Quadratic Equations

Question 1.

Solve the following (1 to 12) equations:

(i) $x^2 - 11x + 30 = 0$

(ii) $4x^2 - 25 = 0$

Solution:

$(i)\ x^2 - 11x + 30 = 0$

$$x^2 - 5x - 6x + 30 = 0 \quad \left\{\begin{array}{c}\because 30 = -5 \times (-6) \\ -11 = -5 - 6\end{array}\right\}$$

$\Rightarrow x(x-5) - 6(x-5) = 0$

$\Rightarrow (x-5)(x-6) = 0$

Either, $x - 5 = 0$, then $x = 5$

or $x - 6 = 0$, then $x = 6$

$\therefore\ x = 5, 6$

$(ii)\ 4x^2 - 25 = 0 \Rightarrow 4x^2 = 0 + 25$

$$\Rightarrow x^2 = \frac{25}{4}$$

$$\therefore\ x = \pm\sqrt{\frac{25}{4}} = \pm\frac{5}{2}$$

$$\therefore\ x = \frac{5}{2}, \frac{-5}{2}$$

Question 2.

(i) $2x^2 - 5x = 0$

(ii) $x^2 - 2x = 48$

Solution:

$(i)\ 2x^2 - 5x = 0$

$x(2x - 5) = 0$

Either, $x = 0$

or $2x - 5 = 0$, then $2x = 5$

$$\Rightarrow x = \frac{5}{2}$$

$$\therefore x = 0, \frac{5}{2}$$

$(ii)\ x^2 - 2x = 48$

$\Rightarrow x^2 - 2x - 48 = 0$

$\Rightarrow x^2 - 8x + 6x - 48 = 0 \quad \left\{\because -48 = -8 \times 6,\ -2 = -8 + 6\right\}$

$\Rightarrow x(x - 8) + 6(x - 8) = 0$

$\Rightarrow (x - 8)(x + 6) = 0$

Either, $x - 8 = 0$, then $x = 8$

or $x + 6 = 0$, then $x = -6$

$\therefore x = 8, -6$

Question 3.

(i) $6 + x = x^2$

(ii) $2x^2 + 3x + 1 = 0$

Solution:

$(i)\ 6 + x = x^2$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0 \quad \left\{\because -6 = -3 \times 2,\ -1 = -3 + 2\right\}$

$\Rightarrow x(x - 3) + 2(x - 3) = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

Either, $x - 3 = 0$, then $x = 3$

or $x + 2, = 0$, then $x = -2$

$\therefore x = 3, -2$

$(ii)\ 2x^2 - 3x + 1 = 0$

$\Rightarrow 2x^2 - 2x - x + 1 = 0$

$\Rightarrow 2x(x - 1) - 1(x - 1) = 0$

$\Rightarrow (x - 1)(2x - 1) = 0$

Either, $x - 1 = 0$, then $x = 1$

or $2x - 1 = 0$, then $2x = 1$

$\Rightarrow x = \frac{1}{2}$.

$\therefore x = 1, \frac{1}{2}$

Question 4.

(i) $3x^2 = 2x + 8$

(ii) $4x^2 + 15 = 16x$

Solution:

$(i)\ 3x^2 = 2x + 8$

$\Rightarrow 3x^2 - 2x - 8 = 0$

$$\Rightarrow 3x^2 - 6x + 4x - 8 = 0 \qquad \left\{\begin{array}{l}\because -8 \times 3 = -24 \\ -24 = -6 \times 4 \\ -2 = -6 + 4\end{array}\right\}$$

$\Rightarrow 3x(x - 2) + 4(x - 2) = 0$

$\Rightarrow (x - 2)(3x + 4) = 0$

Either, $x - 2 = 0$, then $x = 2$

or $3x + 4 = 0$, then $3x = -4$

$$\Rightarrow x = \frac{-4}{3}$$

$$\therefore x = 2, \frac{-4}{3}$$

$(ii)\ 4x^2 + 15 = 16x$

$\Rightarrow 4x^2 - 16x + 15 = 0$

$\Rightarrow 4x^2 - 6x - 10x + 15 = 0$

$$\left\{\begin{array}{l}\because 4 \times 15 = 60 \\ -16 = -6 + (-10) \\ -16 = -6 - 10\end{array}\right\}$$

$\Rightarrow 2x(2x - 3) - 5(2x - 3) = 0$

$\Rightarrow (2x - 3)(2x - 5) = 0$

Either, $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \frac{3}{2}$

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \frac{5}{2}$

$$\therefore x = \frac{3}{2}, \frac{5}{2}$$

Question 5.

