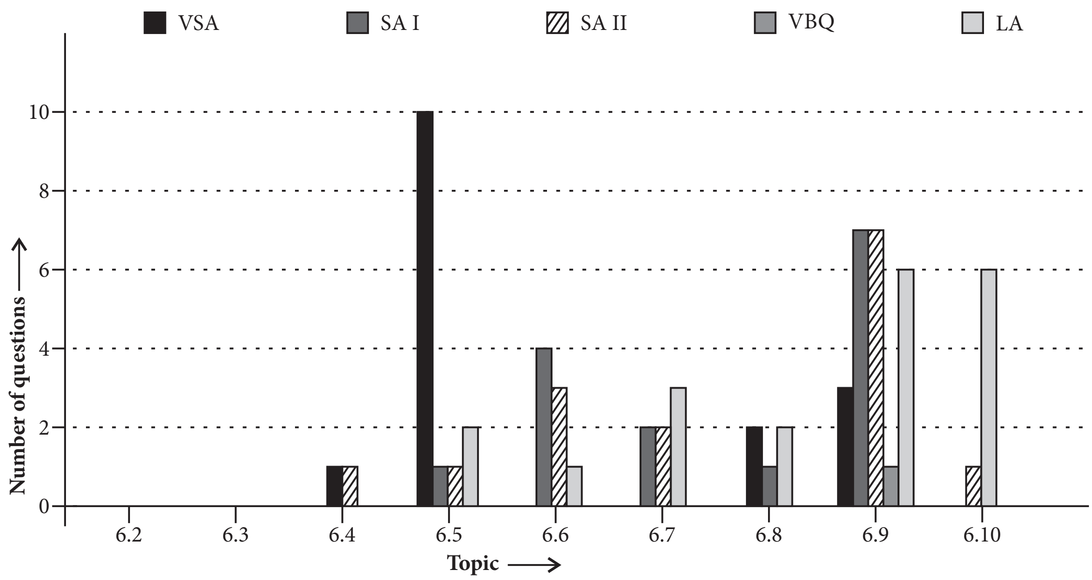
06 Electromagnetic Induction

Topicwise Analysis of Last 10 Years' CBSE Board Questions



- Maximum weightage is of *inductance.*
- Maximum VSA and SI type questions were asked from *Lenz's Law and Conservation of Energy.*
- Maximum SA I and SA II type questions were asked from *inductance.*
- Maximum LA type questions were asked from *inductance and AC Generator.*

QUICK RECAP

- **Magnetic flux :** The number of magnetic lines of force crossing a surface is known as magnetic flux linked with that surface.

It is given by $\phi = \vec{B} \cdot \vec{A} = BA \cos\theta$
where B is the strength of magnetic field, A is the area of surface and θ is the angle between normal to area and field direction.

- When $\vec{B}$ is perpendicular to the surface *i.e.* $\theta = 0°$, $\phi = NBA$ (maximum value).
- When $\vec{B}$ is parallel to the surface *i.e.* $\theta = 90°$, $\phi = 0$ (minimum value).
- In case of a coil of area A having N turns $\phi = NBA\cos\theta$
- Magnetic flux is a scalar quantity. It can be positive, negative or zero.
- The dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$.
- The SI unit of magnetic flux is weber.
- The CGS unit of magnetic flux is maxwell.
- 1 weber = 10^8 maxwell

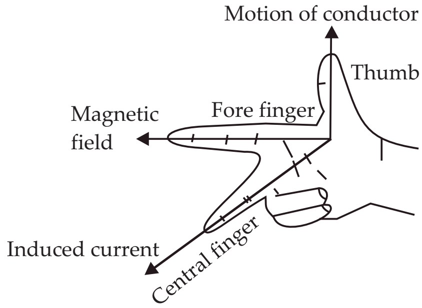
Electromagnetic induction : It is the phenomenon of generating an emf by changing the number of magnetic lines of force (*i.e.* magnetic flux) associated with the circuit. The emf so generated is known as induced emf. If the circuit is closed the current which flows in it due to induced emf is known as induced current.

Faraday's law of electromagnetic induction

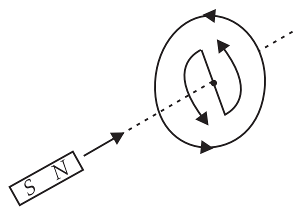
- **First law :** Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in the circuit. This induced emf persists as long as the change in magnetic flux continues.
- **Second law :** The magnitude of the induced emf is equal to the time rate of change of magnetic flux. Mathematically, induced emf is given by $\varepsilon = -\frac{d\phi}{dt}$ where negative sign indicates the direction of ε.

Lenz's law : This law gives us the direction of induced emf. According to this law, the direction of induced emf in a circuit is such that it opposes the change in magnetic flux responsible for its production. Lenz's law is in accordance with the principle of conservation of energy.

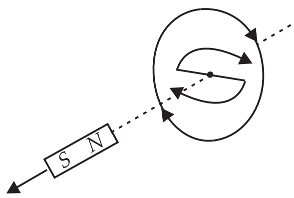
- **Fleming's right hand rule :** Fleming's right hand rule also gives us the direction of induced emf or current, in a conductor moving in a magnetic field. According to this rule, if we stretch the fore finger, central finger and thumb of our right hand in mutually perpendicular directions such that fore finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current or emf.



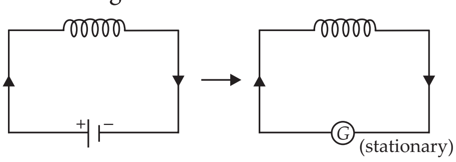
- **Applications of Lenz's law**
  - When a north pole of a bar magnet is moved towards a coil, the current induced in the coil will be in anticlockwise direction as shown in the figure.



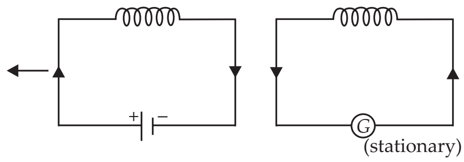
  - When a north pole of a bar magnet is moved away from the coil, the current induced in the coil will be in clockwise direction as shown in the figure.



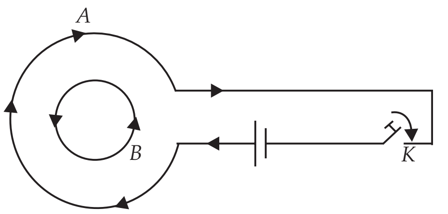
  - When a current carrying coil is moved towards a stationary coil, the direction of current induced in stationary coil is as shown in figure.



– When a current carrying coil is moved away from a stationary coil, the direction of current induced in stationary coil is as shown in figure.

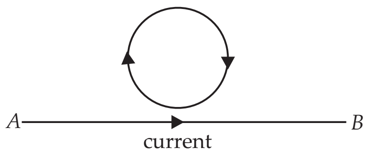


– When two coils A and B are arranged as shown in figure, then on pressing K, current in A increases in clockwise direction. Therefore, induced current in B will be in anticlockwise direction.

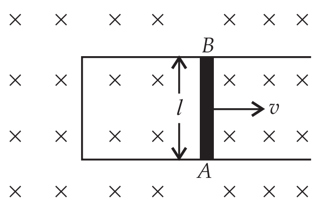


– However, when key K is released, current in A decreases in clockwise direction. Therefore, induced current in B will be in clockwise direction.

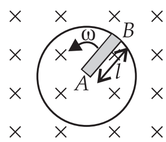
– When current in a straight conductor AB is increased, induced current in loop will be in clockwise direction as shown in the figure. If current in AB is decreasing, the induced current in the loop will be in anticlockwise direction.



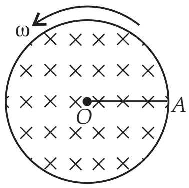
Motional emf : When a conducting rod of length l, moves with a velocity v perpendicular to a uniform magnetic field B, the induced emf across its ends is $|\varepsilon| = Blv$. This emf is known as motional emf.



- If the rod makes an angle θ with the direction of the field, then induced emf is
$|\varepsilon| = Blv \sin\theta$
- When a conducting rod of length l is rotated perpendicular to a uniform magnetic field B, then induced emf between the ends of the rod is $|\varepsilon| = \dfrac{B\omega l^2}{2} = \dfrac{B(2\pi\upsilon)l^2}{2}$
$|\varepsilon| = B\upsilon\,(\pi l^2) = B\upsilon A$
where, ω is angular frequency and υ is frequency of rod, $A = \pi l^2$.



- When a conducting solid disc of radius r is rotating with a uniform angular velocity ω with its plane perpendicular to a uniform magnetic field B, the emf induced between the centre and rim of disc is
$|\varepsilon| = \dfrac{B\omega r^2}{2} = B\upsilon A$



Eddy currents : Eddy currents are basically the currents induced in the body of a conductor due to change in magnetic flux linked with the conductor.

