DPP - Daily Practice Problems

Date : ________ **Start Time :** ________ **End Time :** ________

PHYSICS

CP05

SYLLABUS : System of Particles and Rotational Motion

Max. Marks : 74 **Time : 60 min.**

GENERAL INSTRUCTIONS

- The Daily Practice Problem Sheet contains 20 Questions divided into 5 sections.
  Section I has **5** MCQs with ONLY 1 Correct Option, **3** marks for each correct answer and **–1** for each incorrect answer.
  Section II has **4** MCQs with ONE or MORE THAN ONE Correct options.
  For each question, marks will be awarded in one of the following categories:
  Full marks: **+4** If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
  Partial marks: **+1** For darkening a bubble corresponding to each correct option provided NO INCORRECT option is darkened.
  Zero marks: If none of the bubbles is darkened.
  Negative marks: **–2** In all other cases.
  Section III has **4** Single Digit Integer Answer Type Questions, **3** marks for each Correct Answer and 0 marks in all other cases.
  Section IV has Comprehension/Matching Cum-Comprehension Type Questions having **5** MCQs with ONLY ONE correct option, **3** marks for each Correct Answer and 0 marks in all other cases.
  Section V has **2** Matching Type Questions, **2** mark for the correct matching of each row and 0 marks in all other cases.
- You have to evaluate your Response Grids yourself with the help of Solutions.

Section I - Straight Objective Type

This section contains 5 multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

1. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}\right)L$ from A, then a and b are related as :

(a) $a = 2b$ (b) $2a = b$ (c) $a = b$ (d) $3a = 2b$

2. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct?

(a) $\vec{L}$ (angular momentum) is conserved about the centre

(b) Only direction of angular momentum $\vec{L}$ is conserved

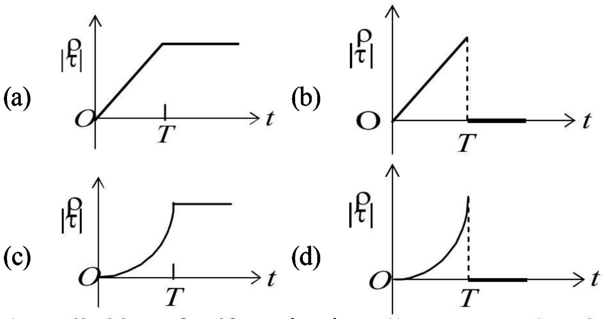
(c) It spirals towards the centre

(d) Its acceleration is towards the centre.

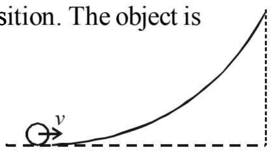
RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ

Space for Rough Work

3. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ about O, as a function of time is best represented by plot

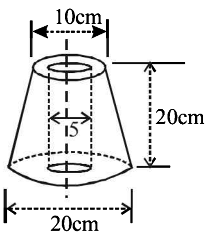


4. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



(a) a ring
(b) a solid sphere
(c) a hollow sphere
(d) a disc

5. A frustum of a solid right circular cone has a base diameter of 20cm, top diameter of 10cm and height 20cm. It has an axial cylindrical hole of diameter 5cm. Determine the position of centre of gravity of this body



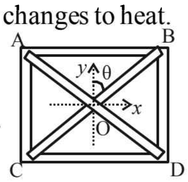
(a) 7.6cm (b) 4.3cm
(c) 12.6cm (d) 15.3 cm

Section II - Multiple Correct Answer Type

This section contains 4 multiple correct answer(s) type questions. Each question has 4 choices (a), (b), (c) and (d), out of which **ONE OR MORE** is/are correct.

6. A horizontal disc rotates freely about a vertical axis through it centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc. After some time, the two rotate with a common angular velocity. Select the correct statements from the following.
(a) Some friction exists between the disc and the ring
(b) The angular momentum of the 'disc plus ring' is conserved.
(c) The final common angular velocity is (2/3) rd of the initial angular velocity of the disc
(d) (2/3) rd of the initial kinetic energy changes to heat.

