Chapter -3

PROPERTIES OF A TRIANGLE

You know that a figure enclosed by three lines, having three sides and three angles is known as a triangle. The sum of three angles of a triangle is always 180°. We have learnt to classify triangles on the basis of their angles and their sides. Let us first revise those properties of a triangle that we know:-

Opposite sides and opposite angles

See the triangle given below:-

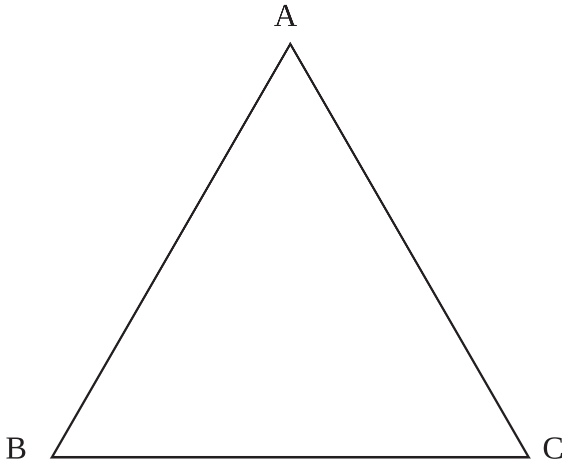


Figure 3.1

Here the opposite angle of sides AB is $\angle C$.

This angle is not located at the end points of AB (i.e. A and B). Just as the opposite angle of side AB is $\angle C$, similarly, the opposite side of $\angle C$ is AB.

In the same way, write two other pairs of opposite sides and opposite angles.

Have you ever thought of a relation between angles and their opposite sides and vice-versa? Let us find a relation between them with the help of an activity.

Activity 1

Given are some triangles with different measurements are given. Measure the lengths of the sides and angles opposite to them in the triangles. Fill them in the given table as shown.

