(COMPOUND ANGLES)

1. BASIC TRIGONOMETRIC IDENTITIES :

(a) $\sin^2\theta + \cos^2\theta = 1$;
$-1 \le \sin\theta \le 1; -1 \le \cos\theta \le 1 \ \forall \ \theta \in R$

(b) $\sec^2\theta - \tan^2\theta = 1$;
$|\sec\theta| \ge 1 \quad \forall \quad \theta \in R$

(c) $\text{cosec}^2\theta - \cot^2\theta = 1$;
$|\text{cosec}\,\theta| \ge 1 \quad \forall \quad \theta \in R$

2. IMPORTANT T′ RATIOS:

(a) $\sin n\pi = 0$; $\cos n\pi = (-1)^n$;
$\tan n\pi = 0$ where $n \in I$

(b) $\sin\dfrac{(2n+1)\pi}{2} = (-1)^n$ &
$\cos\dfrac{(2n+1)\pi}{2} = 0$ where $n \in I$

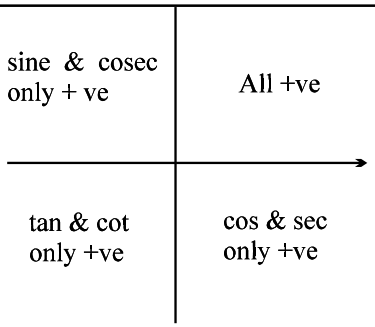
(c) $\sin 15^\circ$ or $\sin\dfrac{\pi}{12} = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$
$= \cos 75^\circ$ or $\cos\dfrac{5\pi}{12}$;
$\cos 15^\circ$ or $\cos\dfrac{\pi}{12} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$
$= \sin 75^\circ$ or $\sin\dfrac{5\pi}{12}$;
$\tan 15^\circ = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} = \cot 75^\circ$;
$\tan 75^\circ = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = 2+\sqrt{3} = \cot 15^\circ$

(d) $\sin\dfrac{\pi}{8} = \dfrac{\sqrt{2-\sqrt{2}}}{2}$; $\cos\dfrac{\pi}{8} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$;
$\tan\dfrac{\pi}{8} = \sqrt{2}-1$; $\tan\dfrac{3\pi}{8} = \sqrt{2}+1$

(e) $\sin\dfrac{\pi}{10}$ or $\sin 18^\circ = \dfrac{\sqrt{5}-1}{4}$ &
$\cos 36^\circ$ or $\cos\dfrac{\pi}{5} = \dfrac{\sqrt{5}+1}{4}$

3. TRIGONOMETRIC FUNCTIONS OF ALLIED ANGLES

If θ is any angle, then $-\theta$, $90 \pm \theta$, $180 \pm \theta$, $270 \pm \theta$, $360 \pm \theta$ etc. are called **ALLIED ANGLES**.



(a) $\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$

(b) $\sin(90^\circ - \theta) = \cos\theta$;
$\cos(90^\circ - \theta) = \sin\theta$

(c) $\sin(90^\circ + \theta) = \cos\theta$;
$\cos(90^\circ + \theta) = -\sin\theta$

(d) $\sin(180^\circ - \theta) = \sin\theta$;
$\cos(180^\circ - \theta) = -\cos\theta$

(e) $\sin(180^\circ + \theta) = -\sin\theta$;
$\cos(180^\circ + \theta) = -\cos\theta$

(f) $\sin(270^\circ - \theta) = -\cos\theta$;
$\cos(270^\circ - \theta) = -\sin\theta$

(g) $\sin(270^\circ + \theta) = -\cos\theta$;
$\cos(270^\circ + \theta) = \sin\theta$

4. TRIGONOMETRIC FUNCTIONS OF SUM OR DIFFERENCE OF TWO ANGLES :

(a) $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(b) $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

