Q1: NTA Test 01 (Single Choice)

Number of roots of the equation $\cos^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$ which lie in the interval $[-\pi, \pi]$ is

(A) 2

(B) 4

(C) 6

(D) 8

Q2: NTA Test 02 (Single Choice)

If the equation $x^2 + 4 + 3\sin(ax+b) - 2x = 0$ has atleast one real solution, where $a, b \in [0, 2\pi]$, then one possible value of $(a+b)$ can be equal to

(A) $\frac{7\pi}{2}$

(B) $\frac{5\pi}{2}$

(C) $\frac{9\pi}{2}$

(D) None of these

Q3: NTA Test 03 (Numerical)

The number of solutions, the equation $\sin^4 x + \cos^4 x = \sin x \cos x$ has, in $[\pi, 5\pi]$ is/are

Q4: NTA Test 04 (Single Choice)

If $1 + \sin\theta + \sin^2\theta + \sin^3\theta + \ldots\infty = 4 + 2\sqrt{3},\ 0 < \theta < \pi$, then

(A) $\theta = \frac{\pi}{3}$

(B) $\theta = \frac{\pi}{6}$

(C) $\theta = \frac{\pi}{3}$ or $\frac{\pi}{6}$

(D) $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

Q5: NTA Test 05 (Single Choice)

If $\frac{1}{6}\sin\theta, \cos\theta$ and $\tan\theta$ are in geometric progression, then the complete solution set of θ is

(A) $\{\theta : \theta = 2n\pi \pm (\frac{\pi}{6}),\ n \in I\}$

(B) $\{\theta : \theta = 2n\pi \pm (\frac{\pi}{3}),\ \mathrm{n} \in I\}$

(C) $\{\theta : \theta = n\pi + (-1)^n (\frac{\pi}{3}),\ \mathrm{n} \in I\}$

(D) $\{\theta : \theta = n\pi + \frac{\pi}{3},\ \mathrm{n} \in I\}$

Q6: NTA Test 06 (Single Choice)

The number of values of $\theta \in \left[\frac{-3\pi}{2}, \frac{4\pi}{3}\right]$ which satisfies the system of equations $2\sin^2\theta + \sin^2 2\theta = 2$ and $\sin 2\theta + \cos 2\theta = \tan\theta$ is

(A) 2

(B) 4

(C) 6

(D)
8

Q7: NTA Test 07 (Single Choice)

If $\tan(\mathrm{k}+1)\theta = \tan\theta$, then the set of all the values of θ is

(A) $\{\mathrm{n}\pi : \mathrm{n} \in I\}$

(B) $\left\{\frac{\mathrm{n}\pi}{2} : \mathrm{n} \in I\right\}$

(C) $\left\{\frac{\mathrm{n}\pi}{\mathrm{k}} : \mathrm{n} \in I\right\}$

(D) $\left\{\frac{\mathrm{n}\pi}{4} : \mathrm{n} \in I\right\}$

Q8: NTA Test 08 (Single Choice)

General solution of the equation $\tan^2\theta + \sec 2\theta = 1$ is

(A) $m\pi, n\pi + \frac{\pi}{3},\ m \in I,\ n \in I$

(B) $m\pi, n\pi \pm \frac{\pi}{3},\ m \in I,\ n \in I$

(C) $m\pi, n\pi \pm \frac{\pi}{6},\ m \in I,\ n \in I$

(D) None of these

Q9: NTA Test 09 (Single Choice)

The general solution of the system of equations $\sin^3 \mathrm{x} + \sin^3\left(\frac{2\pi}{3} + \mathrm{x}\right) + \sin^3\left(\frac{4\pi}{3} + \mathrm{x}\right) + \frac{3}{4}\cos 2\mathrm{x} = 0$ and $\cos \mathrm{x} \neq 0$ is

(A) $\mathrm{x} = \frac{(2\mathrm{k}+1)\pi}{10}, \mathrm{k} \in \mathrm{Z}$

(B) $\mathrm{x} = \frac{(2\mathrm{k}-1)\pi}{5}, \mathrm{k} \in \mathrm{Z}$

(C) $\mathrm{x} = \frac{(4\mathrm{k}+1)\pi}{10}, \mathrm{k} \in \mathrm{Z}$

(D) $x=\left(\frac{4k+1}{5}\right)\pi, k \in Z$

Q10: NTA Test 10 (Single Choice)

The set $\{x \in R : \cos 2x + 2\cos^2 x = 2\}$ is equal to

(A) $\{2n\pi + \frac{\pi}{3} : n \in Z\}$

(B) $\{n\pi \pm \frac{\pi}{6} : n \in Z\}$

(C) $\{n\pi + \frac{\pi}{3} : n \in Z\}$

(D) $\{2n\pi - \frac{\pi}{3} : n \in Z\}$

Q11: NTA Test 11 (Numerical)

