Indefinite Integration

DPP

Questions

1*. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$ is equal to (where C is an arbitrary constant)

(1) $\frac{1}{2}\sin 2x + C$ (2) $-\frac{1}{2}\sin 2x + C$

(3) $-\frac{1}{2}\sin x + C$ (4) $-\sin^2 x + C$

2. $\int (3\operatorname{cosec}^2 x + 2\sin 3x) dx$ is equal to

(1) $-\left(3\cot x + \frac{2}{3}\cos 3x\right) + C$ (2) $3\cot x - \frac{2}{3}\cos 3x + C$

(3) $3\cot x + \frac{1}{3}\cos 3x + C$ (4) None of the above

3. The value of indefinite integral $\int \frac{1}{\sqrt{x} + \sqrt{x+1}} dx$ is equal to (where C is constant of integration)

(1) $\frac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + C$ (2) $\frac{2x}{3}\left[\sqrt{x+1} - \sqrt{x}\right] + C$

(3) $\ln\left[\sqrt{x} + \sqrt{x+1}\right] + C$ (4) $\ln\left[\sqrt{x+1} - \sqrt{x}\right] + C$

4*. $\int \frac{\sec x \cdot \operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x} dx$ is equal to-

(1) $\frac{1}{2}\ln|\sec 2x + \tan 2x| + C$ (2) $\ln|\sec x + \operatorname{cosec} x| + C$

(3) $\ln|\sec x + \tan x| + C$ (4) $\frac{1}{2}\ln|\sec x + \operatorname{cosec} x| + C$

5. If $I = \int \frac{x^8 + 4}{x^4 - 2x^2 + 2} dx$, then I is equal to

(1) $\frac{x^5}{5} - \frac{2}{3}x^3 + 2x + C$ (2) $\frac{x^5}{5} - \frac{2}{3}x^3 - 2x + C$

(3) $\frac{x^5}{5} + \frac{2}{3}x^3 - 2x + C$ (4) $\frac{x^5}{5} + \frac{2}{3}x^3 + 2x + C$

6. The value of $\int \cos 2x \cos 4x \, dx$ is

(1) $\frac{-\tan x}{7} + \frac{\cos 3x}{5} + C$ (2) $\frac{\sin 6x}{12} + \frac{\sin 2x}{4} + C$

(3) $\frac{\sin 3x}{4} + \frac{\cos 3x}{7} + C$ (4) None of these.

7. If $I = \int \frac{dx}{x^2 - 2x + 5} = \frac{1}{2}\tan^{-1}(f(x)) + C$ (where, C is the constant of integration) and $f(2) = \frac{1}{2}$, then the maximum value of $y = f(\sin x)\ \forall x \in R$ is

(1) 4 (2) 2

(3) 0 (4) -1

8*. The indefinite integral $\int \frac{dx}{(x^2 + 4x + 5)^2}$ equals:

(1) $\frac{1}{2}\tan^{-1}(x+2) + \frac{x+2}{2(x^2+4x+5)} + c$ (2) $\tan^{-1}(x+2) + \frac{x+2}{x^2+4x+5} + c$

(3) $\tan^{-1}(x+2) - \left(\frac{x+2}{x^2+4x+5}\right) + c$ (4) $\frac{1}{2}\tan^{-1}(x+2) - \left(\frac{x+2}{2(x^2+4x+5)}\right) + c$

9*. $\int \frac{\ln(x+1) - \ln x}{x(x+1)} dx$ is equal to (where C is an arbitarary constant)

(1) $-\frac{1}{2}\left[\ln\left(\frac{x+1}{x}\right)\right]^2 + C$ (2) $C - \left[\{\ln(x+1)\}^2 - (\ln x)^2\right]$

(3) $-\ln\left[\ln\left(\frac{x+1}{x}\right)\right] + C$ (4) $-\ln\left(\frac{x+1}{x}\right) + C$

10. $\int \operatorname{cosec}^4 x \, dx$ is equal to

(1) $\cot x + \frac{\cot^3 x}{3} + c$ (2) $\tan x + \frac{\tan^3 x}{3} + c$

(3) $-\cot x - \frac{\cot^3 x}{3} + c$ (4) $-\tan x - \frac{\tan^3 x}{3} + c$

11*. $\int \frac{\sin^4 x}{\cos^8 x} dx$ is equal to (where C is an arbitrary constant)

(1) $\frac{(1 + \tan^5 x)}{5} + \frac{\tan^5 x}{7} + C$ (2) $\frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C$

(3) $\frac{\tan^7 x}{5} + \frac{\tan^5 x}{7} + C$ (4) None of these

12*. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to: (where C is a constant of integration)

(1) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$ (2) $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

(3) $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$ (4) $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{13}{7}} + C$

13. If $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B \log \sin(x - \alpha) + C$, then find value of (A, B).

(1) $(-\sin\alpha, \cos\alpha)$ (2) $(\cos\alpha, \sin\alpha)$

(3) $(\sin\alpha, \cos\alpha)$ (4) $(-\cos\alpha, \sin\alpha)$

14*. If $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx = ax + b\, ln|2\sin x + 3\cos x| + C$, then

(1) $a = -\frac{12}{13}, b = \frac{15}{39}$ (2) $a = -\frac{7}{13}, b = \frac{6}{13}$