(i) x (2x + 5) = 25

(ii) (x +3) (x – 3) = 40

Solution:

$(i)\ x(2x + 5) = 25$

$\Rightarrow 2x^2 + 5x - 25 = 0$

$\Rightarrow 2x^2 + 10x - 5x - 25 = 0$

$$\left\{\begin{array}{l}\because -25 \times 2 = -50 \\ -50 = 10 \times (-5) \\ 5 = 10 - 5\end{array}\right\}$$

$\Rightarrow 2x(x + 5) - 5(x + 5) = 0$

$\Rightarrow (x + 5)(2x - 5) = 0$

Either, $x + 5 = 0$, then $x = -5$

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \frac{5}{2}$

$\therefore\ x = -5, \frac{5}{2}$

$(ii)\ (x + 3)(x - 3) = 40$

$\Rightarrow x^2 - 9 = 40 \Rightarrow x^2 - 9 - 40 = 0$

$\Rightarrow x^2 - 49 = 0 \Rightarrow (x)^2 - (7)^2 = 0$

$\Rightarrow (x + 7)(x - 7) = 0$

Either, $x + 7 = 0$, then $x = -7$

or $x - 7 = 0$, then $x = 7$

$\therefore\ x = 7, -7$

Question 6.

(i) (2x + 3) (x – 4) = 6

(ii) (3x + 1) (2x + 3) = 3

Solution:

$(i)\ (2x + 3)(x - 4) = 6$

$\Rightarrow 2x^2 - 8x + 3x - 12 - 6 = 0$

$\Rightarrow 2x^2 - 5x - 18 = 0$

$\Rightarrow 2x^2 - 9x + 4x - 18 = 0$

$$\left\{\begin{array}{l}\because -18 \times 2 = -36 \\ \therefore -36 = -9 \times 4 \\ \quad -5 = -9 + 4\end{array}\right\}$$

$\Rightarrow x(2x - 9) + 2(2x - 9) = 0$

$\Rightarrow (2x - 9)(x + 2) = 0$

Either, $2x - 9 = 0$, then $2x = 9 \Rightarrow x = \frac{9}{2}$

or $x + 2 = 0$, then $x = -2$

$\therefore\ x = \frac{9}{2}, -2$

$(ii)\ (3x + 1)(2x + 3) = 3$

$\Rightarrow 6x^2 + 9x + 2x + 3 - 3 = 0$

$\Rightarrow 6x^2 + 11x = 0$

$\Rightarrow x(6x + 11) = 0$

Either, $x = 0$,

or $6x + 11 = 0$, then $6x = -11 \Rightarrow x = \frac{-11}{6}$

$\therefore\ x = 0, \frac{-11}{6}$

Question 7.

(i) $4x^2 + 4x + 1 = 0$

(ii) $(x - 4)^2 + 5^2 = 132$

Solution:

$(i)\ 4x^2 + 4x + 1 = 0$

$\Rightarrow 4x^2 + 2x + 2x + 1 = 0$

$\Rightarrow 2x(2x + 1) + 1(2x + 1) = 0$

$\Rightarrow (2x + 1)(2x + 1) = 0$

Either, $2x + 1 = 0$, then $x = \frac{-1}{2}$

$\therefore\ x = \frac{-1}{2}, \frac{-1}{2}$

$(ii)\ (x - 4)^2 + 5^2 = 13^2$

$\Rightarrow x^2 - 8x + 16 + 25 = 169$

$x^2 - 8x + 16 + 25 - 169 = 0$

$x^2 - 8x - 128 = 0$

$x^2 - 16x + 8x - 128 = 0 \left\{\begin{array}{l}\because -128 = -16 \times 8 \\ -8 = -16 + 8\end{array}\right\}$

$x(x - 16) + 8(x - 16) = 0$

$(x - 16)(x + 8) = 0$

Either, $x - 16 = 0$, then $x = 16$

or $x + 8 = 0$, then $x = -8$

$\therefore\ x = 16, -8$

Question 8.