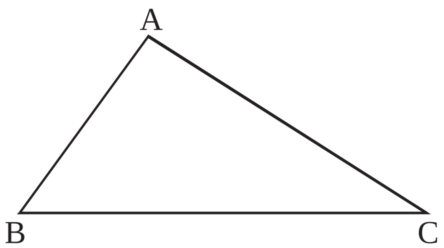


Figure 3.2

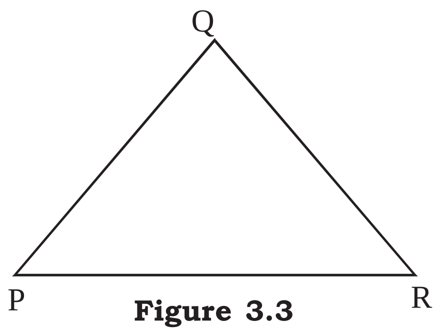


Figure 3.3

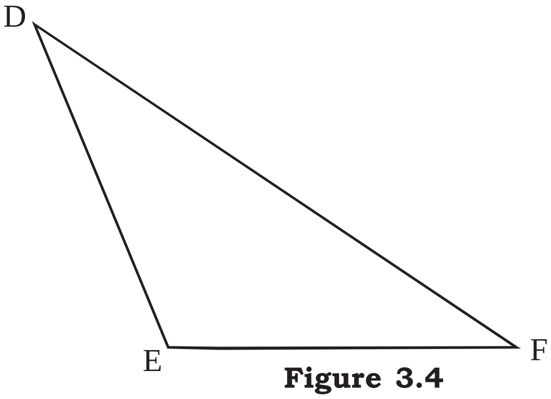


Figure 3.4

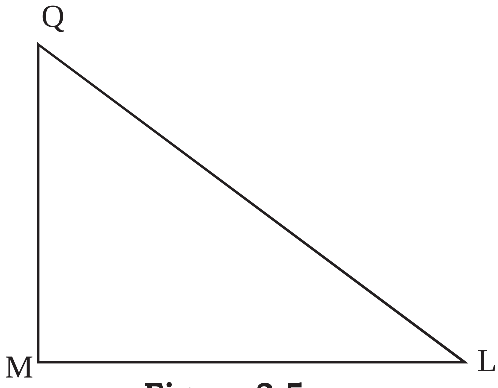


Figure 3.5

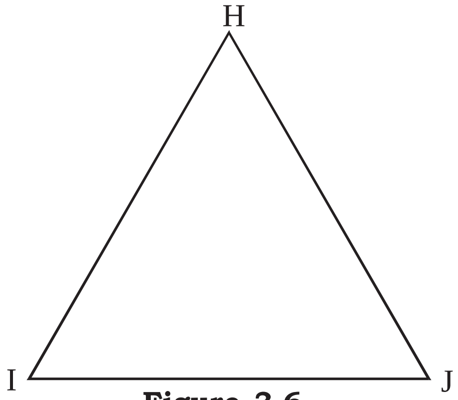


Figure 3.6

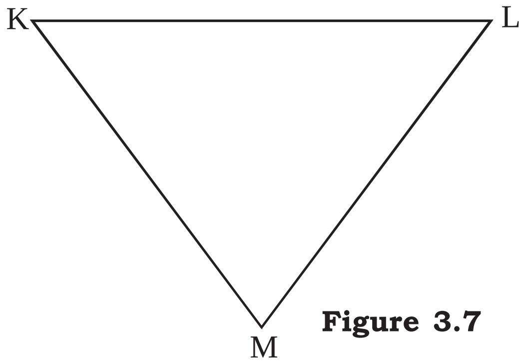


Figure 3.7

Table 1

Fig. No.	**Name of Triangle**	**Measure of sides** Write the length of the sides of fig. 3.2	**Measure of the opposite angle**	**Sides in descending order**	**Angles in descending order**
3.2	Δ ABC	AB = 2.9 CM BC = 5.4 CM CA = 4.4 CM	$\angle C = 30^0$ $\angle A = 95^0$ $\angle B = 55^0$	BC, CA, AB	$\angle A$, $\angle B$, $\angle C$
3.3	Δ PQR	-------------	-------------	----------------------	---------------------------
3.4	Δ DEF	-------------	-------------	----------------------	---------------------------
3.5	Δ QLM	-------------	-------------	----------------------	---------------------------
3.6	Δ HIJ	-------------	-------------	----------------------	---------------------------
3.7	Δ KLM	-------------	-------------	----------------------	---------------------------

Observe the above table, and answer the following questions:-

(i) Is the angle opposite to the longest side, the largest?

(ii) Is the angle opposite to the smallest side, the smallest?

(iii) Is the side opposite to the largest angle, the largest?

(iv) Is the side opposite to the smallest angle, the smallest?

(v) In figure 3.6, are the angles opposite to the two equal sides, equal?

(vi) In figure 3.7, what is the relation between the sides and their respective opposite angles?

You find that the largest side has the largest angle opposite to it and vice versa. Similarly, the smallest angle has the smallest side opposite to it and vice versa.

In ΔHIJ (figure 3.6) equal sides have equal angles opposite to them. In the same manner sides opposite to equal angles are also equal. In figure 3.7, all three sides are of equal lengths and have equal opposite angles. Does this mean opposite sides to the equal opposite angles are always equal in a triangle?

Draw two equilateral and two isosceles triangles of different measures and check the relation and between their sides and their angles.

Examples 1: An isosceles triangle has one angle of 80°. Determine the other two equal angles?

Solution: Isosceles triangle is a triangle in which two sides are of equal length this means, these equal sides has equal angles opposite to them. Let each angle be x.

We known, that sum of three angles of a triangle is 180°.

$\Rightarrow \quad x + x + 80° = 180°$

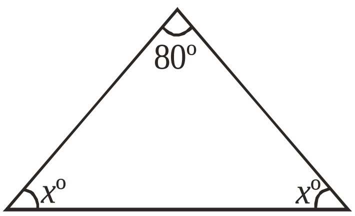
$\Rightarrow \quad 2x + 80° = 180°$

$\Rightarrow \quad 2x = 180° - 80°$

$\Rightarrow \quad 2x = 100°$

$\Rightarrow \quad x = \dfrac{100°}{2}$

$\Rightarrow \quad x = 50°$



⇒ The two angles are of 50° each.

Examples 2: Determine the angles of an equilateral triangle.

Solution: We know that in an equilateral triangle, all three angles are equal. Let each angle be x°. The sum of the three angles of the triangle is 180°.

$\Rightarrow \quad x° + x° + x° = 180°$

$\Rightarrow \quad x° = \dfrac{180°}{3}$

$\Rightarrow \quad x = 60°$

⇒ In an equilateral triangle, each angle is of 60°.

Example 3: Determine all angles of the triangle given below.

