9. FLUID MECHANICS

1. INTRODUCTION

Fluid is a collective term for liquid and gas. A fluid cannot sustain shear stress when at rest. We will study the dynamics of non-viscous, incompressible fluid. We will be learning about pressure variation, Archemides principle, equation of continity, Bernoulli's Theorem and its applications and surface tension, Stoke's Law and Terminal velocity of a spherical body.

2. DEFINITION OF A FLUID

A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.

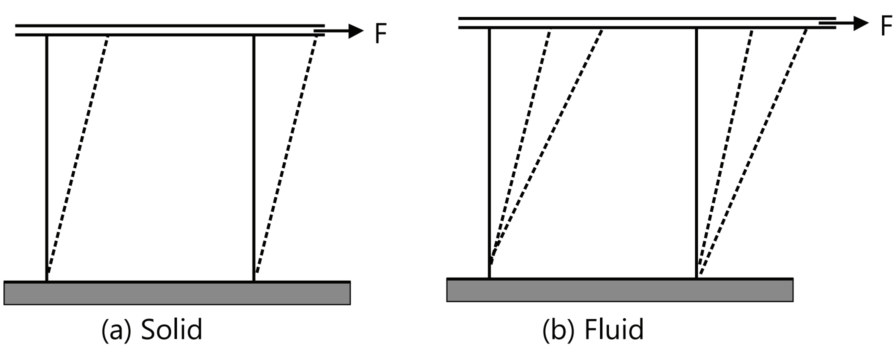


Figure 9.1: Behavior of a solid and a fluid, under the action of a constant shear force.

3. FLUID STATICS

It refers to the state when there is no relative velocity between fluid elements. In this section we will learn some of the properties of fluid statics.

3.1 Density

The density ρ of a substance is defined as the mass per unit volume of a sample of the substance. If a small mass element Δm occupies a volume ΔV, the density is given by $\rho = \frac{\Delta m}{\Delta V}$

In general, the density of an object depends on position, so that $\rho = f(x, y, z)$

If the object is homogeneous, its physical parameters do not change with position throughout its volume. Thus for a homogeneous object of mass M and volume V, the density is defined as $\rho = \frac{M}{V}$

Thus SI units of density are kg m^{-3}.

PLANCESS CONCEPTS

Note: As pressure is increased, volume decreases and hence density will increase.

As the temperature of a liquid is increased, mass remains the same while the volume is increased.

Vaibhav Krishnan (JEE 2009, AIR 22)

3.2 Specific Gravity

The specific gravity of a substance is the ratio of its density to that of water at 4°C, which is 1000 kg/m^3. Specific gravity is a dimensionless quantity numerically equal to the density quoted in g/cm^3. For example, the specific gravity of mercury is 13.6, and the specific gravity of water at 100°C is 0.999. $RD = \frac{\text{Density of substance}}{\text{Density of water at }4^\circ C}$

Illustration 1: Find the density and specific gravity of gasoline if 51 g occupies 75 cm^3? **(JEE MAIN)**

Sol: Density is mass per unit volume, and specific gravity is the ratio of density of substance and density of water.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{0.051\text{kg}}{75\times10^{-6}\text{m}^3} = 680 \text{ kg/m}^3$$

$$\text{Sp. gr} = \frac{\text{density of gasoline}}{\text{density of water}} = \frac{680\text{kg}/\text{m}^3}{1000\text{kg}/\text{m}^3} = 0.68 \text{ or Sp. gravity} = \frac{\text{mass of 75 cm}^3\text{gasoline}}{\text{mass of 75 cm}^3\text{water}}$$

$$= \frac{51\text{g}}{75\text{g}} = 0.68$$

Illustration 2: The mass of a liter of milk is 1.032 kg. The butterfat that it contains has a density of 865 kg/m^3 when pure, and it constitutes 4 percent of the milk by volume. What is the density of the fat-free skimmed milk? **(JEE MAIN)**

Sol: Find the mass of butterfat present in the milk. Subtract this from total mass to get mass of fat-free milk. The density of fat-free milk is equal to its mass divided by its volume.

Volume of fat in 1000 cm^3 of milk = 4% × 1000 cm^3 = 40 cm^3

Mass of 40 cm^3 fat = $V\rho$ = $(40 \times 10^{-6}\text{ m}^3)(865\text{ kg/m}^3)$ = 0.0346 kg

$$\text{Density of skimmed milk} = \frac{\text{mass}}{\text{volume}} = \frac{(1.032-0.0346)\text{kg}}{(1000-40)\times10^{-6}\text{m}^3}$$

3.3 Pressure

The pressure exerted by a fluid is defined as the force per unit area at a point within the fluid. Consider an element of area ΔA as shown in the figure and an external force ΔF is acting normal to the surface. The average pressure in the fluid at the position of the element is given by $P_{av} = \frac{\Delta F}{\Delta A}$ [A normal force ΔF acts on a small cylindrical element of cross-section area ΔA.]

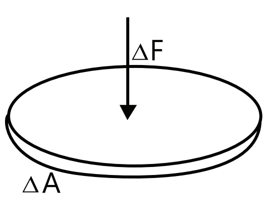


Figure 9.2

As $\Delta A \to 0$, the element reduces to a point, and thus, pressure at a point is defined as

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

When the force is constant over the surface A, the above equation reduces to $p = \frac{F}{A}$

The SI unit of pressure is Nm^{-2} and is also called Pascal (Pa). The other common pressure units are atmosphere and bar.

1 atm = 1.01325×10^5 Pa; 1 bar = 1.00000×10^5 Pa; 1 atm = 1.01325 bar

3.3.1 Pressure Is Isotropic

Imagine a static fluid and consider a small cubic element of the fluid deep within the fluid as shown in the figure. Since this fluid element is in equilibrium therefore, forces acting on each lateral face of this element must also be equal in magnitude. Because the areas of each face are equal, therefore, the pressure on each face is equal in magnitude. Therefore the pressure on each of the lateral faces must also be the same. In the limit as the cube element to a point, the forces on top and bottom surfaces also become equal. Thus, the pressure exerted by a fluid at a point is the same in all directions – pressure is isotropic.

Note: Since the fluid cannot support a shear stress, the force exerted by a fluid pressure must also be perpendicular to the surface of the container that holds it.

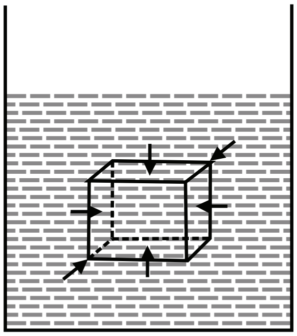


Figure 9.3: A small cubical element is in equilibrium inside a fluid

3.3.2 Atmospheric Pressure (P_0)

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is 1.013×10^5 Pa. Thus,

1 atm = 1.013×10^5 Pa=1.013 Bar

3.3.3 Absolute Pressure and Gauge Pressure

The excess pressure above atmospheric pressure is usually called gauge pressure and the total pressure is called absolute pressure. Thus, Gauge pressure = absolute pressure – atmospheric pressure. Aboslute pressure is always greater than or equal to zero. While gauge pressure can be negative also.

Illustration 3: Atmospheric pressure is about 1.01×10^5 Pa. How large a force does the atmosphere exert on a 2 cm^2 area on the top of your head? **(JEE MAIN)**

Sol: Force = Pressure × Area

Because p = F/A, where F is perpendicular to A, we have F = pA. Assuming that 2 cm^2 of your head is flat (nearly correct) and that the force due to the atmosphere is perpendicular to the surface (as it is), we have $F = pA = (1.01 \times 10^5 \, N/m^2)(2 \times 10^{-4} \, m^2) \approx 20N$

3.3.4 Variation of Pressure with Depth

Weight of a fluid element of mass Δm, $\Delta W = (\Delta m)g$. The force acting on the lower face of the element is pA and that on the upper face is $(p + \Delta p)A$. The figure (b) shows the free body diagram of the element. Applying the condition of equilibrium, we get, $pA - (p + \Delta p)A - (\Delta m)g = 0$

if ρ is the density of the fluid at the position of the element, then $\Delta m = \rho A(\Delta y)$

and pA – (p + Δp) A – ρgA(Δy) = 0

or $\frac{\Delta p}{\Delta y} = -\rho g$

In the limit Δy approaches to zero, $\frac{\Delta p}{\Delta y}$ becomes $\frac{dp}{dy} = -\rho g$. The above equation indicates that the slope of p versus y is negative. That is, the pressure p decreases with height y from the bottom of the fluid. In other words, the pressure p increases with depth h, i.e., $\frac{dp}{dh} = \rho g$

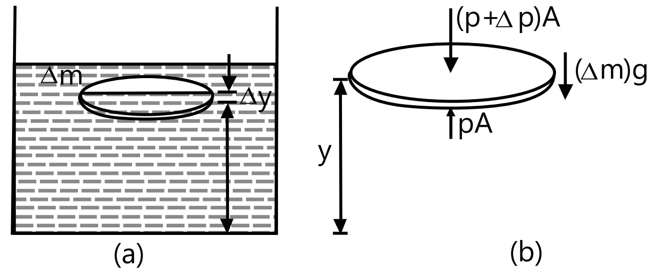


Figure 9.4

3.4 The Incompressible Fluid Model

For an incompressible fluid, the density ρ of the fluid remains constant throughout its volume. It is a good assumption for liquids. To find pressure at the point A in a fluid column as shown in the figure, is obtained by integrating the following equation:

$$dp = \rho g dh \text{ or } \int_{p_0}^{p} dp = \rho g \int_{0}^{h} dh \text{ or } p - p_0 = \rho gh \text{ or } p = p_0 + \rho gh \qquad \text{...(xvi)}$$

where ρ is the density of the fluid, and p_0 is the atmospheric pressure at the free surface of the liquid.

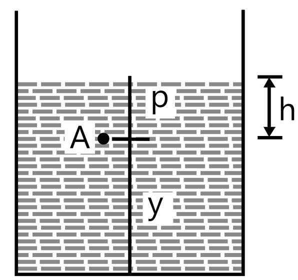


Figure 9.5: A point A is located in a fluid at a height from the bottom and at a deth h from the free surface

Note: Further, the pressure is the same at any two points at the same level in the fluid. The shape of the container does not matter.

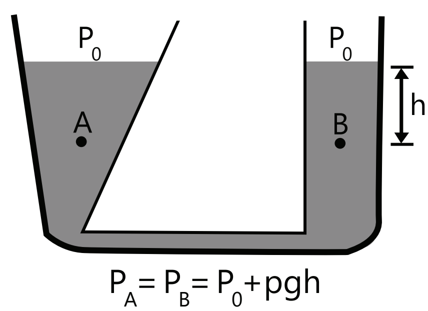


Illustration 4: Find the absolute pressure and gauge pressure at point A, B and C as shown in the Fig. 9.6 (1 atm = 10^5 Pa) **(JEE MAIN)**

Sol: Gauge Pressure = ρgh, Absolute Pressure is sum of gauge pressure and atmospheric pressure.

$P_{atm} = 10^5$ Pa.

Absolute Pressure **A** -> $P_A + P_{atm} = r_1 g h_A$ = (800)(10)1 = 8 kPa

$p'_A = p_A + p_{atm} = 108$ kPa

Gauge Pressure = 8 kPa.

B -> $p_B = \rho_1 g(2) + \rho_2 g(1.5)$

$p'_B = p_B + p_{atm}$ = 131 kPa = (800)(10)(2) + (10^3)(10)(1.5) = 131 kPa

Gauge Pressure = 31 kPa.

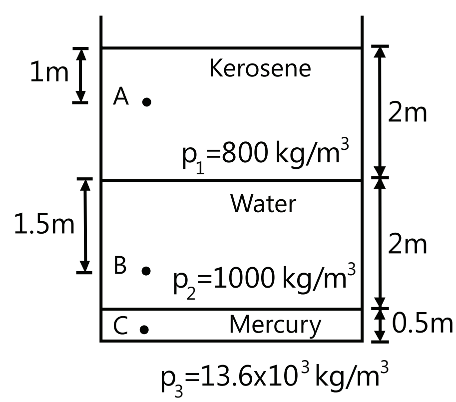


Figure 9.6

C-> $p_c = p_1 g(2) + \rho_2 g(2) + \rho_3 g(0.5)$

$\rho'_C = p_C + p_{atm} = 204\,\text{kPa}$

$= (800)(10)(2) + (10)^3(10)(2) + 1(13.6 \times 10^3)(10)(0.5) = 204$ kPa

Gauge Pressure = 104 kPa.

Illustration 5: A glass full of water of a height of 10 cm has a bottom of area 10 cm^2, top of area 30 cm^2 and volume 1 litre. **(JEE ADVANCED)**

(a) Find the force exerted by the water on the bottom.

(b) Find the resultant force exerted by the side of the glass on the water.

(c) If the glass is covered by a jar and the air inside the jar is completely pumped out, what will be the answer to parts (a) and (b).

(d) If a glass of different shape is used provided the height, the bottom area and the volume are unchanged, will the answers to parts (a) and (b) change.

Take $g = 10\text{m/s}^2$, density of water = 10^3 kg/m^3 and atmospheric pressure = 1.01×10^5 N/m^2.

Sol: Pressure at the bottom depends on the height of water in the container. Force = Pressure × Area. The force on water surface due to atmospheric pressure plus the weight of water are balanced by the force on water by the container bottom and its walls.

(a) Force exerted by the water on the bottom $F_1 = (P_0 + \rho gh)A_1$ ... (i)

Here, P_0 = atmospheric pressure = 1.01×10^5 N/m^2; ρ = density of water = 10^3 kg/m^3

$g = 10$ m/s^2, h = 10 cm = 0.1 m and A_1 = area of base 10 cm^2 = 10^{-3} m^2 . Substituting in Eq. (i), we get $F_1 = (1.01 \times 10^5 + 10^3 \times 10 \times 0.1) \times 10^{-3}$ or $F_1 = 102$ N (downwards)

(b) Force exerted by atmosphere on water $F_2 = (P_0)A_2$

Here, A_2 = area of top = 30 cm^2 = 3×10^{-3} m^2 ; $F_2 = (1.01 \times 10^5)(3 \times 10^{-3}) = 303$ N (downwards)

Force exerted by bottom on the water $F_3 = -F_1$ or $F_3 = 102$ N (upwards)

Weight of water W = (volume)(density)(g) = $(10^{-3})(10^3)(10) = 10$ N (downwards)

Let F be the force exerted by side walls on the water (upwards). Then, from equilibrium of water

Net upward force = net downward force or $F + F_3 = F_2 + W$

$F - F_2 + W - F_3 = 303 + 10 - 102$ or F = 211 N (upwards)

(c) If the air inside of the Jar is completely pumped out,

$F_1 = (\rho gh)A_1$ (as $P_0 = 0$) = $(10^3)(10)(0.1)(10^{-3}) = 1$ N (downwards). In this case $F_2 = 0$ and $F_3 = 1$ N (upwards)
$\therefore \quad F = F_2 + W - F_3 = 0 + 10 - 1 = 9$ N (upwards)

(d) No, the answer will remain the same. Because the answers depend upon P_0, ρ, g, h , A_1 and A_2.

Illustration 6: Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to the same height give different reading on a weighing scale? **(JEE MAIN)**

Sol: Force on the base of the vessel depends on the pressure on it, and pressure depends on the height of the liquid in the vessel. On the other hand the normal reaction from the surface on which the vessel is kept, depends on both the pressure at the base as well as the weight of the liquid in the vessel.

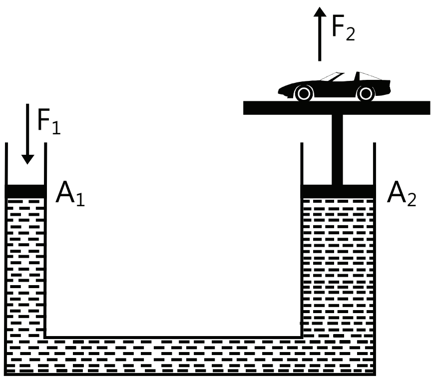
Pressure (and therefore force) on the two equal base areas are identical. But force is exerted by water on the sides of the vessels also, which has non-zero vertical component when the sides of the vessel are not perfectly normal to the base. This net vertical component of force by water on the side of the vessel is greater for the first vessel than the second. Hence, the vessels weigh different when the force on the base is the same in the two cases.

3.4.1 Pascal's Laws

According to the equation $p = p_0 + \rho gh$. Pressure at any depth h in a fluid may be increased by increasing the pressure p_0 at the surface. Pascal recognized a consequence of this fact that we now call Pascal's Law. A pressure applied to a confined fluid at rest is transmitted equally undiminished to every part of the fluid and the walls of the container.

This principle is used in a hydraulic jack or lift, as shown in the figure.

The pressure due to a small force F_1 applied to a piston of area A_1 is transmitted to the large piston of area A_2. The pressure at the two pistons is the same because they are at the same level.



A hydraulic jack

Figure 9.7

$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$ Or $F_2 = \left(\frac{A_2}{A_1}\right)F_1$. Consequently, the force on the larger piston is large.

Thus, a small force F_1 acting on a small area A_1 results in a larger force F_2 acting on a larger area A_2.

PLANCESS CONCEPTS

Since energy is always conserved, $F_1x_1 = F_2x_2$ where x_1 and x_2 are the distances moved by the pistons.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 7: Find the pressure in the air column at which the piston remains in equilibrium. Assume the pistons to be massless and frictionless. **(JEE MAIN)**

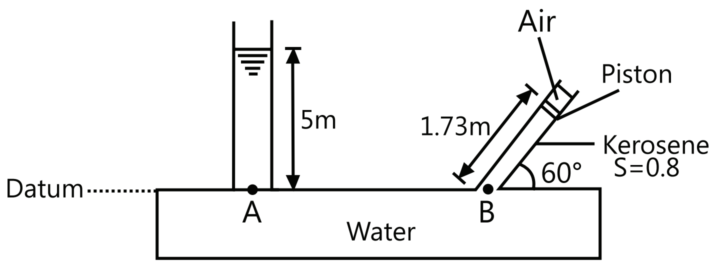


Figure 9.8

Sol: Apply Pascal's law at two points at equal height from a common datum.

Let p_a be the air pressure above the piston.

Applying Pascal's law at point A and B.

$P_{atm} + r_w g(5) = p_a + r_k g(1.73)\frac{\sqrt{3}}{2}$; P_a = 138 kPa

Illustration 8: A weighted piston confines a fluid of density ρ in a closed container, as shown in the figure. The combined weight of piston and container is W = 200 N, and the cross-sectional area of the piston is A = 8 cm^2. Find the total pressure at point B if the fluid is mercury and h = 25 cm (p_m = 13600 kgm^{-3}). What would be an ordinary pressure gauge reading at B? **(JEE ADVANCED)**

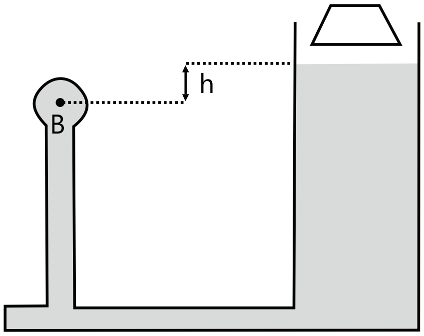


Figure 9.9

Sol: Pressure difference between two points at different heights is equal to ρgh, where h is difference in heights of two points. Apply Pascal's law at two points at different heights from a common datum.

Pascal's principle tells us about the pressure applied to the fluid by the piston and atmosphere. This added pressure is applied at all points within the fluid. Therefore the total pressure at B is composed of three parts: Pressure of atmosphere = 1.0×10^5 Pa

Pressure due to piston and weight = $\frac{W}{A} = \frac{200N}{8\times10^{-4}m^2} = 2.5 \times 10^5$ Pa

Pressure due to height h of fluid = $h\rho g = 0.33 \times 10^5$ Pa

In this case, the pressure of the fluid itself is relatively small. We have

Total pressure at B = 3.8×10^5 Pa = 383 kPa. The gauge pressure does not include atmospheric pressure. Therefore,

Gauge pressure at B = 280 kPa

Illustration 9: For the system shown in the figure, the cylinder on the left, at L, has a mass of 600 kg and a cross-sectional area of 800 cm^2. The piston on the right at S, has cross-sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil (ρ=0.78 g/cm^3), find the force F required to hold the system in equilibrium as shown in figure. **(JEE ADVANCED)**

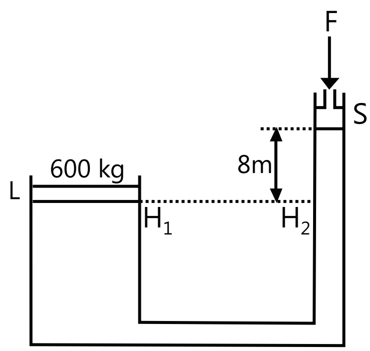


Figure 9.10

Sol: Apply Pascal's law at two points at different heights from a common datum.

The pressures at point H_1 and H_2 are equal because they are at the same level in the single connected fluid. Therefore, Pressure at H_1 = pressure at H_2 = (pressure due to F plus pressure due to liquid column above H_2)

$$\frac{(600)(9.8)N}{0.08m^2} = \frac{F}{25\times10^{-4}m^2} + (8m)(780\ kg/m^{-3})(9.8)$$

After solving, we get, F = 31 N

Illustration 10: As shown in the figure, as column of water 40 cm high supports 31 cm of an unknown fluid. What is the density of the unknown fluid? **(JEE MAIN)**

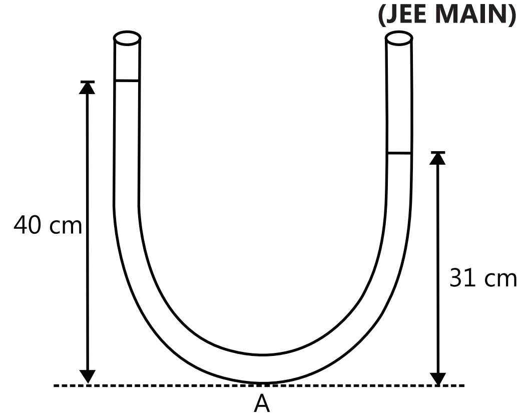


Figure 9.11

Sol: Find the hydrostatic pressure at the bottom most point A due to both the water column and the unknown fluid column.

The pressure at point A due to the two fluids must be equal (or the one with the higher pressure would push lower pressure fluid away). Therefore, pressure due to water = pressure due to known fluid; $h_1r_1g = h_2r_2g$, from which $r_2 = \frac{h_1}{h_2}p_1 = \frac{40}{31}$ (1000 kg/m^2) = 1290 kg/m^3

For gases, the constant density assumed in the compressible model is often not adequate. However, an alternative simplifying assumption can be made that the density is proportional to the pressure, i.e., $\rho = kp$. Let r_0 be the density of air at the earth's surface where the pressure is atmospheric p_o, then $r_0 = kp_0$; After eliminating k, we get $\rho = \frac{\rho_0}{p_0}p$

Putting the value of ρ in equation $dp = -\rho gdy$ or $dp = -\left(\frac{\rho_0}{p^0}p\right)gdy$

On rearranging, we get $\int_{p_0}^{p}\frac{dp}{p} = -\frac{\rho_0}{p_0}g\int_0^h dy$ where p is the pressure at a height y = h above the earth's surface.

After integrating, we get $\ln\left|\frac{p}{p_0}\right| = -\frac{\rho_0}{p_0}gh$ or $p = p_0^{\frac{-\rho_0}{p_0}gh}$

Note: Instead of a linear decrease in pressure with increasing height as in the case of an incompressible fluid, in this case pressure decreases exponentially.

4. PRESSURE MEASURING DEVICES

4.1 Manometer

A manometer is a tube open at both ends and bent into the shape of a "U" and is partially filled with mercury. When one end of the tube is subjected to an unknown pressure p, the mercury level drops on that side of the tube and rises on the other so that the difference in mercury level is h as shown in the figure.

When we move down in a fluid, pressure increases with depth and when we move up the pressure decreases with height. When we move horizontally in a fluid, pressure remains constant. Therefore, $p + r_0gh_0 - r_mgh = p_0$ where p_0 is atmospheric pressure, and r_m is the density of the fluid inside the vessel.

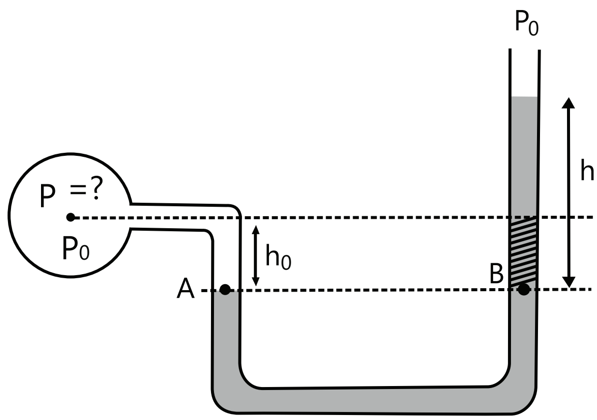
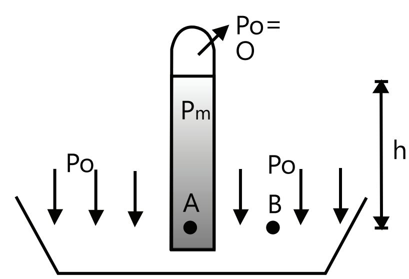


Figure 9.12: An U-shaped manometer tube connected to a vessel

4.2 The Mercury Barometer

It is a straight glass tube (closed at one end) completely filled with mercury and inserted into a dish which is also filled with mercury as shown in the figure. Atmospheric pressure supports the column of mercury in the tube to a height h. The pressure between the closed end of the tube and the column of mercury is zero, p = 0. Therefore, pressure at points A and B are equal and thus $p_0 = 0 + r_mgh$. Hence, $p_0 = (13.6 \times 10^3)(9.8)(0.76) = 1.01 \times 10^5\ Nm^{-2}$ for Pa.



A mercury barometer

Figure 9.13

Illustration 11: What must be the length of a barometer tube used to measure atmospheric pressure if we are to use water instead of mercury? **(JEE MAIN)**

Sol: The length of the barometer tube will be inversely proportional to the density of fluid used in it.

We know that $p_0 = r_mgh_m = r_wgh_w$ where r_w and h_w are the density and height of the water column supporting the atmospheric pressure p_0.

$\therefore \quad h_w = \frac{\rho_m}{\rho_w}h_w$; Since $\frac{\rho_m}{\rho_w} = 13.6$; $h_w = 0.76$ m = (13.6)(0.76) = 10.33 m.

5. PRESSURE DIFFERENCE IN ACCELERATING FLUIDS

Consider a beaker filled with some liquid of density p accelerating upwards with an acceleration a_y along positive y-direction. Let us draw the free body diagram of a small element of fluid of area A and length dy as shown in figure. Equation of motion for this fluid element is, $PA - W - (P + dP)A = (\text{mass})(a_y)$ or $-W - (dP)A = (A\rho\, dy)(a_y)$

or $(A\rho g\, dy) - (dP)A = (A\rho\, dy)(a_y)$ or $\frac{dP}{dy} = -\rho(g + a_y)$

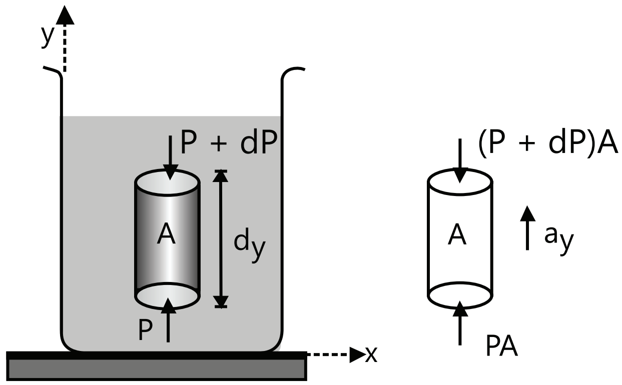


Figure 9.14

Similarly, if the beaker moves along positive x-direction with acceleration a_x, the equation of motion for the fluid element shown in the figure is, PA – (P + dP) A = (mass)(a_x)

or (dP)A = (Aρ dx)a_x Or $\frac{dP}{dx} = -\rho a_x$

But suppose the beaker is accelerated and it has components of acceleration a_x and a_y in x and y directions respectively, then the pressure decreases along both x and y directions. The above equation

in that case reduces to,

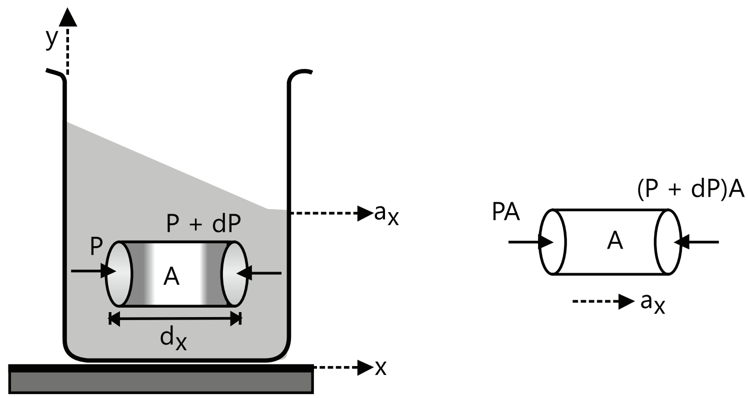


Figure 9.15

$$\frac{dP}{dx} = -\rho a_x \quad \text{and} \quad \frac{dP}{dy} = -\rho(g + a_y) \qquad \text{..... (i)}$$

For surface of a Liquid Accelerated in Horizontal Direction.

Consider a liquid placed in a beaker which is accelerating horizontally with an acceleration 'a'. Let A and B be two points in the liquid at a separation x in the same horizontal line. As we have seen in this case.

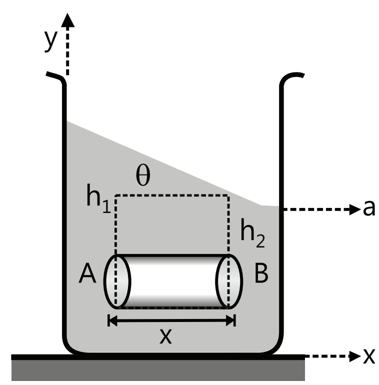


Figure 9.16

$\frac{dP}{dx} = -\rho a$ or dP = -ra dx. Integrating this with proper limits, we get

$P_A - P_B$ = pax ..... (ii)

Further, $P_A = P_0 + \rho g h_1$ And $P_B = P_0 + \rho g h_2$

Substituting in Eq. (ii), we get pg($h_1 - h_2$) = pax ∴ $\frac{h_1 - h_2}{x}$

$= \frac{a}{g} = \tan\theta$ ∴ $\boxed{\tan\theta = \frac{a}{g}}$

Note: When a_y is not equal to zero then the angle of inclination is given by

$$\tan\theta = \frac{dy}{dx} = \left(\frac{\overline{(dp)}}{\left(\frac{dp}{dy}\right)}\right) = \frac{a_x}{g + a_y}$$

Illustration 12: A liquid of density ρ is in a bucket that spins with angular velocity ω as shown is the figure. Show that the pressure at a radial distance r from the axis is

$P = P_0 + \frac{\rho\omega^2 r^2}{2}$ where P_0 is the atmospheric pressure. **(JEE ADVANCED)**

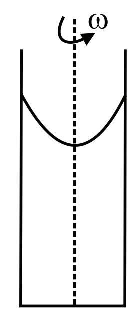


Figure 9.17

Sol: The net force on the liquid surface in equilibrium is always perpendicular to it as the liquid surface cannot sustain shear stress.

Consider a fluid particle P of mass m at coordinates (x, y). From a non-inertial rotating frame of reference, two forces are acting on it.

(i) Pseudo force $(mx\omega^2)$

(ii) Weight (mg) in the direction shown in figure.

Net force on it should be perpendicular to the free surface (in equilibrium). Hence,

$$\tan\theta = \frac{mx\omega^2}{mg} = \frac{x\omega^2}{g} \text{ or } \frac{dy}{dx} = \frac{x\omega^2}{g}$$

$$\therefore \int_0^y dy = \int_0^x \frac{x\omega^2}{g} \cdot dx \therefore y = \frac{x^2\omega^2}{2g}$$

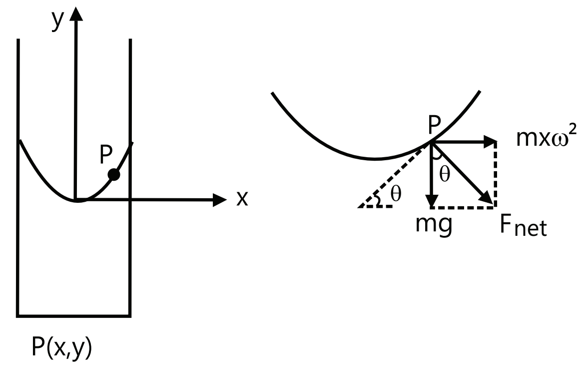


Figure 9.18

This is the equation of the free surface of the liquid, which is a parabola.

As x = r, $y = \frac{r^2\omega^2}{2g} \therefore P(r) = P_0 + \rho g y$ or $P(r) = P_0 + \frac{\rho\omega^2 r^2}{2}$

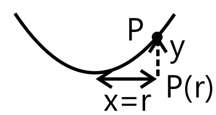


Figure 9.19

Hence proved.

Illustration 13: An open rectangular tank 5 m × 4 m × 3 m high containing water up to a height of 2 m is accelerated horizontally along the longer side.

(a) Determine the maximum acceleration that can be given without spilling the water.

(b) Calculate the percentage of water split over, if this acceleration is increased by 20%.

