Real Numbers

Exercise 1.1

Q. 1 A. Use Euclid's division algorithm to find the HCF of

900 and 270

Answer : Euclid's Division is a method for finding the HCF (highest common factor) of two given integers. According to Euclid's Division Algorithm, For any two positive integers, 'a' and 'b', there exists a unique pair of integers 'q' and 'r' which satisfy the relation:

$a = bq + r$, $0 \le r \le b$

Given integers 900 and 270. Clearly 900>270.

By applying division lemma

$\Rightarrow 900 = 270 \times 3 + 90$

Since remainder $\neq$ 0, applying division lemma on 270 and 90

$\Rightarrow 270 = 90 \times 3 + 0$

$\because$ remainder = 0,

$\therefore$ the HCF of 900 and 270 is 90.

Q. 1 B. Use Euclid's division algorithm to find the HCF of

196 and 38220

Answer : Euclid's Division is a method for finding the HCF (highest common factor) of two given integers. According to Euclid's Division Algorithm, For any two positive integers, 'a' and 'b', there exists a unique pair of integers 'q' and 'r' which satisfy the relation:

$a = bq + r$, $0 \le r \le b$

Given integers 196 and 38220. Clearly 38220>196.

By applying division lemma

$\Rightarrow$ 38220 = 196×195 + 0

Since remainder $=$ 0

∴ the HCF of 196 and 38220 is 195.

Q. 1 C. Use Euclid's division algorithm to find the HCF of

1651 and 2032

Answer : Euclid's Division is a method for finding the HCF (highest common factor) of two given integers. According to Euclid's Division Algorithm, For any two positive integers, 'a' and 'b', there exists a unique pair of integers 'q' and 'r' which satisfy the relation:

$a = bq + r$, $0 \le r \le b$

Given integers 1651 and 2032. Clearly 2032>1651.

By applying division lemma

$\Rightarrow$ 2032 = 1651×1 + 381

Since remainder $\neq$ 0, applying division lemma on 1651 and 381

$\Rightarrow$ 1651 = 381×4 + 127

Since remainder $\neq$ 0, applying division lemma on 381 and 127

$\Rightarrow$ 381 = 127×3 + 0

Since remainder = 0,

∴ the HCF of 1651 and 2032 is 127.

Q. 2. Use Euclid division lemma to show that any positive odd integer is of form 6q + 1, or 6q + 3 or 6q + 5, where q is some integers.

Answer : Let a be any odd positive integer and b = 6

Then using Euclid's algorithm, we get $a = 6q + r$ here r is remainder and value of q is more than or equal to 0 and r = 0,1,2,3,4,5 because $0 \le r < b$ and the value of b is 6

So total form available will be 6q, 6q + 1, 6q + 2, 6q + 3, 6q + 4, 6q + 5 ,6q + 6 is divisible by 2, so it is an even number.

6q + 1 , 6 is divisible by 2 but 1 is not divisible by 2, so it is an odd number.

6q + 2 , 6 is divisible by 2 but 2 is also divisible by 2, so it is an even number.

6q + 3 , 6 is divisible by 2 but 3 is not divisible by 2, so it is an odd number.

6q + 4 , 6 is divisible by 2 but 4 is also divisible by 2, so it is an even number.

6q + 5 , 6 is divisible by 2 but 5 is not divisible by 2, so it is an odd number.

∴ so odd numbers will in form of 6q + 1 or 6q + 3 or 6q + 5.

Q. 3. Use Euclid's division lemma to show that the square of any positive integer is of the form 3p, 3p + 1 or 3p + 2.

Answer : Let a be any positive integer. Then, it is form 3q or, 3q + 1 or, 3q + 2

So, we hae the following cases:

Case I. When a = 3q

In this case, we have

$a^2 = (3q)^2 = 9q^2 = 3q(3q) = 3p$, where $p = 3q^2$

Case II. When a = 3q + 1

In this case, we have

$a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3q(3q + 2) + 1 = 3p + 1$,

where p = q(3q + 2)

Case III. When a = 3q + 2

In this case, we have

$a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1 = 3p + 1$

where $p = 3q^2 + 4^2 + 1$

Hence, a is the form of 3p or 3p + 1 or 3p + 2

Q. 4. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Answer : Let a be any positive integer. Then, it is of the form 3q or, 3q + 1 or, 3q + 2.

We know that according to Euclid's division lemma:

a = bq + rSo, we have the following cases:

Case I When a = 3q

In this case, we have

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 3q^3$

Case II When a = 3q + 1

In this case, we have

$a^3 = (3q + 1)^3$

$\Rightarrow 27q^3 + 27q^2 + 9q + 1$

$\Rightarrow 9q(3q^2 + 3q + 1) + 1$

$\Rightarrow a^3 = 9m + 1$, where $m = q(3q^2 + 3q + 1)$

Case III When a = 3q + 2

In this case, we have

$a^3 = (3q + 1)^3$

$\Rightarrow 27q^3 + 54q^2 + 36q + 8$

$\Rightarrow 9q(3q^2 + 6q + 4) + 8$

$\Rightarrow a^3 = 9m + 8$, where $m = q(3q^2 + 6q + 4)$

Hence, a^3 is the form of 9m or, 9m + 1 or, 9m + 8.

