NAME : ....

RACE

Regular Analysis through Continuous Exercise

GRP 1.0 MATHEMATICS

REVISION ASSINGMENT# 09 (FUNCTION & ITF) TIME : 60 MIN.

SECTION–I(i)

Straight Objective Type (3 Marks each, –1 for wrong answer)

1. Number of solutions of the equation [y + [y]] = 2cosx is

(where $y = \frac{1}{3}\left[\sin x + \left[\sin x + \left[\sin x\right]\right]\right]$ and [.] is greatest integer function)-

(A) 0 (B) 1 (C) 2 (D) infinite

2. $f : R \to R$ be a function satisfying $f(x) + 2f(1-x) = x^2 + 1$, $\forall\ x \in R$, then range of values of 'm' for which equation $f(|x|) = m$ have two solutions -

(A) $(1, \infty)$ (B) $\left(-\frac{1}{3}, 1\right)$ (C) $(1, \infty) \cup \left\{-\frac{1}{3}\right\}$ (D) $\left(-\infty, -\frac{1}{3}\right)$

3. If $f(x) = 3 - x^2$, $1 \le x \le 4$, then the domain of $\log_e(f(2x))$ is

(A) $\left(\frac{-\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$ (B) $\left[\frac{1}{2}, 2\right]$ (C) $\left(0, \frac{\sqrt{3}}{2}\right)$ (D) $\left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

4. Let $f(x) = x - [x]$, $x \ne 0$, $x \in R$ and [.] is greatest integer function, then the number of solutions of equation $f(x) + f\left(\frac{1}{x}\right) = 1$ is :-

(A) 1 (B) 0 (C) 2 (D) infinite

5. $\lim\limits_{n\to\infty} \frac{\sum\limits_{r=0}^{n} \tan^{-1}\left(1 + r + r^2\right)}{n}$ is equal to-

(A) 1 (B) 2 (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Space for Rough Work

6. If p and q are roots of $6x^2 + 10x + 1 = 0$, then the value of $[\tan^{-1}p + \tan^{-1}q]$ is
{where [x] denotes greatest integer less than or equal to x}
(A) 1 (B) –2 (C) 0 (D) –1

7. Let $f(x) = \cos^{-1}([e^x] - 1) + \sin^{-1}([e^x])$, then $f(x)$ is -
(where [.] denotes greatest integer function)
(A) many one and its range contain one element
(B) many one and its range contain two elements
(C) one-one and its range contain one element
(D) one-one and its ange contain two elements

8. If x_1, x_2, x_3, x_4 are positive roots of equation $x^4 - 4x^3 + ax^2 - bx + 1 = 0$, then $\sum_{i=1}^{4}\tan^{-1}(x_i)$ is equal to -
(A) π (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) 0

SECTION–I(ii)

Multiple Correct Answer Type (4 Marks each, –1 for wrong answer)

9. Let $f(x)$ be a bijective function such that $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$, where $fofof(\beta) = 1$ and $f^{-1}(2) = 1$, then which of the following option(s) is/are correct ?
(A) $\alpha = 4$ (B) $f(3) = 3$
(C) $\alpha + \frac{1}{\beta} = 47$ (D) if $|f(x)| < 1$, then $x \in \left(-\frac{1}{17}, \frac{1}{15}\right)$

10. Let $f : A \to R$, $f(x) = \sqrt{(2 - 2^{|x-1|})\log_x(x^2 - 1)}$, then
(where set A denotes domain and R denotes set of all real numbers)
(A) number of integers in domain of $f(x)$ is 1
(B) $f(x)$ is many-one function
(C) domain of function is (1, 2]
(D) $f(x)$ is into function

Space for Rough Work

11. Consider the angle $A = 2\tan^{-1}\left(2\sqrt{2}-1\right)$ and $B = 3\sin^{-1}\frac{1}{3}$, then choose the correct option(s)-