(c) $\sin^2A - \sin^2B = \cos^2B - \cos^2A = \sin(A+B)\cdot\sin(A-B)$

(d) $\cos^2A - \sin^2B = \cos^2B - \sin^2A = \cos(A+B)\cdot\cos(A-B)$

(e) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

(f) $\cot(A \pm B) = \dfrac{\cot A\cot B \mp 1}{\cot B \pm \cot A}$

5. FACTORISATION OF THE SUM OR DIFFERENCE OF TWO SINES OR COSINES :

(a) $\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(b) $\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

(c) $\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

(d) $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

6. Transformation Of Products Into Sum Or Difference Of sines & cosines :

(a) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(b) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(c) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(d) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

7. Multiple Angles And Half Angles :

(a) $\sin 2A = 2\sin A\cos A$; $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(b) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$; $\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$.

$2\cos^2 A = 1 + \cos 2A$, $2\sin^2 A = 1 - \cos 2A$; $\tan^2 A = \frac{1-\cos 2A}{1+\cos 2A}$

$2\cos^2\frac{\theta}{2} = 1 + \cos\theta$, $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$.

(c) $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$; $\tan\theta = \frac{2\tan(\theta/2)}{1-\tan^2(\theta/2)}$

(d) $\sin 2A = \frac{2\tan A}{1+\tan^2 A}$, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$

(e) $\sin 3A = 3\sin A - 4\sin^3 A$

(f) $\cos 3A = 4\cos^3 A - 3\cos A$

(g) $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

8. Three Angles:

(a) $\tan(A+B+C) = \frac{\tan A+\tan B+\tan C-\tan A\tan B\tan C}{1-\tan A\tan B-\tan B\tan C-\tan C\tan A}$

Note If :

(i) $A+B+C = \pi$ then $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(ii) $A+B+C = \frac{\pi}{2}$ then $\tan A\tan B + \tan B\tan C + \tan C\tan A = 1$

(b) If $A + B + C = \pi$ then :

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A\sin B\sin C$

(ii) $\sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

9. Maximum & Minimum values of Trigonometric Functions:

(a) Min. value of $a^2\tan^2\theta + b^2\cot^2\theta = 2ab$ where $\theta \in R$

(b) Max. and Min. value of $a\cos\theta + b\sin\theta$ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$

(c) If $f(\theta) = a\cos(\alpha+\theta) + b\cos(\beta+\theta)$ where a, b, α and β are known quantities then

$-\sqrt{a^2+b^2+2ab\cos(\alpha-\beta)} \le f(\theta) \le \sqrt{a^2+b^2+2ab\cos(\alpha-\beta)}$

(d) If A, B, C are the angles of a triangle then maximum value of $\sin A + \sin B + \sin C$ and $\sin A\sin B\sin C$ occurs when $A = B = C = 60^0$

(e) In case a quadratic in $\sin\theta$ or $\cos\theta$ is given then the maximum or minimum values can be interpreted by making a perfect square.

10. Sum of sines or cosines of n angles,

$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots\ldots + \sin\left(\alpha + \overline{n-1}\,\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left(\alpha + \frac{n-1}{2}\beta\right)$

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + \ldots\ldots + \cos\left(\alpha + \overline{n-1}\,\beta\right) = \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha + \frac{n-1}{2}\beta\right)$

1. Which is correct one ?
(A) $\sin 1° < \sin 1$ (B) $\sin 1° = \sin 1$
(C) $\sin 1° > \sin 1$ (D) $\sin 1° = \sin\frac{\pi}{180}$

2. The difference between two acute angles of a right angle triangle is $\frac{3\pi}{10}$ rad. Find the angles in degree.
(A) 54°, 36° (B) 75°,15°
(C) 72°, 18° (D) None of these

3. A regular hexagon & a regular dodecagon are inscribed in the same circle. If the side of the dodecagon is $(\sqrt{3}-1)$, then the side of the hexagon is
(A) $\sqrt{2}+1$ (B) $\frac{\sqrt{3}+1}{2}$
(C) 2 (D) $\sqrt{2}$