Q. 5. Show that one and only one out of n, n + 2 or n + 4is divisible by 3, where n is any positive integer.

Answer : We know that any positive integer is of the form 3q or, 3q + 1 or,

3q + 2 for some integer q and one and only one of these possibilities can occur.

So, we have following cases:

<u>Case I When n = 3q</u>

In this case, we have

n = 3q, which is divisible by 3

Now, n = 3q

⇒ n + 2 = 3q + 2,

⇒ n + 2 leaves remainder 2 when divided by 3

⇒ n + 2 is not divisible by 3

Again, n = 3q

⇒ n + 4 = 3q + 4 = 3(q + 1) + 1

⇒ n + 4 leaves remainder 1 when divided by 3

⇒ n + 4 is not divisible by 3

Thus, n is divisible by 3 but n + 2 and n + 4 are not divisible by 3.

<u>Case II When n = 3q + 1</u>

In this case, we have

n = 3q + 1

⇒ n leaves remainder 1 when divided by 3

⇒ n is not divisible by 3

Now, n = 3q + 1

$\Rightarrow n + 2 = (3q + 1) + 2 = 3(q + 1)$,

$\Rightarrow n + 2$ is divisible by 3

Again, $n = 3q + 1$

$\Rightarrow n + 4 = (3q + 1) + 4 = 3q + 5 = 3(q + 1) + 2$

$\Rightarrow n + 4$ leaves remainder 2 when divided by 3

$\Rightarrow n + 4$ is not divisible by 3

Thus, $n + 2$ is divisible by 3 but n and $n + 4$ are not divisible by 3.

Case III When n = 3q + 2

In this case, we have

$n = 3q + 2$

$\Rightarrow n$ leaves remainder 2 when divided by 3

$\Rightarrow n$ is not divisible by 3

Now, $n = 3q + 2$

$\Rightarrow n + 2 = 3q + 2 + 2 = 3(q + 1) + 1$,

$\Rightarrow n + 2$ leaves remainder 1 when divided by 3

$\Rightarrow n + 2$ is not divisible by 3

Again, $n = 3q + 2$

$\Rightarrow n + 4 = 3q + 2 + 4 = 3(q + 2)$

$\Rightarrow n + 4$ is divisible by 3

Thus, $n + 4$ is divisible by 3 but n and $n + 2$ are not divisible by 3.

Exercise 1.2

Q. 1. Express each of the following number as a product of its prime factors.

(i) 140
(ii) 156

(iii) 3825
(iv) 5005
(v) 7429

Answer : I. 140 = 2×2×5×7 = 2^2×5×7

II. 156 = 2×2×3×13 = 2^2×3×13

III. 3825 = 3×3×5×5×17 = 3^2×5^2×17

IV. 5005 = 5×7×11×13

V. 7429 = 17×19×23

Q. 2. Find the LCM and HCF of the following integers by the prime factorization method.

(i) 12, 15 and 21
(ii) 17, 23 and 29
(iii) 8, 9 and 25
(iv) 72 and 108
(v) 306 and 657

Answer : I. 12,15 and 21

12 = 2^2×3

15 = 3×5

21 = 3×7

LCM = 2^2×3×5×7 = 420

HCF = 3

II. 17,23 and 29

17 = 1×17

23 = 1×23

29 = 1×29

LCM = 1×17×23×29 = 11339

HCF = 1

III. 8, 9 and 5

$8 = 2^3$

$9 = 3^2$

$5 = 1\times5$

LCM = $2^3\times3^2\times5 = 360$

HCF = 1

IV. 72 and 108

$72 = 2^3\times3^2$

$108 = 2^2\times3^3$

LCM = $2^5\times3^5 = 7776$

HCF = $2^2\times3^2 = 4\times9 = 36$

V. 306 and 657

$306 = 2\times3^2\times17$

$657 = 3^2\times73$

LCM = $2\times3^2\times17\times73 = 22338$

HCF = $3^2 = 9$

Q. 3. Check whether 6^n can end with the digit 0 for any natural number n.

Answer : If any number end with digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorization of $6^n = (2\times3)^n$

It can be observed that 5 is not in the prime factorization of 6^n.

Hence, for an value of n, 6^n will not visible by 5.

$\therefore$ 6^n cannot end with the digit 0 for any natural number n.

