Mock Test 3

Time : 3 hrs. **Max. Marks : 300**

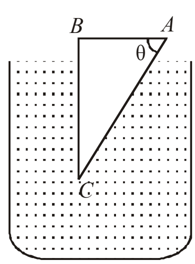
INSTRUCTIONS

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. There are three parts in the question paper consisting of Physics (Q.no.1 to 30), Chemistry (Q.no.31 to 60) and Mathematics (Q. no.61 to 90). Each part is divided into two sections, Section A consists of 20 multiple choice questions & Section B consists of 10 Numerical value answer Questions.
   In Section B, candidates have to attempt **only 5 questions out of 10**.
5. There will be only one correct choice in the given four choices in Section A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for unattempted question. For Section B 4 marks will be awarded for correct answer and zero for marked for each review / unattempted/incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations / written work should be done in the rough sheet provided.

PHYSICS

Section - A

1. A glass prism of refractive index 1.5 is immersed in water (refractive index $\frac{4}{3}$) as shown in figure. A light beam incident normally on the face *AB* is totally reflected to reach the face *BC*, if



(1) $\sin\theta \geq \frac{5}{9}$

(2) $\sin\theta \geq \frac{2}{3}$

(3) $\sin\theta \geq \frac{8}{9}$

(4) $\sin\theta \geq \frac{1}{}$

2. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

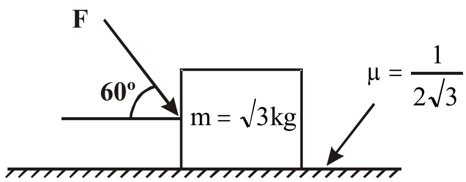
(1) $\frac{5GmM}{6R}$

(2) $\frac{2GmM}{3R}$

(3) $\frac{GmM}{2R}$

(4) $\frac{3GmM}{2R}$

3. What is the maximum value of the force F such that the block shown in the arrangement, does not move?



(1) 20 N (2) 10 N (3) 12 N (4) 15 N

4. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground. Then pickup the incorrect alternative

(1) The magnitude of angular momentum of the particle about O is increasing

(2) The magnitude of torque of the gravitational force on the particle about O is decreasing

(3) The moment of inertia of the particle about O is decreasing

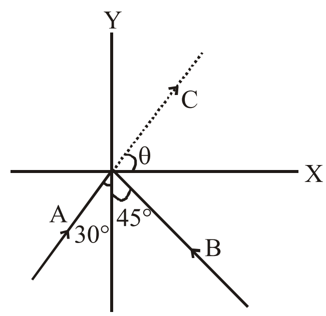
(4) The magnitude of angular velocity of the particle about O is increasing

5. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is :

(1) $T \propto \frac{1}{R}$ (2) $T \propto \frac{1}{R^3}$

(3) $T \propto e^{-R}$ (4) $T \propto e^{-3R}$

6. Two particles A and B of equal mass M are moving with the same speed v as shown in the figure. They collide completely inelastically and move as a single particle C. The angle θ that the path of C makes with the X-axis is given by:



(1) $\tan\theta = \frac{\sqrt{3}+\sqrt{2}}{1-\sqrt{2}}$

(2) $\tan\theta = \frac{\sqrt{3}-\sqrt{2}}{1-\sqrt{2}}$

(3) $\tan\theta = \frac{1-\sqrt{2}}{\sqrt{2}(1+\sqrt{3})}$

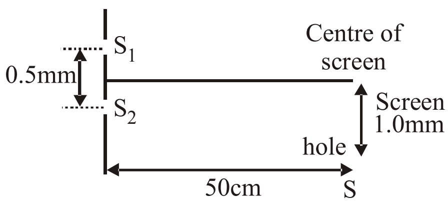
(4) $\tan\theta = \frac{1-\sqrt{3}}{1+\sqrt{2}}$

7. The fundamental frequency of a sonometer wire of length ℓ is n_0. A bridge is now introduced at a distance of $\Delta\ell$ ($<< \ell$) from the centre of the wire. The lengths of wire on the two sides of the bridge are now vibrated in their fundamental modes. Then, the beat frequency nearly is –

(1) $n_0\Delta\ell/\ell$ (2) $8n_0\Delta\ell/\ell$

(3) $2n_0\Delta\ell/\ell$ (4) $n_0\Delta\ell/2\ell$

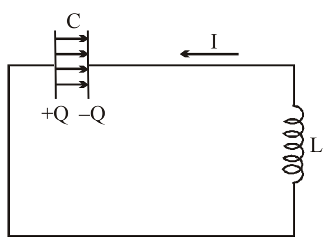
8. In Young's double slit experiment shown in figure S_1 and S_2 are coherent sources and S is the screen having a hole at a point 1.0mm away from the central line. White light (400 to 700nm) is sent through the slits. Which wavelength passing through the hole has strong intensity?



(1) 400 nm (2) 700 nm

(3) 500 nm (4) 667 nm

9. In the LC circuit, the current in the direction shown and the charges on the capacitor plates have the signs shown. At this time



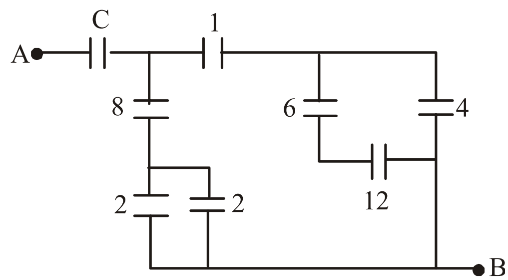
(1) I is increasing and Q is increasing

(2) I is increasing and Q is decreasing

(3) I is decreasing and Q is increasing

(4) I is decreasing and Q is decreasing

10. Figure shows a network of capacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C if the equivalent capacitance between point A and B is to be 1 μF is:



(1) $\frac{32}{23}$ μF (2) $\frac{31}{23}$ μF

(3) $\frac{33}{23}$ μF (4) $\frac{34}{23}$ μF

11. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = \left(40\hat{i} + 50\hat{j}\right)$ m at $t = 2$s. If the projectile was launched at an angle θ from the horizontal, then θ is

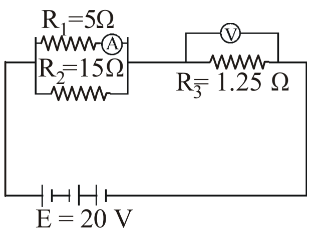
(take $g = 10$ ms^{-2})

(1) $\tan^{-1}\frac{2}{3}$ (2) $\tan^{-1}\frac{3}{2}$

(3) $\tan^{-1}\frac{7}{4}$ (4) $\tan^{-1}\frac{4}{5}$

12. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc
(1) continuously decreases
(2) continuously increases
(3) first increases and then decreases
(4) remains unchanged

13. An ideal ammeter (zero resistance) and an ideal voltmeter (infinite resistance) are connected as shown. The ammeter and voltmeter reading for $R_1 = 5\,\Omega$, $R_2 = 15\,\Omega$, $R_3 = 1.25\,\Omega$ and $E = 20$ V are given as

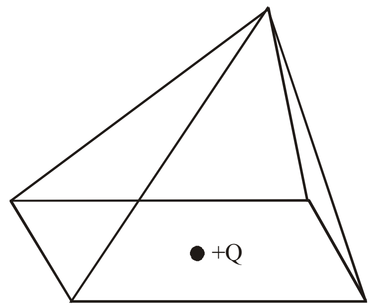


(1) 6.25 A, 3.75 V (2) 3.00 A, 5 V
(3) 3.75 A, 3.75 V (4) 6.25 A, 6.25 V

14. A monoatomic ideal gas is filled in a nonconducting container. The gas can be compressed by a movable nonconducting piston. The gas is compressed slowly to 12.5% of its initial volume.

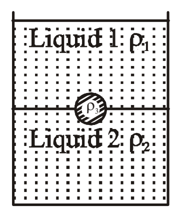
Find final temperature of the gas if it is T_0 initially–
(1) $4T_0$ (2) $3T_0$
(3) $2/3\ T_0$ (4) T_0

15. A point charge +Q is positioned at the center of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is –



(1) $\frac{Q}{16\varepsilon_0}$ (2) $\frac{Q}{4\varepsilon_0}$
(3) $\frac{Q}{8\varepsilon_0}$ (4) None of these

16. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and, ρ_2 respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure. Which of the following is true for ρ_1, ρ_1 and ρ_3?



(1) $\rho_3 < \rho_1 < \rho_2$
(2) $\rho_1 > \rho_3 > \rho_2$
(3) $\rho_1 < \rho_2 < \rho_3$
(4) $\rho_1 < \rho_3 < \rho_2$

17. A point particle of mass 0.1 kg is executing SHM of amplitude of 0.1m. When the particle passes through the mean position, its kinetic energy is 18×10^{-3} J. The equation of motion of this particle when the initial phase of oscillation is 45° can be given by –

(1) $y = 0.1 \cos\left(6t + \frac{\pi}{4}\right)$

(2) $y = 0.1 \sin\left(6t + \frac{\pi}{4}\right)$

(3) $y = 0.4 \sin\left(t + \frac{\pi}{4}\right)$

(4) $y = 0.2 \sin\left(\frac{\pi}{2} + 2t\right)$

18. What happens when the applied load increases and upto breaking stress in the experiment to determine the Young's modulus of elasticity ?

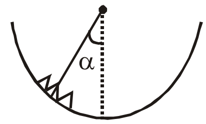
(1) The area of wire goes on decreasing and wire extends and breaks.

(2) The area of wire goes on increasing and wire breaks.

(3) The wire extends and area remains constant.

(4) The area remains same and wire length is also same.

19. An insect crawls up a hemispherical surface very slowly. The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α so that the insect does not slip is given by



(1) $\cot \alpha = 3$ (2) $\sec \alpha = 3$

(3) $\text{cosec}\, \alpha = 3$ (4) $\cos \alpha = 3$

20. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along its length into three equal parts and these parts are then placed on each other with their like poles together. The time period of this combination will be

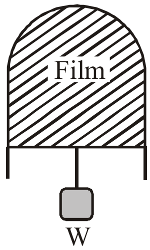
(1) $2\sqrt{3}$ s (2) $\frac{2}{3}$ s

(3) 2 s (4) $\frac{2}{\sqrt{3}}$ s

Section - B

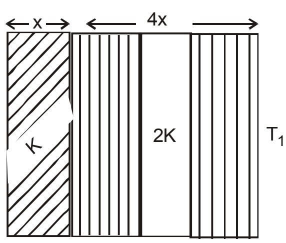
21. A uniform wire of length l and radius r has a resistance of 100 Ω. It is recast into a wire of radius $\frac{r}{2}$. The resistance of new wire will be ________ Ω.

22. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is _________Nm^{-1}.



23. A piece of wood from a recently cut tree shows 20 decays per minute. A wooden piece of same size placed in a museum (obtained from a tree cut many years back) shows 2 decays per minute. If half life of C^{14} is 5730 years, then age of the wooden piece placed in the museum is approximately ________ years.

24. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$, respectively, are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$. Find the value of f.



25. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities?

26. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S , then $S = \frac{ft^2}{x}$. Find the value of x.

27. Consider an optical communication system operating at a wavelength of 800 nm. Suppose, only 1% of the optical source frequency is the available channel bandwidth for optical communication. How many channels can be accommodated for transmitting audio signals requiring a bandwidth of 8 kHz ?

28. The current voltage relation of a diode is given by $I = \left(e^{1000V/T} - 1\right)$ mA, where the applied voltage V is in volts and the temperature T is in degree kelvin. If a student makes an error measuring ± 0.01 V while measuring the current of 5 mA at 300 K, what will be the error in the value of current in ________ mA?

29. Currents of a 10 ampere and 2 ampere are passed through two parallel thin wires A and B respectively in opposite directions. Wire A is infinitely long and the length of the wire B is 2 m.

The force acting on the conductor B, which is situated at 10 cm distance from A will be $x \times 10^{-5}$ N. Find the value of x.

30. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is ______ °C.

CHEMISTRY

Section - A

31. Let ν_1 be the frequency of the series limit of the Lyman series, ν_2 be the frequency of the first line of the Lyman series, and ν_3 be the frequency of the series limit of the Balmer series, then –

(1) $\nu_3 = \frac{1}{2}(\nu_1 - \nu_3)$ (2) $\nu_2 - \nu_1 = \nu_3$

(3) $\nu_1 - \nu_2 = \nu_3$ (4) $\nu_1 + \nu_2 = \nu_3$

32. Match List I with List II and select the correct answer :

List I (Ions)	**List II (Shapes)**
A. ICl_2^-	1. Linear
B. BrF_2^+	2. Pyramidal
C. ClF_4^-	3. Tetrahedral
D. $AlCl_4^-$	4. Square planar
	5. Angular

	A	B	C	D
(1)	1	2	4	5
(2)	4	5	2	3
(3)	1	2	4	3
(4)	5	1	3	4

33. Which one of the following is the correct statement?

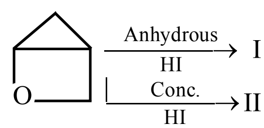
(1) Boric acid is a protonic acid.

(2) Beryllium exhibits coordination number of six.

(3) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase.

(4) $B_2H_6.2NH_3$ is known as 'inorganic benzene'.

34. Choose the correct option for



(1) I and II are identical

(2) I and II are different

(3) Mechanism of formation of I and II are not known

(4) None of these

35. Reducing the pressure from 1.0 atm to 0.5 atm would change the number of molecules in one mole of ammonia to

(1) 25% of its initial value
(2) 50% of its initial value
(3) 75% of its initial value
(4) None of the above

36. Structure of some important polymers are given. Which one represents Buna-S?

(1) $\left(CH_2 - \underset{}{\overset{CH_3}{\overset{|}{C}}} = CH - CH_2 \right)_n$

(2) $\left(CH_2 - CH = CH - CH_2 - \underset{\underset{C_6H_5}{|}}{CH} - CH_2 \right)_n$

(3) $\left(CH_2 - CH = CH - CH_2 - \underset{\underset{CN}{|}}{CH} - CH_2 \right)_n$

(4) $\left(CH_2 - \overset{Cl}{\overset{|}{C}} = CH - CH_2 \right)_n$

37. In electrolysis of NaCl when Pt electrode is taken, then H_2 is liberated at cathode while with Hg cathode it forms sodium amalgam. This is because

(1) Hg is more inert than Pt
(2) more voltage is required to reduce H^+ at Hg than at Pt
(3) Na is dissolved in Hg while it does not dissolve in Pt
(4) conc. of H^+ ions is larger when Pt electrode is taken

38. Which of the following option is having maximum number of unpaired electrons –

(1) A tetrahedral d^6 ion
(2) $[Co(NH_3)_6]^{3+}$
(3) A square planar d^7 ion
(4) A co-ordination compound with magnetic moment of 5.92 B.M.

39. Which of the following carbide does not release any hydrocarbon on reaction with water?

(1) SiC (2) Be_2C
(3) CaC_2 (4) Mg_2C_3

40. An ether (A), $C_5H_{12}O$, when heated with excess of hot concentrated HI produced two alkyl halides which when treated with NaOH yielded compounds (B) and (C). Oxidation of (B) and (C) gave a propanone and an ethanoic acid respectively. The IUPAC name of the ether (A) is:

(1) 2-ethoxypropane
(2) ethoxypropane
(3) methoxybutane
(4) 2-methoxybutane

41. Aqueous solution of (M) + $(NH_4)_2S \rightarrow$ yellow ppt (B) $\xrightarrow{(NH_4)_2S_2}$ insoluble.

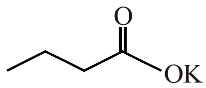
The cation present in (M) is –

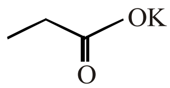
(1) CdS (2) SnS_2
(3) Cd^{2+} (4) Sn^{2+}

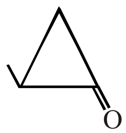
42. Both geometrical and optical isomerisms are shown by

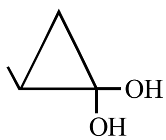
(1) $[Co(en)_2Cl_2]^+$ (2) $[Co(NH_3)_5Cl]^{2+}$
(3) $[Co(NH_3)_4Cl_2]^+$ (4) $[Cr(ox)_3]^{3-}$

43. $Me-CH=CH_2+CHCl_3 \xrightarrow[\text{excess, }\Delta]{\text{aq. KOH}}$ A (Major products) is –

(1) 

(2) 

(3) 

(4) 

44. The decomposition of N_2O_5 in carbon tetrachloride was followed by measuring the volume of O_2 gas evolved :

$$2N_2O_5(CCl_4) \rightarrow 2N_2O_4(CCl_4)+O_2(g).$$

The maximum volume of O_2 gas obtained was 100 cm^3. In 500 minutes, 90 cm^3 of O_2 were evolved. The first order rate constant (in min^{-1}) for the disappearance of N_2O_5 is :

(1) $\frac{2.303}{500}$ (2) $\frac{2.303}{500}\log\frac{100}{90}$

(3) $\frac{2.303}{500}\log\frac{90}{100}$ (4) $\frac{100}{10\times 500}$

45. The basic character of the transition metal monoxides follows the order

(Atomic Nos., Ti = 22, V = 23, Cr = 24, Fe = 26)

(1) TiO > VO > CrO > FeO

(2) VO > CrO > TiO > FeO

(3) CrO > VO > FeO > TiO

(4) TiO > FeO > VO > CrO

46. $NH_4ClO_4 + HNO_3\,(\text{dilute}) \longrightarrow X + HClO_4$

$X \xrightarrow{\text{heat}} Y\,(\text{gas})$ Gas (Y) is –

(1) O_2 (2) N_2

(3) NO_2 (4) N_2O

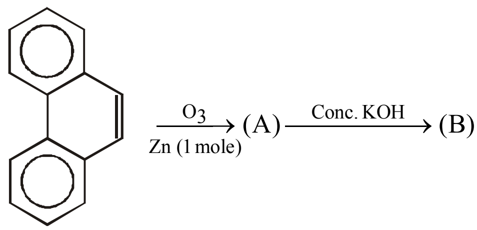
47. Which of the following is/are formed when ozone reacts with the unburnt hydrocarbons in polluted air ?

(i) Formaldehyde (ii) Acrolein

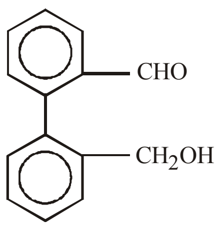
(iii) Peroxyacetyl nitrate (iv) Formic acid

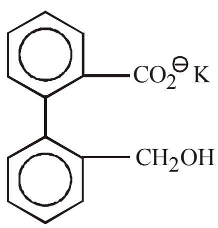
(1) (i) and (iv) (2) (ii) only

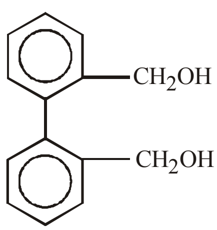
(3) (iii) only (4) (i), (ii) and (iii)

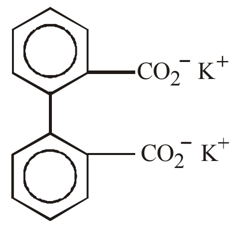
48. 

End product (B) of above reaction is –

(1) 

(2) 

(3) 

(4) 

49. Precautions to be taken in the study of reaction rate for the reaction between potassium iodate (KIO_3) and sodium sulphite (Na_2SO_3) using starch solution as indicator at different concentrations and temperature –

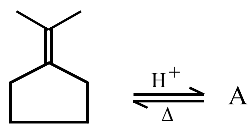
(1) The concentration of sodium thiosulphate solution should always be less than the concentration of the potassium iodide solution.

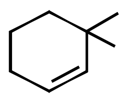
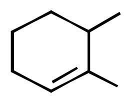
(2) Freshly prepared starch solution should be used.

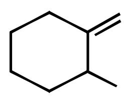
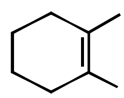
(3) Experiments should be performed with the fresh solutions of H_2O_2 and KI.

(4) All of these

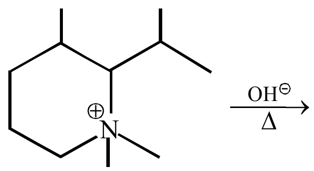
50. Predict the product (A) of the following reaction



(1) 
(2) 

(3) 
(4) 

Section - B

51. 

Find the number of α-H in alkene which is major product in this reaction?

52. At 675K, $H_2(g)$ and $CO_2(g)$ react to form CO(g) and H_2O (g), K_p for the reaction is 0.16. If a mixture of 0.25 mole of $H_2(g)$ and 0.25 mol of CO_2 is heated at 675K, calculate the mole % of CO(g) in equilibrium mixture

53. In the spinel structure, oxides ions are cubical-closest packed whereas 1/8th of tetrahedral voids are occupied by A^{2+} cation and 1/2 of octahedral voids are occupied by B^{3+} cations. The general formula of the compound having spinel structure is AB_nO_{2n}. Find the value of n.

