Class X Session 2024-25
Subject - Mathematics (Basic)
Sample Question Paper - 7

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment carrying 04 marks each.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

Section A

1. Cards marked with numbers 2 to 101 are placed in a box and mixed thoroughly. One card is drawn from the box. The probability that the number on the card is a prime number less than 20 is **[1]**

a) $\frac{8}{25}$ b) $\frac{4}{25}$

c) $\frac{12}{25}$ d) $\frac{2}{25}$

2. The roots of the quadratic equation $\frac{x^2-8}{x^2+20}=\frac{1}{2}$ are **[1]**

a) ± 3 b) ± 4

c) ± 2 d) ± 6

3. A medicine capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each of its ends. The length of the entire capsule is 2 cm. The capacity of the capsule is **[1]**

a) 0.33 cm^3 b) 0.34 cm^3

c) 0.36 cm^3 d) 0.35 cm^3

4. The positive value of k for which the equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$ will both have real roots, is **[1]**

a) 12 b) 4

c) 8 d) 16

5. If the nth term of an A.P. is 2n + 1, then the sum of first n terms of the A.P. is **[1]**

a) n (n + 1) b) n (n – 2)

c) n (n – 1) d) n (n + 2)

6. If the point (x, 4) lies on a circle whose centre is at the origin and radius is 5 then x = **[1]**

a) 0

b) ± 3

c) ± 4

d) ± 5

7. If one zero of the polynomial $6x^2 + 37x - (k - 2)$ is reciprocal of the other, then what is the value of k? **[1]**

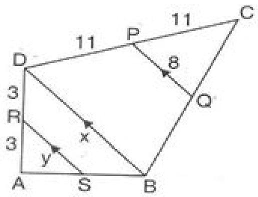
a) 6

b) -4

c) -6

d) 4

8. In the adjoining figures $RS || DB || PQ. If\ CP = PD = and\ DR = RA = 3. Then,$ **[1]**



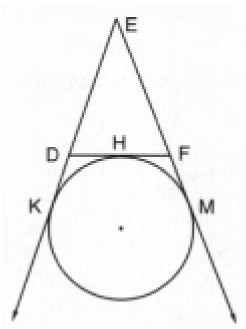
a) x = 14, y = 6.

b) x = 16, y = 8.

c) x = 10, y = 7.

d) x = 12, y = 10.

9. In Figure, a circle touches the side DF of $\triangle$EDF at H and touches ED and EF produced at K and M respectively. If EK = 9 cm, then the perimeter of $\triangle$EDF is **[1]**



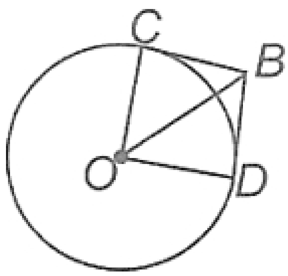
a) 18 cm

b) 13.5 cm

c) 9 cm

d) 12 cm

10. Two tangents BC and BD are drawn to a circle with centre O such that $\angle$CBD = 120^o. Then OB = **[1]**



a) 2BC

b) $\frac{BC}{2}$

c) BC

d) 3BC

11. $\sec^4 A - \sec^2 A$ is equal to **[1]**

a) $\tan^2 A - \tan^4 A$

b) $\tan^4 A - \tan^2 A$

c) $\tan^2 A + \tan^3 A$

d) $\tan^4 A + \tan^2 A$

12. $2 - \sqrt{3}$ is **[1]**

a) an irrational number
b) an integer
c) a rational number
d) a whole number

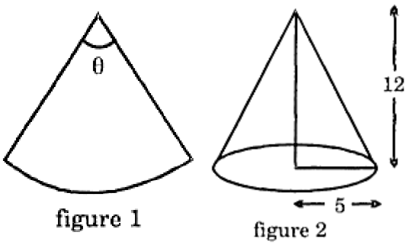
13. The angles of depression of two ships from the top of a lighthouse are 45° and 30° towards east. If the ships are 100 m apart, the height of the lighthouse is **[1]**

a) $50 (\sqrt{3} - 1)$ m
b) $50 (\sqrt{3} + 1)$ m
c) $\frac{50}{\sqrt{3}+1}$ m
d) $\frac{50}{\sqrt{3}-1}$ m

14. A chord of a circle of radius 10 cm subtends a right angle at the centre. The area of the minor segments (given, π = 3.14) is **[1]**

a) 32.5 cm^2
b) 34.5 cm^2
c) 30.5 cm^2
d) 28.5 cm^2

15. A piece of paper in the shape of a sector of a circle (see figure 1) is rolled up to form a right-circular cone (see figure 2). The value of angle θ is: **[1]**



a) $\frac{5\pi}{13}$
b) $\frac{6\pi}{13}$
c) $\frac{10\pi}{13}$
d) $\frac{9\pi}{13}$

16. In a family of 3 children, the probability of having at least one boy is **[1]**

a) $\frac{1}{8}$
b) $\frac{7}{8}$
c) $\frac{3}{4}$
d) $\frac{5}{8}$

17. Two dice are rolled together. What is the probability of getting a sum greater than 10? **[1]**

a) $\frac{5}{18}$
b) $\frac{1}{9}$
c) $\frac{1}{6}$
d) $\frac{1}{12}$

18. The marks obtained by 9 students in Mathematics are 59, 46, 30, 23, 27, 40, 52, 35 and 29. The median of the data is **[1]**

a) 29
b) 35
c) 40
d) 30

19. **Assertion (A):** Two identical solid cubes of side a are joined end to end. Then the total surface area of the resulting cuboid is 10 a^2. **[1]**

Reason (R): The total surface area of a cube having side a = 6 a^2.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The sum of series with the nth term t_n = (9 - 5n) is 220 when no. of terms n = 6. **[1]**

Reason (R): Sum of first n terms in an A.P. is given by the formula: Sn = 2 n $\times$ [2a + (n - 1)d]

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

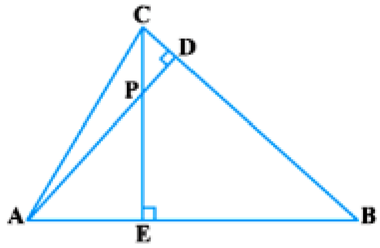
c) A is true but R is false.

d) A is false but R is true.