Q. 4. Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer : Numbers are of two types – composite and prime. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7\times11\times13 + 13 = 13\times(7\times11 + 1) = 13\times(77 + 1) = 13\times78$

$= 13\times13\times6$

The given expression has 6 and 13 as its factors.

∴ , it is a composite factor.

$7\times6\times5\times4\times3\times2\times1 + 5 = 5\times(7\times6\times4\times3\times2\times1 + 1)$

$= 5\times(1008 + 1) = 5\times1009$

1009 cannot be factorized further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Q. 5. How will you show that (17 × 11 × 2) + (17 × 11 × 5) is a composite number? Explain.

Answer : Numbers are of two types – composite and prime. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$(17\times11\times2) + (17\times11\times5) = 17\{(11\times2) + (11\times5)\}$

$= 17\times\{11\times\{(2) + (5)\}\} = 17\times11\times7$

The given expression has 17, 11and 7 as its factors.

∴ it is a composite factor.

Q. 6. What is the last digit of 6^{100}.

Answer : This is related to concept of numbers in the unit digits place of the powers of natural number. The power of 6 any index repet ition 6 i.e. $(6)^n$ the last digit is 6 only.

Example:

i: $6^1 = 6$

ii: $6^2 = 36$

iii: $6^3 = 216$

The last digit in the expansion of 6^{100} is 6.

Exercise 1.3

Q. 1 A. Write the following rational numbers in their decimal form and also state which are terminating and which have non-terminating, repeating decimal.

$$\frac{3}{8}$$

Answer :

$$\frac{3}{8} = \frac{3}{2 \times 2 \times 2}$$

$$= \frac{3}{2^3}$$

$$= \frac{3 \times 5^3}{2^3 \times 5^3}$$

$$= \frac{3 \times 125}{(2 \times 5)^3}$$

$$= \frac{375}{(10)^3}$$

$$= 0.375$$

Since, this decimal has finite number of digits

∴ it is terminating

Q. 1 B. Write the following rational numbers in their decimal form and also state which are terminating and which have non-terminating, repeating decimal.

$$\frac{229}{400}$$

Answer :

$$\frac{229}{400}$$

$$= \frac{229}{2^2 \times 5^2 \times 2^2}$$

$$= \frac{229}{2^4 \times 5^2}$$

$$= \frac{229 \times 5^2}{2^4 \times 5^2 \times 5^2}$$

$$= \frac{229 \times 25}{2^4 \times 5^4}$$

$$= \frac{5725}{(2 \times 5)^4}$$

$$= \frac{5725}{10^4}$$

$$= 0.5725$$

Since, this decimal has finite number of digits

∴ it is terminating.

Q. 1 C. Write the following rational numbers in their decimal form and also state which are terminating and which have non-terminating, repeating decimal.

$$4\frac{1}{5}$$

Answer :

$$\frac{21}{5}$$

$$= \frac{21 \times 2}{5 \times 2}$$

$$= \frac{42}{10}$$

$$= 4.2$$

Since, this decimal has finite number of digits

∴ it is terminating.

Q. 1 D. Write the following rational numbers in their decimal form and also state which are terminating and which have non-terminating, repeating decimal.

$$\frac{2}{11}$$

Answer :

$$\frac{2}{11}$$

$$= 0.\overline{18}$$

Since the decimal continues endlessly, it is non-terminating and repeating.

∴ it is non-terminating and repeating.

Q. 1 E. Write the following rational numbers in their decimal form and also state which are terminating and which have non-terminating, repeating decimal.

$$\frac{8}{125}$$

Answer :

$$\frac{8}{125}$$

$$= \frac{2^3}{5^3}$$

$$= \frac{(2^3 \times 2^3)}{5^3 \times 2^3}$$

$$= \frac{64}{(5 \times 2)^3}$$

$$= \frac{64}{10^3}$$

$$= 0.064$$

Since, this decimal has finite number of digits

∴ it is terminating.

Q. 2 A. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{13}{3125}$$

Answer :

$$\frac{13}{3125}$$

⇒ 13 and 3125 are co-prime.

Now, We have to write the denominator 3125 in the form of 2^n5^m

where, n and m are the non-negative numbers.

$3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$

$3125 \Rightarrow 1325 = 1 \times 5^5 = 2^0 \times 5^5$

∴ denominator is of the form 2^n5^m where, n = 0 and m = 5

Thus, $\frac{13}{3125}$ is a Terminating decimal

Q. 2 B. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

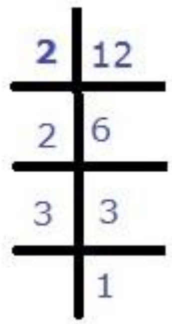
$$\frac{11}{12}$$

Answer :

$\frac{11}{12}$

⇒ 11 and 12 are co-prime.

Now, We have to write the denominator 12 in the form of 2^n5^m where, n and m are the non-negative numbers.



