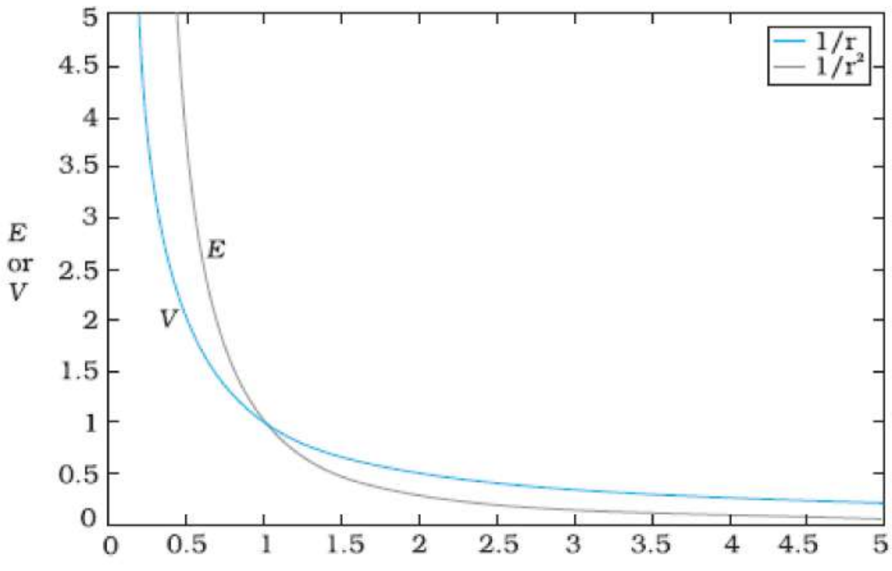
CHAPTER-2 ELECTROSTATIC POTENTIAL AND CAPACITANCE

(1-MARK QUESTION)

1. Where does the energy of capacitor reside?
Ans: Electric field
2. Do electrons tend to go to region of low or high potential?
Ans: Low potential
3. Can electric potential at any point in space be zero while intensity of electric field at that point is not zero?
Ans: Yes
4.What is the electric potential of earth?
Ans. Zero
5.What is the capacitance of the earth?
Ans. Infinity

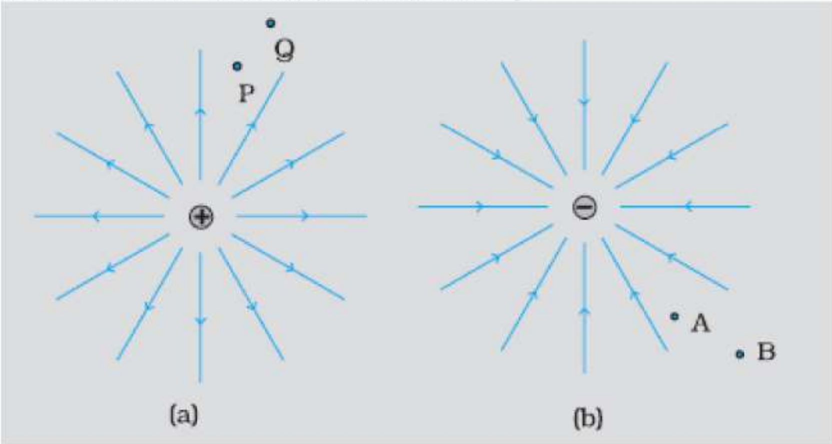
(2-MARKS QUESTION)

6.Graphically show the variation of electric potential and electric field with distance for a point charge.



7.The two charged conductors are touched mutually and then separated. What will be the charge on them?
Ans. The charge on them will be divided in the ratio of their capacitances.We know that q=CV.When the charged conductors touch they acquire the same potential.Hence q is proportional to capacitance

8. Figures (a) and (b) show the field lines of a positive and negative point charge respectively



(a) Give the signs of the potential difference $V_P - V_Q$; $V_B - V_A$.
(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

Solution

(a) As V is inversely proportional to distance, $V_P > V_Q$. Thus, $(V_P - V_Q)$ is positive. Also V_B is less negative than V_A. Thus, $V_B > V_A$ or $(V_B - V_A)$ is positive.
(b) A small negative charge will be attracted towards positive charge. The negative charge moves from higher potential energy to lower potential energy. Therefore the sign of potential energy difference of a small negative charge between Q and P is positive.
Similarly, (P.E.)A > (P.E.)B and hence sign of potential energy differences is positive.