4. If $\sin^6\theta + \cos^6\theta - 1 = \lambda\sin^2\theta\cos^2\theta$, then the value of λ is
(A) 1 (B) 2
(C) 3 (D) None of these

5. If $a\cos\theta + b\sin\theta = 3$ & $a\sin\theta - b\cos\theta = 4$ then $a^2 + b^2$ has the value =
(A) 25 (B) 14
(C) 7 (D) None of these

6. $$\frac{\tan\left(x-\frac{\pi}{2}\right)\cos\left(\frac{3\pi}{2}+x\right)-\sin^3\left(\frac{7\pi}{2}-x\right)}{\cos\left(x-\frac{\pi}{2}\right).\tan\left(\frac{3\pi}{2}+x\right)}$$
when simplified reduces to :
(A) $\sin x\cos x$ (B) $-\sin^2 x$
((C) $-\sin x\cos x$ (D) $\sin^2 x$

7. The expression
$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-$$
$$2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right] \text{ is equal to}$$
(A) 0 (B) 1
(C) 3 (D) $\sin 4\alpha + \sin 6\alpha$

8. The value of $\tan 1° \tan 2° \tan 3° \ldots\ldots \tan 89°$ is
(A) 1 (B) 0
(C) ∞ (D) 1/2

9. If $\sin\alpha\sin\beta - \cos\alpha\cos\beta + 1 = 0$, then the value of $1 + \cot\alpha\tan\beta$ is
(A) 1 (B) –1
(C) 2 (D) None of these

10. If $\tan 25° = x$, then $\frac{\tan 155° - \tan 115°}{1 + \tan 155°\tan 115°}$ is equal to
(A) $\frac{1-x^2}{2x}$ (B) $\frac{1+x^2}{2x}$
(C) $\frac{1+x^2}{1-x^2}$ (D) $\frac{1-x^2}{1+x^2}$

11. Let α, β be such that $\pi < \alpha - \beta < 3\pi$. If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$ is
(A) $-\frac{3}{\sqrt{130}}$ (B) $\frac{3}{\sqrt{130}}$
(C) $\frac{6}{65}$ (D) $-\frac{6}{65}$

12. The value of $\frac{\sin 24°\cos 6° - \sin 6°\sin 66°}{\sin 21°\cos 39° - \cos 51°\sin 69°}$ is
(A) –1 (B) 1
(C) 2 (D) None of these

13. The expression
$\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$ is equal to
(A) $\cos 2x$ (B) $2\cos x$
(C) $\cos^2 x$ (D) $1 + \cos x$

14. If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is
(A) $\frac{(4-\sqrt{7})}{3}$ (B) $-\frac{(4+\sqrt{7})}{3}$
(C) $\frac{(1+\sqrt{7})}{4}$ (D) $\frac{(1-\sqrt{7})}{4}$

15. If A lies in the third quadrant and $3\tan A - 4 = 0$, then $5\sin 2A + 3\sin A + 4\cos A$ is equal to
(A) 0 (B) $-\frac{24}{5}$
(C) $\frac{24}{5}$ (D) $\frac{48}{5}$

16. Prove that $\cos36°\cos72°\cos108°\cos144° = \frac{1}{16}$

(A) $\frac{1}{8}$ (B) $\frac{1}{32}$

(C) $\frac{1}{16}$ (D) None of these

17. The value of the expression

$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)$ is

(A) 1/8 (B) 1/16
(C) 1/4 (D) 0

18. Number of solutions of $\sin x = \frac{x}{10}$ are

(A) 5 (B) 6
(C) 8 (D) 7 .

