MATHEMATICS

1. Sum of series $\frac{1}{1!\,50!} + \frac{1}{3!\,48!} + ... + \frac{1}{51!\,0!}$ is

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{49}}{51!}$ (3) $\frac{2^{51}}{51!}$ (4) $\frac{2^{52}}{51!}$

Ans. **(1)**

Sol. $\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)} \frac{1}{51!}$

$= \frac{1}{51!} \{{}^{51}C_1 + {}^{51}C_3 + ... + {}^{51}C_{51}) = \frac{1}{51!}(2^{50})$

2. $\sum_{r=1}^{10} \frac{r}{1+r^2+r^4}$ is equal to

(1) $\frac{56}{111}$ (2) $\frac{57}{111}$ (3) $\frac{55}{111}$ (4) $\frac{58}{111}$

Ans. **(3)**

Sol. $T_r = \frac{(r^2+r+1)-(r^2-r+1)}{2(r^4+r^2+1)}$

$\Rightarrow T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$

$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$

$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$

$T_3 = \frac{1}{2}\left[\frac{1}{7} - \frac{1}{13}\right]$

$\vdots$

$T_{10} = \frac{1}{2}\left[\frac{1}{91} - \frac{1}{111}\right]$

$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$

3. The value of $\lim_{n\to\infty} \left(\frac{1}{1+n} + \frac{1}{2+n} + \frac{1}{3+n} + ... + \frac{1}{2n}\right)$ is

(1) ℓn 4 (2) ℓn 2 (3) ℓn 3 (4) ℓn 5

Ans. **(2)**

Sol. $\lim_{n\to\infty}\left(\frac{1}{1+n}+.....+\frac{1}{n+n}\right) = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$

$= \int_0^1 \frac{1}{1+x}dx = [\ell n(1+x)]_0^1 = \ell n2$

4. If y = f(x) satisfies $\frac{dy}{dx}+y\tan x = x\sec x$ and y(0) = 1, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}$ (2) $\frac{\pi}{12}+\frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}$

(3) $\frac{\pi}{12}+\frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$ (4) $\frac{\pi}{6}+\frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}$

Ans. (3)

Sol. Here I.F. = sec x

Then solution of D.E :

y(sec x) = x tan x – ln(sec x) + c

Given y(0) = 1 ⇒ c = 1

∴ y(sec x) = x tan x – ln(sec x) + 1

At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}$

5. Find the number of ways in which the word ASSASSINATION can be arrange such that all vowels come together.

(1) 50400 (2) 50200 (3) 51400 (4) 50000

Ans. (1)

Sol. Vowels : A,A,A,I,I,O

Consonants : S,S,S,S,N,N,T

∵ Total number of ways in which vowels come together

$= \frac{\underline{|8}}{\underline{|4}\,\underline{|2}}\times\frac{\underline{|6}}{\underline{|3}\,\underline{|2}}$

= 50400

6. If system of equations

λx + y + z = 1

x + λy + z = 1

x + y + λz = 1 is inconsistent then find $\sum\left(|\lambda^2|+|\lambda|\right)$

Ans. (6)

Sol. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$(\lambda + 2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$(\lambda + 2)[1(\lambda^2 - 1) - 1(\lambda - 1) + (1 - \lambda)] = 0$

$(\lambda + 2)[(\lambda^2 - 2\lambda + 1) = 0$

$(\lambda + 2)(\lambda - 1)^2 = 0 \quad \Rightarrow \quad \lambda = -2, \lambda = 1$

at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$

so $\sum\left(|-2^2| + |-2|\right) = 6$

7. Negation of $p \vee \sim (p \vee \sim q)$ is equivalent to

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim (p \wedge q)$ (4) $\sim p \wedge \sim q$