12 = 2×2×3

$12 = 2^2 \times 3$

∴ denominator is not of the form 2^n5^m where, n = 2 and m = 0

Thus, $\frac{11}{12}$ is Non-terminating and repeating decimal

Q. 2 C. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

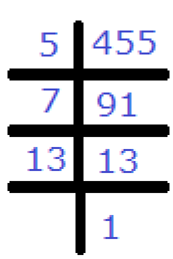
$$\frac{64}{455}$$

Answer :

$$\frac{64}{455}$$

⇒ 64 and 455 are co-prime.

Now, We have to write the denominator 455 in the form of 2^n5^m where, n and m are the non-negative numbers.



455 = 5×7×13

∴ denominator is not of the form 2^n5^m where, n = 0 and m = 1

Thus, $\frac{64}{455}$ is Non-terminating and repeating decimal

Q. 2 D. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{15}{1600}$$

Answer :

$$\frac{15}{1600}$$

$$\frac{15}{1600} = \frac{3}{320}$$

⇒ 3 and 320 are co-prime.

Now, We have to write the denominator 320 in the form of 2^n5^m where, n and m are the non-negative numbers.

2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

$320 = 2\times2\times2\times2\times2\times2\times5 = 2^6\times5$

∴ denominator is in the form 2^n5^m where, n = 6 and m = 1

Thus, $\frac{15}{1600}$ is terminating decimal.

Q. 2 E. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{29}{343}$$

Answer :

$$\frac{29}{343}$$

⇒ 29 and 343 are co-prime.

Now, We have to write the denominator 343 in the form of 2^n5^m where, n and m are the non-negative numbers.

7	343
7	49
7	7
	1

$343 = 7\times7\times7 = 7^3$

∴ denominator is not of the form 2^n5^m where, n = 0 and m = 0

Thus, $\frac{29}{343}$ is Non-terminating and repeating decimal

Thus, $\frac{29}{343}$ is Non-terminating decimal.

Q. 2 F. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{23}{2^3 \cdot 5^2}$$

Answer :

$$\frac{23}{2^3 5^2}$$

⇒ 23 and 2^3 5^2 are co-prime.

Now, we have to write the denominator 2^2 5^7 7^5 in the form of 2^n5^m where, n and m are the non-negative numbers.

2^3 5^2

∴ denominator is in the form 2^n5^m where, n = 3 and m = 2

Thus, $\frac{29}{2^3 5^2}$ is terminating decimal

Q. 2 G. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{129}{2^2 \cdot 5^7 \cdot 7^5}$$

Answer :

$$\frac{129}{2^2 5^7 7^5}$$

⇒ 23 and 2^2 5^7 7^5 are co-prime.

Now, we have to write the denominator 2^2 5^7 7^5 in the form of 2^n5^m where, n and m are the non-negative numbers.

2^3 5^2

$2^2 \times 5^7 \times 7^5$

∴ denominator is not of the form 2^n5^m where, n = 2 and m = 7. Due to one more factor it is not in the form

Thus, $\frac{129}{2^2 5^7 7^5}$ is Non-terminating decimal.

Q. 2 H. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{9}{15}$$

Answer :

$$\frac{6}{15} = \frac{2}{5}$$

⇒ 2 and 5 are co-prime.

Now, we have to write the denominator 5 in the form of 2^n5^m where, n and m are the non-negative numbers.

2^0 5

∴ denominator is in the form 2^n5^m where, n = 0 and m = 1

Thus, $\frac{6}{15}$ is Terminating decimal.

Q. 2 I. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{36}{100}$$

Answer :

$$\frac{35}{50} = \frac{7}{10}$$

⇒ 7 and 10 are co-prime.

Now, we have to write the denominator 10 in the form of 2^n5^m where, n and m are the non-negative numbers.

2	10
5	5
	1

10 = 2×5

∴ denominator is in the form 2^n5^m where, n = 1 and m = 1

Thus, $\frac{35}{50}$ is terminating decimal.

Q. 2 J. Without performing division, state whether the following rational numbers will have a terminating decimal form or non-terminating, repeating decimal form.

$$\frac{77}{210}$$

Answer :

$$\frac{77}{210} = \frac{11}{30}$$

⇒ 11 and 30 are co-prime.

Now, We have to write the denominator 30 in the form of 2^n5^m where, n and m are the non-negative numbers.

2	30
3	15
5	5
	1

30 = 2×3×5

∴ denominator is not of the form 2^n5^m where, n = 1 and m = 1. Due to one more factor it is not in the form

Thus, $\frac{77}{210}$ is Non-terminating and repeating decimal.