- The direction of eddy currents is given by Lenz's law, or Fleming's right hand rule.
- According to Lenz's law, eddy currents set up in a metallic conductor flow in such a direction so as to oppose the change in magnetic flux linked with it.
- Eddy currents cannot be eliminated but can be minimised by
  – laminating the core
  – by taking the metallic core in the form of thin laminated sheets attached together.
- Eddy currents are useful in
  – Electromagnetic damping
  – Induction furnace

– Electric brakes
– Speedometers

Inductor : An inductor is a device for storing energy in a magnetic field. An inductor is generally called as inductance. In usual practice a coil or solenoid is treated as inductor. It is denoted by symbol -ꝏꝏ-.

- **Self induction :** Whenever the current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, an emf is induced in the coil or the circuit which opposes the change that causes it. This phenomenon is known as self induction and the emf induced is known as self induced emf or back emf.
  – When a current I flows through a coil and ϕ is the magnetic flux linked with the coil, then $\phi \propto I$ or $\phi = LI$ where L is coefficient of self induction or self inductance of the coil.
  – The self induced emf is $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$
  – The SI unit of L is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.
  – Self inductance of a solenoid is $L = \mu_0 N^2 lA$ where l is length of the solenoid, N is number of turns per unit length of a solenoid and A is area of cross section of the solenoid.
  – Self inductance of a circular coil is $L = \frac{\mu_0 N^2 \pi R}{2}$

  where R is the radius of a coil and N is the number of turns.
- **Mutual induction :** Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is known as mutual induction. The coil or circuit in which the current changes is known as primary while the other in which emf is set up is known as secondary.
  – Let I_P be the current flowing through primary coil at any instant. If ϕ_S is the flux linked with secondary coil then $\phi_S \propto I_P$ or $\phi_S = MI_P$ where M is coefficient of mutual inductance of the two coils.
  – The emf induced in the secondary coil is given by $\varepsilon_S = -M\frac{dI_P}{dt}$
  – The SI unit of M is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.
- **Coefficient of coupling (K) :** Coefficient of coupling of two coils is a measure of the coupling between the two coils and is given by

  $$K = \frac{M}{\sqrt{L_1 L_2}}$$

  where L_1 and L_2 are coefficients of self inductance of the two coils and M is coefficient of mutual inductance of the two coils.
- The coefficient of mutual inductance of two long co-axial solenoids, each of length l, area of cross section A, wound on air core is

  $$M = \frac{\mu_0 N_1 N_2 A}{l}$$

  where N_1, N_2 are total number of turns of the two solenoid.
- **Combination of inductances**
  – Two inductors of self-inductances L_1 and L_2 are kept so far apart that their mutual inductance is zero. These are connected in series. Then the equivalent inductance is $L = L_1 + L_2$
  – Two inductors of self-inductance L_1 and L_2 are connected in series and they have mutual inductance M. Then the equivalent inductance of the combination is $L = L_1 + L_2 \pm 2M$
  – The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.
  – Two inductors of self-inductors L_1 and L_2 are connected in parallel. The inductors are so far apart that their mutual inductance is negligible. Then their equivalent inductance is

  $$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \quad \text{or} \quad L = \frac{L_1 L_2}{L_1 + L_2}$$

- **Energy stored in an inductor :** When a current I flows through an inductor, the energy stored in it is given by $U = \frac{1}{2} LI^2$
- The energy stored in an inductor is in the form of magnetic energy.
- **AC Generator :** A generator produces electrical energy from mechanical work, just the opposite of what a motor does.
- An AC generator is based on the phenomena of electromagnetic induction, which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil. Here ε is the emf induced in the coil, then

$\varepsilon = NBA\omega \sin \omega t$ or $\varepsilon = \varepsilon_0 \sin \omega t$

where $\varepsilon_0 = NBA\,\omega$ is the maximum or peak value of induced EMF.

The instantaneous EMF ε produced in coil varies sinusoidally with time and hence is also known as alternating EMF.

where, N = number of turns in the coil, B = strength of magnetic field, A = Area of each turns of the coil and ω = angular velocity of rotation.

Previous Years' CBSE Board Questions

6.4 Faraday's Law of Induction

VSA (1 mark)

1. On what factors does the magnitude of the emf induced in the circuit due to magnetic flux depend? *(Foreign 2013)*

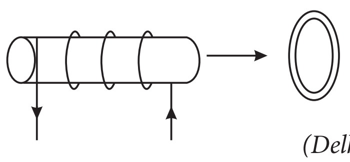
SA II (3 marks)

2. State Faraday's law of electromagnetic induction. *(1/3, AI 2009)*

6.5 Lenz's Law and conservation of Energy

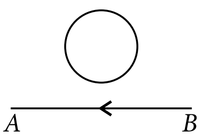
VSA (1 mark)

3. Figure shows a current carrying solenoid moving towards a conducting loop. Find the direction of the current induced in the loop.



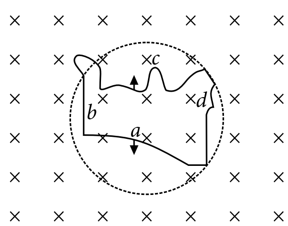
(Delhi 2015C)

4. The electric current flowing in a wire in the direction from B to A is decreasing. Find out the direction of the induced current in the metallic loop kept above the wire as shown.



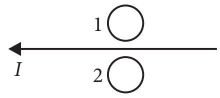
(AI 2014)

5. A flexible wire of irregular shape, abcd, as shown in the figure, turns into a circular shape when placed in a region of magnetic field which is directed normal to the plane of the loop away from the reader. Predict the direction of the induced current in the wire.



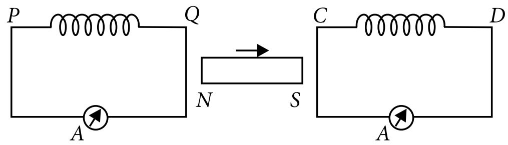
(Foreign 2014)

6. Predict the direction of induced current in metal rings 1 and 2 when current I in the wire is steadily decreasing?



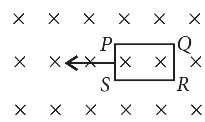
(Delhi 2012)

7. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil.



(AI 2012)

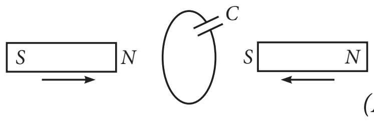
8. The closed loop ($PQRS$) of wire is moved into a uniform magnetic field at right angles to the plane of the paper as shown in the figure. Predict the direction of the induced current in the loop.



(Foreign 2012)

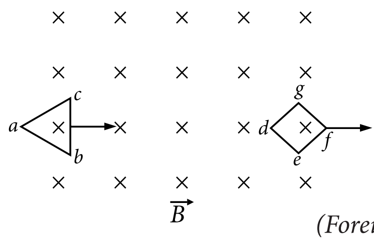
9. State Lenz's law. *(AI 2012C)*

10. Two bar magnets are quickly moved towards a metallic loop connected across a capacitor 'C' as shown in the figure. Predict the polarity of the capacitor.



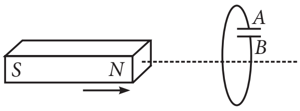
(AI 2011)

11. Two loops of different shapes are moved in a region of uniform magnetic field in the directions marked by arrows as shown in the figure. What is the direction of the induced current in each loop?



(Foreign 2010)

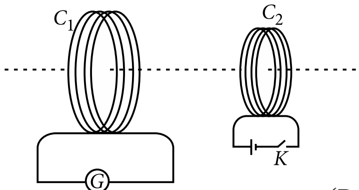
12. In the given figure, a bar magnet is quickly moved towards a conducting loop having a capacitor. Predict the polarity of the plates A and B of the capacitor.



(AI 2010C)

SA I (2 marks)

13. A current is induced in coil C_1 due to the motion of current carrying coil C_2. (a) Write any two ways by which a large deflection can be obtained in the galvanometer G. (b) Suggest an alternative device to demonstrate the induced current in place of a galvanometer.



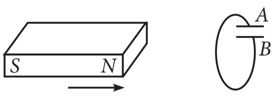
(Delhi 2011)

SA II (3 marks)

14. State Lenz's law. Using this law indicate the direction of the current in a closed loop when a bar magnet with north pole in brought close to it. Explain briefly how the direction of the current predicted wrongly results in the violation of the law of conservation of energy. *(AI 2011C)*

LA (5 marks)

15. State Lenz's law. Use it to predict the polarity of the capacitor in the situation given below :



(2/5, AI 2015C)

16. Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it. *(2/5, Delhi 2014)*

17. State Lenz's law. Give one example to illustrate this law. "The Lenz's law is a consequence of the principle of conservation of energy". Justify this statement. *(3/5, AI 2014)*

6.6 Motional Electromotive Force

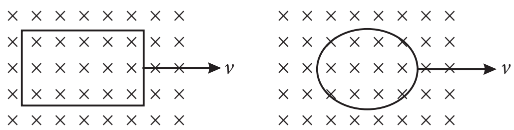
SA I (2 marks)

18. State Lenz's law. A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends? Justify your answer. *(Delhi 2013)*

19. A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the place is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased? *(AI 2013)*

20. A metallic rod of L length is rotated with angular frequency of ω with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius L, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring. *(Delhi 2012)*

21. A rectangular loop, and a circular loop having the same area, are moved out of a uniform magnetic field region, to a field free region, with a constant velocity $\vec{v}$. Would the induced emf remain constant in the two loops as they move out of the field region? Justify your answer.