(c) If initially, the tank is closed at the top and is accelerated horizontally by 9 m/s², find the gauge pressure at the bottom of the front and rear walls of the tank. (Take g = 10 m/s²) **(JEE MAIN)**

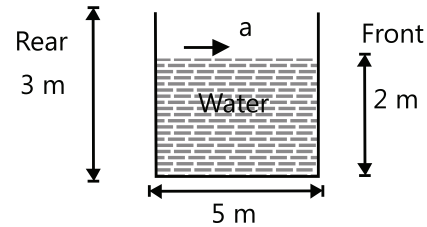


Figure 9.20

Sol: As the water column is accelerated towards right in horizontal direction, the free surface will not be horizontal but will be inclined at an angle with the θ horizontal, such that the left edge of the surface is at a higher level than the right edge. This is because the pressure at the left of water column will be more than the pressure at the right of it.

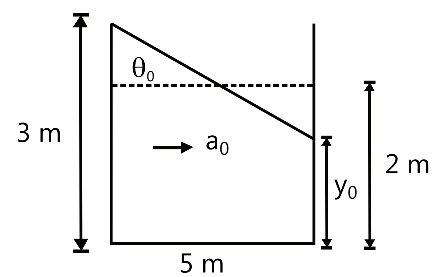


Figure 9.21

(a) Volume of water inside the tank remains constant

$$\left(\frac{3+y_0}{2}\right)5 \times 4 = 5 \times 2 \times 4 \text{ or } y_0 = 1\text{m} \therefore \tan q_0 = \frac{3-1}{5} = 0.4$$

Since, $\tan q_0 = \frac{a_0}{g}$, therefore $a_0 = 0.4\text{ g} = 4\text{ m/s}^2$

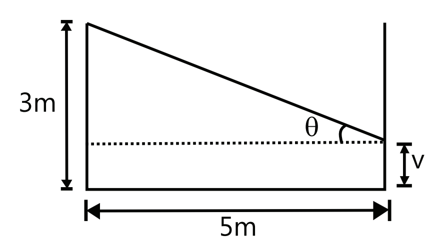


Figure 9.22

(b) When acceleration is increased by 20%

$a = 1.2\ a_0 = 0.48\ g \therefore \tan\theta = \frac{a}{g} = 0.48$

Now, $y = 3 - 5\tan\theta = 3 - 5(0.48) = 0.6$ m

Fraction of water split over = $\frac{4\times2\times5 - \frac{(3+0.6)}{2}\times5\times4}{2\times5\times4} = 0.1$

Percentage of water split over = 10%

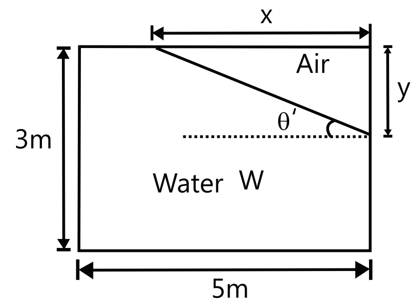


Figure 9.23

(c) $a' = 0.9\ g;\ \tan\theta' = \frac{a'}{g} = 0.9$

Volume of air remains constant -> $4\times\frac{1}{2}yx = (5)(1)\times4 \Rightarrow$ Pressure does not change in the air.

Since $y = x\tan\theta' \therefore\ \frac{1}{2}x^2\tan\theta' = 5$ or $x = 3.33$ m; $y = 3.0$m

Gauge pressure at the bottom of the

(i) Front wall p_f = zero

(ii) Rear wall $p_r = (5\tan\theta')rwg = 5(0.9)(10^3)(10) = 4.5\times10^4$ Pa

Illustration 14: A vertical U-tube with the two limbs 0.75 m apart with water and rotated about a vertical axis 0.5 m from the left limb, as shown in the figure. Determine the difference in elevation of the water levels in the two limbs, when the speed of rotation is 60 rpm. **(JEE MAIN)**

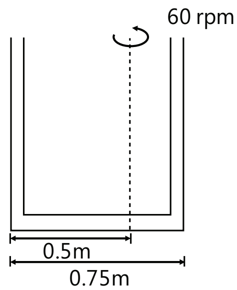


Figure 9.24

Sol: Each element of water in the tube is accelerated towards the axis. Along the horizontal part of the tube, the pressure will increase gradually as one moves radially away from the axis. The extra pressure provides the required centripetal acceleation.

Consider a small element of length dr at a distance r from the axis of rotation. Considering the equilibrium of this element.

$(p + dp) - p = r\omega^2 r\,dr$ or $dp = r\omega^2 r\,dr$

On integrating between 1 and 2

$$p_1 - p_2 = \rho\omega^2\int_{r_2}^{r_1} r\,dr = \frac{\rho\omega^2}{2}(r_1^2 - r_2^2)$$

or $h_1 - h_2 = \frac{\omega^2}{2g}[r_1^2 - r_2^2] = \frac{(2\pi)^2}{2(10)}[(0.5)^2 - (0.25)^2] = 0.37$ m.

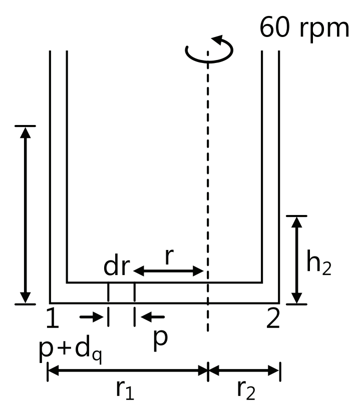


Figure 9.25

6. BUOYANCY

If a body is partially or wholly immersed in a fluid, it experiences an upward force due to the fluid surrounding it.

The phenomenon of force exerted by fluid on the body is called buoyancy and the force is called buoyant force. A body experiences buoyant force whether it floats or sinks, under its own weight or due to other forces applied on it.

Note: The buoyant force is due to the fact that the hydrostatic pressure at different depths is not the same. Buoyant force is independent of:

(a) Total volume and shape of the body.

(b) Density of the body.

6.1 Archimedes Principle

A body immersed in a fluid experiences an upward buoyant force equivalent to the weight of the fluid displaced by it. The proof of this principle is very simple. Imagine a body of arbitrary shape completely immersed in a liquid of density ρ. A body is being acted upon by the forces from all directions. Let us consider a vertical element of height h and cross-sectional area dA.The force acting on the upper surface of the element is F_1 (downward) and that on the lower surface is F_2 (upward). Since $F_2 > F_1$, therefore, the net upward force acting on the element is $dF = F_2 - F_1$. It can be easily seen that

$F_1 = (\rho g h_1)dA$ and $F_2 = (\rho g h_2)dA$. So $dF = \rho g(h)\, dA$

Also, $h_2 - h_1 = h$ and $h(dA) = dV$ $\therefore$ The net upward force is $F = \int \rho g dV = \rho V g$

Hence, for the entire body, the buoyant force is the weight of the volume of the fluid displaced.

Note: Buoyant force acts on the centre of gravity of the displacement liquid. This point is called centre of Buoyancy.

PLANCESS CONCEPTS

The fluid exerts force on the immersed part of the body from all directions.

The net force experienced by every vertical element of the body is in the upward direction.

A uniform body floats in a liquid if density of the body is less than or equal to the density of the liquid and sinks if density of the uniform body is greater than that of the liquid.

B Rajiv Reddy (JEE 2012, AIR 11)

6.1.1 Detailed Explanation

An object floats on water if it can displace a volume of water whose weight is greater than that of the object. If the density of the material is less than that of the liquid, it will float even if the material is a uniform solid, such as a block of wood floats on water surface. If the density of the material is greater than that of water, such as iron, the object can be made to float provided it is not a uniform solid. An iron built ship is an example to this case

Apparent weight of a body immersed in a liquid = $w - w_0$, where 'w' is the true weight of the body and w_0 is the apparent loss in weight of the body, when immersed in the liquid.

6.1.2 Buoyant Force in Accelerating Fluids

Suppose a body is dipped inside a liquid of density ρ_L placed in an elevator moving with acceleration $\vec{a}$. The buoyant force F in this case becomes, $F = V\rho_L g_{eff}$;

Here, $g_{eff} = |\vec{g} - \vec{a}|$

Illustration 15: An iceberg with a density of 920 kgm^{-3} floats on an ocean of density 1025 kgm^{-3}. What fraction of the iceberg is visible? **(JEE MAIN)**

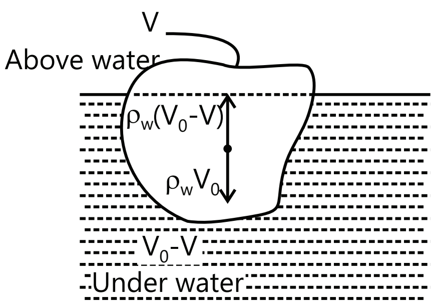


Figure 9.26

Sol: The buoyant force on the iceberg will be equal to its weight. The buoyant force is equal to the weight of water displaced by the iceberg, i.e. the weight of volume of water equal to the volume of iceberg immersed.

Let V be the volume of the iceberg above the water surface, then the volume under inside is $V_0 - V$. Under floating conditions, the weight $(\rho_I V_0 g)$ of the iceberg is balanced by the buoyant force $\rho_w(V_0 - V)g$.

Thus, $\rho_I V_0 g = \rho_w(V_0 - V)g$

or $\rho_w V = (\rho_w - \rho_I)V_0$

or $\frac{V}{V_0} = \frac{\rho_w - \rho_I}{\rho_w}$

Since, r_w = 1025 kg m^{-3} and r_i = 920 kg m^3, therefore, $\frac{V}{V_0} = \frac{1025-920}{1025}$ = 0.10

Hence 10% of the total volume is visible.

Illustration 16: When a 2.5 kg crown is immersed in water, it has an apparent weight of 22 N. What is the density of the crown? **(JEE MAIN)**

Sol: Apply Archemides principle.

Let W = actual weight of the crown and W′ = apparent weight of the crown

ρ = density of crown, ρ_0 = density of water. The buoyant force is given by F_E = W – W′ or

$\rho_0 Vg$ = W – W′. Since W = ρVg, therefore, V = $\frac{W}{\rho g}$. Eliminating V from the above equation, we get

$\rho = \frac{\rho_0 W}{W - W'}$. Here W = 25 N; W′ = 22 N; $\rho_0 = 10^3$ kg m^{-3} ; $\rho = \frac{(10)^3 (25)}{25-22}$ = 9.3 × 10^3 kg m^{-3}.

Illustration 17: The tension in a string holding a solid block below the surface of a liquid (of density greater than that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension in the string if the system has an upward acceleration a? **(JEE MAIN)**

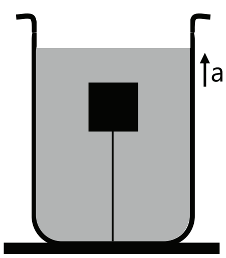


Figure 9.27

Sol: The weight and tension force on the block are balanced by the buoyant force on it. When the system is accelerated upwards, the effective value of g is increased.

Let m be the mass of block.

Initially for the equilibrium of block, F = T_0 + mg ....(i)

Here, F is the up thrust on the block.

When the lift is accelerated upwards, g_{eff} becomes g + a instead of g.

Hence $F' = F\left(\frac{g+a}{g}\right)$ ...(ii)

From Newton's second law, F′ – T – mg = ma ...(iii)

Solving equations (i), (ii) and (iii), we get $T = T_0\left(1+\frac{a}{g}\right)$

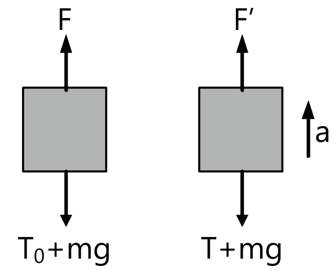


Figure 9.28

Illustration 18: An ice cube of side 1 cm is floating at the interface of kerosene and water in beaker of base area 10 cm^2. The level of kerosene is just covering the top surface of the ice cube.

(a) Find the depth of submergence in the kerosene and that in the water.

(b) Find the change in the total level of the liquid when the whole ice melts into water. **(JEE ADVANCED)**

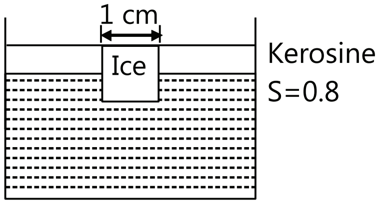


Figure 9.29

Sol: Apply Archemedes principle. Sum of the buoyant forces by kerosene and water will be equal to the weight of the ice cube.

(a) Condition of floating $0.8\ \rho_w g h_k + \rho_w g h_w = 0.9\ \rho_w g h$

or $0.8\ h_k + h_w = (0.9)h$ ... (i)

Where h_k and h_w are the submerged depths of the ice in the kerosene and water, respectively.

Also $h_k + h_w = h$ ... (ii)

Here it is given that h = 1 cm

Solving equations (i) and (ii), we get

$h_k = 0.5$ cm, $h_w = 0.5$ cm

(b) $\underset{\text{Ice}}{1\text{ cm}^3} \xrightarrow{\text{m heat}} \underset{\text{(water)}}{0.9\text{ cm}^3}$

Fall in the level of kerosene $\Delta h_k = \frac{0.5}{A}$; Rise in the level of water $\Delta h_w = \frac{0.9-0.5}{A} = \frac{0.4}{A}$

Net fall in the overall level $\Delta h = \frac{0.1}{A} = \frac{0.1}{10} = 0.01$ cm = 0.1 mm.

6.2 Stability of a Floating Body

The stability of a floating body depends on the effective point of application of the buoyant force. The weight of the body acts at its centre of gravity. The buoyant force acts at the centre of gravity of the displaced liquid. This is called the centre of buoyancy. Under equilibrium condition, the centre of gravity G and the centre of buoyancy B lie along the vertical axis of the body as shown in the figure(s).

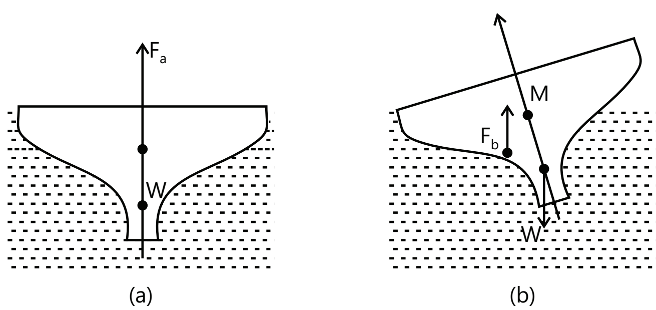


Figure 9.30

(a) The buoyant force acts at the centre of gravity of the displaced fluid.

(b) When the boat tilts, the line of action of the buoyant force intersects the axis of the boat at the metacentre M. In a stable boat, M is above the centre of gravity of the boat. When the body tilts to one side, the centre of buoyancy shifts relative to the centre of gravity as shown in the figure (b). The two forces act along different vertical lines. As a result, the buoyant force exerts a torque about the centre of gravity. The line of action of the buoyant force crosses the axis of the body at the point M, called metacentre. If G is below M, the torque will tend to restore the body to its equilibrium position. If G is above M, the torque will tend to rotate the body away from its equilibrium position and the body will be unstable.

Illustration 19: A wooden plank of length 1 m and uniform cross section is hinged at one end to the bottom of a tank as shown in the figure. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case $\theta = 0$) **(JEE ADVANCED)**

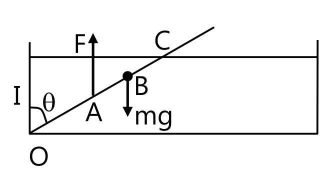


Figure 9.31

Sol: The net torque about the hinge due the weight of the plank and due to the buoyant force acting on the plank should be zero.

The forces acting on the plank are shown in the figure. The height of water level is 0.5m. The length of the plank is $1.0 = 2\ell$. We have OB = ℓ. The buoyant force F acts through the mid-point of the dipped part OC of the plank.

We have $OA = \frac{OC}{2} = \frac{\ell}{2\cos\theta}$; Let the mass per unit length of the plank be ρ.

Its weight mg = 2 ℓ ρg; The mass of the part OC of the plank = $\left(\frac{\ell}{\cos\theta}\right)\rho$.

The mass of water displaced = $\frac{1}{0.5}\frac{1}{\cos\theta}\rho = \frac{2\ell\rho}{\cos\theta}$; The buoyant force F is, therefore, F = $\frac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, mg (OB) sin θ = F(OA) sin θ or (2 ℓ ρ) ℓ = $\left(\frac{2\ell\rho}{\cos\theta}\right)\left(\frac{\ell}{2\cos\theta}\right)$ or $\cos^2\theta = \frac{1}{2}$ or $\cos\theta = \frac{1}{\sqrt{2}}$, or θ = 45°

6.3 Forces on Fluid Boundaries

Whenever a fluid comes in contact with solid boundaries, it exerts a force on it. Consider a rectangular vessel of base size l × b filled with water to a height H as shown in figure The force acting at the base of the container is given by F_b = p × (area of the base)

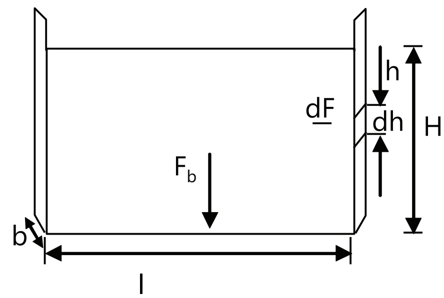


Figure 9.32

Pressure is same everywhere at the base and is equal to ρgH. Therefore, F_b = ρgH(lb) = ρ glb H Since, lbH = V (volume of the liquid) .Thus, F_b = ρgV = weight of the liquid inside the vessel.

A fluid contained in a vessel exerts forces on the boundaries. Unlike the base, the pressure on the vertical wall of the vessel is not uniform but increases linearly with depth from the free surface. Therefore, we have to perform the integration to calculate the total force on the wall. Consider a small rectangular element of width b and thickness dh at depth h from the free surface. The liquid pressure at this position is given by p = ρgh. The force at the element is dF = p(dbh) = ρgbh dh;

The total force is F = $\rho g b\int_O^H h\ dh = \frac{1}{2}\rho g b H^2$. The total force acting per unit width of the critical walls is $\frac{F}{b} = \frac{1}{2}\rho g H^2$

The point of application (the centre of force) of the total force from the free surface is given by $h_c = \frac{1}{F}\int_0^H h\, dF$

Where $\int_0^H h\, dF$ is the moment of force about the free surface.

Here $\int_0^H h\, dF = \int_0^H h(\rho\ gbh\ dh) = \rho g b\int_0^H h^2 dh = \frac{1}{3}\rho g H^3$;

Since F = $\frac{1}{2}\rho g b H^2$, therefore, $h_c = \frac{2}{3}H$

Illustration 20: Find the force acting per unit width on a plane wall inclined at an angle θ with the horizontal as shown in the figure. **(JEE MAIN)**

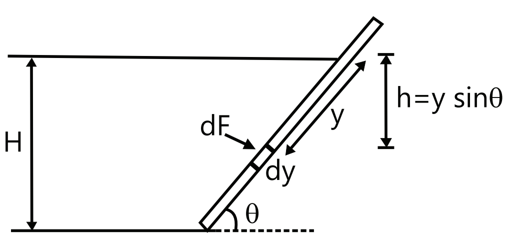


Figure 9.33

Sol: The pressure at each point on the wall will be different, depending on the height. Find pressure on a small element, and use the method of integration.

Consider a small element of thickness dy at a distance y measured along the wall from the free surface. There pressure at the position of the element is p = ρgh = ρgy sin θ. The force given by dF = p(b dy) = ρgb(y dy) sin θ

The total force per unit width b is given by $\frac{F}{b} = \rho g \sin\theta . \int_0^{H/\sin\theta} y\ dy = \rho g \sin\theta\left[\frac{y^2}{2}\right]_0^{H/\sin\theta}$

Or $\frac{F}{b} = \frac{1}{2}\rho g \frac{H^2}{\sin\theta}$

Note: That the above formula reduces to $\frac{1}{2}\rho g H^2$ for a vertical wall (θ = 90º)

6.4 Oscillations of a Fluid Column

The initial level of liquid in both the columns is the same. The area of cross-section of the tube is uniform. If the liquid is depressed by x in one limb, it will rise by x along the length of the tube is the other limb. Here, the restoring force is provided by the hydrostatic pressure difference.

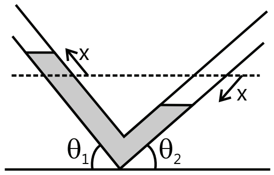


Figure 9.34

$$\therefore \quad F = -(\Delta P)A = -(h_1 + h_2)\rho g A = -\rho g A(\sin\theta_1 + \sin\theta_2)x$$

suppose, m is the mass of the liquid in the tube. Then, $ma = -\rho g A(\sin\theta_1 + \sin\theta_2)x$

Since, F or a is proportional to –x, the motion of the liquid column is simple harmonic in nature, time period of which is given by,

$$T = 2\pi\sqrt{\left|\frac{x}{a}\right|} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{\rho g A(\sin\theta_1 + \sin\theta_2)}}$$

6.5 Oscillations of a Floating Cylinder

Consider a wooden cylinder of mass m and cross-sectional area A floating in a liquid of density ρ. At equilibrium, the cylinder is floating with a depth h submerged [See Fig. 8.35]. If the cylinder is pushed downwards by a small distance y and then released, it will move up and down with SHM. It is desired to find the time period and the frequency of oscillations.

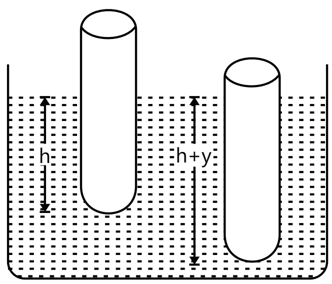


Figure 9.35

According to the principle of flotation, the weight of the liquid displaced by the immersed part of the body is equal to the weight of the body. Therefore, at equilibrium,

Weight of cylinder = Weight of liquid displaced by the immersed part of cylinder

or $mg = (\rho A h)g$ $\therefore$ Mass of cylinder, $m = \rho A h$

When the cylinder is pushed down to an additional distance y, the restoring force F (upward) equal to the weight of additional liquid displaced acts on the cylinder.

$\therefore$ Restoring force, F= - (weight of additional liquid displaced) or $F = -(\rho A y)g$

The negative sign indicates that the restoring force acts opposite to the direction of the displacement.

Acceleration a of the cylinder is given by $a = \frac{F}{m} = \frac{-(\rho A y)g}{\rho A h} = -\left(\frac{g}{h}\right)y$ ... (i)

Since g/h is constant, $a \alpha - y$ Thus the acceleration a of the body (wooden cylinder) is directly proportional to the displacement y and its direction is opposite to the displacement. Therefore, motion of the cylinder is simple harmonic.

$\therefore$ Time period $T = 2\pi\sqrt{\frac{h}{g}}$ ... (ii)

$\therefore$ Frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{h}}$ ... (iii)

These very interesting results show that time period and frequency have the same form as that of simple pendulum. The submerged depth at equilibrium takes the place of the length of the pendulum.

7. FLUID DYNAMICS

In the order to describe the motion of a fluid, in principle, one might apply Newton's laws to a particle (a small volume element of fluid) and follow its progress in time. This is a difficult approach. Instead, we consider the properties of the fluid, such as velocity, pressure, at fixed points in space. In order to simplify the discussion we take several assumptions:

(i) The fluid is non viscous (ii) The flow is steady

(iii) The flow is non rotational (iv) The fluid is incompressible.

7.1 Equation of Continuity

It states that for streamlined motion of the liquid, the volume of liquid flowing per unit time is constant through different cross-sections of the container of the liquid. Thus, if v_1 and v_2 are velocities of fluid at respective points A and B of areas of cross-sections a_1 and a_2 and r_1 and r_2 be the densities respectively. Then the equation of continuity is given by $r_1 a_1 v_1 = r_2 a_2 v_2$ ... (i)

If the same liquid is flowing, then $\rho_1 = \rho_2$; then the equation (i) can be written

As $a_1 v_1 = a_2 v_2$ ...(ii)

$\Rightarrow$ av = constant $\Rightarrow$ $v \propto 1/a$

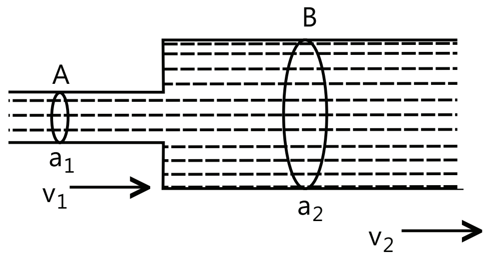


Figure 9.36

PLANCESS CONCEPTS

Equation of continunity repersents the law of conservation of mass of moving fluids.

$a_1 v_1 r_1 = a_2 v_2 r_2$ (General equation of continuity)

This equation is applicable to actual liquids or to other fluids which are not incompressible.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Illustration 21: Water is flowing through a horizontal tube of non-uniform cross-section. At a place, the radius of the tube is 1.0 cm and the velocity of water is 2 m/s. What will be the velocity of water, where the radius of the pipe is 2.0 cm? **(JEE MAIN)**

Sol: Apply the equation of continuity. Where area of cross-section is larger, the velocity of water is lesser and vice-versa.

Using equation of continuity, $A_1 v_1 = A_2 v_2$; $v_2 = \left(\frac{A_1}{A_2}\right) v_1$ or $v_2 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right) v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1$

Substituting the value, we get $v_2 = \left(\frac{1.0\times10^{-2}}{2.0\times10^{-2}}\right)$ or $v_2 = 0.5$ m/s

Illustration 22: Figure shows a liquid being pushed out of a tube by pressing a piston. The area of cross-section of the piston is 1.0 cm^2 and that of the tube at the outlet is 20 mm^2 If the piston is pushed at a speed of 2 $cm\text{-}s^{-1}$, what is the speed of the outgoing liquid?

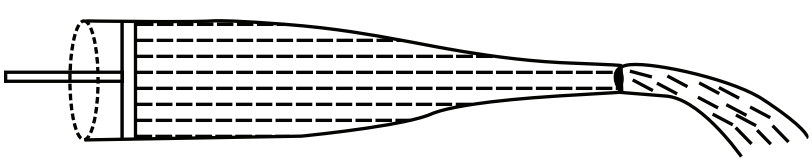


Figure 9.37

Sol: Apply the equation of continuity. Where area of cross-section is larger, the velocity of liquid is lesser and vice-versa.

From the equation of continuity $A_1v_1 = A_2v_2$

or (1.0 cm^2) (2 cm s^{-1}) = (20 mm^2) v_2

or $v_2 = \frac{1.0\text{ cm}^2}{20\text{ mm}^2} \times 2\text{ cm s}^{-1}$

$= \frac{100\text{ mm}^2}{20\text{ mm}^2} \times 2\text{ cm s}^{-1} = 10\text{ cm s}^{-1}$

SHM of fluids in tubes:

Tubes form angles θ_1 and θ_2 with the horizontal.

$$T = 2\pi\sqrt{\frac{m}{\rho gA\left(\sin\theta_1 + \sin\theta_2\right)}}$$

m is total mass of fluid in tubes, A is area of cross – section ρ is density of fluid.

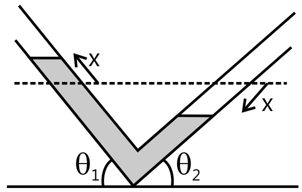


Figure 9.38

8. BERNOULLI'S THEOREM

When a non-viscous and an incompressible fluid flows in a streamlined motion from one place to another in a container, then the total energy of the fluid per unit volume is constant at every point of its path. Total energy = pressure energy + Kinetic energy + Potential energy

$= PV + \frac{1}{2}Mv^2 + Mgh$

Where P is pressure, V is volume, M is mass and h is height from a reference level.

$\therefore$ The total energy per unit volume = $P + \frac{1}{2}\rho v^2 + \rho gh$

Where ρ is density. Thus if a liquid of density ρ, pressure P_1 at a height h_1 which flows with velocity v_1 to another point in streamline motion where the liquid has pressure P_2, at height h_2 which flows with velocity v_2,

then $P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$

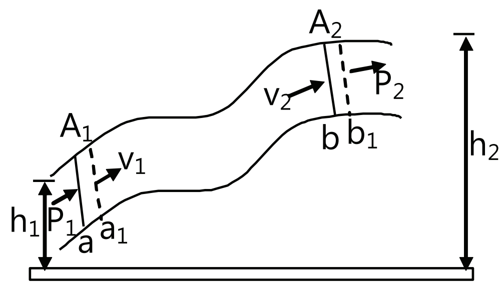


Figure 9.39

8.1 Derivations

8.1.1 Pressure Energy

If P is the pressure on the area A of a fluid, and the liquid moves through a distance due to this pressure, then Pressure energy of liquid = work done = force × displacement = PAl

The volume of the liquid is Al.

$\therefore$ Pressure energy per unit volume of liquid = $\frac{PAl}{Al} = P$

8.1.2 Kinetic Energy

If a liquid of mass m and volume V is flowing with velocity v, then the kinetic energy is $\frac{1}{2}mv^2$.

$\therefore$ Kinetic energy per unit volume of liquid. = $\frac{1}{2}\left(\frac{m}{V}\right)v^2 = \frac{1}{2}\rho v^2$. Here, ρ is the density of liquid.

8.1.3 Potential energy

If a liquid of mass m is at a height h from the reference line (h = 0), then its potential energy is mgh. $\therefore$ Potential energy per unit volume of the liquid = $\left(\frac{m}{v}\right)gh = \rho gh$

Thus, the Bernoulli's equation $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

This can also be written as: Sum of total energy per unit volume (pressure + kinetic + potential) is constant for an ideal fluid.

PLANCESS CONCEPTS

$\frac{P}{\rho g}$ is called the 'pressure head', $\frac{v^2}{2g}$ the velocity head and h the gravitational head.

GV Abhinav JEE 2012, AIR 329

Intresting takeaway is the SI unit of each of these is meter (m).

Illustration 23: Calculate the rate of flow of glycerin of density 1.25×10^3 kg/m^3 through the conical section of a pipe, if the radii of its ends are 0.1 m and 0.04 m and the pressure drop across its length is 10 N/m^2. **(JEE MAIN)**

Sol: Apply the equation of continuity. Where area of cross-section is larger, the velocity of fluid is lesser and vice-versa.

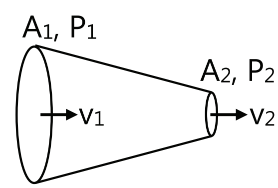


Figure 9.40

From continuity equation, $A_1v_1 = A_2v_2$

or $\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{0.04}{0.1}\right) = \frac{4}{25}$ ... (i)

From Bernoulli's equation , $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

or $v_2^2 - v_1^2 = \frac{2\times10}{1.25\times10^3} = 1.6\times10^{-2} m^2/s^2$ ... (ii)

Solving equations (i) and (ii), we get $v_2 = 0.128$ m/s

$\therefore$ Rate of volume flow through the tube

$Q = A_2v_2 = (\pi r_2^2)\, v_2 = \pi\,(0.04)^2(0.128) = 6.43\times10^{-4}$ m^3/s

Illustration 24: Figure shows a liquid of density 1200 kg m^{-3} flowing steadily in a tube of varying cross section. The cross section at a point A is 1.0 cm^2 and that at B is 20 mm^2, the points A and B are in the same horizontal plane. The speed of the liquid at A is 10 cm s^{-1}. Calculate the difference in pressure at A and B. **(JEE ADVANCED)**

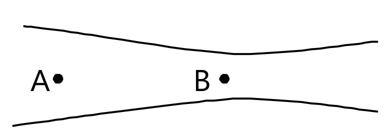


Figure 9.41

Sol: Apply the equation of continuity. Where area of cross-section is larger, the velocity of fluid is lesser and vice-versa.

From equation of continuity. The speed v_2 at B is given by, $A_1v_1 = A_2v_2$

or $(1.0\ cm^2)\ (10\ cm\ s^{-1}) = (20\ mm^2)v_2$ or $v_2 = \frac{1.0cm^2}{20mm^2} \times 10cm\ s^{-1} = 50\,cm\ s^{-1}$

By Bernoulli equation, $P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2$

Here $h_1 = h_2$. Thus $P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = \frac{1}{2} \times (1200\ kg\ m^{-2})(2500\ cm^2\ s^{-2} - 100\ cm^2\ s^{-2})$

$= 600\ kg\ m^{-3} \times 2400\ cm^2\ s^{-2} = 144\ Pa$

8.2 Application Based on Bernoulli's Equation

8.2.1 Venturimeter

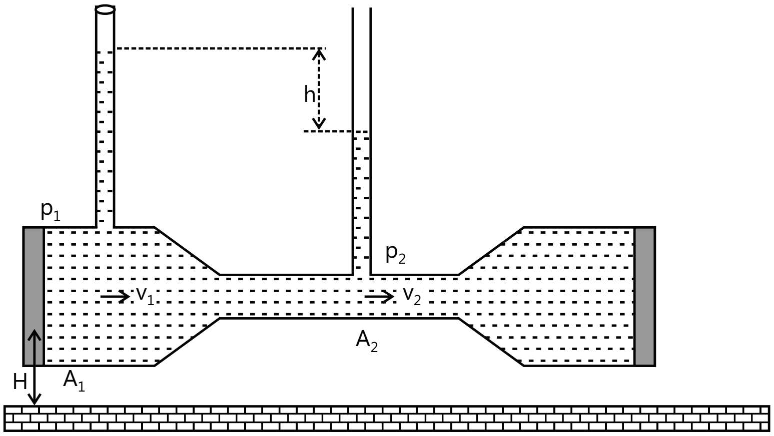
Figure shows a venturimeter used to measure flow speed in a pipe of non-uniform cross-section. We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of the pipe, with $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

From the continuity equation $v_2 = \frac{A_1 v_1}{A_2}$

Substituting and rearranging,

we get $P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$ ...(i)



Venturimeter

Figure 9.42

The pressure difference is also equal to ρgh, where h is the difference in liquid level in the two tubes.

