CHAPTER 4

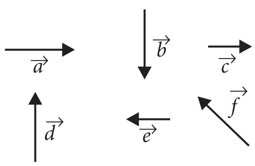
Motion in a Plane

4.2 Scalars and Vectors

1. Identify the vector quantity among the following.
(a) Distance (b) Angular momentum
(c) Heat (d) Energy *(1997)*

4.4 Addition and Subtraction of Vectors-Graphical Method

2. Six vectors, $\vec{a}$ through $\vec{f}$ have the magnitudes and directions indicated in the figure. Which of the following statements is true?



(a) $\vec{b}+\vec{c}=\vec{f}$
(b) $\vec{d}+\vec{c}=\vec{f}$
(c) $\vec{d}+\vec{e}=\vec{f}$
(d) $\vec{b}+\vec{e}=\vec{f}$ *(2010)*

4.5 Resolution of Vectors

3. If a unit vector is represented by $0.5\hat{i}-0.8\hat{j}+c\hat{k}$ then the value of c is
(a) $\sqrt{0.01}$ (b) $\sqrt{0.11}$
(c) 1 (d) $\sqrt{0.39}$ *(1999)*

4.6 Vector Addition-Analytical Method

4. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is
(a) 45° (b) 180°
(c) 0° (d) 90° *(NEET-I 2016)*

5. The vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle between the two vectors is
(a) 45° (b) 90°
(c) 60° (d) 75° *(2006, 1996, 1991)*

6. If $|\vec{A}+\vec{B}| = |\vec{A}|+|\vec{B}|$ then angle between A and B will be
(a) 90° (b) 120°
(c) 0° (d) 60°. *(2001)*

7. The magnitude of vectors $\vec{A}, \vec{B}$ and $\vec{C}$ are 3, 4 and 5 units respectively. If $\vec{A}+\vec{B}=\vec{C}$, the angle between $\vec{A}$ and $\vec{B}$ is
(a) $\pi/2$ (b) $\cos^{-1}(0.6)$
(c) $\tan^{-1}(7/5)$ (d) $\pi/4$. *(1988)*

4.7 Motion in a Plane

8. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in metres and t in seconds. The acceleration of the particle at $t = 2$ s is
(a) 5 m s^{-2} (b) -4 m s^{-2}
(c) -8 m s^{-2} (d) 0 *(NEET 2017)*

9. The position vector of a particle $\vec{R}$ as a function of time is given by $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$ where R is in meters, t is in seconds and $\hat{i}$ and $\hat{j}$ denote unit vectors along x-and y-directions, respectively. Which one of the following statements is wrong for the motion of particle?
(a) Magnitude of the velocity of particle is $8\,\pi$ meter/second.
(b) Path of the particle is a circle of radius 4 meter.
(c) Acceleration vector is along $-\vec{R}$.
(d) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle. *(2015)*

10. A particle is moving such that its position coordinates (x, y) are (2 m, 3 m) at time $t = 0$, (6 m, 7 m) at time $t = 2$ s and (13 m, 14 m) at time $t = 5$ s. Average velocity vector $(\vec{v}_{av})$ from $t = 0$ to $t = 5$ s is
(a) $\frac{1}{5}(13\hat{i}+14\hat{j})$ (b) $\frac{7}{3}(\hat{i}+\hat{j})$
(c) $2(\hat{i}+\hat{j})$ (d) $\frac{11}{5}(\hat{i}+\hat{j})$ *(2014)*

11. A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is

(a) $1\ m/s^2$ (b) $7\ m/s^2$
(c) $\sqrt{7}\ m/s^2$ (d) $5\ m/s^2$ *(2011)*

12. A particle moves in x-y plane according to rule $x = a\sin\omega t$ and $y = a\cos\omega t$. The particle follows
(a) an elliptical path (b) a circular path
(c) a parabolic path
(d) a straight line path inclined equally to x and y-axes *(Mains 2010)*

13. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x-axis an angle of
(a) 45° (b) 60° (c) 0° (d) 30°. *(2007)*

14. A bus is moving on a straight road towards north with a uniform speed of 50 km/hour then it turns left through 90°. If the speed remains unchanged after turning, the increase in the velocity of bus in the turning process is
(a) 70.7 km/hr along south-west direction
(b) zero
(c) 50 km/hr along west
(d) 70.7 km/hr along north-west direction *(1989)*