Section B

21. Find the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively. **[2]**

22. In the figure, altitudes AD and CE of $\triangle$ABC intersect each other at the point P. Show that: $\triangle AEP \sim \triangle ADB$ **[2]**



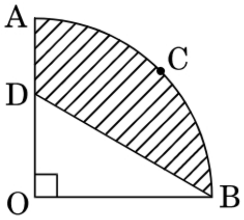
23. A circle touches all the four sides of a quadrilateral ABCD. Prove that AB + CD = BC + DA. **[2]**

24. If sin θ + cos θ = $\sqrt{3}$, then find the value of sin $\theta \cdot$ cos θ. **[2]**

OR

If tan (A + B) = $\sqrt{3}$ and tan (A - B) = 1, 0°< (A + B) < 90° and A > B then find A and B.

25. In Figure, OACB is a quadrant of a circle with centre O and radius 7 cm. If OD = 3 cm, then find the area of the shaded region. **[2]**



OR

What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 6π cm?

Section C

26. Mr. Patil has three classes. Each class has 28, 42 and 56 students respectively. Mr Patil wants to divide each class into groups so that every group in every class has the same number of students and there are no students left over. What is the maximum number of students Mr. Patil can put into each group? **[3]**

27. If P(2,-1), Q(3,4), R(-2,3) and S (-3,-2) be four points in a plane, show that PQRS is a rhombus but not a square. Find the area of the rhombus. **[3]**

28. Solve the quadratic equation by factorization: **[3]**

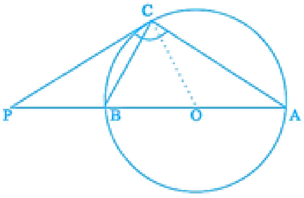
$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

OR

The hypotenuse of a grassy land in the shape of a right triangle is 1 metre more than twice the shortest side. If the third side is 7 metres more than the shortest side, find the sides of the grassy land.

29. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle$PCA = 110^{o}, find $\angle$CBA. **[3]**

[Hint: Join C with centre O].



OR

If ABC is isosceles with AB = AC, prove that the tangent at A to the circumcircle of ABC is parallel to BC.

30. Prove that: $\frac{1}{\operatorname{cosec} A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\operatorname{cosec} A+\cot A}$. **[3]**

31. The pilot of an aircraft flying horizontally at a speed of 1200 km/hr. observes that the angle of depression of a point on the ground changes from 30° to 45° in 15 seconds. Find the height at which the aircraft is flying. **[3]**

Section D

32. A train covered a certain distance at a uniform speed. If it were 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey. **[5]**

OR

A man travels 370 km, partly by train and partly by car. If he covers 250 km by train and the rest by car, it takes him 4 hours. But, if he travels 130 km by train and the rest by car, he takes 18 minutes longer. Find the speed of the train and that of the car.

33. The base BC of an equilateral triangle ABC lies on y-axis. The co-ordinates of point C are (0, - 3). The origin is the mid-point of the base. Find the co-ordinates of the point A and B. Also find the co-ordinates of another point D such that BACD is a rhombus. **[5]**

34. A solid is in the form of a right circular cone mounted on a hemisphere. The radius of the hemisphere is 3.5 cm and the height of the cone is 4 cm. The solid is placed in a cylindrical tub, full of water, in such a way that the whole solid is submerged in water. If the radius of the cylinder is 5 cm and its height is 10.5 cm, find the volume of water left in the cylindrical tub. (Use$\pi=\frac{22}{7}$) **[5]**

OR

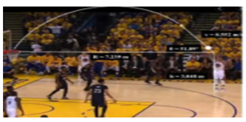
A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of cone is 4 cm and the diameter of the base is 8 cm. Determine the volume of the toy. If a cube circumscribes the toy, then find the difference of the volumes of cube and the toy. Also, find the total surface area of the toy.

35. The sum of the first 9 terms of an AP is 81 and that of its first 20 terms is 400. Find the first term and the common difference of the AP. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

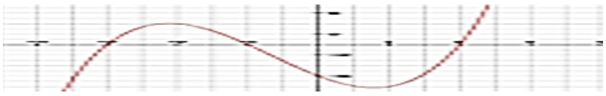
Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial.




i. Which type the shape of the path traced shown in given figure? (1)

ii. Why the graph of parabola opens upwards? (1)

iii. In the below graph, how many zeroes are there? (2)



OR

What is the condition for the graph of parabola to open downwards? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

India meteorological department observes seasonal and annual rainfall every year in different sub-divisions of our country.