(A) $\sin\left(A+\frac{B}{3}\right) = +ve$

(B) tan 2B = –ve

(C) cosA = –ve

(D) cot(A – B) = +ve

12. $f(x) = \begin{cases} \sin^{-1}\sin x & x \in [-\pi, 0) \\ \tan^{-1}\tan x & x \in [0,\pi] - \left\{\frac{\pi}{2}\right\} \end{cases}$, then correct option(s) is/are -

(A) if $x \in [-\pi, 0]$, then $fof(x) = f(x)$

(B) if $x \in \left[0, \frac{\pi}{2}\right)$, then $fof(x) = x$

(C) if $x \in \left(\frac{\pi}{2}, \pi\right]$, then $fof(x) = x - \pi$

(D) if $x \in \left(\frac{\pi}{2}, \pi\right]$, then $fof(x) = \pi - x$

SECTION–I(iii)

Linked Comprehension Type (Single Correct Answer Type) (3 Marks each, –1 for wrong answer)

Paragraph for Question 13 to 14

Consider $f : N \to N$, where $f(n) = \frac{4}{\pi}\sum_{i=1}^{n}\sum_{j=1}^{n}\cot^{-1}\left(\frac{i}{j}\right)$.

13. Function f will be -

(A) one one onto (B) one one into (C) many one onto (D) many one into

14. Value of $f(5) + f(10)$ is -

(A) 75 (B) 100 (C) 125 (D) 150

Paragraph for Question 15 to 16

Let $f : [a, c] \to [b, d]$, $f(x) = x^2 + 4x - |x^2 - 4|$ is an onto function, where $a, b \in I$ & $c, d \in R$.

15. If $f(x)$ is bijective function and $c + d > 0$, then minimum value of a + b is -

(A) –4 (B) –5 (C) –6 (D) –7

16. If $f(k + x) = f(k - x)$ for some $k \in R$, then value of a + b + c + d is -

(A) –12 (B) –9 (C) –10 (D) –11

Space for Rough Work

SECTION–I(v)

Matching list type (4 × 4) (Single option correct) (3 Marks each, –1 for wrong answer)

17. **List-I** | **List-II**

	List-I		List-II
(P)	The number of integral values of k for which the equation $\sin^{-1}x^2+\tan^{-1}x^2=2k+1$ has a solution	(1)	1
(Q)	Let $\sum_{K=1}^{\infty}\cot^{-1}\left(\frac{k^2}{8}\right)=\frac{p}{q}\pi$ where $\frac{p}{q}$ is rational in its lowest form, then find $\lvert q-p\rvert$	(2)	2
(R)	If $\tan^{-1}\left(\sin^2\theta-2\sin\theta+3\right)+\cot^{-1}\left(5^{\sec^2 y}+1\right)=\frac{\pi}{2}$, then the value of $\cos^2 y-\sin\theta$	(3)	3
(S)	Minimum positive integral value of x such that f(x) is defined, $f(x)=\sqrt{\sec^{-1}\left(\frac{1-\lvert x\rvert}{2}\right)}$	(4)	0

Code :

	P	Q	R	S
(A)	3	1	2	2
(B)	4	3	1	4
(C)	1	3	2	3
(D)	1	4	3	2

18. Match List-I with List-II and select the correct answer using the code given below the list.

	List-I		List-II
(P)	$f : R \to (-1, 1),\ f(x)=\frac{10^x-10^{-x}}{10^x+10^{-x}}$	(1)	one-one & onto
(Q)	$f : R \to R,\ f(x) = [x] + x$, where [x] denotes greatest integer function	(2)	many one & into
(R)	$f : R \to (0, \infty),\ f(x)=\begin{cases} e^x, & x<0 \\ x^2+\frac{1}{2}, & x\ge 0\end{cases}$	(3)	one-one & into
(S)	$f : [-1,1]\to\left[-\pi, \frac{\pi}{2}\right],\ f(x) = x\cos^{-1}x$	(4)	many one & onto

Codes :