(i) $21x^2 = 4(2x + 1)$

(ii) $\frac{2}{3}x2 - \frac{1}{3}x - 1 = 0$

Solution:

$(i)\ 21x^2 = 4(2x + 1)$

$\Rightarrow 21x^2 = 8x + 4$

$\Rightarrow 21x^2 - 8x - 4 = 0$

$\Rightarrow 21x^2 - 14x + 6x - 4 = 0$ $\left\{\begin{array}{l}\because 21 \times (-4) = -84 \\ \therefore -84 = -14 \times 6 \\ -8 = -14 + 6\end{array}\right\}$

$\Rightarrow 7x(3x - 2) + 2(3x - 2) = 0$

$\Rightarrow (3x - 2)(7x + 2) = 0$

Either, $3x - 2 = 0$, then $3x = 2 \Rightarrow x = \frac{2}{3}$

or $7x + 2 = 0$, then $7x = -2 \Rightarrow x = \frac{-2}{7}$

$\therefore x = \frac{2}{3}, \frac{-2}{7}$

$(ii)\ \frac{2}{3}x^2 - \frac{1}{3}x - 1 = 0$

$\Rightarrow 2x^2 - x - 3 = 0$

$\Rightarrow 2x^2 - 3x + 2x - 3 = 0$

$\Rightarrow x(2x - 3) + 1(2x - 3) = 0$

$\Rightarrow (2x - 3)(x + 1) = 0$

Either, $2x - 3 = 0$, then $2x = 3 \Rightarrow x = \frac{3}{2}$

or $x + 1 = 0$, then $x = -1$

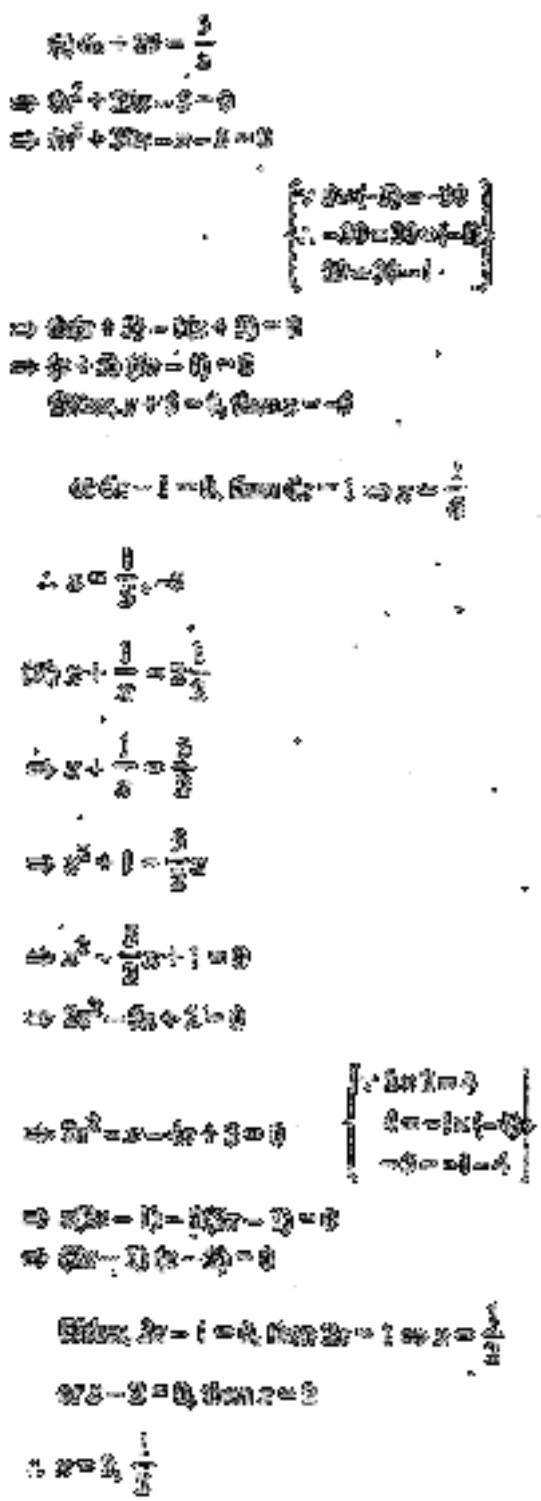
$\therefore x = \frac{3}{2}, -1$

Question 9.