It helps them to compare and analyse the results. The table given below shows sub-division wise seasonal (monsoon) rainfall (mm) in 2018:

Rainfall (mm)	Number of Sub-divisions
200-400	2
400-600	4
600-800	7
800-1000	4
1000-1200	2
1200-1400	3
1400-1600	1
1600-1800	1

i. Write the modal class. (1)

ii. Find the median of the given data. (1)

iii. If sub-division having at least 1000 mm rainfall during monsoon season, is considered good rainfall sub-division, then how many subdivisions had good rainfall? (2)

OR

Find the mean rainfall in this season. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Statue of a Pineapple: The Big Pineapple is a heritage-listed tourist attraction at Nambour Connection Road, Woombye, Sunshine Coast Region, Queensland, Australia. It was designed by Peddle Thorp and Harvey, Paul Luff, and Gary Smallcombe and Associates. It is also known as Sunshine Plantation. It was added to the Queensland Heritage Register on 6 March 2009.

Kavita last year visited Nambour and wanted to find the height of a statue of a pineapple. She measured the pineapple's shadow and her own shadow. Her height is 156 cm and casts a shadow of 39 cm. The length of shadow of pineapple is 4 m.

i. What is the height of the pineapple? (1)

ii. What is the height Kavita in metres? (1)

iii. Write the type of triangles used to solve this problem. (2)

OR

Which similarity criterion of triangle is used? (2)

Solution

Section A

1.

(d) $\frac{2}{25}$

Explanation: Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19 = 8

Number of possible outcomes = 8

Number of total outcomes = 100

$\therefore$ Required Probability = $\frac{8}{100} = \frac{2}{25}$

2.

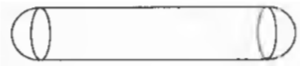
(d) ± 6

Explanation: We have, $\frac{x^2-8}{x^2+20}$ = $\frac{1}{2}$

$\Rightarrow$ $2x^2 - 16 = x^2 + 20 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$

3.

(c) 0.36 cm^3

Explanation: 

Radius of capsule = $\frac{0.5}{2}$ cm = 0.25 cm

Let the length of cylindrical part be x cm.

Then, 0.25 + x + 0.25 = 2 $\Rightarrow$ x + 0.5 = 2 $\Rightarrow$ x = 1.5 cm.

Capacity of the capsule = $\left(\frac{2}{3}\pi r^3 \times 2\right) + \pi r^2 h$

= $\frac{4}{3} \times \frac{22}{7} \times (0.25)^3$ + $\frac{22}{7} \times (0.25)^2 \times 1.5$

= $\left(\frac{4}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$ + $\left(\frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times \frac{15}{10}\right)$

= $\frac{11}{168} + \frac{33}{112} = \left(\frac{22+99}{336}\right)$ = $\frac{121}{336}$ = 0.36 cm^3

4.

(d) 16

Explanation: In the equation $x^2 + kx + 64 = 0$

a = 1, b = k, c = 64

D = b^2 - 4ac = k^2 - 4 $\times$ 1 $\times$ 64

= k^2 - 256

$\because$ The roots are real

$\therefore$ D $\geq$ 0 $\Rightarrow$ $k^2 \geq (\pm 16)2$

$\Rightarrow$ k $\geq$ 16 .....(i)

Only positive value is taken.

Now in second equation

x2 - 8x + k = 0

D =(-8)2 - 4 $\times$ 1 $\times$ k = 64 - 4k

$\because$ Roots are real

$\therefore D \geq 0 \Rightarrow 64 - 4k \geq 0 \Rightarrow 64 \geq 4k$

16 $\geq$ k ....(ii)

From (i) and

16 $\geq$ k $\geq$ 16 $\Rightarrow$ k = 16

5.

(d) n (n + 2)

Explanation: $a_n = 2n + 1$

a or $a_1 = 2 \times 1 + 2 = 2 + 1 = 3$

$a_2 = 2 \times 2 + 1 = 4 + 1 = 5$
$\therefore d = a_2 - a_1 = 5 - 3 = 2$
$\therefore \mathrm{S}_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[2 \times 3 + (n-1) \times 2]$
$= \frac{n}{2}[6 + 2n - 2] = \frac{n}{2}[2n + 4]$
= n(n + 2)

6.

(b) ± 3

Explanation: Point A(x, 4) is on a circle with centre O(0, 0) and radius = 5
$\therefore \mathrm{OA} = \sqrt{(x-0)^2 + (4-0)} = \sqrt{x^2 + 16}$
$\therefore \sqrt{x^2 + 16} = 5 \Rightarrow x^2 + 16 = 25$
Squaring on both sides, we get
$\Rightarrow x^2 = 25 - 16 = 9 = (\pm 3)^2$
$\therefore\ x = \pm 3$

7.

(b) -4

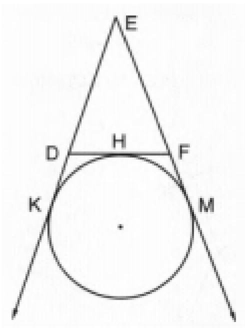
Explanation: Let one zero be x and other zero be $\frac{1}{x}$
$\therefore$ Product of zeroes = $\frac{c}{a}$
$\Rightarrow$ x $\times$ $\frac{1}{x}$ = $\frac{-(k-2)}{6}$
$\Rightarrow$ 1 = $\frac{2-k}{6}$
$\Rightarrow$ 6 = 2 - k
$\Rightarrow$ k = 2 - 6 = -4

8.

(b) x = 16, y = 8.