	P	Q	R	S
(A)	1	3	2	4
(B)	1	4	3	2
(C)	3	2	4	1
(D)	1	3	4	2

Space for Rough Work

19. Match List-I with List-II and select the correct answer using the code given below the list.

	List-I		**List-II**
(P)	If $f:[2,\infty)\to(-\infty,4]$, where $f(x) = x(4-x)$, then $f^{-1}(3)$ is equal to	(1)	0
(Q)	Given $g(x) = \ell n(a^2 - a - 1)\lvert\cos 2x\rvert + \left[\dfrac{a^3}{3}\right]\sin(3\pi x)\ \forall\ x \in R$ is a periodic function with period to be a rational number, then a can be (where [.] denotes greatest integer function)	(2)	2
(R)	Minimum value of $h(x) = \lvert x+1\rvert + \lvert x-2\rvert + \lvert x-3\rvert$ is	(3)	3
(S)	Number of solution of equation $1 + \sin x = -e^{-x}$ is	(4)	4

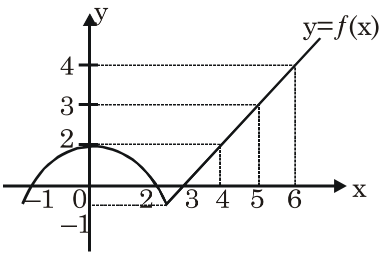
Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	4	3	1	2
(C)	1	2	4	3
(D)	3	1	4	2

Space for Rough Work

SECTION–II (iii)

Numerical Grid Type (Upto Second Decimal place) (4 Marks each, –1 for wrong answer)

1. 

Consider the graph of real valued function $f(x)$ defined $\forall$ x $\in$ R as shown above. The number of real solution of equation $f(f(x)) = 2$ is equal to

2. Let $f : R \to R$ be defined $f(x) = \begin{cases} 2x + 3 & x \le 1 \\ p^2x + 1 & x > 1 \end{cases}$ where $f(x)$ is onto function. If p_1 and p_2 are minimum and maximum value of p respectively, then $\left|\dfrac{p_2 + 1}{p_1}\right|$ is equal to

3. Let $z = \sec^{-1}\left(x + \dfrac{1}{x}\right) + \sec^{-1}\left(y + \dfrac{1}{y}\right)$, where xy < 0. If range of z is (a,b), then value of $\dfrac{b}{a}$ is equal to

4. If x and y are positive integers satisfying $\tan^{-1}\left(\dfrac{1}{x}\right) + \tan^{-1}\left(\dfrac{1}{y}\right) = \tan^{-1}\left(\dfrac{1}{7}\right)$, then the number of ordered pairs of (x, y) is

Space for Rough Work

REVISION ASSINGMENT # 08 (GRP 1.0) (DETERMINANT AND MATRIX) MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	D	C	D	A	B	A	C	C	C
	Q.	11	12	13	14	15	16	17	18		
	A.	A	D	C	A	C	A	C	A		
SECTION-II	Q.	1	2	3	4						
	A.	0.50	0.33 or 0.34	9.00	0.25						

SOLUTION

SECTION–I

1. Ans. (A)

$[y] + [y] = 2\cos x \Rightarrow [y] = \cos x$

Now, $y = \frac{1}{3}\left[\sin x + \left[\sin x + \left[\sin x\right]\right]\right]$

$= \frac{1}{3}\left(\left[\sin x\right] + \left[\sin x\right] + \left[\sin x\right]\right) = \left[\sin x\right]$

$\Rightarrow [\sin x] = \cos x$

Has no solutions because

$[\sin x] = -1$

$\Rightarrow \quad \cos x = -1 \Rightarrow x = \pi$ (not satisfied)

$[\sin x] = 0$

$\Rightarrow \quad \cos x = 0 \Rightarrow x = \frac{\pi}{2}$ (not satisfied)

$[\sin x] = 1$

$\Rightarrow \quad \cos x = 1 \Rightarrow x = 0$ (not satisfied)

2. Ans. (C)

$f(x) + 2f(1 - x) = x^2 + 1$ ...(1)