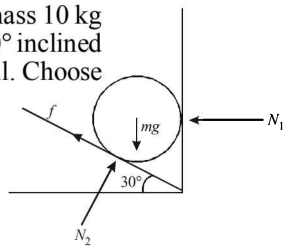
7. As shown in figure, a planner assembly, having six rods, each of mass m is lying in x-y plane with O at origin, lengths of AD and BC are ℓ. If I_z denotes the moment of inertia of the assembly about z-axis and I_y denotes moment of inertia about y-axis then



(a) I_z will have its highest value for $\theta = 45°$
(b) I_z will have its highest value for $\theta = 90°$
(c) I_z will have its highest value for $\theta = 0°$
(d) I_y will have its highest value for $\theta = 90°$

8. The torque τ on a body about a given point is found to be equal to $\boldsymbol{A} \times \boldsymbol{L}$ where $\boldsymbol{A}$ is a constant vector, and $\boldsymbol{L}$ is the angular momentum of the body about that point. From this it follows that
(a) $\frac{d\boldsymbol{L}}{dt}$ is perpendicular to $\boldsymbol{L}$ at all instants of time.
(b) the component of $\boldsymbol{L}$ in the direction of $\boldsymbol{A}$ does not change with time.
(c) the magnitude of $\boldsymbol{L}$ does not change with time.
(d) $\boldsymbol{L}$ does not change with time

9. A sphere of radius 0.10m and mass 10 kg rests in the corner formed by a 30° inclined plane and a smooth vertical wall. Choose the correct options

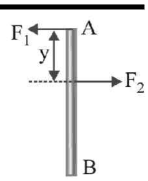


(a) $N_1 = 56.5$ N
(b) $N_2 = 113$ N
(c) $f = 0$
(d) $f \neq 0$

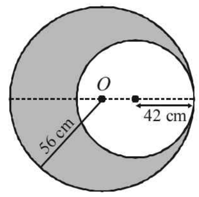
Section III - Integer Type

This section contains 4 questions. The answer to each of the questions is a single digit integer ranging from 0 to 9.

10. A thin uniform rod AB of mass $m = 1.0$ kg moves translationally with acceleration $a = 2.0$ m/s^2 due to two antiparallel forces F_1 and F_2 as shown in figure. The distance between the points at which these forces are applied is equal to $y = 20$ cm. Besides, it is known that $F_2 = 5.0$ N. Find the length of the rod.



11. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown. At what distance (in cm) to the left from the centre of the disc is the centre of mass of the remaining portion ?



RESPONSE GRID	
	3. ⓐⓑⓒⓓ 4. ⓐⓑⓒⓓ 5. ⓐⓑⓒⓓ 6. ⓐⓑⓒⓓ 7. ⓐⓑⓒⓓ
	8. ⓐⓑⓒⓓ 9. ⓐⓑⓒⓓ 10. ⓪①②③④⑤⑥⑦⑧⑨
	11. ⓪①②③④⑤⑥⑦⑧⑨

Space for Rough Work

12. A homogeneous disc with a radius 0.2m and mass 5 kg rotates around an axis passing through its centre. The angular velocity of the rotation of the disc as a function of time is given by the formula $\omega = 2 + 6t$. What will be the tangential force applied to the rim of the disc ?

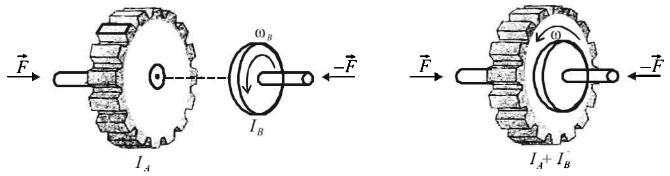
13. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If, $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

Section IV - Comprehension/Matching Cum-Comprehension Type

Directions (Qs. 14 and 15) : Based upon the given paragraph, 2 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** is correct.

PARAGRAPH

The figure has two disks : one an engine flywheel, and the other a clutch plate attached to a transmission shaft. Their moments of inertia are I_A and I_B; initially, they are rotating with constant angular speeds ω_A and ω_B, respectively. We then push the disks together with forces acting along the axis, so as not to apply any torque on either disk. The disks rub against each other and eventually reach a common final angular speed ω.



Suppose flywheel A has a mass of 2.0 kg, a radius of 0.20 m and an initial angular speed of 50 rad/sec. (about 500 rpm) and that clutch plate B has a mass of 4.0 kg, a radius of 0.10 m, and an initial angular speed of 200 rad/sec.