Explanation: In ΔPCQ$\sim\Delta$DCB (AA similarity) $\Rightarrow$ $\frac{11}{22} = \frac{8}{x}$ $\Rightarrow$ x = 16
In ΔSAR$\sim\Delta$BAD (AA similarity) $\Rightarrow$ $\frac{3}{6} = \frac{y}{16} \Rightarrow$ y = 8
or
mid-point Theorem

9. **(a)** 18 cm

Explanation: 

In $\triangle\mathrm{DEF}$
DF touches the circle at H
and circle touches ED and EF Produced at K and M respectively
EK = 9 cm
EK and EM are the tangents to the circle
EM = EK = 9 cm
Similarly DH and DK are the tangent
DH = DK and FH and FM are tangents
FH = FM
Now, perimeter of $\Delta\mathrm{DEF}$
= ED + DF + EF
= ED + DH + FH + EF
= ED + DK + FM + EF

= EK + EM
= 9 + 9
= 18 cm

10. **(a)** 2BC

Explanation: Since, tangents from an external point B to a circle are equally inclined to OB.

$\therefore \angle CBO = \frac{1}{2} \angle CBD = \frac{1}{2} \times 120^o = 60^o$

Also, $\angle OCB\ \ 90^o$ [$\because OC \perp CB$]

In $\triangle OCB$, $\frac{BC}{OB} = \cos 60^o = \frac{1}{2} \Rightarrow OB = 2\ BC$

11.

(d) $\tan^4 A + \tan^2 A$

Explanation: We have, $\sec^4 A - \sec^2 A = \sec^2 A (\sec^2 A - 1)$

$= (1 + \tan^2 A) \tan^2 A$

$= \tan^2 A + \tan^4 A$

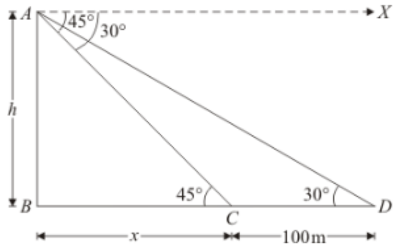
$= \tan^4 A + \tan^2 A$

12. **(a)** an irrational number

Explanation: Let $2 - \sqrt{3}$ be rational number
$2 - \sqrt{3} = \frac{p}{q}$ where p and q are composite numbers
$\sqrt{3} = \frac{p}{q} + 2$
$\sqrt{3} = \frac{(p+2q)}{q}$
since p, q are integers, so $\frac{(p+2q)}{q}$ is rational
$\therefore \sqrt{3}$ is an irrational number
it shows our supposition was wrong
hence $2-\sqrt{3}$ is an irrational number.

13.

(b) $50 (\sqrt{3} + 1)$ m
Explanation: Let AB = h be the lighthouse.
The given situation can be represented as,

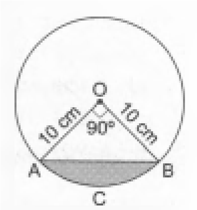


It is clear that $\angle C = 45^o$ and $\angle D = 30^o$
Again, let BC = x and CD = 100 m is given.
Here, we have to find the height of lighthouse.
So we use trigonometric ratios.
In a triangle ABC,
$\Rightarrow \tan C = \frac{AB}{BC}$
$\Rightarrow \tan 45^\circ = \frac{h}{x}$
$\Rightarrow 1 = \frac{h}{x}$
$\Rightarrow$ h = x
Again in a triangle *ABD*,
$\Rightarrow \tan D = \frac{AB}{BC+CD}$
$\Rightarrow \tan 30^\circ = \frac{h}{x+100}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+100}$
$\Rightarrow \sqrt{3}h = x + 100$
Put x = h

$\Rightarrow \sqrt{3}$ h = h + 100
$\Rightarrow$ h ($\sqrt{3}$ - 1) = 100
$\Rightarrow h = \frac{100}{\sqrt{3}-1}$
$\Rightarrow h = \frac{100}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
$\Rightarrow$ h = 50 ($\sqrt{3}$ + 1)

14.

(d) 28.5 cm^2
Explanation:
ar(minor segment A C B A)=ar(sector O A C B O) - ar(ΔOAB)
$= \left(\frac{\pi r^2 \theta}{360} - \frac{1}{2} \times r \times r\right)$

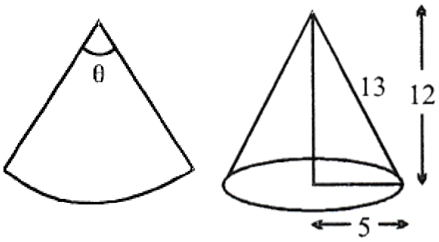


$= \left(\frac{3.14 \times 10 \times 10 \times 90}{360} - \frac{1}{2} \times 10 \times 10\right) \mathrm{cm}^2$
$= (78.5 - 50)\mathrm{cm}^2 = 28.5\mathrm{cm}^2$

15.

(c) $\frac{10\pi}{13}$

Explanation:



$\therefore$ Slant height = 13
As, $\theta = \frac{S}{r}$
$\Rightarrow$ S = rθ
$\Rightarrow$ 2π(5) = 13 θ
$\Rightarrow \theta = \frac{10\pi}{13}$

16.

(b) $\frac{7}{8}$
Explanation: All possible outcomes are BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG.
Number of all possible outcomes = 8.
Let E be the event of having at least one boy.
Then, E contains GGB, GBG, BGG, BBG, BGB, GBB, BBB.
Number of cases favourable to E = 7.
Therefore,required probability = P(E) =$\frac{7}{8}$

17.

(d) $\frac{1}{12}$
Explanation: Total number of outcomes =36
Favorable outcomes for sum greater than 10 are {(5,6),(6,5),(6,6)}
Number of favorable outcomes =3
P = $\frac{3}{36}$ = $\frac{1}{12}$

18.