4.8 Motion in a Plane with Constant Acceleration

15. When an object is shot from the bottom of a long smooth inclined plane kept at an angle 60° with horizontal, it can travel a distance x_1 along the plane. But when the inclination is decreased to 30° and the same object is shot with the same velocity, it can travel x_2 distance. Then $x_1 : x_2$ will be
(a) $1:2\sqrt{3}$ (b) $1:\sqrt{2}$
(c) $\sqrt{2}:1$ (d) $1:\sqrt{3}$ *(NEET 2019)*

16. A particle has initial velocity $(2\hat{i}+3\hat{j})$ and acceleration $(0.3\hat{i}+0.2\hat{j})$. The magnitude of velocity after 10 seconds will be
(a) $9\sqrt{2}$ units (b) $5\sqrt{2}$ units
(c) 5 units (d) 9 units *(2012)*

17. A particle has initial velocity $(3\hat{i}+4\hat{j})$ and has acceleration $(0.4\hat{i}+0.3\hat{j})$. Its speed after 10 s is
(a) 7 units (b) $7\sqrt{2}$ units
(c) 8.5 units (d) 10 units *(2010)*

18. A man is slipping on a frictionless inclined plane and a bag falls down from the same height. Then the velocity of both is related as
(v_m = velocity of man and v_B = velocity of bag)
(a) $v_B > v_m$ (b) $v_B < v_m$
(c) $v_B = v_m$
(d) v_B and v_m can't be related. *(2000)*

4.9 Relative Velocity in Two Dimensions

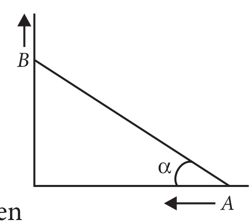
19. The speed of a swimmer in still water is 20 m/s. The speed of river water is 10 m/s and is flowing due east. If he is standing on the south bank and wishes to cross the river along the shortest path, the angle at which he should make his strokes w.r.t. north is, given by
(a) 45° west (b) 30° west
(c) 0° (d) 60° west *(NEET 2019)*

20. Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1. The boy at A starts running simultaneously with velocity v and catches the other in a time t, where t is
(a) $\dfrac{a}{\sqrt{v^2+v_1^2}}$ (b) $\dfrac{a}{v+v_1}$
(c) $\dfrac{a}{v-v_1}$ (d) $\sqrt{\dfrac{a^2}{v^2-v_1^2}}$ *(2005)*

21. The width of river is 1 km. The velocity of boat is 5 km/hr. The boat covered the width of river in shortest time 15 min. Then the velocity of river stream is
(a) 3 km/hr (b) 4 km/hr
(c) $\sqrt{29}$ km/hr (d) $\sqrt{41}$ km/hr
(2000, 1998)

22. A person aiming to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water in the stream, is
(a) 0.25 m/s (b) 0.5 m/s
(c) 1.0 m/s (d) 0.433 m/s *(1999)*

23. Two particles A and B are connected by a rigid rod AB. The rod slides along perpendicular rails as shown here. The velocity of A to the left is 10 m/s. What is the velocity of B when angle $\alpha = 60°$?



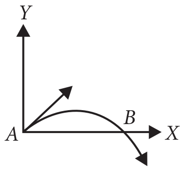
(a) 10 m/s (b) 9.8 m/s
(c) 5.8 m/s (d) 17.3 m/s. *(1998)*

24. A boat is sent across a river with a velocity of 8 km h^{-1}. If the resultant velocity of boat is 10 km h^{-1}, then velocity of river is
(a) 12.8 km h^{-1}
(b) 6 km h^{-1}
(c) 8 km h^{-1}
(d) 10 km h^{-1} *(1994, 1993)*

4.10 Projectile Motion

25. A projectile is fired from the surface of the earth with a velocity of 5 m s^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 m s^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in m s^{-2}) is (Given $g = 9.8$ m s^{-2})
(a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8 *(2014)*

26. The velocity of a projectile at the initial point A is $(2\hat{i}+3\hat{j})$ m/s. Its velocity (in m/s) at point B is
(a) $2\hat{i}-3\hat{j}$
(b) $2\hat{i}+3\hat{j}$
(c) $-2\hat{i}-3\hat{j}$
(d) $-2\hat{i}+3\hat{j}$



(NEET 2013)

27. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is
(a) $\theta=\tan^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \tan^{-1}(4)$
(c) $\theta = \tan^{-1}(2)$ (d) $\theta = 45°$ *(2012)*

28. A missile is fired for maximum range with an initial velocity of 20 m/s. If g = 10 m/s^2, the range of the missile is
(a) 40 m (b) 50 m
(c) 60 m (d) 20 m *(2011)*

29. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is
(a) 45° (b) 60°
(c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *(Mains 2011)*

30. The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is
(a) 60° (b) 15°
(c) 30° (d) 45° *(Mains 2010)*

31. A particle of mass m is projected with velocity v making an angle of 45° with the horizontal. When the particle lands on the level ground the magnitude of the change in its momentum will be
(a) $mv\sqrt{2}$ (b) zero
(c) $2mv$ (d) $mv/\sqrt{2}$ *(2008)*

32. For angles of projection of a projectile at angle $(45°-\theta)$ and $(45°+\theta)$, the horizontal range described by the projectile are in the ratio of
(a) 2 : 1 (b) 1 : 1
(c) 2 : 3 (d) 1 : 2. *(2006)*

33. A particle A is dropped from a height and another particle B is projected in horizontal direction with speed of 5 m/s from the same height then correct statement is
(a) particle A will reach at ground first with respect to particle B
(b) particle B will reach at ground first with respect to particle A
(c) both particles will reach at ground simultaneously
(d) both particles will reach at ground with same speed. *(2002)*

34. Two projectiles of same mass and with same velocity are thrown at an angle 60° and 30° with the horizontal, then which will remain same
(a) time of flight
(b) range of projectile
(c) maximum height acquired
(d) all of them. *(2000)*

35. If a body A of mass M is thrown with velocity v at an angle of 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal, the ratio of horizontal range of A to B will be
(a) 1 : 3 (b) 1 : 1
(c) $1:\sqrt{3}$ (d) $\sqrt{3}:1$. *(1992, 1990)*

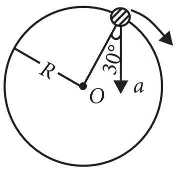
36. The maximum range of a gun of horizontal terrain is 16 km. If g = 10 m s^{-2}, then muzzle velocity of a shell must be
(a) 160 m s^{-1} (b) $200\sqrt{2}$ m s^{-1}
(c) 400 m s^{-1} (d) 800 m s^{-1} *(1990)*

4.11 Uniform Circular Motion

37. Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their time period of rotation is the same. The ratio of angular speed of A to that of B will be
(a) 1 : 1 (b) $r_A : r_B$
(c) $v_A : v_B$ (d) $r_B : r_A$ *(NEET 2019)*

38. A particle starting from rest, moves in a circle of radius 'r'. It attains a velocity of V_0 m/s in the n^{th} round. Its angular acceleration will be
(a) $\frac{V_0}{n}$ rad/s^2 (b) $\frac{V_0}{2\pi nr^2}$ rad/s^2
(c) $\frac{V_0^2}{4\pi nr^2}$ rad/s^2 (d) $\frac{V_0^2}{4\pi nr}$ rad/s^2
(Odisha NEET 2019)

39. In the given figure, a = 15 m s^{-2} represents the total acceleration of a particle moving in the clockwise direction in a circle of radius R = 2.5 m at a given instant of time. The speed of the particle is



(a) 4.5 m s^{-1} (b) 5.0 m s^{-1}
(c) 5.7 m s^{-1} (d) 6.2 m s^{-1} *(NEET-II 2016)*

40. A particle moves in a circle of radius 5 cm with constant speed and time period 0.2π s. The acceleration of the particle is
(a) 15 m/s^2 (b) 25 m/s^2
(c) 36 m/s^2 (d) 5 m/s^2 *(2011)*

41. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 seconds, what is the magnitude and direction of acceleration of the stone?
(a) π^2 m s^{-2} and direction along the radius towards the centre
(b) π^2 m s^{-2} and direction along the radius away from the centre
(c) π^2 m s^{-2} and direction along the tangent to the circle
(d) $\pi^2/4$ m s^{-2} and direction along the radius towards the centre. *(2005)*

42. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is
(a) 40 m/s^2 (b) 640π m/s^2
(c) 160π m/s^2 (d) 40π m/s^2 *(2003)*

43. Two particles having mass M and m are moving in a circular path having radius R and r. If their time period are same then the ratio of angular velocity will be
(a) $\frac{r}{R}$ (b) $\frac{R}{r}$ (c) 1 (d) $\sqrt{\frac{R}{r}}$ *(2001)*

44. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is
(a) $r_1 : r_2$ (b) $m_1 : m_2$
(c) 1 : 1 (d) $m_1\, m_2 : r_1\, r_2$ *(1999)*

