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Mathematical Reasoning

TOPIC 1 Statement, Truth value of a statement, Logical Connectives, Truth Table, Logical Equivalance, Tautology & Contradiction, Duality

1. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to : **[Sep. 06, 2020 (I)]**

(a) $p \wedge \sim q$ (b) $\sim p \wedge \sim q$
(c) $\sim p \vee \sim q$ (d) $\sim p \vee q$

2. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to: **[Sep. 05, 2020 (I)]**

(a) $(x \wedge y) \vee (\sim x \wedge \sim y)$ (b) $(x \wedge y) \wedge (\sim x \vee \sim y)$
(c) $(x \wedge \sim y) \vee (\sim x \wedge y)$ (d) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

3. Given the following two statements :

$(S_1) : (q \vee p) \to (p \leftrightarrow \sim q)$ is a tautology.

$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy. Then : **[Sep. 04, 2020 (I)]**

(a) both (S_1) and (S_2) are correct
(b) only (S_1) is correct
(c) only (S_2) is correct
(d) both (S_1) and (S_2) are not correct

4. The proposition $p \to \sim (p \wedge \sim q)$ is equivalent to : **[Sep. 03, 2020 (I)]**

(a) q (b) $(\sim p) \vee q$
(c) $(\sim p) \wedge q$ (d) $(\sim p) \vee (\sim q)$

5. Let p, q, r be three statements such that the truth value of $(p \wedge q) \to (\sim q \vee r)$ is F. Then the truth values of p, q, r are respectively : **[Sep. 03, 2020 (II)]**

(a) T, F, T (b) T, T, T (c) F, T, F (d) T, T, F

6. If $p \to (p \wedge \sim q)$ is false, then the truth values of p and q are respectively: **[Jan. 9, 2020 (II)]**

(a) F, F (b) T, F (c) T, T (d) F, T

7. Which one of the following is a tautology? **[Jan. 8, 2020 (I)]**

(a) $(p \wedge (p \to q)) \to q$ (b) $q \to (p \wedge (p \to q))$
(c) $p \wedge (p \vee q)$ (d) $p \vee (p \wedge q)$

8. Which of the following statements is a tautology? **[Jan. 8, 2020 (II)]**

(a) $p \vee (\sim q) \to p \wedge q$ (b) $\sim(p \wedge \sim q) \to p \vee q$
(c) $\sim(p \vee \sim q) \to p \wedge q$ (d) $\sim(p \vee \sim q) \to p \vee q$

9. The logical statement

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to: **[Jan. 7, 2020 (I)]**

(a) p (b) q (c) $\sim p$ (d) $\sim q$

10. If the truth value of the statement $p \to (\sim q \vee r)$ is false (F), then the truth values of the statements p, q, r are respectively. **[April 12, 2019 (I)]**

(a) T, T, F (b) T, F, F (c) T, F, T (d) F, T, T

11. The Boolean expression $\sim (p \Rightarrow (\sim q))$ is equivalent to : **[April 12, 2019 (II)]**

(a) $p \wedge q$ (b) $q \Rightarrow \sim p$ (c) $p \vee q$ (d) $(\sim p) \Rightarrow q$

12. Which one of the following Boolean expressions is a tautology ? **[April 10, 2019 (I)]**

(a) $(p \wedge q) \vee (p \wedge \sim q)$ (b) $(p \vee q) \vee (p \vee \sim q)$
(c) $(p \vee q) \wedge (p \vee \sim q)$ (d) $(p \vee q) \wedge (\sim p \vee \sim q)$

13. If $p \Rightarrow (q \vee r)$ is false, then the truth values of p, q, r are respectively: **[April 09, 2019 (II)]**

(a) F, T, T (b) T, F, F (c) T, T, F (d) F, F, F

14. Which one of the following statements is not a tautology? **[April 08, 2019 (II)]**

(a) $(p \vee q) \to (p \vee (\sim q))$ (b) $(p \wedge q) \to (\sim p) \vee q$
(c) $p \to (p \vee q)$ (d) $(p \wedge q) \to p$

15. The Boolean expression

$((p \wedge q) \vee (p \vee \sim q)) \wedge (\sim p \wedge \sim q)$ is equivalent to : **[Jan. 12, 2019 (I)]**

(a) $p \wedge q$ (b) $p \wedge (\sim q)$

(c) $(\sim p) \wedge (\sim q)$ (d) $p \vee (\sim q)$

16. The expression $\sim(\sim p \to q)$ is logically equivalent to : **[Jan. 12, 2019 (II)]**

(a) $\sim p \wedge \sim q$ (b) $p \wedge \sim q$

(c) $\sim p \wedge q$ (d) $p \wedge q$

17. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology? **[Jan. 11, 2019 (I)]**

(a) $(p \vee r) \to (p \wedge r)$ (b) $(p \wedge r) \to (p \vee r)$

(c) $p \wedge r$ (d) $p \vee r$

18. Consider the following three statements:
P : 5 is a prime number.
Q : 7 is a factor of 192.
R : L.C.M. of 5 and 7 is 35.
Then the truth value of which one of the following statements is true? **[Jan. 10, 2019 (II)]**

