COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 28 June, 2022 (SHIFT-2) | TIME : (3.00 p.m. to 06.00 p.m)
Duration: 3 Hours | Max. Marks: 300

PART : MATHEMATICS

1. Let f(x) be a quadratic polynomial such that f(–2) + f(3) = 0. If one root of f(x) = 0 is –1, then the sum of roots of f(x) = 0 is

(1) $\frac{11}{3}$ (2) $\frac{11}{6}$ (3) $\frac{14}{3}$ (4) $\frac{17}{6}$

Ans. **(1)**

Sol. Let $f(x) = ax^2 + bx + c = a(x+1)(x-\alpha)$

$f(-2) = a(-1)(-2-\alpha) = a(2+\alpha)$

$f(3) = a(4)(3-\alpha) = 4a(3-\alpha)$

$f(-2) + f(3) = 0 \Rightarrow a(2 + \alpha + 12 - 4\alpha) = 0$

$\Rightarrow a \neq 0,\ -3\alpha + 14 = 0 \Rightarrow \alpha = \frac{14}{3}$

roots are $= -1, \frac{14}{3}$

sum of roots $= -1 + \frac{14}{3} = \frac{11}{3}$

2. If n, A.M. are inserted between a and 100 such that the ratio of first mean and last mean equal to $\frac{1}{7}$ and a + n = 33 then value of n is

(1) 20 (2) 21 (3) 22 (4) 23

Ans. **(4)**

Sol. If d is common difference then 100 = a + (n +1)d

$d = \frac{100-a}{n+1}$

$\frac{A_1}{A_n} = \frac{a+d}{100-d} = \frac{1}{7}$

$\Rightarrow \frac{a + \frac{100-a}{n+1}}{100 - \frac{100-a}{n+1}} = \frac{1}{7}$

$\Rightarrow \frac{an+100}{100n+a} = \frac{1}{7}$

$\Rightarrow 7an + 700 = 100n + a$

$\Rightarrow 7(33-n)n + 700 = 100n + 33 - n$

$\Rightarrow 7n^2 - 132n - 667 = 0$

$\Rightarrow n = 23$

3. If e and ℓ are respectively the eccentricity and length of latus rectum of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that $e^2 = \frac{11}{14}\ell$. If e' & ℓ' are the eccentricity and latus rectum of the conjugate hyperbola and $(e')^2 = \frac{11}{8}\ell'$ then the value of 77a + 44b is

(1) 130 (2) 140 (3) 145 (4) 155

Ans. **(1)**

Sol. $e^2 = \frac{11}{14}\ell \Rightarrow 1+\frac{b^2}{a^2}=\frac{11}{14}.\frac{2b^2}{a}$

$= a^2+b^2=\frac{11b^2.a}{7}$

$\Rightarrow 7a^2 + 7b^2 = 11ab^2$ ....(1)

$\because (e')^2 = \frac{11}{8}\ell' \Rightarrow 1+\frac{a^2}{b^2}=\frac{11}{8}.\frac{2a^2}{b}$

$\Rightarrow a^2 + b^2 = \frac{11}{4}a^2b$

$\Rightarrow 4a^2 + 4b^2 = 11a^2b$ ....(2)

equation (1) and (2)

$\frac{7}{4}=\frac{b}{a}$

$\because 1+\frac{b^2}{a^2} = \frac{11}{7}\frac{b^2}{a} \Rightarrow 1+\frac{49}{16}=\frac{11}{7}\times\frac{7}{4}\times b$

$\Rightarrow b=\frac{65}{44} \Rightarrow 44b = 65$

$\because 1+\frac{a^2}{b^2}=1+\frac{16}{49}=\frac{11}{4}\times\frac{4}{7}\times a$

$\Rightarrow 65 = 77a$

77a + 44b = 130

4. The term independent of x in $(1 - x^2 - 3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is

(1) $\frac{33}{45}$ (2) $\frac{33}{90}$ (3) $\frac{33}{200}$ (4) $\frac{33}{180}$

Ans. **(3)**

Sol. $(1 - x^2 - 3x^3)\ \left({}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r\right.$

$(1 - x^2 - 3x^3)\left({}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r(x)^{33-5r}\right)$