45. A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s. What is its linear velocity at any point on circular path ?
(a) 20 m/s (b) $\sqrt{2}$ m/s
(c) 10 m/s (d) 2 m/s *(1996)*

46. The angular speed of a flywheel making 120 revolutions/minute is
(a) 4π rad/s (b) 4π^2 rad/s
(c) π rad/s (d) 2π rad/s *(1995)*

47. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is
(a) 1600 m s^{-2} (b) 47.4 m s^{-2}
(c) 23.7 m s^{-2} (d) 50.55 m s^{-2} *(1990)*

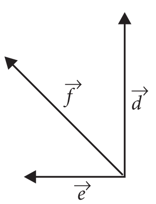
ANSWER KEY

1. (b)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (b)	**9.** (a)	**10.** (d)
11. (d)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (d)
21. (a)	**22.** (a)	**23.** (d)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (a)
31. (a)	**32.** (b)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (c)	**40.** (d)
41. (a)	**42.** (a)	**43.** (c)	**44.** (c)	**45.** (d)	**46.** (a)	**47.** (b)			

Hints & Explanations

1. (b) : Since the angular momentum has both magnitude and direction, it is a vector quantity.

2. (c) :



From figure, $\vec{d}+\vec{e}=\vec{f}$

3. (b) : For a unit vector $\hat{n}$, $|\hat{n}| = 1$

$$\left|0.5\hat{i}-0.8\hat{j}+c\hat{k}\right|^2 = 1^2 \Rightarrow 0.25+0.64+c^2=1$$

or $c=\sqrt{0.11}$

4. (d) : Let the two vectors be $\vec{A}$ and $\vec{B}$.

Then, magnitude of sum of $\vec{A}$ and $\vec{B}$,

$$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta}$$

and magnitude of difference of $\vec{A}$ and $\vec{B}$

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta},$$

$$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}| \text{ (given)}$$

or $\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$

$\Rightarrow \quad 4\,AB\cos\theta = 0$

$\because \quad 4\,AB \neq 0, \quad \therefore \quad \cos\theta = 0$ or $\theta = 90°$

5. (b) : Let θ be angle between $\vec{A}$ and $\vec{B}$

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$, then $|\vec{A}+\vec{B}|^2 = |\vec{A}-\vec{B}|^2$

$$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$$

or $\quad 4AB\cos\theta = 0$ or $\cos\theta = 0$ or $\theta = 90°$

6. (c) : $|\vec{A}+\vec{B}| = |\vec{A}|+|\vec{B}|$ if $\vec{A} \parallel \vec{B}$. $\theta = 0°$

7. (a) : Let θ be angle between $\vec{A}$ and $\vec{B}$.

Given: $A = |\vec{A}| = 3$ units

$B = |\vec{B}| = 4$ units

$C = |\vec{C}| = 5$ units

$|\vec{A}+\vec{B}| = |\vec{C}|$

$A^2 + 2AB\cos\theta + B^2 = C^2$

$9 + 2AB\cos\theta + 16 = 25$ or $2AB\cos\theta = 0$

or $\quad \cos\theta = 0 \therefore \theta = 90°$.

8. (b) : $x = 5t - 2t^2, y = 10t$

$$\frac{dx}{dt} = 5-4t, \frac{dy}{dt} = 10 \quad \therefore \quad v_x = 5-4t, v_y = 10$$

$$\frac{dv_x}{dt} = -4, \frac{dv_y}{dt} = 0 \quad \therefore \quad a_x = -4, a_y = 0$$

Acceleration, $\vec{a} = a_x\hat{i} + a_y\hat{j} = -4\hat{i}$

$\therefore$ The acceleration of the particle at $t = 2$ s is -4 m s^{-2}.

9. (a) : Here, $\vec{R} = 4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}$

The velocity of the particle is

$$\vec{v} = \frac{d\vec{R}}{dt} = \frac{d}{dt}[4\sin(2\pi t)\hat{i} + 4\cos(2\pi t)\hat{j}]$$

$$= 8\pi\cos(2\pi t)\hat{i} - 8\pi\sin(2\pi t)\hat{j}$$

Its magnitude is

$$|\vec{v}| = \sqrt{(8\pi\cos(2\pi t))^2 + (-8\pi\sin(2\pi t))^2}$$

$$= \sqrt{64\pi^2\cos^2(2\pi t) + 64\pi^2\sin^2(2\pi t)}$$

$$= \sqrt{64\pi^2[\cos^2(2\pi t) + \sin^2(2\pi t)]}$$

$$= \sqrt{64\pi^2} \qquad (\text{As } \sin^2\theta + \cos^2\theta = 1)$$

$$= 8\pi \text{ m/s}$$

10. (d) : At time $t = 0$, the position vector of the particle is $\vec{r}_1 = 2\hat{i} + 3\hat{j}$.