(3) $a = \frac{12}{13}, b = -\frac{15}{39}$ (4) $a = -\frac{7}{13}, b = -\frac{6}{13}$

15. $\int \frac{x^2 \tan^{\ 1} x^3}{1 + x^6} dx$ is equal to

(1) $\tan^{-1}(x^3) + c$ (2) $\frac{1}{6}\left(\tan^{-1} x^3\right)^2 + c$

(3) $-\frac{1}{2}\left(\tan^{-1} x^3\right)^2 + c$ (4) $\frac{1}{2}\left(\tan^{-1} x^3\right)^2 + c$

16*. $\int \sec^2 x \cdot \cot^{\frac{4}{3}} x dx$ is equal to

(1) $3\tan^{-\frac{1}{3}} x + C$

(2) $-\frac{3}{4}\tan^{-\frac{4}{3}} x + C$

(3) $-3\tan^{-\frac{1}{3}} x + C$

(4) $-3\cot^{-\frac{1}{3}} x + C$

17*. If $I = \int \frac{dx}{\sqrt[3]{x^{\frac{5}{2}}(1+x)^{\frac{7}{2}}}} = kf(x) + c$, where c is the integration constant and $f(1) = \frac{1}{2^{\frac{1}{6}}}$, then the value of $f(2)$ is equal to

(1) $6\left(\frac{2}{3}\right)^{\frac{1}{6}}$

(2) $6\left(\frac{3}{2}\right)^{\frac{1}{6}}$

(3) $\left(\frac{2}{3}\right)^{\frac{1}{6}}$

(4) $\left(\frac{2}{3}\right)^{6}$

18. If the integral $I = \int \frac{x^5}{\sqrt{1+x^3}} dx = K\sqrt{x^3+1}\left(x^3 - 2\right) + C$, (where, C is the constant of integration), then the value of $9K$ is equal to

(1) 4

(2) 2

(3) 6

(4) 10

19. $\int \frac{(x^3-1)}{(x^4+1)(x+1)} dx$

(1) $\frac{1}{4}\ln(1+x^4) + \frac{1}{3}\ln(1+x^3) + C$

(2) $\frac{1}{4}\ln(1+x^4) - \frac{1}{3}\ln(1+x^3) + C$

(3) $\frac{1}{4}\ln(1+x^4) - \ln(1+x) + C$

(4) $\frac{1}{4}\ln(1+x^4) + \ln(1+x) + C$

20. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx,\ x > 0$, is equal to

(where c is a constant of integration)

(1) $\log_e|x^2 + 5x - 7| + c$

(2) $4\log_e|x^2 + 5x - 7| + c$

(3) $\frac{1}{4}\log_e|x^2 + 5x - 7| + c$

(4) $\log_e \sqrt{x^2 + 5x - 7} + c$

21*. If $\int \frac{x+5}{x^2+4x+5} dx = a\log(x^2 + 4x + 5) + b\tan^{-1}(x + k) +$ constant, then ordered pair of (a, b, k) is equal to

(1) $\left(\frac{1}{2}, 3, 2\right)$

(2) $\left(\frac{1}{2}, 1, 2\right)$

(3) $\left(\frac{1}{2}, 3, 1\right)$

(4) $(1, 3, 2)$

22*. Evaluate: $\int \left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$.

(1) $\frac{x}{x^2+2} + C$

(2) $\frac{\log x}{(\log x)^2 + 1} + C$

(3) $\frac{x}{(\log x)^2 + 1} + C$

(4) $\frac{xe^x}{1+x^2} + C$

23*. If $\int \frac{dx}{(x^2-2x+10)^2} = A\left(\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right) + C$, then (where C is a constant of integration)

(1) $A = \frac{1}{27}$ and $f(x) = 9(x-1)$

(2) $A = \frac{1}{81}$ and $f(x) = 3(x-1)$

(3) $A = \frac{1}{54}$ and $f(x) = 9(x-1)^2$

(4) $A = \frac{1}{54}$ and $f(x) = 3(x-1)$

24. The value obtained on $\int \sqrt{4 - x^2} dx$ would be

(1) $x\sqrt{\frac{4+x^2}{2}} + 2\sin^{-1} x + C$

(2) $x\sqrt{4 + x^2} + 2\sin^{-1} x + C$

(3) $x\sqrt{4 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$

(4) $\frac{x\sqrt{4-x^2}}{2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$

25. If $\int \frac{x}{x+1+e^x} dx = px + q\ln|x + 1 + e^x| + c$, where c is the constant of integration, then $p + q$ is equal to

(1) 0

(2) 1

(3) 2

(4) 3

26*. If $\int \frac{\cos x dx}{\sin^3 x\left(1+\sin^6 x\right)^{\frac{2}{3}}} = f(x)\left(1 + \sin^6 x\right)^{\frac{1}{\lambda}} + c$, where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to

(1) $-\frac{9}{8}$

(2) 2

(3) $\frac{9}{8}$

(4) -2

27. If $I_n = \int (\log x)^n dx$, then $I_n + n\, I_{n-1} = ?$

(1) $x(\log x)^n$

(2) $(x\log x)^n$

(3) $(\log x)^{n-1}$

(4) $n(\log x)^n$

28*. If the integral $\int \frac{\ln x}{x^3} dx = \frac{f(x)}{4x^2} + C$, where $f(e) = -3$ and C is the constant of integration, then the value of $f(e^2)$ is equal to