Number of solutions of $2^{\sin(|x|)} = 3^{|\cos x|}$ in $[-\pi, \pi]$, is equal to

Q12: NTA Test 12 (Numerical)

Consider the equation $\log_{\sqrt{2}\sin x}(1+\cos x) = 2,\ x \in \left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$. If the sum of the roots is $\frac{p\pi}{q}$, where G.C.D $(p, q) = 1$, then the value of $p^2 + q^2$ is

Q13: NTA Test 14 (Single Choice)

If the expression $\left(1 + \tan x + \tan^2 x\right)\left(1 - \cot x + \cot^2 x\right)$ is positive, then the complete set of values of x is

(A) $\left[0, \frac{\pi}{2}\right]$

(B) $[0, \pi]$

(C) $R - \left\{x = \frac{n\pi}{2},\ n \in I\right\}$

(D) $[0,\ \infty]$

Q14: NTA Test 15 (Single Choice)

The number of ordered pairs (x, y) satisfying the equations $y = 2\sin x$ and $y = 5x^2 + 2x + 3$ is/are

(A) 0

(B) 1

(C) 2

(D) Infinite

Q15: NTA Test 16 (Single Choice)

If x and y are the solutions of the equation $5 + 8\sin^2 x = 2y^2 - 8y + 21$, then the least possible value of x^2y^3 is

(A) $2\pi^2$

(B) $4\pi^2$

(C) $9\pi^2$

(D) π^2

Q16: NTA Test 17 (Single Choice)

In the interval $(0, 2\pi)$, sum of all the roots of the equation $\sin\left(\pi \log_3\left(\frac{1}{x}\right)\right) = 0$ is

(A) $\frac{3}{2}$

(B) 4

(C) $\frac{9}{2}$

(D) $\frac{13}{3}$

Q17: NTA Test 18 (Numerical)

If $0 \le x \le 2\pi$, then the number of real values of x satisfying the equation $81^{\sin^2 x} + 81^{\cos^2 x} = 30$ is

Q18: NTA Test 23 (Single Choice)

If $\cos x - \sin x = -\frac{5}{4}$, where $\frac{\pi}{2} < x < \frac{3\pi}{4}$, then $\cot\frac{x}{2}$ is equal to

(A) $\frac{4-\sqrt{7}}{9}$

(B) 8

(C) -8

(D) $\frac{4+\sqrt{7}}{9}$

Q19: NTA Test 24 (Numerical)

The number of solutions of the equation $|\cot x| = \cot x + \operatorname{cosec} x$ in $[0, 10\pi]$ is/are

Q20: NTA Test 27 (Single Choice)

The set of all values of a for which the equation $\cos 2x + a\sin x = 2a - 7$ has a solution is

(A) $(-\infty, 2)$

(B) $[2,6]$

(C) $(6, \infty)$

(D) $(-\infty, \infty)$

Q21: NTA Test 29 (Single Choice)

If $\cot(\alpha + \beta) = 0$, then $\sin(\alpha + 2\beta)$ is equal to

(A) $\sin\alpha$

(B) $\cos\alpha$

(C) $\sin\beta$

(D) $\cos 2\beta$

Q22: NTA Test 31 (Numerical)

The sum of the roots of the equation $\left|\sqrt{3}\cos x - \sin x\right| = 2$ in $[0,4\pi]$ is $k\pi$, then the value of $6k$ is

Q23: NTA Test 32 (Single Choice)

The total number of solution(s) of the equation $2x + 3\tan x = \frac{5\pi}{2}$ in $x \in [0,2\pi]$ is/are equal to

(A) 1

(B) 2

(C) 3

(D) 4

Q24: NTA Test 34 (Single Choice)

If $\log_{\cos x}\sin x \geq 2$ and $0 \leq x \leq 3\pi$, then the value of $\sin x$ lies in the interval

(A) $\left[\frac{\sqrt{5}-1}{2}, 1\right]$

(B) $\left(0, \frac{\sqrt{5}-1}{2}\right]$

(C) $\left[0, \frac{1}{2}\right]$

(D) None of these

Q25: NTA Test 35 (Single Choice)

The number of roots of the equation $\tan x + \sec x = 2\cos x$ in $[0, 4\pi]$ is

(A) 2

(B) 4

(C) 6

(D) 0

Q26: NTA Test 37 (Single Choice)

For $-\frac{\pi}{2} \leq \mathrm{x} \leq \frac{\pi}{2}$, the number of points of intersection of curves $\mathrm{y} = \cos\mathrm{x}$ and $\mathrm{y} = \sin 3\mathrm{x}$ is

(A) 0

(B) 1

(C) 2

(D) 3

Q27: NTA Test 38 (Single Choice)

If $0 < \mathrm{A} < \mathrm{B} < \pi$, $\sin\mathrm{A} - \sin\mathrm{B} = \frac{1}{\sqrt{2}}$ and $\cos\mathrm{A} - \cos\mathrm{B} = \sqrt{\frac{3}{2}}$, then the value of $\mathrm{A} + \mathrm{B}$ is equal to

