin t & y = cos t

If x = f (t) and y = g (t) then

$$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{g'(t)}{f'(t)}$$

(C) DERIVATIVE OF f (x) w.r.t. g (x) :

If y = f (x) and z = g (x) then derivative of f (x) w.r.t. g (x) is given by

$$\frac{dy}{dz} = \frac{dy}{dx}\cdot\frac{dx}{dz} = \frac{f'(x)}{g'(x)}$$

$\therefore$ Differential coefficient of f (x) w.r.t. g(x) $= \dfrac{\textit{derivative of } f(x) \textit{ w.r.t. } x}{\textit{derivative of } g(x) \textit{ w.r.t. } x} = \dfrac{f'(x)}{g'(x)}$

(D) DERIVAIVE OF IMPLICIT FUNCTION : $\phi(x, y) = 0$

(i) In order to find dy/dx, in the case of implicit functions, we differentiate each term w.r.t. x regarding y as a functions of x & then collect terms in dy/dx together on one side to finally find dy/dx.

(ii) In answers of dy/dx in the case of implicit functions, both x & y are present.

Corresponding to every curve represented by an implicit equation, there exist one or more explicit functions representing that equation. It can be shown that dy/dx at any point on the curve remains the same whether the process of differentiation is done explictly or implicitly.

(E) DERIVATIVE OF INVERSE FUNCTION :

Theorem : If the inverse functions f & g are defined by y = f(x) & x = g(y) & if $f'(x)$ exists & $f'(x) \neq 0$ then $g'(y) = \dfrac{1}{f'(x)}$. This result can also be written as,

if $\dfrac{dy}{dx}$ exists & $\dfrac{dy}{dx} \neq 0$, then $\dfrac{dx}{dy} = \dfrac{1}{\frac{dy}{dx}}$ or $\dfrac{dy}{dx}.\dfrac{dx}{dy} = 1$ or $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$ $[\dfrac{dx}{dy} \neq 0]$

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DERIVATIVE OF I.T.F.

Direct Differentiation | Elementary Transformation

(i) $y = f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & |x|\le 1 \\ \pi-2\tan^{-1}x & x>1 \\ -(\pi+2\tan^{-1}x) & x<-1 \end{cases}$

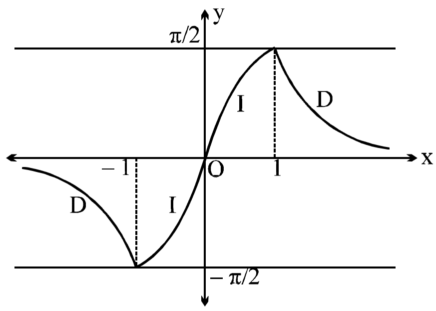
HIGHLIGHTS :

(a) Domain is $x \in R$ & range is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Note: f is odd, aperiodic bound

(b) f is continuous for all x but not diff. at x = 1 , - 1

(c) $\frac{dy}{dx} = \begin{cases} \frac{2}{1+x^2} & \text{for } |x|<1 \\ \text{nonexistent} & \text{for } |x|=1 \\ -\frac{2}{1+x^2} & \text{for } |x|>1 \end{cases}$



(d) I in $(-1, 1)$ & D in $(-\infty, -1) \cup (1, \infty)$

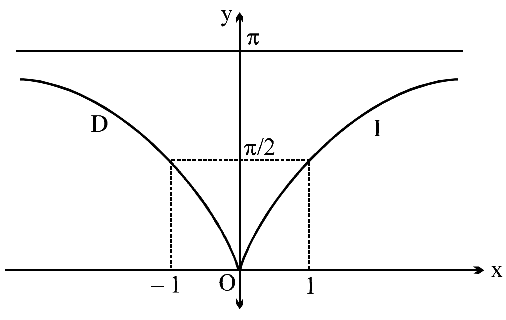
(ii) Consider $y = f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & \text{if } x\ge 0 \\ -2\tan^{-1}x & \text{if } x<0 \end{cases}$