Q. 3. Write the following rationales in decimal form using Theorem 1.1.

(i) $\frac{13}{25}$

(ii) $\frac{15}{16}$

(iii) $\frac{23}{2^3 \cdot 5^2}$

(iv) $\frac{7218}{3^2 \cdot 5^2}$

(v) $\frac{143}{110}$

Answer : According to Euclid's Division Algorithm,

For any two positive integers, 'a' and 'b', there exists a unique pair of integers 'q' and 'r' which satisfy the relation:

a = bq + r , 0 ≤ r ≤ b

(i)

$\frac{13}{25}$

$$= \frac{13}{5 \times 5}$$

$$= \frac{13}{5^2} = \frac{13 \times 2^2}{2^2 \times 5^2}$$

$$= \frac{13 \times 4}{(2 \times 5)^2}$$

$$= \frac{52}{(10)^2} = 0.52$$

(ii)

$\frac{15}{16}$

$$= \frac{15}{2 \times 2 \times 2 \times 2}$$

$$= \frac{15}{2^4}$$

$$= \frac{15 \times 5^4}{2^4 \times 5^4}$$

$$= \frac{15 \times 625}{(2 \times 5)^4}$$

$$= \frac{9375}{10^4} = 0.9375$$

(iii)

$$\frac{23}{2^3.5^2}$$

$$= \frac{23 \times 5}{2^3.5^3}$$

$$= \frac{115}{(2 \times 5)^3}$$

$$= \frac{115}{10^3} = 0.115$$

(iv)

$$\frac{7218}{3^2 5^2}$$

$$= \frac{802}{5^2}$$

$$= \frac{802 \times 2^2}{2^2 \times 5^2}$$

$$= \frac{3208}{(2 \times 5)^2}$$

$$= \frac{3208}{10^2} = 32.08$$

(v)

$$\frac{143}{110}$$

$$= \frac{143}{2 \times 5 \times 11}$$

$$= \frac{13}{2 \times 5}$$

$$= \frac{13}{10} = 1.3$$

Q. 4. The decimal form of some real numbers is given below. In each case, decide whether the number is rational or not. If it is rational, and expressed in form p/q, what can you say about the prime factors of q?

(i) 43.123456789
(ii) 0.120120012000120000….
(iii) 43.$\overline{123456789}$

Answer : (i) 43.123456789

43.123456789 is terminating.

So, it would be a rational number

$$43.123456789 = \frac{43123456789}{1000000000}$$

$$= \frac{43123456789}{10^9}$$

$$= \frac{43123456789}{(2 \times 5)^9}$$

$$= \frac{43123456789}{2^9 \times 5^9}$$

Hence, 43.123456789 is now in the form of $\frac{p}{q}$.

And the prime factors of q are in terms of 2 and 5.

(ii) 0.120120012000120000….

0.120120012000120000…. is non-terminating and non-repeating.

So, it is not a rational number.

(iii)

(iii) $43.\overline{123456789}$

$43.\overline{123456789}$ is non- terminating but terminating.

So, it would be a rational number.

In a non-terminating, repeating expansion of $\frac{p}{q}$

q will have factors other than 2 or 5.

Exercise 1.4

Q. 1 A. Prove that the following are irrational.

$$\frac{1}{\sqrt{2}}$$

Answer :

Let $\frac{1}{\sqrt{2}}$ be rational. Then, there exists positive co-primes a and b such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\Rightarrow \frac{b}{\sqrt{2}} = a$$

$\Rightarrow b = a\sqrt{2}$

$\Rightarrow \sqrt{2} = \frac{b}{a}$

$\frac{b}{a}$ is rational as a and b are integers

∴√2 is rational which contradicts to the fact that √2 is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

Q. 1 B. Prove that the following are irrational.

$\sqrt{3} + \sqrt{5}$

Answer :

Let us suppose that $\sqrt{3} + \sqrt{5}$ is rational.

Let $\sqrt{3} + \sqrt{5}$ be rational equal to $\frac{a}{b}$, where a and b are integers and a≠0 and b≠0
Then,

$$\sqrt{3} + \sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \sqrt{5} = \frac{a}{b} - \sqrt{3}$$

$$\Rightarrow \left(\sqrt{5}\right)^2 = \left(\frac{a}{b} - \sqrt{3}\right)^2 \text{ [squaring both sides]}$$

$$\Rightarrow 5 = \frac{a^2}{b^2} - \frac{2a\sqrt{3}}{b} + 3$$

$\Rightarrow \frac{a^2-2b^2}{b^2} = 2\sqrt{3}\frac{a}{b}$

$\Rightarrow \sqrt{3} = \frac{a^2-2b^2}{2ab}$

Since a and b are integers, $\frac{a^2-2b^2}{2ab}$ is rational. So, √3 is rational

Now, this contradicts the fact that √3 is rational.

Hence √3 + √5 is irrational.