$33 - 5r \neq 0$

$33 - 5r = -2$

$r = 7$

$33 - 5r \neq -3$

term independent of x is = $-\ {}^{11}C_7 \left(\frac{5}{2}\right)^4 \left(\frac{-1}{5}\right)^7$

$= \frac{11 \times 10 \times 9 \times 8}{24} \times \frac{5^4}{16} \times \frac{1}{5^7}$

$= \frac{33}{200}$

5. If a curve y = f(x) satisfies the differential equation $2y^2 e^{\frac{x}{y^2}} dx + (y^2 - 4xe^{\frac{x}{y^2}} y)\, dy = 0$, then the equation of curve is

(1) $2e^{\frac{x}{y^2}} = \frac{2}{y} + c$ (2) $e^{\frac{x}{y^2}} = \frac{1}{y} + c$ (3) $2e^{\frac{2x}{y^2}} = \frac{1}{y} + c$ (4) $2e^{\frac{x}{y^2}} = \frac{1}{y} + c$

Ans. **(4)**

Sol. $2y^2 e^{\frac{x}{y^2}} dx + (y^2 - 4xe^{\frac{x}{y^2}} y)\, dy = 0$

$\Rightarrow (2y^2 dx - 4xydy)\ e^{\frac{x}{y^2}} + y^2\, dy = 0$

$\Rightarrow 2e^{\frac{x}{y^2}} \left(\frac{y^2 dx - 2xydy}{y^4}\right) = -\frac{1}{y^2} dy$

$\Rightarrow \int 2e^{\frac{x}{y^2}} d\left(\frac{x}{y^2}\right) = \int -\frac{1}{y^2} dy$

$\Rightarrow 2e^{\frac{x}{y^2}} = \frac{1}{y} + c$

6. The slope of tangent at a point (x,y) to the curve y = f(x) is equal to 2tanx(cosx – y), if the curve passes through point (π/4 ,0) then the value of $\int_0^{\pi/2} f(x)dx$ is

(1) $2 - \frac{\pi}{\sqrt{2}}$ (2) $2 + \frac{\pi}{\sqrt{2}}$ (3) $4 - \frac{\pi}{\sqrt{2}}$ (4) $4 + \frac{\pi}{\sqrt{2}}$

Ans. **(1)**

Sol. Slop of tangent $\Rightarrow \frac{dy}{dx} = 2\tan x(\cos x - y)$

$\Rightarrow \frac{dy}{dx} + 2\tan x . y = 2\sin x$

I.F. = $e^{\int 2\tan x dx} = e^{2\ell n \sec x} = e^{\ell n \sec^2 x} = \sec^2 x$

solution of equation

$y.\sec^2x = \int \sec^2 x.2\sin x dx + C$

$\Rightarrow y\sec^2x = 2 \int \sec x \tan x dx + C$

$\Rightarrow y\sec^2x = 2\sec x + C$

$\because$ curve passes through $\left(\frac{\pi}{4}, 0\right)$

$0 = 2\sec\pi/4 + C$

$C = -2\sqrt{2}$

$\Rightarrow$ curve $y\sec^2x = 2\sec x - 2\sqrt{2}$

$\Rightarrow y = 2\cos x - 2\sqrt{2}\cos^2x = 2\cos x - \sqrt{2}\ (1 + \cos 2x)$

$$\int_0^{\pi/2} f(x)dx = \int_0^{\pi/2}(2\cos x - \sqrt{2} - \sqrt{2}\cos 2x)dx = \left(2\sin x - \sqrt{2}x - \frac{\sin 2x}{\sqrt{2}}\right)_0^{\pi/2}$$

$$\Rightarrow \left(2(1) - \sqrt{2}.\frac{\pi}{2} - 0\right) - (0 - 0 - 0) = 2 - \frac{\pi}{\sqrt{2}}$$

7. The value of $\lim\limits_{n\to\infty} 6\tan\left(\sum\limits_{r=1}^{n}\tan^{-1}\frac{1}{r^2+3r+3}\right)$ is

Ans. **(3)**

Sol. $\sum\limits_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right) = \sum\limits_{r=1}^{n}\tan^{-1}\left(\frac{(r+2)-(r+1)}{1+(r+1)(r+2)}\right)$