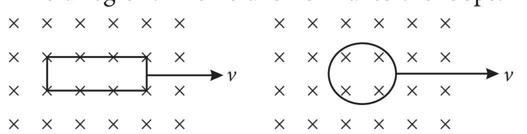


(Delhi 2010C)

SA II (3 marks)

22. A metallic rod of length 'l' is rotated with a frequency υ with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r, about an axis passing through the centre and perpendicular to the plane of the ring. A constant uniform magnetic field B parallel to the axis is present everywhere. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtain the expression for it. *(Delhi 2013)*

23. A rectangular loop and a circular loop are moving out of a uniform magnetic field region with a constant velocity $\vec{v}$ as shown in the figure. In which loop do you expect the induced emf to be constant during the passage out of the field region? The field is normal to the loops.

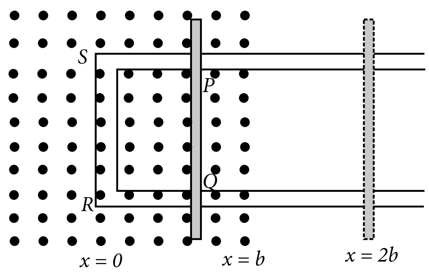


(AI 2011C)

24. A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°? *(2/3, AI 2009)*

LA (5 marks)

25. State Faraday's law of electromagnetic induction. Figure shows a rectangular conductor *PQRS* in which the conductor *PQ* is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from $x = 0$ to $x = b$ and is zero for $x > b$. Assume that only the arm *PQ* possesses resistance r. When the arm *PQ* is pulled outward from $x = 0$ to $x = 2b$ and is then moved backward to $x = 0$ with constant speed v, obtain the expressions for the flux and the induced emf. Sketch the variations of these quantities with distance $0 \le x \le 2b$.

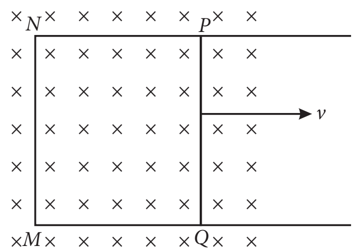


(AI 2010)

6.7 Energy Consideration : A Quantitative Study

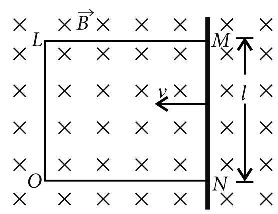
SA I (2 marks)

26. A rectangular loop *PQMN* with movable arm *PQ* of length 10 cm and resistance 2 Ω is placed in a uniform magnetic field of 0.1 T acting perpendicular to the plane of the loop as is shown in the figure. The resistances of the arms *MN*, *NP* and *MQ* are negligible. Calculate the (i) emf induced in the arm *PQ* and (ii) current induced in the loop when arm *PQ* is moved with velocity 20 m/s.



(Delhi 2014C)

27. A rectangular conductor *LMNO* is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor. When the arm *MN* of length of 20 cm is moved towards left with a velocity of 10 m s^{-1}, calculate the emf induced in the arm. Given the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance) find the value of the current in the arm.



(AI 2013)

SA II (3 marks)

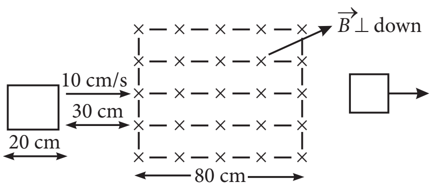
28. A metallic rod of length 'l' is rotated with a uniform angular speed ω, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius $R = l$, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf induced in the rod. If r is the resistance of the rod and the metallic ring has negligible resistance, obtain the expression for the power generated. *(AI 2013C)*

29. A metallic rod of length l is rotated at constant angular speed ω, normal to a uniform magnetic field B. Derive an expression for the current induced in the rod, if the resistance of the rod is R. *(Delhi 2008)*

LA (5 marks)

30. A jet plane is travelling towards west at a speed of 1800 km/h.
(i) Estimate voltage difference developed between the ends of the wing having a span of 25 m if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and dip angle is 30°.
(ii) How will the voltage developed be affected if the jet changes its direction from west to north? *(3/5, AI 2015C)*

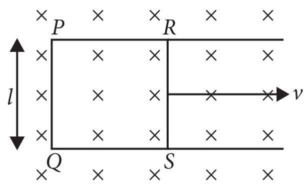
31. A square loop of side 20 cm is initially kept 30 cm away from a region of uniform magnetic field of 0.1 T as shown in the figure. It is then moved towards the right with a velocity of 10 cm s^{-1} till it goes out of the field.
Plot a graph showing the variation of
(i) magnetic flux (ϕ) through the loop with time (t).
(ii) induced emf (ε) in the loop with time t.
(iii) induced current in the loop if it has resistance of 0.1 Ω.



(3/5, AI 2015C)

32. A metallic rod of length l and resistance R is rotated with a frequency υ, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere.
(a) Derive the expression for the induced emf and the current in the rod.
(b) Due to the presence of the current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.
(c) Hence obtain the expression for the power required to rotate the rod. *(AI 2014C)*

33. Figure shows a rectangular conducting loop $PQSR$ in which arm RS of length 'l' is movable. The loop is kept in a uniform magnetic field 'B' directed downward perpendicular to the plane of the loop. The arm RS is moved with a uniform speed 'v'.



Deduce an expression for
(i) the emf induced across the arm 'RS',
(ii) the external force required to move the arm, and
(iii) the power dissipated as heat. *(3/5, AI 2009)*

6.8 Eddy Currents

VSA (1 mark)

34. Give one example of use of eddy currents. *(Foreign 2014)*

35. A metallic piece gets hot when surrounded by a coil carrying high frequency alternating current. Why? *(Delhi 2014C)*

SA I (2 marks)

36. What are eddy currents? Write any two applications of eddy currents. *(AI 2011)*

LA (5 marks)

37. What are eddy currents ? How are they produced? Describe briefly three main useful applications of eddy currents. *(Foreign 2015)*

38. What are eddy currents? Write their two applications. *(2/5, AI 2009)*

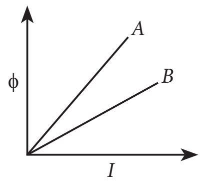
6.9 Inductance

VSA (1 mark)

39. Define the term 'self-inductance' of a coil. Write its S.I. unit. *(AI 2015)*

40. How does the mutual inductance of a pair of coils change when
(i) distance between the coils is increased and
(ii) number of turns in the coils is increased? *(AI 2013)*

41. A plot of magnetic flux (ϕ) versus current (I) is shown in the figure for two inductors A and B. Which of the two has larger value of self inductance?



(Delhi 2010)

SA I (2 marks)

42. Starting from the expression for the energy $W = \frac{1}{2}LI^2$, stored in a solenoid of self-inductance L to build up the current I, obtain the expression for the magnetic energy in terms of the magnetic field B, area A and length l of the solenoid having n number of turns per unit length. Hence, show that the energy density is given by $B^2/2\mu_0$. *(Delhi 2013C)*

43. (i) Define mutual inductance between two long coaxial solenoids.

(ii) Find out the expression for the mutual inductance of inner solenoid of length l having the radius r_1 and the number of turns n_1 per unit length due to the second outer solenoid of same length and n_2 number of turns per unit length. *(Delhi 2012)*

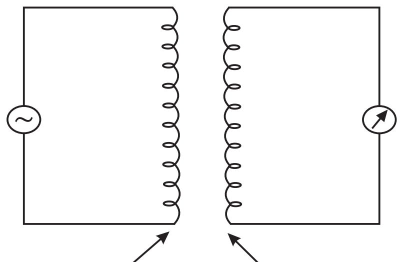
44. Current in a circuit falls steadily from 5.0 A to 0.0 A in 100 ms. If an average e.m.f. of 200 V is induced, calculate the self-inductance of the circuit. *(Foreign 2011)*

45. Two concentric circular coils C_1 and C_2, radius r_1 and r_2 ($r_1 << r_2$) respectively are kept co-axially. If current is passed through C_2, then, find an expression for mutual inductance between the two coils. *(AI 2011C)*

46. Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare (i) the induced emf and (ii) the current produced in the two coils. Justify your answer. *(AI 2010)*

47. Two long co-axial solenoids of the same length but different radii and different number of turns are wound one over the other. Deduce the expression for the mutual inductance of this arrangement. *(Foreign 2010)*

48. The circuit arrangement given below shows that when an a.c. passes through the coil A, the current starts flowing in the coil B.



