

Chapter : 7. TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES

Exercise : 7

Question: 1 A

Without using tri

Solution:

$$\frac{\sin 16^\circ}{\cos 74^\circ} = \frac{\sin 16^\circ}{\cos(90-16)^\circ} = \frac{\sin 16^\circ}{\sin 16^\circ} = 1$$

($\because \cos(90-\theta) = \sin \theta$ and $(90-\theta)$ lies in the first quadrant where all the angles are taken as positive.)

Question: 1 B

Solve

Solution:

$$\frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} = \frac{1/\cos 11^\circ}{1/\sin 79^\circ} = \frac{\sin 79^\circ}{\cos 11^\circ} = \frac{\sin 79^\circ}{\cos(90-79)^\circ} = \frac{\sin 79^\circ}{\sin 79^\circ} = 1$$

Question: 1 C

Solve

Solution:

$$\frac{\tan 27^\circ}{\cot 63^\circ} = \frac{\tan 27^\circ}{\cot(90-27)^\circ} = \frac{\tan 27^\circ}{\tan 27^\circ} = 1 \quad (\because \cot(90-\theta) = \tan \theta)$$

Question: 1 D

Solve

Solution:

$$\frac{\cos 35^\circ}{\sin 55^\circ} = \frac{\cos 35^\circ}{\sin(90-35)^\circ} = \frac{\cos 35^\circ}{\cos 35^\circ} = 1$$

Question: 1 E

Solve

Solution:

$$\frac{\operatorname{cosec} 42^\circ}{\sec 48^\circ} = \frac{1/\sin 42^\circ}{1/\cos 48^\circ} = \frac{\cos 48^\circ}{\sin 42^\circ} = \frac{\cos 48^\circ}{\sin(90-48)^\circ} = \frac{\cos 48^\circ}{\sin 48^\circ} = 1$$

Question: 1 F

Solve

Solution:

$$\frac{\cot 38^\circ}{\tan 52^\circ} = \frac{\cot 38^\circ}{\tan(90-38)^\circ} = \frac{\cot 38^\circ}{\cot 38^\circ} = 1 \quad (\because \tan(90-\theta) = \cot \theta)$$

Question: 2 A

Without using tri

Solution:

$$\begin{aligned} \text{Consider } \cos 81^\circ - \sin 9^\circ &= \cos 81^\circ - \sin(90 - 81)^\circ \\ &= \cos 81^\circ - \cos 81^\circ \\ &= 0 \end{aligned}$$

Hence, proved.

Question: 2 B

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \tan 71^\circ - \cot 19^\circ &= \tan 71^\circ - \cot (90 - 71)^\circ \\ &= \tan 71^\circ - \tan 71^\circ \\ &= 0\end{aligned}$$

Hence, proved.

Question: 2 C

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \operatorname{cosec} 80^\circ - \sec 10^\circ &= \operatorname{cosec} 80^\circ - \sec (90 - 10)^\circ \\ &= \operatorname{cosec} 80^\circ - \operatorname{cosec} 80^\circ \\ &= 0\end{aligned}$$

Hence, proved.

Question: 2 D

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \operatorname{cosec}^2 72^\circ - \tan^2 18^\circ &= \operatorname{cosec}^2 72^\circ - \tan^2 (90 - 72)^\circ \\ &= \operatorname{cosec}^2 72^\circ - \cot^2 72^\circ \\ &= 1\end{aligned}$$

$$(\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta = \operatorname{cosec}^2 \theta - \cot^2 \theta = 1)$$

Hence, proved.

Question: 2 E

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \cos^2 75^\circ + \cos^2 15^\circ &= \cos^2 75^\circ + \cos^2 (90 - 75)^\circ \\ &= \cos^2 75^\circ + \sin^2 75^\circ \\ &= 1\end{aligned}$$

$$(\because \cos^2 \theta + \sin^2 \theta = 1)$$

Hence, proved.

Question: 2 F

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \tan^2 66^\circ - \cot^2 24^\circ &= \tan^2 66^\circ - \cot^2 (90 - 66)^\circ \\ &= \tan^2 66^\circ - \tan^2 66^\circ \\ &= 0\end{aligned}$$

Hence, proved.

Question: 2 G

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \sin^2 48^\circ + \sin^2 42^\circ &= \sin^2 48^\circ + \sin^2 (90 - 48)^\circ \\ &= \sin^2 48^\circ + \cos^2 48^\circ \\ &= 1\end{aligned}$$

$$(\because \cos^2 \theta + \sin^2 \theta = 1)$$

Hence, proved.