Q. 1 C. Prove that the following are irrational.

$6+\sqrt{2}$

Answer : 6 + √2

Let 6 + √2 be a rational number equal to $\frac{a}{b}$, where a,b are positive co-primes. Then,

$6 + \sqrt{2} = \frac{a}{b}$

$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$

$\Rightarrow \sqrt{2} = \frac{a-6b}{b}$

Since a and b are integers, $\frac{a-6b}{b}$ is also rational and hence, √2 should be rational. This is contracdicts the fact that√2 is irrational. Therefore , our assumption is false and hence, 6 + √2 is irrational.

Q. 1 D. Prove that the following are irrational.

$\sqrt{5}$

Answer : √5

Let take √5 as rational number equal to $\frac{a}{b}$, where a, b are positive co-primes. Then,

$$\sqrt{5} = \frac{a}{b}$$

⇒ √5 b = a

⇒ $5b^2 = a^2$[squaring both sides] …I

Therefore, 5 divides a^2 and according to theorem of rational number, for any prime number p divides a^2 then it will divide a also.

∴ a = 5c

Put value of a in Eq. I, we get

$5b^2 = (5c)^2$

⇒ $5b^2 = 25c^2$

⇒ $\frac{b^2}{5} = 5c^2$ [divide by 25 both sides]

Using same theorem we get that b will divide by 5 and we have already get that a is divided by 5. This contradicts our assumption.

Hence, √5 is irrational.

Q. 1 E. Prove that the following are irrational.

$3 + 2\sqrt{5}$

Answer : 3 + 2√5

Let us assume on the contrary that 3 + 2√5 is rational. Then, there exist co-prime positive integers a and b such that

$3 + 2\sqrt{5} = \frac{a}{b}$

$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$

$\Rightarrow \sqrt{5} = \frac{a-3b}{2b}$

$\Rightarrow \sqrt{5}$ is rational [$\because$ a,b are integers $\therefore \frac{a-3b}{2b}$ is a rational]

This contradicts the fact that $\sqrt{5}$ is irrational. So, our supposition is incorrect.

Hence, $3 + 2\sqrt{5}$ is an irrational number.

Q. 2. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.

Answer : Let √p + √q be rational

$\Rightarrow$ √p + √q $= \frac{a}{b}$ [where a, b are co-primes and integers]

Squaring both sides

$\Rightarrow (\sqrt{p} + \sqrt{q})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow p + q + 2\sqrt{pq} = \frac{a^2}{b^2}$

⇒ 2√pq = a^2/b^2 -p-q

$\Rightarrow 2\sqrt{pq} = \frac{a^2 - pb^2 - qb^2}{b^2}$

$\Rightarrow \sqrt{p} = \frac{a^2 - pb^2 - qb^2}{2b^2\sqrt{q}}$ I

∴ from Eq. I our assumption contradicts here, because p is rational

Hence, √p + √q is irrational number.

Exercise 1.5

Q. 1 A. Determine the value of the following.

$\log_{25}5$

Answer : The logarithmic form of $\log_{25}5$

Using property of logarithmic, $\log_b x = \frac{\log_a x}{\log_a b}$ and $\log_a a^x = x$

$= \frac{\log_5 5}{\log_5 25}$

$= \frac{1}{\log_5 5^2}$

$= \frac{1}{2} = 0.5$

Q. 1 B. Determine the value of the following.

$\log_{81}3$

Answer : The logarithmic form of $\log_{81}3$

Using property of logarithmic, $\log_b x = \frac{\log_a x}{\log_a b}$ and $\log_a a^x = x$

$= \frac{\log_3 3}{\log_3 81}$

$= \frac{1}{\log_5 3^4}$

$= \frac{1}{4} = 0.25$

Q. 1 C. Determine the value of the following.

$$\log_2\left(\frac{1}{16}\right)$$

Answer : The logarithmic form of $\log_2 \frac{1}{16}$

Using property of logarithmic,

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$= \log_2 1 – \log_2 16$

$= 0 – \log_2 2^4$

$= 0 – 4 = -4$

Q. 1 D. Determine the value of the following.

$\log_7 1$

Answer : The logarithmic form of $\log_7 1$

$= 0$

Q. 1 E. Determine the value of the following.

$$\log_x \sqrt{x}$$

Answer : The logarithmic form of $\log_x \sqrt{x}$

Using the property of logarithmic, $\log_a a^x = x$

$= \log_x x^{1/2}$

$= \frac{1}{2}$

Q. 1 F. Determine the value of the following.

$\log_2 512$

Answer : The Logarithmic form of $\log_2 512$

Using the property of logarithmic, $\log_a a^x = x$

$= \log_2 2^9$

$= 9$

Q. 1 G. Determine the value of the following.

$\log_{10} 0.01$

Answer : The Logarithmic form of $\log_{10} 0.01$

$= \log_{10} \frac{1}{100}$

Using property of logarithmic, $\log_a \frac{x}{y} = \log_a x - \log_a y$ and $\log_a a^x = x$