Coil A Coil B

(i) State the underlying principle involved.
(ii) Mention two factors on which the current produced in the coil B depends. *(AI 2008)*

SA II (3 marks)

49. (i) Define mutual inductance.
(ii) A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil ? *(Delhi 2016)*

50. (i) Define self-inductance. Write its SI unit.
(ii) A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing? *(Foreign 2016)*

51. Define the term 'mutual inductance' between the two coils. Obtain the expression for mutual inductance of a pair of long coaxial solenoids each of length l and radii r_1 and r_2 ($r_2 >> r_1$). Total number of turns in the two solenoids are N_1 and N_2 respectively. *(AI 2014)*

52. Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance L to build up a current I through it. *(AI 2014)*

53. The currents flowing in the two coils of self-inductance L_1 = 16 mH and L_2 = 12 mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of (i) induced voltages, (ii) the currents and (iii) the energies stored in the two coils at a given instant. *(Foreign 2014)*

54. Define self-inductance of a coil. Write its SI unit.
Derive the expression for self-inductance of a long solenoid of cross-sectional area 'A', length 'l' having 'n' turns per unit length. *(AI 2012C, Delhi 2009)*

55. An air cored solenoid of length 0.3 m, area of cross section is 1.2×10^{-3} m^2 and has 2500 turns. Around its central section, a coil of 350 turns is wound. The solenoid and the coil are electrically insulated from each other. Calculate the emf induced in the coil if the initial current of 3A in the solenoid is reversed in 0.25 s. *(Delhi 2008)*

VBQ (4 marks)

56. Ram is a student of class X in a village school. His uncle gifted him a bicycle with a dynamo fitted in it. He was very excited to get it. While cycling during night, he could light the bulb and see the objects on the road. He, however, did not know how this device works. He asked this question to his teacher. The teacher considered it an opportunity to explain the working to the whole class.
Answer the following questions:
(i) State the principle and working of a dynamo.
(ii) Write two values each displayed by Ram and his school teacher. *(AI 2016)*

LA (5 marks)

57. (a) Explain the meaning of the term mutual inductance. Consider two concentric circular coils, one of radius r_1 and the other of radius $r_2 (r_1 < r_2)$ placed coaxially with centres coinciding with each other. Obtain the expression for the mutual inductance of the arrangement.
(b) A rectangular coil of area A, having number of turns N is rotated at 'f' revolutions per second in a uniform magnetic field B, the field being perpendicular to the coil. Prove that the maximum emf induced in the coil is $2\pi f NBA$. *(AI 2016)*

58. (a) Define mutual inductance and write its S.I. units.
(b) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.
(c) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for emf induced in the coil C_1 due to a change in the current through the coil C_2. *(Delhi 2015)*

59. Define mutual inductance of a pair of coils and write on which factors does it depend. *(2/5, AI 2015C)*

60. The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of
(i) Magnetic flux versus the current
(ii) Induced emf versus dI/dt
(iii) Magnetic potential energy stored versus the current. *(Delhi 2014)*

61. Deduce an expression for the mutual inductance of two long coaxial solenoids but having different radii and different number of turns. *(3/5, AI 2014)*

62. (a) Define the term 'mutual inductance'. Deduce the expression for the mutual inductance of two long coaxial solenoids having different radii and different number of turns. *(Foreign 2013, 3/5, AI 2009)*
(b) A coil is mechanically rotated with constant angular speed ω in a uniform magnetic field which is perpendicular to the axis of rotation of the coil. The plane of the coil is initially held perpendicular to the field. Plot a graph showing variation of (i) magnetic flux ϕ and (ii) the induced emf in the coil as a function of ωt. *(Foreign 2013)*

63. (a) Derive the expression for the mutual inductance of two long coaxial solenoids of same length l having radii r_1 and r_2 $(r_2 > r_1)$.
(b) Show that mutual inductance of solenoid 1 due to solenoid 2, M_{12}, is the same as that of 2 due to 1 *i.e.*, M_{21}. *(Foreign 2011)*

6.10 AC Generator

SA II (3 marks)

64. Draw a labelled diagram of an a.c. generator. Explain briefly its principle and working. *(AI 2012C, Delhi 2007)*

LA (5 marks)

65. (a) Draw a schematic sketch of an ac generator describing its basic elements. State briefly its working principle. Show a plot of variation of
(i) Magnetic flux and
(ii) Alternating emf versus time generated by a loop of wire rotating in a magnetic field.
(b) Why is choke coil needed in the use of fluorescent tubes with ac mains? *(Delhi 2014)*

66. (a) Draw a labelled diagram of a.c. generator and state its working principle.
(b) How is magnetic flux linked with the armature coil changed in a generator?
(c) Derive the expression for maximum value of the induced emf and state the rule that gives the direction of the induced emf.
(d) Show the variation of the emf generated versus time as the armature is rotated with respect to the direction of the magnetic field. *(Delhi 2014C)*

67. (a) State the principle on which AC generator works. Draw a labelled diagram and explain its working.
(b) A conducting rod held horizontally along East-West direction is dropped from rest from a certain height near the Earth's surface. Why should there be an induced emf across the ends of the rod?
Draw a plot showing the instantaneous variation of emf as a function of time from the instant it begins to fall. *(Foreign 2012)*

68. (a) State the working of a.c. generator with the help of a labelled diagram.
(b) The coil of an a.c. generator having N turns, each of area A, is rotated with a constant angular velocity ω. Deduce the expression for the alternating e.m.f. generated in the coil.
(c) What is the source of energy generation in this device? *(AI 2011)*

69. (a) Describe briefly, with the help of a labelled diagram, the basic elements of an A.C. generator. State its underlying principle.
(b) Show diagrammatically how an alternating emf is generated by a loop of wire rotating in a magnetic field. Write the expression for the instantaneous value of the emf induced in the rotating loop. *(Delhi 2010)*

70. (a) Explain briefly, with the help of a labelled diagram, the basic principle of the working of an a.c. generator.
(b) In an a.c. generator, coil of N turns and area A is rotated at υ revolutions per second in a uniform magnetic field B. Write the expression for the emf produced.
(c) A 100-turn coil of area 0.1 m^2 rotates at half a revolution per second. It is placed in a magnetic field 0.01 T perpendicular to the axis of rotation of the coil. Calculate the maximum voltage generated in the coil. *(AI 2008)*

Detailed Solutions

1. The magnitude of the emf induced in the circuit due to magnetic flux depends on the time rate of change of magnetic flux through the circuit.

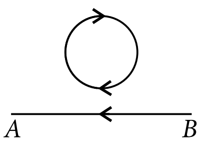
$$|\varepsilon| = \frac{\Delta\phi}{\Delta t}$$

2. Faraday's law of electromagnetic induction states that whenever there is change in magnetic flux linked with the circuit, an emf is induced in it, whose magnitude is directly proportional to the rate of change of magnetic flux linked with the circuit. *i.e.*

$$|\varepsilon| \propto \frac{d\phi}{dt}$$

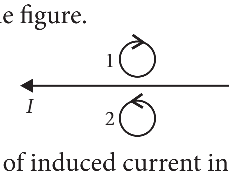
3. The direction of current in the coil is anti-clock wise.

4. Clockwise, so that the magnetic field produced by the induced current is also directed inwards in direction of decreasing magnetic field of current, in straight line.

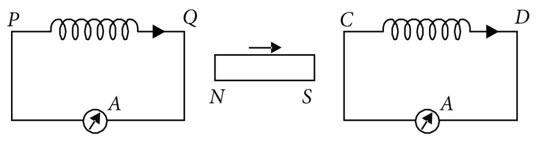


5. The wire is expanding to form a circle, which means that force is acting outwards on each part of the wire because of the magnetic field (acting in the downwards direction). The direction of the induced current should be such that it will produce magnetic field in upward direction (towards the reader). Hence, the force on the wire will be towards inward direction, *i.e.*, induced current is flowing in anticlockwise direction in the loop from c-b-a-d-c.

6. When the current I in the wire is steadily decreasing, the direction of induced current in ring 1 will be clockwise and anticlockwise in ring 2 as shown in the figure.



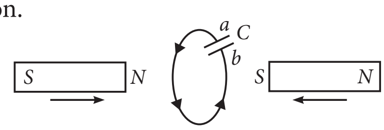
7. Direction of induced current in both the coils is clockwise, $P \rightarrow Q$ and $C \rightarrow D$



8. By Lenz's law, the direction of induced current is such that it opposes its own cause of production. The induced current opposes the increase in magnetic flux. Hence the direction of induced current is *PSRQP* (anticlockwise).

9. Lenz's law states that the direction of the induced emf and the direction of induced current are such that they oppose the cause which produces them.

10. In this situation, a will become positive with respect to b, as current induced is in clockwise direction.



11. The induced current always opposes the change in magnetic flux. Loop *abc* is entering the magnetic field; so magnetic flux linked with loop tends to increase, so current induced in loop *abc* is anticlockwise to produce magnetic field upward to oppose the increase in flux. Loop *defg* is leaving the magnetic field; so flux linked with it tends to decrease, the induced current will be clockwise to produce magnetic field downward to oppose the decrease in magnetic flux.

12. Here, the North pole is approaching the magnet, so the induced current in the face of loop viewed from left side will flow in such a way that it will behave like North pole, so South pole developed on loop when viewed from right hand side of the loop. The flow of induced current is clockwise, hence A acquires positive polarity and B negative.