Question: 2 H

Without using tri

Solution:

$$\begin{aligned}\text{Consider } \cos^2 57^\circ - \sin^2 33^\circ &= \cos^2 57^\circ - \sin^2 (90 - 57)^\circ \\ &= \cos^2 57^\circ - \cos^2 57^\circ \\ &= 0\end{aligned}$$

Hence, proved.

Question: 2 I

Without using tri

Solution:

$$\begin{aligned}\text{Consider } (\sin 65^\circ + \cos 25^\circ)(\sin 65^\circ - \cos 25^\circ) \\ &= \sin^2 65^\circ - \cos^2 25^\circ \\ &= \sin^2 65^\circ - \cos^2 (90 - 65)^\circ \\ &= \sin^2 65^\circ - \sin^2 65^\circ \\ &= 0\end{aligned}$$

Hence, proved.

Question: 3 A

Without using tri

Solution:

$$\begin{aligned}\text{Consider } (\sin 53^\circ \cos 37^\circ) + (\cos 53^\circ \sin 37^\circ) \\ &= (\sin 53^\circ \cos (90-53)^\circ) + (\cos 53^\circ \sin (90-53)^\circ) \\ &= \sin^2 53^\circ + \cos^2 53^\circ \\ &= 1\end{aligned}$$

Hence, proved.

Question: 3 B

$\cos 54^\circ \cos 36^\circ -$

Solution:

$$\begin{aligned}\text{Consider } (\cos 54^\circ \cos 36^\circ) - (\sin 54^\circ \sin 36^\circ) \\ &= (\cos 54^\circ \cos (90-54)^\circ) - (\sin 54^\circ \sin (90-54)^\circ) \\ &= (\cos 54^\circ \sin 54^\circ) - (\sin 54^\circ \cos 54^\circ)\end{aligned}$$

$$= 0$$

Hence, proved.

Question: 3 C

$$\sec 70^\circ \sin 20^\circ +$$

Solution:

Consider L.H.S.

$$\begin{aligned} &= (\sec 70^\circ \sin 20^\circ) + (\cos 20^\circ \operatorname{cosec} 70^\circ) \\ &= (\sec (90-20)^\circ \sin 20^\circ) + (\cos 20^\circ \operatorname{cosec} (90-20)^\circ) \\ &= (\operatorname{cosec} 70^\circ \sin 70^\circ) + (\cos 20^\circ \sec 20^\circ) \\ &= 1 + 1 \quad (\because \operatorname{cosec} \theta = 1/\sin \theta \text{ and } \sec \theta = 1/\cos \theta) \\ &= 2 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 3 D

$$\sin 35^\circ \sin 55^\circ -$$

Solution:

$$\begin{aligned} \text{Consider L.H.S.} &= (\sin 35^\circ \sin 55^\circ) - (\cos 35^\circ \cos 55^\circ) \\ &= (\sin 35^\circ \sin (90-35)^\circ) - (\cos 35^\circ \cos (90-35)^\circ) \\ &= (\sin 35^\circ \cos 35^\circ) - (\cos 35^\circ \sin 35^\circ) \\ &= 0 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 3 E

$$(\sin 72^\circ + \cos 18^\circ)$$

Solution:

$$\begin{aligned} \text{Consider } &(\sin 72^\circ + \cos 18^\circ)(\sin 72^\circ - \cos 18^\circ) \\ &= \sin^2 72^\circ - \cos^2 18^\circ \\ &= \sin^2 72^\circ - \cos^2 (90-72)^\circ \\ &= \sin^2 72^\circ - \sin^2 72^\circ \\ &[\text{since, } \cos(90 - \theta) = \sin \theta] \\ &= 0 \end{aligned}$$

Hence, proved.

Question: 3 F

$$\tan 48^\circ \tan 23^\circ \tan$$

Solution:

Consider L.H.S.

$$\begin{aligned} &= \tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ \\ &= \tan 48^\circ \tan 42^\circ \tan 23^\circ \tan 67^\circ \\ &= \tan 48^\circ \tan (90-48)^\circ \tan 23^\circ \tan (90-23)^\circ \\ &= (\tan 48^\circ \cot 48^\circ) (\tan 23^\circ \cot 23^\circ) \\ &= (1) \times (1) = 1 = \text{R.H.S} \end{aligned}$$

Hence, proved.