(a) $(\sim P) \vee (Q \wedge R)$ (b) $(P \wedge Q) \vee (\sim R)$

(c) $(\sim P) \wedge (\sim Q \wedge R)$ (d) $P \vee (\sim Q \wedge R)$

19. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$ then the ordered pair $(\oplus, \odot)$ is: **[Jan. 09, 2019 (I)]**

(a) $(\vee, \wedge)$ (b) $(\vee, \vee)$ (c) $(\wedge, \vee)$ (d) $(\wedge, \wedge)$

20. The logical statement
$[\sim(\sim p \vee q) \vee (p \wedge r)] \wedge (\sim p \wedge r)$
is equivalent to: **[Jan. 09, 2019 (II)]**

(a) $(\sim p \wedge \sim q) \wedge r$ (b) $\sim p \vee r$

(c) $(p \wedge r) \wedge \sim q$ (d) $(p \wedge \sim q) \vee r$

21. The Boolean expression

$\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to : **[2018]**

(a) p (b) q (c) ~q (d) ~p

22. If $p \to (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively. **[Online April 16, 2018]**

(a) T, F (b) F, F (c) F, T (d) T, T

23. If $(p \wedge \sim q) \wedge (p \wedge r) \to \sim p \vee q$ is false, then the truth values of p, q and r are respectively **[Online April 15, 2018]**

(a) F, T, F (b) T, F, T (c) F, F, F (d) T, T, T

24. Which of the following is a tautology? **[2017]**

(a) $(\sim p) \wedge (p \vee q) \to q$ (b) $(q \to p) \vee \sim(p \to q)$

(c) $(\sim q) \vee (p \wedge q) \to q$ (d) $(p \to q) \wedge (q \to p)$

25. The following statement
$(p \to q) \to [(\sim p \to q) \to q]$ is : **[2017]**

(a) a fallacy (b) a tautology

(c) equivalent to $\sim p \to q$ (d) equivalent to $p \to \sim q$

26. The proposition $(\sim p) \vee (p \wedge \sim q)$ **[Online April 8, 2017]**

(a) $p \to \sim q$ (b) $p \wedge (\sim q)$

(c) $q \to p$ (d) $p \vee (\sim q)$

27. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to: **[2016]**

(a) $p \vee q$ (b) $p \vee \sim q$ (c) $\sim p \wedge q$ (d) $p \wedge q$

28. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[2015]**

(a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$

(c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$

29. The statement $\sim(p \leftrightarrow \sim q)$ is: **[2014]**

(a) a tautology

(b) a fallacy

(c) eqivalent to $p \leftrightarrow q$

(d) equivalent to $\sim p \leftrightarrow q$

30. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is: **[Online April 12, 2014]**

(a) $(p \vee q) \Rightarrow r$ (b) $(p \Rightarrow q) \vee (p \Rightarrow r)$

(c) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$ (d) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

31. The proposition $\sim(p \vee \sim q) \vee \sim(p \vee q)$ is logically equivalent to: **[Online April 11, 2014]**

(a) p (b) q (c) $\sim p$ (d) $\sim q$

32. Consider
Statement-1 : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement-2 : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology. **[2013]**

(a) Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is false; Statement-2 is true.

33. Let p and q be any two logical statements and $r : p \to (\sim p \vee q)$. If r has a truth value F, then the truth values of p and q are respectively : **[Online April 25, 2013]**

(a) F, F (b) T, T (c) T, F (d) F, T

34. For integers m and n, both greater than 1, consider the following three statements :
P : m divides n
Q : m divides n^2
R : m is prime,
then **[Online April 23, 2013]**
(a) $Q \wedge R \rightarrow P$ (b) $P \wedge Q \rightarrow R$
(c) $Q \rightarrow R$ (d) $Q \rightarrow P$

35. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to : **[Online April 22, 2013]**
(a) $p \rightarrow q$ (b) $p \rightarrow (p \vee q)$
(c) $p \rightarrow (p \rightarrow q)$ (d) $p \rightarrow (p \wedge q)$

36. **Statement-1:** The statement $A \rightarrow (B \rightarrow A)$ is equivalent to $A \rightarrow (A \vee B)$.
Statement-2: The statement $\sim [(A \wedge B) \rightarrow (\sim A \vee B)]$ is a Tautology. **[Online April 9, 2013]**
(a) Statement-1 is false; Statement-2 is true.
(b) Statement-1 is true; Statement-2 is true; Statement- 2 is **not** correct explanation for Statement-1.
(c) Statement-1 is true; Statement-2 is false.
(d) Statement-1 is true; Statement-2 is true; Statement- 2 is the correct explanation for Statement-1.