HIGHLIGHTS :

(a) Domain is $x \in R$ & range is $(0, \pi)$

(b) Continuous for all x but not diff. at x = 0

(c) $\frac{dy}{dx} = \begin{cases} \frac{2}{1+x^2} & \text{for } x>0 \\ \text{nonexistent} & \text{for } x=0 \\ -\frac{2}{1+x^2} & \text{for } x<0 \end{cases}$



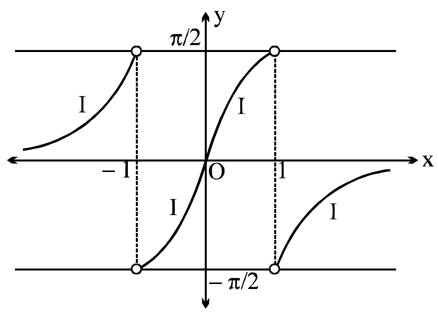
(d) I in $(0, \infty)$ & D in $(-\infty, 0)$

Note: f is even, a periodic bound.

(iii) $y = f(x) = \tan^{-1}\dfrac{2x}{1-x^2} = \begin{cases} 2\tan^{-1}x & |x|<1 \\ \pi+2\tan^{-1}x & x<-1 \\ -\left(\pi-2\tan^{-1}x\right) & x>1 \end{cases}$

HIGHLIGHTS :

(a) Domain is R - {1 , -1} & range is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



(b) f is neither continuous nor diff. at x = 1 , - 1

(c) $\dfrac{dy}{dx} = \begin{cases} \dfrac{2}{1+x^2} & |x|\neq 1 \\ \text{nonexistent} & |x|=1 \end{cases}$

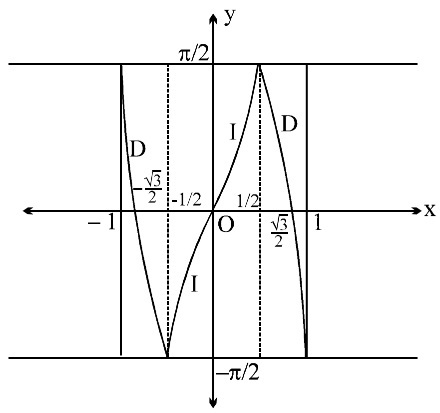
(d) I ∀ x in its domain

(e) It is bound for all x

(iv) $y = f(x) = \sin^{-1}(3x - 4x^3) = \begin{cases} -\left(\pi+3\sin^{-1}x\right) & \text{if } -1\le x\le -\frac{1}{2} \\ 3\sin^{-1}x & \text{if } -\frac{1}{2}\le x\le \frac{1}{2} \\ \pi-3\sin^{-1}x & \text{if } \frac{1}{2}\le x\le 1 \end{cases}$

HIGHLIGHTS :

(a) Domain is $x \in [-1, 1]$ & range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



(b) Not derivable at $|x| = \frac{1}{2}$

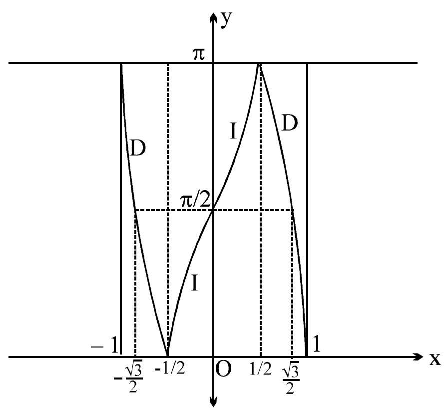
(c) $\dfrac{dy}{dx} = \begin{cases} \dfrac{3}{\sqrt{1-x^2}} & \text{if } x\in\left(-\frac{1}{2},\frac{1}{2}\right) \\ -\dfrac{3}{\sqrt{1-x^2}} & \text{if } x\in\left(-1,-\frac{1}{2}\right)\cup\left(\frac{1}{2},1\right) \end{cases}$