Question: 4 A

Prove that:

Solution:

Consider L.H.S

$$\begin{aligned} &= \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ \\ &= \frac{\sin 70^\circ}{\cos(90-70)^\circ} + \frac{\operatorname{cosec}(90-70)^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec}(90-70)^\circ \\ &= \frac{\sin 70^\circ}{\sin 70^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \sec 70^\circ \\ &= 1 + 1 - 2 = 0 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 4 B

Prove

Solution:

Consider L.H.S.

$$\begin{aligned} &= \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ \\ &= \frac{\cos 80^\circ}{\sin(90-80)^\circ} + \cos 59^\circ \operatorname{cosec}(90-59)^\circ \\ &= \frac{\cos 80^\circ}{\cos 80^\circ} + \cos 59^\circ \sec 59^\circ \\ &= 1 + 1 = 2 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 4 C

Prove

Solution:

Consider L.H.S.

$$\begin{aligned} &= \frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5} \\ &= \frac{2 \sin 68^\circ}{\cos(90-68)^\circ} - \frac{2 \cot 15^\circ}{5 \tan(90-15)^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan(90-50)^\circ \tan(90-20)^\circ}{5} \\ &= \frac{2 \sin 68^\circ}{\sin 68^\circ} - \frac{2 \cot 15^\circ}{5 \cot 15^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \cot 40^\circ \cot 20^\circ}{5} \\ &= 2 - (2/5) - 3(1 \times 1 \times 1)/5 \\ &= 2 - (2/5) - (3/5) \\ &= (10 - 2 - 3)/5 \\ &= 5/5 \\ &= 1 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 4 D

Prove

Solution:

To prove: $\frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ) = 2$ **Proof:** Consider L.H.S.

$= \frac{\sin 18^\circ}{\cos 72^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan 50^\circ \tan 80^\circ)$ Now look for the pairs whose angles give sum of 90° . Here the sum of angles of $\tan 10^\circ$ and $\tan 80^\circ$ gives 90° . Also the sum of angles of $\tan 40^\circ$ and $\tan 50^\circ$ gives 90° . So, now change $\tan 80^\circ$ into $\tan(90-10^\circ)$ and $\tan 50^\circ$ into $\tan(90-40^\circ)$

$= \frac{\sin 18^\circ}{\cos(90-18)^\circ} + \sqrt{3}(\tan 10^\circ \tan 30^\circ \tan 40^\circ \tan(90-40)^\circ \tan(90-10)^\circ)$ We know $\tan(90-\theta) = \cot \theta$ $\tan 30^\circ = 1/\sqrt{3}$

$= \frac{\sin 18^\circ}{\sin 18^\circ} + \sqrt{3}\left(\tan 10^\circ \times \frac{1}{\sqrt{3}} \times \tan 40^\circ \cot 40^\circ \cot 10^\circ\right)$

$= 1 + \sqrt{3}\left(\tan 10^\circ \cot 10^\circ \times \frac{1}{\sqrt{3}} \times \tan 40^\circ \cot 40^\circ\right)$ Since $\tan \theta = 1/\cot \theta$

$= 1 + \sqrt{3}\left(1 \times \frac{1}{\sqrt{3}} \times 1\right)$

$= 1 + 1 = 2 = \text{R.H.S.}$

Hence, proved. **Note:** In such questions take the pairs whose angles give sum of 90° , change one of them in the form of $90-\theta$ and substitute remaining known values. Like in this case $\tan 50^\circ$ is changed into $\tan(90-40)^\circ$ so that it will can be written as $\cot 50^\circ$ and $\tan 50^\circ \times \cot 50^\circ$ gives us value 1.

Question: 4 E

Prove

Solution:

Consider L.H.S.

$= \frac{7 \cos 55^\circ}{3 \sin 35^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec} 20^\circ)}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ)}$

$= \frac{7 \cos 55^\circ}{3 \sin(90-55)^\circ} - \frac{4(\cos 70^\circ \operatorname{cosec}(90-70)^\circ)}{3(\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan(90-25)^\circ \tan(90-5)^\circ)}$

$= \frac{7 \cos 55^\circ}{3 \cos 55^\circ} - \frac{4(\cos 70^\circ \sec 70^\circ)}{3(\tan 5^\circ \tan 25^\circ \times 1 \times \cot 25^\circ \cot 5^\circ)}$

$= \frac{7 \cos 55^\circ}{3 \cos 55^\circ} - \frac{4(\cos 70^\circ \sec 70^\circ)}{3(\tan 5^\circ \cot 5^\circ \times 1 \times \tan 25^\circ \cot 25^\circ)}$

$= (7/3) - (4/3) = 43/3 = 1 = \text{R.H.S.}$

Hence, proved.