$= \sum\limits_{r=1}^{n}\left(\tan^{-1}(r+2) - \tan^{-1}(r+1)\right)$

$= (\tan^{-1}(3) - \tan^{-1}(2)) + (\tan^{-1}(4) - \tan^{-1}(3)) + \text{- - - -- - -- } + \left(\tan^{-1}(n+2) - \tan^{-1}(n+1)\right)$

$= \tan^{-1}(n + 2) - \tan^{-1}(2) = \tan^{-1}\left(\frac{(n+2)-2}{1+2(n+2)}\right)$

$= \tan^{-1}\left(\frac{n}{2n+5}\right)$

$$\lim_{n\to\infty} 6\tan\left(\tan^{-1}\frac{n}{2n+5}\right) = \lim_{n\to\infty}\frac{6n}{2n+5} = 6 \times \frac{1}{2} = 3$$

8. Equation of plane passing through point (2, –1, 0) and perpendicular to planes $2x - 3y + z = 0$ and $2x - y - 3z = 0$ is.

(1) $5x + 4y + 2z + 6 = 0$ (2) $5x + 4y + 2z - 6 = 0$

(3) $5x - 4y + 2z - 6 = 0$ (4) $5x + 4y - 2z - 6 = 0$

Ans. **(2)**

Sol. Normal of required plane

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$= \hat{i}(10) - \hat{j}\ (-6-2) + \hat{k}\ (-2+6)$

$= 10\hat{i} + 8\hat{j} + 4\hat{k}$

equation of plane

$10(x-2) + 8(y+1) + 4(z-0) = 0$

$5x + 4y + 2z - 6 = 0$

9. If $\cot\alpha = 1$, $\alpha \in (\pi, \frac{3\pi}{2})$ and $\sec\beta = \frac{-5}{3}$, $\beta \in \left(\frac{\pi}{2}, \pi\right)$, then the value $\tan(\alpha + \beta)$ and $\alpha + \beta$ lies in quadrant

(1) $\frac{1}{7}$, I^{st} quadrant

(2) $\frac{-1}{7}$, I^{st} quadrant

(3) $\frac{-1}{7}$, IV^{th} quadrant

(4) $\frac{1}{7}$, IV^{th} quadrant

Ans. **(3)**

Sol. $\cot\alpha = 1 \Rightarrow \tan\alpha = 1$

$\sec\beta = \frac{-5}{3} \Rightarrow \tan\beta = \frac{-4}{3}$

Now $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha.\tan\beta}$

$$= \frac{1 - \frac{4}{3}}{1 - 1 \times \left(\frac{-4}{3}\right)} = \frac{-1}{7}$$

also $\pi < \alpha < \frac{3\pi}{2}$

$\frac{\pi}{2} < \beta < \pi$

$\frac{3\pi}{2} < \alpha + \beta < \frac{5\pi}{2}$

Since $\tan(\alpha + \beta)$ is negative so $\alpha + \beta$ lies in IV quadrant

10. Let f(x) be a continuous function such that $f(\pi/4) = \sqrt{2}$, $f(\pi/2) = 0$, $f'(\pi/2) = 1$ and $g(x) = \int_x^{\frac{\pi}{4}} \left(f'(t)\sec t + \sec t.\tan t.f(t)\right) dt$, then the value of $\lim\limits_{x\to\frac{\pi^-}{2}} g(x)$ is

Ans. **(03.00)**

Sol. $g(x) = \int_x^{\frac{\pi}{4}} d(f(t).\sec t) = \left(f(t).\sec t\right)_x^{\frac{\pi}{4}}$

$= f\left(\frac{\pi}{4}\right) . \sec \frac{\pi}{4} - f(x) . \sec x$

$= 2 - \frac{f(x)}{\cos x}$

$\lim\limits_{x\to\frac{\pi^-}{2}} g(x) = \lim\limits_{x\to\frac{\pi^-}{2}} \left(2 - \frac{f(x)}{\cos x}\right) = 2 - \lim\limits_{x\to\frac{\pi^-}{2}} \frac{f(x)}{\cos x} \to \frac{0}{0}$ form

$= 2 - \lim\limits_{x\to\frac{\pi^-}{2}} \frac{f'(x)}{-\sin x} = 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}}$