(A) $\frac{2\pi}{3}$

(B) $\frac{5\pi}{6}$

(C) π

(D) $\frac{4\pi}{3}$

Q28: NTA Test 38 (Single Choice)

$\operatorname{cosec}^2\theta\left(\cos^2\theta - 3\cos\theta + 2\right) \geq 1$, if θ belongs to

(A) $\left(0, \frac{\pi}{3}\right)$

(B) $\left(\frac{\pi}{2}, \pi\right)$

(C) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

(D) $\left(0, \frac{\pi}{4}\right)$

Q29: NTA Test 39 (Single Choice)

Let $\mathrm{f}(\mathrm{x}) = \frac{25^{\mathrm{x}}}{25^{\mathrm{x}}+5}$, then the number of solution(s) of the equation $\mathrm{f}\left(\sin^2\theta\right) + \mathrm{f}\left(\cos^2\theta\right) = \tan^2\theta$, $\theta \in [0,10\pi]$ is/are

(A) 10

(B) 2

(C) 40

(D) 20

Q30: NTA Test 39 (Numerical)

If x and y are the solutions of the equation $12\sin x + 5\cos x = 2y^2 - 8y + 21$, then the value of $12\cot\left(\frac{xy}{2}\right)$ is (Given, $|x| < \pi$)

Q31: NTA Test 45 (Single Choice)

The sum of the roots of the equation $\cos 4x + 6 = 7\cos 2x$ in the interval $[0{,}314]$ is $\lambda\pi$, then the numerical value of λ is

(A) 4950

(B) 2475

(C) 9900

(D) 4945

Q32: NTA Test 46 (Single Choice)

If α and β are the solutions of $\sin x = -\frac{1}{2}$ in $[0{,}2\pi]$ and α and γ are the solutions of $\cos x = -\frac{\sqrt{3}}{2}$ in $[0{,}2\pi]$, then the value of $\frac{\alpha+\beta}{|\beta-\gamma|}$ is equal to

(A) 1

(B) 2

(C) 3

(D) 4

Q33: NTA Test 48 (Single Choice)

If α and β are the solutions of $\cot x = -\sqrt{3}$ in $[0{,}2\pi]$ and α and γ are the solutions of $\operatorname{cosec} x = -2$ in $[0{,}2\pi]$, then the value of $\frac{|\alpha-\beta|}{\beta+\gamma}$ is equal to

(A) $\frac{1}{2}$

(B) 2

(C) $\frac{1}{3}$

(D) 3

Answer Keys

Q1: (B)

Q2: (A)

Q3: 4

Q4: (D)

Q5: (B)

Q6: (C)

Q7: (C)

Q8: (B)

Q9: (C)

Q10: (B)

Q11: 4

Q12: 10

Q13: (C)

Q14: (A)

Q15: (A)

Q16: (C)

Q17: 8

Q18: (D)

Q19: 5

Q20: (B)

Q21: (A)

Q22: 56

Q23: (C)

Q24: (B)

Q25: (B)

Q26: (D)

Q27: (D)

Q28: (C)

Q29: (D)

Q30: 5

Q31: (A)

Q32: (C)

Q33: (A)

Solutions

Q1: (B) 4

Given equation is

$1 - \sin^2 x + \frac{\sqrt{3}+1}{2}\sin x - \frac{\sqrt{3}}{4} - 1 = 0$

$\Rightarrow\ \sin^2 x - \frac{\sqrt{3}+1}{2}\sin x + \frac{\sqrt{3}}{4} = 0; 4\sin^2 x - 2\sqrt{3}\sin x - 2\sin x + \sqrt{3} = 0$

$2\sin x\left(2\sin x - \sqrt{3}\right) - \left(2\sin x - \sqrt{3}\right) = 0$

$\Rightarrow (2\sin x - 1)\left(2\sin x - \sqrt{3}\right) = 0$

On solving we get $\sin x = \frac{1}{2}\ ;\ \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}\ ;\ \frac{\pi}{3}, \frac{2\pi}{3}$

Q2: (A) $\frac{7\pi}{2}$

$(x-1)^2 + 3 + 3\sin(ax+b) = 0$

$(x-1)^2 + 3 = -3\sin(ax+b)$

$L.H.S \geq 3,\ RHS \in [-3, 3]$ now $\sin(ax+b) = -1$

$\therefore x = 1 \quad \sin(b+a) = -1$

Q3: 4

$\sin^4 x + \cos^4 x = \sin x \cos x$
$\Rightarrow \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x\cos^2 x = \sin x\cos x$
$\Rightarrow 2\sin^2 x\cos^2 x + \sin x\cos x - 1 = 0$
$\Rightarrow (2\sin x\cos x - 1)(\sin x\cos x + 1) = 0$
$\Rightarrow \sin 2x = 1$ ...(i)
$x \in [\pi, 5\pi] \Rightarrow 2x \in [2\pi, 10\pi]$
from (i), $2x = \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}$
$\Rightarrow x = \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}$
$\Rightarrow$ The number of solutions $= 4$