(i) $6x + 29 = \frac{5}{x}$

(ii) $x + \frac{1}{x} = 2\frac{1}{2}$

Solution:

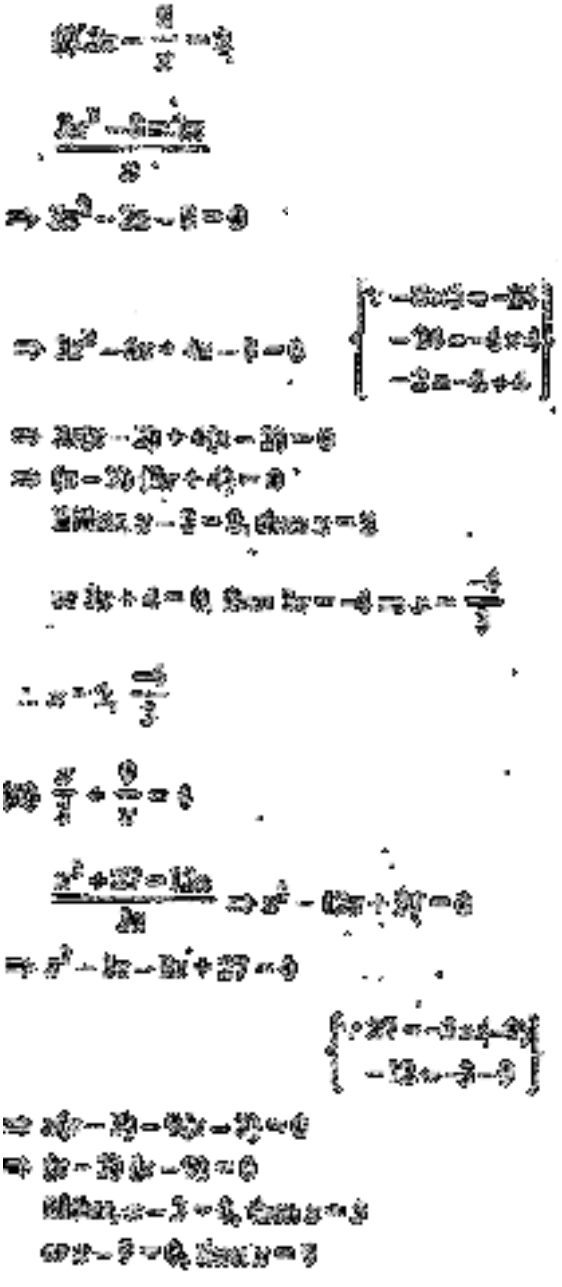


Question 10.

(*i*) $3x - \frac{8}{x} = 2$

(*ii*) $\frac{x}{3} + \frac{9}{x} = 4$

Solution:



Question 11.

(*i*) $\frac{x-1}{x+1} = \frac{2x-5}{3x-7}$

(*ii*) $\frac{1}{x+2} + \frac{1}{x} = \frac{3}{4}$

Solution:

(*i*) $\frac{x-1}{x+1} = \frac{2x-5}{3x-7}$

By cross multiplication,

$(x - 1)(3x - 7) = (x + 1)(2x - 5)$

$\Rightarrow 3x^2 - 7x - 3x + 7 = 2x^2 - 5x + 2x - 5$

$3x^2 - 10x + 7 = 2x^2 - 3x - 5 = 0$

$3x^2 - 10x + 7 - 2x^2 + 3x + 5 = 0$

$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x^2 - 4x - 3x + 12 = 0$

$\Rightarrow x(x-4) - 3(x-4) = 0$

$$\left\{\begin{array}{l}\because 12 = -4 \times (-3) \\ -7 = -4 - 3\end{array}\right\}$$