(d) Continuous everywhere in its domain

(v) $y = f(x) = \cos^{-1}(4x^3 - 3x) = \begin{cases} 3\cos^{-1}x-2\pi & \text{if } -1\le x\le -\frac{1}{2} \\ 2\pi-3\cos^{-1}x & \text{if } -\frac{1}{2}\le x\le \frac{1}{2} \\ 3\cos^{-1}x & \text{if } \frac{1}{2}\le x\le 1 \end{cases}$

HIGHLIGHTS :

(a) Domain is $x \in [-1, 1]$ & range is $[0, \pi]$



(b) Continuous everywhere in its domain but not derivable at $x = \frac{1}{2}, -\frac{1}{2}$

(c) I in $\left(-\frac{1}{2},\frac{1}{2}\right)$ &

D in $\left(\frac{1}{2},1\right]\cup\left[-1,-\frac{1}{2}\right)$

(d) $\dfrac{dy}{dx} = \begin{cases} \dfrac{3}{\sqrt{1-x^2}} & \text{if } x \in \left(-\frac{1}{2}, \frac{1}{2}\right) \\ -\dfrac{3}{\sqrt{1-x^2}} & \text{if } x \in \left(-1, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, 1\right) \end{cases}$

GENERAL NOTE :

Concavity in each case is decided by the sign of 2nd derivative as :

$\dfrac{d^2y}{dx^2} > 0 \Rightarrow$ Concave upwards ; $\dfrac{d^2y}{dx^2} < 0 \Rightarrow$ Concave downwards

D = DECREASING ; I = INCREASING

(F) SUCCESSIVE DIFFERENTIATION :

$y = f(x)$; the popular symbols used to denote the derivatives are $\dfrac{dy}{dx} = Dy = f'(x) = y_1 = y'$. Higher order derivatives are

denoted as $\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d^2y}{dx^2} = D^2y = f''(x) = y_2$ or y'' etc.

(G) DEDUCTION OF NEW IDENTITIES BY DIFFERENTIATING A GIVEN IDENTITY :

(i) If $\cos\dfrac{x}{2} \cdot \cos\dfrac{x}{2^2} \cdot \cos\dfrac{x}{2^3} \ldots\ldots \cos\dfrac{x}{2^n} = \dfrac{\sin x}{2^n \sin\left(x/2^n\right)}$, then prove that

$$\sum_{r=1}^{n} \frac{1}{2^r} \tan\frac{x}{2^r} = \frac{1}{2^n}\cot\frac{x}{2^n} - \cot x$$

[Sol. Taking log on both sides we get

$$\ln\cos\frac{x}{2} \; \ln\cos\frac{x}{2^2} \ldots\ldots \ln\cos\frac{x}{2^n} = \ln(\sin x) - \ln(2^n) - \ln\sin\left(\frac{x}{2^n}\right)$$

differentiating

$$-\left[\frac{1}{2}\tan\frac{x}{2} + \frac{1}{2^2}\tan\frac{x}{2^2} + \ldots\ldots + \frac{1}{2^n}\ln\tan\frac{x}{2^n}\right] = \cot x - \frac{1}{2^n}\cot\frac{x}{2^n} \Rightarrow \text{result]}$$

(ii) $(1 + x)^n = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_nx^n$, prove that

$D(1+x)^n = C_1 + 2C_2 + 3C_3 + \ldots\ldots + nC_n = n2^{n-1}$

$C_0 + 2C_1 + 3C_2 + \ldots\ldots + (n + 1)C_n = (n + 2)2^{n-1}$

(H) DERIVATIVE OF FUNCTIONS EXPRESSED IN THE DETERMINANT FORM :

Let $F(x) = \begin{vmatrix} f & g & h \\ u & v & w \\ l & m & n \end{vmatrix}$ where all functions are differentiable then

$$D'(x) = \begin{vmatrix} f' & g' & h' \\ u & v & w \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u' & v' & w' \\ l & m & n \end{vmatrix} + \begin{vmatrix} f & g & h \\ u & v & w \\ l' & m' & n' \end{vmatrix}$$

This result may be proved by first principle and the same operation can also be done column wise.