Q4: (D) $\theta = \frac{\pi}{3}$ **or** $\frac{2\pi}{3}$

Given, $1 + \sin\theta + \sin^2\theta + \sin^3\theta + \ldots\infty = 4 + 2\sqrt{3}$

$\Rightarrow \quad \frac{1}{1-\sin\theta} = 4 + 2\sqrt{3} \quad \left[\because 0 < \sin\theta < 1\right]$

$\Rightarrow \quad 1 - \sin\theta = \frac{4-2\sqrt{3}}{16-12} = 1 - \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

Q5: (B) $\left\{\theta : \theta = 2n\pi \pm \left(\frac{\pi}{3}\right),\ \text{n} \in I\right\}$
$\cos^2\theta = \left(\frac{1}{6}\sin\theta\right)(\tan\theta)$

$\Rightarrow \cos^2\theta = \frac{1}{6}\left(\frac{\sin^2\theta}{\cos\theta}\right)$

$\Rightarrow 6\cos^3\theta = 1 - \cos^2\theta$

$\Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3},\ \text{n} \in I.$

Q6: (C) 6

First equation is $4\sin^2\theta\cos^2\theta = 2\cos^2\theta$

$\Rightarrow\ \cos^2\theta = 0$ or $\sin^2\theta = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2 = \sin^2\frac{\pi}{4}$

$\Rightarrow\ \theta = (2n+1)\frac{\pi}{2},\ n \in I$ or $\theta = n\pi \pm \frac{\pi}{4},\ n \in I$

Second equation is not satisfied by $\theta = (2n+1)\frac{\pi}{2},\ n \in I$ but satisfied by $\theta = n\pi \pm \frac{\pi}{4},\ n \in I$
So $\theta = \mathrm{n}\pi \pm \frac{\pi}{4}, n \in I$

$\therefore$ In $\left[\frac{-3\pi}{2}, \frac{4\pi}{3}\right]$, the values of θ are

$\frac{-5\pi}{4}, \frac{-3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

Q7: (C) $\left\{\frac{\mathrm{n}\pi}{\mathrm{k}} : \mathrm{n} \in I\right\}$

As we know the general solution of the equation

$\tan x = \tan\alpha$ is given by $x = \mathrm{n}\pi + \alpha,\ \mathrm{n} \in I$

Hence the solution of the equation, $\tan(\mathrm{k}+1)\theta = \tan\theta$

is given by $(\mathrm{k}+1)\theta = \mathrm{n}\pi + \theta \Rightarrow \mathrm{k}\theta = \mathrm{n}\pi,\ \mathrm{n} \in I$

$\therefore\quad \theta \in \frac{\mathrm{n}\pi}{\mathrm{k}}\ :\ \mathrm{n} \in I$

Q8: (B) $m\pi, n\pi \pm \frac{\pi}{3},\ m \in I,\ n \in I$
Using, $\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1+\tan^2\theta}{1-\tan^2\theta}$

$\Rightarrow$ We can write the given equation as $\tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1$

$\Rightarrow \tan^2\theta(1-\tan^2\theta) + 1 + \tan^2\theta = 1 - \tan^2\theta$

$\Rightarrow 3\tan^2\theta - \tan^4\theta = 0 \Rightarrow \tan^2\theta(3-\tan^2\theta) = 0$

$\Rightarrow \tan\theta = 0$ or $\tan^2\theta = 3 = (\sqrt{3})^2$

$\Rightarrow \tan\theta = 0 \Rightarrow \theta = m\pi, m \in I$

or $\tan^2\theta = \tan^2\frac{\pi}{3} \Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in I$

Q9: (C) $\mathrm{x} = \frac{(4\mathrm{k}+1)\pi}{10}, \mathrm{k} \in \mathrm{Z}$

$\because \sin 3\mathrm{x} = 3\sin\mathrm{x} - 4\sin^3\mathrm{x} \ \Rightarrow \sin^3\mathrm{x} = \frac{1}{4}[3\sin\mathrm{x} - \sin 3\mathrm{x}]$

The given equation reduces to

$\frac{1}{4}(3\sin\mathrm{x} - \sin 3\mathrm{x}) + \frac{1}{4}\left(3\sin\left(\frac{2\pi}{3}+\mathrm{x}\right) - \sin(2\pi+3\mathrm{x})\right) + \frac{1}{4}\left(3\sin\left(\frac{4\pi}{3}+\mathrm{x}\right) - \sin(4\pi+3\mathrm{x}) + \frac{3}{4}\cos 2\mathrm{x} = 0\right.$

$3\left\{\sin x+\sin \left(\frac{2\pi}{3}+x\right)+\sin \left(\frac{4\pi}{3}+x\right)\right\}-\{\sin 3x+\sin (2\pi+3x)+\sin (4\pi+3x)\}+3 \cos 2x=0$