(1) 3

(2) -4

(3) -5

(4) 5

29. Let $\int \frac{1-\ln x}{x^2} dx = f(x)$, for all positive x. If $f(e) = \frac{1}{e}$, then $f(2) + f(4)$ is equal to

(1) $\ln 4$

(2) $\ln 2$

(3) 1

(4) 0

30. The value of the integral $\int e^{3\sin\ {}^{1}x}\left(\frac{1}{\sqrt{1-x^2}}+e^{3\cos\ {}^{1}x}\right)dx$ is equal to

(where, c is an arbitrary constant)

(1) $\frac{e^{3\sqrt{\sin\ {}^{1}x}}}{3}+xe^{\frac{3\pi}{2}}+c$

(2) $e^{\sqrt{\sin\ {}^{1}x}}+e^{\pi/2}+c$

(3) $\frac{e^{3\sin\ {}^{1}x}}{3}+xe^{\frac{3\pi}{2}}+c$

(4) $e^{\frac{\pi}{2}}+e^{x}\left(\frac{\pi}{2}\right)+c$

31. $\int \frac{1+\tan x}{e\ {}^{x}\cos x}dx$ is equal to

(1) $e^{-x}\tan x+c$

(2) $e^{-x}\sec x+c$

(3) $e^{x}\sec x+c$

(4) $e^{x}\tan x+c$

32*. $\int x^3\log x dx$ is

(1) $\frac{x^4}{4}\log x-\frac{x^4}{16}+C$

(2) $\frac{x^3}{3}\log x-\frac{x^3}{8}+C$

(3) $\frac{x^3}{3}\log x+\frac{x^3}{8}+C$

(4) none of the above.

33*. $\int \frac{e^x(1+\sin x)}{1+\cos x}$ dx is equal to :

(1) $e^x\ \tan\left(\frac{x}{2}\right)+c$

(2) $e^x\ \tan\ x+c$

(3) $e^x\ \left(\frac{1+\sin x}{1-\cos x}\right)+c$

(4) $c-e^x\ \cot\left(\frac{x}{2}\right)$

34. Given $\int e^x(\tan x+1)\sec x\,dx = e^x f(x)+\mathrm{C}$

A satisfying value of $f(x)$ would be:

(1) sin *x*

(2) cosec *x*

(3) sec *x*

(4) tan *x*

35*. $\int e^{\sin\theta}\left(\log\sin\theta+\operatorname{cosec}^2\theta\right)\cos\theta d\theta$ is equal to

(1) $\int e^{\sin\theta}\left[\log\sin\theta+\operatorname{cosec}^2\theta\right]+c$

(2) $e^{\sin\theta}[\log\sin\theta+\operatorname{cosec}\ \theta]+c$

(3) $e^{\sin\theta}[\log\sin\theta-\operatorname{cosec}\theta]+c$

(4) $e^{\sin\theta}\left[\log\sin\theta-\operatorname{cosec}^2\theta\right]+c$

36*. The value of $\int e^{\tan\ {}^{1}x}.\frac{(1+x+x^2)}{1+x^2}\ dx$ is

(1) $\tan^{-1}x+c$

(2) $e^{\tan\ {}^{1}x}+c$

(3) $e^{\tan\ {}^{1}x}-x+c$

(4) $xe^{\tan\ {}^{1}x}+c$

37. $\int \frac{dx}{(x+1)(x-2)}=A\log(x+1)+B\log(x-2)+c$ then -

(1) $A+B=0\,;A:B=-1$

(2) $AB=1\,;\ A:B=-1$

(3) $A\ -\ B=0\,;\ A:B=-1$

(4) None of these

38*. If $\int \frac{2x^2+3}{(x^2-1)(x^2+4)}dx = A\log\left(\frac{x+1}{x-1}\right)+B\ \tan^{-1}\left(\frac{x}{2}\right)+c$, then (A,B), is

(1) $\left(-\frac{1}{2},\frac{1}{2}\right)$

(2) $\left(\frac{1}{2},-\frac{1}{2}\right)$

(3) $(-1,1)$

(4) $(1,-1)$

39*. Correct evaluation $\int \frac{x}{(x-2)(x-1)}dx$ is (where, P is an arbitrary constant)

(1) $\log_e\frac{(x-2)^2}{(x-1)}+P$

(2) $\frac{x-1}{x-2}+P$

(3) $2\log_e\left(\frac{x-2}{x-1}\right)+P$

(4) $\frac{x-2}{x-1}+P$

40*. $\int \frac{x}{(x^2-a^2)(x^2-b^2)}dx=?$

(1) $\frac{1}{a^2-b^2}\log\left|\frac{x^2-a^2}{x^2-b^2}\right|+C$

(2) $\frac{1}{a^2-b^2}\log\left|\frac{x^2-b^2}{x^2-a^2}\right|+C$

(3) $\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-a^2}{x^2-b^2}\right|+C$

(4) $\frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-b^2}{x^2-a^2}\right|+C$

