Mathematics
Class : Plus Two

CHAPTER 13

PROBABILITY

Worksheet-1

13.2. Conditional Probability

Let E and F be two events associated with the same sample space of a random experiment. Then the probability of occurrence of the event E given that event F has already occurred is called the conditional propability of E given F

It is denoted of by P(E|F)

$$P(E/F)=\frac{P(E\cap F)}{P(F)}, \text{ provided, } P(F)\neq 0$$

Properties of Conditional Probability

(i) Let E and F be events of a sample space S of an experiment

Then $P(S|F)=P(F|F)=1$

(ii) If A and B are two events of a sample space S and F is an event of S such that $P(F)\neq 0$

Then $P((A\cup B)|F)=P(A|F)+P(B|F)-P((A\cap B)|F)$

In particular, if A and B are disjoint events,

Then $P((A\cup B)|F)=P(A|F)+P(B|F)$

(iii) $P(E'|F)=1-P(E|F)$

Activity -1

A family has 2 children. What is the probability that both the children are boys given that atleast one of them is a boy.

Ans: Let 'b' stands for boy and 'g' for girl.

Sample space, $S=\{(b,b),(b,g),.........,..........\}$

Let E and F denote the following events.

E : both the children are boys

F : atleast one of the children is boy.

Then $E = \{(b,b)\}$

$F = \{(b,b), ..., ...\}$

$E \cap F = ...$

$P(F) = \frac{3}{4}$ and $P(E \cap F) = \frac{1}{4}$

$\therefore$ P (both the children are boys given that atleast one of them is a boy)

$= P(E \mid F)$

$= \dfrac{P(E \cap F)}{P(F)}$

$= ....$

$= ....$

Activity -2

Evaluate $P(A \cup B)$, if $2P(A) = P(B) = \dfrac{5}{13}$ and $P(A/B) = \dfrac{2}{5}$

Ans: Given that $2P(A) = P(B) = \dfrac{1}{13}$

Then $2P(A) = \dfrac{5}{13}$ and $P(B) = \dfrac{5}{13}$

i.e. $P(A) = ...$

We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$...............(1)

$P(B) = ...$

$P(A \mid B) = \dfrac{2}{5}$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$\therefore P(A \cap B) = P(B).P(A \mid B)$

$= ....$

(1) becomes

$$P(A \cup B) = ... + \frac{5}{13} - .....$$

$$= ....$$

Activity :3

An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questins, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question.

Ans: Let E and F be the events,
E: getting an easy question
F: getting a multiple choice question.

Total no. of question = ...+........................+...................+...................

=1400

$$N(F) = 500 + 400 = 900$$

$N(E \cap F) = 500$ (easy & multiple choice questions)

$$P(E \cap F) = \frac{500}{---} = \frac{5}{14}$$

$$P(F) = \frac{-}{1400} = ------$$

Required probability,

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$=$$

$$=$$

Activity-4

A black and a red dice are rolled.

(i) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(ii) Find the conditinal probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Ans: (i) Here, the sample space 'S' has---- elements
E and F be events,
E: getting a sum greater than 9
F: black die resulted in 5
We have to find $P(E \mid F)$

$$E = \{(4,6),(5,5),(5,6),(6,4),(6,5),(6,6)\}$$
$$F = \{(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\}$$
$$E \cap F = \{\ldots\ldots,\ldots\ldots\}$$

$$P(E \cap F) = \frac{\cdots}{36} = \ldots\ldots\ldots\ldots\ldots\ldots$$
$$P(F) = \ldots\ldots\ldots\ldots\ldots\ldots$$

Required probabiltiy,

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$
$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

(ii) E: getting sum 8
F: red die resulted in a number less than 4

$$E = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$$
$$F = \{(1,1),(1,2),(1,3),(2,1),(2,2),\ldots\ldots..(6,3)\}$$ (write all outcomes)

$$E \cap F = \{5,3),(6,2)\}$$
$$P(E \cap F) = \ldots\ldots\ldots\ldots$$
$$P(F) = \frac{18}{36}$$
$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$
$$= \ldots\ldots\ldots = \frac{1}{9}$$

Activity 5

If $P(A) = \frac{6}{11}, P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$ .Find

(i) $P(A \cap B)$

(ii) $P(A \mid B)$

(iii) $P(B \mid A)$

Ans: (i) Given $P(A)=\frac{6}{11}$

$$P(B)=\frac{5}{11}$$

$$P(A\cup B)=\frac{7}{11}$$

$$\frac{6}{11}+\frac{5}{11}-P(A\cap B)=\frac{7}{11}$$

$$P(A\cap B)=\frac{6}{11}+\frac{5}{11}-\text{............}$$

$$\therefore\ P(A\cap B)=\frac{4}{11}$$

(ii) $$P(B|A)=\frac{P(A\cap B)}{P(B)}$$

$$=\text{..........}$$

$$=\text{..........}$$

(iii) $$P(B|A)=\frac{P(A\cap B)}{P(A)}$$

$$=\text{..........}$$

$$=\text{..........}$$

Worksheet II

13.4 Independent Events

Two events E and F are said to be independent if the probability of occurrence of any one of them is not affected by the occurrence of the other.