54. The value of P° for benzene is 640 mm of Hg. The vapour pressure of solution containing 2.5g substance in 39g benzene is 600mm of Hg. Find the molecular mass of X.

55. Find the pH of a 2 litre solution which is 0.1 M each with respect to CH_3COOH and $(CH_3OO)_2Ba$. ($K_a = 1.8 \times 10^{-5}$)

56. H_3PO_2 is the molecular formula of an acid of phosphorus. What is its basicity?

57. A gas present in a cylinder fitted with a frictionless piston expands against a constant pressure of 1 atm from a volume of 2 litre to a volume of 6 litre. In doing so, it absorbs 800 J heat from surroundings. Determine increase in internal energy of process.

58. Calculate the number of metamers represented by molecular formula $C_4H_{10}O$.

59. A current of 2.0 A passed for 5 hours through a molten metal salt deposits 22.2 g of metal (At wt. = 177). Find the oxidation state of the metal in the metal salt.

60. The instantaneous rate of disappearance of MnO_4^- ion in the following reaction is $4.56 \times 10^{-3} Ms^{-1}$

$$2MnO_4^- + 10I^- + 16H^+ \rightarrow 2Mn^{2+} + 5I_2 + 8H_2O$$

The rate of appearance I_2 is $x \times 10^{-2}$ Ms^{-1}

MATHEMATICS

Section - A

61. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equation can represent L_1 ?

(1) $x + 7y = 0$ (2) $x - y = 0$

(3) $x - 7y = 0$ (4) Both (1) and (2)

62. Let α_1, α_2 and β_1, β_2 be the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively. If the system of equations $\alpha_1 y + \alpha_2 z = 0$ and $\beta_1 y + \beta_2 z = 0$ has a non-trivial solution, then

(1) $\frac{b^2}{q^2} = \frac{ac}{pr}$ (2) $\frac{c^2}{r^2} = \frac{ab}{pq}$

(3) $\frac{a^2}{p^2} = \frac{bc}{qr}$ (4) None of these

63. If $f''(x) < 0, \forall x \in (a, b)$, then $f'(x) = 0$ occurs

(1) exactly once in (a, b)

(2) atmost once in (a, b)

(3) atleast once in (a, b)

(4) None of these

64. If the function $f : [0, 16] \to R$ is differentiable. If $0 < \alpha < 1$ and $1 < \beta < 2$, then $\int_0^{16} f(t)\,dt$ is equal to–

(1) $4[\alpha^3 f(\alpha^4) - \beta^3 f(\beta^4)]$

(2) $4[\alpha^3 f(\alpha^4) + \beta^3 f(\beta^4)]$

(3) $4[\alpha^4 f(\alpha^3) + \beta^4 f(\beta^3)]$

(4) $4[\alpha^2 f(\alpha^2) + \beta^2 f(\beta^2)]$

65. Three distinct points P $(3u^2, 2u^3)$, Q $(3v^2, 2v^3)$ and R $(3w^2, 2w^3)$ are collinear then –

(1) $uv + vw + wu = 0$ (2) $uv + vw + wu = 3$

(3) $uv + vw + wu = 2$ (4) $uv + ww + wu = 1$

66. Let 'a' denote the roots of equation

$$\cos(\cos^{-1} x) + \sin^{-1} \sin\left(\frac{1+x^2}{2}\right) = 2\sec^{-1}(\sec x)$$

then possible values of $[\,|10a|\,]$ where $[\,.\,]$ denotes the greatest integer function will be

(1) 1 (2) 5

(3) 10 (4) Both (1) and (3)

67. The image of the pair of lines represented by: $ax^2 + 2hxy + by^2 = 0$ by the line mirror $y = 0$ is

(1) $ax^2 - 2hxy - by^2 = 0$

(2) $bx^2 - 2hxy + ay^2 = 0$

(3) $bx^2 + 2hxy + ay^2 = 0$

(4) $ax^2 - 2hxy + by^2 = 0$

68. If $x^2 - 2x\cos\theta + 1 = 0$, then the value of $x^{2n} - 2x^n \cos n\theta + 1, n \in N$ is equal to –

(1) $\cos 2n\theta$

(2) $\sin 2n\theta$

(3) 0

(4) some real number greater than 0

69. If $I_1 = \int_0^1 2^{x^2}dx$, $I_2 = \int_0^1 2^{x^3}dx$, $I_3 = \int_1^2 2^{x^2}dx$ and $I_4 = \int_1^2 2^{x^3}dx$ then

(1) $I_2 > I_1$ (2) $I_1 > I_2$

(3) $I_3 = I_4$ (4) $I_3 > I_4$

70. The value of the definite integral

$$\int_0^{3\pi/4} [(1+x)\sin x + (1-x)\cos x]\,dx \text{ is –}$$

(1) $2(\sqrt{2}+1)$ (2) $\sqrt{2}+1$

(3) $2\sqrt{2}$ (4) 0

71. The locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is –

(1) $9x + 10y - 7 = 0$ (2) $x - y + 2 = 0$

(3) $9x - 10y + 11 = 0$ (4) $9x + 10y + 7 = 0$

72. The sum of the first n terms of the series

$$1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$$

is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is

(1) $\left[\frac{n(n+1)}{2}\right]^2$ (2) $\frac{n^2(n+1)}{2}$

(3) $\frac{n(n+1)^2}{4}$ (4) $\frac{3n(n+1)}{2}$

73. The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is –

(1) $x^2 + 2y^2 - ax = 0$ (2) $2x^2 + y^2 - 2ax = 0$

(3) $2x^2 + 2y^2 - ay = 0$ (4) $2x^2 + y^2 - 2ay = 0$

74. If $z + 1/z = 2 \cos \theta$, then the value of $|(z^{2n} - 1)/(z^{2n} + 1)|$

(1) $|\tan n\theta|$ (2) $\tan n\theta$

(3) $|\cot n\theta|$ (4) $\cot n\theta$

75. The two curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 = 2$

(1) cuts at right angle (2) touch each other

(3) cut at an angle $\frac{\pi}{3}$ (4) cut at an angle $\frac{\pi}{4}$

76. If the pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then

(1) $3a^2 - 10ab + 3b^2 = 0$

(2) $3a^2 - 2ab + 3b^2 = 0$

(3) $3a^2 + 10ab + 3b^2 = 0$

(4) $3a^2 + 2ab + 3b^2 = 0$

77. Which of the following is a contradiction?

(1) $(p \wedge q) \wedge \sim (p \vee q)$ (2) $p \vee (-p \wedge q)$

(3) $(p \Rightarrow q) \Rightarrow p$ (4) None of these

78. If $1, \omega, \omega^2, ...\omega^{n-1}$ are the n roots of unity, then $(1-\omega)(1-\omega^2)........(1-\omega^{n-1})$ equals

(1) 0 (2) 2

(3) n (4) n^2

79. Set of values of m for which two points P and Q lie on the line y = mx + 8 so that $\angle APB = \angle AQB = \frac{\pi}{2}$ where $A \equiv (-4, 0), B \equiv (4, 0)$ is –

(1) $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty) - \{-2, 2\}$

(2) $[-\sqrt{3}, -\sqrt{3}] - \{-2, 2\}$

(3) $(-\infty, -1) \cup (1, \infty) - \{-2, 2\}$

(4) $\{-\sqrt{3}, \sqrt{3}\}$

80. The trace $T_r(A)$ of a 3×3 matrix $A = (a_{ij})$ is defined by the relation $T_r(A) = a_{11} + a_{22} + a_{33}$ (i.e., $T_r(A)$ is sum of the main diagonal elements). Which of the following statements cannot hold ?

(1) $T_r(kA) = kT_r(A)$ (k is a scalar)

(2) $T_r(A + B) = T_r(A) + T_r(B)$

(3) $T_r(I_3) = 3$

(4) $T_r(A^2) = T_r(A)^2$

Section - B

81. The area above the x–axis enclosed by the curves $x^2 - y^2 = 0$ and $x^2 + y - 2 = 0$ is_____.

82. If number of permutations 1, 2, 3, 4, 5, 6, 7, 8 and 9 taken all at a time are such that the digit. 1 appearing somewhere to the left of 2, 3 appearing to the left of 4 and 5 somewhere to the left of 6, is equal to k.7!, then value of k is _____.

(e.g., 815723946 would be one such permutation)

83. Vertices of a parallelogram taken in order are A (2, –1, 4), B (1, 0, –1), C (1, 2, 3) and D. If distance of the point P (8, 2, –12) from the plane of the parallelogram is $\frac{k\sqrt{6}}{9}$, then value of k is__.

84. Given

$\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{C} = \hat{i} + 2\hat{j} + \hat{k}$.

The value of $|\vec{A} \times [\vec{A} \times (\vec{A} \times \vec{B})].\vec{C}|$ is _____.

85. If area of triangle formed by common tangents to the circles $x^2 + y^2 - 6x = 0$ and $x^2 + y^2 + 2x = 0$ is $\lambda\sqrt{3}$, then value of λ is ____.

86. A box contains 6 red, 5 blue and 4 white marbles. Four marbles are chosen at random without replacement. The probability that there is atleast one marble of each colour among the four chosen, is ______.

87. Period of $\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta}$ is $\frac{K\pi}{3}$,then k is ____.

88. If the tangent at the point $\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse $16x^2 + 11y^2 = 256$ is also a tangent to the circle $x^2 + y^2 - 2x = 15$, then the value of θ is $\pm\frac{\pi}{K}$. The value of k is _____.

89. One percent of the population suffers from a certain disease. There is blood test for this disease, and it is 99% accurate, in other words, the probability that it gives the correct answer is 0.99, regardless of whether the person is sick or healthy. A person takes the blood test, and the result says that he has the disease. The probability that he actually has the disease, is _____.

90. Let $a_n = \int_0^{\pi/2} (1 - \sin t)^n \sin 2t \, dt$ then

$\lim_{n \to \infty} \sum_1^n \frac{a_n}{n}$ is ______.