*Note: Question with * denotes it is important.*

ANSWER KEYS

1. (2)	**2.** (1)	**3.** (1)	**4.** (1)	**5.** (4)	**6.** (2)	**7.** (3)	**8.** (1)
9. (1)	**10.** (3)	**11.** (2)	**12.** (1)	**13.** (2)	**14.** (3)	**15.** (2)	**16.** (3)
17. (3)	**18.** (2)	**19.** (3)	**20.** (2)	**21.** (1)	**22.** (3)	**23.** (4)	**24.** (4)
25. (1)	**26.** (4)	**27.** (1)	**28.** (3)	**29.** (2)	**30.** (3)	**31.** (3)	**32.** (1)
33. (1)	**34.** (3)	**35.** (3)	**36.** (4)	**37.** (1)	**38.** (1)	**39.** (1)	**40.** (3)

1. (2)

$\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x\cos^2 x} dx$

$= \int \frac{\left(\sin^4 x - \cos^4 x\right)\left(\sin^4 x + \cos^4 x\right)}{1-2\sin^2 x\cos^2 x} dx$

$= \int \frac{\left(\sin^2 x - \cos^2 x\right)\left(\sin^4 x + \cos^4 x\right)}{1-2\sin^2 x\cos^2 x} dx$

$= \int -\cos 2x\, dx = -\frac{1}{2}\sin 2x + C$

2. (1)

We have,

$\int \left(3\operatorname{cosec}^2 x + 2\sin 3x\right) dx$

$= 3\int \operatorname{cosec}^2 x + 2\int \sin 3x dx$

$= -3\cot x - \frac{2}{3}\cos 3x + C$

$\left[\because \int f(a)dx = g(x)+c \Rightarrow \int f(ax+b)dx = \frac{1}{a}g(ax)+c\right]$

3. (1)

Let $I = \int \frac{1}{\sqrt{x}+\sqrt{x+1}} dx$

Using rationalization, we get

$I = \int \frac{\left(\sqrt{x+1}-\sqrt{x}\right)}{\left(\sqrt{x+1}+\sqrt{x}\right)\left(\sqrt{x+1}-\sqrt{x}\right)} dx$

$\Rightarrow I = \int \left(\sqrt{x+1}-\sqrt{x}\right) dx$

$= \frac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + C$

4. (1)

$\mathrm{I} = \int \frac{\sec x \operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x} dx$

Simplifying the integrand by converting them into sine and cosine function,

$I = \int \frac{dx}{2\cos^2 x - 1}$

Using $\cos 2A = 2\cos^2 A - 1$

$I = \int \sec 2x\, dx$

$I = \frac{1}{2}\ln|\sec 2x + \tan 2x| + C$

5. (4) We have given that

$\int \frac{x^8+4}{x^4-2x^2+2} dx = \int \frac{\left(x^8+4+4x^4\right)-4x^4}{\left(x^4-2x^2+2\right)} dx$

$= \int \frac{\left(x^4+2\right)^2-\left(2x^2\right)^2}{\left(x^4-2x^2+2\right)} dx = \int \frac{\left(x^4+2+2x^2\right)\left(x^4+2-2x^2\right)}{\left(x^4-2x^2+2\right)} dx$

$= \int \left(x^4+2x^2+2\right) dx = \frac{x^5}{5} + \frac{2}{3}x^3 + 2x + C$, where C is the constant of integration.

6. (2)

Let $\mathrm{I} = \int \cos 2x \cos 4x\, dx$

$\because \cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$\because \cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

$= \frac{1}{2}\int (\cos 6x + \cos 2x)\, dx$

$= \frac{1}{2}\left[\frac{\sin 6x}{6} + \frac{\sin 2x}{2}\right] + C$

$= \frac{\sin 6x}{12} + \frac{\sin 2x}{4} + C$

7. (3) The given integral is $I = \int \frac{dx}{(x-1)^2+4}$

$= \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right) + c$

$\therefore f(x) = \frac{x-1}{2}$

i.e. $f(\sin x) = \frac{\sin x - 1}{2}$

Hence, max $(f(\sin x)) = 0$

8. (1)

Let $I=\int \frac{1}{\left[(x+2)^{2}+1\right]^{2}} dx$

Put $x+2=\tan \theta ; d x=\sec ^{2} \theta$

$I=\int \frac{d \theta}{\sec ^{2} \theta}=\int\left(\cos ^{2} \theta\right) d \theta=\int \frac{1+\cos 2 \theta}{2} d \theta=\frac{\theta}{2}+\frac{\sin 2 \theta}{4}+c$

$=\frac{\theta}{2}+\frac{2 \tan \theta}{4\left(1+\tan ^{2} \theta\right)}+c$

$=\frac{\tan ^{-1}(x+2)}{2}+\frac{(x+2)}{2\left(x^{2}+4 x+5\right)}+c$

9. (1) Put $\ln (x+1)-\ln x=t$

$\Rightarrow \frac{1}{x+1}-\frac{1}{x}=\frac{dt}{dx} \Rightarrow \frac{x-(x+1)}{x(x+1)}=\frac{dt}{dx}$

$\Rightarrow \frac{-dx}{x(x+1)}=dt \Rightarrow \frac{dx}{x(x+1)}=-dt$

so question becomes

$-\int t\, dt=-\frac{t^{2}}{2}+C$

10. (3)

$\int \operatorname{cosec}^{4} x\, d x=\int \operatorname{cosec}^{2} x \cdot \operatorname{cosec}^{2} x\, d x$

$=\int \operatorname{cosec}^{2} x\left(1+\cot ^{2} x\right) d x$

$=\int \operatorname{cosec}^{2} x\, d x+\int \cot ^{2} x \cdot \operatorname{cosec}^{2} x\, d x$

$=-\cot x-\frac{\cot ^{3} x}{3}+c$

11.