14. Find an expression for ω

(a) $\frac{I_A\omega_A + I_B\omega_B}{I_A - I_B}$ (b) $\frac{I_A\omega_A - I_B\omega_B}{I_A - I_B}$

(c) $\frac{I_A\omega_A + I_B\omega_B}{I_A + I_B}$ (d) $\frac{I_A\omega_A - I_B\omega_B}{I_A + I_B}$

15. What happens to the final kinetic energy during this process?

(a) 300 J (b) 3 J (c) 30 J (d) 3000 J

Directions (Qs. 16-18) : This passage contains a table having 3 columns and 4 rows. Based on the table, there are three questions. Each question has four options (a), (b), (c) and (d) **ONLY ONE** of these four options is correct.

If moment of inertia of an object about an axis is given by $I = MK^2$, then radius of gyration is given by $K=\left[\frac{I}{M}\right]^{1/2}$. Column II and III represents moment of inertia along the tangent (in the plane of the object) and radius of gyration repectively of different objects of Mass M and radius R.

	Column I		Column II		Column III
I.	Solid sphere	(i)	$\frac{5}{3}MR^2$	(P)	$\sqrt{\frac{3}{2}}R$
II.	Disc	(ii)	$\frac{3}{2}MR^2$	(Q)	$\frac{\sqrt{5}}{2}R$
III.	Thin spherical shell	(iii)	$\frac{7}{5}MR^2$	(R)	$\frac{\sqrt{5}}{\sqrt{3}}R$
IV.	Ring	(iv)	$\frac{5}{4}MR^2$	(S)	$\frac{\sqrt{7}}{\sqrt{5}}R$

16. The correct matching for moment of inertia and radius of gyration of solid sphere of mass M and radius R along the tangent is

(a) I (i) Q(b) (b) (I) (iii) (S) (c) (III) (ii) (R) (d) (I) (iv) (S)

17. What is the correct matching for the disc, if the moment of inertia of the disc about diameter in the plane of the disc is $\frac{MR^2}{4}$

(a) II (ii) Q (b) II (iv) Q (c) II (i) R (d) II (iii) S

18. If radius (R) and mass (M) of thin spherical shell are $\sqrt{15}m$ and 2kg respectively, then the moment of inertia and radius of gyration of the shell along the tangent are 50 kg m^2 and 5m respectively. Which of the following is correct for spherical shell?

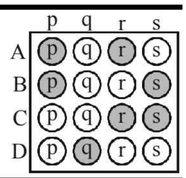
(a) III (i) R (b) I (iii) Q (c) III (ii) R (d) IV (iii) S

RESPONSE GRID	
	12. ⓪①②③④⑤⑥⑦⑧⑨ 13. ⓪①②③④⑤⑥⑦⑧⑨ 14. ⓐⓑⓒⓓ
	15. ⓐⓑⓒⓓ 16. ⓐⓑⓒⓓ 17. ⓐⓑⓒⓓ 18. ⓐⓑⓒⓓ

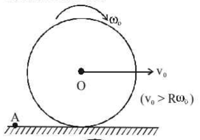
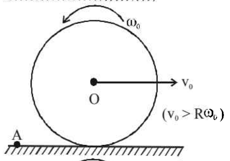
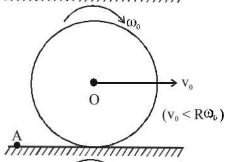
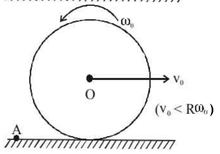
Space for Rough Work

Section V - Matrix-Match Type

This section contains 2 questions. It contains statements given in two columns, which have to be matched. Statements in column I are labelled as A, B, C and D whereas statements in column II are labelled as p, q, r and s. The answers to these questions have to be appropriately bubbled as illustrated in the following example. If the correct matches are A-p, A-r, B-p, B-s, C-r, C-s and D-q, then the correctly bubbled matrix will look like the following:



19. In each situation of column -I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown. At t = 0 (initially) the angular velocity of disc is ω_0 and velocity of centre of mass of disc is v_0 (in horizontal direction). The relation between v_0 and ω_0 for each situation and also initial sense of rotation is given for each situation in column-I. Then match the statements in column-I with the corresponding results in column-II.

Column I	Column II
(A) 	(p) The angular momentum of disc about point A remains conserved.
(B) 	(q) The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy.
(C) 	(r) In the duration dics rolls with slipping, the friction acts on disc towards left.
(D) 	(s) In the duration disc rolls with slipping, the friction acts on disc for sometime to right and for sometime to left.