Mock Test-3

ANSWER KEY											
1.	(3)	**16.**	(4)	**31.**	(3)	**46.**	(4)	**61.**	(4)	**76.**	(4)
2.	(1)	**17.**	(2)	**32.**	(3)	**47.**	(4)	**62.**	(1)	**77.**	(1)
3.	(1)	**18.**	(1)	**33.**	(3)	**48.**	(2)	**63.**	(2)	**78.**	(3)
4.	(2)	**19.**	(1)	**34.**	(1)	**49.**	(4)	**64.**	(2)	**79.**	(1)
5.	(1)	**20.**	(2)	**35.**	(4)	**50.**	(4)	**65.**	(1)	**80.**	(4)
6.	(1)	**21.**	(1600)	**36.**	(2)	**51.**	(2)	**66.**	(4)	**81.**	(2.33)
7.	(2)	**22.**	(0.025)	**37.**	(2)	**52.**	(14.28)	**67.**	(4)	**82.**	(9.00)
8.	(3)	**23.**	(19039)	**38.**	(4)	**53.**	(2)	**68.**	(3)	**83.**	(32.00)
9.	(2)	**24.**	(0.33)	**39.**	(1)	**54.**	(80)	**69.**	(2)	**84.**	(343.00)
10.	(1)	**25.**	(1.25)	**40.**	(1)	**55.**	(5.0)	**70.**	(1)	**85.**	(3.00)
11.	(3)	**26.**	(72)	**41.**	(3)	**56.**	(1)	**71.**	(3)	**86.**	(0.53)
12.	(3)	**27.**	(4.8×10^8)	**42.**	(1)	**57.**	(395)	**72.**	(2)	**87.**	(2.00)
13.	(2)	**28.**	(0.2)	**43.**	(2)	**58.**	(3)	**73.**	(2)	**88.**	(3.00)
14.	(1)	**29.**	(8)	**44.**	(1)	**59.**	(3)	**74.**	(1)	**89.**	(0.50)
15.	(3)	**30.**	(10)	**45.**	(1)	**60.**	(1.14)	**75.**	(1)	**90.**	(0.50)

Solutions

PHYSICS

1. (3) For total internal reflection on face AC
$\theta >$ critical angle (C) and $\sin\theta \geq \sin C$

$$\sin\theta \geq \frac{1}{{}^{w}\mu_g}$$

$$\sin\theta \geq \frac{\mu_w}{\mu_g} \Rightarrow \sin\theta \geq \frac{\frac{4}{3}}{\frac{3}{2}}$$

$$\therefore \sin\theta \geq \frac{8}{9}.$$

2. (1) As we know,

Gravitational potential energy $= \frac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right) = \frac{-GMm}{6R}$$

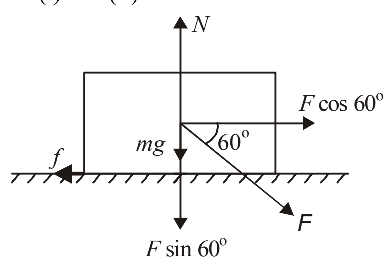
$$E_i = \frac{-GMm}{R} + K$$

$$E_i = E_f$$

Therefore minimum required energy,

$$K = \frac{5GMm}{6R}$$

3. (1) The forces acting on the block are shown. Since the block is not moving forward for the maximum force F applied, therefore
$F\cos 60° = f = \mu N$... (i) (Horizontal Direction)
Note: For maximum force F, the frictional force is the limiting friction $= \mu N$]
and $F\sin 60° + mg = N$... (ii)
From (i) and (ii)

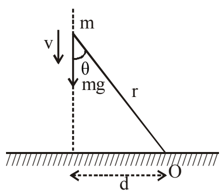


$F\cos 60° = \mu\,[F\sin 60° + mg]$

$$\Rightarrow \quad F = \frac{\mu mg}{\cos 60° - \mu \sin 60°}$$

$$= \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{5}{\frac{1}{4}} = 20\,\text{N}$$

4. (2) If student will use angular momentum $= mvr$.
He/she may conclude answer (1) as r is decreasing angular momentum must decrease hence (1) is incorrect.



But the magnitude of angular momentum of particle about O = mvd
Since speed v of particle increases, its angular momentum about O increases.
Magnitude of torque of gravitational force about O = mgd ⇒ constant
Moment of inertia of particle about O = mr^2
Hence MI of particle about O decreases.

Angular velocity of particle about $O = \frac{v\sin\theta}{r}$

∵ v and sin θ increases and r decreases.
∴ angular velocity of particle about O increases.

5. (1) As, $P = \frac{1}{3}\left(\frac{U}{V}\right)$

But $\frac{U}{V} = KT^4$

So, $P = \frac{1}{3}KT^4$

or $\frac{uRT}{V} = \frac{1}{3}KT^4$ [As PV = u RT]

$\frac{4}{3}\pi R^3 T^3 = \text{constant}$

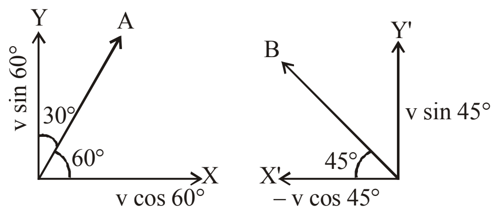
Therefore, $T \propto \frac{1}{R}$

6. (1) For particle C,
According to law of conservation of linear momentum, verticle component,
$2\,mv'\sin\theta = mv\sin 60° + mv\sin 45°$

$$2mv'\sin\theta = \frac{mv}{\sqrt{2}} + \frac{mv\sqrt{3}}{2} \qquad \text{...... (i)}$$

Horizontal component,
$2\,mv'\cos\theta = mv\sin 60° - mv\cos 45°$

$$2mv'\cos\theta = \frac{mv}{2} - \frac{mv}{\sqrt{2}} \qquad \text{...... (ii)}$$



For particle A **For particle B**

Dividing eq^n (i) by eq^n (ii),

$$\tan\theta = \frac{\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}}{\frac{1}{2} - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}+\sqrt{3}}{1-\sqrt{2}}$$

7. (2) $n_0 = \frac{v}{2\ell}$,

$n_1 = \frac{v}{2\,(\ell/2 - \Delta\ell)}$, $n_2 = \frac{v}{2\,(\ell/2 + \Delta\ell)}$

Beat frequency = $n_1 - n_2$

$$\Rightarrow v\left[\frac{1}{\ell - 2\Delta\ell} - \frac{1}{\ell + 2\Delta\ell}\right]$$

$$= v\left[\frac{(\ell + 2\Delta\ell) - (\ell - 2\Delta\ell)}{\ell^2 - 4\Delta\ell^2}\right]$$

$$= v\frac{4\Delta\ell}{\ell^2 - 4\Delta\ell^2} = \frac{8}{\ell}\frac{\Delta\ell v}{2\ell} = \frac{8\Delta\ell n_0}{\ell}$$

8. (3) Wavelength for which maximum obtained at the hole has the maximum intensity on passing.

So, $x = \frac{n\lambda D}{d}$

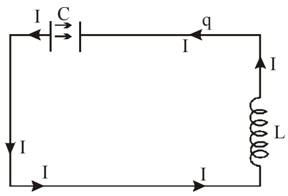
$$\lambda = \frac{xd}{nD} = \frac{1\times10^{-3}\times0.5\times10^{-3}}{n\times50\times10^{-2}}$$

$= \frac{1\times10^{-6}}{n} = \frac{1000\text{nm}}{n}$

$n = 1,\ \lambda = 1000\text{ nm} \rightarrow$ Not in the given range

$n = 2, \lambda = 500\text{nm}$

9. (2)



As it can be easily seen by the direction of I that Q is decreasing thus, energy of capacitor is decreasing and hence, energy of inductance is increasing or $\left(\frac{1}{2}LI^2\right)$ gives that I is increasing.

10. (1) Capacitors 2μF and 2μF are parallel, their equivalent = 4 μF

6μF and 12 μF are in series, their equivalent = 4 μF

Now 4μF (2 and 2 μF) and 8μF in series $= \frac{3}{8}\mu F$

And 4μF (12 & 6 μF) and 4μF in parallel = 4 + 4 = 8μF

8μF in series with $1\mu F = \frac{1}{8} + 1 \Rightarrow \frac{8}{9}\mu F$

Now $C_{eq} = \frac{8}{9} + \frac{8}{3} = \frac{32}{9}$

With C, $\frac{1}{C_{eq}} = \frac{1}{C} + \frac{9}{32} = 1 \Rightarrow C = \frac{32}{23}\mu F$

11. (3) From question,

Horizontal velocity (initial),

$u_x = \frac{40}{2} = 20\text{m/s}$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2} g t^2$

$\Rightarrow \quad u_y \times 2 + \frac{1}{2}(-10)\times 4$

or, $\quad 50 = 2u_y - 20$

or, $\quad u_y = \frac{70}{2} = 35\text{m/s}$

$\therefore \quad \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$

$\Rightarrow \quad$ Angle $\theta = \tan^{-1}\frac{7}{4}$

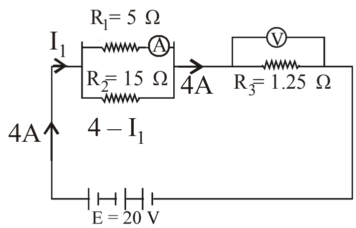
12. (3) As insect moves along a diameter, the effective mass and hence the M.I. first decreases then increases so from principle of conservation of angular momentum, angular speed, first increases then decreases.

13. (2) $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$ (Parallel combination);

$R_{Net} = R_{eg} + R_3$(Series combination)

R_{Net} of the circuit

$= \frac{5\times15}{5+15} + \frac{125}{100} = \frac{75}{20} + \frac{5}{4} = 5\,\Omega$



We know that $I = \frac{E}{R_{eq}} = \frac{20}{5} = 4\text{ A}$

Potential difference across R_1 and R_2 are same (Parallel combination) $I_1R_1 = (4 - I_1)R_2$

$\Rightarrow 5I_1 = (4 - I_1)\times 15 \Rightarrow I_1 = 12 - 3I_1 \Rightarrow I_1 = 3A$

Thus reading of ammeter = 3A

Voltage across $1.25\Omega = I \times R = 4 \times 1.25 = 5V$ (Reading of voltmeter)

14. (1) Let initial temperature and volume be T_0 and V_0.

Since the process is adiabatic, the first temperature and volume is $TV^{\gamma-1} = T_0V_0^{\gamma-1}$ (γ = 5/3 for monoatomic gas)

$\therefore \ T = T_0\left(\frac{V_0}{V_0/8}\right)^{2/3} = 4T_0$

15. (3) Flux going in pyramid $= \frac{Q}{2\varepsilon_0}$

Which is divided equally among all 4 faces

$\therefore$ Flux through one face $= \frac{Q}{8\varepsilon_0}$

16. (4) From the figure it is clear that liquid 1 floats on liquid 2. The lighter liquid floats over heavier liquid. Therefore we can conclude that $\rho_1 < \rho_2$
Also $\rho_3 < \rho_2$ otherwise the ball would have sink to the bottom of the jar.
Also $\rho_3 > \rho_1$ otherwise the ball would have floated in liquid 1. From the above discussion we conclude that $\rho_1 < \rho_3 < \rho_2$.