(2) $\int \frac{\frac{\sin ^{4} x}{\cos ^{4} x}}{\frac{\cos ^{8} x}{\cos ^{4} x}} . dx=\int \tan ^{4} x . \sec ^{4} x\, dx$

$=\int \tan ^{4} x\left(1+\tan ^{2} x\right) \sec ^{2} x\, dx$

$=\int \tan ^{4} x\left(1+\tan ^{2} x\right) d(\tan x)=\int t^{4}\left(1+t^{2}\right)$ dt$=\frac{t^{5}}{5}+\frac{t^{7}}{7}+C$

12. (1)

$\Rightarrow \int\left(\frac{x-3}{x+4}\right)^{-\frac{6}{7}} \frac{1}{(x+4)^{2}} d x \quad \ldots$(i)

Let $\frac{x-3}{x+4}=t^{7}$,

$\frac{7}{(x+4)^{2}} d x=7 t^{6} d t \ldots$ (ii)

In (i) from (ii) $\int t^{-6} t^{6} d t=t+C$

13. (2)

$\int \frac{\sin x}{\sin (x-\alpha)} d x=A x+B \log \sin (x-\alpha)+C$

$\Rightarrow$ Differentiating w.r.t. x both sides

$\Rightarrow \frac{\sin x}{\sin (x-\alpha)}=A+\frac{B \cos (x-\alpha)}{\sin (x-\alpha)}$

$\Rightarrow \sin x=A \sin (x-\alpha)+B \cos (x-\alpha)$

$\sin x=A(\sin x \cos \alpha-\cos x \sin \alpha)+B(\cos x \cos \alpha+\sin x \sin \alpha)$

$\sin x=\sin x(A \cos \alpha+B \sin \alpha)+\cos x(B \cos \alpha-A \sin \alpha)$

Comparing coefficients of $\sin x$ and $\cos x$ both side

$A \cos \alpha+B \sin \alpha=1$ and $B \cos \alpha-A s \operatorname{in} \alpha=0$

$(A, B)=(\cos \alpha, \sin \alpha)$

14. (3) Differentiating both sides, we get

$\frac{3 \sin x+2 \cos x}{3 \cos x+2 \sin x}==a+\frac{b(2 \cos x-3 \sin x)}{(2 \sin x+3 \cos x)}$

$=\frac{\sin x(2a-3b)+\cos x(3a+2b)}{(3 \cos x+2 \sin x)}$

Comparing like terms on both sides, we get

$3=2a-3b, 2=3a+2b \Rightarrow a=\frac{12}{13}, b=-\frac{15}{39}$

15. (2)

Let,

$I=\int \frac{x^{2} \tan ^{-1} x^{3}}{1+x^{6}} d x$

Put $\tan ^{-1} x^{3}=t$

$\frac{1}{1+x^{6}} . 3 x^{2} d x=d t$

$\Rightarrow \frac{x^{2}}{1+x^{6}} d x=\frac{d t}{3}$

$\Rightarrow I=\frac{1}{3} \int t d t=\frac{1}{3} . \frac{t^{2}}{2}+c$

$I=\frac{\left(\tan ^{-1} x^{3}\right)}{6}+c$

16. (3)

Given integral can be written as

$I=\int \frac{\sec^2 x}{(\tan x)^{\frac{4}{3}}}\,dx$

Let $\tan x = t$

$\Rightarrow \sec^2 x dx = dt$

$\Rightarrow I=\int t^{-\frac{4}{3}}\,dt$

Using $\int x^n dx = \frac{x^{n+1}}{n+1}+C$, we get

$I=\frac{t^{-\frac{1}{3}}}{\left(-\frac{1}{3}\right)}+C$

$\Rightarrow I=-\frac{3}{t^{\frac{1}{3}}}+C$

$\Rightarrow I=-3\tan^{-\frac{1}{3}}x+C.$

17. (3) The given integral is $I=\int \frac{dx}{\left[x^6\left(\frac{1+x}{x}\right)^{\frac{7}{2}}\right]^{\frac{1}{3}}}$

$\Rightarrow I=\int \frac{dx}{x^2\left(1+\frac{1}{x}\right)^{\frac{7}{6}}}$

Let, $1+\frac{1}{x}=t$

$\Rightarrow -\frac{dx}{x^2}=dt$

$\Rightarrow I=\int -\frac{dt}{t^{\frac{7}{6}}}$

$=6t^{-\frac{1}{6}}+c$

$=6\left(\frac{x}{x+1}\right)^{\frac{1}{6}}+c$

$\therefore f(x)=\left(\frac{x}{x+1}\right)^{\frac{1}{6}}$

$\Rightarrow f(2)=\left(\frac{2}{3}\right)^{\frac{1}{6}}$

18. (2) Let $1+x^3=t^2$

$\Rightarrow 3x^2dx=2tdt$

$\Rightarrow I=\int \frac{(t^2-1)\,2t\,dt}{3t}$

$\Rightarrow I=\frac{2}{3}\int(t^2-1)dt$

$=\frac{2t^3}{9}-\frac{2}{3}t+C$

$=\frac{2}{9}t(t^2-3)+C$

$=\frac{2}{9}\sqrt{1+x^3}(x^3-2)+C$

$\therefore K=\frac{2}{9}\Rightarrow 9K=2$

19. (3)