13. (a) To obtain large deflection in galvanometer we can take following steps :

(i) Connect the coil C_2 to a powerful battery for large current.

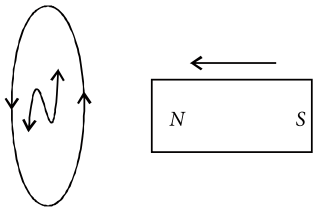
(ii) Switch on and off the key at a rapid rate.

(iii) Develop a relative shift/motion between the two coils.

(iv) Use a ferromagnetic material like iron inside the coil C_2 to increase the magnetic flux.

(b) Galvanometer is replaced by a torch bulb. Now a relative motion between two coils or switch on and off of the key glows the bulb and shows presence of induced current.

14. *Refer to answer 9.*



When the N pole of a magnet is moved towards a coil, the induced current in the coil flows in anticlockwise direction on the side of magnet, so as to acquire north polarity and oppose the motion of the magnet towards the coil, by applying repulsive force on it.

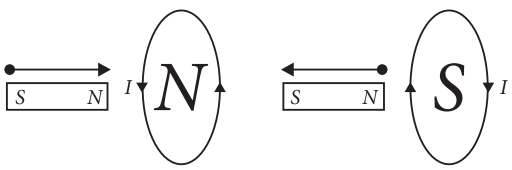
In order to continue the change in magnetic flux linked with the circuit, some work is to be done or some energy is to be spent against the opposition offered by induced EMF. This energy spent by the external source ultimately appears in the circuit in the form of electrical energy.

Suppose that the Lenz's law is not valid. Then the induced current flows through the coil in a direction opposite to one dictated by Lenz's law. The resulting force on the magnet makes it move faster and faster, *i.e.*, the magnet gains speed and hence kinetic energy without expanding an equivalent amount of energy. This sets up a perpetual motion machine, violating the law of conservation of energy. Thus Lenz's law is valid and is a consequence of the law of conservation of energy.

15. *Refer to answers 9 and 12.*

16. Whenever magnetic flux linked with a circuit changes, it induces an EMF in it. The induced current set up in the circuit flows in such a direction that it opposes the change in magnetic flux linked with the circuit.

In order to continue the change in magnetic flux linked with the circuit, some work is to be done or some energy is to be spent against the opposition offered by induced EMF. This energy spent by the external source ultimately appears in the circuit in the form of electrical energy.



This is why a magnet is moved near the closed loop with its N-pole towards the loop, then current is produced in loop on the side of a magnet in anticlockwise direction so as to develop the north pole which applies repulsive force on magnet opposing motion of magnet towards the loop.

Similarly when a magnet is moved away from the closed loop with its N-pole towards the loop, the current is produced in the loop on the side of magnet in clockwise direction, so as to develop the south pole which attracts the bar magnet opposing its motion away from the loop.

17. *Refer to answer 14.*

18. *Refer to answer 9.*

The rod held along east west direction will fall in a perpendicular magnetic field B_H present in N-S direction. Hence an emf will be induced in the rod following the relation for the motional emf $\varepsilon = B_H vl$.

19. Horizontal component $B_H = B\cos\delta$

$= 0.4 \times \cos 60°$

$= 0.2\text{ G} = 0.2 \times 10^{-4}\text{ T}$

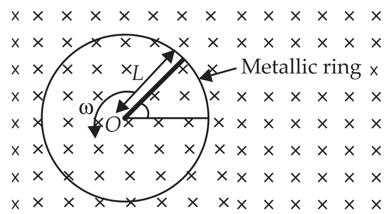
Induced emf $\varepsilon = \frac{1}{2}BL^2\omega$

$= \frac{1}{2} \times 0.2 \times 10^{-4} \times (0.5)^2 \times 2\pi \times 2$

$= 3.14 \times 10^{-5}\text{ V}$

The emf induced between the axle and the rim of the wheel is independent of number of spokes in the wheel.

20.



As the rod is rotated, free electrons in the rod move towards the outer end due to Lorentz force and get distributed over the ring. Thus, the resulting separation of charges produce an emf across the ends of the rod. At a certain value of emf, there is no more flow of electrons and a steady state is reached. The magnitude of the emf generated across a length dL of the rod as it moves at right angle to the magnetic field is given by $d\varepsilon = BvdL$. Hence,

$$\varepsilon = \int d\varepsilon = \int_0^L BvdL = \int_0^L B\omega LdL = \frac{B\omega L^2}{2} \quad (\because\ v = \omega L)$$

21. Magnitude of induced emf is directly proportional to the rate of area moving out of the field, for a constant magnetic field,

$$\varepsilon = -\frac{d\phi}{dt} = -B\frac{dA}{dt}$$

For the rectangular coil, the rate of area moving out of the field remains same while it is not so for the circular coil. Therefore, the induced emf for the rectangular coil remains constant.

22. *Refer to answer 20.*

23. *Refer to answer 21.*

24. EMF induced across the ends of the wings of plane is

or $\varepsilon = vB_v \cdot l = vB \sin\delta \cdot l$

or $\varepsilon = \left(1800 \times \frac{5}{18} \text{m/s}\right) \times (5 \times 10^{-4}\text{ T}) \times \sin 30° \times 25\text{ m}$

or $\varepsilon = 500 \times 5 \times 10^{-4} \times \frac{1}{2} \times 25$

or $\varepsilon = 3.125$ V

25. Faraday's law of electromagnetic induction states that whenever there is a change in the magnetic flux linked with a circuit an induced emf is set up in it, which lasts as long as the magnetic flux linked with it is changing and the magnitude of induced emf 'ε' is directly proportional to the rate of change of magnetic flux linked with it *i.e.*, $|\varepsilon| \propto \frac{d\phi}{dt}$.

According to the data given in question, during the motion from $x = 0$ to $x = b$

Initial flux = 0

Final flux = Blb

Motional emf in the arm PQ, $\varepsilon = -Bvl$ $\left[\because\ \varepsilon = -\frac{d\phi}{dt}\right]$

During the motion from $x = b$ to $x = 2b$,

Flux remains constant which is $\phi = Blb$

$\therefore$ Motional emf, $\varepsilon = 0$

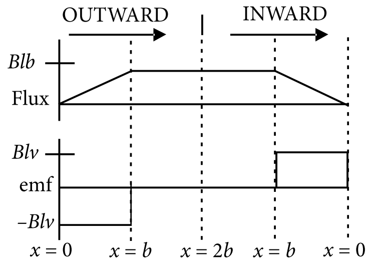
Above values remain same, for the motion of arm PQ, from $x = 2b$ to b.

During motion from $x = b$ to $x = 0$,

initial flux $\phi = Blb$, final flux = 0

$\therefore$ Motional emf $\varepsilon = -\frac{d\phi}{dt} = +Blv$ (direction reversed)

Variation of these quantities is shown in following graph.



26. Induced emf $\varepsilon = Blv$

$= 0.1 \times 0.1 \times 20$

$= 0.2$ V

Current $I = \frac{\varepsilon}{R} = \frac{0.2}{2} = 0.1$ A

27. Induced emf $\varepsilon = Blv$

$= 0.5 \times 0.2 \times 10 = 1$ V

Current $I = \frac{\varepsilon}{R}$

$I = \frac{1}{5} = 0.2$ A

28. In one revolution change of area $dA = \pi l^2$.

$\therefore$ Change in magnetic flux

$d\phi = \vec{B} \cdot \vec{dA} = BdA\cos 0°$

$= B\pi l^2$

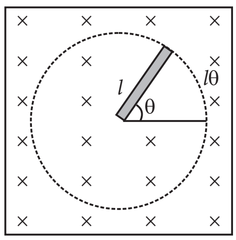
Period of revolution = T

(i) Induced emf $(\varepsilon) = \frac{B\pi l^2}{T} = B\pi l^2 \upsilon$

(ii) Induced current $I = \frac{\varepsilon}{r} = \frac{B\pi l^2 \upsilon}{r}$

Power, $P = \varepsilon I = \frac{B^2\omega^2 l^4}{4r}$ $(\because\ \omega = 2\pi\upsilon)$

29.



If θ is the angle traced by the free electron in time t, then area swept out,

$$A = \pi l^2 \times \left(\frac{\theta}{2\pi}\right) = \frac{1}{2} l^2\theta$$

Magnetic flux linked, $\phi = B\left(\frac{1}{2}l^2\theta\right)\cos 0°$

$[\because\ \phi = BA\cos\theta]$

$\phi = \frac{1}{2}Bl^2\theta$

According to Faraday's laws of electromagnetic induction,

Induced emf, $\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\frac{d\theta}{dt} = \frac{1}{2}Bl^2\omega$

$\therefore$ Induced current, $I = \frac{\varepsilon}{R} = \frac{\frac{1}{2}Bl^2\omega}{R} = \frac{Bl^2\omega}{2R}$

30. (i) *Refer to answer 24.*

(ii) Now B' = horizontal component of earth's magnetic field

$B' = B\cos 30° = B\frac{\sqrt{3}}{2}$

$\therefore$ $V' = \sqrt{3}\,V = 1.732 \times 3.125 \cong 5.4$ V.