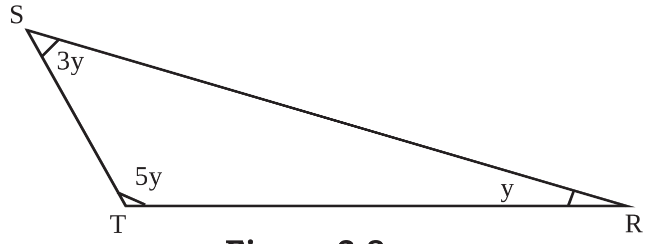


Figure 3.8

Solution: We know that the sum of three angles of triangle is 180°.

∴ In Δ RST, ∠R+ ∠S+ ∠T=180°

⇒ $y + 3y + 5y = 180°$

⇒ $9y = 180°$

⇒ $y = \frac{180°}{9}$

⇒ $y = 20°$

⇒ ∠R = 20°, ∠S = 3×20°= 60°, ∠T = 5 × 20= 100°.

⇒ The three angle of the given triangle are 20°, 60° and 100°.

EXERCISE 3.1

Q1. Fill in the blanks:

(i) In a triangle, equal sides have .... opposite angles.

(ii) If a triangle has two equal angles, then it is called..........

(iii) Equilateral triangle has three ....angles.

(iv) In an isosceles triangle, if one angle is 100°, then the other two equal angles are ......

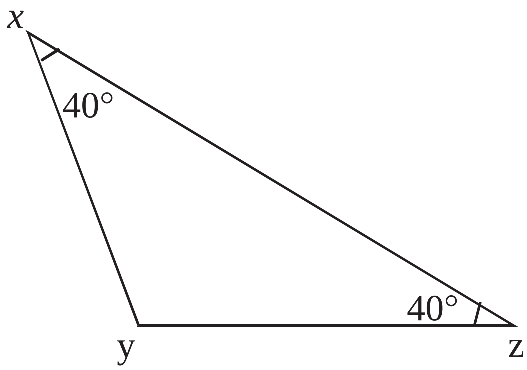
(v) The largest angle in a triangle has the .... opposite side.

(vi) The smallest angle in a triangle has the .... side opposite of it.

Q2. Complete the table given below:-

S. No.	Name of Δ	Length of sides	Angles	Remaining angles
1.	Δ ABC	AB=AC= 4 cm, BC = 5 cm	∠B = 50°	∠C = ----------, ∠A = ------
2.	Δ PQR	PQ=PR= 5 cm, QR = 7 cm	∠R = --------	∠P = -----------, ∠Q = 45°
3.	Δ DEF	DE=DF= 6 cm, FE = 8 cm	∠E = ----------	∠D = 84°, ∠F = ---------
4.	Δ LMN	LM = MN = NL = 5 cm	∠L = ----------	∠M = -------, ∠N = --------

Q3. In the ΔXYZ, given below, write the name of equal sides and determine the third angle ?

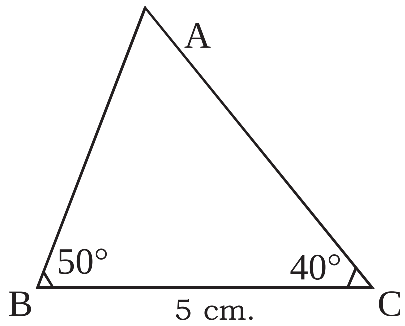


Q4. In the ΔABC given below $BC=5cm$, $\angle C = 40°$ and $\angle B = 50°$, then answer the following questions

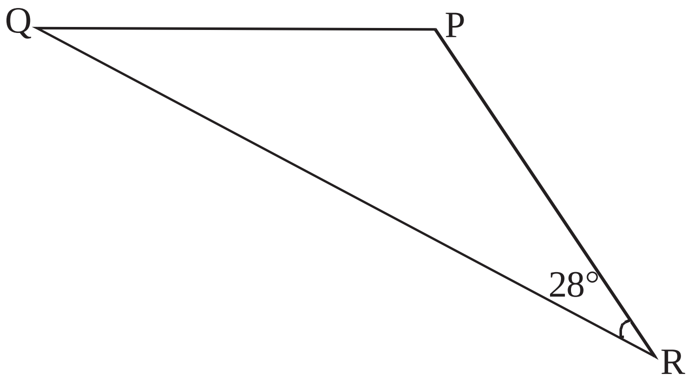
(i) Is AB=AC ? If not, why ?

(ii) Which one is longer side, AB or AC ?

(iii) The largest side is opposite to the smallest angle or to the largest angle?