19. The value of $\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11}$ is

(A) 0 (B) 1

(C) $\frac{1}{2}$ (D) None of these

20 tan 7 cos 11 sin 27 is
(A) negative
(B) positive
(C) neither negative nor positive
(D) None of these

21. If $\cot\alpha + \tan\alpha = m$ and $\frac{1}{\cos\alpha} - \cos\alpha = n$, then
(A) $m\,(mn^2)^{1/3} - n(nm^2)^{1/3} = 1$
(B) $m(m^2n)^{1/3} - n(nm^2)^{1/3} = 1$
(C) $n\,(mn^2)^{1/3} - m(nm^2)^{1/3} = 1$
(D) $n(m^2n)^{1/3} - m(mn^2)^{1/3} = 1$

22. If $2\sec^2\alpha - \sec^4\alpha - 2\,\mathrm{cosec}^2\alpha + \mathrm{cosec}^4\alpha = 15/4$, then $\tan\alpha$ is equal to

(A) $1/\sqrt{2}$ (B) 1/2

(C) $1/2\sqrt{2}$ (D) 1/4

23. If $3\sin x + 4\cos x = 5$ then $4\sin x - 3\cos x$ is equal to
(A) 0 (B) 1
(C) 5 (D) None of these

24. If $x\sin45°\cos^2 60° = \frac{\tan^2 60°\,\mathrm{cosec}\,30°}{\sec 45°\cot^2 30°}$, then x is equal to
(A) 9 (B) 10
(C) 11 (D) None of these

25. If $\sin 2\theta = k$, then the value of $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$ is equal to

(A) $\frac{1-k^2}{k}$ (B) $\frac{2-k^2}{k}$
(C) $k^2 + 1$ (D) $2 - k^2$

26. If $\cos\alpha + \cos\beta = a$, $\sin\alpha + \sin\beta = b$ and $= \beta - \alpha 2\theta$, then $\frac{\cos3\theta}{\cos\theta} =$

(A) $a^2 + b^2 - 2$ (B) $a^2 + b^2 - 3$
(C) $3 - a^2 - b^2$ (D) $(a^2 + b^2)/4$

27. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°}$ is

(A) 1 (B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2}$ (D) 2

28. If $A = \tan 6°\tan 42°$ and $B = \cot 66°\cot 78°$, then
(A) A = 2B (B) A = 1/3 B
(C) A = B (D) 3A = 2B

29. If $0° < x < 90°$ & $\cos x = \frac{3}{\sqrt{10}}$, then the value of $\log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$ is
(A) 0 (B) 1
(C) –1 (D) None of these

30. If $\tan\theta = \cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\ddots_{\infty}}}}$ where $\theta \in (0, 2\pi)$, find the number possible values of θ.

(A) 2 (B) 3
(C) 4 (D) more than 4

31. Let $f(x)$ denote the sum of the infinite trigonometric series, $f(x) = \sum_{n=1}^{\infty} \sin\frac{2x}{3^n}\sin\frac{x}{3^n}$. then the sum of the solutions of the equation $f(x) = 0$ lying in the interval $(0, 629)$ is

(A) 10100π (B) 5050π

(C) 4950π (D) None of these

32. Let **A** and **B** denote the statements **[AIEEE-2009]**

A : $\cos\alpha + \cos\beta + \cos\gamma = 0$

B : $\sin\alpha + \sin\beta + \sin\gamma = 0$

If $\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta) = -\frac{3}{2}$, then :

(A) A is false and B is true

(B) both A and B are true

(C) both A and B are false

(D) A is true and B is false

33. Let $\cos(\beta+\alpha) = \frac{4}{5}$ and let $\sin(\alpha-\beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha$ = **[AIEEE-2010]**

(A) $\frac{25}{16}$ (B) $\frac{56}{33}$

(C) $\frac{19}{12}$ (D) $\frac{20}{7}$

34. If $A = \sin^2 x + \cos^4 x$, then for all real x : **[AIEEE-2011]**

(A) $\frac{3}{4} \le A \le 1$ (B) $\frac{13}{16} \le A \le 1$

(C) $1 \le A \le 2$ (D) $\frac{3}{4} \le A \le \frac{13}{16}$

35 Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \ge 1$. Then $f_4(x) - f_6(x)$ equals : **[JEE MAIN-2014]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$

(C) $\frac{1}{4}$ (D) $\frac{1}{12}$