31. Given $l = 20$ cm $= 0.2$ m,

$B = 0.1$ T, $v = 10$ cm s^{-1} $= 0.1$ m s^{-1}

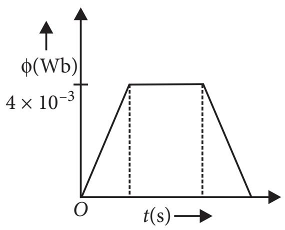
(i) Magnetic flux through loop

$\phi = B \cdot A = Blx$

$\phi_{max} = 0.1 \times 0.2 \times 0.2$

$= 0.004$ Wb

$= 4 \times 10^{-3}$ Wb



(ii) Induced emf,

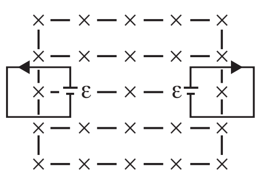
$\varepsilon = \frac{-d\phi}{dt}$

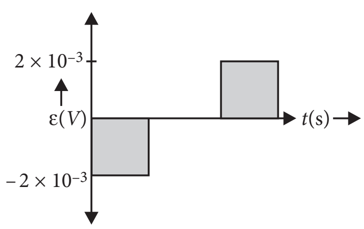
$= -Blv$

$|\varepsilon|_{max} = 0.1 \times 0.2 \times 0.1$

$= 0.002$ V

$= 2 \times 10^{-3}$ V

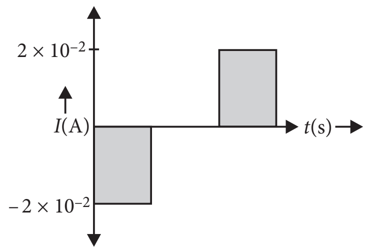




(iii) Induced current,

$I = \frac{|\varepsilon|}{R}$

$= \frac{2 \times 10^{-3}}{0.1} = 2 \times 10^{-2}$ A



32. (a) *Refer to answer 29.*

(b) Force acting on the rod,

$F = IlB = \frac{\pi\upsilon B^2 l^3}{R}$

The external force required to rotate the rod opposes the Lorentz force acting on the rod. External force acts in the direction opposite to the Lorentz force.

(c) Power required to rotate the rod,

$P = Fv = \frac{\pi\upsilon B^2 l^3 v}{R}$

33. Let length $PQ = l$ and width of rod $= dx$ and let area be A, velocity $= v$, resistance $= r$.

(i) $\varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}(BA) = -B\frac{dA}{dt}$

$\varepsilon = Bl\frac{dx}{dt} = Blv$

(ii) Force in the loop

$F = ilB = \frac{\varepsilon}{r}lB = \frac{B^2l^2v}{r}$

(iii) $P = Fv = \frac{B^2l^2v^2}{r}$

34. Eddy current is used for magnetic braking in trains. Strong electromagnets are situated in the train, just above the rails. When the electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train.

35. It becomes hot due to the eddy currents produced in it.

36. When magnetic flux linked with a metallic conductor in the form of solid mass changes, then induced currents are set up in the conductor in the form of closed loops known as 'Eddy currents'.

Two applications of eddy currents are :

(i) Electromagnetic braking in trains

(ii) Induction furnace

37. *Refer to answer 36.*

Applications of eddy currents :

(i) Magnetic brake : Whenever a train is moving, its axle rotates. A drum attached with its axle also rotates. To stop the train, a magnetic field is applied on the rotating drum. Owing to eddy currents set up in the drum, an opposing couple acts on it and as a result of that the train stops. This system is called the magnetic brake.

(ii) Induction furnace : If the resistance of a conductor is very small, eddy currents set up in it are very strong. These currents, therefore, produce a lot of heat in conductor. An induction furnace is based on this principle and is used to prepare alloys by melting the constituent metals. Since the amount of heat produced due to eddy currents is directly proportional to the square of its frequency, currents of high frequency are used in these furnaces.

(iii) Speedometer : This instrument is used for the measurement of speeds of the vehicles and is based on the principle of eddy currents.

38. *Refer to answer 36.*

39. Self inductance : When the current in a coil is changed, a back emf is induced in the same coil. This phenomenon is called self-inductance. If L is self-inductance of coil, then

$N\phi \propto I$

$N\phi = LI$

$$L = \frac{N\phi}{I}$$

The unit of self-inductance is henry (H).

40. (i) With the increase in distance between the coils the magnetic flux linked with the secondary coil decreases and hence, the mutual inductance of the two coils will decrease with the increase in separation between them.

(ii) Since, the mutual inductance of the two coils is given as $M = \mu_0 n_1 n_2 A l$. So, with the increase in number of turns mutual inductance increases.

41. Since $\phi = LI \Rightarrow L = \dfrac{\phi}{I}$ = slope of $\phi - I$ graph

$\because$ Slope of inductor A is greater than slope of inductor B.

Hence, the inductor A has larger value of self inductance.

42. The magnetic energy is,

$$U_B = \frac{1}{2}LI^2$$

$$= \frac{1}{2}L\left(\frac{B}{\mu_0 n}\right)^2 \qquad \text{(since } B = \mu_0 nI \text{, for a solenoid)}$$

$$= \frac{1}{2}(\mu_0 n^2 Al)\left(\frac{B}{\mu_0 n}\right)^2$$

$$= \frac{1}{2\mu_0}B^2 Al$$

The magnetic energy per unit volume is,

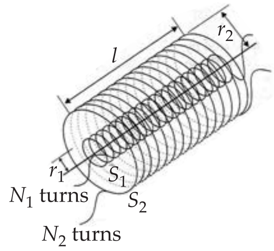
$$u_B = \frac{U_B}{V} \qquad \text{(where } V \text{ is volume that contains flux)}$$

$$= \frac{U_B}{Al}$$

$$= \frac{B^2}{2\mu_0}$$

43. (i) The phenomena of inducing current in a circuit by changing the current or flux in a neighbouring circuit is called mutual induction. S.I. unit of mutual inductance is Henry denoted by H.

(ii) Consider two long coaxial solenoids each of length l. Let n_1 be the number of turns per unit length of inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of outer solenoid S_2 of radius r_2. Let N_1 and N_2 be the total number of turns of solenoids S_1 and S_2 respectively.



When a current I_2 is passed through S_2, the magnetic flux linked with solenoid S_1 is

$$N_1\phi_1 = M_{12}I_2 \qquad \text{...(i)}$$

where M_{12} is called the mutual inductance of solenoid S_1 with respect to solenoid S_2.

It is also referred as the coefficient of mutual induction.

The magnetic field due to current I_2 in S_2 is

$$B_2 = \mu_0 n_2 I_2 \qquad \text{...(ii)}$$

$\therefore$ The magnetic flux linked with S_1 is

$$N_1\phi_1 = B_2(\pi r_1^2)n_1 l = \mu_0 n_1 n_2 \pi r_1^2 l I_2 \qquad \text{...(iii)}$$

where $n_1 l$ is the total number of turns in solenoid S_1.

From (i) and (iii), we get

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l \qquad \text{...(iv)}$$

which is required expression.

Similarly, $M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$ ...(v)

From (iv) and (v), we get

$M_{12} = M_{21} = M$

Hence, coefficient of mutual induction between two coaxial solenoids is

$$M = \mu_0 n_1 n_2 \pi r_1^2 l \text{ or, } M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l}$$

44. Change in current (ΔI) = (0.0 – 5.0) A = – 5.0 A

Time taken (Δt) = 100 × 10^{-3} s

Induced emf (ε) = 200 V

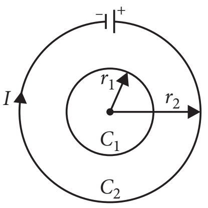
Induced emf (ε) is given by

$$\varepsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{\Delta(LI)}{\Delta t}$$

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

or $L = -\varepsilon \cdot \dfrac{\Delta t}{\Delta I} = -\dfrac{(200).(100\times10^{-3})}{(-5.0)}$

$L = 4.0$ H

45. 

Let a current I flows through the coil having radius r_2. The magnetic field through the coil having radius r_1 is,

$$B_2 = \frac{\mu_0 I}{2r_2}$$

Total flux through smaller coil $\phi_1 = B_2 A_1$

$$\phi_1 = \left(\frac{\mu_0 \pi r_1^2}{2r_2}\right) I \quad \text{...(i)}$$

By definition of mutual induction

$\phi_1 = M_{12} I_2$ ...(ii)

So, by equations (i) and (ii) mutual inductance of the pair of coils is $M = \dfrac{\phi_1}{I} = \dfrac{\mu_0 \pi r_1^2}{2r_2}$.

46. (i) Induced emf in both the loops will be same, as the two loops are of same area A and are rotated with same angular speed ω in same magnetic field B as $\varepsilon = BA\omega\sin\omega t$.

(ii) As $I = \dfrac{\varepsilon}{R}$, so in copper loop with less resistance, induced current will be more.