17. (2) A = 0.1m, m = 0.1 kg, $KE_{max} = 18 \times 10^{-3}$ J, $\phi = \frac{\pi}{4}$

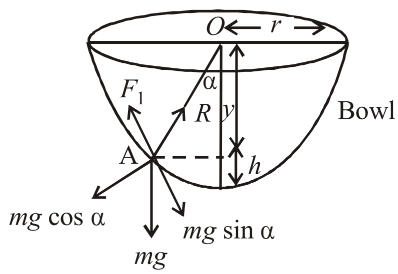
$$k = \frac{36 \times 10^{-3}}{(0.1)^2} = 3.6\ ;$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.6}{0.1}} = 6 \text{ rad/s}$$

$\therefore$ Eqn. $y = 0.1 \sin\left(6t + \frac{\pi}{4}\right)$

18. (1) First, the length of wire goes on increasing ie., area decreases and finally at breaking stress the wire breaks.

19. (1)



The insect crawls up the bowl upto a certain height h only till the component of its weight along the bowl is balanced by limiting frictional force.
For limiting condition at point A

$R = mg\cos\alpha$ ...(i)

$F_1 = mg\sin\alpha$ ...(ii)

Dividing eq. (ii) by (i)

$$\tan\alpha = \frac{1}{\cot\alpha} = \frac{F_1}{R} = \mu\left[As\ F_1 = \mu R\right]$$

$$\Rightarrow \quad \tan\alpha = \mu = \frac{1}{3}\left[\because \mu = \frac{1}{3}(\text{Given})\right]$$

$\therefore \quad \cot\alpha = 3$

20. (2) $T = 2\pi\sqrt{\frac{I}{M \times B}} = 2\pi\sqrt{\frac{I}{MB}}$

where $I = \frac{1}{12}m\ell^2$

When the magnet is cut into three pieces the pole strength will remain the same and

$$\text{M.I. } (I') = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)^2 \times 3 = \frac{I}{9}$$

We have, Magnetic moment (M)
= Pole strength (m) × ℓ
$\therefore$ New magnetic moment,

$$M' = m \times \left(\frac{\ell}{3}\right) \times 3 = m\ell = M$$

$$\therefore\ T' = \frac{T}{\sqrt{9}} = \frac{2}{3}s.$$

21. (1600) Given, $R_1 = 100\,\Omega$, $r' = r/2$, $R_2 = ?$

Resistance of wire, $R = \frac{\rho l}{A}$

$\because$ Area × length = volume

Hence, $R = \frac{\rho V}{A^2}$

Since, $\rho \to$ constant, $V \to$ constant

$$R \propto \frac{1}{A^2}$$

or $\quad R \propto \frac{1}{r^4} \qquad \because A = \pi r^2$

$$\frac{R_2}{R_1} = 16 \Rightarrow R_2 = 16 \times 100 = 1600\,\Omega$$

22. (0.025) At equilibrium, weight of the given block is balanced by force due to surface tension, i.e.,
2L. S = W

or $S = \frac{W}{2L} = \frac{1.5 \times 10^{-2} N}{2 \times 0.3\ m} = 0.025 Nm^{-1}$

23. (19039) Given: $\frac{dN_0}{dt} = 20$ decays/min

$\frac{dN}{dt} = 2$ decays/min

$T_{1/2} = 5730$ years
As we know,

$$N = N_0 e^{-\lambda t}$$

$$\log\frac{N_0}{N} = \lambda t$$

$$\therefore \quad t = \frac{1}{\lambda}\log\frac{N_0}{N}$$

$$= \frac{2.303\times T_{1/2}}{0.693}\times\log_{10}\frac{N_0}{N}$$

But $\dfrac{\frac{dN_0}{dt}}{\frac{dN}{dt}} = \dfrac{N_0}{N} = \dfrac{20}{2} = 10$

$$\therefore \; t = \frac{2.303\times 5730}{0.693}\times 1$$

$= 19039$ years

24. (0.33) The thermal resistance is given by

$$\frac{x}{KA}+\frac{4x}{2KA} = \frac{x}{KA}+\frac{2x}{KA} = \frac{3x}{KA}$$

$$\therefore \frac{dQ}{dt} = \frac{\Delta T}{\frac{3x}{KA}} = \frac{(T_2-T_1)KA}{3x}$$

$$= \frac{1}{3}\left\{\frac{A(T_2-T_1)K}{x}\right\}$$

$$\therefore f = \frac{1}{3}$$

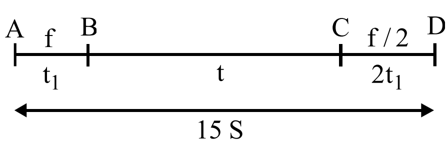
25. (1.25) $\dfrac{I_e}{I_h} = \dfrac{n_e eAv_e}{n_h eAv_h} \Rightarrow \dfrac{7}{4} = \dfrac{7}{5}\times\dfrac{v_e}{v_h} \Rightarrow \dfrac{v_e}{v_h} = \dfrac{5}{4}$

26. (72) Distance from A to B $= S = \dfrac{1}{2}{ft_1}^2$

Distance from B to C $= (ft_1)t$

Distance from C to D $= \dfrac{u^2}{2a} = \dfrac{(ft_1)^2}{2(f/2)}$

$= {ft_1}^2 = 2S$



$\Rightarrow \quad S + f\,t_1 t + 2S = 15S$

$\Rightarrow \; f\,t_1 t = 12S$ .... (i)

$\dfrac{1}{2}f\,t_1^2 = S$ .... (ii)

Dividing (i) by (ii), we get $t_1 = \dfrac{t}{6}$

$$\Rightarrow \; S = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{f\,t^2}{72}$$

27. (4.8×10^8) Optical source frequency

$$f = \frac{c}{\lambda} = 3\times 10^8/(800\times 10^{-9}) = 3.8\times 10^{14}\,\text{Hz}$$

Bandwidth of channel (1% of above)
$= 3.8\times 10^{12}$Hz
Number of channels = (Total bandwidth of channel) / (Bandwidth needed per channel)
Number of channels for audio signal

$$= (3.8\times 10^{12})/(8\times 10^3) \sim 4.8\times 10^8$$

28. (0.2) The current voltage relation of diode is

$I = (e^{1000\,V/T}-1)$ mA (given)

When, $I = 5mA,\; e^{1000\,V/T} = 6mA$

Also, $dI = (e^{1000\,V/T})\times\dfrac{1000}{T}$

(By exponential function)

$= (6\,mA)\times\dfrac{1000}{300}\times(0.01) = 0.2$ mA

29. (8) Force acting on conductor B due to conductor A is given by relation

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

l-length of conductor B
r-distance between two conductors

$$\therefore \; F = \frac{4\pi\times 10^{-7}\times 10\times 2\times 2}{2\times\pi\times 0.1} = 8\times 10^{-5}\text{ N}$$

30. (10) By Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t} = -K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

where θ_0 is the temperature of surrounding.
Now, hot water cools from 60°C to 50°C in 10 minutes,

$$\frac{60-50}{10} = -K\left[\frac{60+50}{2}-\theta_0\right] \quad ...(i)$$

Again, it cools from 50°C to 42°C in next 10 minutes.

$$\frac{50-42}{10} = -K\left[\frac{50+42}{2} - \theta_0\right] \quad \text{...(ii)}$$

Dividing equations (i) by (ii) we get

$$\frac{1}{0.8} = \frac{55-\theta_0}{46-\theta_0}$$

$$\frac{10}{8} = \frac{55-\theta_0}{46-\theta_0}$$

$$460 - 10\theta_0 = 440 - 8\theta_0$$

$$2\theta_0 = 20$$

$$\theta_0 = 10°C$$

CHEMISTRY

31. (3) $\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\nu_1 = RcZ^2\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = RcZ^2$$

$$\nu_2 = RcZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3RcZ^2}{4}$$

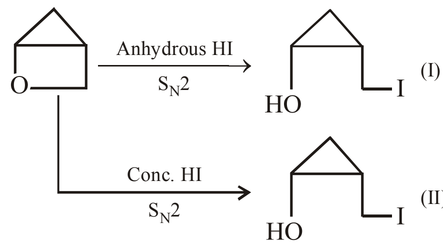
$$\nu_3 = RcZ^2\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{RcZ^2}{4}$$

$\therefore\ \nu_1 - \nu_2 = \nu_3$

32. (3) $ICl_2^- \Rightarrow 2\,bp + 3\,lp$
(thus, sp^3d hybridisation) = linear
$BrF_2^+ \Rightarrow 2\,bp + 2\,lp$
(thus, sp^3 hybridisation) = pyramidal
$ClF_4^- \Rightarrow 4\,bp + 2\,lp$
(thus, sp^3d^2 hybridisation) = square planar
$AlCl_4^- \Rightarrow 4\,bp + 0\,lp$
(thus sp^3 hybridisation) = tetrahedral

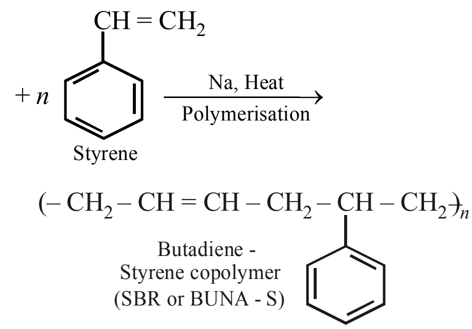
33. (3) The correct formula of inorganic benzene is $B_3N_3H_6$ so (4) is incorrect statement.
Boric acid (H_3BO_3 or $\begin{array}{c} OH \\ | \\ B-OH \\ | \\ OH \end{array}$) is a Lewis acid so (1) is incorrect statement.
The coordination number exhibited by beryllium is 4 and not 6 so statement (2) is incorrect.
Both $BeCl_2$ and $AlCl_3$ exhibit bridged structures in solid state so (3) is correct statement.

34. (1)



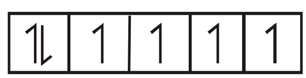
35. (4) One mole of a substance contains the number of molecules which is independent of pressure.

36. (2) $nCH_2 = CH - CH = CH_2$
1, 3-Butadiene

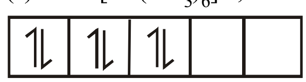


37. (2) In electrolysis of NaCl when Pt electrode is taken, then H_2 liberated at cathode, while with Hg cathode it forms sodium amalgam because more voltage is required to reduce H^+ at Hg than at Pt.