20. A particle moves with position given by $\vec{r} = 3t\hat{i} + 4\hat{j}$. Where $\vec{r}$ is measured in meters and $t\ (>0)$ in seconds

Column I	Column II
(A) Rate of change of distance from origin	(p) Increasing with time
(B) Magnitude of linear acceleration of particle	(q) Decreasing with time
(C) Magnitude of angular velocity of particle about origin	(r) Constant
(D) Magnitude of angular momentum of particle about origin	(s) Zero

RESPONSE GRID	
	19. A - ⓟⓠⓡⓢ; B - ⓟⓠⓡⓢ; C - ⓟⓠⓡⓢ; D - ⓟⓠⓡⓢ
	20. A - ⓟⓠⓡⓢ; B - ⓟⓠⓡⓢ; C - ⓟⓠⓡⓢ; D - ⓟⓠⓡⓢ

DAILY PRACTICE PROBLEM DPP CP05 - PHYSICS			
Total Questions	20	Total Marks	74
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	24	Qualifying Score	35

$$\text{Net Score} = \sum_{i=1}^{V}\left[\left(\text{correct}_i \times MM_i\right) - \left(In_i - NM_i\right)\right]$$

Space for Rough Work

1. **(b)** Centre of mass of the rod is given by:

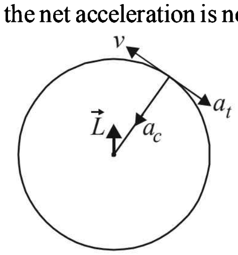
$$x_{cm} = \frac{\int_0^L (ax + \frac{bx^2}{L})dx}{\int_0^L (a + \frac{bx}{L})dx}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}} = \frac{L\left(\frac{a}{2} + \frac{b}{3}\right)}{a + \frac{b}{2}}$$

Now $\frac{7L}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}}$

On solving we get, $b = 2a$

2. **(b)** Since v is changing (decreasing), L is not conserved in magnitude. Since it is given that a particle is confined to rotate in a circular path, it cannot have spiral path. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.

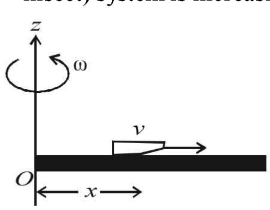


The direction of $\vec{L}$ remains same even when the speed decreases.

3. **(b)** We know that $|\vec{\tau}| = \left|\frac{d\vec{L}}{dt}\right|$ where $L = I\omega$

$$\therefore \quad \tau = \frac{d}{dt}(I\omega) = \omega\frac{dI}{dt} \quad ...(i)$$

From the situation it is clear that the moment of inertia for (rod + insect) system is increasing.



Let at any instant of time 't', the insect is at a distance x from O. At this instant, the moment of inertia of the system is

$$I = \frac{1}{3}ML^2 + mx^2 \quad ...(ii)$$

From (i) & (ii)

$$\tau = \omega\frac{d}{dt}\left[\frac{1}{3}ML^2 + mx^2\right] = \omega\, m\frac{d}{dt}(x^2)$$

$$= 2\omega mx\frac{dx}{dt} = 2\omega mxv$$

$$= 2\omega mv^2 t \quad [\because x = vt]$$

$$\therefore \quad \tau \propto t \quad (\text{till } t = T)$$

When the insect stops moving, $\vec{L}$ does not change and therefore τ becomes constant.

4. **(d)** By the concept of energy conservation

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mg\left(\frac{3v^2}{4g}\right)$$

For rolling motion $v = R\omega$

$$\therefore \quad \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}mv^2$$

$$\therefore \quad \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{1}{4}mv^2$$

$$\frac{1}{2}I\frac{v^2}{R^2} = \frac{1}{4}mv^2$$

$$\Rightarrow \quad I = \frac{1}{2}mR^2$$

This is the formula of the moment of inertia of the disc.

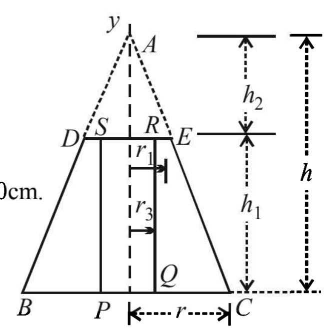
5. **(a)** The given body can be considered as a right circular cone ABC from which a cone ADE and a cylindrical $PQRS$ have been cut out as shown.