Replace $x \to 1-x$

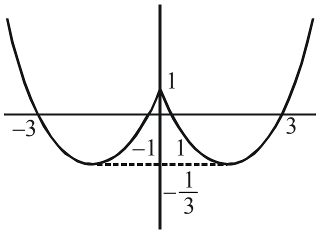
$f(1 - x) + 2f(x) = (1 - x)^2 + 1$ ...(2)

By (1) & (2)

$f(x) + 2\{x^2 - 2x + 2 - 2f(x)\} = x^2 + 1$

$3f(x) = x^2 - 4x + 3 = (x - 1)(x - 3)$

Graph of $f(|x|)$ will be as follows :



so for two solutions of $f(|x|) = m$

$m \in (1, \infty) \cup \left\{-\frac{1}{3}\right\}$

3. Ans. (D)

$f(x) = 3 - x^2,\ 1 \le x \le 4$

$f(2x) = 3 - 4x^2,\ 1 \le 2x \le 4$

for $\log_e(f(2x))$, $3 - 4x^2 > 0$ & $2x \in [1, 4]$

$x \in \left[\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

4. Ans. (D)

$f(x) + f\left(\frac{1}{x}\right) = 1$

$\Rightarrow x - [x] + \frac{1}{x} - \left[\frac{1}{x}\right] = 1$

$\Rightarrow \left(x + \frac{1}{x}\right) - \left([x] + \left[\frac{1}{x}\right]\right) = 1$

$x + \frac{1}{x} = [x] + \left[\frac{1}{x}\right] + 1 = \text{integer}$ ......(1)

$\Rightarrow x + \frac{1}{x} = n$

$x^2 - nx + 1 = 0 \Rightarrow x = \frac{n \pm \sqrt{n^2 - 4}}{2}$

But $n \ne 2, -2$ as it does not satisfy (i)

$\Rightarrow$ n can be any integer in $(-\infty, -2) \cup (2, \infty)$

So infinite solutions.

5. Ans. (D)

$\sum_{r=0}^{n} \tan^{-1}\left(1 + r + r^2\right)$

$= \sum_{r=0}^{n} \frac{\pi}{2} - \tan^{-1}\frac{1}{1 + r + r^2}$

$= \frac{n\pi}{2} - \sum_{r=0}^{n} \tan^{-1}\frac{(r+1) - r}{1 + r(r+1)}$

$= \frac{n\pi}{2} - \sum_{r=0}^{n}\left(\tan^{-1}(r+1) - \tan^{-1} r\right)$

6. Ans. (B)

$6x^2 + 10x + 1 = 0$

$p + q = -\frac{5}{3}$

$pq = \frac{1}{6}$

$p, q < 0$

Let $p = -a$ & $q = -b$ when $a > 0,\ b > 0$

$\tan^{-1}p + \tan^{-1}q = -(\tan^{-1}a + \tan^{-1}b)$

$= -\tan^{-1}\left[\frac{a + b}{1 - ab}\right]$

$\left(\because a > 0, b > 0, ab = \frac{1}{6} < 1\right)$

$= -\tan^{-1}\left[\frac{5/3}{5/6}\right]$

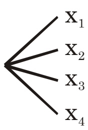
$= -\tan^{-1} 2$ lies in $\left(-\frac{\pi}{2}, -\frac{\pi}{3}\right)$