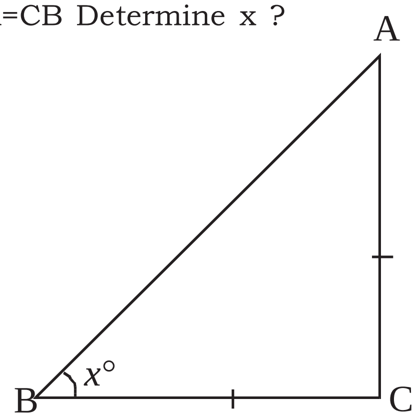
Q5. If ΔPQR has $PQ=PR$ and $\angle R=28°$. Then calculate the remaining angles.



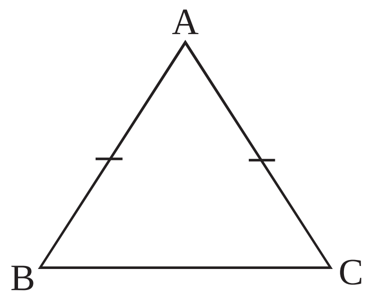
Q6. A triangle has two sides of equal length. If the angles opposite of them are 30°, then determine the third angle ?

Q7. In an isosceles triangle, if the angle at the vertex is 70°, then determine the other two angles opposite to the equal sides ?

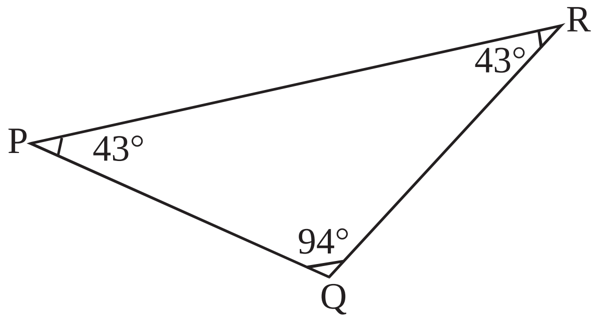
Q8. ΔABC is a right angled triangle with $C=90°$ and $CA=CB$ Determine x ?



Q9. In the figure given below, AB=AC and if ∠B is twice of ∠A, then determine all three angles of the triangle?



Q10. In figure, ΔPQR is given with its three angles. Which are the two equal sides of the triangle? Name the largest side ?



Q11. If three angles of a triangle are in the ratio 2:3:4, find all its three angles ?

MEDIANS OF A TRIANGLE

Cut a triangle from a piece of paper, fold it to keep any two corners of it one over the other. In this way, you have divided one side of the triangle into two equal parts. Label the point from where the side has been folded.

Mark the mid points of all the three sides. Now join all the mid point to the vertices and get the intersection points (fig. 3.9(iii)). Check whether all the points of intersection coincide at the same point? In a triangle, the line segments joining the vertices to the mid point of the opposite sides, are called medians.

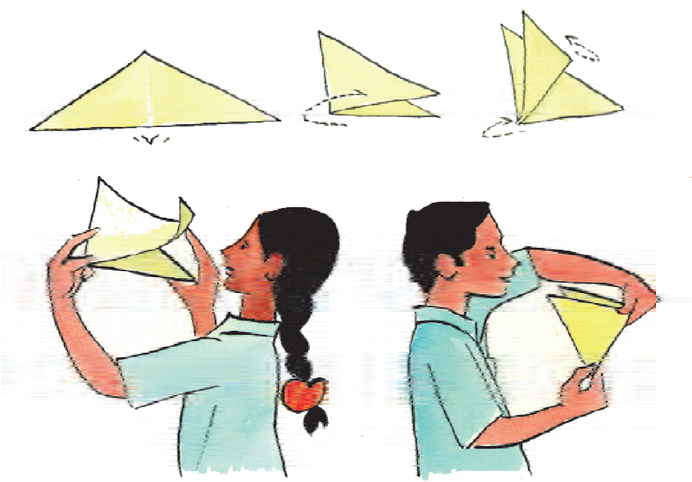


Figure 3.9

You got the mid points of the sides of the triangle made of paper but how will you find out the mid points of a triangle drawn on paper. In class VI, you have learnt to draw perpendicular bisector of a line. Can you find out the mid point of AB? Let us see.

Firstly, open the rounder to more than half of the given line segment AB, place the rounder needle on A and put an arc above and below AB, as shown in figure 3.10 by M and L. Then put the rounder needle on B, and with the same radius mark arcs P and Q above and below AB intersecting the previous arcs.