(b) 35
Explanation: Arranging the given data in ascending order, we get
23, 27, 29, 30, 35, 40, 46, 52, 59

Here, n = 9, which is odd.

$\therefore$ Median = $\left(\frac{n+1}{2}\right)^{th}$ term

= $\left(\frac{9+1}{2}\right)^{th}$ term

= $\left(\frac{10}{2}\right)^{th}$ term

= 5th term

= 35

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. We have to find the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

Let assume that x be the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

So, it means

x divides 85 - 1 = 84

and

x divides 72 - 2 = 70

So, from this we concluded that

= x divides 84 and 70

= x = HCF (84, 70)

Now, to find HCF(84, 70), we use method of prime factorization.

Prime factors of 84 = 2 $\times$ 2 $\times$ 3 $\times$ 7

Prime factors of 70 = 2 $\times$ 5 $\times$ 7

So,

= HCF (84, 70) = 2 $\times$ 7 = 14

= x = 14

Hence, 14 is the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

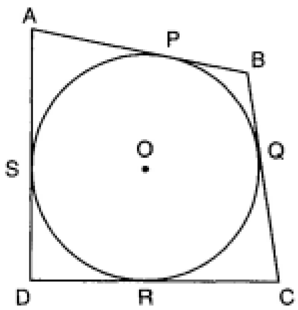
22. In $\triangle$AEP and $\triangle$ADB, we have

AEP= $\angle$ ADB ....(1) [Each equal to 90^0]

$\angle$EAP=$\angle$DAB ..... (2) [Common angle]

In view of (1) and (2),

$\triangle$AEP$\sim$$\triangle$ADB [AA similarity criterion]

23. 

Since tangents drawn from an exterior point to a circle are equal in length.

AP = AS [From A] ...(i)

BP = BQ [FromB] ...(ii)

CR = CQ [From C] ...(iii)

and, DR = DS [From D] ...(iv)

Adding (i), (ii), (iii) and (iv), we get

AP + BP + CR+DR = AS + BQ + CQ + DS

$\Rightarrow$ (AP + BP) + (CR+DR) = (AS + DS)+(BQ + CQ)

$\Rightarrow$ AB + CD = AD + BC

Hence, AB + CD = BC + DA

24. sin θ + cos θ = $\sqrt{3}$

squaring both sides

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow \sin\theta\cos\theta = 1$

OR

tan(A + B) = $\sqrt{3}$

tan (A + B) = tan60°

A + B = 60°. ...(i)

tan (A - B) = 1

tan (A- B) = tan 45°

A - B = 45°.........(ii)

Solving (i) and (ii), we get

A = (52.5)° and B = (7.5)°.

Hence, A = (52.5)° and B = (7.5)°.

25. Area of quadrant = $\frac{1}{4}\pi(7)^2 = \frac{49}{4}\pi$ cm^2

Area of triangle = $\frac{1}{2} \times 7 \times 3$ = $\frac{21}{2}$ cm^2

Area of shaded region = $\frac{49}{4}\pi - \frac{21}{2}$

= $\frac{7}{2}\left(\frac{7}{2}\pi - 3\right)$ cm^2 or 28 cm^2

OR

l = 6π, r = 6 cm,

$l = \frac{\theta\pi r}{180^\circ}$

$\Rightarrow 6\pi = \frac{\theta\times\pi\times6}{180^\circ}$

$\Rightarrow \theta = 180^\circ$

Section C

26. For maximum number of students to put into each group

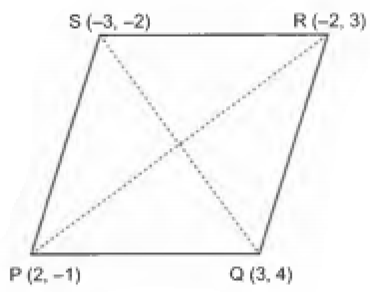
Mr patil sir should have to take H.C.F of 28, 42 and 56

so

maximum number of students Mr. Patil can put into each group is 14.

27. The given points are P (2, -1), Q (3,4), R(-2,3) and S (-3, -2).

We have,



$PQ = \sqrt{(3-2)^2+(4+1)^2} = \sqrt{1^2+5^2} = \sqrt{26}$ units

$QR = \sqrt{(-2-3)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$ units

$RS = \sqrt{(-3+2)^2+(-2-3)^2} = \sqrt{1+25} = \sqrt{26}$ units

$SP = \sqrt{(-3-2)^2+(-2+1)^2} = \sqrt{26}$units

and $QS = \sqrt{(-3-3)^2+(-2-4)^2} = \sqrt{36+36} = 6\sqrt{2}$ units

$\therefore$ PQ = QR = RS = SP = $\sqrt{26}$ units

$PR = \sqrt{(-2-2)^2+(3+1)^2} = \sqrt{16+16} = \sqrt{32}$

Therefore, $PR \neq QS$

This means that PQRS is a quadrilateral whose sides are equal but diagonals are not equal.

Thus, PQRS is a rhombus but not a square.

Now, Area of rhombus PQRS $= \frac{1}{2} \times$ (Product of lengths of diagonals)

$= \frac{1}{2} \times (PR \times QS) = \left(\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}\right)$ sq. units = 24 sq. units

28. Consider $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\Rightarrow$ 2ab (2x - 2a – b – 2x) = (2a + b)2x (2a + b + 2x)

$\Rightarrow$ 2ab (-2a - b) = 2(2a+ b) (2ax + bx + $2x^2$)

$\Rightarrow$ -ab = 2ax + bx + $2x^2$

$\Rightarrow$ $2x^2$ + 2ax + bx + ab = 0

$\Rightarrow$ 2x (x + a) + b (x + a) = 0

$\Rightarrow$ (2x + b) (x + a) = 0

$\Rightarrow \mathrm{x} = -\mathrm{a}, -\frac{\mathrm{b}}{2}$

Hence the roots are $-a, -\frac{b}{2}$.