Ans. **(4)**

Sol. negation of given statement

$\sim (p \vee \sim(p \vee \sim q)) \equiv \sim p \wedge (p \vee \sim q)$

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$

$\equiv f \vee (\sim p \wedge \sim q)$

$\equiv \sim p \wedge \sim q$

8. For a binomial distribution B(n, p), sum and product of mean & variance is 5 & 6 respectively, then find $6(n + p - q)$

Ans. **(52)**

Sol. $np + npq = 5,\ np \,.\, npq = 6$

$np\,(1 + q) = 5,\ n^2p^2q = 6$

$n^2p^2(1 + q)^2 = 25,\ n^2p^2q = 6$

$\frac{6}{q}(1 + q)^2 = 25$

$6q^2 + 12q + 6 = 25q$

$6q^2 - 13q + 6 = 0$

$6q^2 - 9q - 4q + 6 = 0$

$(3q - 2)\,(2q - 3) = 0$

$q = \frac{2}{3}, \frac{3}{2}, q = \frac{2}{3}$ is accepted

$p = \frac{1}{3} \Rightarrow n \,.\, \frac{1}{3} + n \,.\, \frac{1}{3} \,.\, \frac{2}{3} = 5$

$\frac{3n + 2n}{9} = 5$

$n = 9$

so $6(n + p - q) = 6\left(9 + \frac{1}{3} - \frac{2}{3}\right) = 52$

9. If n-numbers $a_1, a_2, \ldots\ldots, a_n$ are in A.P. such that its first term is 8, sum of first four terms is 50 and sum of last four terms is 170, then the value of $(a_7.a_8)$ is equal to

Ans. (754)

Sol. $a_1 + a_2 + a_3 + a_4 = 50$

$\Rightarrow 32 + 6d = 50$

$\Rightarrow d = 3$

and, $a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$

$\Rightarrow 32 + (4n - 10).3 = 170$

$\Rightarrow n = 14$

$a_7 = 26, a_8 = 29$

$\Rightarrow a_7.a_8 = 754$

10. A relation on real numbers defined by R = $\{3a - 3b + \sqrt{7}$ is Irrational , $a, b \in R\}$ then relation R is

(1) Reflexive, Symmetric and Transitive

(2) Reflexive but not Symmetric and Transitive

(3) Reflexive and Transitive but not Symmetric

(4) Equivalence

Ans. (2)

Sol. Reflexive (a, a)

$3a - 3a + \sqrt{7} = \sqrt{7}$ is irrational

so reflexive true

Symmetric $(a, b) \leftrightarrow (b, a)$

$3a - 3b + \sqrt{7}$ is irrational

$3b - 3a + \sqrt{7}$ Irrational, not always true

so not symmetric

Transitive $(a, b), (b, c) \in R \nRightarrow (a, c) \in R$ so not transitive

11 Remainder when $23^{200} + 19^{200}$ is divided by 49

(1) 21 (2) 29 (3) 31 (4) 39

Ans. (2)

Sol. $(21 + 2)^{200} + (21 - 2)^{200}$

$\Rightarrow 2[{}^{200}C_0\, 21^{200} + {}^{200}C_2\, 21^{198} . 2^2 + \ldots\ldots + {}^{200}C_{198}\, 21^2 . 2^{198} + 2^{200}]$

$\Rightarrow 2[49I_1 + 2^{200}] = 49I_1 + 2^{201}$

Now, $2^{201} = (8)^{67} = (1 + 7)^{67} = 49I_2 + {}^{67}C_0 + {}^{67}C_1 . 7 = 49I_2 + 470 = 49I_2 + 49 \times 9 + 29$

$\therefore$ Remainder is 29

12. If 1, 3, 5, a, b have mean 5 and variance 8 then the value of $a^3 + b^3$ is

(1) 1072 (2) 1702 (3) 1027 (4) 1207

Ans. (1)

Sol. $\frac{1+3+5+a+b}{5} = 5$

$a + b = 16$ .... (1)

$\sigma^2 = \frac{\sum x_1^2}{5} - \left(\frac{\sum x}{5}\right)^2$

$8 = \frac{1^2+3^2+5^2+a^2+b^2}{5} - 25$

$a^2 + b^2 = 130$ ...... (2)

by (1), (2)