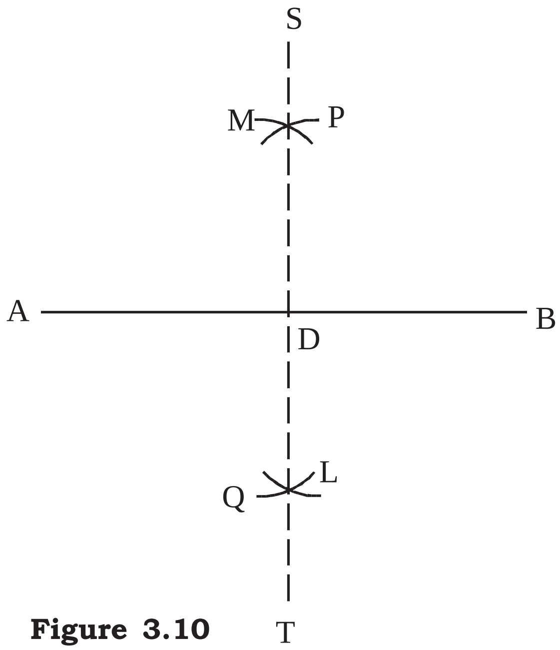


Figure 3.10

Now draw a line joining the points of intersection. This is the perpendicular bisector ST and cuts AB at D. D is the mid point of AB. Similarly, we can get the mid points of the remaining sides of the triangle.

In figure 3.11, ΔABC is given and D is the mid point of BC. A is joined to the mid point D. AD is a median of ΔABC.

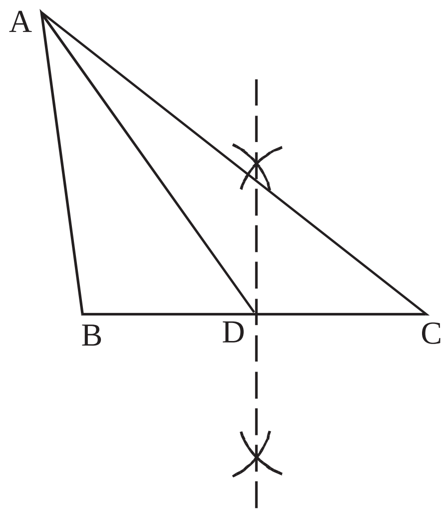


Figure 3.11

In this way draw the three medians of the triangle.

Let us see how we can do this. E and D are mid points of AC and BC respectively. Join these mid points E and D to the vertices B and A respectively.

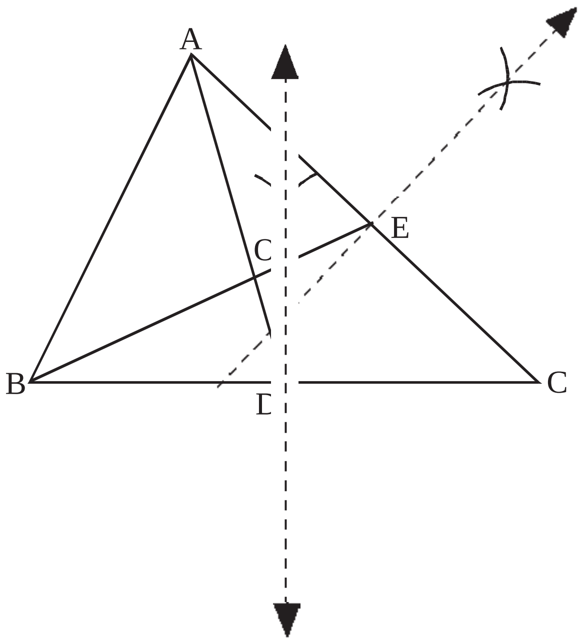


Figure 3.12

The medians AD and BE intersect at the point O. Now you draw a line from C through O and extend it to the point on the side AB. Is that the mid point of AB or not? Does any other median of a triangle exist? You will see that all the three medians of triangle pass through same point or we can say, "The medians of a triangle coincide at a point". The intersecting point of medians is known as **centroid**. O is the centroid of ΔABC.

Now, draw a triangle in your copy and draw its medians to get the centroid. We have seen that if we draw any two medians of a triangle, then the third median also passes through the point where they intersect. So we can say that to find the centroid, only two medians of triangle are required.

Let us learn some more about medians of a triangle.

Activity 2

Some triangles are given below and their medians are drawn. Fill up the blanks in the table accordingly.