OR

Let the length of the shortest side be x metres.

As per given condition

The hypotenuse of a grassy land in the shape of a right triangle is 1 metre more than twice the shortest side.

So, Hypotenuse = (2x + 1) metres

And if the third side is 7 metres more than the shortest side

So, third side = (x + 7) metres.

By Pythagoras theorem, we have

$(\text{Hypotenuse})^2$ = Sum of the squares of the remaining two sides

$\Rightarrow$ $(2x + 1)^2 = x^2 + (x + 7)^2$

$\Rightarrow$ $4x^2 + 4x + 1 = x^2 + x^2 + 14x + 49$

$\Rightarrow$ $4x^2 + 4x + 1 = 2x^2 + 14x + 49$

$\Rightarrow$ $2x^2 - 10x - 48 = 0$

$\Rightarrow$ $x^2 - 5x - 24 = 0$

$\Rightarrow$ $x^2 - 8x + 3x - 24 = 0$

$\Rightarrow x(x-8) + 3(x-8) = 0$

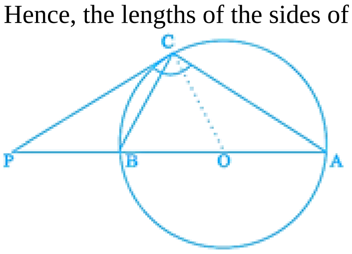
$\Rightarrow$ (x - 8) (x + 3) = 0

$\Rightarrow$ x = 8 , - 3

$\Rightarrow$x = 8 [$\because$ x = -3 is not possible]

Hence, the lengths of the sides of the grassy land are 8 metres, 17 metres and 15 metres.

29.



Let D be the centre of the circle.

A, D, B, P all are on the same line and P and C are points on the tangent.

Now, $\angle$BCA is inscribed in a semi-circle, $\angle BCA = 90^\circ$

C is the point on the circle where the tangent touches the circle.

So, $\angle \mathrm{DCP} = 90^\circ$

$\angle \mathrm{PCA} = \angle \mathrm{PCD} + \angle \mathrm{DCA}$

$\Rightarrow$ $110^o = 90^o + \angle \mathrm{DCA}$

$\Rightarrow \angle DCA = 20^\circ$

In $\triangle$ADC,

AD = DC .... (Radii of the same circle)

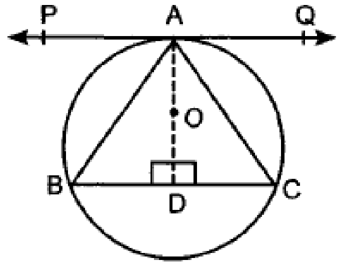
$\Rightarrow \angle DCA = \angle CAD = 20^\circ$

In $\triangle$ABC,

$\angle \mathrm{BCA} = 90^{\circ}, \angle \mathrm{CAB} = 20^{\circ}$
So, $\angle CBA = 70^{\circ}$

OR

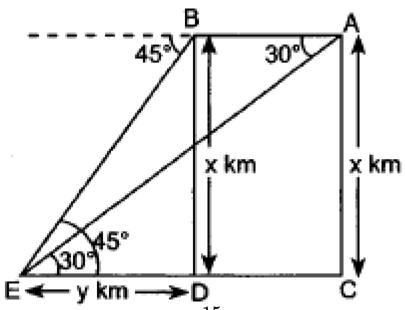
Draw AD $\perp$ BC



In $ADB\,and\,ADC$
$AB = AC$ [Given]
$AD = AD$
$ADB = ADC[Each\,90^{o}]$
$\therefore \triangle \mathrm{ADB} \cong \triangle \mathrm{ADC}$
$\Rightarrow BD = CD$
$\therefore$ AC passes through O,centre of the circle
$\therefore$ Perpendicular bisector of the chord passes through the centre of the circle
Now OA $\perp$ PQ (radius through the point of contact)
$\therefore$ $\angle$PAO = 90^{o}
Also $ADB = 90^{o}$
$\therefore \angle \mathrm{PAO} + \angle \mathrm{ADB} = 180^{\circ}$
$\therefore$ AP||BC

30. We have,
$\Rightarrow \frac{1}{\operatorname{cosec} A - \cot A} - \frac{1}{\sin A} = \frac{1}{\sin A} - \frac{1}{\operatorname{cosec} A + \cot A}$
$\Rightarrow \frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{1}{\sin A} + \frac{1}{\sin A}$
$\Rightarrow \frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A} = \frac{2}{\sin A}$
LHS = $\frac{1}{\operatorname{cosec} A - \cot A} + \frac{1}{\operatorname{cosec} A + \cot A}$
$\Rightarrow \frac{\operatorname{cosec} A + \cot A + \operatorname{cosec} A - \cot A}{(\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)}$
$\Rightarrow \frac{2\operatorname{cosec} A}{\operatorname{cosec}^2 A - \cot^2 A}$
$\Rightarrow \frac{\frac{2}{\sin A}}{1} = \frac{2}{\sin A}$ = RHS.
Hence Proved.