Substituting in equation (i), we get $v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$

PLANCESS CONCEPTS

Because A_1 is greater than A_2, v_2 is greater than v_1 and hence the pressure P_2 is less than P_1.

The discharge or volume flow rate can be obtained as, $\frac{dV}{dt} = A_1 v_1 = A_1\sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$

Anurag Saraf (JEE 2011, AIR 226)

9. TORRICELLI'S THEOREM

It states that the velocity of efflux of a liquid through an orifice is equal to that velocity which a body would attain in falling from a height from the free surface of a liquid to the orifice. If h is the height of the orifice below the free surface of a liquid and g is acceleration due to gravity, the velocity of efflux of liquid = v= $\sqrt{2gh}$. Total energy per unit volume of the liquid at the surface = KE + PE + Pressure energy = 0 + $\rho gh + P_0$ ...(i)

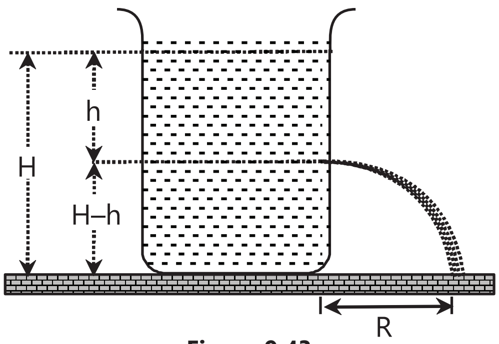


Figure 9.43

and total energy per unit volume at the orifice = KE + PE + Pressure energy = $\frac{1}{2}\rho v^2 + 0 + P_0$

Since total energy of the liquid must remain constant in steady flow, in accordance with Bernoulli's equation,

we have $\rho gh + P_0 = \frac{1}{2}\rho v^2 + P_0$ or $v = \sqrt{2gh}$

Range = velocity × time ; R = V_x × time = $\sqrt{2gh} \times 1$

Now, $H - h = \frac{1}{2}gt^2 \Rightarrow t = \frac{\sqrt{2(H-h)}}{g}$. From equation (i),

$$R = \sqrt{2gh} \times \frac{\sqrt{2(H-h)}}{g} = \sqrt{2h \times 2(H-h)} \times \sqrt{h(H-h)}2$$

$$\therefore \quad \boxed{R = 2\sqrt{h(H-h)}}$$

Range is max. if $\frac{dR}{dh} = 0 \Rightarrow 2 \times \frac{H-2h}{2\sqrt{h(H-h)}} = 0 \Rightarrow H - 2h = 0 \Rightarrow \boxed{h = \frac{H}{2}}$

PLANCESS CONCEPTS

$R_h = R_{H-h}$

$R_h = 2\sqrt{h(H-h)}$

$R_{H-h} = 2\sqrt{h(H-h)}$

i.e. Range would be the same when the hole is at a height h

or at a height H – h from the ground or from the top of the beaker.

R is maximum at h = $\frac{H}{2}$ and R_{max} =H.

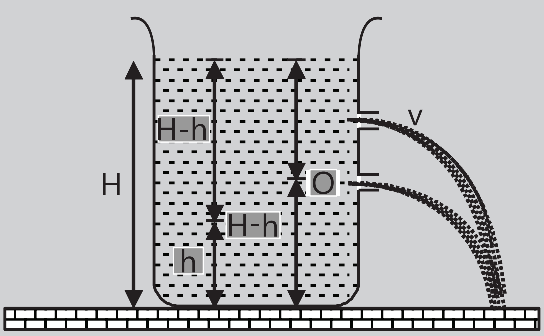


Figure 9.44

Vijay Senapathi (JEE 2011, AIR 71)

9.1 An Expression for the Force Experienced by the Vessel

The force experienced by the vessel from which liquid is coming out.

$F = \frac{dp}{dt}$ (Rate of change of momentum) $= \frac{d}{dt}(mv) = \frac{d}{dt}(\rho A v t v)$

$\boxed{F = \rho A v^2}$ Where ρ = It is the density of the liquid.

A = It is the area of hole through which liquid is coming out.

9.2 Time taken to Empty a Tank

Consider a tank filled with a liquid of density ρ up to a height H. A small hole of area of cross section a is made at the bottom of the tank. The area of cross-section of the tank is A.

Let at some instant of time the level of liquid in the tank be y. Velocity of efflux at this instant of time would be, $v = \sqrt{2gy}$.

At this instant volume of liquid coming out of the hole per second is $\left(\frac{dV_1}{dt}\right)$.

Volume of liquid coming down in the tank per second is $\left(\frac{dV_2}{dt}\right)$.

$$\frac{dV_1}{dt} = \frac{dV_2}{dt};\ \therefore\ av = A\left(-\frac{dy}{dt}\right)\quad \therefore a\sqrt{2gy} = A\left(-\frac{dy}{dt}\right) \text{ Or } \quad \int_0^t dt = -\frac{A}{a\sqrt{2g}}\int_H^0 y^{-1/2}dy$$

$$\therefore t = \frac{2A}{a\sqrt{2g}}[\sqrt{y}]_0^H = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

Illustration 25: A tank is filled with a liquid up to a height H. A small hole is made at the bottom of this tank. Let t_1 be the time taken to empty first half of the tank and t_2 the time taken to empty rest half of the tank.

Then find $\frac{t_1}{t_2}$. **(JEE MAIN)**

Sol: This problem needs to be solved by method of integration.

Substituting the proper limit in equation (i), derived in the theory, we have

$$\int_0^{t_1} dt = -\frac{A}{a\sqrt{2g}}\int_H^{H/2} y^{-1/2}dy \text{ Or } t_1 = \frac{2A}{a\sqrt{2g}}[\sqrt{y}]_{H/2}^H \text{ Or } = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$$

Or $t_1 = \frac{A}{a}\sqrt{\frac{H}{g}}(\sqrt{2} - 1)$ ...(ii)

Similarly $\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_{H/2}^0 y^{-1/2}dy$ Or $t_2 = \frac{A}{a}\sqrt{\frac{H}{g}}$ ... (iii)

From equations (ii) and (iii), we get $\frac{t_1}{t_2} = \sqrt{2} - 1$ Or $\frac{t_1}{t_2} = 0.414$

PLANCESS CONCEPTS

From here we see that $t_1 < t_2$. This is because inititally the pressure is high and the liquid comes out with greater speed.

Ankit Rathore (JEE Advanced 2013, AIR 158)

10. VISCOSITY

When a liquid moves slowly and steadily on a horizontal surface, its layer in contact with the fixed surface is stationary and the velocity of the layers increase with the distance from the fixed surface.

Consider two layers CD and MN of a liquid at distances x and x + dx from the fixed surface AB having velocities v and v + dv respectively as shown in the figure. Here $\left(\frac{dv}{dx}\right)$ denotes the rate of change of velocity with distance and is known as velocity gradient. The tendency of the upper layer is to accelerate the motion and the lower layer tries to retard the motion of upper layer. The two layers together tend to destroy their relative motion as if there is some backward dragging force acting tangentially on the layers. To maintain the motion, an external force is applied to overcome this backward drag.

Hence the property of a liquid virtue of which it opposes the relative motion between its different layers is known as viscosity.

The viscous force is given by $F = -\eta A \frac{dv}{dx}$

Where η is a constant, called the coefficient of viscosity.

The SI unit of η is N-s/m^2. It is also called decapoise or Pascal second. Thus,

1 decapoise = N-s/m^2 = 1 Pa-s = 10 poise.

Dimensions of h are [$ML^{-1}T^{-1}$]

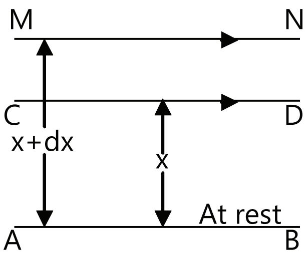


Figure 9.45

PLANCESS CONCEPTS

The negative sign in the above equation shows that the direction of viscous force F is opposite to the direction of relative velocity of the layer.

Viscous force depends upon the velocity gradient whereas the mechanical frictional force is independent of the velocity gradient.

Vaibhav Gupta (JEE 2009, AIR 54)

10.1 Effect of Temperature

In case of liquids, coefficient of viscosity decreases with increase of temperature as the cohesive forces decrease with increase of temperature.

Illustration 26: A plate of area 2 m^2 is made to move horizontally with a speed of 2 m/s by applying a horizontal tangential force over the free surface of a liquid. The depth of the liquid is 1 m and the liquid in contact with the bed is stationary. Coefficient of viscosity of liquid is 0.01 poise. Find the tangential force needed to move the plate. **(JEE MAIN)**

Sol: Apply the Newton's formula for the frictional force between two layers of a liquid.

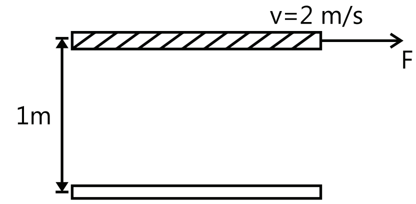


Figure 9.46

Velocity gradient = $\dfrac{\Delta v}{\Delta y} = \dfrac{2-0}{1-0} = 2\dfrac{m/s}{m}$

From Newton's law of viscous force,

$|F| = \eta A\dfrac{\Delta v}{\Delta y}$ = $(0.01 \times 10^{-1})(2)(2) = 4 \times 10^{-3}$ N.

So, to keep the plate moving, a force of 4×10^{-3} N must be applied.

10.2 Stokes' Law and Terminal Velocity

Stokes established that the resistive force or F, due to the viscous drag, for a spherical body of radius r, moving with velocity V, in a medium of coefficient of viscosity η is given by

$F = 6\ p\eta\ rV$

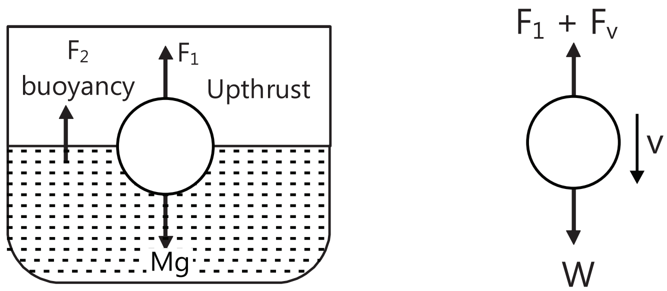


Figure 9.47

10.3.1 An Experiment for Terminal Velocity

Consider an established spherical body of radius r and density ρ falling freely from rest under gravity through a fluid of density σ and coefficient of viscosity η. When the body acquires the terminal velocity V

$W = F_t + 6\pi\eta rV$;

$$6\pi\eta rV = \frac{4}{3}\pi pr^3(\rho-\sigma)g \Rightarrow \boxed{V = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}}$$

Note: From the above expression we can see that terminal velocity of a spherical body is directly proportional to the densities of the body and the fluid $(\rho - \sigma)$. If the density of the fluid is greater than that of the body (.i.e. $\sigma > \rho$), the terminal velocity is negative. This means that the body instead of falling, moves upward. This is why air bubbles rise up in water.

Illustration 28: Two spherical raindrops of equal size are falling vertically through air with a terminal velocity of 1 m/s. What would be the terminal speed if these two drops were to coalesce to form a large spherical drop?

(JEE MAIN)

Sol: Use the formula for terminal velocity for spherical body.

$v_T \propto r^2$. Let r be the radius of small rain drops and R the radius of large drop.

Equating the volume, we have $\dfrac{4}{3}\pi R^2 = 2\left(\dfrac{4}{3}\pi r^3\right)$

$\therefore$ $R = (2)^{1/3}.\ r$ or $\dfrac{R}{r} = (2)^{1/3}\ \backslash$ $\dfrac{v_T'}{v_T} = \left(\dfrac{R}{r}\right)^2 = (2)^{2/3}$

$\therefore$ $v_T' = (2)^{2/3}\ v_T = (2)^{2/3}\ (1.0)$ m/s = 1.587 m/s.

Illustration 29: An air bubble of diameter 2 mm rises steadily through a solution of density 1750 kg m^{-3} at the rate of 0.35 cm s^{-1}. Calculate the coefficient of viscosity of the solution. The density of air is negligible. **(JEE MAIN)**

Sol: As the air bubble rises with constant velocity, the net force on it is zero.

The force of buoyancy B is equal to the weight of the displaced liquid. Thus B = $\frac{4}{3}pr^3\sigma g$.

This force is upward. The viscous force acting downward is F = $6\pi\eta rv$.

The weight of the air bubble may be neglected as the density of air is small. For uniform velocity

$$F = B \text{ or, } 6\pi\eta rv = \frac{4}{3}\pi r^3\sigma g \text{ or, } \eta = \frac{2r^2\sigma g}{9v} = \frac{2\times(1\times10^{-3}\,m)^2\times(1750\text{ kg m}^{-3})(9.8\text{ ms}^{-2})}{9\times(0.35\times10^{-2}\text{ ms}^{-1})} \approx 11\text{ poise}.$$

This appears to be a highly viscous liquid.

10.3 Stream Line Flow

When liquid flows in such a way that the velocity at a particular point is the same in magnitude as well as in direction. As shown in figure every molecule should have the same velocity at A, if it crossed from that point. Notice that the velocity at the point B will be different from that of A. But every molecule which reaches at the point B, gets the velocity of the point B.

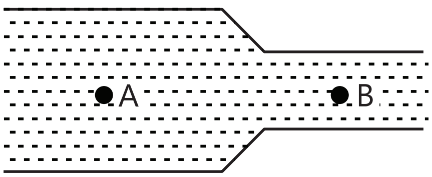


Figure 9.48

10.4 Turbulent Flow

When the motion of a particle at any point varies rapidly in magnitude and direction, the flow is said to be turbulent or beyond critical velocity. If the paths and velocities of particles change continuously and haphazardly, then the flow is called turbulent flow.

10.5 Critical Velocity and Reynolds Number

When a fluid flows in a tube with small velocity, the flow is steady. As the velocity is gradually increased, at one stage the flow becomes turbulent. The largest velocity which allows a steady flow is called the critical velocity.

Whether the flow will be steady or turbulent mainly depends on the density, velocity and the coefficient of viscosity of the fluid as well as the diameter of the tube through which the fluid is flowing. The quantity N = $\frac{\rho vD}{\eta}$ is called the Reynolds number and plays a key role in determining the nature of flow. It is found that if the Reynolds number is less than 2000, the flow is steady. If it is greater than 3000, the flow is turbulent. If it is between 2000 and 3000, the flow is unstable.

11. SURFACE TENSION

The properties of a surface are quite often marked different from the properties of the bulk material. A molecule well inside a body is surrounded by similar particles from all sides. But a molecule on the surface has particles of one type on one side and of a different type on the other side. Figure shows an example: A molecule of water well inside the bulk experiences force from water molecules from all sides, but a molecule at the surface interacts with air molecules from above and water molecules from below. This asymmetric force distribution is responsible for surface tension.

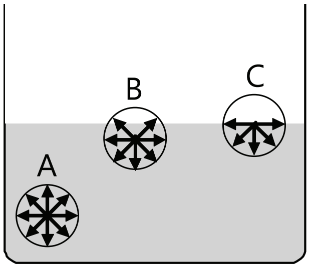


Figure 9.49

A surface layer is approximately 10-15 molecular diameters. The force between two molecules decreases as the separation between them increases. The force becomes negligible if the separation exceeds 10-15 molecular diameters. Thus, if we go 10-15 molecular diameters deep, a molecule finds equal forces from all directions.

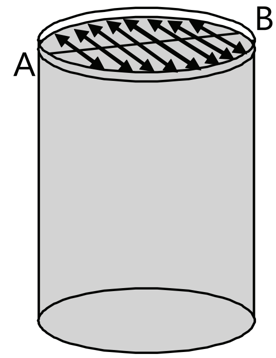


Figure 9.50

Imagine a line AB drawn on the surface of a liquid (figure). The line divides the surface in two parts, surface on one side and the surface on the other side of the line. Let us call them surface to the left of the line and surface to the right of the line. It is found that the two parts of the surface pull each other with a force proportional to the length of the line AB. These forces of pull are perpendicular to the line separating the two parts and are tangential to the surface. In this respect the surface of the liquid behave like a stretched rubber sheet. The rubber sheet which is stretched from all sides is in the state of tension. Any part of the sheet pulls the adjacent part towards itself.

Let F be the common magnitude of the forces exerted on each other by the two parts of the surface across a line of length ℓ. We define the surface tension T of the liquid as $T = F/\ell$

The SI unit of surface tension is N/m.

Note: The surface tension of a particular liquid usually decreases as temperature increases. To wash clothing thoroughly, water must be forced through the tiny spaces between the fibers. This requires increasing the surface area of the water, which is difficult to do because of surface tension. Hence, hot water and soapy water is better for washing.

PLANCESS CONCEPTS

Surface tension acts over the free surface of a liquid only and not within the interior of the liquid.

Due to surface tension the insects can walk on liquid surface.

Vaibhav Krishnan (JEE 2009, AIR 22)

Illustration 30: Calculate the force required to take away a flat circular plate of radius 4 cm from the surface of water, surface tension of water being 75 dyne cm^{-1}. **(JEE MAIN)**

Sol: Force = Surface tension×length of the surface

Length of the surface = circumference of the circular plate = $2\pi r = (8\pi)$ cm

Required force = $T \times L = 72 \times 8\pi = 1810$ dyne.

12. SURFACE ENERGY

When the surface area of a liquid is increased, the molecules from the interior rise to the surface. This requires work against force of attraction of the molecules just below the surface. This work is stored in the form of potential energy. Thus, the molecules in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called 'surface energy'. The surface energy is related to the surface tension as discussed below:

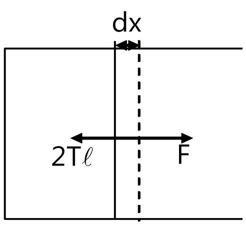


Figure 9.51

Let a liquid film be formed on a wire frame and a straight wire of length ℓ can slide on this wire frame as shown in figure. The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force $T\ell$. Thus, net force on the wire due to both the surfaces is $2T\ell$. One has to apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus, $F = 2T\ell$

Now, suppose the wire is moved through a small distance dx, the work done by the force is,

$dW = F\,dx = (2T\ell)dx$

But $(2\ell)(dx)$ is the total increase in the area of both the surfaces of the film. Let it be dA. Then,

$dW = T\,da$ or $T = \frac{dW}{dA}$

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity. Further, since there is no change in kinetic energy, the work done by the external force is stored as the potential energy of the new surface.

$\therefore T = \frac{dU}{dA}$ (as dW = dU)

Thus, the surface tension of a liquid is equal to the surface energy per unit surface area.

Illustration 31: How much work will be done in increasing the diameter of a soap bubble from 2 cm to 5 cm? Surface tension of soap solution is 3.0×10^{-2} N/m. **(JEE MAIN)**

Sol: Work done will be equal to the increase in the surface porential energy, which is surface tension multiplied by increase in area of surface of liquid.

Soap bubble has two surfaces. Hence, $W = T\,\Delta A$

Here, $\Delta A = 2[4\pi\{(2.5\times10^{-2})^2 - (1.0\times10^{-2})^2\}] = 1.32 \times 10^{-2}\ m^2$

$W = (3.0\times10^{-2})(1.32\times10^{-2})J = 3.96\times10^{-4}J$

Illustration 32: Calculate the energy released when 1000 small water drops each of same radius 10^{-7}m coalesce to form one large drop. The surface tension of water is 7.0×10^{-2} N/m. **(JEE MAIN)**

Sol: Energy released will be equal to the loss in surface potential energy.

Let r be the radius of smaller drops and R of bigger one.

Equating the initial and final volumes, we have $\frac{4}{3}\pi R^3 = (1000)\left(\frac{4}{3}\pi r^3\right)$

$R = 10r = (10)(10^{-7})\ m = 10^{-6}$ m. Further, the water drops have only one free surface. Therefore,

$\Delta A = 4\pi R^2 - (1000)(4\pi r^2) = 4\pi[(10^{-6})^2 - (10^3)(10^{-7})^2] = -36\pi(10^{-12})m^2$

Here, negative sign implies that surface area is decreasing. Hence, energy is released in the process.

$U = T[\Delta A] = (7\times10^{-2})(36\pi\times10^{-12})J = 7.9\times10^{-12}J$

13. EXCESS PRESSURE

The pressure inside a liquid drop or a soap bubble must be in excess of the pressure outside the bubble drop because without such pressure difference, a drop or a bubble cannot be in stable equilibrium. Due to the surface tension, the drop or bubble has got the tendency to contract and disappear altogether. To balance this, there must be excess of pressure inside the bubble.

13.1 Excess Pressure Inside a Drop

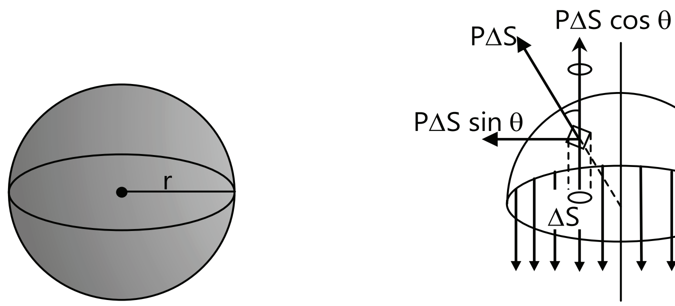


Figure 9.52

To obtain a relation between the excess of pressure and the surface tension, consider a water drop of radius r and surface tension T. Divide the drop into two halves by a horizontal passing through its centre as shown in figure and consider the equilibrium of one-half, say, the upper half. The force acting on it are:

(a) Force due to surface tension distributed along the circumference of the section.

(b) Outward thrust on elementary areas of it due to excess pressure.

Obviously, both the types of forces are distributed. The first type of distributed forces combine into a force of magnitude $2\pi r \times T$. To find the resultant of the other type of distributed forces, consider an elementary area ΔS of the surface. The outward thrust on $\Delta S = p\Delta S$ where p is the excess of the pressure inside the bubble. If this thrust makes an angle θ with the vertical, then it is equivalent to $\Delta Sp \cos \theta$ along the vertical and $\Delta Sp \sin \theta$ along the horizontal. The resolved component $\Delta Sp \sin \theta$ is infective as it is perpendicular to the resultant force due to surface tension. The resolved component $\Delta Sp \cos \theta$ is equal to balancing the force due to surface tension

The resultant outward thrust = $\Sigma\Delta Sp \cos \theta = p\Sigma\Delta S \cos \theta = p\Sigma\Delta S \cos \theta \quad = p\Sigma\Delta S'$

where $\Delta S' = \Delta S \cos \theta$ = area of the projection of ΔS on the horizontal dividing plane

$= p \times \pi r^2$ ($\ell\, \Delta S' = \pi r^2$)

For equilibrium of the bubble we have $\pi r^2\, p = 2\pi r\, T$ or $p = \frac{2T}{r}$

PLANCESS CONCEPTS

If we have an air bubble inside a liquid, a single surface is formed.

There is air on the concave side and liquid on the convex side.

The pressure in the concave side (that is in the air) is greater than

the pressure in the convex side (that is in the liquid) by an amount $\frac{2T}{R}$.

$\therefore \boxed{P_2 - P_1 = \frac{2T}{R}}$

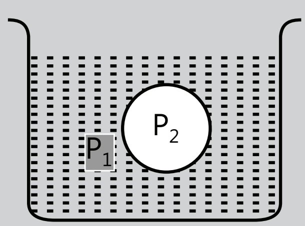


Figure 9.53

Nivvedan (JEE 2009, AIR 113)

13.2 Excess Pressure Inside Soap Bubble

A soap bubble consists of two spherical surface films with a thin layer of liquid between them. $P' - P_1 = 2S/R$ where R is the radius of the bubble.

As the thickness of the bubble is small on a macroscopic scale, the difference in the radii of the two surfaces will be negligible.

Similarly, looking at the inner surface, the air is on the concave side of the surface, hence $P_2 - P' = 2S/R$. Adding the two equations, $P_2 - P_1 = 4S/R$

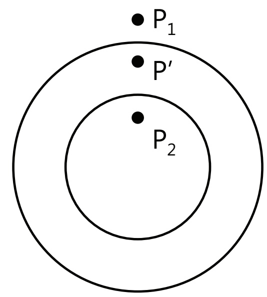


Figure 9.54

Illustration 33: What should be the pressure inside a small air bubble of 0.1 mm radius situated just below the water surface? Surface tension of water = 7.2×10^{-2} N/m and atmospheric pressure = 1.013×10^{5} N/m^2. **(JEE MAIN)**

Sol: Pressure inside the air bubble is larger than that outside it by amount 2T/R, where T is surface tension and R is its radius.

Surface tension of water T = 7.2×10^{-2} N/m; Radius of air bubble R = 0.1 mm = 10^{-4} m

The excess pressure inside the air bubble is given by, $P_2 - P_1 = \frac{2T}{R}$

$\therefore$ Pressure inside the air bubble, $P_2 = P_1 + \frac{2T}{R}$; Substituting the values, we have,

$$P_2 = (1.013\times10^5) + \frac{(2\times7.2\times10^{-2})}{10^{-4}} = 1.027\times10^5\ \text{N/m}^2$$

Illustration 34: A 0.02 cm liquid column balances the excess pressure inside a soap bubble of radius 7.5 mm. Determine the density of the liquid. Surface tension of soap solution = 0.03 Nm^{-1}. **(JEE MAIN)**

Sol: Pressure inside the soap bubble is larger than that outside it by amount 4T/R, where T is surface tension and R is its radius. Gauge pressure of liquid column is ρgh where symbols have the usual meaning.

The excess pressure inside a soap bubble is DP = 4S/R = $\frac{4\times0.03\text{Nm}^{-1}}{7.5\times10^{-3}\text{m}}$ = 16 Nm^{-2}

The pressure due to 0.02 cm of the liquid column is P = $h\rho g$ = $(0.02\times10^{-2}\ \text{m})\ \rho\ (9.8\ \text{ms}^{-2})$

Thus, 16 N m^{-2} = $(0.02\times10^{-2}\ \text{m})\ \rho\ (9.8\ \text{ms}^{-2})$; $\rho = 9.2\times10^3$ kg m^{-3}.

14. CAPILLARY ACTION

When a glass tube of very fine bore called a capillary tube is dipped in a liquid (like water), the liquid immediately rises into it due to the surface tension. The phenomenon of rise of a liquid in a narrow tube is known as capillarity.

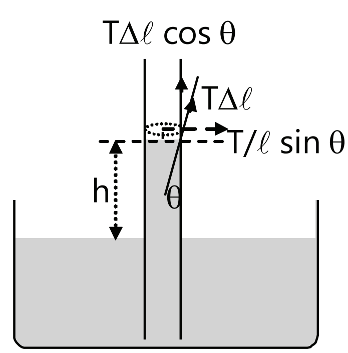


Figure 9.55

Suppose that a capillary tube of radius r is dipped vertically in a liquid. The liquid surface meets the wall of the tube at some inclination θ called the angle of contact. Due to surface tension, a force, $\Delta\ell T$ acts on an element $\Delta\ell$ of the circle of contact along which the liquid surface meets the solid surface and it is tangential to the liquid surface at inclination θ to the wall of the tube. (The liquid on the wall of the tube exerts this force. The tube also exerts the same force on the liquid in the opposite direction.) Resolving this latter force along and perpendicular to the wall of the tube, we have $\Delta\ell T\cos\theta$ along the tube vertically upwards and $\Delta\ell T\sin\theta$ perpendicular to the wall. The latter component is ineffective. It simply comes the liquid against the wall of the tube. The vertical component $\Delta\ell T\cos\theta$ pulls the liquid up the tube.

The total vertical upward force = $\Sigma\Delta\ell T\cos\theta = T\cos\theta\Sigma\Delta\ell = T\cos\theta.2\pi r$ (ℓ $\Sigma\Delta\ell$ = 2pr). Because of this upward pull liquid rises up in the capillary tube till it is balanced by the downward gravitational pull. If h is the height of the liquid column in the tube up to the bottom, the gravitational pull, i.e. weight of the liquid inside the tube is $(\pi r^2 h + V)\rho g$, where V is the volume of the liquid in meniscus. For equilibrium of the liquid column in the tube $2\pi rT\cos\theta = (\pi r^2 h + V)\rho g$

If value of the liquid in meniscus is negligible then, $2\pi rT\cos\theta = (\pi r^2 h)\rho g$; $h = \frac{2T\cos\theta}{r\rho g}$

The small volume of the liquid above the horizontal plane through the lowest point of the meniscus can be calculated if θ is given or known. For pure water and glass $\theta = 0°$ and hence the meniscus is hemispherical.

∴ V = volume of the cylinder of height r – volume of hemisphere.

$$= \pi r^3 - \frac{1}{2}\frac{4\pi}{3}r^3 = \pi r^3 - \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^3$$

∴ For water and glass $2\pi rT = \left(\pi r^2 h + \frac{\pi r}{3}\right)^3 \rho g$

$$2T = r\left(h + \frac{r}{3}\right)\rho g \quad \Rightarrow \quad h = \frac{2T}{r\rho g} - \frac{r}{3}$$

For a given liquid and solid at a given place as ρ, T, θ and g are constant, ∴ hr = constant

i.e. lesser t the radius of capillary greater will be the rise and vice-versa.

Illustration 36: A capillary tube of radius 0.20 mm is dipped vertically in water. Find the height of the water column raised in the tube. Surface tension of water = 0.075 N m^{-1} and density of water = 1000 kg m^{-3}. Take g = 10 m s^{-2}. **(JEE MAIN)**

Sol: Use the formula for height of the liquid in the capillary.

We have, $h = \frac{2S\cos\theta}{r\rho g} = \frac{2\times 0.075\ \text{N m}^{-1}\times 1}{(0.20\times 10^{-3}\text{m})\times(1000\ \text{kg m}^{-3})(10\ \text{m s}^{-2})}$ = 0.075 m = 7.5 cm.

PROBLEM SOLVING TACTICS

(a) Suppose two liquids of densities r_1 and r_2 having masses m_1 and m_2 are mixed together.

Then the density of the mixture will be = $\frac{(m_1 + m_2)}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)}$

If two liquids of densities r_1 and r_2 having volume V_1 and V_2 are mixed, then the density of the mixture will be

$$\frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}.$$

(b) When solving questions on Bernoulli's always assume a reference level and calculate the heights from the reference level.

FORMULAE SHEET

Fluid Statics:

1. Density = $\frac{\text{mass}}{\text{volume}}$, S.I. units: kg/m^3

2. Specific gravity / Relative density / Specific density = $\frac{\text{Ratio of its density}}{\text{Ratio of density of water at } 4°C}$,

S.I. units: No units

3. If two liquids of volume V_1 and V_2 and densities d_1 and d_2 respectively are mixed then the density d of the mixture is $d = \frac{V_1 d_1 + V_2 d_2}{V_1 + V_2}$; If $V_1 = V_2$ then $d = \frac{d_1 + d_2}{2}$

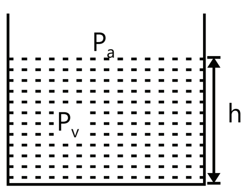
4. If two liquids of densities d_1 and d_2 and masses m_1 and m_2 respectively are mixed together,

then the density d of the mixture is $d = \frac{m_1 + m_2}{\frac{m_1}{d_1} + \frac{m_2}{d_2}}$; if $m_1 = m_2$ then $d = \frac{2d_1 d_2}{d_1 + d_2}$

5. Pressure = $\frac{\text{Normal component of force}}{\text{Area on which force acts}} = \frac{f}{A}$, S.I. units: N/m^2, Pa

6. Pressure P acting at the bottom of an open fluid column of height h and density d is

$= 1.013 \times 10^5$ Pa $= 1.013 \times 10^5$ Pa $= 1.013 \times 10^6$ dynes/cm^2 = 76 cm of Hg = 760 torr = 1.013 bars.



$P = P_a + hdg$ (P: Absolute pressure; P_a: Atmospheric pressure)

$\frac{dp}{dh} = sg$ (As h increases P increases)

Figure 9.56

$P - P_a = hdg$

gauge pressure = absolute – atmospheric pressure.

7.

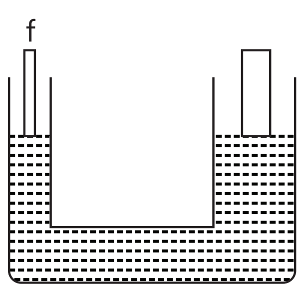


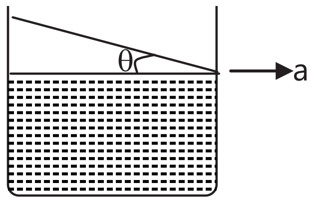
Figure 9.57

Area of smaller piston, a; area of larger piston, A, f is applied on the smaller piston

Force F developed on the larger piston $\frac{F}{A} = \frac{f}{a}$

$\therefore \quad F = \dfrac{fA}{a}$

8. Beaker is accelerated in horizontal direction



$\tan\theta = \dfrac{a}{g}$

a is the acceleration of the beaker in horizontal direction.

Figure 9.58

9. Beaker is accelerated and it has components of acceleration a_x, and a_y in x and y directions respectively.

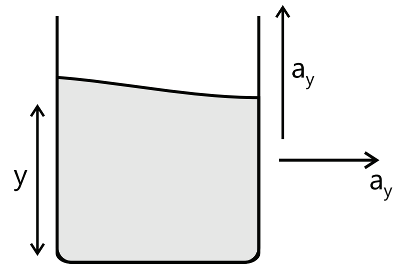


Figure 9.59

P increases with depth $\rightarrow \dfrac{dP}{dy} = p(g + ay)$

P is the density of the fluid.