$a = 7, b = 9$

or $a = 9, b = 7$

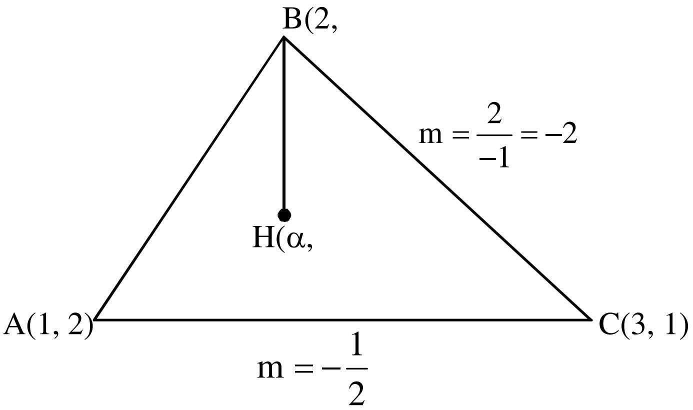
13. Let A(1,2), B(2, 3) and C(3, 1) are vertices of ΔABC and orthocentre of ΔABC is (α, β) then quadratic equation whose roots are $(\alpha + 4\beta)$ and $(\beta + 4\alpha)$ is

(1) $x^2 - 9x + 11 = 0$ (2) $x^2 - 11x + 9 = 0$

(3) $x^2 - 20x + 99 = 0$ (4) $x^2 - 9x + 20 = 0$

Ans. **(3)**

Sol.



Here $m_{BH}\, m_{AC} = -1$

$\left(\frac{\beta-3}{\alpha-2}\right)\left(\frac{1}{-2}\right) = -1$

$\beta - 3 = 2\alpha - 4$

$\beta = 2\alpha - 1$ …(i)

$m_{AH}\, m_{BC} = -1$

$\Rightarrow \left(\frac{\beta-2}{\alpha-1}\right)(-2) = -1$ $\Rightarrow 2\beta - 4 = \alpha - 1$

$\Rightarrow 2(2\alpha - 1) = \alpha + 3$

$\Rightarrow 3\alpha = 5$

$\alpha = \frac{5}{3},\ \beta = \frac{7}{3} \Rightarrow H\left(\frac{5}{3}, \frac{7}{3}\right)$

$\alpha + 4\beta = \frac{5}{3} + \frac{28}{3} = \frac{33}{3} = 11$

$\beta + 4\alpha = \frac{7}{3} + \frac{20}{3} = \frac{27}{3} = 9$

$x^2 - 20x + 99 = 0$

14. If $y = f(x) = x|x-3|$; $x \in [-1, 2]$ and area bounded by $y = f(x)$ in $x \in [-1, 2]$ is A, then 12A is

Ans. **(62)**

Sol. $A = \int_{-1}^{0} (x^2 - 3x)\,dx + \int_{0}^{2} (3x - x^2)\,dx$

$\Rightarrow A = \left.\frac{x^3}{3} - \frac{3x^2}{2}\right|_{-1}^{0} + \left.\frac{3x^2}{2} - \frac{x^3}{3}\right|_{0}^{2}$

$\Rightarrow A = \frac{11}{6} + \frac{10}{3} = \frac{31}{6}$

$\therefore$ 12A = 62

15. For the circle $\left|\frac{z-1}{z-3}\right| = 2$ if center of circle is (α, β) & radius is γ find $\alpha + \beta + \gamma$.