37. Let p and q be two Statements. Amongst the following, the Statement that is equivalent to $p \rightarrow q$ is **[Online May 19, 2012]**
(a) $p \wedge \sim q$ (b) $\sim p \vee q$ (c) $\sim p \wedge q$ (d) $p \vee \sim q$

38. The logically equivalent preposition of $p \Leftrightarrow q$ is **[Online May 12, 2012]**
(a) $(p \Rightarrow q) \wedge (q \Rightarrow p)$ (b) $p \wedge q$
(c) $(p \wedge q) \vee (q \Rightarrow p)$ (d) $(p \wedge q) \Rightarrow (q \vee p)$

39. The only statement among the following that is a tautology is **[2011RS]**
(a) $A \wedge (A \vee B)$ (b) $A \vee (A \wedge B)$
(c) $[A \wedge (A \rightarrow B)] \rightarrow B$ (d) $B \rightarrow [A \wedge (A \rightarrow B)]$

40. **Statement-1 :** $\sim (p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.
Statement-2 : $\sim (p \leftrightarrow \sim q)$ is a tantology **[2009]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is false.
(c) Statement-1 is false, Statement-2 is true.
(d) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement -1

41. The statement $p \rightarrow (q \rightarrow p)$ is equivalent to **[2008]**
(a) $p \rightarrow (p \rightarrow q)$ (b) $p \rightarrow (p \vee q)$
(c) $p \rightarrow (p \wedge q)$ (d) $p \rightarrow (p \leftrightarrow q)$

42. Let p be the statement "x is an irrational number", q be the statement "y is a transcendental number", and r be the statement "x is a rational number iff y is a transcendental number". **[2008]**
Statement-1 : r is equivalent to either q or p
Statement-2 : r is equivalent to $\sim(p \leftrightarrow \sim q)$.
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

TOPIC 2 Converse, Inverse & Contrapositive of the Conditional Statement, Negative of a Compound Statement, Algebra of Statement

43. Consider the statement: "For an integer n, if $n^3 - 1$ is even, then n is odd." The contrapositive statement of this statement is: **[Sep. 06, 2020 (II)]**
(a) For an integer n, if n is even, then $n^3 - 1$ is odd.
(b) For an intetger n, if $n^3 - 1$ is not even, then n is not odd.
(c) For an integer n, if n is even, then $n^3 - 1$ is even.
(d) For an integer n, if n is odd, then $n^3 - 1$ is even.

44. The statement $(p \rightarrow (q \rightarrow p)) \rightarrow (p \rightarrow (p \vee q))$ is : **[Sep. 05, 2020 (II)]**
(a) equivalent to $(p \wedge q) \vee (\sim q)$
(b) a contradiction
(c) equivalent to $(p \vee q) \wedge (\sim p)$
(d) a tautology

45. Contrapositive of the statement :
'If a function f is differentiable at a, then it is also continuous at a', is : **[Sep. 04, 2020 (II)]**
(a) If a function f is continuous at a, then it is not differentiable at a.
(b) If a function f is not continuous at a, then it is not differentiable at a.
(c) If a function f is not continuous at a, then it is differentiable at a
(d) If a function f is continuous at a, then it is differentiable at a.

46. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is : **[Sep. 02, 2020 (I)]**
(a) If I do not reach the station in time, then I will catch the train.
(b) If I do not reach the station in time, then I will not catch the train.
(c) If I will catch the train, then I reach the station in time.
(d) If I will not catch the train, then I do not reach the station in time.

47. Negation of the statement:

$\sqrt{5}$ is an integer of 5 is irrational is: **[Jan. 9, 2020 (I)]**

(a) $\sqrt{5}$ is not an integer or 5 is not irrational

(b) $\sqrt{5}$ is not an integer and 5 is not irrational

(c) $\sqrt{5}$ is irrational or 5 is an integer.

(d) $\sqrt{5}$ is an integer and 5 is irrational

48. Let A, B, C and D be four non-empty sets. The contrapositive statement of "If $A \subseteq B$ and $B \subseteq D$, then $A \subseteq C$" is: **[Jan. 7, 2020 (II)]**

(a) If $A \not\subseteq C$, then $A \subseteq B$ and $B \subseteq D$

(b) If $A \subseteq C$, then $B \subset A$ or $D \subset B$

(c) If $A \not\subseteq C$, then $A \not\subseteq B$ and $B \subseteq D$

(d) If $A \not\subseteq C$, then $A \not\subseteq B$ or $B \not\subseteq D$

49. The negation of the Boolean expression $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[April 10, 2019 (II)]**

(a) $\sim s \wedge \sim r$ (b) r

(c) $s \vee r$ (d) $s \wedge r$

50. For any two statements p and q, the negation of the expression $p \vee (\sim p \wedge q)$ is: **[April 9, 2019 (I)]**

(a) $\sim p \wedge \sim q$ (b) $p \wedge q$

(c) $p \leftrightarrow q$ (d) $\sim p \vee \sim q$

51. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is : **[April 8, 2019 (I)]**

(a) If you are not a citizen of India, then you are not born in India.

(b) If you are a citizen of India, then you are born in India.

(c) If you are born in India, then you are not a citizen of India.

(d) If you are not born in India, then you are not a citizen of India.

52. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal". is : **[Jan. 11, 2019 (II)]**

(a) If the squares of two numbers are not equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are equal, then the numbers are equal.

(d) If the squares of two numbers are not equal, then the numbers are not equal.