$\therefore \quad [-\tan^{-1}2] = -2$

7. Ans. (A)

$f(x) = \pi$, for $[e^x] = 0$ or $[e^x] = 1$

8. Ans. (A)

Equation $x^4 - 4x^3 + ax^2 - bx + 1 = 0$ 

A.M. of roots = $\frac{x_1 + x_2 + x_3 + x_4}{4} = 1$

G.M. of roots = $[x_1 x_2 x_3 x_4]^{1/4} = 1$

$\Rightarrow$ AM = GM

So all roots are equal $x_1 = x_2 = x_3 = x_4 = 1$

$\therefore \sum \tan^{-1}(x_i) = 4\left(\frac{\pi}{4}\right) = \pi$

9. Ans. (A,B,C)

$\because f^{-1}(2) = 1 \Rightarrow f(1) = 2$

$\Rightarrow \frac{\alpha}{2} = 2 \Rightarrow \alpha = 4$

$\Rightarrow f(x) = \frac{4x}{x+1}$

$fofof(\beta) = 1$

$\Rightarrow fof(\beta) = f^{-1}(1) = \frac{1}{3}$

$\Rightarrow f(\beta) = f^{-1}\left(\frac{1}{3}\right) = \frac{1}{11}$

$\beta = f^{-1}\left(\frac{1}{11}\right) = \frac{1}{43}$

10. Ans. (A,B,D)

$f(x) = \sqrt{2 - 2^{|x-1|}\log_x(x^2 - 1)}$

for $\log_x(x^2 - 1)$ to be defined

$x > 0, x \neq 1$ and $x^2 - 1 > 0$

$\Rightarrow x > 1$ ...(1)

So, for $x > 1$

$f(x) = \sqrt{(2 - 2^{x-1})(\log_x(x^2 - 1))}$

Case-I : If $2 - 2^{x-1} \geq 0$

$\Rightarrow 2 \geq 2^{x-1} \Rightarrow 1 \geq x - 1$

$\Rightarrow x \leq 2$ ...(2)

$\log_x(x^2 - 1) \geq 0 \Rightarrow x^2 - 1 \geq 1$

$\Rightarrow x^2 \geq 2 \Rightarrow x \in (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$ ...(3)

from (1), (2) and (3)

$x \in [\sqrt{2}, 2]$

Case-II : If $2 - 2^{x-1} < 0 \Rightarrow 2^{x-1} > 2$

$\Rightarrow x > 2$

but for $x > 2$, $\log x(x^2 - 1) > 0$

So, $(2 - 2^{x-1})(\log_x(x^2 - 1)) < 0, \forall x > 2$

$\Rightarrow f(x)$ is not defined for $x > 2$

So, Domain of the function $f(x)$ is $[\sqrt{2}, 2]$

11. Ans. (A,B,C,D)

$A = 2\tan^{-1}(2\sqrt{2} - 1)$

$= 2(\tan^{-1}(1.828.....))$

$= 2 \times$ greater that $60° \quad \Rightarrow A > 120°$

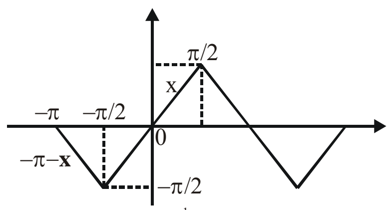
& $\quad \frac{1}{3} < \frac{1}{2} < \frac{1}{\sqrt{2}}$

$\therefore \quad \sin^{-1}\frac{1}{3} < \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

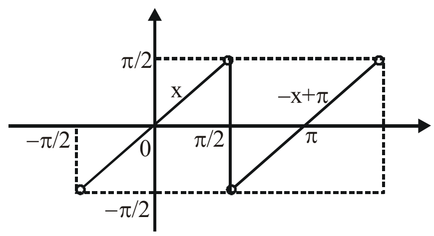
$\therefore \quad B = 3\sin^{-1}\frac{1}{3} < 90°$

Now check the answer.

12. Ans. (A,B,C)



y=sin^{-1}sinx



y=tan^{-1}(tanx)

$$f(x)=\begin{cases}-\pi-x & x\in\left[-\pi,\frac{-\pi}{2}\right)\\ x & x\in\left[\frac{-\pi}{2},0\right)\\ x & x\in\left[0,\frac{\pi}{2}\right)\\ -\pi+x & x\in\left(\frac{\pi}{2},\pi\right]\end{cases}$$