$= \log_{10} 1 - \log_{10} 100$

$= 0 - \log_{10} 10^2$

$= 0 - 2 = -2$

Q. 1 H.Determine the value of the following.

$\log_{\frac{2}{3}} \left(\frac{8}{27} \right)$

Answer : The Logarithmic form of

$\log_{\frac{2}{3}} \frac{8}{27}$

Using the property of logarithmic, $\log_a a^x = x$

$= \log_{\frac{2}{3}} \frac{2^3}{3^3}$

$= \log_{\frac{2}{3}} \left(\frac{2}{3}\right)^3$

$= 3$

Q. 1 I. Determine the value of the following.

$2^{2 + \log_2 3}$

Answer : The Logarithmic form of $2^{2 + \log_2 3}$

Using property of logarithmic, $a^{m+n} = a^m \times a^n$ and $a^{\log_a x} = x$

$= 2^{2 + \log_2 3}$

$= 2^2 \times 2^{\log_2 3}$

$= 4 \times 3$

$= 12$

Q. 2. Write the following expressions as log N and find their values.

(i) log2 + log5
(ii) $\log_2 16 - \log_2 2$
(iii) 3 $\log_{64} 4$
(iv) 2 log3 – 3 log 2
(v) log 10 + 2 log 3 – log 2

Answer : Some basic logarithmic formulas are

1. $a^{\log_a b} = b$
2. $\log_a 1 = 0$
3. $\log_a a = 1$
4. $\log_a(x \cdot y) = \log_a x + \log_a y$
5. $\log_a xy = \log_a x - \log_a y$
6. $\log_a 1x = -\log_a x$
7. $\log_a x^p = p \log_a x$
8. $\log_{a^k} x = 1k \log_a x$,   for $k \neq 0$

I. log 2 + log 5

using the property of logarithm, $\log_a xy = \log_a x + \log_a y$

= log(2 × 5)

= log 10

II. $\log_2 16 - \log_2 2$

using the property of logarithm,

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

and

$$\log_a x^m = m\log_a x$$

= $\log_2 \frac{16}{2}$

= $\log_2 8$

= $\log_2 2^3$

= $3\log_2 2$ [$\because \log_a x^m = m\log_a x$]

= 3 [$\log_2 2 = 1$]

III. $3 \log_{64} 4$

Using the property of logarithm,
$$\log_a x^m = m\log_a x$$

= $\log_{64} 4^3$

= $\log_{64} 64$

= 1

IV. 2 log 3 - 3 log 2

using the property of logarithm,

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

and

$$\log_a x^m = m\log_a x$$

= $\log 3^2 - \log 2^3$

= $\log \frac{3^2}{2^3}$

= $\log \frac{9}{8}$

V. log 10 + 2 log 3 - log 2

using the property of logarithm, $\log_a xy = \log_a x + \log_a y$

= log 5 + log 2 + $\log 3^2$ - log 2

= log 5 + log 9

= log (5×9)

= log 45

Q. 3. Evaluate each of the following in terms of x and y, if it is given x = $\log_2 3$ and y = $\log_2 5$

(i) $\log_2 15$ (ii) $\log_2 7.5$

(iii) $\log_2 60$ (iv) $\log_2 6750$

Answer :

I. $\log_2 15$

$\Rightarrow \log_2 5 + \log_2 3$ 〗 $[\because \log_a xy = \log_a x + \log_a y]$

⇒ x + y

II. $\log_2 7.5$

$\Rightarrow \log_2 \frac{15}{2}$

$\Rightarrow \log_2 15 - \log_2 2$ [$\because \log_a \frac{x}{y} = \log_a x - \log_a y$]

$\Rightarrow \log_2 5 + \log_2 3 - \log_2 2$ [$\because \log_a xy = \log_a x + \log_a y$]

$\Rightarrow x + y - 1$

III. $\log_2 60$

$\Rightarrow \log_2 12 + \log_2 5$ [$\because \log_a xy = \log_a x + \log_a y$]

$\Rightarrow \log_2 4 + \log_2 3 + \log_2 5$ [$\because \log_a xy = \log_a x + \log_a y$]

$\Rightarrow 2\log_2 2 + \log_2 3 + \log_2 5$

$\Rightarrow 2(1) + x + y = x + y + 2$

IV. $\log_2 6750$

$\Rightarrow \log_2 54 + \log_2 125$〗 [$\because \log_a xy = \log_a x + \log_a y$]

$\Rightarrow \log_2 27 + \log_2 2 + \log_2 125$ 〗 〗

$\Rightarrow \log_2 3^3 + \log_2 5^3 + \log_2 2$

$\Rightarrow 3 \log_2 3 + 3 \log_2 5 + \log_2 2$

$\Rightarrow 3x + 3y + 1$

Q. 4. Expand the following.

(i) $\log 1000$

(ii) $\log\left(\frac{128}{625}\right)$

(iii) $\log x^2 y^3 z^4$

(iv) $\log \frac{p^2 q^3}{r}$

(v) $\log \sqrt{\frac{x^3}{y^2}}$

Answer :

i.