31. Distance covered in 15 seconds $= AB$
Speed $= 1200\ km/hr.$



$\therefore \mathrm{AB} = 1200 \times \frac{15}{3600} = 5\mathrm{km}$
$AB = DC = 5km$
Let height = x km
In rt. $\triangle$BDE,
$\frac{BD}{ED} = \tan 45^{\circ} \Rightarrow \frac{x}{y} = 1 \Rightarrow x = y$
In rt. $\triangle$ACE,
$\frac{AC}{EC} = \tan 30^{\circ} \Rightarrow \frac{x}{y+5} = \frac{1}{\sqrt{3}}$
$\Rightarrow \quad \frac{x}{x+5} = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \sqrt{3}x = x + 5 \Rightarrow (\sqrt{3} - 1)x = 5$

$\therefore \quad x = \frac{5}{\sqrt{3}-1} = \frac{5(\sqrt{3}+1)}{2} = 6.83\text{km}$

Section D

32. Let the actual speed of the train be x km/hr and the actual time taken be y hours. Then,

Distance covered = (xy) km ... (i) [$\because$ Distance = Speed $\times$ Time]

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e., when speed is (x + 6) km/hr, time of journey is (y - 4) hours.

$\therefore$ Distance covered = (x + 6) (y - 4)

$\Rightarrow$ xy = (x + 6) (y - 4) [Using (i)]

$\Rightarrow$ -4x + 6y - 24 = 0

$\Rightarrow$ -2.x + 3y - 12 = 0 ... (ii)

When the speed is reduced by 6 km/hr, then the time of journey is increased by 6 hours i.e., when speed is (x - 6) km/hr, time of journey is (y + 6) hours.

$\therefore$ Distance covered = (x - 6) (y + 6)

$\Rightarrow$ xy = (x - 6) (y + 6) [Using (i)]

$\Rightarrow$ 6x - 6y - 36 = 0

$\Rightarrow$ x - y - 6 = 0 ... (iii)

Thus, we obtain the following system of equations:

- 2x + 3y - 12 = 0

x-y- 6 = 0

By using cross-multiplication, we have,

$\frac{x}{3\times-6-(-1)\times-12} = \frac{-y}{-2\times-6-1\times-12} = \frac{1}{-2\times-1-1\times3}$

$\Rightarrow \frac{x}{-30} = \frac{-y}{24} = \frac{1}{-1}$

$\Rightarrow$ x = 30 and y = 24.

Putting the values of x and y in equation (i), we obtain

Distance = (30 $\times$ 24) km = 720 km.

Hence, the length of the journey is 720 km.

OR

Let the speed of the train be x km/hr and that of the car be y km/hr.

Case I Distance covered by train = 250 km.

Distance covered by car = (370 - 250) km = 120 km.

Time taken to cover 250 km by train = $\frac{250}{x}$ hours

Time taken to cover 120 km by car = $\frac{120}{y}$hours

Total time taken =4 hours

$\therefore \quad \frac{250}{x} + \frac{120}{y} = 4 \Rightarrow \frac{125}{x} + \frac{60}{y} = 2$ ....(i)

Case II Distance covered by train = 130 km.

Distance covered by car = (370 -130) km = 240 km.

Time taken to cover 130 km by train = $\frac{130}{x}$ hours

Time taken to cover 240 km by car =$\frac{240}{y}$ hours

Total time taken = $4\frac{18}{60}$ hours $= 4\frac{3}{10}$ hours $= \frac{43}{10}$ hours

$\therefore \quad \frac{130}{x} + \frac{240}{y} = \frac{43}{10} \Rightarrow \frac{1300}{x} + \frac{2400}{y} = 43$ ......(ii)

Putting $\frac{1}{x}$ = u and $\frac{1}{y}$=v, equations (i) and (ii) become

$125u + 60v = 2$ ... (iii) and $1300u + 2400v = 43$ . ... (iv)

On multiplying (iii) by 40 and subtracting (iv) from the result, we get

5000u - 1300v = 80 - 43 $\Rightarrow$ 3700u = 37

$\Rightarrow u = \frac{37}{3700} = \frac{1}{100} \Rightarrow \frac{1}{x} = \frac{1}{100} \Rightarrow x = 100$

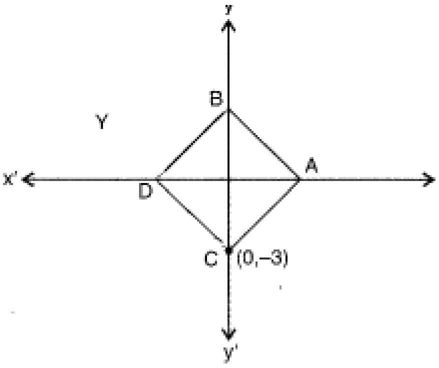
Putting u = $\frac{1}{100}$ in (iv), we get

$\left(1300 \times \frac{1}{100}\right) + 2400v = 43 \Rightarrow 2400v = 43 - 13 = 30$

$\Rightarrow v = \frac{30}{2400} = \frac{1}{80} \Rightarrow \frac{1}{y} = \frac{1}{80} \Rightarrow y = 80$

$\therefore$ x = 100 and y = 80.

Hence, the speed of the train is 100 km/hr and that of the car is 80 km/hr

33. 