At time $t = 5$ s, the position vector of the particle is $\vec{r}_2 = 13\hat{i} + 14\hat{j}$.

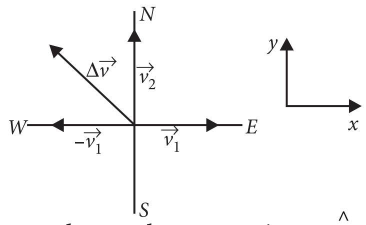
Displacement from $\vec{r}_1$ to $\vec{r}_2$ is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (13\hat{i} + 14\hat{j}) - (2\hat{i} + 3\hat{j}) = 11\hat{i} + 11\hat{j}$$

$\therefore$ Average velocity,

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{11\hat{i} + 11\hat{j}}{5-0} = \frac{11}{5}(\hat{i} + \hat{j})$$

11. (d) :



Velocity towards east direction, $\vec{v}_1 = 30\hat{i}$ m/s

Velocity towards north direction, $\vec{v}_2 = 40\hat{j}$ m/s

Change in velocity, $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1 = (40\hat{j} - 30\hat{i})$

$\therefore \quad |\Delta\vec{v}| = |40\hat{j} - 30\hat{i}| = 50$ m/s

$$\text{Average acceleration, } \vec{a}_{av} = \frac{\text{Change in velocity}}{\text{Time interval}}$$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$$

$$|\vec{a}_{av}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{50 \text{ m/s}}{10 \text{ s}} = 5 \text{ m/s}^2$$

12. (b) : $x = a\sin\omega t$ or $\dfrac{x}{a} = \sin\omega t$

$y = a\cos\omega t$ or $\dfrac{y}{a} = \cos\omega t$

Squaring and adding, we get

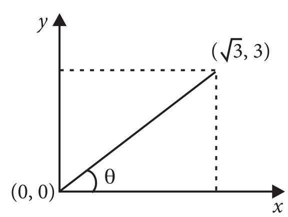
$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \qquad (\because \cos^2\omega t + \sin^2\omega t = 1)$$

or $\quad x^2 + y^2 = a^2$

This is the equation of a circle. Hence particle follows a circular path.

13. (b) : Let θ be the angle which the particle makes with an x-axis.

From figure,



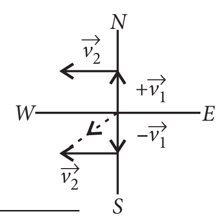
$$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3}$$

or, $\theta = \tan^{-1}(\sqrt{3}) = 60°$

14. (a) : $\vec{v}_1 = 50$ km/hr due north

$\vec{v}_2 = 50$ km/hr due west

$-\vec{v}_1 = 50$ km/hr due south



Magnitude of change in velocity

$$= |\vec{v}_2 - \vec{v}_1| = |\vec{v}_2 + (-\vec{v}_1)| = \sqrt{{v_2}^2 + (-v_1)^2}$$

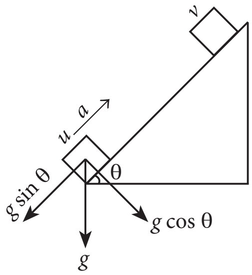
$$= \sqrt{(50)^2 + (50)^2} = 70.7 \text{ km/hr}$$

$\vec{v} =$ 70.7 km/hr along south-west direction

15. (d) : $v^2 - u^2 = 2ax$

For case I : $v^2 = u^2 - 2(g \sin \theta_1)\, x_1$

$[\because\ a = -g \sin \theta]$



$$x_1 = \frac{u^2}{2g \sin \theta_1} \quad [\because\ v = 0]$$

For case II :

$v^2 = u^2 - (2g \sin \theta_2)\, x_2$

$$x_2 = \frac{u^2}{2g \sin \theta_2}$$

$$\therefore\ \frac{x_1}{x_2} = \frac{\sin \theta_2}{\sin \theta_1} = \frac{\sin 30°}{\sin 60°} = \frac{1}{\sqrt{3}}$$

16. (b) : Here, $\vec{u} = 2\hat{i} + 3\hat{j},\ \vec{a} = 0.3\hat{i} + 0.2\hat{j}, t = 10$ s

As $\vec{v} = \vec{u} + \vec{a}t$

$$\therefore\ \vec{v} = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j})(10)$$

$$= 2\hat{i} + 3\hat{j} + 3\hat{i} + 2\hat{j} = 5\hat{i} + 5\hat{j}$$

$$|\vec{v}| = \sqrt{(5)^2 + (5)^2} = 5\sqrt{2} \text{ units}$$

17. (b) : Here, Initial velocity, $\vec{u} = 3\hat{i} + 4\hat{j}$

Acceleration, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$; time, t = 10 s

Let $\vec{v}$ be velocity of a particle after 10 s.