Let $I=\int \frac{(x^3-1)}{(x^4+1)(x+1)}dx.$

Add and subtract x^4 in the numerator, we get

$I=\int \frac{(x^4+x^3)-(x^4+1)}{(x^4+1)(x+1)}dx$

$\Rightarrow I=\int \frac{x^3}{(x^4+1)}dx-\int \frac{1}{x+1}dx\ \ldots(i)$

$\Rightarrow I_1=\int \frac{x^3}{(x^4+1)}dx$ and $I_2=\int \frac{1}{x+1}dx=\ln(x+1)+c'$

In I_1 put $x^4+1=t\Rightarrow 4x^3dx=dt$

$\Rightarrow I_1=\frac{1}{4}\int \frac{1}{t}dt=\frac{1}{4}\ln t+c$

So by (i), we get

$I=\frac{1}{4}\ln(x^4+1)-\ln(x+1)+C$

20. (2)

$\int \frac{e^{3\log_e 2x}+5e^{2\log_e 2x}}{e^{4\log_e x}+5e^{3\log_e x}-7e^{2\log_e x}}dx,\ x>0$

$=\int \frac{(2x)^3+5(2x)^2}{x^4+5x^3-7x^2}dx=\int \frac{4x^2(2x+5)}{x^2(x^2+5x-7)}dx$

$=4\int \frac{d(x^2+5x-7)}{(x^2+5x-7)}=4\log_e|x^2+5x-7|+c$

option (2)

21. (1) $\int \frac{x+5}{x^2+4x+5} dx = \frac{1}{2} \int \frac{2x+10}{x^2+4x+5} dx$

$= \frac{1}{2}\left[\int \frac{2x+4}{x^2+4x+5} dx + \int \frac{6}{x^2+4x+5} dx\right]$

$= \frac{1}{2}\left[\log\left(x^2+4x+5\right) + 6\tan^{-1}(x+2)\right]$

$= \frac{1}{2}\log\left(x^2+4x+5\right) + 3\tan^{-1}(x+2) + c$

$a = \frac{1}{2},\ b = 3,\ k = 2$

22. (3)

Method1: by cross checking the options

Consider $f(x) = \frac{x}{(\log x)^2+1}$

$\therefore\ f'(x) = \frac{1+(\log x)^2 - \frac{2x\log x}{x}}{\left(1+(\log x)^2\right)^2}$

$\therefore\ f'(x) = \frac{1+(\log x)^2 - 2\log x}{\left(1+\log^2 x\right)^2} = \left(\frac{(\log x - 1)}{(1+\log x)^2}\right)^2$

$\therefore\ \int\left(\frac{(\log x-1)^2}{1+(\log x)^2}\right) dx = \int f'(x) dx = f(x) + C$

$\therefore\ \int\left(\frac{\log x - 1}{1+(\log x)^2}\right)^2 dx = \frac{x}{1+(\log x)^2} + C$

Hence option 3 is the correct answer and we can check the other choices by the similar argument.

<u>**Alternate solution**</u>

$\int\left\{\frac{\log(x) - 1}{1+(\log x)^2}\right\}^2 dx$

Put $\log(x) = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$

$= \int e^t\left\{\frac{(t-1)^2}{(t^2+1)^2}\right\} dt$

$\int e^t\left\{\frac{t^2+1-2t}{(t^2+1)^2}\right\} dt$

$= \int e^t\left\{\frac{1}{t^2+1} + \left(\frac{-2t}{(t^2+1)^2}\right)\right\} dt$

$= \frac{e^t}{t^2+1} + C \quad \left[\because\ \int e^x(f(x)+f'(x))dx = e^x f(x) + C\right]$

$= \frac{x}{(\log x)^2+1}$

23. (4)

We have $I = \int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{\left[(x-1)^2+3^2\right]^2}$

Let $x-1 = 3\tan\theta \Rightarrow dx = 3\sec^2\theta d\theta$

$\Rightarrow I = \int \frac{3\sec^2\theta d\theta}{81\sec^4\theta} = \frac{1}{27}\int \cos^2\theta d\theta$

$= \frac{1}{54}\int(1+\cos 2\theta) d\theta$

$= \frac{1}{54}\left(\theta + \frac{\sin 2\theta}{2}\right) + C$

$= \frac{1}{54}(\theta + \sin\theta\cos\theta) + C$, where C is the constant of integration.