$\Rightarrow (x-4)(x-3) = 0$

Either, $x - 4 = 0$, then $x = 4$

or $x - 3 = 0$, then $x = 3$

$\therefore x = 3, 4$

(*ii*) $\dfrac{1}{x+2} + \dfrac{1}{x} = \dfrac{3}{4}$

$$\frac{x+x+2}{x(x+2)} = \frac{3}{4} \Rightarrow \frac{2x+2}{x(x+2)} = \frac{3}{4}$$

By cross multiplication,

$\Rightarrow 3x(x+2) = 4(2x+2)$

$\Rightarrow 3x^2 + 6x = 8x + 8$

$\Rightarrow 3x^2 + 6x - 8x - 8 = 0$

$\Rightarrow 3x^2 - 2x - 8 = 0$

$\Rightarrow 3x^2 - 6x + 4x - 8 = 0$

$$\left\{\begin{array}{l}\because 3 \times (-8) = -24 \\ \therefore -24 = -6 \times 4 \\ -2 = -6 + 4\end{array}\right\}$$

$\Rightarrow 3x(x-2) + 4(x-2) = 0$

$\Rightarrow (x-2)(3x+4) = 0$

Either, $x - 2 = 0$, then $x = 2$

or $3x + 4 = 0$, then $3x = -4 \Rightarrow x = \dfrac{-4}{3}$

Hence, $x = 2, \dfrac{-4}{3}$

Question 12.

(*i*) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

(*ii*) $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{6}$

Solution:

(*i*) $\frac{8}{x+3} - \frac{3}{2-x} = 2$

$\Rightarrow \frac{8(2-x)-3(x+3)}{(x+3)(2-x)} = \frac{2}{1}$

$\Rightarrow \frac{16-8x-3x-9}{2x-x^2+6-3x} = \frac{2}{1}$

$\Rightarrow \frac{7-11x}{-x^2-x+6} = \frac{2}{1}$

$\Rightarrow 7 - 11x = -2x^2 - 2x + 12$

(By cross multiplication)

$\Rightarrow 2x^2 + 2x - 12 + 7 - 11x = 0$

$\Rightarrow 2x^2 - 9x - 5 = 0$

$\Rightarrow 2x^2 - 10x + x - 5 = 0$

$\left\{\begin{array}{l}\because 2\times(-5) = -10 \\ \therefore -10 = -10\times 1 \\ \quad -9 = -10 + 1\end{array}\right\}$

$\Rightarrow 2x(x - 5) + 1(x - 5) = 0$

$\Rightarrow (x - 5)(2x + 1) = 0$

Either, $x - 5 = 0$, then $x = 5$

or $2x + 1 = 0$, then $2x = -1 \Rightarrow x = \frac{-1}{2}$

or $2x + 1 = 0$, then $2x = -1 \Rightarrow x = \frac{-1}{2}$

$\therefore\ x = 5, \frac{-1}{2}$

(*ii*) $\frac{x}{x+1} + \frac{x+1}{x} = 2\frac{1}{6}$

$$\frac{x^2 + (x+1)^2}{x(x+1)} = \frac{13}{6}$$

$$\Rightarrow \frac{x^2 + x^2 + 2x + 1}{x^2 + x} = \frac{13}{6}$$

$$\Rightarrow \frac{2x^2 + 2x + 1}{x^2 + x} = \frac{13}{6}$$

$13x^2 + 13x = 12x^2 + 12x + 6$

$\Rightarrow 13x^2 + 13x - 12x^2 - 12x - 6 = 0$

$\Rightarrow x^2 + x - 6 = 0$

$\Rightarrow x^2 + 3x - 2x - 6 = 0 \qquad \left\{\begin{array}{c}\because -6 = 3 \times (-2) \\ 1 = 3 - 2\end{array}\right\}$

$\Rightarrow x(x + 3) - 2(x + 3) = 0$

$\Rightarrow (x + 3)(x - 2) = 0$

Either, $x + 3 = 0$, then $x = -3$

or $x - 2 = 0$, then $x = 2$

$\therefore\ x = 2, -3$

Multiple Choice Questions

Choose the correct Solution from the given four options (1 to 5):

Question 1.