Question: 5 A

Prove that:

Solution:

Consider L.H.S.

$= \sin \theta \cos(90^\circ - \theta) + \sin(90^\circ - \theta) \cos \theta$

$$= \sin \theta \sin \theta + \cos \theta \cos \theta$$

$$= \sin^2 \theta + \cos^2 \theta$$

$$= 1 = \text{R.H.S.}$$

Hence, proved.

Question: 5 B

Prove

Solution:

Consider L.H.S.

$$= \frac{\sin \theta}{\cos(90^\circ - \theta)} + \frac{\cos \theta}{\sin(90^\circ - \theta)}$$

$$= \frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\cos \theta}$$

$$= 1 + 1 = 2 = \text{R.H.S.}$$

Hence, proved.

Question: 5 C

Prove

Solution:

Consider L.H.S.

$$= \frac{\sin \theta \cos(90^\circ - \theta) \cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos \theta \sin(90^\circ - \theta) \sin \theta}{\cos(90^\circ - \theta)}$$

$$= \frac{\sin \theta \sin \theta \cos \theta}{\cos \theta} + \frac{\cos \theta \cos \theta \sin \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta \cos \theta}{\cos \theta} + \frac{\cos^2 \theta \sin \theta}{\sin \theta}$$

$$= \frac{\sin^2 \theta \cos \theta + \cos^2 \theta \sin \theta}{\cos \theta \sin \theta}$$

$$= \frac{\cos \theta \sin \theta (\cos^2 \theta + \sin^2 \theta)}{\cos \theta \sin \theta}$$

$$= \frac{\cos \theta \sin \theta (1)}{\cos \theta \sin \theta} = 1 = \text{R.H.S.}$$

Hence, proved.

Question: 5 D

Prove

Solution:

Consider L.H.S.

$$= \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta}$$

$$= \frac{(\sin \theta \operatorname{cosec} \theta) \tan \theta}{(\sec \theta \cos \theta) \tan \theta} + \frac{\cot \theta}{\cot \theta}$$

$$= \frac{1 \times \tan \theta}{1 \times \tan \theta} + 1 = 1 + 1 = 2 = \text{R.H.S.}$$

Hence, proved.

Question: 5 E

Prove

Solution:

Consider L.H.S.

$$\begin{aligned} &= \frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} \\ &= \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)} \\ &= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{(\sin^2 \theta + \cos^2 \theta) + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} \\ &= \frac{1 + 1 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} \\ &= \frac{2}{\sin \theta} = 2 \operatorname{cosec} \theta = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 5 F

Prove

Solution:

Consider L.H.S.

$$\begin{aligned} &= \frac{\sec(90 - \theta) \operatorname{cosec} \theta - \tan(90 - \theta) \cot \theta + \cos^2 25^\circ + \cos^2 65^\circ}{3 \tan 27^\circ \tan 63^\circ} \\ &= \frac{\operatorname{cosec} \theta \operatorname{cosec} \theta - \cot \theta \cot \theta + \cos^2 25^\circ + \cos^2 (90 - 25)^\circ}{3 \tan 27^\circ \tan (90 - 27)^\circ} \\ &= \frac{(\operatorname{cosec}^2 \theta - \cot^2 \theta) + (\cos^2 25^\circ + \sin^2 25^\circ)}{3 \tan 27^\circ \cot 27^\circ} \\ &= \frac{1 + 1}{3 \times 1} = 2/3 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 5 G

$\cot \theta \tan (90^\circ - \theta)$

Solution:

Consider L.H.S.