(A) for $x \in [-\pi,0]$, $f(x) = \sin^{-1}(\sin x)$

$$fof=\begin{bmatrix}-\pi-f(x) & -\pi\le f(x)<-\frac{\pi}{2}\\ f(x) & -\frac{\pi}{2}\le f(x)<0\end{bmatrix}$$

since $f(x)=\sin^{-1}(\sin x)\notin\left[-\pi,-\frac{\pi}{2}\right]$

$\Rightarrow fof(x) = f(x)$

(B) for $x\in\left[0,\frac{\pi}{2}\right), f(x)=\tan^{-1}(\tan x)=x$

$\Rightarrow fof(x) = f(x)$

(C) for $x\in\left(\frac{\pi}{2},\pi\right], f(x)=\tan^{-1}(\tan x)=-\pi+x$

$\Rightarrow fof(x) = -\pi + f(x),\ \frac{\pi}{2}<f(x)\le\pi$

since $f(x)\notin\left(\frac{\pi}{2},\pi\right]\Rightarrow$ rejected

Paragraph for Question 13 to 14

Consider $\sum_{i=1}^{n}\sum_{j=1}^{n}\cot^{-1}\frac{i}{j}$

If will be sum of following terms :

$$S=\begin{cases}\cot^{-1}\frac{1}{1}+\cot^{-1}\frac{1}{2}+\cot^{-1}\frac{1}{3}+.....+\cot^{-1}\frac{1}{x}\\ \cot^{-1}\frac{2}{1}+\cot^{-1}\frac{2}{2}+\cot^{-1}\frac{2}{3}+.....+\cot^{-1}\frac{2}{x}\\ \vdots\\ \vdots\\ \vdots\\ \cot^{-1}\frac{n}{1}+\cot^{-1}\frac{n}{2}+\cot^{-1}\frac{n}{3}+.....+\cot^{-1}\frac{n}{n}\end{cases}$$

There will be n diagonal elements, each having value $\cot^{-1}1$ and other $\underbrace{\left(\frac{n^2-n}{2}\right)}_{\because \text{No.of off-diagonal elements of } n\times n \text{ matrix} = n^2-n}$ elements in pair of the form

$\cot^{-1}k+\cot^{-1}\frac{1}{k}, k=1,2...n$

$\therefore\ S=n\left(\frac{\pi}{4}\right)+\frac{(n^2-n)}{2}\left(\frac{\pi}{2}\right)=\frac{n^2\pi}{4}$

$\therefore$ Required sum $f(n)=\frac{4}{\pi}.\frac{n^2\pi}{4}=n^2$

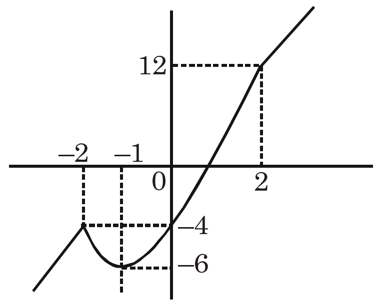
13. Ans. (B)

for $f : N \to N$, function will be one one into

14. Ans. (C)

$f(5) + f(10) = 125$

Paragraph for Question 15 to 16



$$f(x)=\begin{cases}4x+4 & x\in[-\infty,-2]\cup[2,\infty)\\ 2x^2+4x-4 & x\in(-2,2)\end{cases}$$

15. Ans. (D)

As c + d > 0 $f(x)$ is bijective for $x \ge -1$ and range of $f(x)$ is $[-6,\infty)$

$\Rightarrow$ Minimum value of a + b is –7

16. Ans. (A)

$f(x)$ is symmetric about $x = -1$ for $x \in [-2,0]$ and its range is $[-6,-4] \Rightarrow a + b + c + d = -12$

17. Ans. (C)

(P) Let $f(x)=\sin^{-1}x^2+\tan^{-1}x^2$ is even function with domain is [–1. 1]

$$=\begin{cases}\text{decreasing in}[-1,0]\\ \text{increasing in}[0,1]\end{cases}$$

$\Rightarrow\ f(x)|_{min}=f(0)=0$

$f(x)|_{max} = f(1) = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} = f(-1)$

$f(x) \in [0, \frac{3\pi}{4}]$

If k = 0, 2k + 1 = 1, possible

If k = 1, 2k + 1 = 3 > $\frac{3\pi}{4}$ Not possible

i.e. only one integral value of k is possible.