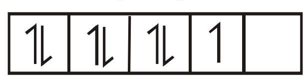
38. (4)
(1) For tetrahedral d^6 ion,



4 unpaired electrons
(2) For $[Co(NH_3)_6]^{3+}$,



0 unpaired electrons
(3) For square planar d^7 ion,



1 unpaired electrons

(4) B.M. = $\sqrt{n(n+2)}$, n = unpaired electrons

$5.92\ \text{B.M.} = \sqrt{n(n+2)}$

n = 5 unpaired electrons

39. (1) (1) $SiC \Rightarrow$ Covalent carbide

(2) $Be_2C + 4H_2O \longrightarrow 2Be(OH)_2 + CH_4\uparrow$

(3) $CaC_2 + 2H_2O \longrightarrow HC{\equiv}CH\uparrow + Ca(OH)_2$

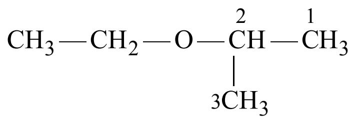
(4) $Mg_2C_3 + 2H_2O \longrightarrow 2Mg(OH)_2 + H_3C{-}C \equiv CH\uparrow$

40. (1) $\underset{(A)}{CH_3CH_2OCH(CH_3)_2} \xrightarrow{\text{Conc. HI}} \underset{(B)}{CH_3CH_2I} + \underset{(C)}{(CH_3)_2CHI}$

$\underset{(B)}{CH_3CH_2I} \xrightarrow{NaOH} CH_3CH_2OH \xrightarrow{[O]} CH_3COOH$

$\underset{(C)}{(CH_3)_2CHI} \xrightarrow{NaOH} (CH_3)_2CHOH \xrightarrow{[O]} (CH_3)_2CO$

hence the IUPAC name of compound is



2–ethoxy propane

41. (3)

(1) $\underset{\text{Yellow}}{CdS\downarrow} + (NH_4)_2S \longrightarrow \underset{\text{Yellow}}{CdS\downarrow} \xrightarrow{(NH_4)_2S_2} \underset{\text{Insoluble}}{CdS\downarrow}$

(2) $SnS_2\downarrow + (NH_4)_2S \longrightarrow \underset{\text{Yellow}}{CdS\downarrow} \xrightarrow{(NH_4)_2S_2} \underset{\text{Soluble}}{(NH_4)_2\,SnS_3}$

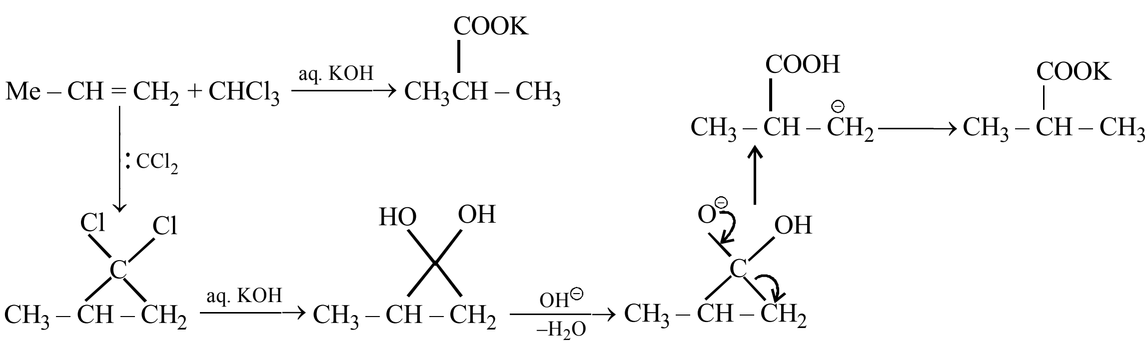
(3) $Cd^{2+} + (NH_4)_2S \longrightarrow \underset{\text{Yellow}}{CdS\downarrow} \xrightarrow{(NH_4)_2S_2} \underset{\text{Insoluble}}{CdS\downarrow}$

(4) $Sn^{2+} + (NH_4)_2S \longrightarrow \underset{\text{Brown}}{SnS\downarrow} \xrightarrow{(NH_4)_2S_2} (NH_4)_2SnS_3$

(Amm. thiostannate)
Soluble

42. (1) The compounds of the type $M(AA)_2B_2$ exhibit both geometrical and optical isomerism.

43. (2)



44. (1)

$$2N_2O_5(CCl_4) \longrightarrow 2N_2O_4(CCl_4) + O_2(g)$$

$t = 0$	a mol	0
$t = 500$ min.	$a - x$	$\frac{x}{2}$ mol
$t = \infty$	0	$\frac{a}{2}$ mol

$a = 100$
$x = 90$

$$k = \frac{1}{t} \ln \frac{a}{a-x} = \frac{2.303}{500}$$

45. (1) The order of basic character of the transition metal monoxide is TiO > VO > CrO > FeO because basic character of oxides decreases with increase in atomic number.

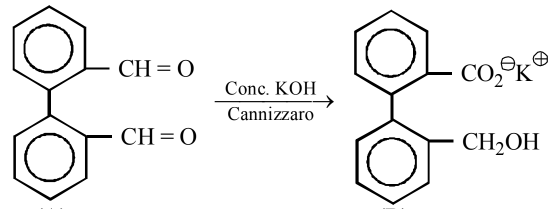
46. (4) $NH_4ClO_4 + HNO_3 \text{(dilute)} \longrightarrow \underset{(X)}{NH_4NO_3} + HClO_4$

$$\underset{(X)}{NH_4NO_3} \xrightarrow{\text{heat}} \underset{(Y)}{N_2O} + 2H_2O$$

47. (4) $3CH_4 + 2O_3 \longrightarrow \underset{\text{Formaldehyde}}{3CH_2{=}O} + 3H_2O$

$\underset{\text{Acrolein}}{CH_2 = CHCH = O}$ $\quad \underset{\text{Peroxyacetyl nitrate (PAN)}}{CH_3COONO_2}$

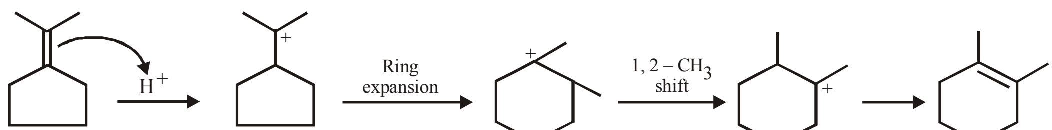
48. (2)



(A) (B)

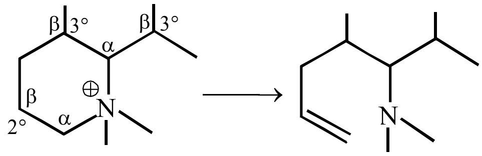
49. (4) (1) The concentration of sodium thiosulphate solution should always be less than the concentration of the potassium iodide solution.
(2) Freshly prepared starch solution should be used
(3) Experiments should be performed with the fresh solutions of H_2O_2 and KI.

50. (4)



(3° Carbocation stable)

51. (2)



(2αH)

52. (14.28) $H_2(g) + CO_2(g) \rightleftharpoons CO(g) + H_2O(g)$

At eq^m $\quad 0.25-x \quad 0.25-x \quad x \quad x$

$$K_p = 0.16 = \frac{x^2}{(0.25-x)^2}$$

$$\Rightarrow 0.4 = \frac{x}{0.25-x} \Rightarrow 0.1 - 0.4x = x$$

$x = 0.0714$

$$\text{Mole\% of CO (g)} = \frac{0.0714}{0.50} \times 100 = 14.28$$

53. (2) No. of $A^{2+} = \frac{1}{8} \times 8 = 1$

No. of $B^{3+} = \frac{1}{2} \times 4 = 2$

No. of $O^{2-} = 8 \times \frac{1}{8} \times 6 \times \frac{1}{2} = 4$

(AB_2O_4)
$\therefore$ value of n = 2

54. (80) $\frac{P^\circ - P}{P^\circ} = \frac{n_2}{n_1 + n_2}$

$$\frac{640 - 600}{640} = \frac{2.5/m}{39/78}$$

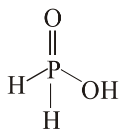
$$m = \frac{640 \times 78 \times 2.5}{39 \times 40} = 80$$

55. (5.0) $pH = pK_a + \log \frac{[CH_3COO^-]}{[CH_3COOH]}$

$pK_a = -\log(1.8 \times 10^{-5}) = 4.7447$
$[CH_3COO^-] = 2 \times [(CH_3COO)_2Ba] = 0.2$ M
$[CH_3COOH] = 0.1$ M

$pH = 4.7447 + \log \frac{0.2}{0.1} = 5.046 \approx 5.0$

56. (1) H_3PO_2 is named as hypophosphorous acid. It is monobasic as it contains only one P – OH bond, its basicity is one.



57. (395) Since, work is done against constant pressure and thus, irreversible.

Given, $\Delta V = (6-2) = 4$ L; P = 1 atm

$\therefore \quad W = -1 \times 4 \text{ L-atm} = -\frac{1 \times 4 \times 1.987}{0.0821}$ cal

(since 0.0821 L-atm = 1.987 cal)

$= -96.81 \text{ cal} = -96.81 \times 4.184$ J

$(\because 1 \text{ cal} = 4.184 \text{ J})$

$= -405.05$ J

Now from I[st] law of thermodynamics

$q = \Delta U - W$

$800 = \Delta U + 405.05$

$\therefore \quad \Delta U = 395$ J

58. (3) Three, these are $CH_3CH_2OCH_2CH_3$ (I), $CH_3OCH_2CH_2CH_3$ (II) and $CH_3OCH(CH_3)_2$ (III). Here I and II, I and III are pairs of metamers.

59. (3) $m = \frac{E.wt \times Q}{96500}$;

$\therefore E.wt = \frac{m \times 96500}{Q} = \frac{22.2 \times 96500}{2 \times 5 \times 60 \times 60} = 60.3$

$\text{Oxidation state} = \frac{\text{At wt.}}{\text{Eq. wt.}} = \frac{177}{60.3} = 3$

60. (1.14) Given $-\frac{dMnO_4^-}{dt} = 4.56 \times 10^{-3}\ Ms^{-1}$

From the reaction given,

$$-\frac{1}{2}\frac{dMnO_4^-}{dt} = \frac{4.56 \times 10^{-3}}{2} Ms^{-1}$$

$$-\frac{1}{2}\frac{dMnO_4^-}{dt} = \frac{1}{5}\frac{dI_2}{dt}$$

$$\therefore \quad -\frac{5}{2}\frac{dMnO_4^-}{dt} = \frac{dI_2}{dt}$$

On substituting the given value

$$\therefore \quad \frac{dI_2}{dt} = \frac{4.56 \times 10^{-3} \times 5}{2} = 1.14 \times 10^{-2} \text{ M/s}$$

MATHEMATICS

61. (4) Centre of the circle is $\left(\frac{1}{2}, -\frac{3}{2}\right)$.