$$\begin{aligned} &= \cot \theta \tan (90^\circ - \theta) - \sec (90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12^\circ \tan 60^\circ \tan 78^\circ \\ &= \cot \theta \cot \theta - \operatorname{cosec} \theta \operatorname{cosec} \theta + \sqrt{3} \tan 60^\circ \tan 12^\circ \tan 78^\circ \\ &= \cot^2 \theta - \operatorname{cosec}^2 \theta + \sqrt{3} \tan 60^\circ \tan 12^\circ \tan (90 - 12)^\circ \end{aligned}$$

$$\begin{aligned}
&= -(\operatorname{cosec}^2 \theta - \cot^2 \theta) + \sqrt{3} \tan 60^\circ \tan 12^\circ \cot 12^\circ \\
&= -1 + \sqrt{3}(\sqrt{3} \times \tan 12^\circ \cot 12^\circ) \\
&= -1 + \sqrt{3}(\sqrt{3} \times 1) \\
&= -1 + 3 \\
&= 2 = \text{R.H.S.}
\end{aligned}$$

Hence, proved.

Question: 6 A

Prove that:

Solution:

$$\begin{aligned}
&\text{Consider L.H.S.} \\
&= \tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 65^\circ \tan 85^\circ \\
&= \tan 5^\circ \tan 85^\circ \tan 25^\circ \tan 65^\circ \tan 30^\circ \\
&= \tan 5^\circ \tan (90-5)^\circ \tan 25^\circ \tan (90-25)^\circ \tan 30^\circ \\
&= (\tan 5^\circ \cot 5^\circ) (\tan 25^\circ \cot 25^\circ) \tan 30^\circ \\
&= 1 \times 1 \times (1/\sqrt{3}) \\
&= 1/\sqrt{3} = \text{R.H.S.}
\end{aligned}$$

Hence, proved.

Question: 6 B

$$\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ$$

Solution:

$$\begin{aligned}
&\text{Consider L.H.S.} \\
&= \cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 60^\circ \cot 78^\circ \\
&= (\cot 12^\circ \cot 78^\circ) (\cot 38^\circ \cot 52^\circ) \cot 60^\circ \\
&= (\cot 12^\circ \cot (90-12)^\circ) (\cot 38^\circ \cot (90-38)^\circ) \cot 60^\circ \\
&= (\cot 12^\circ \tan 12^\circ) (\cot 38^\circ \tan 38^\circ) \cot 60^\circ \\
&= 1 \times 1 \times (1/\sqrt{3}) \\
&= 1/\sqrt{3} = \text{R.H.S.}
\end{aligned}$$

Hence, proved.

Question: 6 C

$$\cos 15^\circ \cos 35^\circ \cos 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ$$

Solution:

$$\begin{aligned}
&\text{Consider L.H.S.} = \cos 15^\circ \cos 35^\circ \cos 55^\circ \cos 60^\circ \operatorname{cosec} 75^\circ \\
&= \cos 15^\circ \operatorname{cosec} 75^\circ \cos 35^\circ \cos 55^\circ \cos 60^\circ \\
&= \cos 15^\circ \operatorname{cosec} (90-15)^\circ \cos 35^\circ \operatorname{cosec} (90-35)^\circ \cos 60^\circ \\
&= (\cos 15^\circ \sec 15^\circ) \times (\cos 35^\circ \sec 35^\circ) \times \cos 60^\circ \\
&= (1) \times (1) \times (1/2) \\
&= 1/2 = \text{R.H.S.}
\end{aligned}$$

Hence, proved.

Question: 6 D

$$\cos 1^\circ \cos 2^\circ \cos$$

Solution:

$$\text{Consider L.H.S.} = \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 180^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \times \cos 90^\circ \times \dots \cos 180^\circ$$

$$= \cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \times 0 \times \cos 180^\circ$$

$$= 0 \quad (\because \cos 90^\circ = 0)$$

Hence, proved.

Question: 6 E

Prove

Solution:

Consider L.H.S.

$$= \left(\frac{\sin 49^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ} \right)^2$$

$$= \left(\frac{\sin (90 - 41)^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\sin (90 - 41)^\circ} \right)^2$$

$$= \left(\frac{\cos 41^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\cos 41^\circ} \right)^2$$

$$= 1^2 + 1^2$$

$$1 + 1 = 2 = \text{R.H.S.}$$

Hence, proved.

Question: 7 A

Prove that:

Solution:

Consider L.H.S.

$$= \sin (70^\circ + \theta) - \cos (20^\circ - \theta)$$

$$= \sin (70^\circ + \theta) - \cos [90^\circ - (70^\circ + \theta)]$$

$$= \sin (70^\circ + \theta) - \sin (70^\circ + \theta)$$

$$= 0 = \text{R.H.S.}$$

Hence, proved.