53. Consider the following two statements.

Statement p**:**

The value of sin 120° can be divided by taking $\theta = 240°$ in the equation $2 \sin \frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$.

Statement q**:**

The angles A, B, C and D of any quadrilateral $ABCD$ satisfy the equation $\cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$

Then the truth values of p and q are respectively. **[Online April 15, 2018]**

(a) F, T (b) T, T (c) F, F (d) T, F

54. Contrapositive of the statement

'If two numbers are not equal, then their squares are not equal', is : **[Online April 9, 2017]**

(a) If the squares of two numbers are equal, then the numbers are equal.

(b) If the squares of two numbers are equal, then the numbers are not equal.

(c) If the squares of two numbers are not equal, then the numbers are not equal.

(d) If the squares of two numbers are not equal, then the numbers are equal.

55. The contrapositive of the following statement,

"If the side of a square doubles, then its area increases four times", is : **[Online April 10, 2016]**

(a) If the area of a square increases four times, then its side is not doubled.

(b) If the area of a square increases four times, then its side is doubled.

(c) If the area of a square does not increases four times, then its side is not doubled.

(d) If the side of a square is not doubled, then its area does not increase four times.

56. Consider the following two statements :

P : If 7 is an odd number, then 7 is divisible by 2.

Q : If 7 is a prime number, then 7 is an odd number.

If V_1 is the truth value of the contrapositive of P and V_2 is the truth value of contrapositive of Q, then the ordered pair (V_1, V_2) equals: **[Online April 9, 2016]**

(a) (F, F) (b) (F, T) (c) (T, F) (d) (T, T)

57. Consider the following statements :

P : Suman is brilliant

Q : Suman is rich.

R : Suman is honest

the negation of the statement

"Suman is brilliant and dishonest if and only if suman is rich" can be equivalently expressed as : **[Online April 11, 2015]**

(a) $\sim Q \leftrightarrow \sim P \vee R$ (b) $\sim Q \leftrightarrow \sim P \wedge R$

(c) $\sim Q \leftrightarrow P \vee \sim R$ (d) $\sim Q \leftrightarrow P \wedge \sim R$

58. The contrapositive of the statement “If it is raining, then I will not come”, is : **[Online April 10, 2015]**
(a) If I will not come, then it is raining.
(b) If I will not come, then it is not raining.
(c) If I will come, then it is raining.
(d) If I will come, then it is not raining.

59. The contrapositive of the statement “if I am not feeling well, then I will go to the doctor” is **[Online April 19, 2014]**
(a) If I am feeling well, then I will not go to the doctor
(b) If I will go to the doctor, then I am feeling well
(c) If I will not go to the doctor, then I am feeling well
(d) If I will go to the doctor, then I am not feeling well.

60. The contrapositive of the statement “I go to school if it does not rain” is **[Online April 9, 2014]**
(a) If it rains, I do not go to school.
(b) If I do not go to school, it rains.
(c) If it rains, I go to school.
(d) If I go to school, it rains.

61. The negation of the statement "If I become a teacher, then I will open a school", is : **[2012]**
(a) I will become a teacher and I will not open a school.
(b) Either I will not become a teacher or I will not open a school.
(c) Neither I will become a teacher nor I will open a school.
(d) I will not become a teacher or I will open a school.

62. Let p and q denote the following statements
p : The sun is shining
q: I shall play tennis in the afternoon
The negation of the statement “If the sun is shining then I shall play tennis in the afternoon”, is **[Online May 26, 2012]**
(a) $q \Rightarrow \sim p$
(b) $q \wedge \sim p$
(c) $p \wedge \sim q$
(d) $\sim q \Rightarrow \sim p$

63. The Statement that is TRUE among the following is **[Online May 7, 2012]**
(a) The contrapositive of $3x+2=8 \Rightarrow x=2$ is $x \neq 2 \Rightarrow 3x+2 \neq 8$.
(b) The converse of $\tan x=0 \Rightarrow x=0$ is $x \neq 0 \Rightarrow \tan x=0$.
(c) $p \Rightarrow q$ is equivalent to $p \vee \sim q$.
(d) $p \vee q$ and $p \wedge q$ have the same truth table.

64. Let S be a non-empty subset of R. Consider the following statement :
P : There is a rational number $x \in$ S such that $x > 0$.
Which of the following statements is the negation of the statement P ? **[2010]**
(a) There is no rational number $x \in$ S such than $x \leq 0$.
(b) Every rational number $x \in$ S satisfies $x \leq 0$.
(c) $x \in$ S and $x \leq 0 \Rightarrow x$ is not rational.
(d) There is a rational number $x \in$ S such that $x \leq 0$.