L'Hospital's Rule (0^0 / ∞^0): e.g. $f(x) = x^x$ or $\left(-\frac{1}{x}\right)^{\sin x}$

If *f* (x) and *g* (x) are two functions such that

(i) $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$

(ii) *f* and g are derivable / continuous at x = a

i.e. $\lim_{x\to a} f(x) = f(a) = 0$; $\lim_{x\to a} g(x) = g(a) = 0$

(iii) f ' (x) and g ' (x) are continuous at x = a, then

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} = \lim_{x\to a}\frac{f''(x)}{g''(x)} \text{ till the indeterminant form vanishes}$$

L'Hospital's Rule for $\left(\frac{\infty}{\infty}\right)$

If $\lim_{x\to a} f(x) \to \infty$ and $\lim_{x\to a} g(x) \to \infty$ then also $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$

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1. If $y = \frac{a + bx^{3/2}}{x^{5/4}}$ & $\frac{dy}{dx}$ vanishes when x = 5 then $\frac{a}{b} =$

(A) $\sqrt{3}$ (B) 2

(C) $\sqrt{5}$ (D) None of these

2. If $y = \sqrt{\sin x + y}$, then $\frac{dy}{dx} =$

(A) $\frac{\sin x}{2y - 1}$ (B) $\frac{\sin x}{1 - 2y}$

(C) $\frac{\cos x}{1 - 2y}$ (D) $\frac{\cos x}{2y - 1}$

3. If $x\sqrt{y} + y\sqrt{x} = 1$, then $\frac{dy}{dx}$ equals-

(A) $-\frac{y + 2\sqrt{xy}}{x + 2\sqrt{xy}}$ (B) $-\sqrt{\frac{x}{y}}\left(\frac{y + 2\sqrt{xy}}{x + 2\sqrt{xy}}\right)$

(C) $-\sqrt{\frac{y}{x}}\left(\frac{y + 2\sqrt{xy}}{x + 2\sqrt{xy}}\right)$ (D) None of these

4. If $e^x \sin y - e^y \cos x = 1$, then $\frac{dy}{dx}$ equals–

(A) $\frac{e^x \sin y + e^y \sin x}{e^y \cos x - e^x \cos y}$ (B) $\frac{e^x \sin y + e^y \sin x}{e^y \cos x + e^x \cos y}$

(C) $\frac{e^x \sin y - e^y \sin x}{e^y \cos x - e^x \cos y}$ (D) None of these

5. If $f(x) = |x|^{|\sin x|}$ then $f\pi('/4)$ equals

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

(B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$

(D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\ln\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

6. The derivative of $\sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$ w.r.t. $\sqrt{1 - x^2}$ at $x = \frac{1}{2}$ is

(A) 4 (B) 1/4

(C) 1 (D) None of these

7. If $y = \sin^{-1}\frac{2x}{1 + x^2}$ then $\left.\frac{dy}{dx}\right]_{x = -2}$ is

(A) $\frac{2}{5}$ (B) $\frac{2}{\sqrt{5}}$

(C) $-\frac{2}{5}$ (D) None of these

8. If $y = x - x^2$, then the derivative of y^2 w.r.t. x^2 is

(A) $2x^2 + 3x - 1$ (B) $2x^2 - 3x + 1$

(C) $2x^2 + 3x + 1$ (D) none of these

9. The differential coefficient of $a^{\sin^{-1} x}$ w.r.t. $\sin^{-1} x$ is -

(A) $a^{\sin^{-1} x} \log_e a$ (B) $a^{\sin^{-1} x}$

(C) $\frac{a^{\sin^{-1} x}}{\sqrt{(1 - x^2)}}$ (D) $a^{\sin^{-1} x}\sqrt{(1 - x^2)}$