In this case, $P\left(E \mid F\right) = P(E), h\text{ere } P(F) \neq 0$

$$P\left(F \mid E\right) = P(F), \text{here } P(E) \neq 0$$

Thus, if E and F are independent events,

$$P(E \cap F) = P(E).P(F).$$

* The term 'independent' is defined in terms of 'probability of events' where as 'mutually exclusive' is defined in tems of events (subsets of sample space).
* Independent events may have common outcome
* Mutually exclusive ents never have a common outcome
* Two mutually exclusive events having non-zero probabilities of occurrence cannot be independent
* Three events A, B and C are said to be mutually independent, if

$$P(A \cap B) = P(A).P(B)$$
$$P(A \cap C) = P(A).P(C)$$
$$P(B \cap C) = P(B).P(C)$$
$$P(A \cap B \cap C) = P(A).P(B).P(C)$$

* If the events E & F are independent, then

  (a) E`and F are independent

  (b) E and F` are independent

  (c) E` and F` are independent

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Activity -1

A die is thrown E and F are events such that
E: the number appearing is a multiple of 3
F: the number appearing is even.
Find whether E and F are independent.

Ans: Here sample space $S = \{1, 2, 3, 4, 5, 6\}$

Now $E = \{3, 6\}$

$F = \{\_\_\_\_, \_\_\_\_, \_\_\_\_\}$
$E \cap F = \{6\}$

$$P(F) = \frac{3}{6} = \frac{1}{2}$$

$P(E \cap F) = \_\_\_\_\_\_$ ....(1)

$P(E).P(F) = \frac{1}{3}.\frac{1}{2} = \frac{1}{6}$ ....(2)

from (1) & (2)

$\therefore E \quad and \quad F$ are independent

Activity -2

Three coins are tossed simultaneously.
E, F & G are events such that
E: three heads or three tails
F: alterast 2 heads
G: atmost 2 heads.
Of the pairts (E,F), (E,G) and (F,G), which are independent, which are dependent?

Ans: Here Sample space $S = \{HHH, HHT, HTH, THH, HTT, TTH, THT, TTT\}$

$$E = \{HHH, TTT\}$$
$$F = \{HHH, HHT, HTH, THH\}$$
$$G = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

$E \cap F = \_\_\_\_\_\_\_\_\_\_$
$E \cap G = \_\_\_\_\_\_\_\_\_\_$
$F \cap G = \_\_\_\_\_\_\_\_\_\_$

$$P(E)=\frac{2}{8}=\frac{1}{4},\ P(F)=\frac{4}{8}=\frac{1}{2},\ P(G)=\frac{7}{8}$$

$$P(E\cap F)=\frac{1}{8}$$

$$P(E\cap G)=\_\_\_\_\_\_\_$$

$$P(F\cap G)=\_\_\_\_\_\_\_\_\_\_$$

$$P(F).P(G)=\_\_\_\_\_\_\_\_\_\_$$

$$P(E).P(F)=\frac{1}{4}.\frac{1}{2}=\frac{1}{8}=P(F\cap E)$$

Hence, the events E and F are independent events.

The pairs of events (E,G) and (F,G) are dependent.

Activity -3

Events A and B are such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and $P(\text{not A or not B})=\frac{1}{4}$ state whether A and B are independent.

Ans:

$$P(\text{not A or not B})=\frac{1}{4}$$

$$P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$$

$$P(A'\cup B')=\frac{1}{4}$$

$$\Rightarrow P(A\cap B)'=\frac{1}{4}$$

$$\Rightarrow 1-\_\_\_\_\_\_\_\_=\frac{1}{4}$$

$$\Rightarrow P(A\cap B)=1-\frac{1}{4}=\frac{3}{4}$$

Now, P(A). P(B) =____________×____________

= ....

$\therefore$ A and B are not indepedent

Activity -4

(1) The probability of solving a problem independently by A and B are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability that exactly one of them solves the problem

(2) A and B try to solve a problem independently.

The probability that A solves a problem is $\frac{1}{2}$ and that B solves the problem is $\frac{1}{3}$.

Find the probability that

(a) Both of them solves the problem.

(b) Problem is solved.

Ans:

(1) $P(A) = \frac{1}{3}$ $P(B) = \frac{1}{4}$

$$P(A^1) = 1 - P(A) = \_\_\_\_\_\_\_\_\_\_$$
$$P(B^1) = 1 - P(B) = \_\_\_\_\_\_\_\_\_\_$$

Probability of .... one of them solves the problem

$$= P(A).P(B^1) + P(B).P(A^1)$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_ = \frac{5}{12}$$

(2) Let A: problem is solved by A

B: Problem is solved by B

$$P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$$

(a) P (both solve the problem) $= P(A \cap B)$

$$= P(A).P(B) = \_\_\_\_\_\_\_\_ = \frac{1}{6}$$

(b) P(problem is solved) $= 1 - P(\text{Problem not solved})$

$$= 1 - P(A' \cap B')$$
$$= 1 - P(A^1)(B^1)$$
$$= 1 - \ldots\ldots\ldots\ldots\ldots$$
$$= \frac{2}{3}$$

$P(A^1) =$ ______
$P(B^1) =$ ________

Activity -6

Rani and Joy appear in an interview for 2 vacancies in the same post. The probability of Rani's selection is $\frac{1}{7}$ and that of Joy's selection is $\frac{1}{5}$. What is the probability that

(a) Rani will not be selected

(b) Both of them will be selected

(c) None of them will be selected

Ans:

Let Rani's selection be the event A & Joy's selection be the event B.

$$P(A) = \frac{1}{7}, P(B) = \frac{1}{5}$$

(a) P(Rani will not be selected) $= P(A^1)$

$$= 1 - P(A)$$
$$= \_\_\_\_\_\_\_$$
$$= \frac{6}{7}$$

(b) P(Both of them will be selected)

$$= P(A \cap B) = P(A).P(B)$$
$$= \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$
$$= \frac{1}{35}$$

(iii) P(None of them will be selected)

$$= P(A' \cap B')$$
$$= P(A').P(B')$$
$$= \ldots\ldots \times \ldots\ldots\ldots\ldots$$
$$= \ldots\ldots\ldots\ldots\ldots$$