Which of the following is not a quadratic equation :

(a) $2x^2 = 3x - 5$

(b) $(2x- 1)(x- 1) = 2x^2 - 7x + 2$

(c) $(2x - 1)(x + 2) = (x - 1)(x + 1)$

(d) $(x+ 1), = x, + 2x+2$

Solution:

$(2x - 1)(x - 1) = 2x^2 - 7x + 2$ is not a quadratic equation. **(b)**

Question 2.

If 2 is a root of the quadratic equation $2x^2 - kx + 1 = 0$, then the value of k is

(a) 9 **(b) –9**

(c) $\frac{9}{2}$ **(d) $-\frac{9}{2}$**

Solution:

$\because$ 2 is root of $2x^2 - kx + 1 = 0$

$\therefore\ 2(2)^2 - k(2) + 1 = 0$

$8 - 2k + 1 = 0 = 9 = 2k$

$\Rightarrow k = \frac{9}{2}$ (c)

Question 3.

If -3 is a root of the quadratic equation $kx^2 + 2x - 3 = 0$, then the value of k is

(a) 1

(b) -1

(c) $\frac{1}{9}$

(d) $\frac{1}{-9}$

Solution:

-3 is a root of quadratic equation.

$kx^2 + 2x - 3 = 0$

$\Rightarrow k(-3)^2 + 2(-3) - 3 = 0$

$\Rightarrow 9k - 6 - 3 = 0 \Rightarrow 9k = 9$

$\Rightarrow k = \frac{9}{9} = 1$ (a)

Question 4.

Which of the following quadratic equations has -1 as a root?

(a) $x^2 + 5x + 6 = 0$

(b) $2x^2 - 3x + 1 = 0$

(c) $2x^2 + x - 3 = 0$

(d) $2x^2 - x - 3 = 0$

Solution:

$\because -1$ is a root of the quadratic equation.

$\therefore$ It will satisfy it

$2x^2 - x - 3 = 0$

$\Rightarrow 2(-1)^2 - (-1) - 3 = 0$

$\Rightarrow 2 + 1 - 3 = 0$

$\Rightarrow 3 - 3 = 0$ (d)

Question 5.

The root of the quadratic equation $x^2 - 3x - 4 = 0$ are

(a) -4, 1

(b) 4, -1

(c) 4, 1

(d) -4, -1

Solution:

$x^2 - 3x - 4 = 0$

$\Rightarrow x^2 - 4x + x - 4 = 0 \Rightarrow x(x - 4) + 1(x - 4)$

$\Rightarrow (x - 4)(x + 1) = 0$

If $x - 4 = 0$, then $x = 4$

$x + 1 = 0$, then $x = -1$

$\therefore x = 4, -1$ (b)

Chapter Test

Solve the following (1 to 3) equations:

Question 1.

(i) x(2x+ 5) = 3

(ii) $3x^2 - 4x - 4 = 0$

Solution:

$(i)\ x(2x + 5) = 3$

$$\Rightarrow 2x^2 + 5x - 3 = 0 \qquad \left\{\begin{array}{l}\because 2\times(-3) = -6\\ \therefore -6 = 6\times(-1)\\ \quad 5 = 6 - 1\end{array}\right\}$$

$$\Rightarrow 2x^2 + 6x - x - 3 = 0$$

$$\Rightarrow 2x(x + 3) - 1(x + 3) = 0$$

$$\Rightarrow (x + 3)(2x - 1) = 0$$

Either, $x + 3 = 0$, then $x = -3$

or $2x - 1 = 0$, then $2x = 1 \Rightarrow x = \frac{1}{2}$

$$\therefore x = -3, \frac{1}{2}$$

$(ii)\ 3x^2 - 4x - 4 = 0$

$$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$$

$$\left\{\begin{array}{l}\because 3\times(-4) = -12\\ \therefore -12 = -6\times 2\\ \quad -4 = -6 + 2\end{array}\right\}$$

$$\Rightarrow 3x(x - 2) + 2(x - 2) = 0$$

$$\Rightarrow (x - 2)(3x + 2) = 0$$

Either, $x - 2 = 0$, then $x = 2$

or $3x + 2 = 0$, then $3x = -2 \Rightarrow x = \frac{-2}{3}$

$$\therefore x = 2, \frac{-2}{3}$$

Question 2.