10. If $x = e^{y + e^{y + \ldots \text{to } \infty}}$, x > 0, then $\frac{dy}{dx}$

(A) $\frac{x}{1 + x}$ (B) $\frac{1}{x}$

(C) $\frac{1 - x}{x}$ (D) $\frac{1 + x}{x}$

11. If $8 f(x) + 6 f\left(\frac{1}{x}\right) = x + 5$ and $y = x^2 f(x)$, then $\frac{dy}{dx}$ at x = –1 is equal to

(A) 0 (B) $\frac{1}{14}$

(C) $-\frac{1}{14}$ (D) none of these

12. If $x = \frac{1 + t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$ then, $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} =$

(A) 0 (B) – 1

(C) 1 (D) 2

13. If $x = at^2$, $y = 2at$, then $\frac{d^2y}{dx^2}$ is

(A) $-\frac{1}{t^2}$ (B) $\frac{1}{2at^2}$

(C) $-\frac{1}{t^3}$ (D) $-\frac{1}{2at^3}$

14. If g is inverse of f and $f'(x) = \frac{1}{1+x^n}$, then g'(x) equals -

(A) $1 + x^n$ (B) $1 + (f(x))^n$

(C) $1 + (g(x))^n$ (D) None of these

15. Derivative of $\log_e(\log_e |\sin x|)$ with respect to x at $x = \frac{\pi}{6}$ is

(A) $-\frac{\sqrt{3}}{\log_e 2}$ (B) $\frac{\sqrt{3}}{\log_e 2}$

(C) $-\frac{\sqrt{3}}{2\log 2}$ (D) does not exist

16. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, then the value of dy/dx is -

(A) $\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (B) $\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

(C) $-\frac{\sqrt{1-x^2}}{\sqrt{1-y^2}}$ (D) $-\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

17. If $x^m . y^n = (x + y)^{m+n}$, then $\frac{dy}{dx}$ is

(A) $\frac{x+y}{xy}$ (B) xy

(C) $\frac{x}{y}$ (D) $\frac{y}{x}$

18. $d/dx \left[\sin^2 \cot^{-1} \frac{1}{\sqrt{\frac{1+x}{1-x}}}\right]$, is equal to -

(A) 0 (B) 1/2

(C) – 1/2 (D) –1

19. Find derivative of f(tan x) w.r.t. g(sec x) at $x = \frac{\pi}{4}$ where $f'(1) = 2$ and $g'(\sqrt{2}) = 4$ is

(A) 3 (B) –4

(C) $\frac{2}{19}$ (D) $\frac{1}{\sqrt{2}}$

20. The derivative of the function,

$$f(x) = \cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x - 3\sin x)\right\} + \sin^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x + 3\sin x)\right\}$$

w.r.t, $\sqrt{1+x^2}$ is :

(A) 2x (B) $2\sqrt{1+x^2}$

(C) $\frac{2}{x}\sqrt{1+x^2}$ (D) $\frac{2x}{\sqrt{1+x^2}}$

21. If y = a cos (*l*n x) + b sin (*l*n x), then $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx}$

(A) 0 (B) y

(C) –y (D) None of these

22. If $x = \frac{1+t}{t^3}$, $y = \frac{3}{2t^2} + \frac{2}{t}$ then, $x\left(\frac{dy}{dx}\right)^3 - \frac{dy}{dx} =$

(A) 0 (B) – 1

(C) 1 (D) 2

23. If $\sin x = \frac{2t}{1+t^2}$ and $\cot y = \frac{1-t^2}{2t}$. Then value of $\frac{d^2x}{dy^2}$ is equal to

(A) 0 (B) 1 (C) –1 (D) $\frac{1}{2}$

24. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then y'(1) equals-

[AIEEE 2009]