Question: 7 B

$$\tan (55^\circ - \theta) - \cot$$

Solution:

Consider L.H.S.

$$= \tan (55^\circ - \theta) - \cot (35^\circ + \theta)$$

$$= \tan (90^\circ - (35^\circ + \theta)) - \cot (35^\circ + \theta)$$

$$= \cot (35^\circ + \theta) - \cot (35^\circ + \theta)$$

$$= 0$$

Hence, proved.

Question: 7 C

$$\operatorname{cosec} (67^\circ + \theta) -$$

Solution:

Consider L.H.S.

$$\begin{aligned} &= \operatorname{cosec} (67^\circ + \theta) - \sec (23^\circ - \theta) \\ &= \operatorname{cosec} (67^\circ + \theta) - \sec (90^\circ - (23^\circ + \theta)) \\ &= \operatorname{cosec} (67^\circ + \theta) - \operatorname{cosec} (67^\circ + \theta) \\ &= 0 \end{aligned}$$

Hence, proved.

Question: 7 D

$$\operatorname{cosec} (65^\circ + \theta) -$$

Solution:

Consider L.H.S.

$$\begin{aligned} &= \operatorname{cosec} (65^\circ + \theta) - \sec (25^\circ - \theta) - \tan (55^\circ - \theta) + \cot (35^\circ + \theta) \\ &= \operatorname{cosec} (65^\circ + \theta) - \sec (90^\circ - (65^\circ + \theta)) - \tan (90^\circ - (35^\circ + \theta)) + \cot (35^\circ + \theta) \\ &= \operatorname{cosec} (65^\circ + \theta) - \operatorname{cosec} (65^\circ + \theta) - \cot (35^\circ + \theta) + \cot (35^\circ + \theta) \\ &= 0 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 7 E

$$\sin (50^\circ + \theta) - \cos$$

Solution:

Consider L.H.S.

$$\begin{aligned} &= \sin (50^\circ + \theta) - \cos (40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 80^\circ \tan 89^\circ \\ &= \sin ((90^\circ - (40^\circ - \theta))) - \cos (40^\circ - \theta) + (\tan 1^\circ \tan 89^\circ)(\tan 10^\circ \tan 80^\circ) \\ &= \cos (40^\circ - \theta) - \cos (40^\circ - \theta) + [\tan 1^\circ \tan (90^\circ - 1^\circ)][\tan 10^\circ \tan (90^\circ - 10^\circ)] \\ &= 0 + [(\tan 1^\circ \cot 1^\circ)] [\tan 10^\circ \cot 10^\circ] \\ &= 0 + [1] \times [1] \\ &= 0 + 1 = 1 = \text{R.H.S.} \end{aligned}$$

Hence, proved.

Question: 8 A

Express each of t

Solution:

$$\begin{aligned} \text{Consider } \sin 67^\circ + \cos 75^\circ &= \sin (90 - 23)^\circ + \cos (90 - 15)^\circ \\ &= \cos 23^\circ + \sin 15^\circ \end{aligned}$$

Question: 8 B

Express each of t

Solution:

$$\text{Consider } \cot 65^\circ + \tan 49^\circ = \cot (90 - 25)^\circ + \tan (90 - 41)^\circ$$

$$= \tan 25^\circ + \cot 41^\circ$$

Question: 8 C

Express each of t

Solution:

$$\text{Consider } \sec 78^\circ + \operatorname{cosec} 56^\circ = \sec (90-12)^\circ + \operatorname{cosec} (90-34)^\circ$$

$$= \operatorname{cosec} 12^\circ + \sec 34^\circ$$

Question: 8 D

Express each of t

Solution:

$$\text{Consider } \operatorname{cosec} 54^\circ + \sin 72^\circ = \operatorname{cosec} (90-36)^\circ + \sin (90-18)^\circ$$

$$= \sec 36^\circ + \cos 18^\circ$$

Question: 9

If A, B and C are

Solution:

Since A, B and C are the angles of a triangle, therefore sum of the angles equals 180° .

$$\therefore A + B + C = 180^\circ \Rightarrow C + A = 180^\circ - B$$

$$\text{Now, consider L.H.S.} = \tan\left(\frac{C+A}{2}\right)$$

$$= \tan\left(\frac{180^\circ - B}{2}\right)$$

$$= \tan\left(90^\circ - \frac{B}{2}\right)$$

$$= \cot\left(\frac{B}{2}\right) = \text{R.H.S.}$$

Hence, proved.