(1) 4 (2) 5 (3) $\frac{5}{2}$ (4) $\frac{5}{3}$

Ans. **(2)**

Sol. $\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$x^2 + y^2 - 2x + 1 = 4x^2 + 4y^2 - 24x + 36$

$x^2 + y^2 - \frac{22}{3}x + \frac{35}{3} = 0$

$\alpha + \beta + \gamma = \frac{11}{3} + 0 + \frac{4}{3} = 5$

16. If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = 2x + f'(1)$, then $f(4) - g(4)$ is equal to

Ans. **(12)**

Sol. $g'(x) = 2$, $g''(x) = 0$

$\Rightarrow f(x) = x^2 + 2x$

$\Rightarrow f'(x) = 2x + 2$

$\Rightarrow f'(1) = 4$

$\therefore g(x) = 2x + 4$

$f(4) - g(4) = 12$

17. Let $S = \left\{x : \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} + \left(\sqrt{3}-\sqrt{2}\right)^{x^2-4} = 10\right\}$, then n(s) is

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **(3)**

Sol. Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow x^2 - 4 = 2, -2$ or $x^2 = 6, 2$

$\Rightarrow x = \pm\sqrt{2}, \pm\sqrt{6}$

18. If $f(x) + f'(x) = \int_0^2 f(t)\,dt$ and $f(0) = e^{-2}$, then the value of $f(2) - 2f(0)$ is -

(1) 0 (2) 1 (3) –1 (4) 2

Ans. **(3)**

Sol. $\frac{dy}{dx} + y = k$

$y \cdot e^x = k \cdot e^x + c$

$f(0) = e^{-2}$

$\Rightarrow c = e^{-2} - k$

$\therefore y = k + \left(e^{-2} - k\right)e^{-x}$

now $k = \int_0^2 \left(k + \left(e^{-2} - k\right)e^{-x}\right)dx$

$\Rightarrow k = e^{-2} - 1$

$\therefore y = \left(e^{-2} - 1\right) + e^{-x}$

$f(2) = 2e^{-2} - 1,\ f(0) = e^{-2}$

$f(2) - 2f(0) = -1$

19. The number of 3-digit numbers divisible by 2 or 3 but not by 7 is

Ans. **(514)**

Sol. Divisible by 2 $\rightarrow$ 450

Divisible by 3 $\rightarrow$ 300

Divisible by 7 $\rightarrow$ 128

Divisible by 2 & 7 $\rightarrow$ 64

Divisible by 3 & 7 $\rightarrow$ 43

Divisible by 2 & 3 $\rightarrow$ 150

Divisible by 2, 3 & 7 $\rightarrow$ 21

$\therefore$ Total numbers = 450 + 300 – 150 – 64 – 43 + 21 = 514

20. A triangle is given such that cos2A + cos2B + cos2C is minimum. If radius of incenter is 3 cm then which of the following is incorrect ? (where O is incenter)

(1) $\overrightarrow{OB}.\overrightarrow{OA} = -18$

(2) Area of $\Delta ABC = \dfrac{27\sqrt{3}}{2}$

(3) sin A + sin B + sin C = sin 2A + sin 2B + sin 2C

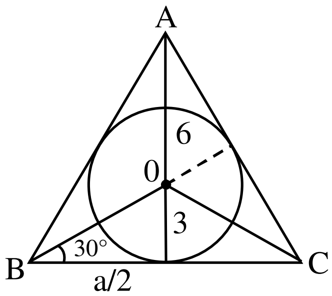
(4) Length of altitude from vertex A is 9

Ans. **(B)**

Sol. If cos 2A + cos 2B + cos 2C is minimum then

$A = B = C = 60°$

so ΔABC is equilateral now



(A) $\overrightarrow{OA}.\overrightarrow{OB} = |OA|\,|OB| \cos 120$

$6.6.\left(\dfrac{1}{2}\right) = -18$ correct option

(B) $\Delta ABC = \dfrac{\sqrt{3}}{4}.a^2 = \dfrac{\sqrt{3}}{4}.(6\sqrt{3})^2 = \dfrac{\sqrt{3}.36.3}{4} = 27\sqrt{3}$ incorrect option

(C) sin A + sin B + sin C = sin 2A + sin 2B + sin C correct option

(D) length of altitude = 9 correct option