Let the x-axis be along the base and y-axis as the axis of symmetry. For the cone ABC of height h is :

$$\frac{h_2}{r_1} = \frac{h}{r}$$

or $\frac{h-20}{5} = \frac{h}{10}$

$\Rightarrow 2h - 40 = h \Rightarrow h = 40\text{cm}.$



Volume $V_1 = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(10)^2 \times 40 = \frac{4000}{3}\pi \text{ cm}^3$$

Position of CG on y-axis,

$$y_1 = \frac{1}{4} \times 40 = 10\text{cm}.$$

For cone ADE, Volume

$$V_2 = \frac{1}{3}\pi r_1^2 h_2 = \frac{1}{3}\pi(5)^2 \times 20 = \frac{500}{3}\pi \text{ cm}^3$$

Position of CG on y-axis,

$$y_2 = 20 + \frac{1}{4} \times 20 = 25\text{cm}.$$

For cylindrical hole

Volume, $V_3 = \pi r_3^2 h_1 = \pi\left(\frac{5}{2}\right)^2 \times 20 = 125\,\pi\ \text{cm}^3$

Position of CG on y-axis, $y_3 = \frac{h_1}{2} = \frac{20}{2} = 10$ cm.

The given body has a volume = $V_1 - V_2 - V_3$

$\therefore$ Position of CG of given body on y-axis

$$\bar{y} = \frac{\Sigma V_y}{\Sigma V} = \frac{V_1 y_1 - V_2 y_2 - V_3 y_3}{V_1 - V_2 - V_3}$$

$$= \frac{\frac{4000}{3}\pi \times 10 - \frac{500}{3}\pi \times 25 - 125\pi \times 10}{\frac{4000}{3}\pi - \frac{500}{3}\pi - 125\pi}$$

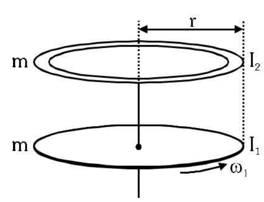
$$\bar{y} = \frac{40000 - 12500 - 3750}{4000 - 500 - 375} = \frac{23750}{3125} = 7.6\text{cm}$$

6. **(a, b, d)** Let ω_1 = the initial angular velocity of the disc.
ω_2 = the final common angular velocity of the disc and the ring.

For the disc, $I_1 = \frac{1}{2}mr^2$

For the ring, $I_2 = mr^2$

By conservation of angular momentum,



$$L = I_1\omega_1 - (I_1 + I_2)\omega_2$$

or $\omega_2 = \frac{I_1\omega_1}{I_1 + I_2} = \omega_1 / 3$

Initial kinetic energy $= E_1 = \frac{1}{2}I_1\omega_1^2$

Final kinetic energy $= E_2 = \frac{1}{2}(I_1 + I_2)\omega_2^2$

Heat produced = loss in kinetic energy = $E_1 - E_2$.

Ratio of heat produced to initial kinetic energy

$$= \frac{E_1 - E_2}{E_1} = \frac{2}{3}.$$

7. **(a, b, c, d)** $I_z = \frac{m(\ell\sin\theta)^2}{12} + m\left(\frac{\ell}{2}\cos\theta\right)^2 + m\frac{(\ell\cos\theta)^2}{12}$

$$+ m\left(\frac{\ell}{2}\sin\theta\right)^2 + \frac{m\ell^2}{12} + \frac{m\ell^2}{12}$$

$$= \frac{m\ell^2}{12} + \frac{m\ell^2}{4} + \frac{m\ell^2}{12} + \frac{m\ell^2}{12}$$

$$= \frac{m\ell^2}{2} \text{ (constant independent of } \theta)$$

[I_z will be maximum for any value of θ (Obviously)]

8. **(a, b, c)** $\vec{\tau} = \frac{d\vec{L}}{dt}$

Given that

$$\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{d\vec{L}}{dt} = \vec{A} \times \vec{L}$$

From cross-product rule, $\frac{d\vec{L}}{dt}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{L}$.