Using, $\vec{v} = \vec{u} + \vec{a}t$

$$\therefore\ \vec{v} = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j})(10)$$

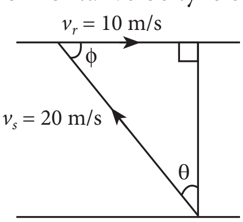
$$= 3\hat{i} + 4\hat{j} + 4\hat{i} + 3\hat{j} = 7\hat{i} + 7\hat{j}$$

Speed of the particle after 10 s $= |\vec{v}|$

$$= \sqrt{(7)^2 + (7)^2} = 7\sqrt{2} \text{ units}$$

18. (c) : Vertical acceleration in both the cases is g, whereas horizontal velocity is constant.

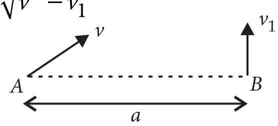
19. (b) :



$$\cos \phi = \frac{10}{20} = \frac{1}{2} \text{ or } \phi = 60°$$

$\Rightarrow\ \theta = 180° - (90° + 60°) = 30°$

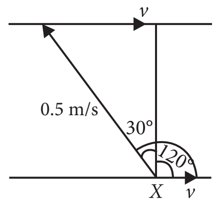
20. (d) : $t = \dfrac{a}{v'} = \dfrac{a}{\sqrt{v^2 - v_1^2}}$



21. (a) : $v_{\text{Resultant}} = \dfrac{1 \text{ km}}{1/4 \text{ hr}} = 4$ km/hr

$\therefore\ v_{\text{River}} = \sqrt{5^2 - 4^2} = 3$ km/hr

22. (a) : Let v be the velocity of river water. As shown in figure,



$$\sin 30° = \frac{v}{0.5}$$

or, $v = 0.5 \sin 30°$

$= 0.5 \times (1/2) = 0.25$ m/s

23. (d) : Let particle B move upwards with velocity v, then

$$\tan 60° = \frac{v}{10};\ v = \sqrt{3} \times 10 = 17.3 \text{ m/s}$$

24. (b) : Let the velocity of river be v_R and velocity of boat is v_B.

$\therefore$ Resultant velocity $= \sqrt{v_B^2 + v_R^2 + 2v_B v_R \cos \theta}$

$(10) = \sqrt{v_B^2 + v_R^2 + 2v_B v_R \cos 90°}$

$(10) = \sqrt{(8)^2 + v_R^2}$ or $(10)^2 = (8)^2 + v_R^2$

$v_R^2 = 100 - 64$ or $v_R = 6$ km/h

25. (a) : The equation of trajectory is

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

where θ is the angle of projection and u is the velocity with which projectile is projected.

For equal trajectories and for same angles of projection,

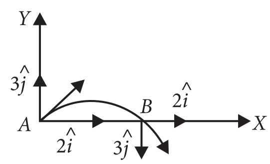
$$\frac{g}{u^2} = \text{constant}$$

As per question, $\dfrac{9.8}{5^2} = \dfrac{g'}{3^2}$

where g' is acceleration due to gravity on the planet.

$$g' = \frac{9.8 \times 9}{25} = 3.5 \text{ m s}^{-2}$$

26. (a) : At point B, X component of velocity remains unchanged while Y component reverses its direction.



$\therefore$ The velocity of the projectile at point B is $2\hat{i} - 3\hat{j}$ m/s.

27. (b) : Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

where u is the velocity of projection and θ is the angle of projection.