$= 2 + 1 = 3$

11. Vertex of parabola is (2,–1), and equation of its directrix is $4x - 3y = 21$ then length of latus rectum of parabola is

(1) 2 (2) 8 (3) 3 (4) 4

Ans. **(2)**

Sol. Distance between directrix and vertex is $a = \left|\frac{8+3-21}{5}\right| = 2$

Now length of latus rectum = 4a = 8

12. If z = x + iy is a complex number satisfies $\bar{z} = iz^2 + z - i$ then sum of square of all values of z satisfies given equation is

(1) 1 (2) 2 (3) 3 (4) 4

Ans. **(1)**

Sol. $\bar{z} = iz^2 + z - i$

$x - iy = i(x + iy)^2 + (x + iy - i)$

$x - iy = i(x^2 - y^2 + 2xyi) + x + iy - i$

compare real and imaginary part

$x = -2xy + x \Rightarrow x = 0$ or $y = 0$

$-y = x^2 - y^2 + y - 1 \Rightarrow x^2 - y^2 + 2y - 1 = 0$

when $x = 0 \Rightarrow y^2 - 2y + 1 = 0 \Rightarrow (y - 1)^2 = 0 \Rightarrow y = 1$

when $y = 0 \Rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

Hence complex number z = i, 1, – 1

sum of square is = $(i)^2 + (1)^2 + (-1)^2 = 1$

13. If $\lim_{x \to 1} \frac{\sin(3x^2 + x - 4) - x^2 + 1}{2x^3 - 7x + ax + b} = -2$, then the value of b – a is

Ans. **(08.00)**

Sol. For finite non-zero limit it should be $\frac{0}{0}$ form

So, by denominator 2 – 7 + a + b = 0

$\Rightarrow$ a + b = 5

Now

$$\lim_{x \to 1} \frac{\sin(3x^2 + x - 4) - x^2 + 1}{2x^3 - 7x + ax + b} = -2$$

by L–H rule

$$\lim_{x \to 1} \frac{\cos(3x^2 + x - 4) \times (6x + 1) - 2x}{6x^2 - 7 + a} = -2$$

$$\Rightarrow \frac{1 \times (7) - 2}{6 - 7 + a} = -2$$

5 = –2(a – 1)

$$\Rightarrow -\frac{5}{2} + 1 = a$$

$$a = -\frac{3}{2}$$

and b = 5 – a

$$= 5 + \frac{3}{2} = \frac{13}{2}$$

so $b - a = \frac{13}{2} + \frac{3}{2} = 8$

14. The area bounded by x-axis and $y = 3 - |x + 1| - |x - \frac{1}{2}|$ is

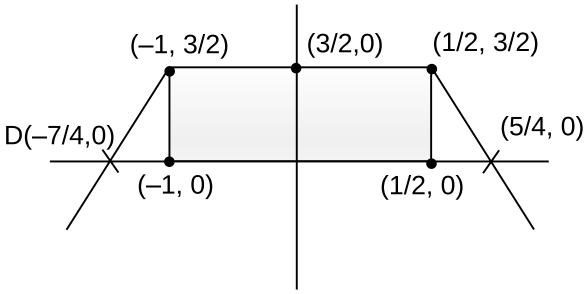
(1) $\frac{27}{7}$ (2) $\frac{27}{8}$ (3) $\frac{29}{7}$ (4) $\frac{25}{7}$

Ans. **(2)**

Sol. $y = 3 - |x + 1| - |x - \frac{1}{2}|$

$$y = \begin{cases} 3x + x + 1 + x - \frac{1}{2} & x < -1 \\ 3 - x - 1 + x - \frac{1}{2} & -1 \le x \le \frac{1}{2} \\ 3 - x - 1 - x + \frac{1}{2} & x \ge \frac{1}{2} \end{cases}$$

$$y = \begin{cases} 2x + \frac{7}{2} & x < -1 \\ \frac{3}{2} & -1 \le x \le \frac{1}{2} \\ -2x + \frac{5}{2} & x \ge \frac{1}{2} \end{cases}$$



Required area = $\frac{3}{2} \times \frac{3}{2} + \frac{1}{2}\left(\frac{3}{2} \times \frac{3}{4}\right) = \frac{1}{2}\left(\frac{3}{2} \times \frac{3}{4}\right)$