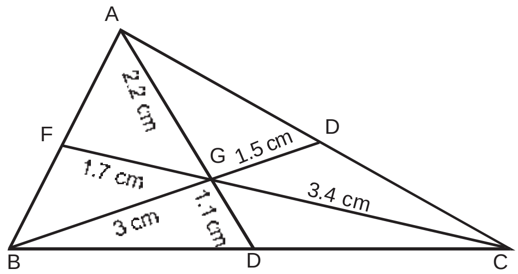


Figure 3.13

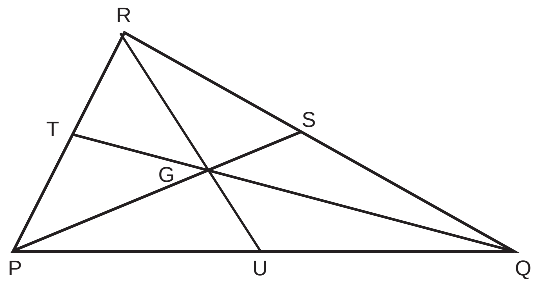


Figure 3.14

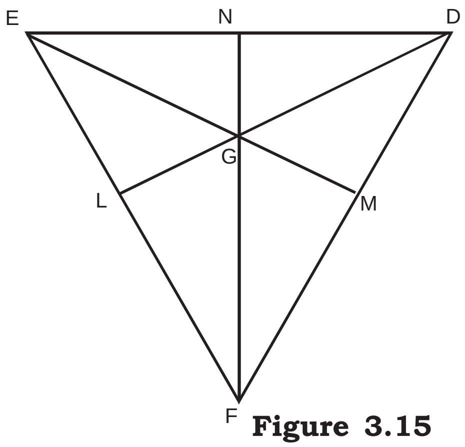


Figure 3.15

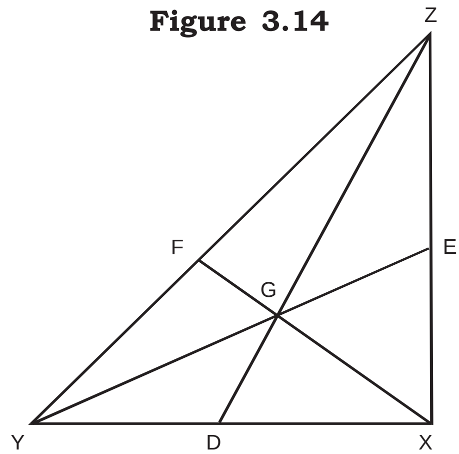


Figure 3.16

Fig.	Name	Distance of centroid G from the vertex	Distance of mid point of opposite side	Ratio
3.13	Δ ABC	AG = 2.2 cm	GD = 1.1 cm	AG = GD 2 : 1
		BG = 3 cm	GE = 1.5 cm	BG = GE 2 : 1
		CG = 3.4 cm	GF = 1.7 cm	CG = GF 2 : 1
3.14	Δ PQR	PG = ----------------	GS = ----------------	PG :GS = ---------------------
		QG = ----------------	GT = ----------------	QG :GT = ---------------------
		RG = ----------------	GU = ----------------	RG :GU = ---------------------
3.15	Δ DEF	DG = ----------------	GL = ----------------	DG :GL = ---------------------
		EG = ----------------	GM = ----------------	EG :GM = ---------------------
		FG = ----------------	GN = ----------------	FG :GN = ---------------------
3.16	Δ XYZ	XG = ----------------	GF = ----------------	XG :GF = ---------------------
		YG = ----------------	GE = ----------------	YG :GE = ---------------------
		ZG = ----------------	GD = ----------------	ZG :GD = -----------------

From the above table, find out the ratio of the line segments from the vertices to the centroid and from the centroid to the respective mid points? Is the ratio same for all the ratios in a triangle? Is it the same for all triangles? You will find out that the ratio remains 2:1 for all triangles and for all ratios in a triangle. Draw more triangles of different measurements in your copy. Construct their medians and find out whether the centroid divides all the medians in the ratio 2:1.

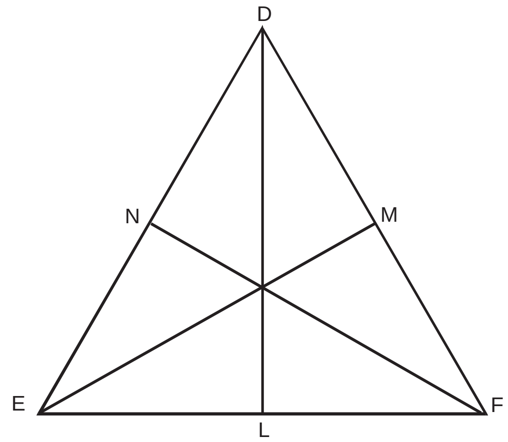


Figure 3.17

Now, consider an equilateral triangle DEF whose medians are DL, EM and FN respectively. Measure the medians and consider the relation between them. Also find out the relation between the angles made with the respective sides by the medians of an equilateral triangle.