By the dot product definition

$$\vec{L}.\vec{L} = L^2$$

Differentiating with respect to time

$$\vec{L}\cdot\frac{d\vec{L}}{dt} + \vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt} \Rightarrow 2\vec{L}\cdot\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

Since, $\frac{d\vec{L}}{dt}$ i.e. $\vec{\tau}$ is perpendicular to $\vec{L}$

$$\therefore\ \vec{L}\cdot\frac{d\vec{L}}{dt} = 0 \Rightarrow \frac{dL}{dt} = 0$$

$\Rightarrow$ L = constant

Thus, the magnitude of L always remains constant.

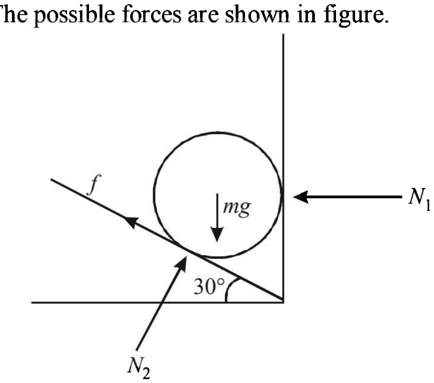
As $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$,

Also, $\vec{L}$ is perpendicular to $\vec{A}$

$\because\ \vec{L} \perp \vec{A}$ $\quad\therefore\ \vec{L}.\vec{A} = 0$

Thus, it can be concluded that component of $\vec{L}$ along $\vec{A}$ is zero i.e., always constant.

9. **(a, b, c)** The possible forces are shown in figure.



If we take moments about an axis through the center of the sphere, only f can have a torque and $\Sigma\,\tau = 0$ implies $f = 0$.

Then $\Sigma F_y = 0$ yields

$N_2 \cos 30° = mg = (10\text{ kg})\,(9.8\text{ m/s}^2)$

$\Sigma F_x = 0$ yields $N_2 \sin 30° - N_1 = 0$,

or $N_1 = 56.5$ N, $N_2 = 113$ N

10. **1** By Newton's second law

$$F_2 - F_1 = ma$$

$$\therefore\quad F_1 = F_2 - ma$$

$$= 5 - 1 \times 2 = 3\text{ N}.$$

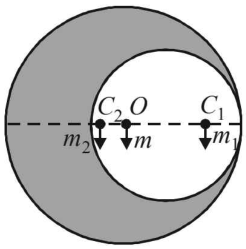
For rotational equilibrium, taking moment of forces about centre of mass, we get

$$F_1 \times \frac{l}{2} - F_2\left(\frac{l}{2} - y\right) = 0$$

$$3 \times \frac{l}{2} - 5\left(\frac{l}{2} - 0.2\right) = 0$$

$$\therefore \quad l = 1\text{m.}$$

11. 9



Area of whole plate = $\pi(56/2)^2 = 784\,\pi$ sq. cm.

Area of cutout portion = $\pi(42/2)^2 = 441\,\pi$ sq. cm. ;

Area of remaining portion = $784\pi - 441\pi = 343\,\pi\,\text{cm}^2$;

As mass $\propto$ area.

$$\therefore \frac{\text{mass of cutout portion}}{\text{mass of remaining portion}} = \frac{m_1}{m_2} = \frac{441\pi}{343\pi} = \frac{9}{7}$$

Let C_2 be centre of mass of remaining portion and C_1 be centre of mass of cutout portion.

O is centre of mass of the whole disc.;

$OC_1 = r_1 = 28 - 21 = 7$ cm.

$OC_2 = r_2 = ?$;

Equating moments of masses about O,

we get $m_2 \times r_2 = m_1 \times r_1 \Rightarrow r_2 = \frac{m_1}{m_2} \times r_1 = \frac{9}{7} \times 7 = \mathbf{9}$

$\therefore$ Centre of mass of remaining portion is at 9 cm to the left of centre of disc.