47. *Refer to answer 43 (ii).*

48. (i) The principle involved in the given circuit arrangement is mutual induction. Mutual induction is the property of two coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced emf.

(ii) The current produced in the coil B depends on

(a) number of turns in the coil

(b) nature of material

(c) distance between two coils.

49. (i) *Refer to answer 43 (i).*

(ii) Here, $M = 1.5$ H, $\Delta I_1 = 20$ A, $\Delta t = 0.5$ s, $\Delta\phi = ?$

We know, emf induced in the second coil,

$$\varepsilon = -\frac{(\Delta\phi)_2}{\Delta t} = -\frac{M\Delta I_1}{\Delta t}$$

$\therefore$ $(\Delta\phi)_2 = M\Delta I_1 = 1.5 \times 20 = 30$ Wb

50. (i) *Refer to answer 39.*

(ii) Mutual inductance of solenoid coil system

$$M = \frac{\mu_0 N_1 N_2 A_2}{l}$$

Here, $N_1 = 15$, $N_2 = 1$, $l = 1\text{cm} = 10^{-2}\text{m}$,

$A = 2.0 \text{ cm}^2 = 2.0 \times 10^{-4} \text{ m}^2$

$$\therefore \ M = \frac{4\pi \times 10^{-7} \times 15 \times 1 \times 2.0 \times 10^{-4}}{10^{-2}}$$

$= 120\pi \times 10^{-9}$ H

Induced emf in the loop

$$\varepsilon_2 = M\frac{\Delta I_1}{\Delta t} \text{ (numerically)}$$

$$= 120\pi \times 10^{-9} \frac{(4-2)}{0.1}$$

$$= 120 \times 3.14 \times 10^{-9} \times \frac{2}{0.1}$$

$= 7.5 \times 10^{-6}$ V

$= 7.5$ µV.

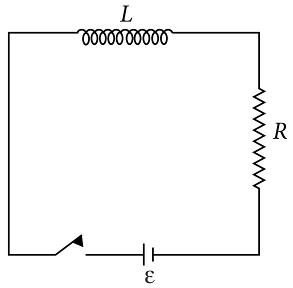
51. *Refer to answer 43.*

52. The phenomena of induced emf in a solenoid due to change in current or magnetic flux linked with the solenoid is called self inductance of the solenoid.

The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ is given by

$L = \mu I_0 n^2 A l$

where, A is the area of cross-section of the solenoid, l is the length and n is the number of turns per unit length.



Consider the circuit shown here, consisting of a inductor L and a resistor R, connected to a source of emf ε. As the connections are made, the current grows in the circuit and the magnetic field increases in the inductor. Part of the work done by the battery during the process is stored in the inductor as magnetic field energy and the rest appears as thermal energy in the resistor. After sufficient time, the current, and hence the magnetic field, becomes constant and further work done by the battery appears completely as thermal energy. If, I be the current in the circuit at time t, we have

Self induced emf $\varepsilon = L\frac{dI}{dt}$

$dW = \varepsilon I dt$

$dW = L\frac{dI}{dt}\, I dt$

$dW = LIdI$

Work done by source of emf to supply current I for a small time dt.

Now total work done by cell to establish current I_0 in inductor

$$W = \int dW = L\int_0^{I_0} I\, dI = \frac{1}{2}LI_0^2$$

Total work done is stored as magnetic energy in the solenoid.

53. (i) Induced voltage $V = L\frac{dI}{dt}$

$\frac{V_1}{V_2} = \frac{L_1}{L_2}$ (as $\frac{dI}{dt}$ is same)

$\Rightarrow \frac{V_1}{V_2} = \frac{16}{12} = \frac{4}{3}$

(ii) Power $P = IV$

$\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{3}{4}$ (as P is same)

$\Rightarrow \frac{I_1}{I_2} = \frac{3}{4}$

(iii) Energy stored $E = \frac{1}{2}LI^2$

$\frac{E_1}{E_2} = \frac{L_1 I_1^2}{L_2 I_2^2} = \frac{16}{12} \times \frac{9}{16} = \frac{3}{4}$

$\Rightarrow \frac{E_1}{E_2} = \frac{3}{4}$

54. *Refer to answer 39.*

Consider a long air solenoid having n number of turns per unit length. If current in solenoid is I, then magnetic field within the solenoid,

$B = \mu_0 nI$ ...(i)

where $\mu_0 = 4\pi \times 10^{-7}$ henry / metre is the permeability of free space.

If A is cross-sectional area of solenoid, then effective flux linked with solenoid of length 'l',

$\phi = NBA$

where $N = nl$ is the number of turns

$\phi = (nlBA)$

Substituting the value of B from (i)

$\phi = nl\,(\mu_0\, nI)A = \mu_0\, n^2 AlI$

$\therefore$ Self-inductance of air solenoid

$L = \frac{\phi}{I} = \mu_0\, n^2 Al$

If N is total number of turns in length l, then

$n = \frac{N}{l}$

$\therefore$ Self-inductance, $L = \mu_0\left(\frac{N}{l}\right)^2 Al$

$L = \frac{\mu_0 N^2 A}{l}$

55. $N_1 = 2500$, $N_2 = 350$, $A = 1.2 \times 10^{-3}$ m^2, $l = 0.3$ m.

$dI = 3 - (-3) = 3 + 3 = 6$ A

$dt = 0.25$ s

Since mutual inductance,

$M = \frac{\mu_0 N_1 N_2 A}{l}$

$= \frac{4\pi \times 10^{-7} \times 2500 \times 350 \times 1.2 \times 10^{-3}}{0.3}$

$= \frac{4 \times 3.14 \times 1.05 \times 10^{-4}}{0.3} = 4.39 \times 10^{-3}$ H

Induced emf $|\varepsilon| = M\frac{dI}{dt}$

$= \frac{4.39 \times 10^{-3} \times 6}{0.25} = 105.36 \times 10^{-3} = 0.10536$ V

56. (i) Principle : Dynamo works on the principle of electromagnetic induction.

Whenever a coil is rotated in a magnetic field, an emf is induced in it due to the change in magnetic flux linked with it.

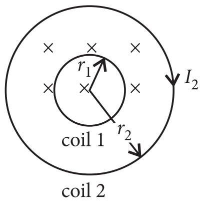
Working : As the coil of the dynamo rotates in the magnetic field, the angle θ between its area vector $\vec{A}$ and magnetic field $\vec{B}$ changes continuously with time, thereby changing the magnetic flux linked with the coil. Hence, a time varying sinusoidal emf $\varepsilon = \varepsilon_0 \sin \omega t$ is obtained across the coil of dynamo.

(ii) Ram has scientific aptitude, curiosity, keenness to learn, positive approach, etc (any two).

Teacher has dedication, concern for students, depth of knowledge, generous, positive attitude towards queries, motivational approach, etc (any two).

57. (a) Mutual inductance of a pair of coils is defined as the emf induced in one of the coils, when the rate of change of current is unity in the other coil.

When current I_2 flows through the outer coil-2, magnetic field produced at its centre is given by $B_2 = \frac{\mu_0 I_2}{2r_2}$ directed normal to the plane of coils. As $r_1 < r_2$, so this magnetic field is almost uniform over the plane of coil-1. So, magnetic flux linked with coil-1 is

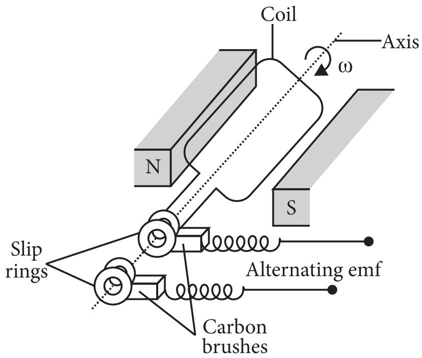


$$\phi_{12} = B_2 A_1 \cos 0° \quad \text{or} \quad \phi_{12} = \frac{\mu_0 I_2}{2r_2} \times \pi r_1^2 \times 1$$

$$\text{or} \quad \frac{\phi_{12}}{I_2} = \frac{\mu_0 \pi r_1^2}{2r_2} \quad \text{or} \quad M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2}$$

(b) The magnetic flux linked with coil at any instant of time t is

$\phi = NBA \cos\theta = NBA \cos \omega t$



From Faraday's law of electromagnetic induction, the induced emf across the coil is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos \omega t)$$

or $\varepsilon = NBA\omega \sin \omega t$ or $\varepsilon = \varepsilon_0 \sin \omega t$

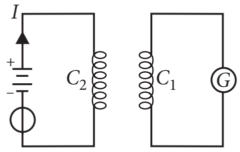
where, the maximum emf induced in the coil is when $\sin \omega t = +1$ and is given by

$\varepsilon_0 = NBA\omega$

or $\varepsilon_0 = 2\pi f NBA$ $[\because \omega = 2\pi f]$

58. (a), (b) *Refer to answer 43.*

(c)



Mutually induced emf in coil C_1 is

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI)$$

$$\text{or} \quad \varepsilon = -M\frac{dI}{dt}$$

The rate of change of current in the neighbouring coil is $\frac{dI}{dt}$.