(i) $4x^2 - 2x + \frac{1}{4} = 0$

(ii) $2x^2 + 7x + 6 = 0$

Solution:

$$(i)\ 4x^2 - 2x + \frac{1}{4} = 0$$

$$\Rightarrow 16x^2 - 8x + 1 = 0$$

$$\Rightarrow 16x^2 - 8x + 1 = 0$$

$$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$$

$$\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$$

$$\Rightarrow (4x - 1)(4x - 1) = 0 \Rightarrow (4x - 1)^2 = 0$$

$$\Rightarrow 4x - 1 = 0 \Rightarrow 4x = 1$$

$$\therefore x = \frac{1}{4}, \frac{1}{4}$$

$$(ii)\ 2x^2 + 7x + 6 = 0$$

$$\Rightarrow 2x^2 + 4x + 3x + 6 = 0 \qquad \left\{\begin{array}{l}\because 2 \times 6 = 12 \\ \therefore 12 = 3 \times 4 \\ 7 = 3 + 4\end{array}\right\}$$

$$\Rightarrow 2x(x + 2) + 3(x + 2) = 0$$

$$\Rightarrow (x + 2)(2x + 3) = 0$$

Either, $x + 2 = 0$, then $x = -2$

or $2x + 3 = 0$, then $2x = -3 \Rightarrow x = \frac{-3}{2}$

$$\therefore x = -2, \frac{-3}{2}$$

Question 3.

(*i*) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$

(*ii*) $\frac{6}{x}-\frac{2}{x-1}=\frac{1}{x-2}$

Solution:

(*i*) $\frac{x-1}{x-2}+\frac{x-3}{x-4}=3\frac{1}{3}$

$$\frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)}=\frac{10}{3}$$

$$\Rightarrow \frac{x^2-5x+4+x^2-5x+6}{x^2-6x+8}=\frac{10}{3}$$

$$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8}=\frac{10}{3}$$

$\Rightarrow 10x^2-60x+80=6x^2-30x+30$

$\Rightarrow 10x^2-60x+80-6x^2+30x-30=0$

$\Rightarrow 4x^2-30x+50=0$

$\Rightarrow 2x^2-15x+25=0$

$\Rightarrow 2x^2-10x-5x+25=0$

$$\left\{\begin{array}{l}\because 2\times 25=50\\ \therefore 50=-10\times(-5)\\ \quad -15=-10-5\end{array}\right\}$$

$\Rightarrow 2x(x-5)-5(x-5)=0$

$\Rightarrow (x-5)(2x-5)=0$

Either, $x - 5 = 0$, then $x = 5$

or $2x - 5 = 0$, then $2x = 5 \Rightarrow x = \frac{5}{2}$

$\therefore\ x = 5, \frac{5}{2}$

(*ii*) $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x-2}$

$$\frac{6x-6-2x}{x(x-1)} = \frac{1}{x-2}$$

$$\Rightarrow \frac{4x-6}{x^2-x} = \frac{1}{x-2}$$

$\Rightarrow (4x - 6)(x - 2) = x^2 - x$

(By cross multiplication)

$\Rightarrow 4x^2 - 8x - 6x + 12 = x^2 - x$

$\Rightarrow 4x^2 - 14x + 12 - x^2 + x = 0$

$\Rightarrow 3x^2 - 13x + 12 = 0$

$\Rightarrow 3x^2 - 4x - 9x + 12 = 0$

$$\left\{\begin{array}{l}\because 3\times 12 = 36\\ \therefore 36 = (-4)\times(-9)\\ -13 = -4 - 9\end{array}\right\}$$

$\Rightarrow x(3x - 4) - 3(3x - 4) = 0$

$\Rightarrow (3x - 4)(x - 3) = 0$

Either, $3x - 4 = 0$, then $3x = 4 \Rightarrow x = \frac{4}{3}$

or $x - 3 = 0$, then $x = 3$

$\therefore\ x = 3, \frac{4}{3}$