$= \frac{9}{4} + \frac{1}{2}\frac{9}{8} + \frac{1}{2}\frac{9}{8}$

$= \frac{9}{4} + \frac{9}{10} + \frac{9}{16} = \frac{27}{8}$

15. A chord of circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$ is diameter of $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$, then the value of r is

(1) $\sqrt{10}$ (2) $\sqrt{12}$ (3) $\sqrt{15}$ (4) $\sqrt{20}$

Ans. **(1)**

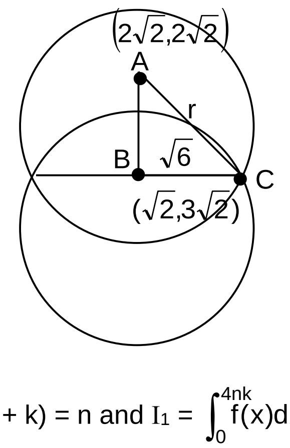
Sol. AC = r

AB = $\sqrt{(2\sqrt{2} - \sqrt{2})^2 + (2\sqrt{2} - 3\sqrt{2})^2}$

= $\sqrt{2 + 2} = 2$

AC = $\sqrt{AB^2 + BC^2}$

= $\sqrt{4 + 6} = \sqrt{10}$



16. If f(x) satisfying the relation f(x) + f(x + k) = n and $I_1 = \int_0^{4nk} f(x)dx$, $I_2 = \int_{-k}^{3k} f(x)dx$ then

(1) $I_1 + I_2 = nk(2n + 2)$ (2) $I_1 + I_2 = (2n + 2)$

(3) $I_1 + I_2 = (2n + 2)k$ (4) $I_1 + I_2 = (2n + 2)k$

Ans. **(1)**

Sol. f(x) + f(x + k) = n …(1)

put $x \to x + k$

f(x + k) + f(x + 2k) = n …(2)

subtract f(x) – f(x + 2k) = 0

period is 2k

Now, $I_1 = \int_0^{4nk} f(x)dx$

$= 2n\int_0^{2k} f(x)dx$

$I_2 = \int_{-k}^{3k} f(x)dx = 2\int_0^{2k} f(x)dx$

$I_1 + I_2 = (2n + 2)\int_0^{2k} f(x)dx$

$= (2n + 2)\left[\int_0^{k} f(x)dx + \int_k^{2k} f(x)dx\right]$

$= (2n + 2)\left[\int_0^{k} f(x)dx + \int_0^{k} f(x+k)dx\right]$

$= (2n + 2)\left[\int_0^{k} f(x) + f(x+k)dx\right]$

$= (2n + 2)\, nk$

17. In how many ways 30 identical candies are distributed among four persons X_1, X_2, X_3, X_4, such that X_2 gets atleast 3 and atmost 7 and X_3 gets atleast 2 and atmost 6

(1) 551 (2) 550 (3) 552 (4) 549

Ans. **(2)**

Sol. $x_1 + x_2 + x_3 + x_4 = 30$

$3 \le x_2 \le 7$, $2 \le x_3 \le 6$, $x_1, x_2 \ge 0$,

Total number of ways = coefficient of x^{30} is

$(x^0 + x^1 + x^2 + - - - -)^2 . (x^3 + x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$

$\Rightarrow \left(\frac{1}{1-x}\right)^2 . x^3\left(\frac{1-x^5}{1-x}\right) . x^2\left(\frac{1-x^5}{1-x}\right) = x^5 (1 - x^5)^2 (1 - x)^{-4}$

$\Rightarrow$ coefficient of x^{25} in $(1 - x^5)^2 . (1 - x)^{-4}$

$= (1 - 2x^5 + x^{10})(1 - x)^{-4}$

$\Rightarrow {}^{4+25-1}C_{25} - 2.\,{}^{4+20-1}C_{20} + {}^{4+15-1}C_{15}$

$\Rightarrow {}^{28}C_{25} - 2.\,{}^{23}C_{20} + {}^{18}C_{15}$ $\Rightarrow \frac{28.27.26}{3.2} - \frac{2.23.22.21}{3.2} + \frac{18.17.16}{3.2} = 550$