Hints & Solutions

1. **(b)** Negation of given statement $= \sim(p \vee (\sim p \wedge q))$

$= \sim p \wedge \sim(\sim p \wedge q) = \sim p \wedge (p \vee \sim q)$

$= (\sim p \wedge q) \vee (\sim p \wedge \sim q)$

$= F \vee (\sim p \wedge \sim q) = \sim p \wedge \sim q$

2. **(a)** $p : x \leftrightarrow \sim y = (x \to \sim y) \wedge (\sim y \to x)$

$= (\sim x \vee \sim y) \wedge (y \vee x)$

$= \sim(x \wedge y) \wedge (x \vee y)$ $\quad (\because \sim(x \wedge y) = \sim x \vee \sim y)$

Negation of p is

$\sim p = (x \wedge y) \vee \sim(x \vee y) = (x \wedge y) \vee (\sim x \wedge \sim y)$

3. **(d)** The truth table of both the statements is

p	q	~p	~q	q ∨ p	p↔~q	(S1)	~p↔q	(S2)
T	T	F	F	T	F	F	F	F
T	F	F	T	T	T	T	T	T
F	T	T	F	T	T	T	T	F
F	F	T	T	F	F	T	F	F

∴ S_1 is not tautology and
S_2 is not fallacy.
Hence, both the statements (S_1) and (S_2) are not correct.

4. **(b)**

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$p \to \sim(p \wedge \sim q)$	$\sim p \vee q$
T	T	F	F	F	T	T
T	F	T	T	F	F	F
F	T	F	F	T	T	T
F	F	T	F	T	T	T

$\therefore p \to \sim(p \wedge \sim q)$ is equivalent to $\sim p \vee q$

5. **(d)** $(p \wedge q) \to (\sim q \vee r)$

$= \sim(p \wedge q) \vee (\sim q \vee r)$

$= (\sim p \vee \sim q) \vee (\sim q \vee r)$

$= (\sim p \vee \sim q \vee r)$

$\because (\sim p \vee \sim q \vee r)$ is false, then $\sim p$, $\sim q$ and r all these must be false.

$\Rightarrow p$ is true, q is true and r is false.

6. **(c)**

p	q	$\sim q$	$p \wedge \sim q$	$p \to (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

7. **(a)**

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$	$q \to p \wedge (p \to q)$	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T	T	T
F	T	T	F	T	F	F	F	T	F
F	F	T	F	T	T	F	F	F	F

8. **(d)** $(\sim p \wedge q) \to (p \vee q)$

$\Rightarrow \sim\{(\sim p \wedge q) \wedge (\sim p \wedge \sim q)\}$

$\Rightarrow \sim\{\sim p \wedge f\}$

9. **(c)**

p	q	$p \Rightarrow q$	$\sim p$	$q \Rightarrow \sim p$	$(p \Rightarrow q) \wedge (p \Rightarrow \sim q)$
T	T	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	T	T

Clearly $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to $\sim p$

10. **(a)** Given statement $p \to (\sim q \vee r)$ is False.

$\Rightarrow$ p is True and $\sim q \vee r$ is False

$\Rightarrow$ p is True and $\sim q$ is False and r is False

$\therefore$ truth values of p, q r are T, T, F respectively.

11. **(a)** Given Boolean expression is,

$\sim(p \Rightarrow (\sim q))$ $\quad \{\because p \Rightarrow q$ is same as $\sim p \vee q\}$

$\equiv \sim((\sim p) \vee (\sim q)) \equiv p \wedge q$

12. **(b)** $(p \vee q) \vee (p \vee \sim q) = p \vee (q \vee p) \vee \sim q$

$= (p \vee p) \vee (q \vee \sim q) = p \vee T = T$

Hence first statement is tautology.

13. **(b)** For $p \Rightarrow q \vee r$ to be F.

r should be F and $p \Rightarrow q$ should be F

for $p \Rightarrow q$ to be F, $p \Rightarrow$ T and $q \Rightarrow$ F

$p, q, r \equiv$ T, F, F

14. **(a)** By truth table :

p	q	$\sim q$	$pv \sim q$	$\sim p$	$p \wedge \sim q$	pvq	$p \rightarrow pvq$	$p \wedge q$	$(p \wedge q) \rightarrow p$	$\sim pvq$
T	T	F	T	F	F	T	T	T	T	T
T	F	T	T	F	T	T	T	F	T	F
F	T	F	F	T	F	T	T	F	T	T
F	F	T	T	T	F	F	T	F	T	T

$(p \wedge q) \rightarrow (\sim p) \vee q$	$(p \vee q) \rightarrow (p \vee (\sim q))$
T	T
T	T
T	F
T	T

15. **(c)** Consider the Boolean expression

$((p \wedge q) \vee (pv \sim q)) \wedge (\sim p \wedge \sim q)$

$= (p \vee \sim q) \wedge (\sim p \wedge \sim q)$

$= ((p \vee \sim q) \wedge \sim p) \wedge ((p \vee \sim q) \wedge \sim q)$

$= ((p \wedge \sim p) \vee (\sim q \wedge \sim p)) \wedge \sim q$

$= (\sim p \wedge \sim q) \wedge \sim q = (\sim p \wedge \sim q)$

16. **(a)** $\sim (\sim p \rightarrow Q) \equiv \sim (p \vee q) \equiv \sim p \wedge \sim q$

17. **(b)** q is false and $[(p \wedge q) \leftrightarrow r]$ is true

As $(p \wedge q)$ is false

[False $\leftrightarrow r$] is true

Hence r is false

Option (a): says $p \vee r$,

Since r is false

Hence $(p \vee r)$ can either be true or false

Option (b): says $(p \wedge r) \rightarrow (p \vee r)$

$(p \wedge r)$ is false

Since, $F \rightarrow T$ is true and

$F \rightarrow F$ is also true

Hence, it is a tautology

Option (c): $(p \vee r) \rightarrow (p \wedge r)$

i.e. $(p \vee r) \rightarrow F$

It can either be true or false

Option (d): $(p \wedge r)$,

Since, r is false

Hence, $(p \wedge r)$ is false.