$= \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2}\right] + C$

$= \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + C$

Hence, $A = \frac{1}{54}$ & $f(x) = 3\left(x-1\right)$

24. (4) $\int\sqrt{4-x^2}dx$

We know that:

$\int\sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

Here,

$a^2 = 4,\ a = 2$

$\therefore\ \int\sqrt{4-x^2}dx = \frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right) + C$

$= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$

25. (1)

$\int \frac{(x+1+e^x)-(e^x+1)}{x+1+e^x}dx$

$= \int\left(1-\frac{(e^x+1)}{x+1+e^x}\right)dx$

$= x - \ln|x+1+e^x| + c$

On comparing,

$p=1,\ q=-1$

Hence, $p+q=0$

26. (4)

Let, $\sin x = t$

$\cos x dx = dt$

$I = \int \frac{dt}{t^3(1+t^6)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$

Put $1+\frac{1}{t^6}=r^3$

$\Rightarrow \frac{dt}{t^7} = \frac{-1}{2}r^2dr$

$I = -\frac{1}{2}\int\frac{r^2dr}{r^2} = -\frac{1}{2}r + c$

$\Rightarrow I = -\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}} + c$

$\Rightarrow I = -\frac{1}{2\sin^2 x}\left(1+\sin^6 x\right)^{\frac{1}{3}} + c$

So, $f(x) = -\frac{1}{2}\text{cosec}^2 x$ and $\lambda = 3$

$\lambda f\left(\frac{\pi}{3}\right) = -2$

27. (1) $I_n = \int(\log x)^n.1dx$

$= x(\log x)^n - \int n(\log x)^{n-1}.\frac{1}{x}.xdx$

$= (\log x)^n x - nI_{n-1}$

$\Rightarrow I_n + nI_{n-1} = x(\log x)^n$

28. (3) $\int \underbrace{\ln x}_{I}\underbrace{\frac{1}{x^3}}_{II}dx = (\ln x)\left(\frac{-1}{2x^2}\right) - \int\frac{1}{x}\left(\frac{-1}{2x^2}\right)dx$

(Using integration by parts)

$= -\frac{\ln x}{2x^2} + \frac{1}{2}\left(\frac{x^{\ 2}}{-2}\right) + C$

$= -\frac{1}{4x^2}(2\ln x+1) + C$

$\Rightarrow f(x) = -2\ln x - 1$

$\therefore f(e^2) = -2\ln(e^2) - 1$

$= -4-1 = -5$

29. (2)

$f(x) = \int\frac{1}{x^2}dx - \int \ln x \cdot \frac{1}{x^2}dx$

$= -\frac{1}{x} - \left\{\ln x\left(-\frac{1}{x}\right) + \int\frac{1}{x^2}dx\right\}$

$= -\frac{1}{x} + \frac{\ln x}{x} + \frac{1}{x} + c = \frac{\ln x}{x} + c$

Now, $f(e) = \frac{1}{e} \Rightarrow c = 0$

Hence, $f(2)+f(4) = \frac{\ln 2}{2} + \frac{\ln 4}{4} = \ln 2$

30. (3) $\int\frac{e^{3\sin\ 1_x}}{\sqrt{1-x^2}}dx + \int e^{3(\sin\ ^1x+\cos\ ^1x)}dx$

$= \int e^{3t}dt + \int e^{\frac{3\pi}{2}}dx$ (Put $\sin^{-1}x = t$)

$= \frac{e^{3t}}{3} + e^{\frac{3\pi}{2}}\cdot x + c$

31. (3) Let $I = \int e^x \sec x dx + \int e^x \sec x \tan x dx$

$= e^x \sec x + c$ $\quad\{\int e^x(f(x))+f'(x)dx = e^x f(x)\ \text{ where }\ f(x=\sec x)\}$

$= e^x \sec x + c$

32. (1)

Integration by parts

$\int uvdx = u\int vdx - \int\left(\frac{du}{dx}\left(\int vdx\right)\right)dx$

Let $I = \int \underset{II}{x^3}\underset{I}{\log x}dx$

$\Rightarrow I = \log x\left(\frac{x^4}{4}\right) - \int\left(\frac{1}{x}\right)\cdot\left(\frac{x^4}{4}\right)dx$

$\Rightarrow I = \log x\left(\frac{x^4}{4}\right) - \frac{1}{4}\int x^3dx$

$\Rightarrow I = \log x\left(\frac{x^4}{4}\right) - \frac{1}{16}x^4 + C$

33. **(1)** Let $I=\int e^{x}\left(\frac{1+\sin\ x}{1+\cos\ x}\right)\mathrm{dx}$

$=\int e^{x}\frac{\left(1+2\sin\frac{x}{2}\cos\frac{x}{2}\right)}{2\cos^{2}\frac{x}{2}}\mathrm{dx}$

$=\int\frac{1}{2}e^{x}\sec^{2}\frac{x}{2}\mathrm{dx}+\int e^{x}\tan\frac{x}{2}\mathrm{dx}$

$=\frac{1}{2}\left[2e^{x}\tan\frac{x}{2}-\int 2e^{x}\tan\frac{x}{2}dx\right]+\int e^{x}\tan\frac{x}{2}dx$

$=e^{x}\tan\frac{x}{2}-\int e^{x}\tan\frac{x}{2}dx+\int e^{x}\tan\frac{x}{2}dx+c$

$=e^{x}\tan\frac{x}{2}+c$

34. **(3)**

Given : $\int e^{x}(\tan x+1)\sec x\,dx=e^{x}f(x)+\mathrm{C}\qquad\ldots(1)$

L.H.S

$=\int e^{x}(\tan x+1)\sec x\,dx$

$=\int e^{x}(\sec x+\sec x\tan x)\,dx$

$=\int e^{x}\sec x\,dx+\int e^{x}\sec x\ \tan x\,dx$

(Integrating first by parts)

$=e^{x}(\sec x+\tan x)\ +\ e^{x}\sec x-e^{x}(\sec\ x\tan x)\ +\mathrm{C}$

$=e^{x}\sec x+\mathrm{C}$

$=e^{x}f(x)+\mathrm{C}\qquad$ ...[Using (1)]

On comparing , we get $f(x)=\sec x$.