Co-ordinates of point B are (0,3)

$\therefore$BC = 6 unit

Let the co-ordinates of point A be (x, 0)

or, AB = $\sqrt{x^2+9}$

$\because$ AB = BC

$\therefore$ x^2 + 9 = 36

or, x^2 = 27 or, x = $\pm 3\sqrt{3}$

Co-ordinates of point A = ($3\sqrt{3}$,0)

Since ABCD is a rhombus

or, AB = AC = CD = DB

$\therefore$ Co-ordinate of point D = ($-3\sqrt{3}$,0)

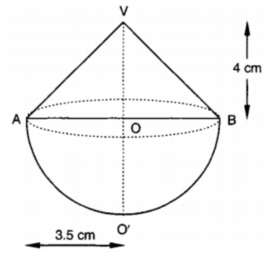
34. We have, radius of the hemisphere = 3.5 cm

Height of the cone = 4 cm

Radius of the cylinder = 5 cm

|Height of the cylinder = 10.5 cm

We have to find out the volume of water left in the cylindrical tub



$\therefore$ Volume of the solid = Volume of its conical part + Volume of its hemispherical part

$= \left\{\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 4 + \frac{2}{3} \times \frac{22}{7} \times (3.5)^3\right\} \mathrm{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times (3.5)^2\{4 + 2 \times 3.5\}\mathrm{cm}^3 = \left\{\frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \mathrm{cm}^3$

Clearly, when the solid is submerged in the cylindrical tub the volume of water that flows out of the cylinder is equal to the volume of the solid.

Hence,

Volume of water left in the cylinder = Volume of cylinder - Volume of the solid

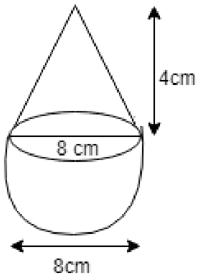
$= \left\{\frac{22}{7} \times (5)^2 \times 10.5 - \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 11\right\} \mathrm{cm}^3$

$= \left\{\frac{22}{7} \times 25 \times \frac{21}{2} - \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 11\right\} \mathrm{cm}^3$

$= \left(11 \times 25 \times 3 - \frac{1}{3} \times 11 \times \frac{7}{2} \times 11\right) \mathrm{cm}^3$

= (825 - 141.16) cm^3 = 683.83 cm^3

OR



Volume of toy = volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4(4 + 2 \times 4)$

= 201.14cm^3

If a cube circumscribes the toy then,

Volume of cubi = $(side)^3$

Volume = 512cm^3

Difference of the volume of cube and toy

= 512 - 201.14

= 310.86 cm^3

Total surface Area of toy = Curved surface area of cone + Curved surface area of hemisphere

l = $\sqrt{h^2 + r^2}$

l = $\sqrt{4^2 + 4^2}$

l = $\sqrt{32}$

l = $4\sqrt{2}$

l = 5.64 cm

Total surface area of toy = πrl + $2\pi r^2$

= πr(l + 2r)

$= \frac{22}{7} \times 4\,(5.64 + 2 \times 4)$

Total surface area of toy = 171.47 cm^2

35. Let a be the First term and d be the common difference of given AP.

Then, we have

$\Rightarrow \frac{9}{2}[2a + 8d] = 81$

$\Rightarrow \frac{9 \times 2}{2}[a + 4d] = 81$

$\Rightarrow$a+4d=9...(i)

Also, S_{20}=400

$\Rightarrow \frac{20}{2}[2a + 19d] = 400$

$\Rightarrow$10[2a+19d]=400

$\Rightarrow$2a+19d=40...(ii)

Multiplying equation (i) by 2, we get

2a+8d=18 ....(iii)

Subtracting (iii) from (ii), we get

11d=22

$\Rightarrow$d=2

$\Rightarrow$a=9-4(2)=9-8=1

Thus, the first term is 1 and the common difference is 2.

Section E

36. i. Parabola

ii. a > 0

iii. $\because$ The graph cut the x-axis thrice

$\therefore$ No of zeroes = 3

OR

a < 0

37. i. Modal Class is 600 - 800

ii. $\frac{N}{2}$ = 12, median class is 600 - 800

Rainfall	x_i	f_i	**ef.**
200 - 400	300	2	2
400 - 600	500	4	6
600 - 800	700	7	13
800 - 1000	900	4	17
1000 - 1200	1100	2	19
1200 - 1400	1300	3	22
1400 - 1600	1500	1	23
1600 - 1800	1700	1	24
		24	

Median = 600 + $\frac{200}{7}$(12 - 6)

= $\frac{5400}{7}$ or 771.4

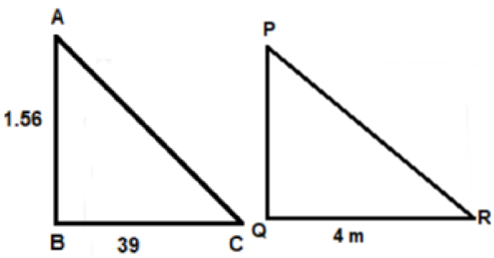
iii. Sub-divisions having good rainfall = 2 + 3 + 1 + 1 = 7.

OR

Rainfall	x_i	f_i	$f_i x_i$
200 - 400	300	2	600
400 - 600	500	4	2000
600 - 800	700	7	4900
800 - 1000	900	4	3600
1000 - 1200	1100	2	2200
1200 - 1400	1300	3	3900
1400 - 1600	1500	1	1500
1600 - 1800	1700	1	1700
		24	20400

Mean = $\frac{20400}{24}$ = 850

38. i.



$\triangle ABC \sim \triangle PQR$

$\frac{1.56}{0.39} = \frac{PQ}{4}$

$\frac{1.56 \times 4}{0.39}$ = PQ

PQ = 16 m

$\therefore$ height of Pine apple = 16 m.

ii. Height of Kavita = 1.56 m

iii. Right triangle

OR

AA criteria