$\log 1000 = \log 10^3$

$= 3\log 10$

ii. $\log\frac{128}{625} = \log\frac{2^7}{5^4}$

⇒ $\log 2^7 - \log 5^4$

⇒ $7\log 2 - 4\log 5$

iii. $\log x^2 y^3 z^4$

⇒ $\log x^2 + + \log y^3 + \log z^4$

⇒ $2\log x + 3\log y + 4\log z$

Q. 5. If $x^2 + y^2 = 25xy$, then prove that $2\log(x + y) = 3\log 3 + \log x + \log y$.

Answer : $x^2 + y^2 = 25xy$

Adding 2xy on both sides

$x^2 + y^2 + 2xy = 27xy$

$(x + y)^2 = 27xy$

Taking log both sides

$\log (x + y)^2 = \log 27xy$

$2 \log (x + y) = \log 27 + \log x + \log y$

$2 \log(x + y) = \log 3^3 + \log x + \log y$

$2 \log (x + y) = 3 \log 3 + \log x + \log y$

Hence proved.

Q. 6.

If $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$, **then find the value of** $\frac{x}{y} + \frac{y}{x}$.

Answer :

$\log\frac{x+y}{3} = \frac{1}{2}(\log x + \log y)$

$\Rightarrow 2 \log\frac{x+y}{3} = \log x + \log y$

$\Rightarrow \log\left(\frac{x+y}{3}\right)^2 = \log xy$ [$\because \log_a xy = \log_a x + \log_a y$] [$\because \log_a x^m = m\log_a x$]

Remove log from both sides

$\Rightarrow \left(\frac{x+y}{3}\right)^2 = xy$

$\Rightarrow \frac{(x+y)^2}{3^2} = xy$

$\Rightarrow x^2 + y^2 + 2xy = 9xy$

$\Rightarrow x^2 + y^2 = 7xy$

Divide by xy both sides

Q. 7. If (2.3) ^{x} = (0.23) ^{y} = 1000 then find the value of

$\frac{1}{x} - \frac{1}{y}$.

Answer : $2.3^x = 0.23^y = 1000$

Consider $2.3^x = 1000$

$\Rightarrow \log 2.3^x = \log 10^3$

$\Rightarrow x \log 2.3 = 3 \log 10$

$\Rightarrow \log 2.3 = \frac{3}{x}$ [$\because \log 10 = 1$] …..I

Consider $0.23^y = 1000$

$\Rightarrow \log 0.23^y = \log 1000$

$\Rightarrow y \log 0.23 = \log 10^3$

$\Rightarrow y \log 0.23 = 3 \log 10$

$\Rightarrow \log 0.23 = \frac{3}{y}$ ……..II

Subtract eq. II from I

$\Rightarrow \log 2.3 - \log 0.23 = \frac{3}{x} - \frac{3}{y}$

$\Rightarrow \log \frac{2.3}{0.23} = \frac{3}{x} - \frac{3}{y}$

$\Rightarrow \log 10 = \frac{3}{x} - \frac{3}{y}$

$\Rightarrow 1 = \frac{3}{x} - \frac{3}{y}$

$\Rightarrow 3\left(\frac{1}{x} - \frac{1}{y}\right) = 1$

$\Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{3}$

Q. 8. If $2^{x+1} = 3^{1-x}$ then find the value of x.

Answer : $2^{(x + 1)} = 3^{(1-x)}$

Taking log base both sides

$\log(2^{(x + 1)}) = \log(3^{(1-x)})$

(x + 1) log 2 = (1 - x) log 3x log 2 + x log 3 = log 3 - log 2x(log 2 + log 3) = log 3 - log 2

x = (log 3 - log 2)/(log 2 + log 3)

Q. 9 A. Is log2 rational or irrational? Justify your answer.

Answer : Assume that log 2 is rational, that is,

$\log 2 = \frac{p}{q}$ ....(1)

where p, q are integers.

Since, $\log 1 = 0 \text{ and} \log 10 = 1, 0 < \log 2 < 1$ and therefore, p<q

From Eq. 1,

$2 = 10^{\frac{p}{q}}$

$2^q = (2\times5)^p$

$2^{(q-p)} = 5^p$

Where p-q is an integer greater than 0.

Now, it can be seen that the L.H.S is even and the R.H.S is odd.

Hence, there is contradiction and log 2 is irrational.

Q. 9 B. log 100 rational or irrational? Justify your answer.

Answer : Let us assume log 100 is rational

$\log 100 = \log_{10} 100$

$\log_{10} 10^2 = 2 \log_{10} 10 = 2$

As 2 is rational number, ∴ log 100 is also rational.