9.n small drops of the same size are charged to V volt. They coalesce to form a bigger drop. Calculate the potential of the bigger drop.

Solution: Let r be the radius of each small drop and R the radius of the bigger drop. Since bigger drop is equal to that of n small drops,

$$\frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$$

$$R = n^{1/3} r$$

If q is the charge on each small drop,then

$$\text{Potential on the bigger drop} = \frac{\text{total charge}}{\text{capacitance}} = \frac{nq}{4\pi\epsilon n^{1/3} r}$$

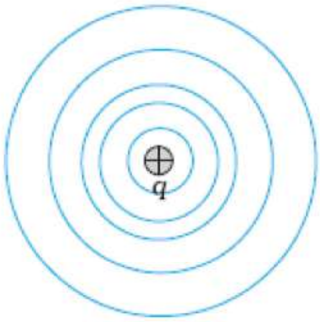
$$= n^{2/3}\frac{q}{4\pi\epsilon r} = n^{2/3} \text{ x potential on each drop}$$

(3-MARKS QUESTION)

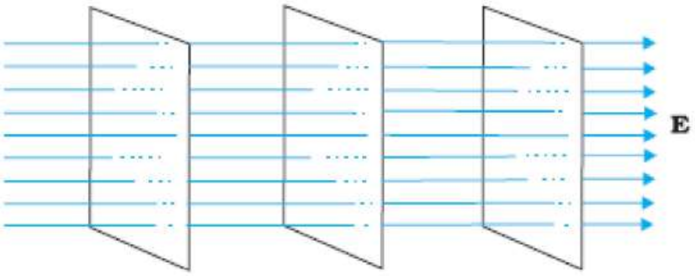
10.Draw the equipotential surfaces for (i) point charge (ii) for uniform electric field (iii) electric dipole

Ans

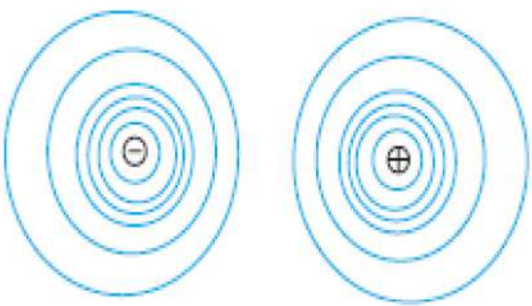
(i)



(ii)

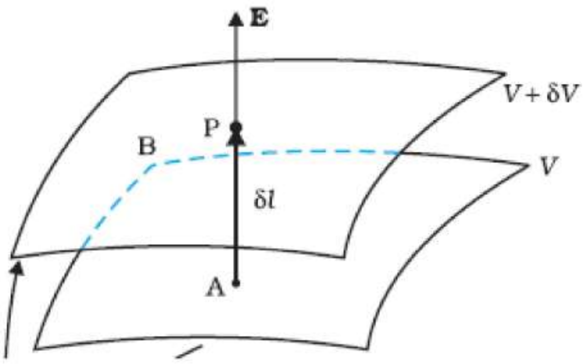


(iii)



11.Derive a relation between electric field and potential

Ans.

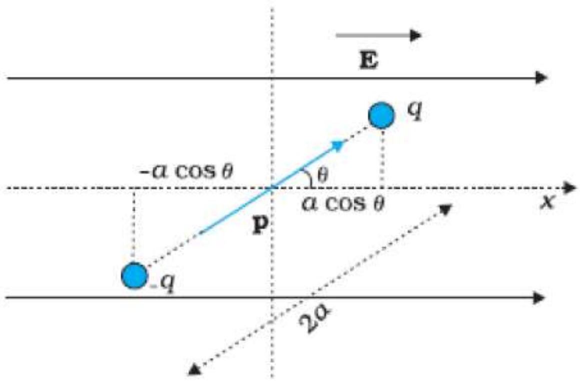


Consider two closely spaced equipotential surfaces A and B with potential values V and $V + \delta V$, where δV is the change in V in the direction of the electric field **E**. Let P be a point on the
surface B. δl is the perpendicular distance of the surface A from P. Imagine that a unit positive charge is moved along this perpendicular from the surface B to surface A against the electric field.The work done in this process is $|\mathbf{E}|\delta l$.
This work equals the potential difference
$V_A - V_B$.
Thus, $|\mathbf{E}|\delta l = V - (V + \delta V) = -\delta V$
i.e., $|\mathbf{E}| = -\frac{\delta V}{-\delta l}$
Since δV is negative, $\delta V = -|\delta V|$. we can rewrite

$$|\mathbf{E}| = -\frac{\delta V}{-\delta l} = \frac{\delta V}{\delta l}$$

Q12.Find the potential energy of a dipole in an electric field.