ρ is the density of the fluid. $\dfrac{dP}{dx} = -pax$

10. Buoyant force $F = V_1\rho_1(\vec{g} - \vec{a})$

V_1 = immersed volume of liquid

ρ_1 = density of liquid

g = acceleration due to gravity

a = acceleration of body dipped inside liquid.

11. Body floats when Buoyant force balances the weight of the body.

$\underset{\text{(Buoyant force)}}{V_i\rho_2 g} = \underset{\text{(Weight of body)}}{V_b\rho_b g}$

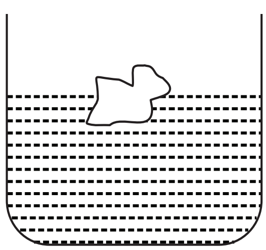


Figure 9.60

V_b, ρ_b = volume and density of body.

V_i = Volume of the immersed part of body.

ρ_2 = density of liquid.

Fraction of volume immersed $\frac{V_i}{V_b} = \frac{\rho_b}{\rho_2}$

% of volume immersed $\frac{V_i}{V_b} \times 100 = \frac{\rho_b}{\rho_2} \times 100.$

12. Apparent weight of a body inside a fluid is $W_{app} = W_{act}$ – Upthrust

$$W_{app} = V_b g\,(\rho_b - \rho_2)$$

V_b, δ_b = volume and density of body.

V_i = Volume of the immersed part of body.

ρ_2 = density of liquid.

13. General equation of continuity

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ Generally $\rho_1 = \rho_2$ i.e., density is uniform.

A_1 & A_2 are area of cross-section at point P and Q.

V_1 & V_2 are velocities of the fluid at point P and Q.

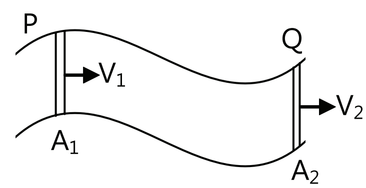


Figure 9.61

14. Bernoulli's Equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$$

i.e., $$P + \rho g h + \frac{1}{2}\rho v^2 = \text{constant}$$

Pressure Height Velocity at the point

at that point from the reference level

$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \text{constant}$$

Pressure head Velocity head gravitational head

15. Volumetric flow $Q = A\nu = \frac{dV}{dt}$ A – Area of cross section; ν – Velocity; V– Volume

S.I. unit = $\frac{m^3}{S}$

16. Torricelli Theorem:

$V = \sqrt{2gh}$ → height

V → velocity of efflux

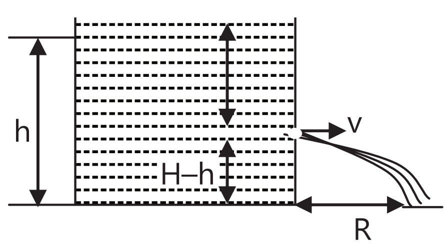


Figure 9.62

Range $R = 2\sqrt{h(H-h)}$

Range is maximum at $h = \frac{H}{2}$ and $R_{max} = H$

A_b – Area of orifice

A – Area of cross-section of the container.

Time taken to fall from H_1 to H_2 = $t \times \frac{A}{A_0}\sqrt{\frac{2}{g}}$

17. Viscous Force $F = \eta A \frac{dv}{dy}$

↓

coefficient of viscosity

L – Length of pipe

P_1 and P_2 are pressure at two ends of pipe.

R – Radius of pipe.

When liquid is flowing through a tube, velocity of flow of a liquid at distance from the axis.

$V = \frac{P}{4\eta L}\left(r^2 - x^2\right)$. Velocity distribution curve is a parabola.

18. Stoke's Law: Formula for the viscous force on a sphere

$F = 6\pi\eta rv$ (η – coefficient of viscosity)

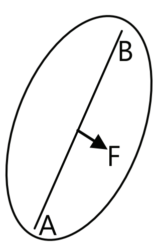
(r – radius of sphere)

(v – velocity of sphere)

$V_T = \frac{2}{9}r^2\frac{(\rho-\sigma)g}{\eta}$ (ρ – density of sphere)

($\propto$ – density of fluid)

19. Surface Tension



$T = \frac{F}{L}$

F is the total force acting on either side of AB.

L is length of AB.

Figure 9.63

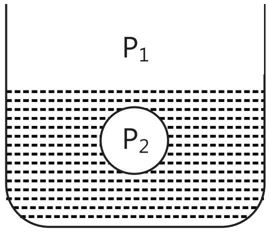
20. Surface Energy: dW = TdA

Surface Tension $T = \frac{dV}{dA} = \frac{\text{Surface energy}}{\text{Area}}$

21. Pressure inside the soap bubble is P, then

$P - P_0 = \frac{4T}{R}$

22. Air Bubble Inside a Liquid



$$P_2 - P_1 = \frac{2T}{R}$$

Figure 9.64

R – radius of bubble

T – surface tension force

23. Capillary Rise

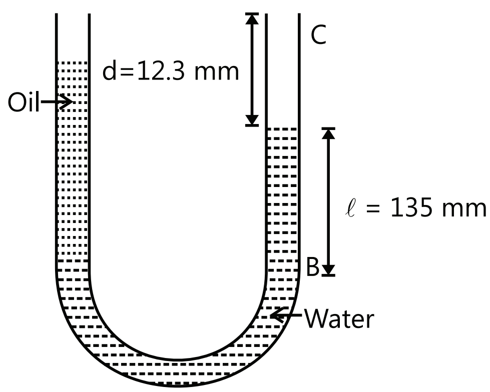
$h = \dfrac{2T\cos\theta}{r\rho g}$ r = is the radius of capillary tube

θ = angle of contact

Solved Examples

JEE Main/Boards

Example 1: For the arrangement shown in the figure. What is the density of oil?



Sol: Pressure will be same at all points at the same height in the same liquid.

$P_0 + \rho_w gl = P_0 + \rho_{oil}(\ell + d)g$

$$\Rightarrow \rho_{oil} = \frac{\rho_w \ell}{\ell + d} = \frac{1000.(135)}{(135+12.3)} = 916\ kg/m^3$$

Example 2: A solid floats in a liquid of different material. Carry out an analysis to see whether the level of liquid in the container will rise or fall when the solid melts.

Sol: Level of liquid will rise or fall depending on the density of the solid.

Let M = Mass of the floating solid.

ρ_1 = density of liquid formed by the melting of the solid.

ρ_2= density of the liquid in which the solid is floating. The mass of liquid displaced by the solid is M. Hence, the volume of liquid displaced is $\frac{M}{\rho_2}$. When the solid melts, the volume occupied by it is $\frac{M}{\rho_1}$. Hence, the level of liquid in container will rise or fall according as

$\frac{M}{\rho_2} - \frac{M}{\rho_1}$ is less than or greater than zero.

$\Rightarrow$ rises for $\rho_1 < \rho_2$

$\Rightarrow$ falls for $\rho_1 > \rho_2$

There will be no change in the level if the level if $\rho_1 = \rho_2$. In case of ice floating in water $\rho_1 = \rho_2$ and hence, the level of water remains unchanged when ice melts.

Example 3: An iron casting containing a number of cavities weighs 6000 N in air and 4000 N in water. What is the volume of the cavities in the casting? Density of iron is 7.87 g/cm^3.

Take g = 9.8 m/s^2 and density of water = 10^3 kg/m^3.

Sol: Apply Archemides principal. The volume of iron without the cavity is easily found. The total volume is found from the upthrust. The difference in volumes is the volume of cavity.

Let ν be the volume of cavities and V the volume of solid iron. Then,

$$V = \frac{\text{mass}}{\text{density}} = \left(\frac{6000/9.8}{7.87\times10^3}\right) = 0.078\text{m}^3$$

Further, decrease in weight = upthrust

$\therefore$ (6000 – 4000) = $(V + \nu)\rho_w g$

or 2000 = $(0.078 + \nu) \times 10^3 \times 9.8$

or $0.078 + \nu \approx 0.2$

$\therefore$ $\nu = 0.12$ m^3

Example 4: A boat floating in a water tank is carrying a number of stones. If the stones were unloaded into water, what will happen to the water level?

Sol: When the stones are in boat they will displace more water as compared to the case when they are out of the boat and inside water.

Let weight of boat = W and weight of stone = w.

Assuming density of water = 1 g/cc

Volume of water displaced initially= $(w + W)/\rho_w$

Later, Volume displaced = $\left(\frac{W}{\rho_w} + \frac{w}{\rho}\right)$

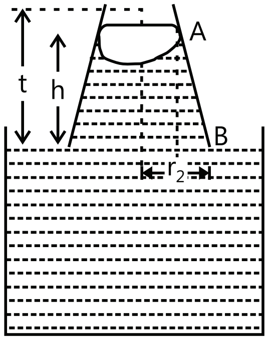
(ρ = density of stones)

$\Rightarrow$ Water level comes down.

Example 5: A conical glass capillary tube A of length 0.1 m has diameters 10^{-3}m and 5×10^{-4}m at the ends. When it is just immersed in a liquid at 0°C with larger radius in constant contact with it, the liquid rises to 8×10^{-2}m in the tube. In another cylindrical glass capillary tube B, when immersed in the same liquid at 0°C, the liquid rises to 6×10^{-2}m height. The rise of liquid in tube B is only 5.5×10^{-2}m when the liquid is at 50°C. Find the rate at which the surface tension changes with temperature considering the change to be linear. The density of liquid is (1/4) × 10^4 kg/m^3 and the angle of contact is zero. Effect of temp on the density of liquid and glass is negligible.

Sol: Use the formula for height of the liquid in the capillary.

Let r_1 and r_2 be radii of upper and lower ends of the conical capillary tube. The radius r at the meniscus is given by



$$r = r_1 + (r_2 - r_1)\left(\frac{\ell - h}{\ell}\right)$$

$$= (2.5\times10^{-4}) + (2.5\times10^{-4})\left(\frac{0.1-0.08}{0.1}\right)$$

= 3.0×10^{-4} m

The surface tension at 0°C is given by

$$T_0 = \frac{rh\rho g}{2}$$

$$= \frac{(3.0\times10^{-4})(8\times10^{-2})(1/4\times10^4)\times9.8}{2} = 0.084$$

For tube B, N/m $\frac{T_0}{T_{50}} = \frac{h_0}{h_{50}} = \frac{6\times10^{-2}}{5.5\times10^{-2}} = \frac{12}{11}$

$$\Rightarrow T_0 = \frac{11}{12}\times T_0 = \frac{11}{12}\times0.084 = 0.077 \text{ N/m}$$

Considering the change in the surface tension as linear, the change in surface tension with temp is given by

$$\alpha = \frac{T_{50} - T_0}{T_0 - T_{50}} = \frac{0.077 - 0.084}{0.084\times0.077} = -\frac{1}{60}k.$$

Negative sign shows that with rise in temp surface tension decreases.

Example 6: A piece of copper having an internal cavity weighs 264 gm in air and 221 gm when it is completely immersed in water. Find the volume of the cavity. The density of copper is 9.8 gm/cc.

Sol: Apply Archemides principal. The volume of copper without the cavity is easily found. The total volume is found from the upthrust. The difference in volumes is the volume of cavity.

Mass of copper in air = 264 gm

Mass of copper in water = 221 gm

Apparent loss of mass in water

= 264 – 221 = 43 gm

∴ Mass of water displaced by copper piece when completely immersed in water is equal to 43 gm.

Volume of water displaced $= \frac{\text{mass of displaced}}{\text{density of water}}$

$= \frac{43}{1.0} = 43.0$ cc

∴ Volume of copper piece including volume of cavity = 43.0 cc. Volume of copper block only

$= \frac{\text{mass}}{\text{density}} = \frac{264}{8.8} = 30.0$cc

Volume of cavity = 43.0 – 30.0 = 13.0 cc

Example 7: A cubical block of each side equal to 10 cm is made of steel of density 7.8 gm/cm^3. It floats on mercury surface in a vessel with its sides vertical. The density of mercury is 13.6 gm/cm^3.

(a) Find the length of the block above mercury surface.

(b) If water is poured on the surface of mercury, find the height of the water column when water just covers the top of the steel block.

Sol: Apply Archemides principal. The weight of the block will be equal to the weight of the liquid displaced.

(a) Volume of steel block

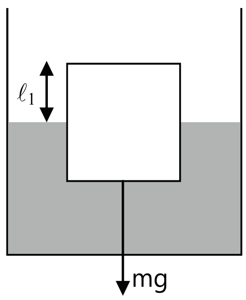
= $(10)^3$ = 1000 cm^3

Mass of steel block = 1000 × 7.8 = 7800 gm

Let ℓ_1 be the height of steel block above the surface of mercury. Height of block under mercury = $10 - \ell_1$. Weight of mercury displaced by block

$=(10 - \ell_1) \times 100 \times 13.6 \times g$ gm

Archimedes' principle shows that upward thrust is equal to the weight of mercury displaced by block is equal to the weight of the block.



$\therefore (10 - \ell_1) \times 100 \times 13.6 \times g = 7800$ g

$10 - \ell_1 = \frac{7800}{100 \times 13.6} = 5.74$

∴ length of block above mercury surface

= 10 – 5.74 = 4.26 cm

(b) Let ℓ_2 be the height of water column above mercury surface so that water just covers the top of the steel block. The upward thrust due to mercury and water displaced is equal to the weight of the body

∴ weight of block = wt. of water displaced + wt. of mercury displaced

$\therefore 7800 \text{ g} = \ell_2 \times 1000 \times 1 \times g$

$+ (10 - \ell_2) \times 100 \times 13.6 \times g$

$7800 = 100\ \ell_2 + 13600 - 1360\ \ell_2$

$1260\ \ell_2 = 13600 - 7800 = 5800$

∴ Height of water column above mercury=

$l_2 = \frac{5800}{1260} = 4.6$ cm

Example 8: A cubical block of wood of each side 10 cm long floats at the interface between oil and water with its lower surface 2 cm below the interface. The height of oil and water column is 10 cm each. The density of oil is 0.8 g cm^{-3}.

(a) What is the mass of the block?

(b) What is the pressure at the lower side of surface of block?

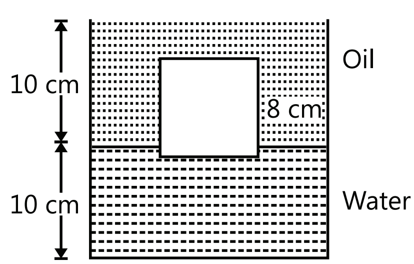
Sol: Apply Archemides principle. The weight of the block will be equal to the weight of the liquid displaced.

(a) Buoyant force = (mass of liquid displaced) × g

= [10 × 10 × 8 × 0.8 + 10 × 10 × 1]g= 840 g

If m is mass of block

mg = 840 g or m = 840 gm



(b) Pressure at the lower surface of block

= pressure at any point on the same level.

10 × 0.8 × g + 2 × 1 × g

= 10g = 10 × 9.81 = 99.1 Newton/meter^2

Example 9: A massless smooth piston forces water with a velocity of 8 m/s out of a tube shaped container with radii 4.0 cm and 1.0 cm respectively as shown in the figure. Assume that the water leaving the container enters air at 1 atmospheric pressure. Find

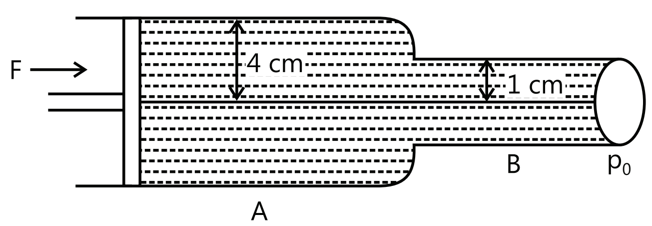
(a) The velocity of the piston

(b) Force F applied to the piston.

Sol: Apply Bernoulli's Theorem at two points, one near the piston and the other at the end of the tube.

(a) Let F be the force applied horizontally such that v_1 is the velocity of water in tube A of radius 4.0 cm and v_2 equal to 8 m/s is the velocity of water out of tube B of radius 1.0 cm.

Let p_0 be atmospheric pressure.



∴ At A, v_1 = ?, r_1 = 4.0 cm, $p_1 = p_0 + \frac{F}{a}$ Where a is area of cross-section of piston or tube A. At B, v_2 = 8 m/s, r_2 = 1.0 cm, $p_2 = p_0$

Bernoulli's theorem sat A and B gives,

$$p_1 + \frac{1}{2}\rho v_1 + h\rho g = p_2 + \frac{1}{2}\rho v_2^2 + h\rho ga$$

where ρ is density of water and h is height of axis of both tubes from ground level

$$\therefore p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{F}{a} + p_0 = p_0 + \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$\frac{F}{a} = \frac{\rho}{2}(v_2^2 - v_1^2) \qquad ...(i)$$

Equation of continuity at A and B gives

$v_1 a_1 = v_2 a_2$

or $v_1 = v_2 \times \frac{\pi r_2^2}{\pi r_1^2} = 8 \times \left(\frac{1}{4}\right)^2 = 0.5\text{ m / s}$

(b) Equation (i) gives $F = \frac{a\rho}{2}(v_2^2 - v_1^2) =$

$$\frac{\pi \times (4)^2 \times 1000}{2} \times \left[64 - \frac{1}{4}\right]$$

$$= \frac{1}{2} \times \frac{22}{7} \times 16 \times 1000 \times \frac{255}{4} = 160.3\text{N}$$

Example 10: A horizontal tube has different cross-sections at two points A and B. The diameter at A is 4.0 cm and that at B is 2 cm/ The two manometer arms are fixed at A and B. When a liquid of density 800 kg/m^3 flows through the tube, the difference of pressure between the arms of two manometers is 8 cm. Calculate the rate of flow of tube liquid.

Sol: Apply Bernoulli's Theorem and equation of continuity.

From Bernoulli's principle:

$$p_1 + \frac{1}{2}\rho\, v_1^2 = p_2 + \frac{1}{2}\rho\, v_2^2$$

From the equation of continuity: $A_1 v = A_2 v_2$

pressure difference: $p_1 - p_2 = h\rho g$

These equations give $v_1 = A_2\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}$

Rate of flow of volume

$$V = A_1 v_1 = A_1 A_2\sqrt{\frac{2gh}{(A_1^2 - A_2^2)}}$$

$$= \pi^2(4 \times 10^{-4})(1 \times 10^{-2})\sqrt{\frac{2 \times 9.8 \times 8 \times 10^{-2}}{(4\pi \times 10^{-4}) - (\pi \times 10^4)^2}}$$

= 4.06 × 10^{-4} m^3/s

JEE Advanced/Boards

Example 1: Under isothermal condition two soap bubbles of radii a and b coalesce to form a single bubble of radius c. If the external pressure is p_0

show that surface tension $T = \frac{p_0(c^3 - a^3 - b^3)}{4(a^2 + b^2 + c^2)}$

Sol: Pressure inside the soap bubble is larger than that outside it by amount 4T/R, where T is surface tension and R is its radius.

As we know that for a soap bubble, the excess pressure is = $\frac{4T}{r}$. External pressure is p_0

$\therefore p_a = p_0 + \frac{4T}{a} \therefore p_b = p_0 + \frac{4T}{b}$ and

$p_c = p_0 + \frac{4T}{c}$

and $v_a = \frac{4}{3}\pi a^3$, $v_b = \frac{4}{3}\pi b^3$ & $v_c = \frac{4}{3}\pi c^3$ ....(i)

Applying conservation of mass

$$n_a + n_b = n_c$$

$$\Rightarrow \frac{p_a v_a}{RT_a} + \frac{p_b v_b}{RT_b} = \frac{p_c v_c}{RT_c} \ [\because pv = nRT \Rightarrow n = \frac{Pv}{RT}]$$

Since, temp is constant

i.e.$T_a = T_b = T_c$, so the expression reduces to

$p_a v_a + p_b v_b = p_c v_c$

with the help of equation (i), we have

$$\left(p_0 + \frac{4T}{a}\right)\left(\frac{4}{3}\pi a^3\right) + \left(p_0 + \frac{4T}{b}\right)\left(\frac{4}{3}\pi b^3\right)$$

$$= \left(p_0 + \frac{4T}{c}\right)\left(\frac{4}{3}\pi c^3\right)$$

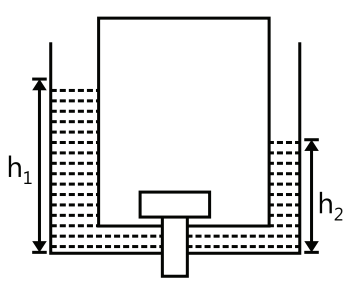
$$\Rightarrow 4T(a^2 + b^2 - c^2) = p_0(c^3 - a^3 - b^3)$$

$$\Rightarrow T = \frac{p_0(c^3 - a^3 - b^3)}{4(a^2 + b^3 - c^3)}$$

Example 2: Two identical cylindrical vessels with their bases at the same level contain a liquid of density ρ. The height of liquid in one vessel is h_1 and that in the other vessel is h_2. The areas of either base is A. What is the work done by gravity in equalizing the levels when the two vessels are connected.

Sol: Work done by gravity is equal to the loss in the gravitational potential energy.

The center of gravity of liquid column would be at height h_1 and h_2 respectively. A is area of cross-section.



Total P.E. when they are not connected

$$Ah_1\rho g\left(\frac{h_1}{2}\right) + Ah_2\rho g\left(\frac{h_2}{2}\right) = A\rho g\left[\frac{h_1^2}{2} + \frac{h_2^2}{2}\right]$$

When the levels are equal, the potential energy is given as

$$= A\left(\frac{h_1 + h_2}{2}\right)\rho g\left(\frac{h_1 + h_2}{4}\right) + A\left(\frac{h_1 + h_2}{2}\right)\rho g\left(\frac{h_1 + h_2}{4}\right)$$

$$= 2A\rho g\frac{(h_1 + h_2)^2}{2 \times 4} = A\rho g\frac{(h_1 + h_2)^2}{4}$$

The change in potential energy

$$= \frac{A\rho g}{2}\left[\frac{(h_1 + h_2)^2}{2} - \left(h_1^2 - h_2^2\right)\right]$$

$$= \frac{A\rho g}{2}\left[\frac{h_1^2 + h_2^2 - 2h_1^2 - 2h_2^2 + 2h_1h_2}{2}\right]$$

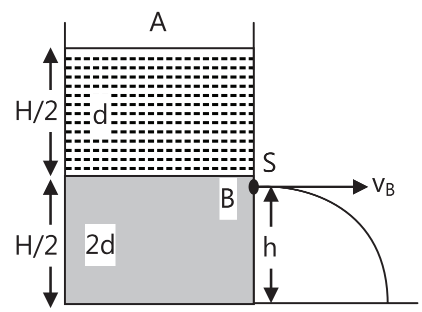
$$= \frac{A\rho g}{2}\left[\frac{-(h_1^2 + h_2^2 - 2h_1h_2)}{2}\right]$$

$$\text{Work done due to gravity} = -A\rho g\left[\frac{h_1 - h_2}{2}\right]^2$$

The negative sign shows that the work is done by the gravitational field on the liquid.

Example 3: A container of large uniform cross-section area A resting on a horizontal surface holds two immiscible, non-viscous and incompressible liquids of densities d and 2d, each of height H/2 as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0.

(a) A homogeneous solid cylinder of length L (L < H/2) and cross-section area A/5 is immersed such that, it floats with its axis vertical at the liquid-liquid interface with length L/4 in the dense liquid.



Determine:

(i) The density D of the solid.

(ii) The total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area S(S<<A) is punched on the vertical side of the container at a height h(h<H/2).

Determine:

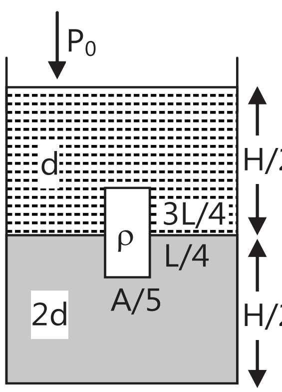
(i) The initial speed of efflux of liquid at the hole.

(ii) The horizontal distance x travelled by the liquid initially.

(iii) The height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m: (Neglect the air-resistance in these calculations)

Sol: Apply the principles of hydrostatic pressure, Archemedes and Bernoulli's Theorem.

(A) (i) As per Archimedes' principle, the buoyant force on a body is equal to the weight of the fluid displaced by the body.



Weight of solid cylinder = $L \times \frac{A}{5} \times D \times g = F\downarrow$

$F\uparrow$= Buoyant force = weight of liquid displaced $= \frac{L}{4} \times \frac{A}{5} \times 2dg + \frac{3L}{4} \times \frac{A}{5} \times d \times g$

Equating: $L \times \frac{A}{5} \times D \times g$

$$= \frac{L}{4} \times \frac{A}{5} \times 2dg + \frac{3L}{4} \times \frac{A}{5} \times d \times g$$

$$D = \frac{d}{2} + \frac{3d}{4} = \frac{2d + 3d}{4} = \frac{5d}{4}$$

(ii) Pressure at the bottom of the cylinder

$$= P_{atmosphere} + P_{dense\ liquid} + P_{light\ liquid}$$

Pressure due to liquid $= \frac{Force}{Area}$

$$= \frac{1}{A}\left[Adg\left(\frac{H}{2}\right) + A(2d)g\left(\frac{H}{2}\right)\right] = dg\left(\frac{3H}{2}\right)$$

Pressure due to buoyancy reaction

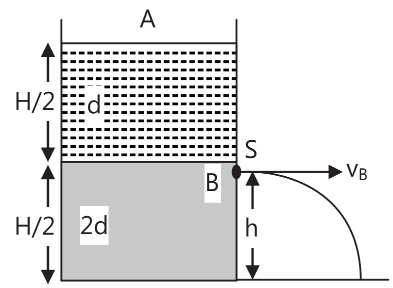
$$= \frac{\text{Buoyancy reaction force}}{\text{area}} \qquad = \left(\frac{A}{5}\right)\frac{LDg}{A}$$

$$= \frac{A}{5} \times L \times \frac{5d}{4} \times \frac{l}{A} \times g = \frac{Lgd}{4}$$

$\therefore$ Total pressure $= P_0 + dg\left(\frac{3H}{2}\right) + \frac{Ldg}{4}$

$$= P_0 + dg\left[\frac{3H}{2} + \frac{L}{4}\right]$$

(b) (i) Let v_A and v_B be velocity of fluids at points A and B.



$Av_A = sv_B$

$$\because \quad v_A = \left(\frac{s}{A}\right)v_B \simeq 0 \qquad (\ell\ A>>H/2)$$

Bernoulli's Equation: $p + \frac{1}{2}\rho v_2 + \rho gh = \text{constant}$

At A, $P_0 + \frac{1}{2}dv_A^2 + dg\frac{H}{2} + 2d(g)\left(\frac{H}{2}\right) = \text{constant or}$

$P_0 + \frac{3}{2}dgH = \text{constant}$ $(\because\ V_A = 0)$

At point B,
$$P_0 + \frac{1}{2}dv_A^2 + \frac{1}{2}(2d)v_B^2 + 2dgh = \text{cosntant}$$

or $\quad P_0 + dv_B^2 + 2dgh = \text{constant}$

Equating: $P_0 + dv_B^2 + 2dgh = P_0 + \frac{3}{2}dgH$

$$dv_B^2 + 2dgh = \frac{3}{2}dgH$$

$$v_B^2 = g\left[\frac{3}{2}H - 2h\right]; \ v_B = \sqrt{g\left(\frac{3}{2}H - 2h\right)}$$

(ii) Time t taken by liquid to fall through height h under g with zero initial velocity. $t = \sqrt{\frac{2h}{g}}$

Horizontal distance

$$x = v_B t = \sqrt{\frac{2h}{g}} \times \sqrt{g\left(\frac{3}{2}H - 2h\right)}$$

$$\sqrt{h(3H-4h)} = 2 \times \sqrt{h} \times \sqrt{\frac{3H}{4} - h}$$

(iii) To find height h at which x is max, $\frac{dx}{dh} = 0$.

$$\frac{d}{dh}\left[3Hh - 4h^2\right]^{1/2} = 0 ; \frac{d}{dh}\left[h(3H-4h)\right]^{1/2} = 0$$

$$\frac{d}{dh}\left[2 \times h\sqrt{\frac{3H}{4} - h}\right] = 0 .$$

$$2 \times \frac{1}{2h}\left(\frac{3H}{4} - h\right)^{1/2} + 2\sqrt{h} \times \frac{1}{2}\left(\frac{3H}{4} - h\right)^{-1/2}(-1) = 0$$

$$\frac{1}{h}\left(\frac{3H}{4} - h\right)^{1/2} = \frac{\sqrt{h}}{\left[\frac{3H}{4} - h\right]^{1/2}}$$

or $\frac{3H}{4} - h = h$ or $h = \frac{3}{8}H$

$$\therefore \quad x_m = 2 \times \sqrt{\frac{3H}{8}}\left(\frac{3H}{4} - \frac{3H}{8}\right)^{1/2}$$

$$= 2 \times \sqrt{\frac{3H}{8}} \times \sqrt{\frac{3H}{8}} = \frac{3H}{4}$$

Example 4: A tube of length ℓ and radius R carries a steady flow of liquid whose density is ρ and viscosity η. The velocity v of flow is given by $V = V_0\left(1 - \frac{r^2}{R^2}\right)$, where r is the distance of flowing fluid from the axis. Find

(a) Volume of fluid, flowing across the section of the tube, in unit time.

(b) Kinetic energy of the fluid within the volume of the tube.

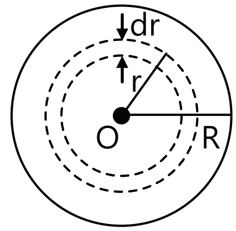
(c) The frictional force exerted on the tube by the fluid, and

(d) The difference of pressures at the ends of the tube.

Sol: The cross section of tube can be thought of made-up of elementary rings of infinitesimal thickeness. Find the volume flow rate and kinetic energy of one ring. Use the method of integration to find the flow rate and energy for the tube.

(a) Let us consider a cylindrical section at a distance of r and having thickness dr. The volume of fluid flowing through this section per second. $dv = (2\pi r dr)v_0\left(1 - \frac{r^2}{R^2}\right)$

So, the volume of fluid flowing across the section of the tube in unit time.



$$v = \int_0^R (2\pi r dr)v_0\left(1 - \frac{r^2}{R^2}\right) = 2\pi v_0 \int_0^R r\left(1 - \frac{r^2}{R^2}\right)dr$$

$$= 2\pi v_0\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = 2\pi v_0\left(\frac{R^2}{4}\right)$$

(b) The kinetic energy of the fluid within the volume element of thickness dr

$$\frac{1}{2}(dm)v^2 = \frac{1}{2}(2\pi r dr \ell)\rho v_0^2\left(1 - \frac{r^2}{R^2}\right)^2$$

So, the K.E. of fluid within the tube

$$= \frac{1}{2}(2\pi\ell)\rho v_0^2 \int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 r dr$$

Integrating, we get

$$K.E. = \pi r \ell \ \ell \ v_0^2\left(\frac{R^2}{6}\right) \ell$$

(c) The viscous drag exerts a force on the tube $F = \eta A\left(\frac{dv}{dx}\right)_{r=R}$

Hence $\left(\frac{dv}{dr}\right)_{r=R} = v_0\left(-\frac{2r}{R^2}\right)_{r=R} = \frac{-2v_0}{R}$

$\therefore F = -\eta(2\pi R\ell)(-2v_0\ell R) = 4\pi\eta h l v_0$

(d) The pressure difference ΔP is given by

$$\Delta P = P_2 - P_1 = P$$

Where $P_1 = O$ and $P_2 = P$

As we know that $P = \dfrac{\text{Force(F)}}{\text{area of section of tube}}$

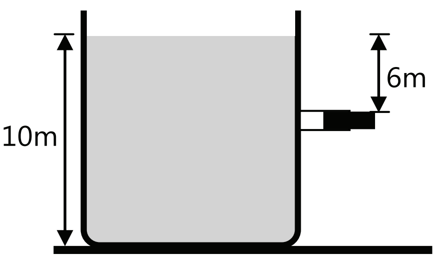
$$P = \frac{F}{\pi R^2} = \frac{4\pi\eta\ell v_0}{\pi R^2}$$

$$P = \frac{4\eta\ell v_0}{R^2}$$

Example 5: A fresh water reservoir is 10 m deep. A horizontal pipe 4.0 cm in diameter passes through the reservoir 6.0 m below the water surface as shown in figure. A plug secures the pipe opening.

(a) Find the friction force between the plug and pipe wall.

(b) The plug is removed. What volume of water flows out of the pipe in 1 h? Assume area of reservoir to be too large.



Sol: Force of friction will balance the force due to pressure difference on the plug. Use the formula for velocity of efflux for part (b)

(a) Force of friction

= pressure difference on the sides of the plug × area of cross section of the plug

$= (\rho gh)\,A = (10)^3(9.8)(6.0)(\pi)(2 \times 10^{-2})^2$

= 73.9 N

(b) Assuming the area of the reservoir to be too large.