59. The flux (ϕ_B) linked with the secondary coil is directly proportional to the strength of the current (I) flowing through the primary coil, *i.e.*,

$\phi_B \propto I$ or $\phi_B = MI$

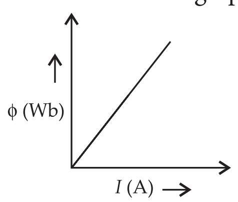
where M is a constant of proportionality and is termed as the mutual inductance of one circuit with respect to the other.

The value of M depends upon : (i) the characteristics of both the circuits, and (ii) their orientation with respect to each other.

60. (i) Suppose current I is flowing through an inductor of self inductance L. Then magnetic flux linked with the inductor is given by

$\phi = LI$

Magnetic flux versus the current graph,

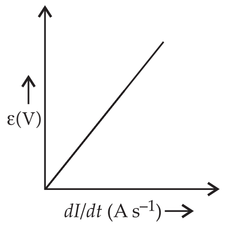


(ii) Induced emf is given by,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$$

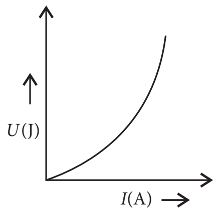
$|\varepsilon| = L\dfrac{dI}{dt}$

Induced emf versus dI/dt graph,



(iii) Magnetic potential energy stored versus the current graph,

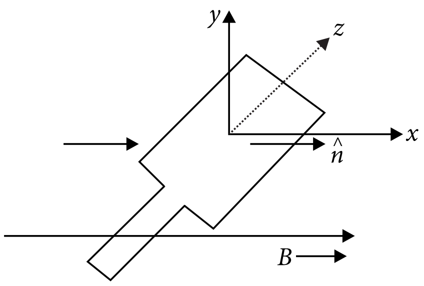
$U = \dfrac{1}{2}LI^2$



61. *Refer to answer 43 (ii).*

62. (a) *Refer to answer 43.*

(b) The plane of the coil is in yz plane and perpendicular to the x-axis *i.e.,* direction of magnetic field.



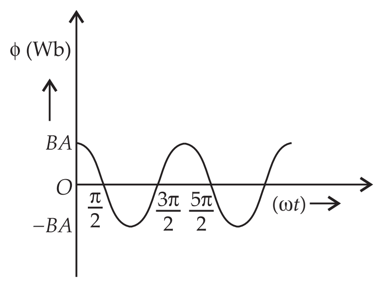
Maximum magnetic flux $\phi_{max} = B|A|$.

As the coil rotates with angular speed ω, magnetic flux at any instant t, (or at angle ωt)

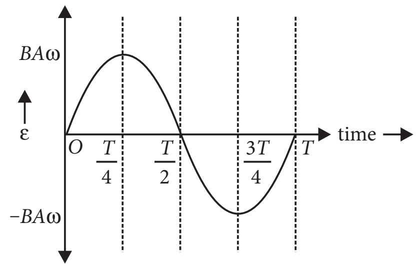
$\phi = |B||A| \cos \omega t$

(i) Magnetic flux

$\phi = BA \cos \omega t$

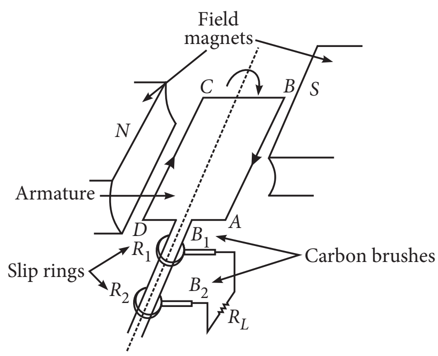


(ii) Alternating emf $\varepsilon = BA\omega \sin \omega t$



63. *Refer to answer 43 (ii).*

64. Principle : A.C. generator is based on the principle of electromagnetic induction. It converts mechanical energy into electrical energy.



It consists of

(i) Armature coil of large number of turns of copper wire wound over soft iron core. Soft iron core is used to increase magnetic flux.

(ii) Field magnets are used to apply magnetic field, in which armature coil is rotated with its axis perpendicular to field lines.

(iii) Slip rings are used to provide movable contact of armature coil with external circuit containing load.

(iv) Brushes are the metallic pieces used to pass on electric current from armature coil to the external circuit containing load.

When armature is rotated in the magnetic field, due to change in orientation of the coil magnetic flux through it changes. Due to change in flux an e.m.f. is induced.

$\varepsilon = -N\dfrac{d\phi}{dt}$

$\varepsilon = NBA\omega \sin \omega t$ $\quad [\because \ \phi = BA \cos \omega t]$

$i = \dfrac{\varepsilon}{R} = \dfrac{NBA\omega}{R}\sin\omega t$

Direction of induced current is given by Fleming's right hand rule.

65. (a) *Refer to answers 64 and 62 (b).*

(b) A choke reduces current in an a.c. circuit without dissipating any power. A rheostat also reduces current but it dissipates energy in the form of heat.

66. (a) *Refer to answer 64.*

(b) In an ac generator we keep the armature coil fixed and rotate the field magnet so as to produce induced emf. It is because the flux linked with the coil will change and an induced emf is set up in it.

(c) The maximum value of the induced emf is called peak value. If $f = \left(\frac{\omega}{2\pi}\right)$ is frequency of a current, then

$I = I_0 \sin \omega t$

Similarly alternating voltage (emf) is

$V = V_0 \sin \omega t$.

Emf generated versus time : If N is the number of turns in coil, f is the frequency of rotation, A area of coil and B the magnetic induction, then induced emf,

$$\varepsilon = \frac{d\phi}{dt} = \frac{d}{dt}(2\pi NBAf \cos 2\pi ft)$$

$= 2\pi NBAf \sin 2\pi ft$

$\varepsilon = \varepsilon_0 \sin 2\pi ft$, where $\varepsilon_0 = 2\pi NBAf$, the maximum emf induced.

The emf produced is alternating and hence the current is also alternating. Current produced by an ac generator cannot be measured by an moving coil ammeter, because the average value of ac over a complete cycle is zero.

(d) *Refer to answer 62 (b).*

67. (a) *Refer to answer 64.*

(b) As the earth's magnetic field lines are cut by the falling rod, the change in magnetic flux takes place. This change in flux induces an emf across the ends of the rod.

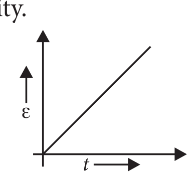
Since the rod is falling under gravity.

$v = gt \qquad (\because u = 0)$

Induced emf, $\varepsilon = Blv$

$\varepsilon = Blgt$

$\therefore \quad \varepsilon \propto t$



68. (a) *Refer to answer 64.*

(b) To calculate the magnitude of emf induced,

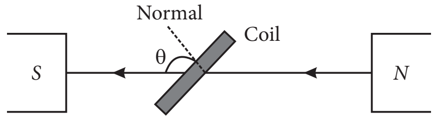
Suppose

N = number of turns in the coil,

A = area enclosed by each turn of coil,

$\vec{B}$ = strength of magnetic field,

θ = angle which normal to the coil makes with $\vec{B}$ at any instant t.



$\therefore$ Magnetic flux linked with the coil in this position

$\phi = N(\vec{B}\cdot\vec{A}) = NBA \cos \theta$

$= NBA \cos \omega t \qquad$ ...(i)

where ω be the angular velocity of the coil. At this instant t, if ε is the emf induced in the coil, then

$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d}{dt}(NAB \cos \omega t)$$

$$= -NAB\frac{d}{dt}(\cos \omega t) = -NAB(-\sin \omega t)\omega$$

$\varepsilon = NAB\omega \sin \omega t \qquad$ ...(ii)

The induced emf will be maximum, when

$\sin \omega t$ = maximum = 1

$\therefore \quad \varepsilon_{max} = \varepsilon_0 = NAB\omega \times 1 \qquad$ ...(iii)

From equations (ii) and (iii),

$\varepsilon = \varepsilon_0 \sin \omega t$

(c) The source of energy generation in this device is the mechanical energy consumed in rotating the coil.

69. *Refer to answer 64.*

70. (a) *Refer to answer 64.*

(b) The magnetic flux linked with the coil at any instant is

$\phi = NBA \cos \omega t$

Induced emf will be

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(NBA \cos \omega t)$$

$= NBA\omega \sin \omega t$

or $\quad \varepsilon = \varepsilon_0 \sin \omega t$

where $\varepsilon_0 = NBA\omega$ = peak value of induced emf

(c) Given, $N = 100$, $A = 0.1$ m^2, $B = 0.01$ T

$\upsilon = \frac{1}{2}$ revolution per sec = 0.5 r.p.s.

$\therefore$ Maximum voltage generated $\varepsilon_0 = NBA\omega$

$= NBA(2\pi\upsilon)$

$$\therefore \quad \varepsilon_0 = 100 \times 0.01 \times 0.1 \times 2 \times \frac{22}{7} \times 0.5$$

$$= \frac{2.2}{7} = 0.314 \text{ V}$$

$$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{0.314}{1.414} = 0.22 \text{ V.}$$