$3\left\{\ \sin x+2 \sin x \ \ \cos \ \frac{2\pi}{3}\right\}-3 \sin 3x+3 \cos 2x=0$

$\Rightarrow \quad \sin 3x=\cos 2x \ \Rightarrow \ \cos \left(\frac{\pi}{2}-3x\right)=\cos 2x \Rightarrow 2x=2k\pi \pm\left(\frac{\pi}{2}-3x\right)$

$\Rightarrow \quad 5x=2k\pi+\frac{\pi}{2}=(4k+1) \frac{\pi}{2} \ \forall k \in Z$ or $x=-2\ k\pi+\frac{\pi}{2}, \ \forall k \in Z$

$\Rightarrow \quad x=(4k+1) \frac{\pi}{10}, \forall k \in Z \quad \{\cos x \neq 0\}$

Q10: (B) $\left\{n\pi \pm \frac{\pi}{6}: n \in Z\right\}$

Given, $2\cos^2 x-1+2\cos^2 x=2$

$\Rightarrow \cos^2 x=\frac{3}{4}$

$\cos^2 x=\cos^2 \frac{\pi}{6}$

$\therefore x=n\pi \pm \frac{\pi}{6} \ : n \in Z$

Q11: 4

In the interval $[-\pi, \ \pi]$ all the values of $\sin |x|$ are positive as well as $|\cos x|$

Hence in $[-\pi, \ \pi]$, equation gets reduced to,

$2^{\sin x}=3^{\cos x}$

Taking log on both sides,

$\sin x \log 2=\cos x \log 3$

$\Rightarrow \quad |\tan x|=\log 3 / \log 2$, value of $\tan x$ are positive in 1^{st} and 3^{rd} quadrant and negative in 2^{nd} and 4^{th} quadrant but positive for $|\tan x|$ in all the values of x in $[-\pi, \ \pi]$.

Hence, $\tan x$ will repeat its value in all 4 quadrants, so the number of solutions are 4 .

Q12: 10

$\log_{\sqrt{2}\sin x}(1+\cos x)=2$ ....(i)

$\sqrt{2}\sin x \neq 1, \ \sqrt{2}\sin x>0, \ 1+\cos x>0$

$\Rightarrow \sin x \neq \frac{1}{\sqrt{2}}, \ \sin x>0$ and $x \neq 0$ or a multiple of π

$\Rightarrow x \in(0, \pi)-\left\{\frac{\pi}{4}, \frac{3\pi}{4}\right\}$ (feasible region)

Hence, from (i), we get,

$(\sqrt{2}\sin x)^2=1+\cos x$

$\Rightarrow 2\sin^2 x=1+\cos x$

$\Rightarrow 2\cos^2 x+\cos x-1=0$

$\Rightarrow(2\cos x-1)(\cos x+1)=0$

$\Rightarrow \cos x=\frac{1}{2} \ldots[\cos x+1>0]$

$\Rightarrow x=\frac{\pi}{3}$

$\Rightarrow p=1, q=3$

$\Rightarrow p^2+q^2=10$

Q13: (C) $R-\left\{x=\frac{n\pi}{2}, \ n \in I\right\}$

$\frac{(1+\tan x+\tan^2 x)(1+\tan^2 x-\tan x)}{\tan^2 x}>0$

$\Rightarrow \ \frac{(1+\tan^2 x)^2-\tan^2 x}{\tan^2 x} > 0$

Since, $1+\tan^2 x > \tan^2 x,\ \forall x \in R-\left\{x=\frac{n\pi}{2},\ n\in I\right\}$

Hence, given expression is positive for all values of $x \in R-\left\{x=\frac{n\pi}{2},\ n\in I\right\}$

Q14: (A) 0

$2\sin x = 5x^2+2x+3$

$\Rightarrow \ 2\sin x = 4x^2+(x+1)^2+2$

But, $2\sin x \le 2$

and $4x^2+(x+1)^2+2 > 2$, so it has no solution

Q15: (A) $2\pi^2$

$\because 5+8\sin^2 x \in [5,13]$
Also, $2y^2-8y+21 = 2(y-2)^2+13 \ge 13$
So, equality should hold true if,
$5+8\sin^2 x = 13$ and $2y^2-8y+21=13$
$\Rightarrow \sin x = \pm 1,\ y=2$
$\Rightarrow \sin x = (2n+1)\frac{\pi}{2},\ y=2$
For least value, $x=\pm\frac{\pi}{2}$
Hence, $x^2y^3 = \frac{\pi^2}{4}\cdot 8 = 2\pi^2$

Q16: (C) $\frac{9}{2}$

$\pi \log_3\left(\frac{1}{x}\right) = k\pi, k \in I$

$\log_3\left(\frac{1}{x}\right) = k \ \Rightarrow \ x = 3^{-k}$

Possible values of k are $-1,\ 0,\ 1,\ 2,\ 3,\ \ldots\ldots.$

$S = (3+1)+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots..\infty\right)$