Velocity of efflux $v = \sqrt{2gh} = \text{constant}$

$\therefore \quad v = \sqrt{2 \times 9.8 \times 6} = 10.84\,\text{m/s}$

Volume of water coming out per sec,

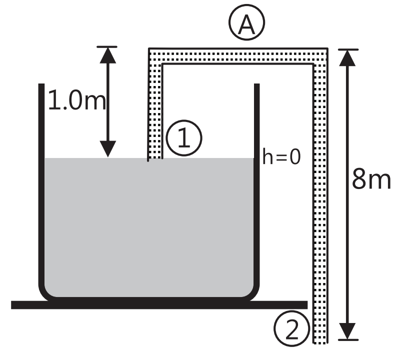
$$\frac{dV}{dt} = Av = \pi(2\times10^{-2})^2(10.84) = 1.36\times10^{-2}\ \text{m}^3/\text{s}$$

$\therefore$ The volume of water flowing through the pipe in 1 h.

$$V = \left(\frac{dV}{dt}\right)t = (1.36\times10^{-2})(3600) = 49.96\ \text{m}^3$$

Example 6: The U-tube acts as a water siphon. The bend in the tube is 1m above the water surface. The tube outlet is 7 m below the water surface. The water issues from the bottom of the siphon as a free jet at atmospheric pressure. Determine the speed of the free jet and the minimum absolute pressure of the water in the bend. Given atmospheric pressure = 1.01×10^5 N/m^2.

$g = 9.8$ m/s^2 and density of water = 10^3 kg/m^3.



Sol: Apply Bernoulli's Theorem at points 1, A and 2.

(a) Applying Bernoulli's equation between point (1) and (2)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho h_1 + P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Since, area of reservoir >> area of pipe

$v_1 \approx 0,\quad$ also $P_1 = P_2$ = atmospheric pressure

So, $v_1 = \sqrt{2g(h_1 - h_2)} = \sqrt{2\times9.8\times7} = 11.7\ \text{m/s}$

(b) The minimum pressure in the bend will be at A. Therefore, applying Bernoulli's equation between (1) and (A)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_A + \frac{1}{2}\rho v_A^2 + \rho g h_A$$

Again, $v_1 \approx 0$ and from conservation of mass $v_A = v_2$;

$$P_A = P_1 + \rho g(h_1 - h_A) - \frac{1}{2}\rho v_2^2$$

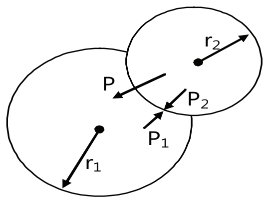
Therefore, substituting the values, we have

$P_A = (1.01 \times 10^5) + (1000)(9.8)(-1)$

$-\dfrac{1}{2}\times(1000)(11.7)^2 2 = 2.27 \times 10^4$ N/m^2

Example 7: Two separate air bubbles (radii 0.004 m and 0.002 m) formed of the same liquid (surface tension 0.07 N/m) come together to form a double bubble. Find the radius and the sense of curvature of the internal film surface common to both the bubbles.

Sol: Pressure inside the soap bubble is larger than that outside it by amount 4T/R, where T is surface tension and R is its radius.



For the two bubbles,

$$P_1 = P_0 + \frac{4T}{r_1};$$

$$P_2 = P_0 + \frac{4T}{r_2},\ r_2 < r_1$$

$\therefore \quad P_2 > P_1$

i.e. pressure inside the smaller bubble will be more. The excess pressure

$$P = P_2 - P_1 = 4T\left(\frac{r_1 - r_2}{r_1 r_2}\right) \qquad \text{...(i)}$$

This excess pressure acts from concave to convex side, the interface will be concave towards smaller bubble and convex towards larger bubble. Let R be the radius of interface then,

$$P = \frac{4T}{R} \qquad \text{....(ii)}$$

From equations (i) and (ii)

$$R = \frac{r_1 r_2}{r_1 - r_2} = \frac{(0.004)(0.002)}{(0.004 - 0.002)} = 0.004\ \text{m}$$

Example 8: A cylindrical tank of base area A has a small hole of area 'a' at the bottom. At time t = 0, a tap starts to supply water into the tank at a constant rate α m^3/s.

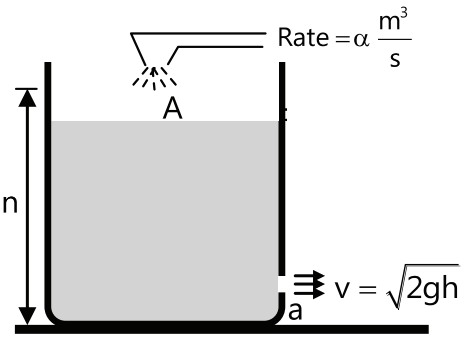
(a) What is the maximum level of water h_{max} in the tank?

(b) Find the time when level of water becomes $h (< h_{max})$.

Sol: The height of water level will increase till the rate of inflow is greater than the rate of outflow. Use method of integration to find the time taken by water level to reach height h.

(a) Level will be maximum level when

Rate of inflow of water = rate of outflow of water



i.e., $\alpha = av$ or $\alpha = a\sqrt{2gh_{max}}$

$$\Rightarrow \quad h_{max} = \frac{\alpha^2}{2ga^2}$$

(b) Let at time t, the level of water be h. Then,

$$A\left(\frac{dh}{dt}\right) = \alpha - a\sqrt{2gh} \quad \text{or} \quad \int_0^h \frac{dh}{\alpha - a\sqrt{2gh}} = \int_0^t \frac{dt}{A}$$

Solving this, we get

$$t = \frac{A}{ag}\left[\frac{\alpha}{a}\ln\left\{\frac{\alpha - a\sqrt{2gh}}{\alpha}\right\} - \sqrt{2gh}\right]$$

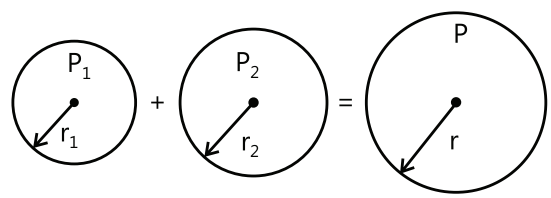
Example 9: Under isothermal condition, two soap bubbles of radii r_1 and r_2 coalesce to form a single bubble of radius r. The external pressure is P_0. Find the surface tension of the soap in terms of the given parameters.

Sol: Pressure inside the soap bubble is larger than that outside it by amount 4T/R, where T is surface tension and R is its radius. Use ideal gas equation and the condition that he total number of moles of air is conserved.

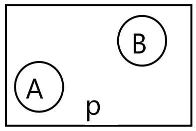
As mass of the air is conserved,

$\therefore \quad n_1 + n_2 = n \qquad$ (as PV = nRT)

$$\therefore \quad \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{PV}{RT}$$



Although not given in the question, but we will have to assume that temperature of A and B are the same.



$p = 8\ Nm^{-2}$

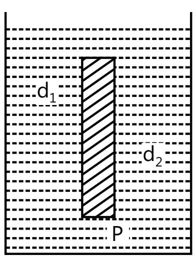
$$\frac{n_B}{n_A} = \frac{p_B V_B / RT}{p_A V_A / RT} = \frac{p_B V_B}{p_A V_A}$$

$$= \frac{(p + 4s/r_A) \times 4/3\pi(r_A)^3}{(p + 4s/r_B) \times 4/3\pi(r_B)^3}$$

(s = surface tension)

Substituting the values, we get $\frac{n_B}{n_A} = 6$

Example 10: A thin rod of length L and area of cross section S is pivoted at its lowest point P inside a stationary, homogeneous and non-viscous liquid as shown in the figure. The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the density d_2 of the liquid. The rod is displaced by a small angle. From its equilibrium position and then released, show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



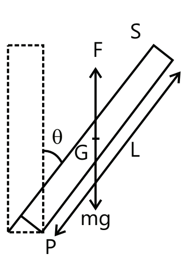
Sol: Use the restoring torque method to find the angular frequency.

Consider the rod be displaced through an angle θ. The different forces on the rod are shown in the figure.

Weight of rod acting downward=S L d_1 g=mg

Buoyant force acting upwards =S L d_2 g

Net thrust acting on the rod upwards; F=S L (d_2-d_1)g



Restoring torque $\tau = Fx\frac{L}{2}\sin\theta = SL(d_2 - d_1)g\frac{L}{2}\sin\theta$

$\sin\theta \approx \theta$ (θ is small)

$$\therefore\ \tau = \frac{1}{2}SL^2(d_2 - d_1)g\theta$$

$$\tau = I\alpha = \left(\frac{ML^2}{3}\right)\frac{d^2\theta}{dt^2} = \left(\frac{SLd_1 xL^2}{3}\right)\frac{d^2\theta}{dt^2}$$

$$\therefore\ \frac{d^2\theta}{dt} = \frac{3}{SL^3 d_1} x \frac{1}{2} SL^2(d_2 - d_1)g\theta$$

or $\frac{d^2\theta}{dt} = \frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)\theta$; so motion is S.H.M;

comparing with differential equation of S.H.M.

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0;\ \omega = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)};$$

$$\text{Time period, } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2Ld_1}{3g(d_2 - d_1)}}$$

Example 11: Two non-viscous, incompressible and immiscible liquids of density ρ and 1.5ρ are poured into two limbs of a circular tube of radius R and small cross-section kept fixed in a vertical plane as shown in the figure.

Each liquid occupies one fourth the circumference of the tube.

(a) Find the angle that the radius vector to the interface makes with the vertical in the equilibrium position.

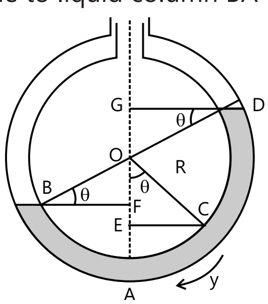
(b) If the whole liquid is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

Sol: Use the restoring torque method to find the angular frequency.

(a) Density of liquid column BC = 1.5 ρ;

Density of liquid column CD = ρ

Pressure at A due to liquid column BA= ρ AB



$= AFx1.5\rho xg = (AO - OF)1.5\rho gxg\rho$
$= (R - R\sin\theta)1.5g\rho$

Pressure at A due to liquid column AD = ρAD
$= AEx1.5\rho xg + EG\rho g$

$\therefore\ \rho AD - (AO - OE)1.5\rho g + (EO + OG)\rho g$

$-(R - R\cos\theta)1.5\rho g + R(\cos\theta + \sin\theta)\rho g$

In equilibrium $P_{AB} = P_{AD}$

$R(1-\sin\theta)1.5\rho g = R(1-\cos\theta)1.5\rho g + R(\cos\theta + \sin\theta)\rho g$

$\tan\theta = \frac{0.5}{2.5} = \frac{1}{5}.$ or $\tan^{-1}\left(\frac{1}{5}\right)$

(b) If a is area of cross – section,

length of each column $= \frac{2\pi R}{4} = \frac{\pi R}{2}$

Volume of each column $= \frac{\pi Ra}{2}$

Mass of column BC $= \frac{\pi Ra}{2}x1.5\rho$

Mass of column CD $= \frac{\pi Ra}{2}xp$

M.I. of whole liquid about O $= \left(\frac{\pi Rap}{2}\right)(1.5+1)R^2$

or $I = \frac{2.5\pi R^3 ap}{2}$

Let y be small displacement toward left and θ be the angular displacement,

$\theta = \frac{y}{R}$ or $y = R\theta$, Angular acceleration $= \frac{d^2\theta}{dt^2}$,

Torque about A $= I\frac{d^2\theta}{dt^2} = \frac{2.5\pi R^3 ap}{2}\left(\frac{d^2\theta}{dt^2}\right)$

Restoring torque due to displaced liquid.

$\tau_{rest} = -\left[ay\,x1.5\rho g + ay\rho g\right]xR\cos\theta$
$= -2.5ay\rho g\,xR\cos\theta = -2.5ap gR^2\cos\theta.\theta$

[R $\cos\theta$ is perpendicular distance of gravitational force from axis of rotation]

Equating $\left(\frac{2.5\pi R^3 ap}{2}\right)\frac{d^2\theta}{dt^2} = -\left(2.5apgR^2\cos\theta\right)\theta$

$\frac{d^2\theta}{dt^2} = -\left(\frac{2g\cos\theta}{\pi R}\right)\theta = -\omega^2\theta$

As $\frac{2g\cos\theta}{\pi R}$ Acceleration is proportional to angular displacement and is directed towards mean position, the liquid undergoes SHM

$T = \frac{2\pi}{\omega} = 2\pi x\sqrt{\frac{\pi R}{2g\cos\theta}}$

As $\tan\theta = \frac{1}{5}.\cos\theta = \frac{5}{\sqrt{26}}.$

$T = 2\pi\sqrt{\frac{\pi R}{2xgx\frac{5}{\sqrt{26}}}} = 2(\pi)^{\frac{3}{2}}\sqrt{\frac{R}{\left(\frac{10g}{\sqrt{26}}\right)}}$

JEE Main/Boards

Exercise 1

Q.1 If water in one flask and castor oil in other are violently shaken and kept on a table, then which one will come to rest earlier?

Q.2 What is the acceleration of a body falling through a viscous medium after terminal velocity is reached?

Q.3 The liquid is flowing steadily through a tube of varying diameter. How are the velocity of liquid flow (V) in any portion and the diameter (D) of the tube in that portion related?

Q.4 How does the viscosity of gases depend upon temperature?

Q.5 Explain the effect of (i) density (ii) temperature and (iii) pressure on the viscosity of liquids and gases.

Q.6 Two equal drops of water falling through air with a steady velocity v. If the drops coalesced, what will be the new steady velocity?

Q.7 What is the viscous force on a drop of liquid of radius 0.2 mm moving with a constant velocity 4 cm s^{-1} through a medium of viscosity 1.8×10^{-1} Nm^{-2} s.

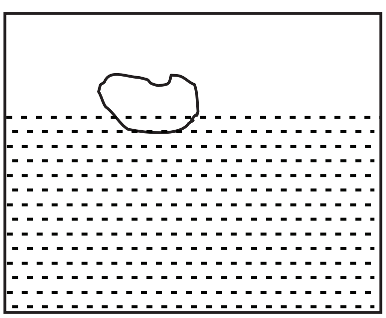
Q.8 Eight rain drops of radius 1 mm each falling downwards with a terminal velocity of 5 cm s^{-1} coalesce to form a bigger drop. Find the terminal velocity of bigger drop.

Q.9 The flow rate of water from a tap of diameter 1.25 cm is 0.48 L/min. The coefficient of viscosity of water is 10^{-3} Pa-s. After sometime, the flow rate is increased to 3 L/min. The coefficient of viscosity of water is 10^{-3} Pa-s. Characterize the flow.

Q.10 A block of wood is floating in a lake? What is apparent weight of the floating block?

Q.11 A block of wood is floating in a lake. What is apparent weight of the floating block?

Q.12 A body floats in a liquid contained in a beaker. The whole system shown in the figure falls freely under gravity. What is the up thrust on the body due to the liquid?



Q.13 A force of 60 N is applied on a nail, where tip has an area of cross-section of 0.0001 cm^2. Find the pressure on the tip.

Q.14 If the water pressure gauge shows the pressure at ground floor to be 270 kPa, how high would water rise in the pipes of a building?

Q.15 A metal cube is 5 cm side and relative density 9, suspended by a thread is completely immersed in a liquid of density 1.2×10^{3} kg m^{-3}. Find the tension in the thread.

Q.16 A boat having a length of 3 m and breadth 2 m is floating on a lake. The boat sinks by one cm, when a man gets on it. What is the mass of the man?

Q.17 Calculate the force required to take away a flat plate of radius 5 cm from the surface of water. Given surface tension of water = 72×10^{-3} Nm^{-1}.

Q.18 A square wire frame of side 10 cm is dipped in a liquid of surface tension 28×10^{-3} Nm^{-1}. On taking out, a membrane is formed. What is the force acting on the surface of wire frame?

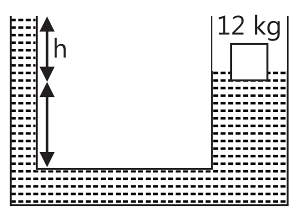
Q.19 The air pressure inside a soap bubble of diameter 3.5 mm is 8 mm of water above the atmosphere. Calculate the surface tension of soap solution.

Q.20 What should be the radius of the capillary tube so that water will rise to a height of 8 cm in it? Surface tension of water 70×10^{-3} Nm^{-1}.

Exercise 2

Single Correct Choice Type

Q.1 The area of cross-section of the wider tube shown in figure is 800 cm^2. If a mass of 12 kg is placed on the massless piston, the difference in heights h in the level of water in the two tubes is:



(A) 10 cm (B) 6 cm (C) 15 cm (D) 2 cm

Q.2 Two cubes of size 1.0 m side, one of relative density 0.60 and another of relative density = 1.15 are connected by weightless wire and placed in a large tank of water. Under equilibrium the lighter cube will project above the water surface to a height of:

(A) 50 cm (B) 25 cm (C) 10 cm (D) Zero

Q.3 A cuboidal piece of wood has dimensions a, b and c. Its relative density is d. It is floating in a large body of water such that side a is vertical. It is pushed down a bit and released. The time period of SHM executed by it is:

(A) $2\pi\sqrt{\frac{abc}{g}}$ (B) $2\pi\sqrt{\frac{h}{da}}$

(C) $2\pi\sqrt{\frac{bc}{dg}}$ (D) $2\pi\sqrt{\frac{da}{g}}$

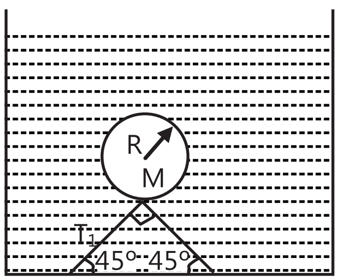
Q.4 The frequency of a sonometer wire is f, but when the weights producing the tensions are completely immersed in water the frequency becomes f/2 and on immersing the weights in a certain liquid the frequency becomes f/3. The specific gravity of the liquid is:

(A) $\frac{4}{3}$ (B) $\frac{16}{9}$ (C) $\frac{15}{12}$ (D) $\frac{32}{27}$

Q.5 A small ball of relative density 0.8 falls into water from a height of 2m. The depth to which the ball will sink is (neglect viscous forces):

(A) 8 m (B) 2 m (C) 6 m (D) 4 m

Q.6 A hollow sphere of mass M and radius r is immersed in a tank of water (density ρ_w). The sphere would float if it were set free. The sphere is tied to the bottom of the tank by two wires which makes angle 45° with the horizontal as shown in figure. The tension T_1 in the wire is:



(A) $\frac{\frac{4}{3}\pi R^3\rho_w g - Mg}{\sqrt{2}}$ (B) $\frac{2}{3}\pi R^3\rho_w g - Mg$

(C) $\frac{\frac{4}{3}\pi R^3\rho_w g - Mg}{2}$ (D) $\frac{4}{3}\pi R^3\rho_w g - Mg$

Q.7 A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 times to decrease the height of water to H η, (η> 1) and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is:

(A) 2 (B) 3 (C) 4 (D) 2.2

Q.8 In the case of a fluid, Bernoulli's theorem exes the application of the principle of conservation of:

(A) Linear momentum (B) Energy

(C) Mass (D) Angular momentum

Q.9 Fountains usually seen in gardens are generated by a wide pipe with an enclosure at one end having many small holes. Consider one such fountain which is produced by a pipe of internal diameter 2 cm in which water flows at a rate 3 ms^{-1}. The enclosure has 100 holes each of diameter 0.05 cm. The velocity of water coming out of the holes is (in ms^{-1}):

(A) 0.48 (B) 96 (C) 24 (D) 48

Q.10 A vertical tank open at the top, is filled with a liquid and rests on a smooth horizontal surface. A small hole is opened at the centre of one side of the tank. The area of cross-section of the tank is N times the area of the hole, where N is a large number. Neglect mass of the tank itself. The initial acceleration of the tank is:

(A) $\frac{g}{2N}$ (B) $\frac{g}{\sqrt{2N}}$

(C) $\frac{g}{N}$ (D) $\frac{g}{2\sqrt{N}}$

Q.11 Two water pipes P and Q having diameters 2×10^{-2}m and 4×10^{-2}m, respectively, are joined in series with the main supply line of water. The velocity of water flowing in pipe P is:

(A) 4 times that of Q (B) 2 times that of Q

(C) 1/2 times that of Q (D)1/4times that of Q

Q.12 A rectangular tank is placed on a horizontal ground and is filled with water to a height H above the base. A small hole is made on one vertical side at a depth D below the level of the water in the tank. The distance x from the bottom of the tank at which the water jet from the tank will hit the ground is:

(A) $2\sqrt{D(H-D)}$ (B) $2\sqrt{DH}$

(C) $2\sqrt{D(H+D)}$ (D) $\frac{1}{2}\sqrt{DH}$

Q.13 A horizontal pipe line carries water in a streamline flow. At a point along the tube where the cross-sectional area is 10^{-2} m^2, the water velocity is 2ms^{-1} and the pressure is 8000 Pa . The pressure of water at another point where the cross-sectional area is $0.5\times10^{-2}m^2$ is:

(A) 4000 Pa (B) 1000 Pa

(C) 2000 Pa (D) 3000 Pa

Q.14 Which of the following is not an assumption for an ideal fluid flow for which Bernoulli's principle is valid:

(A) Steady flow (B) Incompressible

(C) Viscous (D) Irrotational

Q.15 A solid metallic sphere of radius r is allowed to fall freely through air. If the frictional resistance due to air is proportional to the cross-sectional area and to the square of the velocity, then the terminal velocity of the sphere is proportional to which of the following?

(A) r^2 (B) r (C) $r^{3/2}$ (D) $r^{1/2}$

Q.16 If two soap bubbles of different radii are connected by a tube.

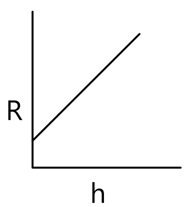
(A) Air flows from the bigger bubble to the smaller bubble till the sizes become equal

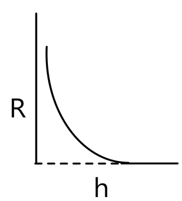
(B) Air flows from bigger bubble to the smaller bubble till the sizes are interchanged

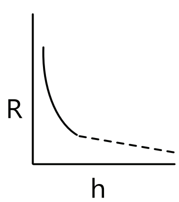
(C) Air flows from the smaller bubble to the bigger

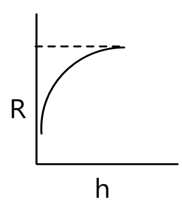
(D) There is no flow of air

Q.17 A long capillary of radius r is initially just vertically completely immerged inside a liquid of angle of contact 0º. If the tube is slowly raised, then relation between radius of curvature of meniscus inside the capillary tube and displacement (h) of tube can be represented by:

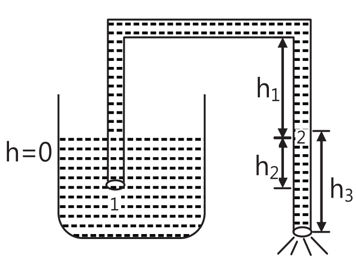
(A) 

(B) 

(C) 

(D) 

Q.18 Figure shows a siphon. Choose the wrong statement:



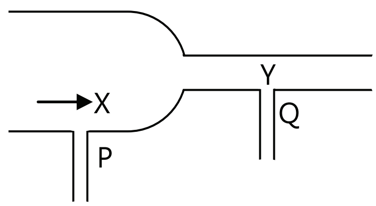
(A) Siphon works when $h_3 > 0$

(B) Pressure at point 2 is $P_2 = p_0 - \rho g h_3$

(C) Pressure at point 3 is P_0

(D) None of the above

Q.19 A steady flow of water passes along horizontal tube from a wide section X to the narrower section Y, see figure. Manometers are placed at P and Q of the sections. Which of the statements A,B,C,D is most correct?



(A) water velocity at X is greater than at Y

(B) the manometer at P shows lower pressure than at Q

(C) kinetic energy per m^3 of water at X = kinetic energy per m^3 at Y

(D) the manometer at P shows greater pressure than at Y

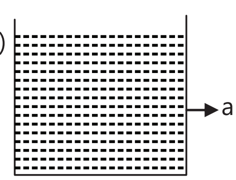
Previous Years' Questions

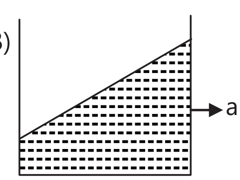
Q.1 A metal ball immersed in alcohol weighs W_1 at 0º C and W_2 at 50ºC. The coefficient of cubical expansion of the metal is less than that of the alcohol. Assuming that the density of the metal is large compared to that of alcohol, it can be shown that: ***(1980)***

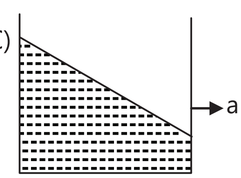
(A) $W_1 > W_2$ (B) $W_1 = W_2$

(C) $W_1 < W_2$ (D) All of these

Q.2 A vessel containing water is given a constant acceleration a towards the right along a straight horizontal path. Which of the following diagrams represent the surface of the liquid? ***(1981)***

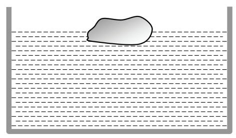
(A) 

(B) 

(C) 

(D) None of these

Q.3 A body floats in a liquid contained in a beaker. The whole system as shown in figure falls freely under gravity. The upthrust on the body due to the liquid is: ***(1982)***



(A) Zero

(B) Equal to the weight of the liquid displaced

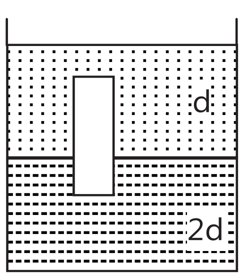
(C) Equal to the weight of the body in air

(D) Equal to the weight of the immersed position of the body

Q.4 A U-tube of uniform cross-section is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If specific gravity of liquid I is 1.1, the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be: ***(1983)***

(A) 1.12 (B) 1.1 (C) 1.05 (D) 1.0

Q.5 A homogeneous solid cylinder of length L. Cross-sectional area A/5 is immersed such that it floats with its axis vertical at the liquid-liquid interface with length L/4 in the denser liquid as shown in the fig. The lower density liquid is open to atmosphere having pressure p_0. Then density D of solid is given by: ***(1995)***



(A) $\frac{5}{4}d$ (B) $\frac{4}{5}d$ (C) 4d (D) $\frac{d}{5}$

Q.6 Water from a tap emerges vertically downwards with an initial speed of 1.0 m/s. The cross-section area of the tap is $10^{-4}m^2$. Assume that the pressure is constant throughout the steam of water and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is: ***(1998)***

(A) 5.0×10^{-4} m^2 (B) 1.0×10^{-4} m^2

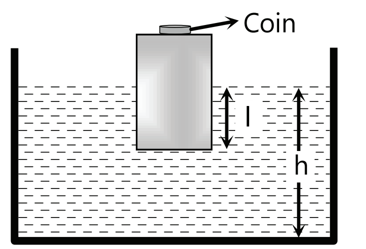
(C) 5.0×10^{-5} m^2 (D) 2.0×10^{-4} m^2

Q.7 A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth 4y from the top. When the tank is completely filled with water the quantities of water flowing out per second from both the holes are the same. Then R is equal to ***(2000)***

(A) $L/\sqrt{2\pi}$ (B) $2\pi L$

(C) L (D) L/2p

Q.8 A wooden block, with a coin placed on its top, floats in water as shown in fig. The distance l and h are shown there. After some time the coin falls into the water. Then: ***(2002)***



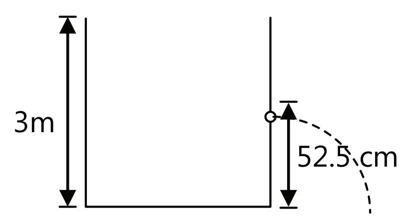
(A) *l* Decreases and h increases

(B) Increases and h decreases

(C) Both *l* and h increase

(D) Both *l* and h decrease

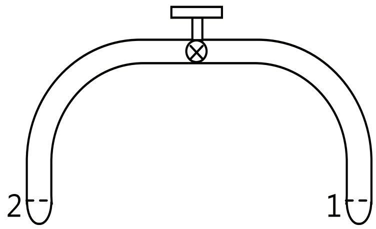
Q.9 Water is filled in a cylindrical container to a height of 3 m. The ratio of the cross-sectional area of the orifice and the beaker is 0.1. The square of the speed of the liquid coming out from the orifice is (g = 10 m/s^2) ***(2005)***



(A) 50 m^2/s^2 (B) 50.5 m^2/s^2

(C) 51 m^2/s^2 (D) 52 m^2/s^2

Q.10 A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. ***(2008)***



Just after opening the valve:

(A) air from end 1 flow towards end 2. No change in the volume of the soap bubbles.

(B) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

(C) no change occurs

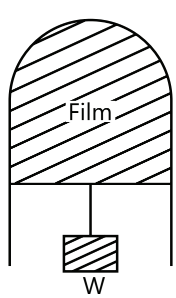
(D) air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

Q.11 A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density p at equilibrium position. When the cylinder is given a small downward push and released it starts oscillating vertically with a small amplitude. If the force constant of the spring is $k_1$the frequency of oscillation of the cylinder is ***(1990)***

(A) $\frac{1}{2\pi}\left(\frac{k-A\rho g}{M}\right)^{1-2}$ (B) $\frac{1}{2\pi}\left(\frac{k+A\rho g}{M}\right)^{1/2}$

(C) $\frac{1}{2\pi}\left(\frac{k+\rho g L^2}{M}\right)^{1/2}$ (D) $\frac{1}{2\pi}\left(\frac{k+A\rho g}{A\rho g}\right)^{1/2}$

Q.12 A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is ***(2012)***



(A) 0.0125 Nm^{-1} (B) 0.1 Nm^{-1}

(C) 0.05 Nm^{-1} (D) 0.025 Nm^{-1}

Q.13 A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: ***(2013)***

(A) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (B) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$

(C) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$ (D) $\frac{Mg}{k}$

(Here k is spring constant)

Q.14 Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperatureremains unchanged. What should be the minimum radius of thedrop for this to be possible? The surface tensionis T, density of liquid is ρ and L is its latent heat of vaporization. ***(2013)***

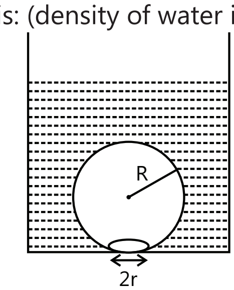
(A) $\sqrt{T/\rho L}$ (B) $T/\rho L$

(C) $2T/\rho L$ (D) $\rho L/T$

Q.15 An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) ***(2014)***

(A) 38 cm (B) 6 cm (C) 16 cm (D) 22 cm

Q.16 On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r < < R, and the surface tension of water is T, value of r just before bubbles detach is: (density of water is ρ_w) ***(2014)***



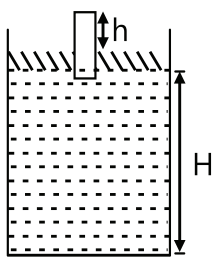
(A) $R^2\sqrt{\frac{\rho_w g}{T}}$ (B) $R^2\sqrt{\frac{3\rho_w g}{T}}$

(C) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (D) None of these

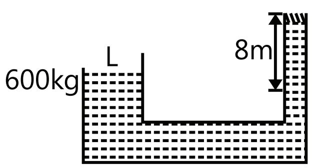
JEE Advanced/Boards

Exercise 1

Q.1 A piston of mass M = 3 kg and radius R=4 cm has a hole into which a thin pipe of radius r = 1 cm is inserted. The piston can enter a cylinder tightly and without friction, and initially it is at the bottom of the cylinder. 750 gm of water is now poured into the pipe so that the piston and pipe are lifted up as shown. Find the height H of water in the cylinder and height h of water in pipe.



Q.2 A solid ball of density half of that of water falls freely under gravity from a height of 19.6m and then enters the water. Upto what depth will the ball go? How much time will it take to come again to the water surface? Neglect air resistance & velocity effects in water.



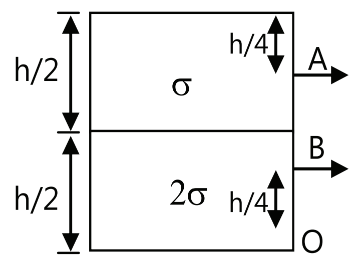
Q.3 For the system shown in the figure, the cylinder on left at L has a mass of 600 kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil (ρ = 0.75 gm/cm^3). Find the force F required to hold the system in equilibrium.

Q.4 (a) A spherical tank of 1.2 m radius is half filled with oil of relative density 0.9. If the tank is given a horizontal acceleration of 10 m/s^2, calculate the inclination of the oil surface to horizontal and maximum gauge pressure on the tank.

(b) The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. If the atmospheric pressure is H m of mercury & the density of mercury is n times that of lake water, find the depth of the lake.

Q.5 A test tube of thin walls has lead shots in it at its bottom and the system floats vertically in water, sinking by a length l = 10 cm. A liquid of density less than that of water, is poured into the tube till the levels inside and outside the tube are even. If the tube now sinks to a length l = 40 cm, the specific gravity of the liquid is ....

Q.6 A large tank is filled with two liquids of specific gravities 2σ and σ. Two holes are made on the wall of the tank as shown. Find the ratio of distances from O of the points on the ground where the jets from holes A and B strike.



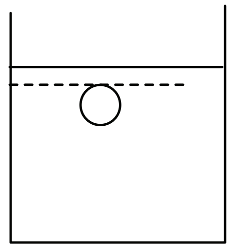
Q.7 A jet of water having velocity = 10 m/s and stream cross-section = 2 cm^2 hits a plate perpendicularly, with the water splashing out parallel to plate. Find the force that the plate experiences.

Q.8 A laminar stream is flowing vertically down from a tap of cross-section area 1 cm^2. At a distance 10 cm below the tap, the cross-section area of the stream has reduced to 1/2m^2. Find the volumetric flow rate of water from the tap.