(Q) $T_K = \tan^{-1}\left(\frac{8}{k^2}\right) = \tan^{-1}\frac{2}{\frac{k^2}{4}}$

$$= \tan^{-1}\frac{2}{1+\left(\frac{k}{2}-1\right)\left(\frac{k}{2}+1\right)}$$

$T_K = \tan^{-1}\left(\frac{k}{2}+1\right) - \tan^{-1}\left(\frac{k}{2}-1\right)$

$T_1 = \tan^{-1}\frac{3}{2} - \tan^{-1}\left(\frac{-1}{2}\right)$

$T_2 = \tan^{-1}\frac{4}{2} - \tan^{-1}0$

$T_3 = \tan^{-1}\frac{5}{2} - \tan^{-1}\frac{1}{2}$

$T_4 = \tan^{-1}\frac{6}{2} - \tan^{-1}\frac{2}{2}$

$T_5 = \tan^{-1}\frac{7}{2} - \tan^{-1}\frac{3}{2}$

:

:

Sum =

$2\pi - \left(-\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{2} + \tan^{-1}1\right) = \frac{7\pi}{4} \Rightarrow p = 7, q = 4$

(2π is sum of last four term)

(R) we know $\tan^{-1}x + \cot^{-1}y = \frac{\pi}{2} \Rightarrow x = y$

according to question

$\underbrace{(\sin\theta - 1)^2 + 2}_{[2,6]} = \underbrace{5\sec^2 y + 1}_{\geq 6}$

Only posssible $\Rightarrow \sin\theta - 1 = -2$

$\sin\theta = -1$

$\sec^2 y = 1 \Rightarrow \cos^2 y = 1$

$\cos^2 y - \sin\theta = 2$

(S) $\sec^{-1}\left(\frac{1-|x|}{2}\right) \geq 0$ always true in

domain { $\because \sec^{-1}x \in [0,\pi] - \left\{\frac{\pi}{2}\right\}$ }

Now, according to domain $\frac{1-|x|}{2} \geq 1$ or

$\frac{1-|x|}{2} \leq -1$

$1 - |x| \geq 2$ or $3 \leq |x|$

$-1 \geq |x|$ or $x \in (-\infty, -3] \cup [3, \infty)$

Not possible

least positive integer is 3

18. Ans. (D)

(P) $f(x) = \frac{10^{2x}-1}{10^{2x}+1} = 1 - \frac{2}{10^{2x}+1}$

$f'(x) > 0 \quad \forall x \in R$

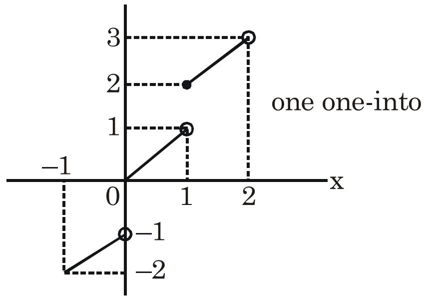
$\Rightarrow f(x)$ is increasing function

$\Rightarrow f(x)$ is one-one function

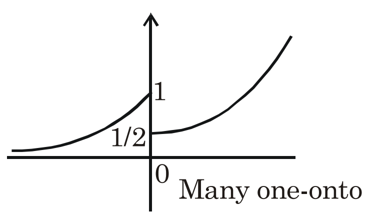
Range of $f(x) = 1 - \frac{2}{(1,\infty)} \equiv 1 - 2(0,1) \equiv (-1, 1)$

$\Rightarrow f(x)$ is onto function

(Q) $f(x) = [x] + x$



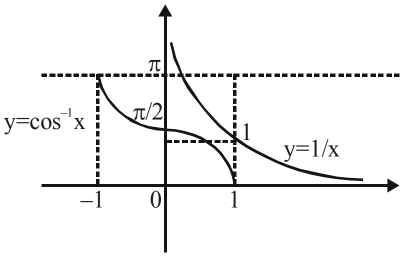
(R)