You will find out that medians of an equilateral triangle are equal and each median is perpendicular to the respective side of triangle.

Now draw an isosceles triangle and construct medians of its equal sides. Can you see a relation between the two medians? If yes! then write that relation.

Perpendicular from a point on a given line segment

We can draw a perpendicular on a line segment from a point under given two conditions.

1 When point is on the line segment, and

2 When point is outside the line segment

Condition1:- When point is on the line segment.

Steps of Construction

(i) First of all draw a line segment AB, having a point P on it.

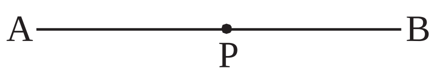


Figure 3.18

(ii) Keep the needle of compass on P and draw a semicircle which cuts AB at Q and R as shown in figure 3.19.

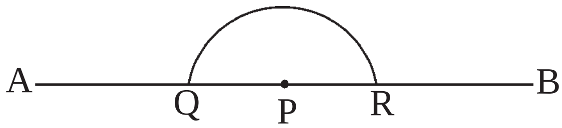


Figure 3.19

(iii) Now keep the needle of compass opened with the same radius first at R. Draw an arc cutting the semicircle QR at T as in figure 3.20. Keeping the needle at T with the same radius draw an arc marking 'S' as shown in figure 3.20.

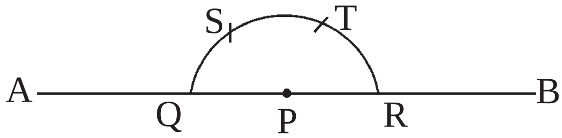


Figure3.20

(iv) Again from points S and T mark arcs of equal radius cutting each other at point M as shown in figure 3.21

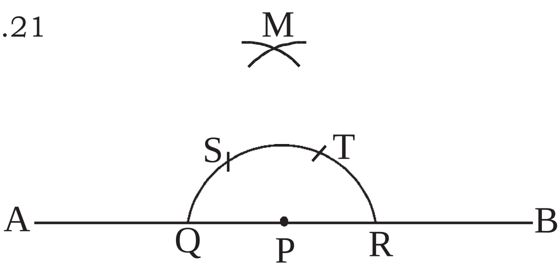


Figure3.21

(v) Join points M and P

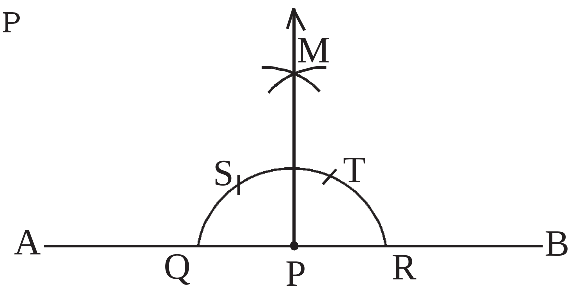


Figure3.22

PM is the required perpendicular on AB from P.

Condition 2: When the point is outside the line segment.

Steps of construction:-

(i) First of all, draw a line AB, take a point P outside AB as shown in figure.

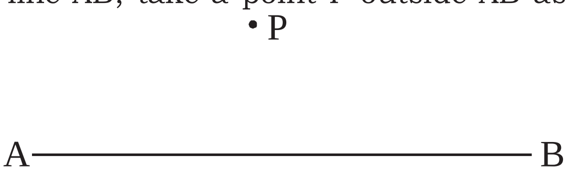


Figure3.23

(ii) Now, from the point P, make an arc QR on AB as shown in figure 3.24

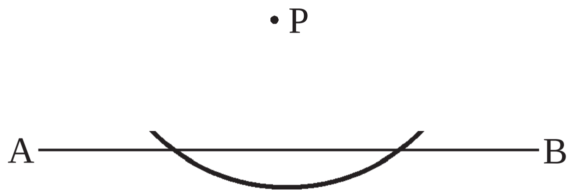


Figure3.24

(iii) Draw two arcs from the point Q and R with equal radius. These arcs intersect at a point S.

(iv) Join S and P and extend. The segments PS and AB meet at M.