Q.9 A cylindrical vessel open at the top is 20 cm high and 10 cm in diameter. A circular hole whose cross-sectional area is 1 cm^2 is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate 100 cm^3 s^{-1}. Find the height of water in the vessel under steady state.

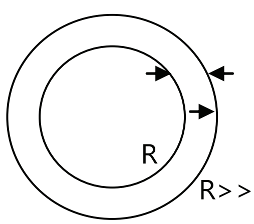
Q.10 Calculate the rate of flow of glycerin of density 1.25×10^3 kg/m^3 through the conical section of a 0.1m and 0.04m and the pressure drop across its length is 10N/m^2.

Q.11 A ball is given velocity v_0 (greater than the terminal velocity v_T) in downward direction inside a highly viscous liquid placed inside a large container. The height of liquid in the container is H. The ball attains the terminal velocity just before striking at the bottom of the container. Draw graph between velocity of the ball and distance moved by the ball before getting terminal velocity.

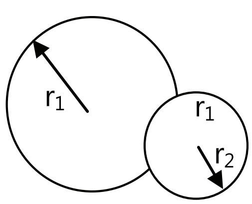


Q.12 A spherical ball of radius 1×10^{-4}m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m^2.

Q.13 Two arms of a U-tube have unequal diameters d_1 = 10 mm and d_2 = 1.0cm. If water (surface tension 7×10^{-2}N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero.



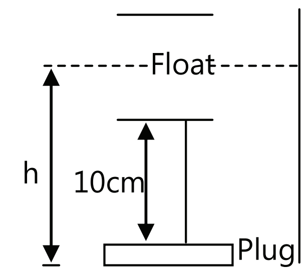
Q.14 A soap bubble has radius R and thickness d(<<R) as shown. It collapses into a spherical drop. Find the ratio of excess pressure in the drop to the excess pressure inside the bubble.



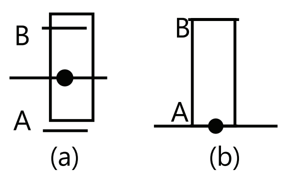
Q.15 Two soap bubbles with radii r_1 and r_2 ($r_1>r_2$) come in contact. Their common surface has a radius curvature r.

Q.16 Place a glass beaker, partially filled with water, in a sink. The beaker has mass 390 gm and an interior volume of 500 cm^3. You now start to fill the sink with water and you find, by experiment, that if the beaker is less than half full, it will float; but if it is more than half full, it remains on the bottom of the sink as the water rises to its rim. What is the density of the material of which the beaker is made?

Q.17 A level controller is shown in the figure. It consists of a thin circular plug of diameter 10 cm and a cylindrical float of diameter 20 cm tied together with a light rigid rod of length 10 cm. The plug fits in snugly in a drain hole at the bottom of the tank which opens into the atmosphere. As water fills up and the level reaches height h, the plug opens. Find h. Determine the level of water in the tank when the plug closes again. The float has a mass 3kg and the plug may be assumed as massless.



Q.18 A cylindrical rod of length l=2m and density $\frac{\rho}{2}$ floats vertically in a liquid of density ρ as shown in fig. (a)

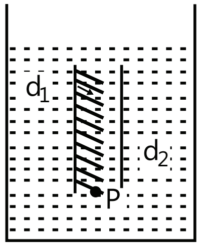


(a) Show that it performs SHM when pulled slightly up & released & find its time period. Neglect change in liquid level.

(b) Find the time taken by the rod to completely immerse when released from position shown in figure (b). Assume that it remains vertical throughout its motion.

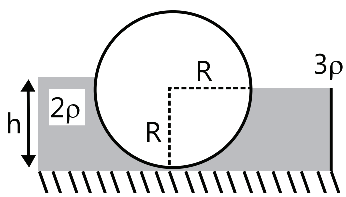
(take g = π^2m/s^2)

Q.19 A thin rod of length L and area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous & non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. the density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



Q.20 A hollow cone floats with its axis vertical up to one-third liquid of its height in a liquid of relative density ρ is filled in it up to one-third of its height, the cone floats up to half its vertical height. The height of the cone is 0.10 m and the radius of the circular base is 0.05m. Find the specific gravity ρ.

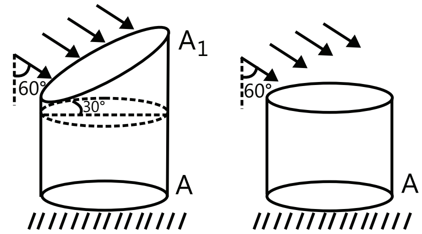
Q.21 In the figure shown, the heavy cylinder (radius R) resting on a smooth surface separates two liquids of densities 2ρ and 3ρ. Find the height 'h' for the equilibrium of cylinder.



Q.22 The vertical limbs of a U shaped tube are filled with a liquid of density ρ up to a height h on each side. The horizontal portion of the U tube having length 2h contains a liquid of density 2ρ. The U tube is moved horizontally with an acceleration g/2 parallel to the horizontal arm. Find the difference in heights in liquid levels in the two vertical limbs, at steady state.

Q.23 A wooden stick of length l and radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ(>ρ).

Q.24 A vertical cylindrical container of base area A and upper cross-section area A_1 making angle 30° with the horizontal placed in an open rainy field as shown near another cylindrical container having same base area A. Find the ratio of rates of collection of water in the two containers.



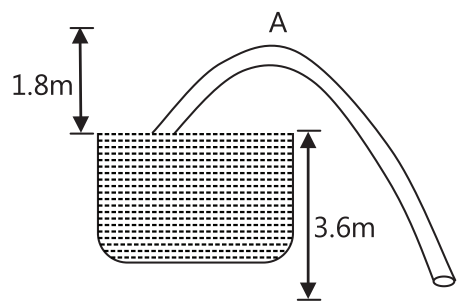
Q.25 A siphon has a uniform circular base of diameter $\frac{8}{\pi}$ cm with its crest A 1.8 m above water level as in figure. Find

(a) Velocity of flow.

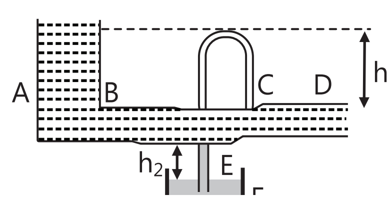
(b) Discharge rate of the flow in m^3/sec.

(c) Absolute pressure at the crest level A.

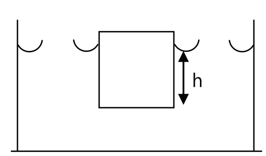
[Use $P_0 = 10^5$ N/m^2& g=10m/s^2]



Q.26 Two very large open tanks A and F both contain the same liquid. A horizontal pipe BCD, having a constriction at C leads out of the bottom of tank A, and a vertical pipe E opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section at C is one half that at D and if D is at a distance h_1 below the level of liquid in A, to what height h_2 (in terms of h_1) will liquid rise in pipe E?



Q.27 A cube with mass 'm' completely wet by water floats on the surface of water. Each side of the cube is 'a'. What is the distance h between the lower face of cube and the surface of the water as ρ_w. Take angle of contact as zero.



Exercise 2

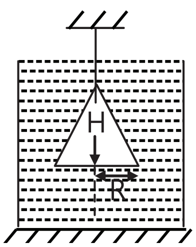
Single Correct Choice Type

Q.1 A bucket contains water filled up to a height = 15cm. The bucket is tied to a rope which is passed on a frictionless light pulley and the other end of the rope is tied to a weight of mass which is half of that of the (bucket + water). The water pressure above atmosphere at the bottom is:

(A) 0.5 kPa (B) 1 kPa

(C) 5 kPa (D) None of these

Q.2 A cone of radius R and height H, is hanging inside a liquid of density ρ by means of a string as shown in the figure. The force, due to the liquid acting on the slant surface of the cone is (Neglect atmosphere pressure)



(A) $prgHR^2$ (B) $p\rho HR^2$

(C) $4/3\ prgHR^2$ (D) $2/3\ prgHR^2$

Q.3 An open cubical tank was initially fully filled with water. When the tank was accelerated on a horizontal plane along one of its side, it was found that one third of volume of water spilled out. The acceleration was:

(A) g/3 (B) 2g/3

(C) 3g/2 (D) None

Q.4 Some liquid is filled in a cylindrical vessel of radius R. Let F_1 be the force applied by the liquid on the bottom of the cylinder. Now the same liquid is poured into a vessel of uniform square cross-section of side R. Let F_2 be the force applied by the liquid on the bottom of this new vessel. (Neglect atmosphere pressure). Then:

(A) $F_1 = \pi F_2$ (B) $F_1 = F_2/p$

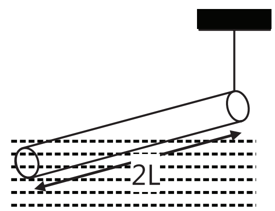
(C) $F_1 = \sqrt{\pi}F_2$ (D) $F_1 = F_2$

Q.5 A heavy hollow cone of radius R and height h is placed on a horizontal table surface, with its base on the table. The whole volume inside the cone is filled with water of density ρ. The circular rim of the cone's base has a water tight seal with the table's surface and the top apex of the cone has a small hole. Neglecting atmospheric pressure, the total upward force exerted by water on the cone is:

(A) $(2/3)pR^2hrg$ (B) $(1/3)pR^2hrg$

(C) pR^2hrg (D) None

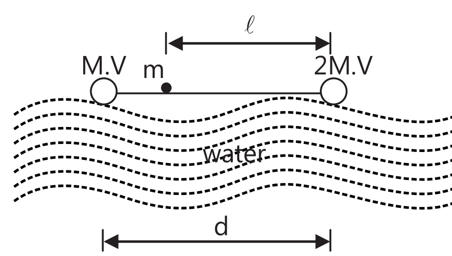
Q.6 A slender homogeneous rod of length 2L floats partly immersed in water, being supported by a string fastened to one of its ends, as shown. The specific gravity of the rod is 0.75. The length of rod that extends out of water is:



(A) L^2 (B) $L^2/2$

(C) $L^2/4$ (D) $3L^2/4$

Q.7 A dumbbell is placed in water of density ρ. It is observed that by attaching a mass m to the rod, the dumbbell floats with the rod horizontal on the surface of water and each sphere exactly half submerged as shown in the figure. The volume of the mass m is negligible. The value of length ℓ is:



(A) $\frac{d(V\rho - 3M)}{2(V\rho - 2M)}$ (B) $\frac{d(V\rho - 2M)}{2(V\rho - 3M)}$

(C) $\frac{d(V\rho + 2M)}{2(V\rho - 3M)}$ (D) $\frac{d(V\rho - 2M)}{2(V\rho + 3M)}$

Q.8 A small wooden ball of density ρ is immersed in water of density σ to depth h and then released. The height H above the surface of water up to which the ball will jump out of water is:

(A) $\frac{\sigma h}{\rho}$ (B) $\left(\frac{\sigma}{\rho} - 1\right)h$

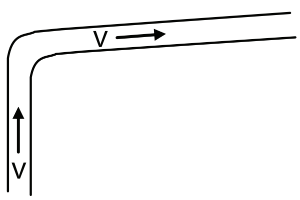
(C) h (D) Zero

Q.9 A sphere of radius R and made of material of relative density σ has a concentric cavity of radius r. It just floats when placed in a tank full of water. The value of the ratio R/r will be:

(A) $\left(\frac{\sigma}{\sigma-1}\right)^{1/3}$ (B) $\left(\frac{\sigma-1}{\sigma}\right)^{1/3}$

(C) $\left(\frac{\sigma+1}{\sigma}\right)^{1/3}$ (D) $\left(\frac{\sigma-1}{\sigma+1}\right)^{1/3}$

Q.10 A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is:



(A) ρVL (B) Zero

(C) $2\rho VL$ (D) $\sqrt{2}\,\rho VL$

Q.11 A cylindrical vessel filled with water up to height of H stands on a horizontal plane. The side wall of the vessel has a plugged circular hole touching the bottom. The coefficient of friction between the bottom of vessel and plane is μ and total mass of water plus vessel is M. What should be the minimum diameter of the hole so that the vessel begins to move on the floor if plug is removed (here density of water is ρ)

(A) $\sqrt{\frac{2\mu M}{\pi\rho H}}$ (B) $\sqrt{\frac{\mu M}{2\pi\rho H}}$

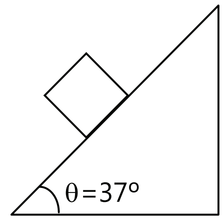
(C) $\sqrt{\frac{\mu M}{\rho H}}$ (D) None

Q.12 A Newtonian fluid fills the clearance between a shaft and a sleeve. When a force of 800N is applied to shift, parallel to the sleeve, the shaft attains of 1.5 cm/sec. If a force of 2.4 kN is applied instead, the shaft would move with a speed of

(A) 1.5 cm/sec (B) 13.5 cm/sec

(C) 4.5 cm/sec (D) None

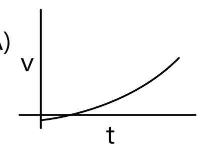
Q.13 A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be:

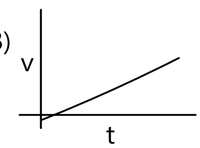


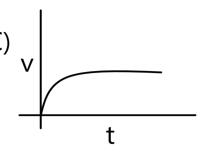
(A) $\frac{3\rho a g t}{5v}$ (B) $\frac{4\rho a g t}{5v}$

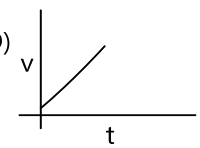
(C) $\frac{\rho a g t}{5v}$ (D) None of these

Q.14 Which of the following graphs best represent the motion of a raindrop?

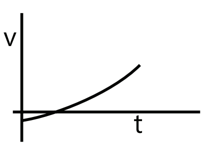
(A)


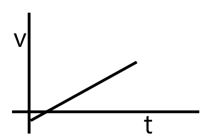
(B)


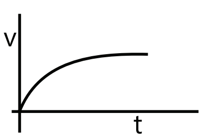
(C)


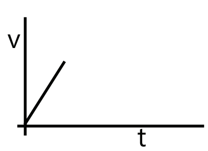
(D)


Q.15 Which of the following is the incorrect graph for a sphere falling in a viscous liquid? (Given at t = 0, velocity v = 0 and displacement x = 0)

(A)


(B)


(C)


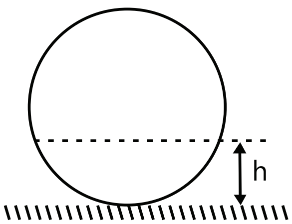
(D)


Q.16 A container, whose bottom has round holes with diameter 0.1 mm is filled with water. The maximum height in cm up to which water can be filled without leakage will be what?

Surface tension=75×10^{-3}N/m and g=10 m/s^2:

(A) 20 cm (B) 40 cm (C) 30 cm (D) 60 cm

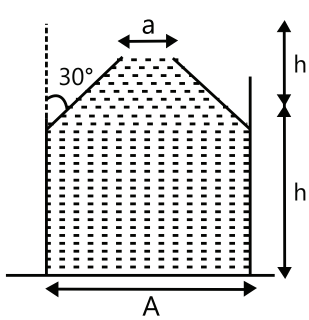
Q.17 A liquid is filled in a spherical container of radius R till a height h. At this position the liquid surface at the edges is also horizontal. The contact angle is:



(A) 0 (B) $\cos^{-1}\left(\frac{R-h}{R}\right)$

(C) $\cos^{-1}\left(\frac{h-R}{R}\right)$ (D) $\sin^{-1}\left(\frac{R-h}{R}\right)$

Q.18 The vessel shown in the figure has two sections. The lower part is a rectangular vessel with area of cross-section A and height h. The upper part is a conical vessel of height h with base area 'A' and top area 'a' and the walls of the vessel are inclined at an angle 30° with the vertical. A liquid of density ρ fills both the sections up to a height 2h. Neglecting atmospheric pressure,



(A) The force F exerted by the liquid on the base of the vessel is $2h\rho g\frac{(A+a)}{2}$

(B) The pressure P at the base of the vessel is $2h\rho g\frac{A}{a}$

(C) The weight of the liquid W is greater than the force exerted by the liquid on the base.

(D) The walls of the vessel exert a downward force (F-W) on the liquid.

Multiple Correct Choice Type

Q.19 A cubical block of wood of edge 10 cm and mass 0.92 kg floats on a tank of water with oil of relative density 0.6 to a depth of 4 cm above water. When the block attains equilibrium with four of its side edges vertical,

(A) 1 cm of it will be above the force of oil.

(B) 5 cm of it will be under water.

(C) 2 cm of it will be above the common surface of oil and water.

(D) 8 cm of it will be under water.

Q.20 Water coming out of a horizontal tube at a speed v strikes normally a vertically wall close to the mouth of the tube and falls down vertically after impact. When is the speed of water increased to 2v.

(A) the thrust exerted by the water on the wall will be doubled.

(B) the thrust exerted by the water on the wall will be four times

(C) the energy lost per second by water striking the wall will also be four times

(D) the energy lost per second by water striking the wall be increased eight times.

Q.21 A beaker filled with water is accelerated a m/s^2 in +x direction. The surface of water shall make on angle:

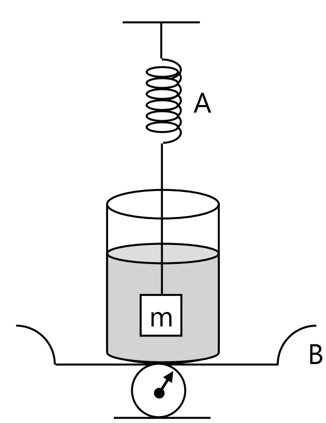
(A) $\tan^{-1}(a/g)$ backwards

(B) $\tan^{-s}$ draw of $(g/a)^1$

(C) $\cot^{-1}(g/a)$ backwards

(D) $\cot^{-1}(a/g)$ backwards

Q.22 The spring balance A read 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure in this situation:



(A) The balance A will read more than 2kg

(B) The balance B will read more than 5 kg

(C) The balance A will read less than 2 kg and B will read more than 5 kg.

(D) The balance A and B will read 2 kg and 5 kg respectively

Q.23 When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble:

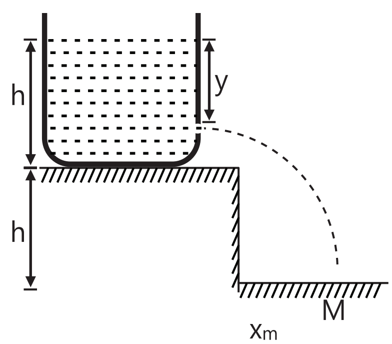
(A) Is greater than the pressure outside it

(B) Is less than the pressure outside it

(C) Increases as the bubble moves up

(D) Decreases as the bubble moves up

Q.24 A tank is filled up to a height h with a liquid and is placed on a platform of height h at a distance of y from the free surface of the liquid. Then



(A) $x_m = 2h$ (B) $x_m = 1.5\ h$

(C) $y = h$ (D) $y = 0.75\ h$

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and Statement-II is the correct explanation for statement-I

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I

(C) Statement-I is true, statement-II is false.

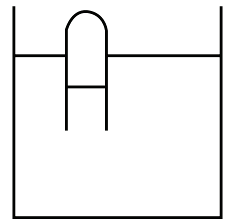
(D) Statement-I is false, statement-II is true

Q.25 Statement-I: A helium filled balloon does not rise indefinitely in air but halts after a certain height.

Statement-II: Viscosity opposes the motion of balloon.

Q.26 Statement-I: A partly filled test tube is floating in a liquid as shown. The tube will remain as if its atmosphere pressure changes.

Statement-II: The buoyant force on a submerged object is independent of atmospheric pressure.

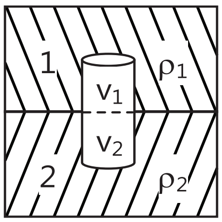


Q.27 Statement-I: Submarine sailors are advised that they should not be allowed to rest on floor of the ocean.

Statement-II: The force exerted by a liquid on a submerged body may be downwards.

Q.28 Statement-I: When a body floats such that it's parts are immersed into two immiscible liquids then force exerted by liquid-1 is of magnitude r_1v_1g.

Statement-II: Total Bouyant force $r_1v_1g + r_2v_2g$.



Q.29 Statement-I: When temperature rises the coefficient of viscosity of gases decreases.

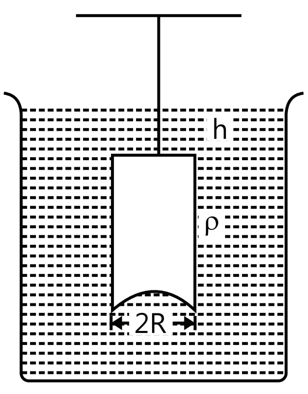
Statement-II: Gases behave more like ideal gases at higher temperature.

Q.30 Statement-I: The free surface of a liquid at rest with respect to stationary container is always normal to the $\vec{g}_{eff}$.

Statement-II: Liquids at rest cannot have shear stress.

Previous Years' Questions

Q.1 A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and mass M. It is suspended by a string in a liquid of density ρ, where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is ***(2001)***



(A) Mg (B) Mg – Vρg

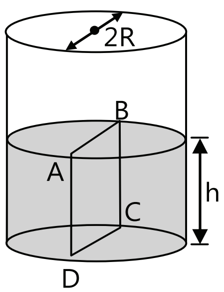
(C) Mg + $\rho R^2 h \rho R$ (D) $\rho g(V + pR^2 h)$

Q.2 When a block of iron floats in mercury at 0°C, fraction k_1 of its volume is submerged, while at the temperature 60°C, a fraction k_2 is seen to be submerged. If the coefficient of volume expansion of iron is γ_{Fe} and that of mercury is γ_{Hg}, then the ratio k_1/k_2 can be expressed as ***(2001)***

(A) $\frac{1+60\gamma_{Fe}}{1+60\gamma_{Hg}}$ (B) $\frac{1-60\gamma_{Fe}}{1+60\gamma_{Hg}}$

(C) $\frac{1+60\gamma_{Fe}}{1-60\gamma_{Hg}}$ (D) $\frac{1+60\gamma_{Hg}}{1+60\gamma_{Fe}}$

Q.3 Water is filled up to a height h in a beaker of radius R as shown in the figure. The density of water is ρ, the surface tension of water is T and the atmospheric pressure is p_0. Consider a vertical section ABCD of the water column through a diameter of the beaker. The force on water on one side of this section by water on the other side of this section has magnitude ***(2007)***



(A) $|2p_0Rh + \pi R^2 \rho gh - 2RT|$

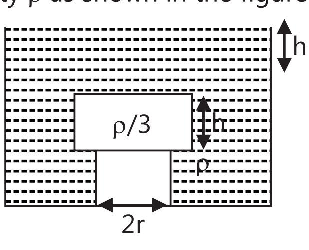
(B) $|2p_0Rh + R\rho gh^2 - 2RT|$

(C) $|p_0\pi R^2 + R\rho gh^2 - 2RT|$

(D) $|p_0\pi R^2 + R\rho gh^2 - 2RT|$

Paragraph 1: (Q.4 - Q.6)

A wooden cylinder of diameter 4r, height h and density ρ/3 is kept on a hole of diameter 2r of a tank, filled with liquid of density ρ as shown in the figure.

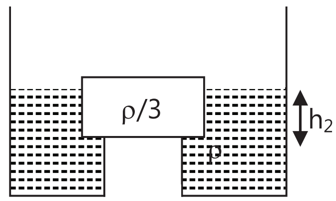


Q.4 Now level of the liquid starts decreasing slowly. When the level of liquid is at a height h_1 above the cylinder the block starts moving up. At what value of h_1, will the block rise? ***(2005)***

(A) 4h/9 (B) 5h/9

(C) 5h/3 (D) Remains same

Q.5 The block in the above question is maintained at the position by external means and the level of liquid is lowered. The height h_2 when this external force reduces to zero is: ***(2006)***



(A) $\frac{4h}{9}$ (B) $\frac{5h}{9}$ (C) Remains same (D) $\frac{2h}{3}$

Q.6 If height h_2 of water level is further decreased, then: ***(2006)***

(A) cylinder will not move up and remains at its original position

(B) for h_2=h/3, cylinder again starts moving up

(C) for h_2=h/4, cylinder again starts moving up

(D) h_2=h/5, cylinder again starts moving up

Paragraph 2: (Q.7 - Q.9)

When a liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop.

We first assume that the drop formed at the opening is spherical because that requires minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper. ***(2010)***

Q.7 If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming r<<R) is

(A) $2\pi rT$ (B) $2\pi RT$ (C) $\frac{2\pi r^2 T}{R}$ (D) $\frac{2\pi R^2 T}{r}$

Q.8 If r = 5×10^{-4}m, r = 10^3 kg m^{-3}, g= $10ms^{-2}$, T =0.11 Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately:

(A) 1.4×10^{-3} m (B) 3.3×10^{-3} m

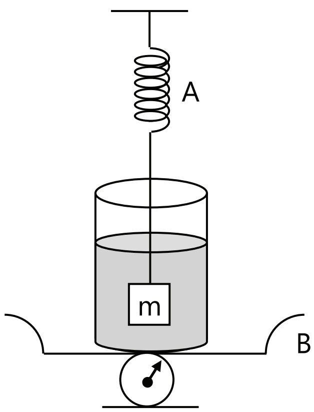
(C) 2.0×10^{-3} m (D) 4.1×10^{-3} m

Q.9 After the drop detaches, its surface energy is:

(A) 1.4×10^{-6} J (B) 2.7×10^{-6} J

(C) 5.4×10^{-6} J (D) 9.1×10^{-9} J

Q.10 The spring A reads 2 kg with a block m suspended from it. A balance reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation: ***(1985)***



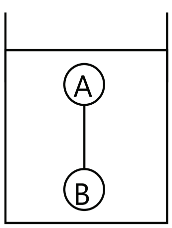
(A) The balance A will read more than 2 kg

(B) The balance A will read more than 5 kg

(C) The balance A will read less than 2 kg and B will read more than 5 kg

(D) The balances A and B will read 2 kg and 5 kg respectively.

Q.11 Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into the equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if ***(2011)***



(A) $d_A < d_F$ (B) $d_B > d_F$

(C) $d_A > d_F$ (D) $d_A + d_B = 2d_F$

Q.12 A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_c is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is – ***(2012)***

(A) More than half-filled if ρ_c is les sthan 0.5

(B) More than half-filled if ρ_c is less than 0.5

(C) Half-filled if ρ_c is more than 0.5

(D) Less than half – filled if ρ_c is less than 0.5

Q.13 A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3 ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) ***(2013)***

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

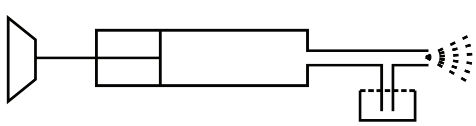
(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

Paragraph for Questions 14 and 15

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



Q.14 If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of ***(2014)***

(A) 0.1 ms^{-1} (B) 1 ms^{-1}

(C) 2 ms^{-1} (D) 8 ms^{-1}

Q.15 If the density of air is ρ_a and that of the liquid ρ_ℓ, for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to ***(2014)***

(A) $\sqrt{\frac{\rho_a}{\rho_\ell}}$ (B) $\sqrt{\rho_a \rho_\ell}$

(C) $\sqrt{\frac{\rho_\ell}{\rho_a}}$ (D) ρ_ℓ

Q.16 A person in a lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person.

In the following, state of the lift's motion is given in List I and the distance where the water jethits the floor of the lift is given in List II. Match the statements from List I with those in List II and select the correct answer using the code given below the lists. ***(2014)***

	List I		List II
1.	Lift is accelerating vertically up.	(p)	d=1.2 m
2.	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration.	(q)	d > 1.2 m
3.	Lift is moving vertically up with constant speed.	(r)	d < 1.2 m
4.	Lift is falling freely.	(s)	No water leaks out of the jar

Code:

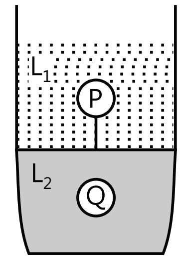
(A) 1 - q, 2 - r, 3 - q, 4-s

(B) 1 - q, 2 - r, 3 - p, 4 - s

(C) 1 - p, 2 - p, 3 - p, 4 - s

(D) 1 - q, 2 - r, 3 - p, 4 - p

Q.17 Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then ***(2015)***



(A) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ (B) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$

(C) $\vec{V}_P . \vec{V}_Q > 0$ (D) $\vec{V}_P . \vec{V}_Q < 0$

Q.18 A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is(are) ***(2015)***

(A) $P(r=0) = 0$ (B) $\frac{P(r=3R/4)}{P(r=2R/3)} = \frac{63}{80}$

(C) $\frac{P(r=3R/5)}{P(r=2R/5)} = \frac{16}{21}$ (D) $\frac{P(r=R/2)}{P(r=R/3)} = \frac{20}{27}$

Q.19 Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1cm and 0.5cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is ***(2016)***

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 7 Q.9 Q.15
Q.16 Q.20

Exercise 2

Q. 1 Q.7 Q.9
Q.13 Q.17

Previous Years' Questions

Q.8 Q.9 Q.10

JEE Advanced/Boards

Exercise 1

Q.3 Q.6 Q.9
Q.17

Exercise 2

Q.1 Q.4 Q.10
Q.11 Q.19 Q.22

Previous Years' Questions

Q.7 Q.8 Q.9

Answer Key

JEE Main/Boards

Exercise 1

Q.2 Zero

Q.6 $(2)^{2/3}$ v_T

Q.7 2.714×10^{-9} m/s

Q.9 Streamline, turbulent

Q.10 Turbulent

Q.13 60×10^{8} Pa

Q.14 27.6 m

Q.15 9.56 N

Q.16 60 kg

Q.17 $72\pi\times10^{-4}$ N

Q.18 0.0224

Q.19 3.5×10^{-2} Nm^{-1}

Q.20 1.785×10^{-4} m

Exercise 2

Single Correct Choice Type

Q.1 C	**Q.2** B	**Q.3** D	**Q.4** D	**Q.5** A	**Q.6** A
Q.7 C	**Q.8** B	**Q.9** D	**Q.10** C	**Q.11** A	**Q.12**A
Q.13 C	**Q.14** C	**Q.15** D	**Q.16** C	**Q.17** B	**Q.18** D
Q.19 D					

Previous Years' Questions

Q.1 C **Q.2** A **Q.3** A **Q.4** B **Q.5** A **Q.6** C

Q.7 A **Q.8** D **Q.9** A **Q.10** B **Q.11** B **Q.12** D

Q.13 B **Q.14** C **Q.15** C **Q.16** D

JEE Advanced/Boards

Exercise 1

Q.1 $h = \frac{2m}{\pi}, H = \frac{11}{32\pi}m$

Q.2 19.6m, 4 sec.

Q.3 37.5 N

Q.4 (a) $9600\sqrt{2}$, (b) nH

Q.5 0.75

Q.6 $\sqrt{3} : \sqrt{2}$

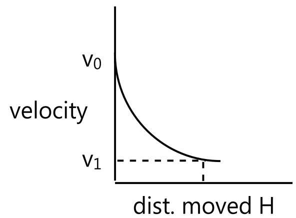
Q.7 20N

Q.8 4.9 litre/min

Q.9 5 cm

Q.10 6.43×10^{-4} m^3/s

Q.11



Q.12 20 m

Q.13 2.5 cm

Q.14 $\left(\frac{R}{24d}\right)^{\frac{1}{3}}$

Q.15 $r = \frac{r_1 r_2}{r_1 - r_2}$

Q.16 2.79 gm/cc

Q.17 $h_1 = \frac{2(3+\pi)}{15\pi} = 0.26;\ h_1 = \frac{3+\pi}{10\pi} = 0.195$

Q.18 2 sec., 1 sec

Q.19 $w = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)}$

Q.20 1.9

Q.21 $R\sqrt{\frac{3}{2}}$

Q.22 $\frac{8h}{7}$

Q.23 $m_{min} = \pi r^2 \ell\left(\sqrt{\rho\sigma} - \rho\right)$; if tilted then it's axis should become vertical, C.M. should be lower than centre of buoyancy.