Ans.



Consider a dipole with charges $q_1 = +q$ and $q_2 = -q$ placed in a uniform electric field **E**, as shown in figure
the dipole experiences no net force; but experiences a torque τ given by
$\tau = \mathbf{p} \times \mathbf{E}$ (1)
which will tend to rotate it (unless **p** is parallel or antiparallel to **E**). Suppose an external torque τ_{ext} is applied in such a manner that it just neutralises this torque and rotates it in the plane of paper from angle θ_0 to angle θ_1 at an infinitesimal angular speed and *without*

angular acceleration. The amount of work done by the external torque will be given by

$$W = \int_{\theta_0}^{\theta_1} \tau_{ext}(\theta)d\theta = \int_{\theta_0}^{\theta_1} pE\sin\theta\, d\theta$$

$$= pE(\cos\theta_0 - \cos\theta_1)$$

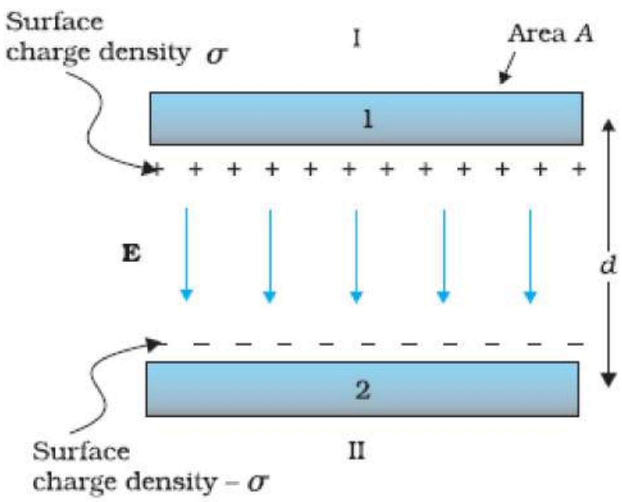
This work is stored as the potential energy of the system. We can then associate potential energy *U*(θ) with an inclination θ of the dipole. Similar to other potential energies, there is a freedom in choosing the angle where the potential energy *U* is taken to be zero. A natural choice is to take θ0 = π / 2. (An explanation for it is provided towards the end of discussion.)

We can then write,

$$U(\theta) = pE\left(\cos\frac{\pi}{2} - \cos\theta\right) = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$$

Q13. Derive an expression for the capacitance of a parallel plate capacitor.

Ans-



Electric field in different region

Outer region I (region above the plate 1).

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

Outer region II (region below the plate 2).

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$|E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$V = E\,d = \frac{1}{\varepsilon_0}\frac{Qd}{A}$$

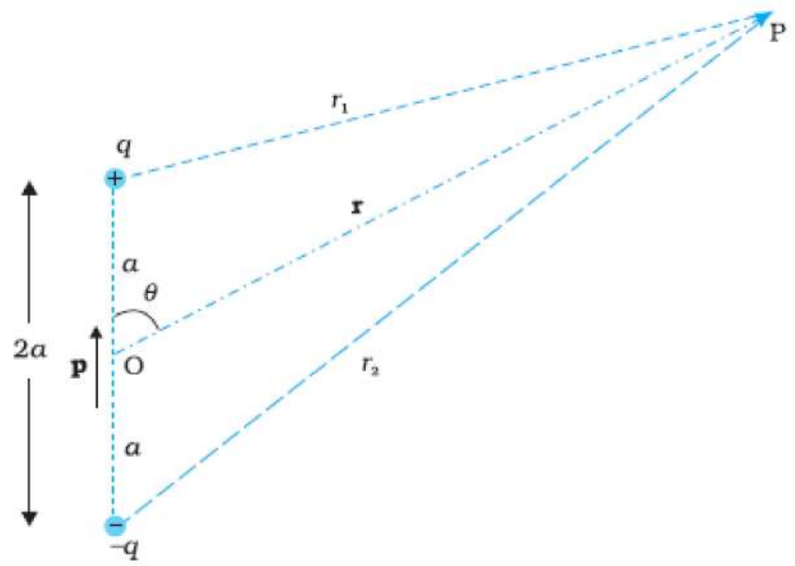
The capacitance C of the parallel plate capacitor is then

$$C = \frac{Q}{V} = = \frac{\varepsilon_0 A}{d}$$

(5-MARKS QUESTION)

13.Derive an expression for the electric potential at a point due to an electric dipole. Mention the contrasting features of electric potential of a dipole at a point as compared to that due to a single charge.