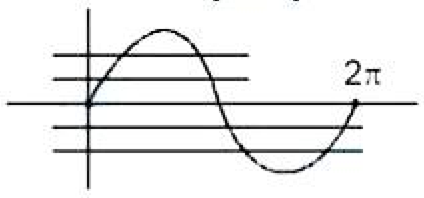
$= 4+\frac{(1/3)}{1-(1/3)} = 4+\frac{1}{2} = \frac{9}{2}$

Q17: 8

Let $81^{\sin^2 x} = t$,

then, $81^{\cos^2 x} = 81^{1-\sin^2 x} = \frac{81}{t}$
So, the given equation is $t+\frac{81}{t} = 30$
$t^2 - 30t + 81 = 0 \Rightarrow t = 3$ or 27
$81^{\sin^2 x} = 3$ or 27
$\Rightarrow 3^{4\sin^2 x} = 3^1$ or 3^3

$\Rightarrow \sin^2 x = \frac{1}{4}$ or $\frac{3}{4} \Rightarrow \sin x = \pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}$



Hence, there are 8 solutions between 0 and 2π.

Q18: (D) $\frac{4+\sqrt{7}}{9}$

Given, $4(\cos x - \sin x) = -5$

$4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} - \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) = -5$

$\Rightarrow 4 - 4\tan^2\frac{x}{2} - 8\tan\frac{x}{2} = -5 - 5\tan^2\frac{x}{2}$

$\Rightarrow \tan^2\frac{x}{2} - 8\tan\frac{x}{2} + 9 = 0$

$\Rightarrow 9\cot^2\frac{x}{2} - 8\cot\frac{x}{2} + 1 = 0$

$\Rightarrow \cot\frac{x}{2} = \frac{4\pm\sqrt{7}}{9}$

Since, $\frac{\pi}{4} < \frac{x}{2} < \frac{3\pi}{8} \Rightarrow \sqrt{2} - 1 < \cot\frac{x}{2} < 1$

$\Rightarrow \cot\frac{x}{2} = \frac{4+\sqrt{7}}{9}$

Q19: 5

If x lies in 1^{st} or 3^{rd} quadrant then $|\cot x| = \cot x$, thus equation becomes

$\cot x = \cot x + \frac{1}{\sin x} \Rightarrow \frac{1}{\sin x} = 0$, not possible

If x lies in 2^{nd} or 4^{th} quadrant, then $|\cot x| = -\cot x$, thus equation becomes

$-\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow -2\cot x = \frac{1}{\sin x} \Rightarrow -2\cos x \sin x = \sin x$

$\Rightarrow \sin x(1 + 2\cos x) = 0 \Rightarrow \cos x = -\frac{1}{2}\ (\because \sin x \neq 0)$

$\Rightarrow x = \frac{2\pi}{3}$ is the only solution

(In 2^{nd} quad.)