12. 3

$FR = I\alpha$

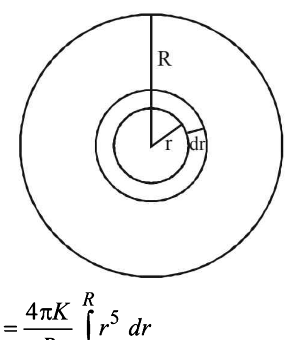
where R is the radius of the disc.

$$F = \frac{1}{2}MR\alpha$$

$$= \frac{1}{2}MR\left(\frac{d\omega}{dt}\right) = \frac{1}{2}MR(6)$$

$$= \frac{1}{2} \times 5 \times 0.2 \times 6 = 3.0\,\text{N}$$

13. 6 $\quad I = \int_0^R (dm)r^2$



$$\therefore \quad I = \int_0^R \rho \times 4\pi r^2\, dr \times r^2$$

$$\therefore \quad I = 4\pi \int_0^R \rho\, r^4\, dr$$

$$\therefore \quad I_A = 4\pi \int_0^R k\frac{r}{R} \times r^4\, dr = \frac{4\pi K}{R}\int_0^R r^5\, dr$$

$$= \frac{4\pi K}{R}\left(\frac{R^6}{6}\right) = 4\pi K\frac{R^5}{6}$$

$$I_B = 4\pi \int_0^R K\left(\frac{r}{R}\right)^5 r^4\, dr = \frac{4\pi K}{R^5} \times \frac{R^{10}}{10} = 4\pi K\frac{R^5}{10}$$

$$\therefore \quad \frac{I_B}{I_A} = \frac{6}{10} \Rightarrow n = 6$$

14. (c) As shown in the figure that all of the angular velocities are in the same direction, so we can regard ω_A, ω_B, and ω as the components of angular velocity along the rotation axis. Conservation of angular momentum then gives

$$I_A\omega_A + I_B\omega_B = (I_A + I_B)\omega$$

$$\omega = \frac{I_A\omega_A + I_B\omega_B}{I_A + I_B}$$

15. (a) The initial kinetic energy is

$$K_1 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2$$

$$= \frac{1}{2}(0.040\,\text{kg.m}^2)(50\,\text{rad/s})^2 + \frac{1}{2}(0.020\,\text{kg.m}^2)(200\,\text{rad/s})^2$$

The final kinetic energy is

$$K_2 = \frac{1}{2}(I_A + I_B)\omega_2 = \frac{1}{2}(0.040\,\text{kg.m}^2 + 0.020\,\text{kg.m}^2)(100\,\text{rad/s})^2 = 300\,\text{J}$$

16. (b) As we know that moment of inertia of solid sphere along its diameter is $\frac{2}{5}MR^2$.

Using parallel axes theorem $I_{tangent} = I_{cm} + MR^2$

$$= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

So the radius of gyration = $\sqrt{\frac{7}{5}}R$

17. (b) MOI along tangent in the plane is

$$\frac{5}{4}MR^2$$

$$I_{\text{along tangent}} = I_{\text{diameter (c.m.)}} + MR^2$$

$$\Rightarrow \quad \frac{5}{4}MR^2 = I_{\text{diameter}} + MR^2$$

$$\Rightarrow \quad I_{\text{diameter}} = \frac{5}{4}MR^2 - MR^2 = \frac{MR^2}{4}$$

18. (a) Moment of inertia of thin spherical shell along the tangent is $\frac{5}{3}MR^2$ and radius of gyration is $\sqrt{\frac{5}{3}}R$

so, $I = \frac{5}{3} \times 2 \times \left(\sqrt{15}\right)^2 = 50\,\text{kg m}^2$,

radius of gyration = $\sqrt{\frac{5}{3}} \times \sqrt{15} = 5\text{m}$

19. **A → p, q, r; B → p, q, r; C → p, q; D → p, q, r;**

(A) Since all forces on disc pass through point of contact with horizontal surface, the angular momentum of disc about point on ground in contact with disc is conserved. Also the angular momentum of disc in all cases is conserved about any point on the line passing through point of contact and parallel to velocity of centre of mass. The K.E. of disc is decreased in all cases due to work done by friction.

From calculation of velocity of lowest point on disc, the direction of friction in case A, B and D is towards left and in case C is towards right.

The direction of frictional force cannot change in any given case.

20. **A-p; B-r, s; C-q; D-r**

$\vec{v} = 3\hat{i}$, $a_t = 0$, $L = mvr_{\perp} = (mr^2)\omega$;

$$\omega = \frac{12}{\sqrt{9t^2 + 16}}$$

distance $= |\vec{r}| = \sqrt{(3t)^2 + (4)^2}$;

$$\frac{d}{dt}(\text{distance}) = \left(\frac{3t}{\sqrt{(3t)^2 + (4)^2}} \right)$$

this is increasing function with time (t).