18. **(d)** P is True, Q is False and R is True

(a) $(\sim P) \vee (Q \wedge R) \equiv F \vee (F \wedge T) \equiv F \vee F = F$

(b) $(P \wedge Q) \vee (\sim R) \equiv (T \wedge F) \vee (F) \equiv F \vee F = F$

(c) $(\sim P) \wedge (\sim Q \wedge R) \equiv F \wedge (T \wedge T) \equiv F \wedge T = F$

(d) $P \vee (\sim Q \wedge R) \equiv T \vee (T \wedge T) \equiv T \vee T = T$

19. **(c)** Check each option

(a) $(p \vee q) \wedge (\sim p \wedge q) = (\sim p \wedge q)$

(b) $(p \vee q) \wedge (\sim p \vee q) = q$

(c) $(p \wedge q) \wedge (\sim p \vee q) = p \wedge q$

(d) $(p \wedge q) \wedge (\sim p \wedge q) = F$

20. **(c)** Logical statement,

$= [\sim (\sim p \vee q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \vee (p \wedge r)] \wedge (\sim q \wedge r)$

$= [(p \wedge \sim q) \wedge (\sim q \wedge r)] \vee [(p \wedge r) \wedge (\sim q \wedge r)]$

$= [p \wedge \sim q \wedge r] \vee [p \wedge r \wedge \sim q]$

$= (p \wedge \sim q) \wedge r$

$= (p \wedge r) \wedge \sim q$

21. **(d)** $\sim (p \vee q) \vee (\sim p \wedge q)$

$\Rightarrow (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\Rightarrow \sim p \wedge (\sim q \vee q)$

$\Rightarrow \sim p \wedge t \equiv \sim p$

22. **(d)**

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \rightarrow (\sim p \vee \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

From the truth table,

$p \rightarrow (\sim p \vee \sim q)$ is false only when p and q both are true.

23. **(b)** As the truth table for the $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then only possible values of (p, q, r) is (T, F, T)

p	q	r	$\sim q$	$p \wedge \sim q$	$p \wedge r$	$\sim p$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (p \wedge r)$	$(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$
T	T	T	F	F	T	F	T	F	T
T	F	T	T	T	T	F	F	T	F
T	T	F	F	F	F	F	T	F	T
F	T	T	F	F	F	T	T	F	T
F	F	T	T	F	F	T	T	F	T
F	T	F	F	F	F	T	T	F	T
T	F	F	T	T	F	F	F	F	T
F	F	F	T	F	F	T	T	F	T

24. **(a)** Truth table

p	q	$\sim p$	$p \vee q$	$(\sim p) \wedge (p \vee q)$	$(\sim p) \wedge (p \vee q) \to q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

$\therefore$ (a) $\sim p \wedge (p \vee q) \to q$ be a tautology

Other options are not tautology.

25. **(b)** We have

p	q	~ p	p → q	~ p → q	(~ p → q) → q	(p → q) → ((~p→q)→ q)
T	F	F	F	T	F	T
T	T	F	T	T	T	T
F	F	T	T	F	T	T
F	T	T	T	T	T	T

$\therefore$ It is tautology.

26. **(b)** $(\sim p) \vee (p \wedge \sim q)$

p	q	~ p	~ q	p∧ ~ q	(~ p)∨(p ∧ ~ q)
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F

27. **(a)** $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \wedge (\sim q \vee q)\} \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \wedge T\} \vee (\sim p \wedge q)$

$\Rightarrow (p \vee q) \vee (\sim p \wedge q)$

$\Rightarrow \{(p \vee q) \vee \sim p\} \wedge (p \vee q \vee q)$

$\Rightarrow T \wedge (p \vee q)$

$\Rightarrow p \vee q$

28. **(b)** $\sim[\sim s \vee (\sim r \wedge s)]$

$= s \wedge \sim(\sim r \wedge s)$

$= s \wedge (r \vee \sim s)$

$= (s \wedge r) \vee (s \wedge \sim s)$

$= (s \wedge r) \vee f$

$= s \wedge r$

29. **(c)**

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim (p \leftrightarrow \sim q)$ (i)	$p \leftrightarrow q$ (ii)
F	F	T	F	T	T
F	T	F	T	F	F
T	F	T	T	F	F
T	T	F	F	T	T

From column (i) and (ii) are equivalent.