Question: 10

If $\cos 2\theta$

Solution:

given: $\cos 2\theta = \sin 4\theta$ **To find:** the value of θ **Solution:** Consider $\cos 2\theta = \sin 4\theta$, Since, $\sin(90^\circ - \theta) = \cos \theta \therefore$ We can rewrite it as: $\sin(90^\circ - 2\theta) = \sin 4\theta$

On comparing both sides, we get,

$$90^\circ - 2\theta = 4\theta$$

$$\Rightarrow 90^\circ = 4\theta + 2\theta$$

$$\Rightarrow 6\theta = 90^\circ$$

$$\Rightarrow \theta = 15^\circ$$

Question: 11

If $\sec 2A = \operatorname{cosec}$

Solution:

We are given that: $\sec 2A = \operatorname{cosec} (A - 42^\circ)$

$$\therefore \text{We can rewrite it as: } \operatorname{cosec} (90^\circ - 2A) = \operatorname{cosec} (A - 42^\circ)$$

On comparing both sides, we get,

$$90^\circ - 2A = A - 42^\circ$$

$$\Rightarrow A + 2A = 90^\circ + 42^\circ$$

$$\Rightarrow 3A = 132^\circ$$

$$\Rightarrow A = 44^\circ$$

Question: 12

If $\sin 3A = \cos ($

Solution:

We are given that: $\sin 3A = \cos (A - 26^\circ)$

$\therefore$ We can rewrite it as: $\cos (90^\circ - 3A) = \cos (A - 26^\circ)$

On comparing both sides, we get,

$$90^\circ - 3A = A - 26^\circ$$

$$\Rightarrow A + 3A = 90^\circ + 26^\circ$$

$$\Rightarrow 4A = 116^\circ$$

$$\Rightarrow A = 29^\circ$$

Question: 13

If $\tan 2A = \cot ($

Solution:

We are given that: $\tan 2A = \cot (A - 12^\circ)$

$\therefore$ We can rewrite it as: $\cot (90^\circ - 2A) = \cot (A - 12^\circ)$

On comparing both sides, we get,

$$90^\circ - 2A = A - 12^\circ$$

$$\Rightarrow A + 2A = 90^\circ + 12^\circ$$

$$\Rightarrow 3A = 102^\circ$$

$$\Rightarrow A = 34^\circ$$

Question: 14

If $\sec 4A = \operatorname{cosec}$

Solution:

We are given that: $\sec 4A = \operatorname{cosec} (A - 15^\circ)$

$\therefore$ We can rewrite it as: $\operatorname{cosec} (90^\circ - 4A) = \operatorname{cosec} (A - 15^\circ)$

On comparing both sides, we get,

$$90^\circ - 4A = A - 15^\circ$$

$$\Rightarrow A + 4A = 90^\circ + 15^\circ$$

$$\Rightarrow 5A = 105^\circ$$

$$\Rightarrow A = 21^\circ$$

Question: 15

Prove that:

Solution:

Consider L.H.S.

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \tan 32^\circ - \frac{5}{3} \tan 13^\circ \tan 37^\circ \tan 45^\circ \tan 53^\circ \tan 77^\circ$$

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \tan (90-58)^\circ - \frac{5}{3} \times [\tan 13^\circ \tan 77^\circ] \times [\tan 37^\circ \tan 53^\circ] \times \tan 45^\circ$$

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot 58^\circ \cot 58^\circ - \frac{5}{3} \times [\tan 13^\circ \tan (90-13)^\circ] \times [\tan 37^\circ \tan (90-37)^\circ] \times 1$$

$$= \frac{2}{3} \operatorname{cosec}^2 58^\circ - \frac{2}{3} \cot^2 58^\circ - \frac{5}{3} \times [\tan 13^\circ \cot 13^\circ] \times [\tan 37^\circ \cot 37^\circ] \times 1$$

$$= \frac{2}{3} [\operatorname{cosec}^2 58^\circ - \cot^2 58^\circ] - \frac{5}{3} \times [1] \times [1] \times 1$$

$$= \frac{2}{3} [1] - \frac{5}{3}$$

$$= (2/3) - (5/3)$$

$$= (2 - 5)/3$$

$$= -3/3$$

$$= -1 = \text{R.H.S.}$$

Hence, proved.