(A) –1 (B) 1

(C) log 2 (D) – log 2

25. Let f : (–1, 1) → R be a differentiable function with f(0) = – 1 and f'(0) = 1. Let $g(x) = [f(2f(x)+2)]^2$, then g'(0) = **[AIEEE 2010]**

(A) 4 (B) –4

(C) 0 (D) –2

26. $\frac{d^2x}{dy^2}$ equals : **[AIEEE 2011]**

(A) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (B) $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

(C) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (D) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

27. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to: **[AIEEE 2013]**

(A) 1 (B) $\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

28. For $x \in \mathbf{R}$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then: **[JEE MAIN 2016]**

(A) $g'(0) = \cos(\log 2)$

(B) $g'(0) = -\cos(\log 2)$

(C) g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$

(D) g is not differentiable at x = 0

29. If $y = \frac{1+x^2+x^4}{1+x+x^2}$ and $\frac{dy}{dx} = ax + b$, then values of a & b are -

(A) a = 2 , b = 1 (B) a = –2 , b = 1

(C) a = 2 , b = –1 (D) a = –2 , b = –1

30. Differential coefficient of

$\left(x^{\frac{\ell+m}{m-n}}\right)^{\frac{1}{n-\ell}} \cdot \left(x^{\frac{m+n}{n-\ell}}\right)^{\frac{1}{\ell-m}} \cdot \left(x^{\frac{n+\ell}{\ell-m}}\right)^{\frac{1}{m-n}}$ w.r.t. x is -

(A) 1 (B) 0

(C) –1 (D) x^{lmn}

31. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$ then $\frac{dy}{dx} =$

(A) $-\frac{x}{y}$ (B) $-\frac{y}{x}$

(C) $\frac{y}{x}$ (D) $\frac{x}{y}$

32. If $f(x) = \prod_{n=1}^{100}(x-n)^{n(101-n)}$; then $\frac{f(101)}{f'(101)} =$

(A) 5050 (B) $\frac{1}{5050}$

(C) 10010 (D) $\frac{1}{10010}$

33. If $y = \frac{x}{a+}\frac{x}{b+}\frac{x}{a+}\frac{x}{b+}\frac{x}{a+}\frac{x}{b+}\ldots\ldots\infty$ then $\frac{dy}{dx}$ -

(A) $\frac{a}{ab+2ay}$ (B) $\frac{b}{ab+2by}$

(C) $\frac{a}{ab+2by}$ (D) $\frac{b}{ab+2ay}$

34. If $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$, then $\frac{dy}{dx}$ equals -

(A) $\frac{1}{(1+x)^2}$ (B) $-\frac{1}{(1+x)^2}$

(C) $-\frac{1}{(1+x)} + \frac{1}{(1+x)^2}$ (D) none of these

35. Let g is the inverse function of f & $f'(x) = \frac{x^{10}}{(1+x^2)}$.

If g (2) = a then g'(2) is equal to -

(A) $\frac{5}{2^{10}}$ (B) $\frac{1+a^2}{a^{10}}$

(C) $\frac{a^{10}}{1+a^2}$ (D) $\frac{1+a^{10}}{a^2}$

36. If $f(x) = x^n$, then the 1 value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots\ldots + \frac{(-1)^n f^{'''''''\ldots\ldots(n\ \text{times})}(1)}{n!}$ -

(A) 2^{n-1} (B) 0

(C) 1 (D) 2^n

37. If f(4) = g(4) = 2 ; f' (4) = 9 ; g' (4) = 6 then

$\lim_{x\to 4}\frac{\sqrt{f(x)} - \sqrt{g(x)}}{\sqrt{x}-2}$ is equal to -

(A) $3\sqrt{2}$ (B) $\frac{3}{\sqrt{2}}$ (C) 0 (D) none