Clearly equivalent to $p \leftrightarrow q$

30. **(b)** Given statement is

$p \Rightarrow (q \vee r)$ which is equivalent to

$(p \Rightarrow q) \vee (p \Rightarrow r)$

31. **(c)** Given $\sim (p \vee \sim q) \vee \sim (p \vee q)$

$\equiv (\sim p \vee q) \vee (\sim p \vee \sim q)$

$\equiv \sim p \vee (q \vee \sim q)$

$\equiv \sim p$

32. **(b)** **Statement-2 :** $(p \to q) \leftrightarrow (\sim q \to \sim p)$

$\equiv (p \to q) \leftrightarrow (p \to q)$

which is always true.

So, statement 2 is true

Statement-1: $(p \wedge \sim q) \wedge (\sim p \wedge q)$

$= p \wedge \sim q \wedge \sim p \wedge q$

$= p \wedge \sim p \wedge \sim q \wedge q$

$= f \wedge f = f$

So statement-1 is true

33. **(c)** $p \to (\sim p \vee q)$ has truth value F.

It means $p \to (\sim p \vee q)$ is false.

It means p is true and $\sim p \vee q$ is false.

$\Rightarrow$ p is true and both $\sim p$ and q are false.

$\Rightarrow$ p is true and q is false.

34. **(a)**

(b) $\frac{8}{4} = 2, \frac{64}{4} = 16$; but 4 is not prime.

Hence P $\wedge$ Q $\to$ R, false

(c) $\frac{(6)^2}{12} = \frac{36}{12} = 3$; but 12 is not prime

Hence Q $\to$ R, false

(d) $\frac{(4)^2}{8} = \frac{16}{8} = 2$; $\frac{4}{8}$ is not an integer

Hence Q $\to$ P, false

35. **(b)**

q	p	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

Since truth value of $p \to (q \to p)$ and $p \to (p \vee q)$ are same, hence $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

36. **(c)**

A	B	$\sim A$	$A \wedge B$	$\sim A \vee B$	$(A \wedge B) \to (\sim A \vee B)$	$\sim[(A \wedge B) \to (\sim A \vee B)]$
T	T	F	T	T	T	F
T	F	F	F	F	T	F
F	T	T	F	T	T	F
F	F	T	F	T	T	F

37. **(b)** Let p and q be two statements.
$p \to q$ is equivalent to $\sim p \vee q$.

38. **(a)** $(p \Rightarrow q) \wedge (q \Rightarrow p)$ means $p \Leftrightarrow q$

39. **(c)** Truth table of all options is as follows.

A	B	$A \vee B$	$A \wedge B$	$A \wedge (A \vee B)$	$A \vee (A \wedge B)$	$A \to B$	$A \wedge (A \to B)$	$[A \wedge (A \to B) \to B]$	$[B \to [A \wedge (A \to B)]]$
T	F	T	F	T	T	F	F	T	T
F	T	T	F	F	F	T	F	T	F
T	T	T	T	T	T	T	T	T	T
F	F	F	F	F	F	T	F	T	T

$\therefore$ It is tautology.

40. **(b)** The truth table for the logical statements, involved in statement 1, is as follows :

p	q	$\sim q$	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$ (i)	$p \leftrightarrow q$ (ii)
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

We observe the columns (i) and (ii) are identical, therefore $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$

But $\sim(p \leftrightarrow \sim q)$ is not a tautology as all entries in its column are not T.

$\therefore$ Statement-1 is true but statement-2 is false.

41. **(b)** The truth table for the given statements, as follows :

p	q	$p \vee q$	$q \to p$	$p \to (q \to p)$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table we observe that

$p \to (q \to p)$ is equivalent to $p \to (p \vee q)$

42. **(None)**

Given that

p : x is an irrational number

q : y is a transcendental number

r : x is a rational number iff y is a transcendental number.

clearly $r : \sim p \leftrightarrow q$

Truth table to check the equivalence of 'r' and 'q or p'; 'r' and $\sim(p \leftrightarrow \sim q)$

p	q	$\sim p$	$\sim q$	$\sim p \leftrightarrow q$ (i)	q or p (ii)	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$ (iii)
T	T	F	F	F	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	F
F	F	T	T	F	F	F	T

From columns (i), (ii) and (iii), we observe, that none of the these statements are equivalent to each other.

$\therefore$ Statement 1as well as statement 2 both are false.

$\therefore$ None of the options is correct.

43. **(a)** Contrapositive statement will be

"For an integer n, if n is not odd then $n^3 - 1$ is not even".

or

"For an integer n, if n is even then $n^3 - 1$ is odd".

44. **(d)** The truth table of $(p \to (q \to p)) \to (p \to (p \vee q))$ is

p	q	$p \vee q$	$p \to (p \vee q)$	$q \to p$	$p \to (q \to p)$	$(p \to (q \to p)) \to (p \to (p \vee q))$
T	T	O	T	T	T	T
T	F	T	T	T	T	T
F	T	T	T	F	T	T
F	F	F	T	T	T	T

Hence, the statement is tautology.

45. **(b)** Contrapositive statement will be "If a function is not continuous at 'a', then it is not differentiable at 'a'.