Maximum height, $H = \dfrac{u^2 \sin^2 \theta}{2g}$

According to question $R = H$

$$\therefore \quad \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2\theta}{2g} \quad \text{or,} \quad \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{u^2 \sin^2\theta}{2g}$$

$\tan\theta = 4$ or $\theta = \tan^{-1}(4)$

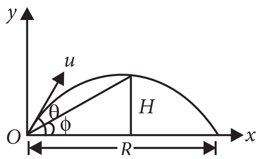
28. (a) : Here, $u = 20$ m/s, $g = 10$ m/s^2

For maximum range, angle of projection is $\theta = 45°$

$$\therefore \ R_{max} = \frac{u^2 \sin 90°}{g} = \frac{u^2}{g} \qquad \left(\because \ R = \frac{u^2 \sin 2\theta}{g}\right)$$

$$= \frac{(20\ \text{m/s})^2}{(10\ \text{m/s}^2)} = 40\ \text{m}$$

29. (c) : Let ϕ be elevation angle of the projectile at its highest point as seen from the point of projection O and θ be angle of projection with the horizontal.



$$\text{From figure, } \tan\phi = \frac{H}{R/2} \qquad \text{...(i)}$$

In case of projectile motion

$$\text{Maximum height, } H = \frac{u^2 \sin^2\theta}{2g}$$

$$\text{Horizontal range, } R = \frac{u^2 \sin 2\theta}{g}$$

Substituting these values of H and R in (i), we get

$$\tan\phi = \frac{\dfrac{u^2 \sin^2\theta}{2g}}{\dfrac{u^2 \sin 2\theta}{2g}} = \frac{\sin^2\theta}{\sin 2\theta} = \frac{\sin^2\theta}{2\sin\theta\cos\theta} = \frac{1}{2}\tan\theta$$

Here, $\theta = 45°$

$$\therefore \quad \tan\phi = \frac{1}{2}\tan 45° = \frac{1}{2} \qquad (\because \ \tan 45° = 1)$$

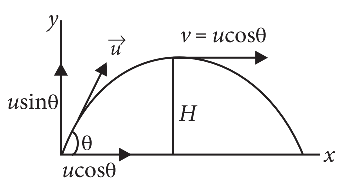
$$\phi = \tan^{-1}\left(\frac{1}{2}\right)$$

30. (a) : Let v be velocity of a projectile at maximum height H.

$v = u\cos\theta$

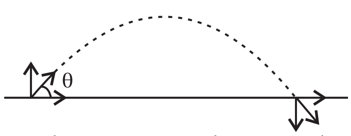
According to given problem,

$$v = \frac{u}{2}$$



$$\therefore \quad \frac{u}{2} = u\cos\theta \ \Rightarrow \ \cos\theta = \frac{1}{2} \ \Rightarrow \ \theta = 60°$$

31. (a) :



The horizontal momentum does not change. The change in vertical momentum is

$$mv\sin\theta - (-mv\sin\theta) = 2mv\frac{1}{\sqrt{2}} = \sqrt{2}mv$$

32. (b) : Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

For angle of projection $(45° - \theta)$, the horizontal range is

$$\therefore \quad R_1 = \frac{u^2 \sin[2(45° - \theta)]}{g} = \frac{u^2 \sin(90° - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$$

For angle of projection $(45° + \theta)$, the horizontal range is

$$R_2 = \frac{u^2 \sin[2(45° + \theta)]}{g} = \frac{u^2 \sin(90° + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{u^2 \cos 2\theta / g}{u^2 \cos 2\theta / g} = \frac{1}{1}$$

33. (c) : Time required to reach the ground is dependent on the vertical motion of the particle. Vertical motion of both the particles A and B are exactly same. Although particle B has an initial velocity, but that is in horizontal direction and it has no component in vertical (component of a vector at a direction of 90° = 0) direction. Hence they will reach the ground simultaneously.

34. (b) : As $\theta_2 = (90 - \theta_1)$,

So range of projectile,

$$R_1 = \frac{v_0^2 \ \sin 2\theta}{g} = \frac{v_0^2 \ 2\sin\theta\cos\theta}{g}$$

$$R_2 = \frac{v_0^2 \ 2\sin(90 - \theta_1)\cos(90 - \theta_1)}{g}$$

$$R_2 = \frac{v_0^2 \ 2\cos\theta_1 \sin\theta_1}{g} = R_1$$

35. (b) : For the given velocity of projection u, the horizontal range is the same for the angle of projection θ and $90° - \theta$.

$$\text{Horizontal range, } R = \frac{u^2 \sin 2\theta}{g}$$

$$\therefore \quad \text{For body } A, \ R_A = \frac{u^2 \sin(2 \times 30°)}{g} = \frac{u^2 \sin 60°}{g}$$

$$\text{For body } B, R_B = \frac{u^2 \sin(2 \times 60°)}{g}$$

$$R_B = \frac{u^2 \sin 120°}{g} = \frac{u^2 \sin(180° - 60°)}{g} = \frac{u^2 \sin 60°}{g}$$

The range is the same whether the angle is θ or $90° - \theta$.