Q.24 2 : 1

Q.25 (a) $6\sqrt{2}$ m/s, (b) $9.6\sqrt{2}\times10^{-3}M^3$ / sec, (c) 4.6×10^4

Q.26 $h_2 = 3h_1$

Q.27 $h = \frac{mg + 4Sa}{\rho_w a^2 g}$

Exercise 2

Single Correct Choice Type

Q.1 B **Q.2** D **Q.3** B **Q.4** D **Q.5** A **Q.6** A

Q.7 B **Q.8** B **Q.9** A **Q.10** D **Q.11** A **Q.12** C

Q.13 A **Q.14** C **Q.15** C **Q.16** C **Q.17** B **Q.18** D

Multiple Correct Choice Type

Q.19 C, D	**Q.20** B, D	**Q.21** A, C	**Q.22** B, C	**Q.23** A, D	**Q.24** A, C

Assertion Reasoning Type

Q.25 B	**Q.26** D	**Q.27** A	**Q.28** D	**Q.29** D	**Q.30** A

Previous Years' Questions

Q.1 D	**Q.2** A	**Q.3** B	**Q.4** C	**Q.5** A	**Q.6** A
Q.7 C	**Q.8** A	**Q.9** B	**Q.10** B, C	**Q.11** A, B, D	**Q.12** A
Q.13 A, D	**Q.14** C	**Q.15** A	**Q.16** C	**Q.17** A, D	**Q.18** B, C

Solutions

JEE Main/Boards

Exercise 1

Sol 1: Castor oil will come to rest first because its viscosity is greater than water

Sol 2: Acceleration is zero as velocity is constant

Sol 3: Flow rate is equal in any part of the body so

$A_1 V_1$ = constant

$\pi\left(\frac{D}{2}\right)^2 V$ = constant

Sol 4: Viscosity of gas increases with increase in temperature

Sol 5: For gas, viscosity of gases are independent of density and pressure but viscosity of gas increases with increase in temperature

For liquids:– Viscosity decreases with increase in temperature. Viscosity increase with increase in density viscosity of liquid is normally independent of pressure, but liquid under extreme pressure after experience an increase in viscosity

Sol 6: Volume remains same so

$2 \times 4/3\ \pi r^3 = 4/3\ \pi R^3$

$R = (2)^{1/3} r$

$V_T \propto r^2$

$\Rightarrow V'_T = k\ 2^{1/3} R^2 = (2)^{2/3} V_T$

Sol 7: $r = 0.2$ mm $= 2 \times 10^{-4}$ m

$v = 4$cm/s $= 4 \times 10^{-2}$ m/s

$F = 6\pi\ \eta\ rv$

$= 6\pi \times 1.8 \times 10^{-5} \times 2 \times 10^{-4} \times 4 \times 10^{-2}$

$= 6\pi \times 14.4 \times 10^{-11} = 2.714 \times 10^{-9}$ m/s

Sol 8: Refer Q-6 Exercise –I JEE Main

Sol 9: Critical velocity $= V_c = \frac{k\eta}{\rho r}$

k = Reynolds's number = 1000

$V_C = \frac{1000\times10^{-3}}{1000\times\frac{1.25}{200}} = \frac{1}{6.25} = 0.16$ m/s

Q = flow rate = 0.48L/min $= \frac{0.8}{100}$ L/sec $= 8 \times 10^{-6}$ m^3/sec

Area $= \pi\left(\frac{1.25}{2}\right)^2 \times 10^{-4} = 1.227 \times 10^{-4}$

Velocity = $\frac{Q}{A} = \frac{8\times10^{-6}}{1.222\times10^{-4}}$

$= 6.5 \times 10^{-2}$ m/s

$V_1 < V_C \Rightarrow$ Streamline

When flow rate is 3L/min

$Q' = 3L/\text{min} = \frac{3}{60}$ L/sec $= \frac{1}{20} \times 10^{-3}$ m^3/sec

$A = 1.227 \times 10^{-4}$

$V_2 = \frac{Q'}{A} = \frac{1\times10^{-3}}{20\times1.227\times10^{-4}}$

$= \frac{1}{2\times1.227} = 0.40$ m/s

$V_2 > V_c \Rightarrow$ turbulent flow

Sol 10: Refer Q – 9 Exercise–I JEE Main

Sol 11: Apparent weight of the floating block is zero.

Sol 12: Up thrust will be zero as body is not exerting any force on water during free fall and there is no buoyant force

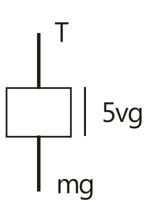
Sol 13: Pressure = $\frac{F}{A} = \frac{60}{10^{-8}} = 60 \times 10^{8}$ Pa

Sol 14: $370\times10^{3} = \rho gh + 10^{5}$

$\rho gh = (3.7–1) \times 10^{5}$

$h = \frac{2.7\times10^{5}}{9.8\times10^{3}} = 27.6$ m

Sol 15:



$T = mg - \rho_\omega vg$

$= 9000 \times125\times10^{-6}\times9.8 - 1200 \times 125 \times 10^{-6} \times 9.8$

$= 7800 \times 125 \times 10^{-6} \times 9.8$

$= 7.8 \times 125 \times 10^{-3} \times 9.8 = 9.56$ N

Sol 16: Change in depth corresponds to mass of man

$\rho \times 3 \times 2 \times \frac{1}{100} \times 10 = m \times 10$

$m = 1000 \times \frac{6}{100} = 60$ kg

Sol 17: Force = $2\pi r S$

$= 2\pi \times \frac{5}{100} \times 72 \times 10^{-3}$

$= \pi \times 72 \times 10^{-4}$

Sol 18: F = 2 × perimeter × S

$= 2 \times 4 \times \frac{1}{10} \times 28 \times 10^{-3}$

$= 8 \times 28 \times 10^{-4}$

$= 0.0224$

Sol 19: Pressure inside above atmospheric pressure

$\rho gh = \frac{4T}{r}$

$10^{4} \times 8 \times 10^{-3} = \frac{4T\times2}{3.5\times10^{-3}}$

$T = 3.5 \times 10^{-2}$ Nm^{-1}

Sol 20: $h = \frac{2T}{r\rho g}$

$r = \frac{2\times70\times10^{-3}}{8\times10^{-2}\times10^{4}}$

$= \frac{70}{4} \times 10^{-5} = 1.785 \times 10^{-4}$ m

Exercise 2

Single Correct Choice Type

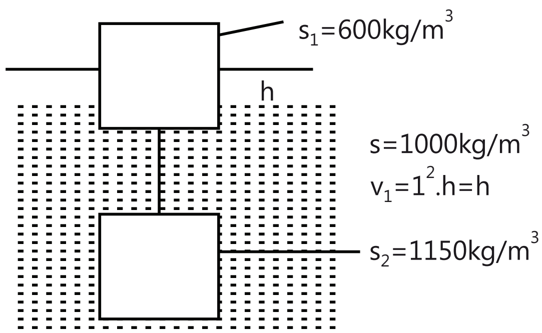
Sol 1: (C) Pressure due to difference in heights will be balanced by pressure due to 12 kg block

$\Rightarrow \rho gh = \frac{120}{800\times10^{-4}}$

$10^{4}\ h = \frac{120}{800}\times10^{4}$

$h = \frac{12}{80} = \frac{3}{20}$ m = 15 cm

Sol 2: (B)



Downward force on the cubes = $(m_1 + m_2)$ g

= ρ_1 Vg + ρ_2 Vg

(1750) × 10

Upward force on the cubes = $\rho(V_1 + V)$ g

= 1000 (h + 1) × 10

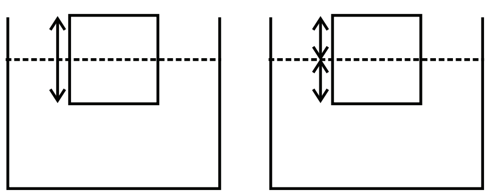
Since cubes are in equilibrium

So 17500 = 10000 (h + 1)

1.75–1 = h

⇒ h = 0.75 m

Sol 3: (D)



In equilibrium   Pushed down by y distance

Initially in equilibrium

When pushed down by y distance, an extra upward force will act on the cube

ρ (ybc) g = d ρ abc A

[A = acceleration of the cube]

$y = \frac{da}{g} A \Rightarrow A = \frac{g}{da} y \Rightarrow \omega^2 = \frac{g}{da} \Rightarrow \omega = \sqrt{\frac{g}{da}} \Rightarrow T = \frac{2\pi}{w} = 2\pi\sqrt{\frac{da}{g}}$

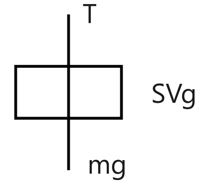
Sol 4: (D) $f \propto \sqrt{T}$, T = tension in the wire

in water frequency becomes f/2

⇒ Tension becomes ¼ of the initial

in liquid frequency becomes f/3

⇒ Tension become 1/9 of the initial



For water $\rho Vg = \frac{3mg}{4}$

for liquid $d\rho Vg = \frac{8}{9}$ mg

$\Rightarrow d = \frac{8}{27} \times 4 = \frac{32}{27}$

Sol 5: (A) By work energy theorem

$W_{water} + W_{gravity} = \Delta KE = 0$

$W_{water} = (\rho Vg)\ h$

$W_{gravity} = -(0.8\ \rho\ Vg)\ (h + 2)$

$\Rightarrow \rho Vgh - 0.8\ \rho Vg\ (h + 2) = 0$

$h - 0.8\ (h + 2) = 0 \Rightarrow \frac{5h}{4} = h + 2$

$\frac{h}{4} = 2 \Rightarrow h = 8m$

Sol 6: (A) The vertical component of tension balances out the net of weight & buoyancy.

Sol 7: (C) We know that time taken for the vessel to empty is $t_o = \sqrt{\frac{2H}{g}}$, H = height of water

Time taken to empty vessel of height $\frac{H}{\eta}$ is t_2

$= \sqrt{\frac{2H}{g\eta}}$

$t_1 = t_0 - t_2$ and $t_1 = t_2$

$\Rightarrow \sqrt{\frac{2H}{g}} - \sqrt{\frac{2H}{g\eta}} = \sqrt{\frac{2H}{g\eta}} \Rightarrow \sqrt{\frac{2H}{g}} = 2\sqrt{\frac{2H}{g\eta}} \Rightarrow \eta = 4$

Sol 8: (B) Bernoulli's theorem is derived by the conservation of energy.

Sol 9: (D) Volume flow rate is same

So $\pi\ (1 \times 10^{-2})^2 \times 3$

$$= 100 \times \pi \left(\frac{0.05 \times 10^{-2}}{2}\right)^2 \times V$$

$\pi \times 10^{-4} \times 3$

$= 100 \times \pi \times ¼ \times 25 \times 10^{-8} \times V$

$$V = \frac{4 \times 3}{25} \times 100 = 48$$

Sol 10: (C) We know that force exerted by fluid coming out on the container is $\rho A v^2$

v = velocity of fluid

$$v = \sqrt{2g\frac{H}{2}}$$

A = area of the hole

$$\text{Acceleration of the tank} = \frac{\rho A v^2}{\rho(NAH)}$$

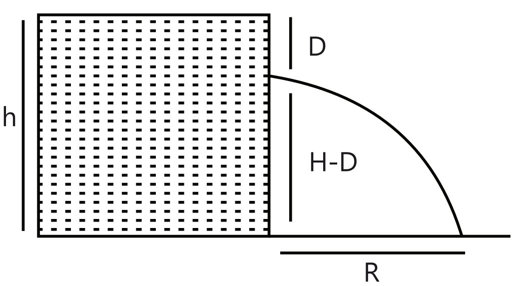
$$= \frac{\rho(AgH)}{\rho NAH} = \frac{g}{N}$$

Sol 11: (A) $A_1V_1 = A_2 V_2$

$\pi (10^{-2})^2 V_P = \pi (2 \times 10^{-2})^2 V_Q$

$V_P = 4V_Q$

Sol 12: (A)



Velocity of water = $\sqrt{2Dg}$

Time taken by water to come to the ground

$$t = \sqrt{\frac{2(H-D)}{g}}$$

Distance where water hit the surface = vt

$$\sqrt{2Dg} \cdot \sqrt{\frac{2(H-D)}{g}} = 2\sqrt{D(H-d)}$$

Sol 13: (C) A = volume flow rate

$= 10^{-2} \times 2 = 2 \times 10^{-2}\ m^3/s$

$Q = A_1 v_1 = A_2 v_2$

$$v_2 = \frac{Q}{A_2} = \frac{2 \times 10^{-2}}{1/2 \times 10^{-2}} = 4 \text{ m/s}$$

By Bernoulli equation

$P_1 + ½ \rho v_1^2 = P_2 + ½ \rho v_2^2$

$8000 + ½ \times 1000 \times 4 = P_2 + ½ \times 1000 \times 16$

$10000 = P_2 + 8000$

$P_2 = 2000$ Pa

Sol 14: (C) Viscosity is not an assumption

Sol 15: (D) Frictional resistance $f \propto Av^2$

$f = kAv^2 = k\pi r^2 v^2$

k = constant

When ball acquires terminal velocity

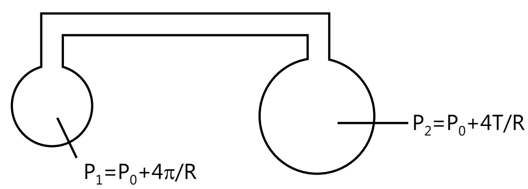
f = mg

$k\pi\, r^2 v^2 = mg$

$k\, \pi\, r^2 v^2 = (4/3\, \pi\, r^3)\, \rho\, g$

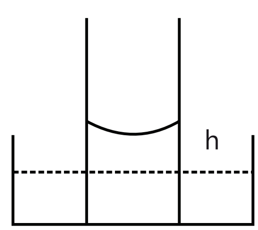
$v^2 \propto r \Rightarrow v \propto r^{1/2}$

Sol 16: (C)



as $P_1 > P_2$ so air will flow out of the small bubble.

Sol 17: (B)



$$h = \frac{2T}{R\rho g}$$

R = radius of curvature

$$hR = \frac{2T}{\rho g} = \text{constant}$$

So graph of R vs. h will be hyperbola

Sol 18: (D) By Bernoulli equation

$P_0 + \rho g h_3 = P_0 + \frac{1}{2}\rho v^2$

$\frac{1}{2}\rho v^2 = \rho g h_3$

$P_0 = P_2 + \frac{1}{2}\rho v^2$

$P_2 = P_0 - \rho g h_3$

Sol 19: (D) By continuity

$A_x V_x = A_y V_y$

$A_x > A_y$

$\Rightarrow v_x < v_y$

By Bernoulli equation

$P_x + \frac{1}{2}\rho v_x^2 = P_y + \frac{1}{2}\rho v_y^2$

$v_x < v_y$

$\Rightarrow P_x > P_y$

KE per m^3 of water = $\frac{1}{2}\rho v^2$

$KE_{x=} = \frac{1}{2}\rho v_x^2$

$KE_y = \frac{1}{2}\rho v_y^2$

$KE_x < KE_y$

Previous Years' Questions

Sol 1: (C) $W_{app} = W_{actual} - \text{Upthrust}$

Upthrust $F = V_s \rho_L g$

Here, V_s = volume of solid,

r_L= density of liquid.

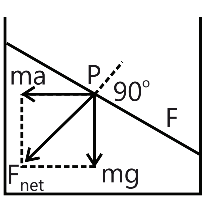
At higher temperature $F' = V'_s \rho'_L g$

$$\therefore \frac{F'}{F} = \frac{V'_s}{V_s} \cdot \frac{\rho'_L}{\rho_L} = \frac{(1+\gamma_s \Delta\theta)}{(1+\gamma_L \Delta\theta)}$$

Since $\gamma_s < \gamma_L$ (given)

$\therefore F' < F$ or $W'_{app} > W_{app}$

Sol 2: (A) Net force on the free surface of the liquid in equilibrium (from accelerate frame) should be perpendicular to it.



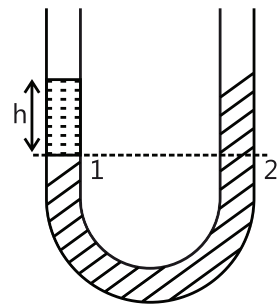
Force on a water particle P on the free surfaces have been shown in the figure. In the figure ma is the pseudo force.

Sol 3: (A) In a freely falling system $g_{eff} = 0$ and since, Upthrust = $V_1 \rho_L g_{eff}$

(V_1 = immersed volume, ρ_L = density of liquid)

Upthrust = 0.

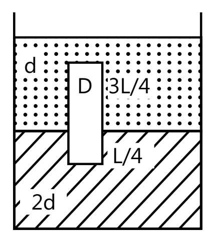
Sol 4: (B)



$p_1 = p_2 \Rightarrow p_0 + \rho_I g h = p_0 + \rho_{II} g h$

$\therefore \rho_I = \rho_{II}$

Sol 5: (A)



Considering vertical equilibrium of cylinder

Weight of cylinder = Upthrust due to upper liquid+ upthrust due to lower liquid.

$\therefore (A/5)(L) D_g = (A/5)(3L/4)(d)g + (A/5)(L/4)(2d)(g)$

$$\therefore D = \left(\frac{3}{4}\right)d + \left(\frac{1}{4}\right)(2d)$$

$$D = \frac{5}{4}d$$

Sol 6: (C) From conservation of energy

$v_2^2 = v_1^2 + 2gh$ .....(i)

[can also be found by applying Bernoulli's theorem]

From continuity equation

$$A_1v_1 = A_2v_2$$

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 \quad .....(ii)$$

Substituting value of v_2 from Eq. (ii) in Eq. (i)

$$\frac{A_1^2}{A_2^2}.v_1^2 = v_1^2 + 2gh$$

$$\text{or } A_2^2 = \frac{A_1^2\ v_1^2}{v_1^2 + 2gh}.$$

$$\therefore A_2 = \frac{A_1v_1}{\sqrt{v_1^2 + 2gh}}$$

Substituting the given value

$$A_2 = \frac{(10^{-4})(1.0)}{\sqrt{(1.0)^2 + 2(10)(0.15)}}$$

$A_2 = 5.0 \times 10^{-5}$ m^2

Sol 7: (A) Velocity of efflux at a depth h is given by $v = \sqrt{2gh}$. Volume of water flowing out per second from both the holes are equal.

$\therefore \quad a_1v_1 = a_2v_2$

or $\quad (L^2)\sqrt{2g(y)} = \pi R^2\sqrt{2g(4y)}$

or $\quad R = \frac{L}{\sqrt{2\pi}}$

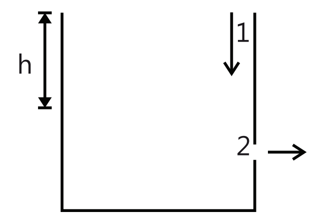
Sol 8: (D) *l* will decreases because the block moves up and h will decrease because the coin will displace the volume of water (V_1) equal to its own volume when it is in the water whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the density of water, $V_1 < V_2$.

Sol 9: (A) Applying continuity equation at 1 and 2, we have

$A_1v_1 = A_2v_2$ .....(i)

Further applying Bernoulli's equation at these two points, we have

$$p_0 + \rho gh + \frac{1}{2}\rho v_1^2 = p_0 + 0 + \frac{1}{2}\rho v_2^2 \quad ....(ii)$$



Solving eq. (i) and (ii), we have

$$v_2^2 = \frac{2gh}{1 - \frac{A_2^2}{A_1^2}}$$

Substituting the values, we have

$$v_2^2 = \frac{2\times10\times2.475}{1-(0.1)^2} = 50\text{m}^2/\text{s}^2$$

Sol 10: (B) $\Delta p_1 = \frac{4T}{r_1}$ and $\Delta p_2 = \frac{4T}{r_2}$

$r_1 < r_2$

$\therefore \quad \Delta p_1 > \Delta p_2$

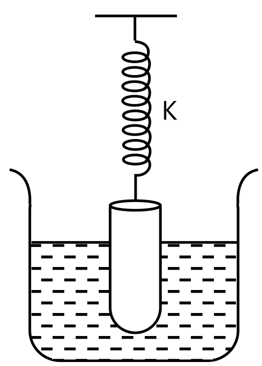
$\therefore$ Air will flow from 1 to 2 and volumes of bubble at end 1 will decrease.

Therefore, correct option is (B).

Sol 11: (B) When cylinder is displaced by an amount x from its mean position, spring force and upthrust both will increase. Hence, Net restoring fore = extra spring force + extra upthrust

or $F = -(kx + Ax\rho g)$ or $a = -\left(\frac{k + \rho Ag}{M}\right)x$

Now, $f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{k+\rho Ag}{M}}$

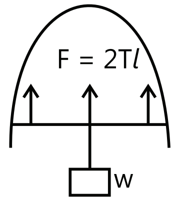


Sol 12: (D) The force of surface tension acting on the slider balances the force due to the weight.

$\Rightarrow F = 2T\ \ell = w$

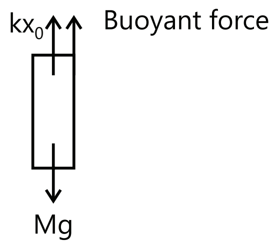
$\Rightarrow 2T(0.3) = 1.5 \times 10^{-2}$

$\Rightarrow T = 2.5 \times 10^{-2}\ N/m$



Sol 13: (B)

At equilibrium $\sum F = 0$



$$kx_0 + \left(\frac{AL}{2}\sigma g\right) - Mg = 0$$

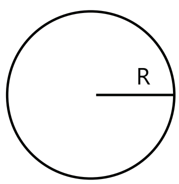
$$x_0 = Mg\left[1 - \frac{LA\sigma}{2M}\right]$$

Sol 14: (C)

$$\rho 4\pi R^2\ \Delta RL = T4\pi\left[R^2 - (R - \Delta R)^2\right]$$

$$\rho R^2\ \Delta RL = T\left[R^2 - R^2 + 2R\Delta R - \Delta R^2\right]$$

$$\rho R^2\ \Delta RL = T2R\Delta R\ \ (\Delta R \text{ is very small})$$



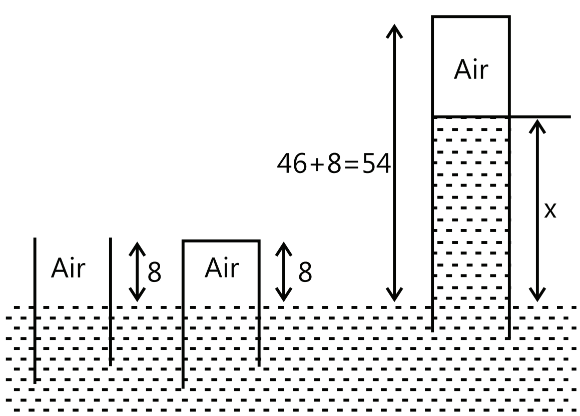
$$R = \frac{2T}{\rho L}$$

Sol 15: (C)

$(76)(8) = (54 - x)(76 - x)$

$x = 38$ cm

Length of air column = 54 – 38 = 16 cm.



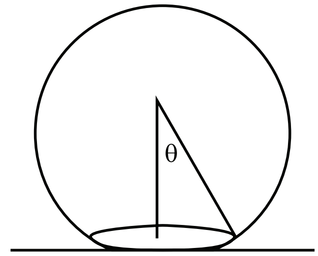
Sol 16: (D)

$$(2\pi r\ T)\sin\theta = \frac{4}{3}\pi R^3\ \rho_w.g$$

$$T \times \frac{r}{R} \times 2\pi r = \frac{4}{3}\pi R^3\ \rho_w\ g$$

$$r^2 = \frac{2}{3}\frac{R^4\ \rho_w g}{T}$$

$$r = R^2\sqrt{\frac{2\rho_w g}{3T}}$$



JEE Advanced/Boards

Exercise 1

Sol 1: Pressure at A = $P_0 + \frac{Mg}{A} = P_0 + \rho gh$

$A = \pi(0.04)^2 - (0.01)^2 = \pi \times 15 \times 10^{-4}$

$\frac{M}{A} = \rho h$

$h = \frac{M}{A\rho} = \frac{3}{\pi \times 15 \times 10^{-4} \times 1000} = \frac{2}{\pi}$ m

mass of water = 750 gm = 0.75 kg

mass of water below piston

$= 0.75 - (1000)(\pi \times (0.01)^2) \times h$

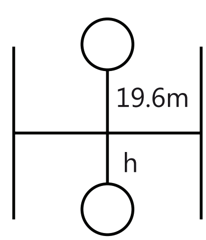
$= (1000) \times \pi \times (0.04)^2 \times H$

$0.75 = \frac{\pi}{10} \times \frac{2}{\pi} + \frac{16\pi}{10} \times H$

$0.55 = \frac{16\pi}{10} H \Rightarrow H = \frac{5.5}{16\pi} = \frac{11}{32\pi}$ m

Sol 2: Net force on the ball in water

$F_B = \rho_w Vg - \frac{\rho_w Vg}{2} = \frac{\rho_w Vg}{2} = mg$



Let us assume that ball will go up to depth h in water. By work energy theorem

$-mg(19.6) + mg\,h = 0 \Rightarrow h = 19.6$ m

Upward force F = mg in water

Acceleration = g

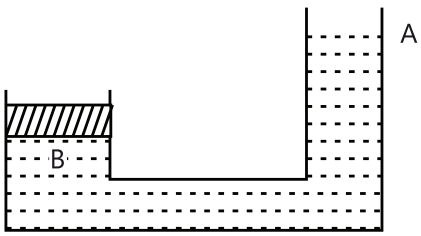
Time required to come on surface

$= \sqrt{\frac{2s}{g}} = \sqrt{\frac{2\times 19.6}{9.8}}$ = 2 sec

Time required to go inside surface is also 2 sec

So total time required = 4 sec

Sol 3:



Pressure at point A = $P_0 + \frac{F}{A_1} = P_A$

Pressure at point B = $P_0 + \frac{mg}{A_2} = P_B$

Difference in pressure = ρg × 8 = $P_B - P_A$

$\Rightarrow P_0 + \frac{F}{A_1} - P_0 - \frac{mg}{A_2} = -\rho g \times 8$

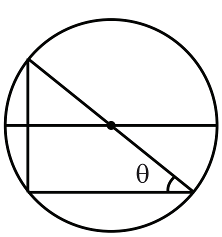
$\frac{F}{A_1} = \frac{6000}{800\times 10^{-4}} - 750 \times 10 \times 8 = \frac{15}{2} \times 10^4 - 6 \times 10^4$

$\frac{F}{A_1} = \frac{3}{2} \times 10^{-4}$

$F = 1.5 \times 10^4 \times 25 \times 10^{-4}$

F = 37.5 N

Sol 4:



(a) $\tan\theta = \frac{a}{g} = 1$

θ = 45°

Maximum gang pressure = $\rho\sqrt{a^2 + g^2}\ r$

$= 800 \times 10\sqrt{2} \times 1.2 = 9600\sqrt{2}$ N/m²

(b) h = depth of lake

When bubble is at bottom pressure inside

$= P_0 + \rho g h$

When bubble is at surface pressure is = P_0

T_0 = surface tension

$P_0 = \frac{2T}{2r} = \frac{T}{r}$

$P_0 + \rho g h = \frac{2T}{r}$

$\frac{T}{r} + \rho g h = \frac{2T}{r}$

$\rho g h = \frac{T}{r} = P_0 = \rho_m\, g\, h_m$

$\rho_m = n\rho$

h = nH

Sol 5: Upward force on test tube initially = $\rho_s A\times 0.1\ g$

Upward force after adding liquid = $\rho_w A\times 0.4g$

Weight of the fluid = $\rho_w A \times 0.4g - \rho_w A \times 0.1g = \rho' A \times 0.4\ g$

$\Rightarrow \rho' = \frac{3\rho_w}{4}$

Sol 6: At point A by Bernoulli equation

$P_0 + \sigma g\frac{h}{4} = \frac{1}{2}\sigma v^2 + P_0 \Rightarrow v = \sqrt{\frac{gh}{2}}$

Time take = $\frac{\sqrt{2(3h/4)}}{g} = \sqrt{\frac{3h}{2g}}$

Distance travelled = vt

$= \sqrt{\frac{gh}{2}} \times \sqrt{\frac{3g}{2g}}$

Distance = $\frac{h}{2}\sqrt{3}$

At point B

$P_0 = \sigma g\frac{h}{2} = P_0 + (+2\sigma\, g\, (-h/4)) + ½\, 2\sigma\, v'^2$

$\frac{gh}{2} = -\frac{gh}{2} + v'^2$

$V' = \sqrt{gh}$

Time $t' = \sqrt{\frac{2(h/4)}{g}} = \sqrt{\frac{h}{2g}}$

Distance travelled = $vt' = \sqrt{gh}\sqrt{\frac{h}{2g}} = \frac{h}{\sqrt{2}}$

Ratio of distance travelled = $\frac{h\sqrt{3}}{2\frac{h}{\sqrt{2}}} = \frac{\sqrt{3}}{\sqrt{2}}$

Sol 7: Force exerted is change in momentum per sec

$= \frac{d(mv)}{dt} = v\frac{dm}{dt}$

$= v\,\rho\,Av$

$= \rho Av^2 = 1000 \times 2 \times 10^{-4} \times 100$

= 20 N

Sol 8: By Bernoulli's equation

$P_0 + \rho gh + ½\,\rho v_1^2 = P_0 + ½\,\rho v_2^2$

By continuity equation

$A_1v_1 = A_2v_2$

$v_1 = \frac{v_2}{2}$

$gh + ½\left(\frac{v_2}{2}\right)^2 = \frac{1}{2}v_2^2$

$gh = \frac{v_2^2}{2}[1-1/4] = 3/8\, v_2^2$

$v_2 = \sqrt{\frac{8gh}{3}} \Rightarrow v_1 = \sqrt{\frac{2gh}{3}} = \sqrt{\frac{2}{3}}$

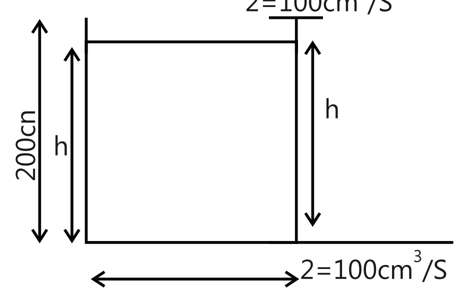
Volume flow rate = $\sqrt{\frac{2}{3}} \times 10^{-4}\ m^3/s$

$= 60\sqrt{\frac{2}{3}} \times 10^{-4}\ m^3/min$

$= 60\sqrt{\frac{2}{3}} \times \frac{1}{10}$ litre/min

= 4.9 litre/min

Sol 9:



By Bernoulli's equation

$P_0 + \rho gh = P_0 + ½\,\rho v^2$

$v = \sqrt{2gh}$

$Q = 100\ cm^3/s$

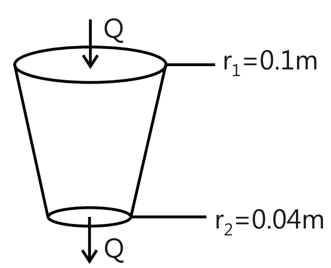
$A = 1\ cm^2$

v = 100 cm/s = 1 m/s

$v = \sqrt{2 \times 10 \times h} = 1$

$h = \frac{1}{20}$ m = 5 cm

Sol 10:



By Bernoulli's equation

$P_1 + ½\,\rho V_1^2 = P_2 + ½\,\rho V_2^2$

$-P_2 + P_1 = 10\ N/m^2$

$10 + ½ \times 1250\, v_1^2 = ½ \times 1250\, v_2^2$ ....(i)

Continuity equation

$A_1v_1 = A_2v_2$

$\pi (0.1)^2 v_1 = \pi (0.04)^2 v_2$

$\frac{1}{100}v_1 = \frac{4 \times 4}{10000}v_2$

$v_1 = 0.16\, v_2$ .....(ii)

By (i) and (ii)

$10 + 625\,(0.16\, v_2)^2 = 625\,(v_2)^2$

$625\,(0.9744)\, v_2^2 = 10$

$v_2 = 0.128$ m/s

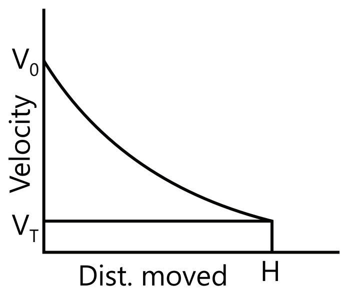
$Q = A_2v_2$

$= \pi\ (0.04)^2 \times 0.128$

$= \pi \times 16 \times 10^{-4} \times 0.128$

$= 6.44 \times 10^{-4}\ m^3/s$

Sol 11: Since velocity is greater than terminal velocity, so it will decrease until velocity reaches terminal velocity



Sol 12: Terminal velocity = $V = \frac{2}{9} r^2 \frac{(\rho - \sigma)}{\eta} g$

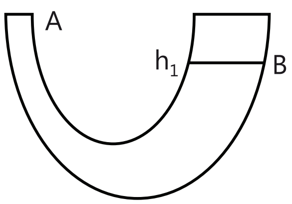
$= \frac{2}{9}\ 10^{-8}\ \frac{(10^4 - 10^3)\times 9.8}{9.8\times 10^{-6}}$

v = 20 m/s

Distance required to reach terminal velocity is

$h = \frac{v^2}{2g} = \frac{(20)^2}{2\times 10} = \frac{400}{20} = 20\ m$

Sol 13:



$P_A = P_0 - \frac{2T}{r_1}$

$P_B = P_0 - \frac{2T}{r_2}$

$P_B - P_A = \rho gh = 2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 2T\ [2000 - 200]$

$\rho gh = 2 \times 2 \times 7 \times 9 = 252$

$h = \frac{252}{10\times 10^3} = 2.5$ cm

Sol 14: Surface energy of bubble = $4\pi r^2\ T_1$

Surface energy of drop = $4\pi r_2^{\,2}\ T_2$

Volume of bubble and drop is same so

$4\pi R^2 d = (4/3)\ \pi r_2^{\,3}$

$r_2^{\,3} = 3R^2 d$

$r_2^{\,2} = (3R^2 d)^{\,2/3}$

$\frac{P_1}{P_2} = \frac{4T}{R2T}\ r_2 = \frac{2r_2}{R} = \sqrt[3]{\frac{24d}{R}}$

$\frac{P_2}{P_1} = \sqrt[3]{\frac{R}{24d}}$

Sol 15: $P_0 + \frac{4T}{r_1} + \frac{4T}{r} = P_0 + \frac{4T}{r_2}$

$\Rightarrow r = \frac{r_1 r_2}{r_1 - r_2}$

Sol 16: Let the volume of bearer be V

Then balancing force on beaker

$\rho_w V.g = (0.39)g + \rho_w \times 250$ g

$10^3 \times V = 0.39 + 0.25$

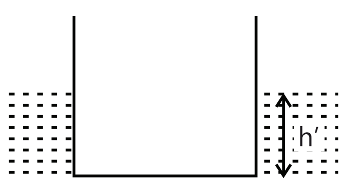
$V = 640 \times 10^{-6}\ m^3$

$V = 60\ cm^3$

Volume of glass = 640 – 500 = 140 cm^3

Density = $\frac{m}{v} = \frac{390}{140} = 2.785$ gm/cc

Sol 17: Plug will open when float is lifted upwards due to buoyant force



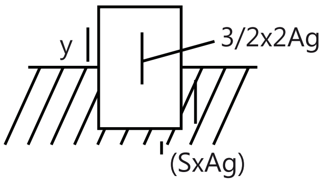
Balancing force we get

$\rho\ h' \times \pi\ (0.1)^2 \times g = 3g$

$h' = \frac{3}{1000\times\pi\times\frac{1}{100}} = \frac{3}{10\pi}$

height $h = h' + 10\ cm = \frac{3}{10\pi} + \frac{1}{10} = \frac{3+\pi}{10\pi}$

Sol 18:



(a) By Newton's second law

a = Upward acceleration

$\frac{-\rho}{2} \times 2\ Ag + \rho(1 - y)\ Ag = (\rho/2)\ 2Aa$

$-\rho Ag + \rho(1-y)\ Ag = \rho Aa$

$-g + g - gy = a$

$a = -gy$

acceleration is density proportional to the displacement so it will perform SHM

$a = -\omega^2\ y$

$\omega^2 = g$

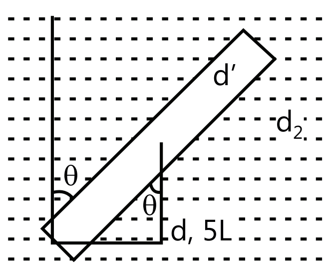
$\omega = \sqrt{g} = \sqrt{\pi^2} = \pi$

Time period = $\frac{2\pi}{\omega}$ = 2 sec

(b) Time taken for rod to go from 1 extreme position to other is half of the time period

So time taken = 2/2 = 1 sec

Sol 19:



Torque on rod = $(d_1\ \ell Sg - d_2\ \ell sg)\ l/2 \sin\theta$

for small θ

$\tau = (d - d_2)\ (\ell^2/2)\ s\ g\ \theta$

Net torque = $I\alpha$

$I\alpha = (d_1 - d_2)\ \ell^2\ s\ g\ \theta$

$I = \frac{(d_1 s\ell)\ell^2}{3}$

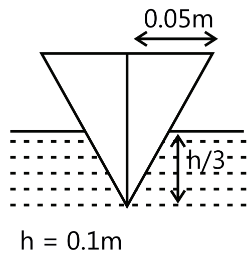
$\frac{d_1 s\ell^3}{3}\ \alpha = (d_1 - d_2)\ \frac{\ell^2}{2}\ \rho\ g\ \theta$

$\frac{d_1 \ell\alpha}{3} = \frac{(d_1 - d_2)}{2}\ g\ \theta$

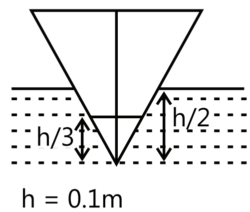
$\alpha = \frac{3(d_1 - d_2)g}{2d_1\ell}\ \theta$

$\omega = \sqrt{\frac{3(d_1 - d_2)g}{2d_1\ell}}$

Sol 20:



$\frac{r}{0.05} = \frac{r}{3h}$



h = 0.1m

By force balance

$mg = 0.8\left(\frac{1}{3}\pi\left(\frac{0.05}{3}\right)^2\left(\frac{h}{3}\right)\right)$ .....(i)

When liquid is added

$Mg + \rho\left(\frac{1}{3}\pi\left(\frac{0.05}{3}\right)^2\left(\frac{h}{3}\right)\right)$

$= 0.8\left(\frac{1}{3}\pi\left(\frac{0.05}{2}\right)^2\left(\frac{h}{2}\right)\right)$ ......(ii)

By (i) and (ii)

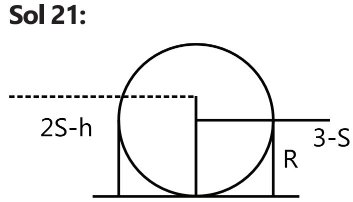
$(0.8)\left(\frac{1}{3}\pi\frac{(0.05)^2 h}{3^3}\right) + \rho\left(\frac{1}{3}\pi\frac{(0.05)^2 h}{3^3}\right)$

$= 0.8\left(\frac{1}{3}\pi\frac{(0.05)^2 h}{2^3}\right)$

$\frac{0.8 + \rho}{3^3} = \frac{0.8}{8} = 0.1 \Rightarrow 0.8 + \rho = 2.7$

$\rho = 1.9$

Sol 21:



Balancing force on both sides

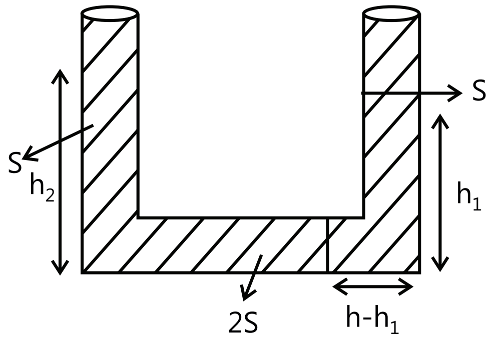
Horizontal force acting on the cylinder can be assumed to be acting on the cross–sectional area in the vertical direction

$$2\rho gh.\frac{h}{2} = \frac{3\rho gR.R}{2}$$

$$h^2 = \frac{3}{2}R^2$$

$$h = \sqrt{\frac{3}{2}}\,R$$

Sol 22:



$h = h_2 - (h - h_1)$

$h = h_2 - h + h_1$

$$h = \frac{h_1 + h_2}{2}$$

$P_0 + \rho gh_1 + \rho a\,(h - h_1) + 2\rho a\,(h + h_1) - 2\rho g\,(h - h_1) - \rho gh = P_0$

$gh_1 + ah - ah_1 + 2ah + 2ah_1 - 2gh + 2gh_1 - gh = 0$

$h_1\,(g - a + 2a + 2g) + h\,(a + 2a - 2g - g) = 0$

$$h_1 = \frac{3(g-a)h}{3g+a}$$

$a = g/2$

$$\Rightarrow h_1 = \frac{\frac{3h}{2}}{3+\frac{1}{2}} = \frac{3h}{7}$$

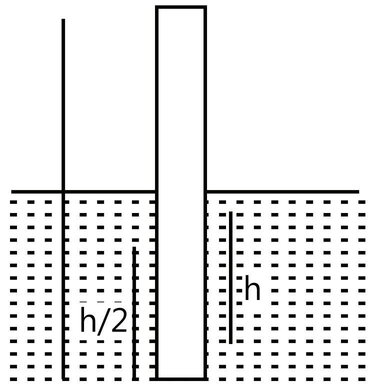
$h_2 = h + h - h_1$

$h_2 = 2h - h_1$

Difference in height $= h_2 - h_1$

$$= 2h - h_1 - h_1 = 2(h - h_1) = \frac{8h}{7}$$

Sol 23:



For the rod to be in equilibrium centre of mass of (rod + mass m) system should be below centre of gravity of the volume displaced by the rod.

For minimum m should coincide so.

Suppose h length of rod is below water then, by force balance

$\sigma(\pi R^2 h)\,g - \rho l\pi R^2\,g - mg = 0$

$(\pi R^2)\,(\sigma h - \rho\,\ell) = m$ .....(i)

Reaction of centre of mass should be at h/2 distance from bottom

$$\Rightarrow h/2 = \frac{\frac{\rho\ell\pi R^2 \times \ell}{2}}{m + \rho\ell\pi R^2} \quad .....(ii)$$

$$\Rightarrow h^2 = \frac{\rho\ell^2}{\sigma} \Rightarrow h = \ell\sqrt{\frac{\rho}{6}} \quad .... (iii)$$

By (i) and (iii)

$m = (\sqrt{\sigma\ell} - \rho)\ \pi R^2\, l$

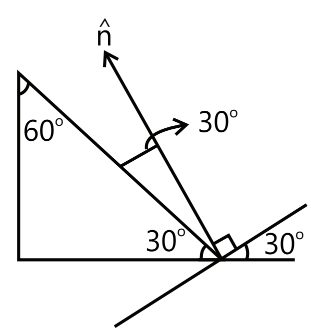
Sol 24: Volume of water collected = A.V

A = cross sectional area perpendicular to the rain.

v = velocity of rain

in 1st beaker $A_2 = A_1 \cos 30°$

in 2nd beaker $A'_3 = A \cos 60°$



$A_1 = \frac{A}{\cos 30°}$

$\Rightarrow A_2 = \frac{A}{\cos 30°} \cos 30° = A$

$\Rightarrow A_3 = \frac{A}{2}$

So $\frac{Q_2}{Q_3} = \frac{A_2 V}{A_3 V} = 2 : 1$

Sol 25: (a) By Bernoulli's equation

$P_0 + \rho g(3.6) = P_0 + ½ \rho v^2$

$v = \sqrt{2g(3.6)} = \sqrt{72}$

$v = 6\sqrt{2}$ m/s

(b) Discharge rate = $\pi r^2 v$

$= \pi \times \frac{16}{\pi} \times 10^{-4} \times 6\sqrt{2}$ m^3/s

$= 9.6 \sqrt{2} \times 10^{-3}$ m^3/s

(c) By Bernoulli equation

$P_A + ½ \rho v^2 + \rho g (5.4) = P_0 + ½ \rho V^2$

$P_A = P_0 - \rho g \times 5.4$

$= 10 \times 10^4 - 5.4 \times 10^4$

$= 4.6 \times 10^{-4}$ Pa

Sol 26: Pressure at C = P_c

By Bernoulli's equation

$P_0 + \rho g h_1 = P_0 + ½ \rho V_D^2$

$½ \rho V_D^2 = \rho g h_1$

$P_0 + \rho g h_1 = P_c + ½ \rho V_C^2$

$A_C V_C = A_D V_D$

$A_C = \frac{A_D}{2}$

$\Rightarrow v_C = 2v_D$

$P_0 + \rho g h_1 = P_C + 2\rho V_D^2$

$\Rightarrow P_c = P_0 + \rho g h_1 - 4\rho g h_1 = P_0 - 3 \rho g h_1$ ....(i)

Pressure at C can also be written as

$P_C + \rho g h_2 = P_0$

$P_C = P_0 - \rho g h_2$ ...... (ii)

By (i) and (ii)

$\rho g h_2 = 3 \rho g h_1$

$h_2 = 3h_1$

Sol 27: By force equilibrium on the cube

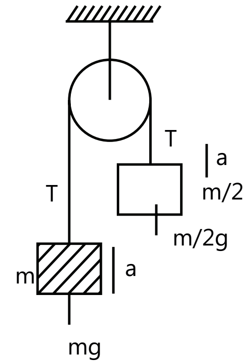
$F_{gravity} + F_{buoyant} + F_{surface\ tension} = 0$

$-mg + \rho_w a^2 hg - S \times 4a = 0$

$h = \frac{mg + 4Sa}{\rho_w a^2 g}$

Exercise 2

Sol 1: (B) By Newton's second law



$T - \frac{mg}{2} = \frac{ma}{2}$

$T = \frac{m}{2}(g + a)$ ..... (i)

$-T + mg = me$ ..... (ii)

By (i) and (ii)

$\frac{-m}{2}(g + a) + mg = ma$

$\frac{-g}{2} \frac{-a}{2} + g = a$

$\frac{g}{2} = \frac{3a}{2}; a = \frac{g}{3}$

Effective acceleration of the bucket is $\left(g - \frac{g}{3}\right)$ downwards water pressure at the bottom above atmospheric pressure is

$P = \frac{2g}{3} h\rho = 1000 \times \frac{2}{3} \times 10 \times \frac{15}{100} = 1$ kPa

Sol 2: (D) Buoyant force = sum of all forces acting on the body

= force acting on the slant surface

+ force acting on the bottom surface

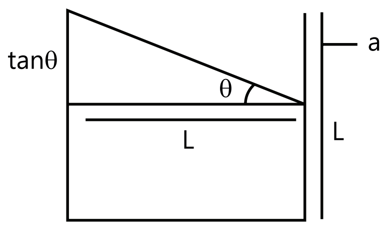
$F_B = F_s + F_b$

$F_B = (1/3)\ \pi R^2\ H\rho g$

$F_b = \pi R^2\ \rho g H$

$\Rightarrow F_s = (-2/3)\ \pi R^2\ \rho g H$

Sol 3: (B)



A = area of the base

$\tan\theta = a/g$

Finally 1/3 rd of the water spilled out

So volume of water spilled out finally

$= V_f = \dfrac{2\tan\theta \times A}{2} = \dfrac{L^3\tan\theta}{2}$

this is 1/3 volume of L^3

$\Rightarrow \dfrac{\tan\theta}{2} = \dfrac{1}{3} \Rightarrow \tan\theta = 2/3 = a/g$

$a = 2g/3$

Sol 4: (D) Force applied by the liquid will be same on both the vessels as the mass of liquid is same in both the vessels

Sol 5: (A) Total force exerted on the base by water and cane's slant surface = mg

$= 1/3\ \pi R^2\ H\rho g$ downwards

Force exerted by the water =

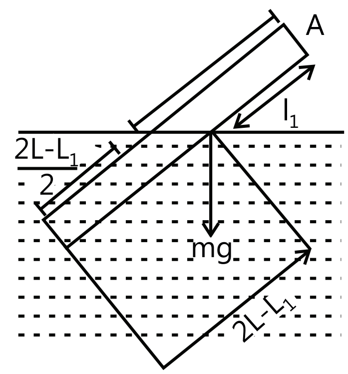
$(\rho g H)\ (\pi R^2)$ downwards

So force exerted by the slant surface =

$2/3\ \rho g H\ \pi R^2$ upwards

So force exerted by water on slant surface = $2/3\ \rho g H\ \pi R^2$

Sol 6: (A)



Let the length of rod that extends out of water is λ_1 since the rod is in equilibrium

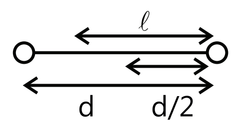
So balancing net torque about point A

we get $(\rho A(2L-L_1)g)\left(\dfrac{2L+L_1}{2}\right)\cos\theta$

$= 0.75\ \rho AL\ g\ L\cos\theta$

$\dfrac{4L^2 - L_1^2}{2} = \dfrac{3}{4}L^2$

Sol 7: (B)



By force equilibrium we get

$-Mg - 2Mg - mg + \dfrac{\rho Vg}{2} + \dfrac{\rho Vg}{2} = 0$

$\Rightarrow m = \rho v - 3M$ .....(i)

By torque equilibrium about mass M we get

$-mg\ (d - \ell) - 2\ Mgd + \dfrac{d\rho Vg}{2} = 0$

$m\ell - d\left(m + 2M - \dfrac{\rho V}{2}\right) = 0$

$\ell = \dfrac{d\left(2M + m - \dfrac{\rho v}{2}\right)}{m}$ ...(ii)

By (i) and (ii) we get $\ell = \dfrac{d(\rho V - 2M)}{2(\rho V - 3M)}$

Sol 8: (B) By work energy theorem

$W_{water} + W_{gravity} = \Delta KE = 0$

$(\sigma\ v\ g\ h) - \rho\ v\ g\ (h + H) = 0$

$\sigma\ h = \rho\ (h + H)$

$H = \dfrac{(\sigma - \rho)h}{\rho} = \left(\dfrac{\sigma}{\rho} - 1\right)h$

Sol 9: (A) Buoyant force = $\rho_w \times 4/3\ \pi R^3\ g$

Gravitational force = $(\sigma\rho_w)\ (4/3\ \pi(R^3 - r^3))\ g$

Sphere is in equilibrium so

$$\rho_w\ 4/3\ \pi\ R^3\ g = (\sigma\rho_w)\ (4/3\ \pi(R^3 - r^3)\ g)$$

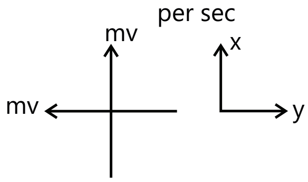
$$R^3 = \sigma(R^3 - r^3)$$

$$\frac{1}{\sigma} = 1 - \frac{r^3}{R^3}$$

$$\frac{r^3}{R^3} = 1 - \frac{1}{\sigma} = \frac{\sigma - 1}{\sigma}$$

$$\frac{R}{r} = \left(\frac{\sigma}{\sigma - 1}\right)^{1/3}$$

Sol 10: (D) Force exerted = change in momentum per sec



$$= \frac{mv\hat{j} - mv\hat{i}}{t} = \rho Lv\hat{j} - \rho Lv\hat{i} = \sqrt{2}\ \rho\ v\ L$$

Sol 11: (A) Force exerted by water = ρAV^2

A = area of hole

V = velocity of water through hole

Friction force = μ Mg

for the vessel to just move

$\rho AV^2 = \mu$ Mg

$$\rho \times \frac{\pi D^2}{4} \times 2\ g\ H = \mu Mg \Rightarrow D = \sqrt{\frac{2\mu M}{\pi\rho H}}$$

Sol 12: (C) We know that force applied is proportional to velocity of shaft. So if the force is increased three times, velocity will also increase three times.

Sol 13: (A) Viscous force $F = -\eta\ A\ \frac{dv}{dx}$

$$F = -\eta\ A\ \frac{v}{t}$$

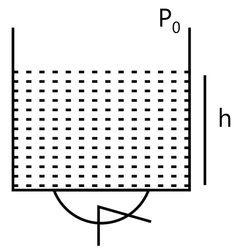
$$F = mg \sin 37° = \frac{3mg}{5}$$

$$\eta = \frac{3mgt}{5AV} = \frac{3\rho a^3 gt}{5a^2 V} = \frac{3\rho agt}{5V}$$

Sol 14: (C) Graph (c) best represents the motion of raindrop because velocity of rain approaches the terminal velocity.

Sol 15: (C) Graph (D) incorrect because at t = 0; x = 0 and graph will not be straight time

Sol 16: (C)



$$P = P_0 + 2T/r$$

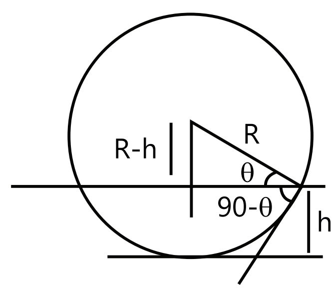
$$P_0 + 2T/r - \rho gh = P_0$$

$$\frac{2T}{r} = \rho gh$$

$$h = \frac{2T}{r\rho g} = \frac{2 \times 75 \times 10^{-3}}{\frac{10^{-4}}{2} \times 1000 \times 10} = 0.30\ m$$

h = 30 cm

Sol 17: (B)



$$\cos(90 - \theta) = \sin\theta = \frac{R - h}{R}$$

$$\text{Angle of contact} = 90 - \theta = \cos^{-1}\left(\frac{R - h}{R}\right)$$

Sol 18: (D) Force exerted by liquid = $\rho g\ (2h).A = F$

weight of liquid is W

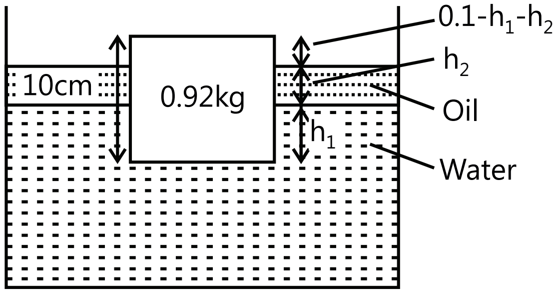
Force exerted by liquid on walls = F – W (upwards)

So force exerted by the walls on the liquid

= (F – W) downwards

Multiple Correct Choice Type

Sol 19: (C, D)



Balancing net force on the block we get

$-0.92\times10 + (1000)\times h_1\times(0.01)\times10$

$+ (600)h_2\times(0.01) \times 10 = 0$

$10\, h_1 + 6h_2 = 0.92$

if $h_2 = 4$ cm

then $10h_1 + 6 \times 0.04 = 0.92$

$10h_1 = 0.68$

$h_1 = \dfrac{0.68}{10} = 0.068$ m = 6.8 cm

$h_1 + h_2$ should be less than 10 cm so

$h_2 < 4$ cm

and $h_1 + h_2 = 10$ cm

$\Rightarrow 10\, h_1 + 6\,(0.1 - h_1) = 0.92$

$4h_1 + 0.6 = 0.92$

$4h_1 = 0.32$

$h_1 = 0.08$ m

$\Rightarrow h_1 = 8$ cm

$h_2 = 2$ cm

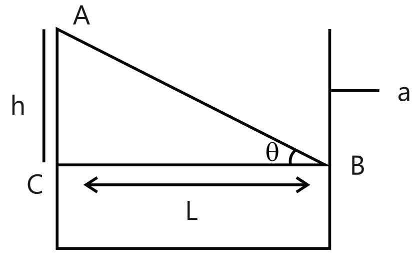
Sol 20: (B, D) Thrust exerted by the water is ρAV^2 if velocity is doubled then thrust will increase 4 times.

Energy lost per second = $\frac{1}{2}\dfrac{dm}{dt}v^2$

$= \frac{1}{2}\rho Av.\, v^2 = \frac{1}{2}\rho Av^3$

If velocity is doubled then energy lost per second will be 8 times

Sol 21: (A, C)



$P_A = P_0$

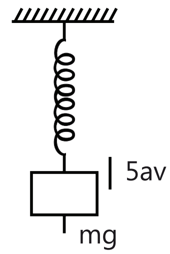
$P_B = P_0$

$P_B = P_A + \rho gh - \rho a\ell = P_0$

$gh = a\ell$

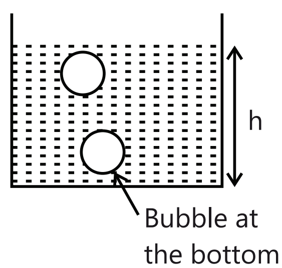
$\tan\theta = \dfrac{h}{L} = \dfrac{a}{g}$

Sol 22: (B, C)



Balance A will read less than 2 kg as an upward buoyant force is acting on the block. Balance B will read more than 5 kg as downward reaction of the block due to buoyant force is acting on beaker.

Sol 23: (A, D)



Pressure inside the bubble at the bottom is

$P_1 = P_0 + \rho gh + \dfrac{2T}{r_1}$

Pressure inside the bubble near the surface is

$P_2 = P_0 + \dfrac{2T}{r_2}$

Pressure inside the bubble near the surface is

$P_2 = P_0 + \dfrac{2T}{r_2}$

So pressure will decrease as we move upwards.

Sol 24: (A, C)

Velocity of fluid coming out of the hole =

$v = \sqrt{2gy}$

time taken by the fluid to collide with surface =

$t = \sqrt{\frac{2(h+h-y)}{g}}$

range = vt

$= \sqrt{2gy} \cdot \sqrt{\frac{2(h+h-y)}{g}}$

$R = \sqrt{4y(2h-y)}$. For maximum R, $\frac{dR}{dy} = 0$

$\Rightarrow \frac{1}{\sqrt{4y(2h-y)}}(2h-2y) = 0$

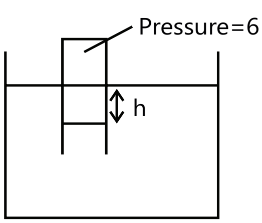
$\Rightarrow y = h$

$R_{max} = \sqrt{4h^2} = 2h$

Assertion Reasoning Type

Sol 25: (B) Pressure of air decreases with increase in height so when pressure outside the balloon is equal to balloon pressure, it will not size up.

Sol 26: (D)



Pressure inside the tube is $P = P_0 + \rho gh$

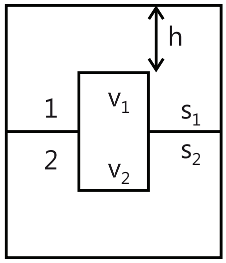
When pressure changes height will also change.

So Statement-I is true.

Buoyant force is independent of atmospheric pressure.

Sol 27: (A) Suppose submarine is resting on the floor, then water is exerting only net downward force on the submarine as lower surface is not available for the upward force.

Sol 28: (D)



Force exerted by liquid – 1 = (ρgH + P_0) A downwards

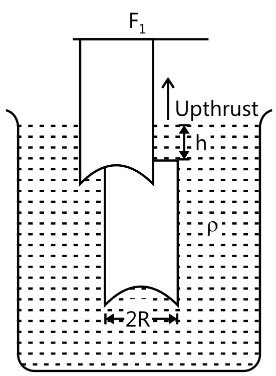
So statement-I is false

Sol 29: (D) Coefficient of viscosity of gases increase with increasing temperature

Sol 30: (A) Free surface is always perpendicular to the g_{eff}. Liquids at rest can have only normal forces.

Previous Years' Questions

Sol 1: (D) $F_2 - F_1$ = upthrust



$\therefore F_2 = F_1$ + upthrust

$F_2 = (p_0 + \rho gh)\pi R^2 + V\rho g$

$= p_0\pi R^2 + \rho g(\pi R^2 h + V)$

$\therefore$ Most appropriate option is (D).

Sol 2: (A) $k_1 = \left(\frac{\rho_{Fe}}{\rho_{Hg}}\right)_{0°C}$ and $k_2 = \left(\frac{\rho_{Fe}}{\rho_{Hg}}\right)_{60°C}$.

Here, ρ = Density

$\therefore \frac{k_1}{k_2} = \frac{(\rho_{Fe})_{0°C}}{(\rho_{Hg})_{0°C}} \times \left(\frac{\rho_{Hg}}{\rho_{Fe}}\right)_{60°C} = \frac{(1+60\gamma_{Fe})}{(1+60\gamma_{Hg})}$

Note: In this problem two concepts are used:

(i) When a solid floats in a liquid, then

Fraction of volume submerged (k) = $\dfrac{\rho_{solid}}{\rho_{liquid}}$

This result comes from the fact that

Weight = Upthrust

$$V\rho_{solid}g = V_{submerged}\ \rho_{liquid}g$$

$$\therefore \quad \frac{V_{submerged}}{V} = \frac{\rho_{solid}}{\rho_{liquid}}$$

(ii) $\dfrac{\rho_{\theta°C}}{\rho_{0°C}} = \dfrac{1}{1+\gamma\cdot\theta}$

This is because $\rho \propto \dfrac{1}{\text{Volume}}$ (mass remaining constant)

$$\therefore \quad \frac{\rho_{\theta°C}}{\rho_{0°C}} = \frac{V_{0°C}}{V_{\theta°C}} = \frac{V_{0°C}}{V_{0°C}+\Delta V}$$

$$= \frac{V_{0°C}}{V_{0°C}+V_{0°C}\gamma\theta} = \frac{1}{1+\gamma\theta}$$

Sol 3: (B) Force from right hand side liquid on left hand side liquid.

(i) Due to surface tension force

=2RT (towards right)

(ii) Due to liquid pressure force

$$= \int_{x=0}^{x=h} (p_0+\rho gh)(2R.x)dx$$

$$= (2p_0Rh + R\rho gh^2)\text{(towards left)}$$

$\therefore$ Net force is $|2p_0Rh + R\rho gh^2 - 2RT|$

Sol 4: (C) Let

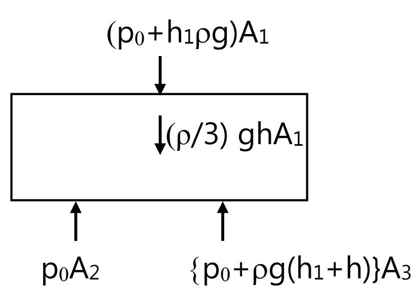
A_1 = Area of cross-section of cylinder = $4\pi r^2$

A_2 = Area of base of cylinder in air = πr^2

and A_3 = Area of base of cylinder in water

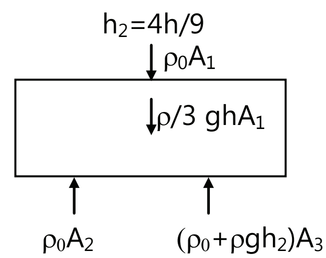
= $A_1 - A_2$ = $3\pi r^2$

Drawing free body diagram of cylinder



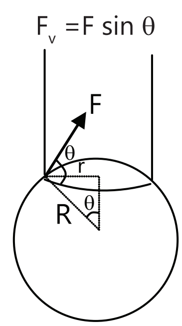
Equating the net downward forces and net upward forces, we get, $h_1 = \dfrac{5}{3}h$.

Sol 5: (A) Again equating the forces, we get



Sol 6: (A) For $h_2 < \dfrac{4h}{9}$, buoyant force will further decrease. Hence, the cylinder remains at its original position.

Sol 7: (C) Vertical force due to surface tension.



= $(T2\pi r)(r/R)$

$$= \frac{2\pi r^2T}{R}$$

Sol 8: (A) $\dfrac{2\pi r^2T}{R} = mg = \dfrac{4}{3}\pi R^3.\ \rho.g$

$$\therefore\ R^4 = \frac{3r^2T}{2\rho g} = \frac{3\times(5\times10^{-4})^2(0.11)}{2\times10^3\times10}$$

$= 4.125 \times 10^{-12}\ m^4$

$\therefore$ R = 1.425×10^{-3} m

$\approx 1.4\times10^{-3}$m

Sol 9: (B) Surface energy,

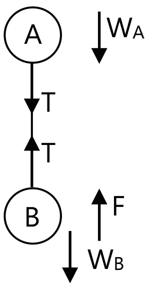
E = $(4\pi R^2)T$

= $(4\pi)\ (1.4\times10^{-3})^2\ (0.11)$

= 2.7×10^{-6} J

Sol 10: (B, C) Liquid will apply an upthrust on m. An equal force will be exerted (from Newton's third law) on the liquid. Hence, A will read less than 2 kg and B more than 5 kg. Therefore, the correct options are (B) and (C).

Sol 11: (A, B, D)



F = upthrust = Vd_Fg

Equilibrium of A

$Vd_Fg = T + W_A$

$= T + Vd_Ag$ ....(i)

Equilibrium of B

$T + Vd_Fg = Vd_Bg$ ....(ii)

Adding eqns. (i) and (ii), we get

$2d_f = d_A + d_B$

∴ Option (D) is correct.

From Eq. (i) we can see that

$d_F > d_A$ (as T>0)

∴ Option (A) is correct.

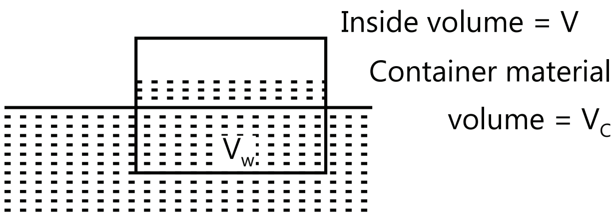
From equation (ii) we can see that,

$d_B > d_F$

∴ Option (B) is correct.

∴ Correct options are (A), (B) and (D).

Sol 12: (A)



$$m_cg + m_wh = F_B$$

$$\rho_cV_cg + 1V_wg = 1\left[\frac{V}{2} + \frac{V_c}{2}\right]g$$

$$V_w = \frac{V}{2} + V_c\left[\frac{1}{2} - \rho_c\right]$$

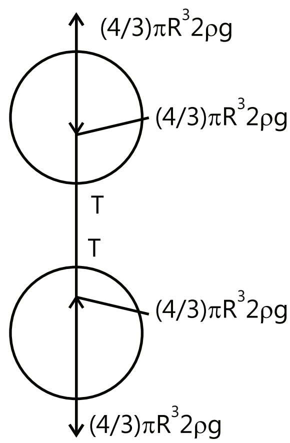
If $\rho_c < \frac{1}{2}$; $V_w > \frac{V}{2}$

Sol 13: (A, D) At equilibrium,

$$\frac{4}{3}\pi R^3\,2\rho g = \frac{4}{3}\pi R^3\rho g + T$$

$$T = \frac{4}{3}\pi R^3\ \rho\ g$$

$$\therefore \Delta\ \ell = \frac{4}{3k}\pi R^3\ \rho\ g$$



For equilibrium of the complete system, net force of buoyancy must be equal to the total weight of the sphere which holds true in the given problem. So both the spheres are completely submerged.

Sol 14: (C) By $A_1V_1 = A_2V_2$

$$\Rightarrow \pi\left(20\right)^2 \times 5 = \pi\left(1\right)^2 V_2 \Rightarrow V_2 = 2\ m/s^2$$

Sol 15: (A) $\frac{1}{2}\rho_a\ V_a^2 = \frac{1}{2}\rho_\ell V_\ell^2$

For given V_a

$$V_\ell \propto \sqrt{\frac{\rho_a}{\rho_\ell}}$$

Sol 16: (C) In P, Q, R no horizontal velocity is imparted to falling water, so d remains same.

In S, since its free fall, $a_{eff} = 0$

∴ Liquid won't fall with respect to lift.

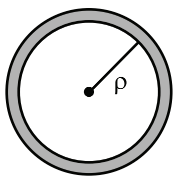
Sol 17: (A, D) From the given conditions,

$\rho_1 < \sigma_1 < \sigma_2 < \rho_2$

$$V_P = \frac{2}{9}\left(\frac{\rho_1 - \sigma_2}{\eta_2}\right)g \text{ and } V_Q = \frac{2}{9}\left(\frac{\rho_2 - \sigma_1}{\eta_1}\right)g$$

So, $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$ and $\vec{V}_P.\vec{V}_Q < 0$

Sol 18: (B, C) $P(r) = K\left(1 - \frac{r^2}{R^2}\right)$



Sol 19: Terminal velocity $v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho - \sigma)g$, where ρ is the density of the solid sphere and σ is the density of the liquid

$$\therefore \frac{v_P}{v_Q} = \frac{(8-0.8)\times\left(\frac{1}{2}\right)^2 \times 2}{(8-1.6)\times\left(\frac{1}{4}\right)^2 \times 3} = 3$$