In $[0,2\pi]$ there is only one solution

In $[0,10\pi]$, we get, five solutions

Q20: (B) $[2,6]$

$\cos 2x + a\sin x = 2a - 7$

i.e. $2\sin^2 x - a\sin x + 2a - 8 = 0$

$\sin x = \frac{a\pm\sqrt{a^2-8(2a-8)}}{4} = \frac{a\pm(a-8)}{4}$

$\sin x = \frac{a-4}{2}$ or 2

Hence, $-1 \leq \frac{(a-4)}{2} \leq 1$

The range of a is $[2,6]$

Q21: (A) $\sin\alpha$

Given, $\cot(\alpha+\beta) = 0 \Rightarrow \cos(\alpha+\beta) = 0$

$\Rightarrow \alpha + \beta = (2n+1)\frac{\pi}{2},\ n \in I$

$\therefore\ \sin(\alpha + 2\beta) = \sin(2\alpha + 2\beta - \alpha)$

$= \sin[(2n+1)\pi - \alpha]$

$= \sin(2n\pi + \pi - \alpha)$

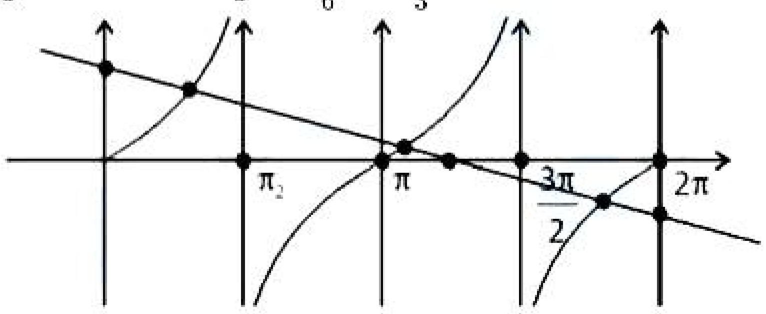
$= \sin(\pi - \alpha) = \sin\alpha$

Q22: 56

$\sqrt{3}\cos x - \sin x = \pm 2$
$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \pm 1$
$\cos\left(x + \frac{\pi}{6}\right) = \pm 1$
$\Rightarrow x + \frac{\pi}{6} = n\pi$
$\Rightarrow x = n\pi - \frac{\pi}{6}$
$\Rightarrow x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}, 3\pi - \frac{\pi}{6}, 4\pi - \frac{\pi}{6}$
$\Rightarrow$ The sum of roots is $10\pi - \frac{2\pi}{3} = \frac{28}{3}\pi$
$\Rightarrow 6k = 6 \times \frac{28}{3} = 56$

Q23: (C) 3

$2x + 3\tan x = \frac{5\pi}{2} \Rightarrow \tan x = -\frac{2}{3}x + \frac{5\pi}{6}$
$y = \tan x$ and $y = \frac{5\pi}{6} - \frac{2x}{3}$



Both the graphs meet exactly three times in $[0, 2\pi]$.
Thus, there are 3 solutions.

Q24: (B) $\left(0, \frac{\sqrt{5}-1}{2}\right]$
Since, $\sin x > 0, \quad \cos x > 0, \quad \cos x \neq 1$
So, $\sin x \leq \cos^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 \leq 0$

$\Rightarrow \left(\sin x + \frac{1}{2}\right)^2 - \frac{5}{4} \leq 0$

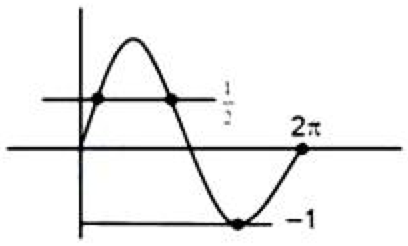
$\Rightarrow \left|\sin x + \frac{1}{2}\right| \leq \frac{\sqrt{5}}{2}$

$\therefore \sin x + \frac{1}{2} \in \left[\frac{-\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

(As, $\sin x > 0$)

$\therefore 0 < \sin x \leq \frac{\sqrt{5}-1}{2}$

Q25: (B) 4
$\tan x + \sec x = 2\cos x$
Multiplying by $\cos x$,
$\sin x + 1 = 2\cos^2 x = 2 - 2\sin^2 x$
$\Rightarrow 2\sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = -1, \frac{1}{2}$



$\sin x = \frac{1}{2}$ at
$x = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, 2\pi]$
$\sin x = \frac{1}{2} \rightarrow$ four times in $[0, 4\pi]$
$\sin x = -1 \Rightarrow x = \frac{3\pi}{2}$ for which $\tan x$ and $\sec x$ are not defined
$\Rightarrow \sin x = -1$ is not possible
$\Rightarrow$ Number of roots in $[0, 4\pi]$ is 4.

Q26: (D) 3

$y = \cos x;\ y = \sin 3x$
$\Rightarrow \cos x = \cos\left(\frac{\pi}{2} - 3x\right) \Rightarrow x = 2n\pi \pm \left(\frac{\pi}{2} - 3x\right)$
$\Rightarrow 4x = 2n\pi + \frac{\pi}{2},\ \ -2x = 2n\pi - \frac{\pi}{2}$
$\Rightarrow x = \frac{n\pi}{2} + \frac{\pi}{8},\ x = k\pi + \frac{\pi}{4}$
$\Rightarrow x = (4n+1)\frac{\pi}{8},\ x = (4k+1)\frac{\pi}{4}$
$n = 0 \Rightarrow x = \frac{\pi}{8}, k = 0 \Rightarrow x = \frac{\pi}{4}$
$n = -1 \Rightarrow x = \frac{-3\pi}{8}$
$\Rightarrow x = \frac{-3\pi}{8}, \frac{\pi}{8}$ and $\frac{\pi}{4}$

Q27: (D) $\frac{4\pi}{3}$

$(\sin A - \sin B)^2 + (\cos A - \cos B)^2 = \frac{1}{2} + \frac{3}{2} = 2$
$\Rightarrow 2 - 2(\sin A \sin B + \cos A \cos B) = 2$
$\Rightarrow \cos(B - A) = 0 \Rightarrow B - A = \frac{\pi}{2} \Rightarrow B = A + \frac{\pi}{2}$
$\sin A - \sin B = \frac{1}{\sqrt{2}} \Rightarrow \sin A - \cos A = \frac{1}{\sqrt{2}}$
$\Rightarrow \sin A. \frac{1}{\sqrt{2}} - \cos A. \frac{1}{\sqrt{2}} = \frac{1}{2}$
$\Rightarrow \sin\left(A - \frac{\pi}{4}\right) = \sin\frac{\pi}{6}$
$\Rightarrow A = \frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$
$A + B = A + A + \frac{\pi}{2} = \frac{5\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{3}$

Q28: (C) $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\cos^2\theta - 3\cos\theta + 2 \geq \frac{1}{\operatorname{cosec}^2\theta} = \sin^2\theta$
$\cos^2\theta - 3\cos\theta + 2 \geq 1 - \cos^2\theta$
$2\cos^2\theta - 3\cos\theta + 1 \geq 0$
$(2\cos\theta - 1)(\cos\theta - 1) \geq 0$
$\Rightarrow \cos\theta \leq \frac{1}{2}$ or $\cos\theta \geq 1$
So, $\theta \in (\pi/3, \pi/2)$ of the given intervals.