Its distance from the line $x + y - 1 = 0$ is $\sqrt{2}$

Let the required line be $mx - y = 0$

$$\therefore \left|\frac{\frac{m}{2} + \frac{3}{2}}{\sqrt{m^2+1}}\right| = \sqrt{2} \Rightarrow m = 1, -1/7$$

$\therefore$ The lines are $x - y = 0$, $x + 7y = 0$

62. (1) Since α_1, α_2 and β_1, β_2 are the roots of $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$ respectively, therefore

$$\alpha_1 + \alpha_2 = \frac{-b}{a}, \alpha_1\alpha_2 = \frac{c}{a} \quad \text{..(1)}$$

$$\text{and } \beta_1 + \beta_2 = \frac{-q}{p}, \beta_1\beta_2 = \frac{r}{p} \quad \text{...(2)}$$

Since the given system of equations has a non-trivial solution

$$\therefore \begin{vmatrix} \alpha_1 & \alpha_2 \\ \beta_1 & \beta_2 \end{vmatrix} = 0 \text{ i.e. } \alpha_1\beta_2 - \alpha_2\beta_1 = 0$$

$$\text{or} \quad \frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\alpha_1 + \alpha_2}{\beta_1 + \beta_2} = \sqrt{\frac{\alpha_1\alpha_2}{\beta_1\beta_2}}$$

$$\Rightarrow \frac{pb}{qa} = \sqrt{\frac{pc}{ra}} \Rightarrow \frac{b^2}{q^2} = \frac{ac}{pr}$$

63. (2) Suppose, there are two points x_1 and x_2 in (a, b) such that $f'(x_1) = f'(x_2) = 0$. By Rolle's theorem applied to f' on $[x_1, x_2]$, there must be a $c \in (x_1, x_2)$ such that $f''(c) = 0$. This contradicts the given condition $f''(x) < 0, \forall x \in (a, b)$.

Hence, our assumption is wrong. Therefore, there can be atmost one point in (a, b) at which $f'(x)$ is zero.

64. (2) $I = \int_0^{16} f(t)\, dt$

Consider $g(x) = \int_0^{x^4} f(t)\, dt \Rightarrow g(0) = 0$

LMVT for g in [0, 1] gives, some $\alpha \in (0, 1)$ such that $\frac{g(1) - g(0)}{1 - 0} = g'(\alpha)$ ..... (1)

Similarly, LMVT in [1, 2] gives, some $\beta \in (1, 2)$ such that $\frac{g(2) - g(1)}{2 - 1} = g'(\beta)$ .... (2)

Eq. (1) + Eq. (2)

$g'(\alpha) + g'(\beta) = g(2) - \underbrace{g(0)}_{\text{zero}}$;

but $g'(x) = f(x^4) \cdot 4x^3$

$\therefore\ 4\left[\alpha^3 f(\alpha^4) + \beta^3 f(\beta^4)\right] = \int_0^{16} f(t)\, dt$

65. (1) $\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$\Rightarrow \begin{vmatrix} u+v & u^2+v^2+vu & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$

$R_1 \to R_1 - R_2$

$\Rightarrow \begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$

$\Rightarrow (v^2 + w^2 + vw) - (v + w)\,[v + w + u] = 0$

$\Rightarrow uv + vw + wu = 0$

66. (4) $x \in [-1, 0]$

$x + \frac{1 + x^2}{2} = -2x$

$x^2 + 6x + 1 = 0$

$x = 2\sqrt{2} - 3 \Rightarrow |10a| = [|20\sqrt{2} - 30|] = 30 - 20\sqrt{2}$

$x \in [0, 1]$

$x + \frac{1 + x^2}{2} = 2x$

$1 + x^2 = 2x \Rightarrow x = 1 \Rightarrow |10a| = 10$

$|10a| = 10,\ |20\sqrt{2} - 30|$

$\Rightarrow [|10a|] = 1, 10$

67. (4) Each point (x, y) has an image in line $y = 0$ as $(x, -y)$. So, replacing y by $-y$ in the given equation, we get the image as

$ax^2 - 2hxy + by^2 = 0.$

68. (3) $x^2 - 2x\cos\theta + 1 = 0,$

$x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2} = \cos\theta \pm i\sin\theta$

Let $x = \cos\theta + i\sin\theta$

$\therefore\ x^{2n} - 2x^n \cos n\theta + 1$

$= \cos 2n\theta + i\sin 2n\theta - 2(\cos n\theta + i\sin n\theta)\cos n\theta + 1$

$= \cos 2n\theta + 1 - 2\cos^2 n\theta +$

$i(\sin 2n\theta - 2\sin n\theta\cos n\theta)$

$= 0 + i\,0 = 0$

69. (2) $I_1 = \int_0^1 2^{x^2} dx,\ I_2 = \int_0^1 2^{x^3} dx,$

$I_3 = \int_1^2 2^{x^2} dx,\ I_4 = \int_1^2 2^{x^3} dx$

$\forall\ 0 < x < 1,\ x^2 > x^3$

$\Rightarrow \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx \Rightarrow I_1 > I_2.$

Also $\forall\ 1 < x < 2$

$x^2 < x^3 \Rightarrow \int_1^2 2^{x^2} dx < \int_1^2 2^{x^3} dx \Rightarrow I_3 < I_4$

70. (1)

$I = \int_0^{3\pi/4} (\sin x + \cos x)\, dx + \int_0^{3\pi/4} \underbrace{x}_{\text{I}}\, \underbrace{(\sin x - \cos x)}_{\text{II}}\, dx$

$$= \int_0^{3\pi/4} (\sin x + \cos x)\, dx + \underbrace{x(-\cos x - \sin x)\Big|_0^{3\pi/4}}_{\text{zero}}$$

$$+ \int_0^{3\pi/4} (\sin x + \cos x)\, dx$$

$$= 2\int_0^{3\pi/4} (\sin x + \cos x)\, dx = 2(\sqrt{2}+1)$$

71. (3) Given circles are $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$
$\therefore$ locus of centres is
$9x - 10y + 11 = 0$

72. (2) If n is odd, the required sum is

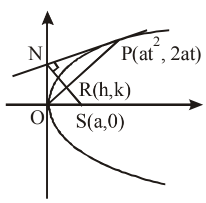
$$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2.(n-1)^2 + n^2$$

$$= \frac{(n-1)(n-1+1)^2}{2} + n^2 \quad [\because (n-1) \text{ is even}$$

$\therefore$ using given formula for the sum of $(n–1)$ terms.]

$$= \left(\frac{n-1}{2}+1\right)n^2 = \frac{n^2(n+1)}{2}$$

73. (2)



$T: ty = x + at^2$ ..... (i)
Line perpendicular to (1) through (a, 0)
$tx + y = ta$ ..... (ii)

Equation of OP : $y - \frac{2}{t}x = 0$ ...... (iii)

From equations (ii) and (iii) eliminating t we get locus
$2x^2 + y^2 - 2ax = 0$

74. (1) $z + \frac{1}{z} = 2\cos\theta$

$\Rightarrow z^2 - 2\cos\theta z + 1 = 0$

$$\Rightarrow z = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$
$$= \cos\theta + i\sin\theta$$

Taking positive sign,

$$\therefore \quad \frac{z^{2n}-1}{z^{2n}+1} = \frac{2i\sin n\theta}{2\cos n\theta} = i\tan n\theta$$

Taking negative sign, we get

$$\frac{z^{2n}-1}{z^{2n}+1} = \frac{-2i\sin n\theta}{2\cos n\theta} = -i\tan n\theta$$

$$\Rightarrow \left|\frac{z^{2n}-1}{z^{2n}+1}\right| = |\pm i\tan n\theta| = |\tan n\theta|$$

75. (1) Two curves cuts at right angle if product of their slopes is –1.
Two given curves are
$x^3 - 3xy^2 + 2 = 0$ ..... (i)
and $3x^2y - y^3 - 2 = 0$ ...... (ii)
Differentiate equ. (i),

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{3(x^2-y^2)}{6xy}$$

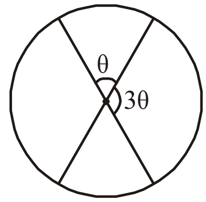
Differentiate equ. (ii),

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-2xy}{(x^2-y^2)}$$

$$\therefore m_1 \times m_2 = \frac{(x^2-y^2)}{2xy} \times \frac{-2xy}{(x^2-y^2)}$$

$$\Rightarrow m_1 \times m_2 = -1$$

76. (4)



Let one angle be θ then other = 3θ

Clearly $\theta + 3\theta = 180 \Rightarrow \theta = 45^\circ$
$\therefore$ Angle between the diameters represented by combined equation

$$ax^2 + 2(a+b)xy + by^2 = 0 \text{ is } 45^\circ$$

$$\therefore \text{ Using } \tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$\Rightarrow 1 = \frac{2\sqrt{a^2 + b^2 + ab}}{a+b}$$

$\Rightarrow (a+b)^2 = 4(a^2+b^2+ab)$

$\Rightarrow 3a^2+3b^2+2ab=0$

77. (1)

p	q	p ∧ q	p ∨ q	~ (p ∨ q)	(p ∧ q) ∧ ~ (p ∨ q)
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\therefore (p \wedge q) \wedge (\sim(p \vee q))$ is a contradiction.

78. (3) Since, $1, \omega, \omega^2, \ldots\ldots\ldots\ldots\ldots \omega^{n-1}$ are the n roots of unity

Consider $x^n = 1$

$\therefore (x^n - 1) = (x-1)(x-\omega)(x-\omega^2)\ldots\ldots(x-\omega^{n-1})$

$\Rightarrow (x-\omega)(x-\omega^2)\ldots\ldots(x-\omega^{n-1}) = \dfrac{x^n-1}{x-1}$

Taking $\lim\limits_{x\to 1}$ on both side

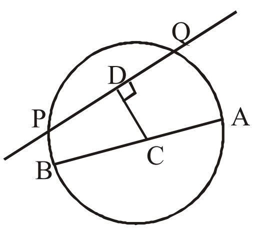
$\lim\limits_{x\to 1}(x-\omega)(x-\omega^2)\ldots\ldots(x-\omega^{n-1})$

$= \lim\limits_{x\to 1}\dfrac{x^n-1}{x-1}$

$\Rightarrow \quad (1-\omega)(1-\omega^2)\ldots\ldots(1-\omega^{n-1}) = n$

79. (1) Since, $\angle APB = \angle AQB = \dfrac{\pi}{2}$ so $y = mx + 8$ intersect the circle whose diameter is AB.