46. **(d)** Contrapositive of $p \to q$ is $\sim q \to \sim p$

i.e. contrapositive of 'if p then q' is 'if not q then not p'.

47. **(b)** Let p and q the statements such that $p = \sqrt{5}$ is an integer $q = 5$ is an irrational number.

Then, negation of the given statement

$\sqrt{5}$ is not an integer and 5 is not an irrational Number

$\sim(p \vee q) = \sim p \wedge \sim q$

48. **(d)** Let $P = A \subseteq B, Q = B \subseteq D, R = A \subseteq C$

Contrapositive of $(P \wedge Q) \to R$ is $\sim R \to \sim (P \wedge Q)$

$\sim R \to \sim P \vee \sim Q$

49. **(d)** $\sim s \vee (\sim r \wedge s) \equiv (\sim s \vee \sim r) \wedge (\sim s \vee s)$

$\equiv (\sim s \vee \sim r)$ $\quad$ ($\because \sim s \vee s$) is tautology)

$\equiv \sim (s \wedge r)$

Hence, its negation is $s \wedge r$.

50. **(d)** $\sim(p \vee (\sim p \wedge q)) = \sim(\sim p \wedge q) \wedge \sim p$
$= (\sim q \vee p) \wedge \sim p$
$= \sim p \wedge (p \vee \sim q)$
$= (\sim q \wedge \sim p) \vee (p \wedge \sim p)$
$= (\sim p \wedge \sim q)$

51. **(a)** S: ''If you are born in India, then you are a citizen of India.''
Contrapositive of $p \to q$ is $\sim q \to \sim p$
So contrapositive of statement S will be :
''If you are not a citizen of India, then you are not born in India.''

52. **(c)** Contrapositive of "If A then B" is "If ~B then ~A". Hence contrapositive of "If two numbers are not equal, then their squares are not equal" is "If squares of two numbers are equal, then the two numbers are equal".

53. **(a)** **Statement p:**

$$\sin 120° = \cos 30° = \frac{\sqrt{3}}{2} \Rightarrow 2\sin 120° = \sqrt{3}$$

So, $\sqrt{1+\sin 240°} - \sqrt{1-\sin 240°}$

$$= \sqrt{\frac{1-\sqrt{3}}{2}} - \sqrt{\frac{1+\sqrt{3}}{2}} \neq \sqrt{3}$$

Statement q:

$$\text{So, } A+B+C+D = 2\pi \Rightarrow \frac{A+C}{2} + \frac{B+D}{2} = \pi$$

$$\Rightarrow \cos\left(\frac{A+C}{2}\right) + \cos\left(\frac{B+D}{2}\right)$$

$$= \cos\left(\frac{A+C}{2}\right) - \cos\left(\frac{A+C}{2}\right) = 0$$

Therefore, statement p is false and statement q is true.

54. **(a)** $p \to q$
then $\sim q \to \sim p$
$\therefore$ If the square of two numbers are equal, then the numbers are equal.

55. **(c)** Contrapositive of $p \to q$ is given by $\sim q \to \sim p$
So (c) is the right option.

56. **(a)** Contrapositive of P :
T is not divisible by 2 $\Rightarrow$ T is not odd number
$T \Rightarrow F : F(V_1)$
Contra positive Q :
T is not odd number $\Rightarrow$ T is not a prime number
$F \Rightarrow F : T(V_2)$

57. **(d)** Suman is brilliant and dishonest can be expressed as $P \wedge \sim R$
therefore given statement is equal to $(P \wedge \sim R) \leftrightarrow Q$
Negation of the above statement is $\sim Q \leftrightarrow P \wedge \sim R$

58. **(d)** The centre positive of the statement is "If i will come, then it is not raining".

59. **(c)** Given statement can be written in implication form as
I am not feeling well $\Rightarrow$ I will go to the doctor.
Contrapositive form :
I will not go to the doctor $\Rightarrow$ I am feeling well.
i.e. If I will not go to the doctor, then I am feeling well.

60. **(b)** let p = If it does not rain
q = I go to school
According to law of contrapositive
$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$
i.e. $\sim q$ = I do not go to school
$\sim p$ = It rains
$\sim q \Rightarrow \sim p$ is If I do not go to school, it rains.

61. **(a)** Let p : I become a teacher.
q : I will open a school
Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$
i.e. I will become a teacher and I will not open a school.

62. **(c)** Let p : The sun is shining.
q : I shall play tennis in the afternoon.
Negation of $p \to q$ is $\sim(p \to q) = p \wedge \sim q$

63. **(a)** Only statement given in option
(a) is true.
(b) The converse of $\tan x = 0 \Rightarrow x = 0$ is
$x = 0 \Rightarrow \tan x = 0$
$\therefore$ Statement (b) is false
(c) $\sim(p \Rightarrow q)$ is equivalent to $p \wedge \sim q$
$\therefore$ Statement given in option (c) is false.
(d) No, $p \vee q$ and $p \wedge q$ does not have the same truth value.

64. **(b)** Given that P : there is a rational number $x \in S$ such that $x > 0$.
$\sim$P : Every rational number $x \in S$ satisfies $x \leq 0$.