Q29: (D) 20

$f\left(\sin^2\theta\right) + f\left(1 - \sin^2\theta\right)$
Let, $\sin^2\theta = t$
$\therefore f(t) + f(1-t)$
$= \frac{25^t}{25^t+5} + \frac{25^{1-t}}{25^{1-t}+5} = \frac{25^t}{25^t+5} + \frac{25}{25+5(25^t)}$
$= \frac{25^t+5}{25^t+5} = 1$
Therefore, the given equation will become $\tan^2\theta = 1$
$\therefore\ \tan\theta = \pm 1 \Rightarrow \theta = n\pi \pm \pi/4,\ n \in Z$
Hence, the number of solutions are

$4 \times 5 = 20$
$\{\because$ 4 solutions in $[0,2\pi]\}$

Q30: 5

$\text{LHS} = 12\sin x + 5\cos x \in \left[-\sqrt{12^2+5^2}, \sqrt{12^2+5^2}\right]$
i.e. $[-13,13]$ i.e.
maximum value of LHS is 13
$\text{RHS} = 2\left(y^2 - 4y + 4\right) + 13$
$= 2(y-2)^2 + 13$
$\text{RHS} \geq 13$
Roots of the equation exist if $\text{LHS} = \text{RHS} = 13$.

$\text{RHS} = 13$ when $y = 2$
$\text{LHS} = 13 \Rightarrow 12\sin x + 5\cos x = 13$
$\Rightarrow \frac{12}{13}\sin x + \frac{5}{13}\cos x = 1$
$\sin(x+\alpha) = 1$, where $\tan\alpha = \frac{5}{12}$
$x + \tan^{-1}\frac{5}{12} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2} - \tan^{-1}\frac{5}{12}$
$\Rightarrow xy = \pi - 2\tan^{-1}\frac{5}{12} \Rightarrow \frac{5}{12} = \tan\left(\frac{\pi}{2} - \frac{xy}{2}\right) = \cot\left(\frac{xy}{2}\right)$
$\Rightarrow 12\cot\left(\frac{xy}{2}\right) = 5$

Q31: (A) 4950

$\left(2\cos^2 2x - 1\right) + 6 = 7\cos 2x$
On putting $\cos 2x = t$, we get,
$2t^2 - 1 + 6 = 7t$
$2t^2 - 7t + 5 = 0$
$(2t-5)(t-1) = 0$
$t = \frac{5}{2}, 1$
$t = \frac{5}{2}$ (not possible)
$t = 1 \Rightarrow \cos 2x = 1 \Rightarrow 2x = 2n\pi$
$\Rightarrow x = n\pi$
The roots in $[0,314]$ are
$\pi, 2\pi, 3\pi, \ldots\ldots, 99\pi$ $\{100\pi > 314\}$
Sum of roots $= \pi + 2\pi + 3\pi + \ldots + 99\pi = 4950\pi$
$\Rightarrow \lambda = 4950$

Q32: (C) 3

$\sin x = -\frac{1}{2} \Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$
$\cos x = -\frac{\sqrt{3}}{2} \Rightarrow x = \frac{5\pi}{6}, \frac{7\pi}{6}$
$\Rightarrow \alpha = \frac{7\pi}{6}, \beta = \frac{11\pi}{6}, \gamma = \frac{5\pi}{6}$
$\Rightarrow \alpha + \beta = \frac{7\pi}{6} + \frac{11\pi}{6} = 3\pi, \beta - \gamma = \frac{11\pi}{6} - \frac{5\pi}{6} = \pi$
$\Rightarrow \frac{\alpha+\beta}{|\beta-\gamma|} = \frac{3\pi}{\pi} = 3$

Q33: (A) $\frac{1}{2}$

$\cot x = -\sqrt{3}$

$\Rightarrow x = \frac{5\pi}{6}, \frac{11\pi}{6}$ and

$\operatorname{cosec} x = -2$

$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6}$

So, $\alpha = \frac{11\pi}{6},\ \beta = \frac{5\pi}{6},\ \gamma = \frac{7\pi}{6}$

$\Rightarrow \alpha - \beta = \pi$ and $\beta + \gamma = 2\pi$

$\Rightarrow \frac{|\alpha - \beta|}{\beta + \gamma} = \frac{\pi}{2\pi} = \frac{1}{2}$