Equation of circle is $x^2 + y^2 = 16$, $CD < 4$



$\Rightarrow \dfrac{8}{\sqrt{1+m^2}} < 4 \Rightarrow 1 + m^2 > 4$

$\Rightarrow m \in (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

If the line passing throw the point A (–4, 0), B(4, 0)

then $\angle APB = \angle AQB = \dfrac{\pi}{2}$ does not formed.

$\therefore m \neq \pm 2$

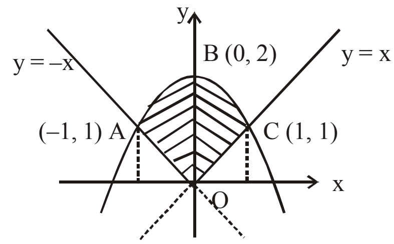
80. (4) (1) $T_r(kA) = k(a_{11} + a_{22} + a_{33}) = kT_r(A)$

(2) $T_r(A+B) = a_{11} + b_{11} + a_{22} + b_{22} + a_{33} + b_{33} = T_r(A) + T_r(B)$

(3) $T_r(I_3) = 1 + 1 + 1 = 3$

(4) $T_r(A^2) = \Sigma a_{11}^2 + \Sigma a_{12}^2 \neq (a_{11} + a_{22} + a_{33})^2$

81. (2.33)



We first draw the given curves

The first curve $x^2 - y^2 = 0 \Rightarrow y = \pm x$ represents a pair of straight lines with slopes 1 and –1 passing through origin. The second curve

$\Rightarrow x^2 + y - 2 = 0 \Rightarrow x^2 = -y + 2$

$\Rightarrow x^2 = -(y-2)$

represents a parabola with vertex (0,2) axis as y–axis and concavity downwards. Both the curves are plotted in the figure and the required area is shown by the shaded region.

The points A and C are the points of intersection of $y^2 = x^2$ with $x^2 + y - 2 = 0$.

Solving the two equations, we get

$y^2 + y - 2 = 0$ [putting value of $x^2 = y^2$]

$\Rightarrow (y+2)(y-1) = 0$

giving $y = -2$ and 1, but $y = -2$ is discarded as the required area is above the x-axis.

$\therefore y = 1 \Rightarrow x = \pm 1$

The points A and C are respectively (–1, 1) and (1, 1) now due to symmetry

Area of the bounded region OABCO

$$= 2 \times \text{Area OBCO} = 2 \times \int_0^1 \left[(2-x^2) - x\right]dx$$

[Since $y = 2 - x^2$ is the upper curve and $y = x$ is the lower curve]

$$= 2\left[2x - \frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = 2\left[2 - \frac{1}{3} - \frac{1}{2}\right] = \frac{7}{3} = 2.33$$

82. (9.00) Number of digits are 9

Select 2 places for the digit 1 and 2 in ${}^9C_2$ ways from the remaining 7 places select any two places for 3 and 4 in ${}^7C_2$ ways and from the remaining 5 places select any two for 5 and 6 in ${}^5C_2$ ways Now, the remaining 3 digits can be filled in 3! ways

$\therefore$ Total ways $= {}^9C_2 \,.\, {}^7C_2 \,.\, {}^5C_2 \,.\, 3!$

$$= \frac{9!}{2!.7!} \cdot \frac{7!}{2!.5!} \cdot \frac{5!}{2!.3!} .3! = 9.7! = k.\,7!$$

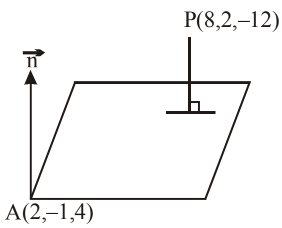
$\Rightarrow k = 9$

83. (32.00) $\vec{n} = 7\hat{i} + 2\hat{j} - \hat{k}$ is normal to plane

(Assuming $n = a\hat{i} + b\hat{j} + c\hat{k}$ and using

$\vec{n}.\overrightarrow{AB} = 0,\ \vec{n}.\overrightarrow{BC} = 0,\ \vec{n}.\overrightarrow{AC} = 0$)

$P = (8, 2, -12)$



$\overrightarrow{AP} = 6\hat{i} + 3\hat{j} - 16\hat{k}$

$\therefore$ Distance

$$d = \left|\frac{\overrightarrow{AP}.\vec{n}}{|\vec{n}|}\right| = \left|\frac{42+6+16}{\sqrt{49+4+1}}\right| = \frac{64}{\sqrt{54}} = \frac{64}{3\sqrt{6}} = \frac{64\sqrt{6}}{18} = \frac{32\sqrt{6}}{9}$$

$$= \frac{k\sqrt{6}}{9} \Rightarrow k = 32.$$

84. (343.00)

$$\vec{V} = \vec{A} \times \left[(\vec{A}.\vec{B})\vec{A} - (\vec{A}.\vec{A})\vec{B}\right].\vec{C}$$

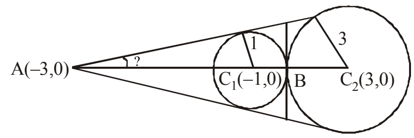
$$= \left(\underbrace{\vec{A}\times(\vec{A}.\vec{B})\vec{A}}_{\text{zero}} - (\vec{A}.\vec{A})\vec{A}\times\vec{B}\right).\vec{C} = -|\vec{A}|^2\,[\vec{A}\ \ \vec{B}\ \ \vec{C}]$$

Now, $|\vec{A}|^2 = 4 + 9 + 36 = 49$

$$[\vec{A}\ \ \vec{B}\ \ \vec{C}] = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix} = 7$$

$$\therefore \left|-|\vec{A}|^2\,[\vec{A}\ \ \vec{B}\ \ \vec{C}]\right| = 49 \times 7 = 343$$

85. (3.00) A divides C_1C_2 externally in the ratio 1 : 3.



$\therefore$ coordinates of A are $(-3, 0)$

We have $\sin\theta = 1/2 \quad \therefore \theta = 30°$

Area $= 3 \times 3 \tan 30° = 3\sqrt{3} = \lambda\sqrt{3}$

$\Rightarrow \lambda = 3$

86. (0.53) Box — 6R, 5B, 4W

P(E) = P(R R B W or B B R W or W W R B)

$n(E) = {}^6C_2 \cdot {}^5C_1 \cdot {}^4C_1 + {}^5C_2 \cdot {}^6C_1 \cdot {}^4C_1 + {}^4C_2 \cdot {}^6C_1 \cdot {}^5C_1$

$n(S) = {}^{15}C_4$

$$\therefore P(E) = \frac{720 \cdot 4!}{15 \cdot 14 \cdot 13 \cdot 12} = \frac{48}{91} = 0.53$$

87. (2.00)

$$\frac{\sin\theta + \sin 2\theta}{\cos\theta + \cos 2\theta} = \frac{2\sin\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos\left(\frac{3\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)} = \tan\left(\frac{3\theta}{2}\right)$$

Hence period $= \dfrac{2\pi}{3} = k\dfrac{\pi}{3}$

$\Rightarrow k = 2.$

88. (3.00) The equation of the tangent at

$\left(4\cos\theta, \dfrac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse

$16x^2 + 11y^2 = 256$ is

$$16x\,(4\cos\theta) + 11y\left(\frac{16}{\sqrt{11}}\sin\theta\right) = 256$$

or $\quad 4x\cos\theta + \sqrt{11}y\sin\theta = 16$

This touches the circle $(x-1)^2 + y^2 = 4^2$ if

$$\left|\frac{4\cos\theta - 16}{\sqrt{16\cos^2\theta + 11\sin^2\theta}}\right| = 4$$

$\Rightarrow (\cos\theta - 4)^2$

$= 16\cos^2\theta + 11\sin^2\theta$

$\Rightarrow \quad 15\cos^2\theta + 11\sin^2\theta + 8\cos\theta - 16 = 0$

$\Rightarrow \quad 4\cos^2\theta + 8\cos\theta - 5 = 0$

$\Rightarrow \quad (2\cos\theta - 1)(2\cos\theta + 5) = 0$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = \pm\frac{\pi}{3} = \pm\frac{\pi}{k}$

$\Rightarrow \quad k = 3$

89. (0.50) A: blood result says positive about the disease

B_1: Person suffers from the disease

$\therefore P(B_1) = \frac{1}{100}$

B_2 : person does not suffer $\therefore P(B_2) = \frac{99}{100}$

$P(A/B_1) = \frac{99}{100}$, $P(A/B_2) = \frac{1}{100}$

$$P(B_1/A) = \frac{P(B_1).P(A/B_1)}{P(B_1).P(A/B_1) + P(B_2).P(A/B_2)}$$

$$= \frac{\frac{1}{100}\cdot\frac{99}{100}}{\frac{1}{100}\cdot\frac{99}{100} + \frac{99}{100}\cdot\frac{1}{100}} = \frac{99}{2\cdot 99} = \frac{1}{2} = 0.50$$

90. (0.50) $a_n = \int_0^{\pi/2} (1-\sin t)^n \sin 2t\, dt$

Let $1 - \sin t = u \Rightarrow -\cos t\, dt = du$

$$= 2\int_0^1 u^n(1-u)\, du = 2\left(\int_0^1 u^n du - \int_0^1 u^{n+1} du\right) = 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

Hence, $\frac{a_n}{n} = 2\left(\frac{1}{n(n+1)} - \frac{1}{n(n+2)}\right)$

$$\lim_{n\to\infty}\sum_1^n \frac{a_n}{n} = 2\left(\sum\left(\frac{1}{n} - \frac{1}{n+1}\right) - \frac{1}{2}\sum\left(\frac{1}{n} - \frac{1}{n+2}\right)\right)$$

$$= 2\left(\sum_1^n\left(\frac{1}{n} - \frac{1}{n+1}\right)\right) - \sum_1^n\left(\frac{1}{n} - \frac{1}{n+2}\right)$$

$$= 2(1) - \left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \ldots\ldots\right] = 2 - \frac{3}{2} = \frac{1}{2}$$

$= 0.50$