(S) $f(x) = x\cos^{-1}x$

$f'(x) = \frac{-x}{\sqrt{1-x^2}} + \cos^{-1}x$

$f'(1^-) \to -\infty$
$f'(-1^+) \to \infty$
$f'(x)$ changes
its sign in its domain
$\Rightarrow f(x)$ is many one
Let for $x\cos^{-1}x = 1$



$\cos^{-1} x = \frac{1}{x}$ no solution

$\Rightarrow x\cos^{-1} x \neq 1$

But codomain is $\left[-\pi, \frac{\pi}{2}\right]$

$\Rightarrow$ into function

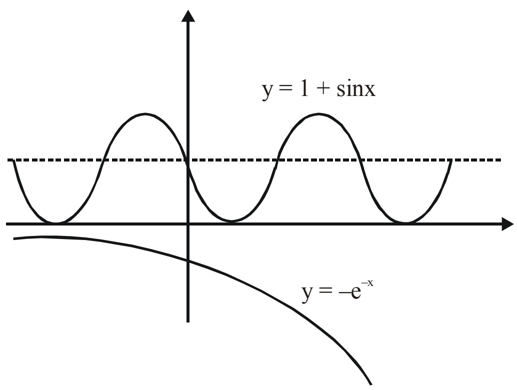
19. Ans. (A)

(P) Put $4x - x^2 = 3 \Rightarrow x^2 - 4x + 3 = 0$
$\Rightarrow x = 1,3$ but $x \neq 1 \Rightarrow f^{-1}(3) = 3$.

(Q) $\ell n(a^2 - a - 1)$ must be zero.
$a^2 - a - 1 = 1 \Rightarrow (a - 2)(a + 1) = 0$
$\Rightarrow a = 2, -1$ but $a \neq -1$ (according to options)
$\therefore$ a = 2 and period $= \frac{2\pi}{3\pi} = \frac{2}{3}$.

(R) minimu value occurs at median of –1,2,3
i.e. at x = 2

(S) By graph, no solution



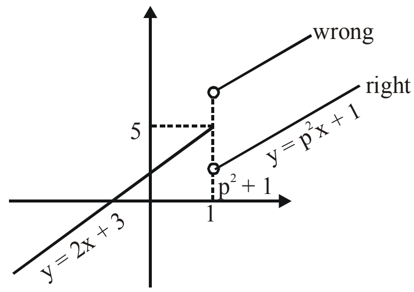
SECTION–II

1. Ans. 4.00

Let $f(x) = t$
$\Rightarrow$ equation become $f(t) = 2$
it has two values of t i.e. t = 0,4
Now, when $t = 0 \Rightarrow f(x) = 0$ has three solutions
i.e. x = –1,2,3
When $t = 4 \Rightarrow f(x) = 4$ has one solution
i.e. x = 6.
$\therefore$ Total solutions = 4.

2. Ans. 1.50

for $x \leq 1 \Rightarrow f(x) \leq 5$
for $x > 1 \Rightarrow f(x) = p^2x + 1 \uparrow$ function



$p^2 + 1 \leq 5$
$p^2 \leq 4$
$-2 \leq p \leq 2$
$p = \{-2, -1, 1, 2\}$
$p \neq 0$ as $f(x) = 1 \ \forall \ x > 1$
$p_2 = 2, p_1 = -2$

3. Ans. 1.40

$\because xy < 0$
$\Rightarrow$ x & y must have opposite sign

let $x + \frac{1}{x} \in (-\infty, -2]$ and $y + \frac{1}{y} \in [2, \infty)$

$\therefore \sec^{-1}\left(x + \frac{1}{x}\right) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

& $\sec^{-1}\left(y + \frac{1}{y}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

$\therefore$ range of z is $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

$\frac{b}{a} = \frac{7}{5}$

4. Ans. 6.00

$$\tan^{-1}\left(\frac{\frac{1}{x} + \frac{1}{y}}{1 - \frac{1}{xy}}\right) = \tan^{-1}\left(\frac{1}{7}\right) \Rightarrow x = 7 + \frac{50}{y - 7}$$

positive integral values of y for which $x \in I^+$ are 8,9,12,17,32,57.