35. **(3)**

Let, $I=\int e^{\sin\theta}(\log\sin\theta)\cos\theta d\theta+\int e^{\sin\theta}\left(\mathrm{cosec}^{2}\,\theta\right)\cos\theta d\theta$

Put, $\sin\theta=t\Rightarrow\cos\theta d\theta=dt$

$\therefore\ I=\int e^{t}\log t\,dt+\int e^{t}t^{-2}dt$

$=\log t e^{t}-\int\frac{e^{t}}{t}dt+\frac{e^{t}t^{-1}}{-1}-\int\frac{e^{t}t^{-1}}{-1}dt$

$=e^{t}\left(\log t-\frac{1}{t}\right)+c$

$=e^{\sin\theta}(\log\sin\theta-\mathrm{cosec}\theta)+c$, where c is constant of integration.

36. **(4)**

Let

$I=\int\left(e^{\tan^{-1}x}\frac{(1+x^{2})}{(1+x^{2})}dx+e^{\tan^{-1}x}.\ \frac{x}{(1+x^{2})}\right)dx$

$I=\int\left(e^{\tan^{-1}x}+\ e^{\tan^{-1}x}\frac{x}{1+x^{2}}\right)dx$

As we know,

$d\left(e^{\tan^{-1}x}\right)=e^{\tan^{-1}x}\left(d\left(\tan^{-1}x\right)\right)$

$\Rightarrow d\left(e^{\tan^{-1}x}\right)=e^{\tan^{-1}x}\left(\frac{1}{1+x^{2}}\right)$

So,

$I=\int\left(e^{\tan^{-1}x}dx\ +\ x\,d\left(e^{\tan^{-1}x}\right)\right)$

$\Rightarrow I=\int\frac{d}{dx}\left(x\ e^{\tan^{-1}x}\right)dx+c$

$I=xe^{\tan^{-1}x}+c$

37. **(1)** $I=\frac{1}{3}\int\frac{(x+1)-(x-2)}{(x+1)(x-2)}dx$

$=\frac{1}{3}\int\left(\frac{1}{(x-2)}-\frac{1}{x+1}\right)dx$

$=\frac{1}{3}\log(x-\ 2)-\frac{1}{3}\log(x+1)+c$

$\Rightarrow\ B=\frac{1}{3}\ \&\ A=\frac{-1}{3}$

38. (1)

$\int \frac{(2x^2+3)}{(x^2-1)(x^2+4)}dx$

$= \int \frac{(x^2+4+x^2-1)}{(x^2-1)(x^2+4)}dx$

$= \int\left(\frac{1}{x^2-1} + \frac{1}{x^2+4}\right)dx$

$= \frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+\frac{1}{2}\ \tan^{-1}\frac{x}{2} + c$

$= -\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\ \tan^{-1}\frac{x}{2} + c$

$= \text{A}\log\left(\frac{x+1}{x-1}\right)+ \text{B}\tan^{-1}\left(\frac{x}{2}\right)+c$, where c is the constant of integration.

$\Rightarrow A =-\frac{1}{2}$ & $\text{B} = \frac{1}{2}$

39. (1)

Let

$I = \int \frac{x}{(x-2)(x-1)}dx$

Put

$\frac{x}{(x-2)(x-1)} = \frac{A}{x-2} + \frac{B}{x-1}$

$\Rightarrow x = A(x-1)+B(x-2)$

$A + B = 1,\ \ -A - 2B = 0$

On solving, we get $A = 2$ and $B = -1$

$\therefore I = \int\left[\frac{2}{x-2} - \frac{1}{x-1}\right]dx$

$\Rightarrow I = 2\log|x-2|-\log|x-1|+P$

$\Rightarrow I = \log\frac{(x-2)^2}{|x-1|} + P$

40. (3) Let, $I = \int\frac{xdx}{(x^2-a^2)(x^2-b^2)}$

Put $x^2 = t \Rightarrow xdx = \frac{1}{2}dt$

$\therefore\ I = \frac{1}{2}\int\frac{dt}{(t-a^2)(t-b^2)}$

$= \frac{1}{2}\int\left\{\frac{dt}{(a^2-b^2)(t-a^2)} - \frac{dt}{(a^2-b^2)(t-b^2)}\right\}$

$= \frac{1}{2(a^2-b^2)}\log\left|\frac{t-a^2}{t-b^2}\right|+C$

$\Rightarrow I = \frac{1}{2(a^2-b^2)}\log\left|\frac{x^2-a^2}{x^2-b^2}\right|+C$