Ans.



Thus, the potential due to the dipole is the sum of potentials due to the charges q and $-q$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) \quad (1)$$

where r_1 and r_2 are the distances of the point P from q and $-q$, respectively.

Now, by geometry,

$$r_1^2 = r^2 + a^2 - 2ar\cos\theta$$

$$r_2^2 = r^2 + a^2 + 2ar\ \cos\theta \quad (2)$$

We take r much greater than a ($r >> a$) and retain terms only upto the first order in a/r

$$r_1^2 = r^2\left(1 - \frac{2a\cos\theta}{r} + \frac{a^2}{r^2}\right)$$

$$\cong r^2\left(1 - \frac{2a\cos\theta}{r}\right)$$

Similarly,

$$r_2^2 \cong r^2\left(1 + \frac{2a\cos\theta}{r}\right) \qquad (3)$$

Using the Binomial theorem and retaining terms upto the first order in a/r ; we obtain,

$$\frac{1}{r_1} \cong \frac{1}{r}\left(1 - \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$$

$$\frac{1}{r_2} \cong \frac{1}{r}\left(1 + \frac{2a\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right) \qquad (4)$$

By equation (1) and (4) and using p=2qa, we get

$$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

Now, $p\cos\theta = \mathbf{p}\cdot\hat{\mathbf{r}}$

where $\hat{\mathbf{r}}$ is the unit vector along the position vector **OP**.

The electric potential of a dipole is then given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}; \qquad (r >> a)$$

Special cases

(i) when point P lies on the axis of dipole, then θ = 0

$$\cos\theta = 1$$

$$V = \frac{1}{4\pi\varepsilon}\frac{p}{r^2}$$

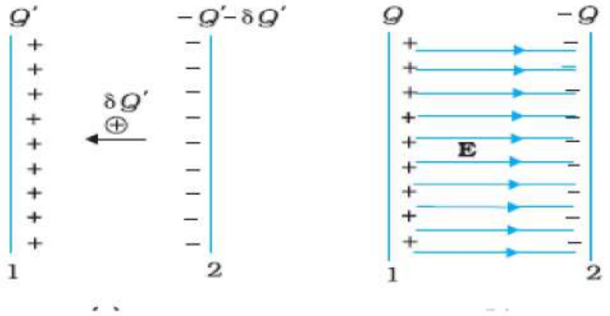
(ii) when point P lies on the equipotential plane of the dipole,then

$$\cos\theta = \cos 90$$

$$v = 0$$

14.Derive an expression for the energy stored in a parallel capacitor C, charged to a potential difference V. Hence derive an expression for the energy density of a capacitor.

Ans



charge $\delta Q'$ is transferred from conductor 2 to 1. Work done in this step ($\delta W'$), resulting in charge Q' on conductor 1 increasing to $Q' + \delta Q'$, is given by

$$\delta W = V'\delta Q' = \frac{Q'}{C}\delta Q'$$

Since $\delta Q'$ can be made as small as we like,

$$\delta W = \frac{1}{2C}[(Q' + \delta Q')^2 - Q'^2]$$

total work done (W) is the sum of the small work (δW) over the very large number of steps involved in building the charge Q' from zero to Q.

$$W = \sum_{\text{sum over all steps}} \delta W$$

$$= \sum_{\text{sum over all steps}} \frac{1}{2C}[(Q' + \delta Q')^2 - Q'^2]$$

$$= \frac{1}{2C}[\{\delta Q'^2 - 0\} + \{(2\delta Q')^2 - \delta Q'^2\} + \{(3\delta Q')^2 - (2\delta Q')^2\} + \ldots$$

$$+ \{Q^2 - (Q - \delta Q)^2\}]$$

$$= \frac{1}{2C}[Q^2 - 0] = \frac{Q^2}{2C}$$

$$W = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Energy stored in the capacitor

$$= \frac{1}{2}\frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\varepsilon_0 A}$$

The surface charge density σ is related to the electric field E between the plates,

$$E = \frac{\sigma}{\varepsilon_0}$$

Energy stored in the capacitor

$$U = (1/2)\varepsilon_0 E^2 \times A d$$

As we know that energy density is defined as energy per unit volume.

Energy density of electric field,

$$u = (1/2)\varepsilon_0 E^2$$