$\therefore$ The ratio of ranges is 1 : 1.

36. (c) : Horizontal range, $R = \dfrac{u^2 \sin 2\theta}{g}$

For maximum horizontal range, $\theta = 45°$

$$R_m = \frac{u^2}{g}$$

where u be muzzle velocity of a shell.

$\therefore \quad (1600\text{ m}) = \dfrac{u^2}{(10\text{ m s}^{-2})^2}$ or $u = 400\text{ m s}^{-1}$.

37. (a) : Time period, $T = \dfrac{2\pi}{\omega}$

As $T_A = T_B$

So, $\dfrac{2\pi}{\omega_A} = \dfrac{2\pi}{\omega_B}$ or $\omega_A : \omega_B = 1 : 1$

38. (c) : Distance travelled in n^{th} rounds = $(2\pi r)n$

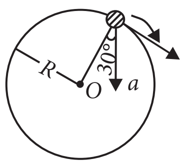
Using $v^2 = u^2 + 2as$ or, $V_0^2 = 0 + 2a\,(2\pi r)n$

$$a = \frac{V_0^2}{4\pi nr}$$

Angular acceleration, $\alpha = \dfrac{a}{r} = \dfrac{V_0^2}{4\pi nr^2}\text{rad/ s}^2$

39. (c) : Here, $a = 15\text{ m s}^{-2}$,

$R = 2.5$ m



From figure,

$$a_c = a\cos 30° = 15 \times \frac{\sqrt{3}}{2}\text{ m s}^{-2}$$

As we know, $a_c = \dfrac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$

$$\therefore \quad v = \sqrt{15 \times \frac{\sqrt{3}}{2} \times 2.5} = 5.69 \simeq 5.7\text{ m s}^{-1}$$

40. (d) : Here, radius, $R = 5$ cm $= 5 \times 10^{-2}$ m

Time period, $T = 0.2\pi$ s

Centripetal acceleration

$$a_c = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5 \times 10^{-2}) = 5\text{ m / s}^2$$

As particle moves with constant speed, therefore its tangential acceleration is zero. So, $a_t = 0$

The acceleration of the particle is

$$a = \sqrt{a_c^2 + a_t^2} = 5\text{ m/s}^2$$

It acts towards the centre of the circle.

41. (a) : $a = r\omega^2$; $\omega = 2\pi\upsilon$

22 revolution = 44 s

1 revolution = 44/22 = 2 s

υ = 1/2 Hz

$$a = r\omega^2 = 1 \times \frac{4\pi^2}{4} = \pi^2\text{ m/s}^2.$$

It is the centripetal acceleration towards the centre.

42. (a) : Given : $r = \dfrac{20}{\pi}$ m, $v = 80$ m/s, $\theta = 2$ rev $= 4\pi$ rad.

From equation $\omega^2 = \omega_0^2 + 2\,\alpha\,\theta$

$$\omega^2 = 2\alpha\theta \quad (\omega_0 = 0)$$

$$a = \frac{v^2}{2r\theta} = 40\text{ m/s}^2 \quad \left(\omega = \frac{v}{r}\text{ and } a = r\alpha\right)$$

43. (c) : $\omega = \dfrac{2\pi}{t}$; As t is same $\therefore \dfrac{\omega_1}{\omega_2} = 1$

44. (c) : $t = \dfrac{2\pi}{\omega_1} = \dfrac{2\pi}{\omega_2} \Rightarrow \dfrac{\omega_1}{\omega_2} = \dfrac{1}{1}$

45. (d) : Radius of circle (r) = 20 cm = 0.2 m and angular velocity (ω) = 10 rad/s

Linear velocity $(v) = r\omega = 0.2 \times 10 = 2$ m/s.

46. (a) : Number of revolutions per minute $(n) = 120$.

Therefore angular speed (ω)

$$= \frac{2\pi n}{60} = \frac{2\pi \times 120}{60} = 4\pi\text{ rad / s}$$

47. (b) : Frequency of rotation υ = 120 rpm

= 2 rps

length of blade r = 30 cm = 0.3 m

Centripetal acceleration $a = \omega^2 r = (2\pi\upsilon)^2 r$

$= 4\pi^2\upsilon^2 r = 4\pi^2(2)^2(0.3) = 47.4\text{ m s}^{-2}$

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