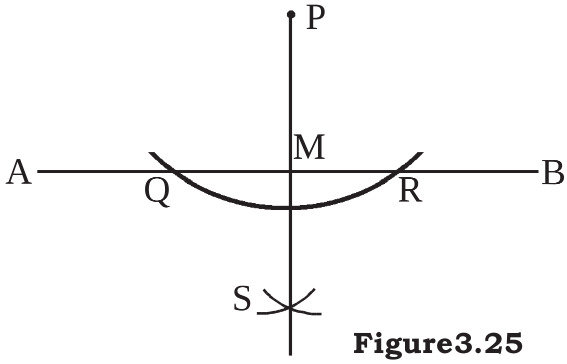


Figure3.25

Thus segment PM is a perpendicular on AB and PM⊥AB.

Altitude of a Triangle

We have learnt to draw a perpendicular to a segment, when the point is on the segment and when the point is out side the segment. Thus we can now easily draw altitudes from the vertices to their opposite side. Altitudes are the perpendiculars drawn from the vertices.

Steps of construction of Altitudes of Triangle

(i) Draw triangle ABC

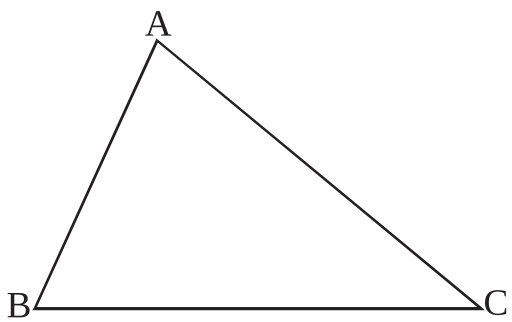


Figure3.26

(ii) We draw perpendiculars from each vertex to their opposite sides. As shown above draw from the point A, a perpendicular on the opposite side. BC meeting it at M and from the point B, a perpendicular on the opposite side AC meeting it at N.

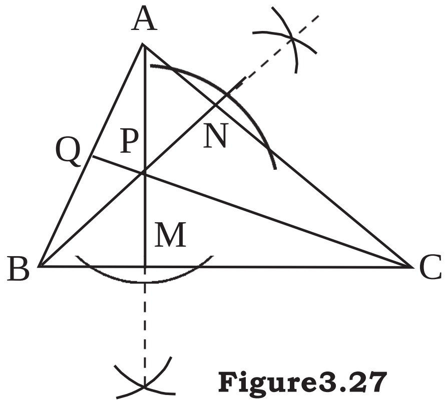


Figure3.27

(iii) Segments AM and BN meet at P

Join P and C. Increase CP to Q at AB. Measure $\angle AQC$. You will get $\angle AQC=90°$ or $CQ \perp AB$. Thus we get the third altitude as CQ. All three altitudes meet at the point P.

Construct some more triangles. Do the altitudes of the triangle meet at one point?

Yes! **The altitudes of all the triangle meet at one point. The point is called the orthocenter.**

We have seen that the third altitude also passes through the point where any two altitudes meet. Can we conclude that for the orthocenter of the triangle, we need only two altitudes?

Activity 3

Construct an obtuse angle triangle and a scalene triangle in your note book. Draw altitudes from each of the vertices to their opposite sides in both triangles. Similarly draw the altitudes of the right angle triangle. What do you conclude? Write in your notebook.

Exercise 3.2

1. Fill the following blanks:

   (i) Median of a triangle is the segment that joins the vertex to the _________ of the opposite side.

(ii) Altitude of a triangle is the segment that is a ______ from a vertex on the opposite side.

(iii) The medians of a triangle are________________

(iv) Medians of a triangle meet at a point. The point is called ________

(v) Altitudes of a triangle meet at a point. The point is called _________.

(vi) Centroid of the triangles divides the median in the ratio ______________

2. Construct two triangles in your notebook and find their centroid?

3. Construct a right angle triangle and find its orthocenter ?

4. Construct a triangle draw its three medians. Do the medians intersect at one point?

We have learnt

1. Angle opposite to the largest side of the triangle is the largest and the angle opposite to the smallest side of the triangle is the smallest.
2. We get medians by joining the mid points of the sides with their opposite vertices. All the medians intersect at one point and divide each other in the ratio 2:1.
3. The segment drawn perpendicular to a side from the opposite vertex is called the altitude. Altitudes of triangle are concurrent.
4. The point where all the altitudes of a triangle meet is called the orthocenter.