CHAPTER › 06

Work, Energy and Power

KEY NOTES

Scalar Product

- The scalar product or dot product of any two vectors **A** and **B** is denoted as $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta$

  where, θ is the angle between the two vectors **A** and **B**.
- Scalar product obeys following laws
  (i) Commutative law; $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
  (ii) Distributive law; $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$
  Further, $\mathbf{A} \cdot (\lambda\mathbf{B}) = \lambda\,(\mathbf{A} \cdot \mathbf{B})$, where λ is a real number.
- For unit vectors, $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, we have

  $$\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 1 \text{ and } \hat{\mathbf{i}} \cdot \hat{\mathbf{j}} = \hat{\mathbf{j}} \cdot \hat{\mathbf{k}} = \hat{\mathbf{k}} \cdot \hat{\mathbf{i}} = 0$$
- If two vectors are given as $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$ and $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$, then their scalar product will be

  $$\mathbf{A} \cdot \mathbf{B} = (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) \cdot (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}})$$
  $$= A_xB_x + A_yB_y + A_zB_z$$
- From the definition of scalar product, we have

  $\mathbf{A} \cdot \mathbf{B} = 0$, if **A** and **B** are perpendicular.

Work

- The work done by a force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement.

  Thus, $W = (F\cos\theta)d = \mathbf{F} \cdot \mathbf{d}$.

  Work is done by a force on the body over a certain displacement.
- **No work is done, if**
  (i) the displacement is zero.
  (ii) the force is zero.
  (iii) the force and displacement are mutually perpendicular, i.e. for $\theta = \frac{\pi}{2}\,\text{rad}(= 90^\circ)$.
- Work can be both positive and negative. If θ is between 0° and 90°, $\cos\theta$ is positive and if θ is between 90° and 180°, $\cos\theta$ is negative.
- If the displacement Δx is small, we can take the force $F(x)$ as approximately constant and the work done is $\Delta W = F(x)\Delta x$.
- If the displacement are allowed to approach zero, then the work done is

  $$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\,\Delta x = \int_{x_i}^{x_f} F(x)$$
  $$= \text{Area under the force-displacement curve.}$$

  Thus, for a varying force, the work done can be expressed as a definite integral of force over displacement.

Energy

- It is defined as the capacity or ability of a body of doing work.
- Some commonly used units of energy are

  $1\text{ erg} = 10^{-7}\text{ J}$, $1\text{ electron-volt (eV)} = 1.6 \times 10^{-19}\text{ J}$,
  $1\text{ cal} = 4.186\text{ J}$ and $1\text{ kilowatt-hour} = 3.6 \times 10^{6}\text{ J}$.

Kinetic Energy

- The kinetic energy of an object is a measure of the work an object can do by virtue of its motion.

  If an object of mass m has velocity v, its kinetic energy

$$(\text{KE}) = \frac{1}{2}mv^2$$

  It is a scalar quantity.

Note As momentum, $p = mv$

$$\text{KE} = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \Rightarrow p = \sqrt{2m\,(\text{KE})}$$

Work-Energy Theorem

- According to work-energy theorem, the change in kinetic energy of a particle is equal to the work done on it by the net force.

  i.e. Work done, $W = K_f - K_i = \frac{1}{2}m(v^2 - u^2)$
- When a force acts in the direction of displacement on the body, then kinetic energy increases. In this case, work done on the body is equal to increase in kinetic energy.
- When a force acts in the opposite direction of displacement on the body, then its kinetic energy decreases.

  In this case, work done on the body is equal to decrease in kinetic energy.
- When kinetic energy of a moving body increases, then work done on the body is positive and when kinetic energy of a moving body decrease, then work done on the body is negative.
- When a body moves along the circular path with uniform speed (constant speed), then change in kinetic energy of the body is zero, hence by work-energy theorem, work done on the body by centripetal force is zero.
- **The work-energy theorem for a variable force** is given by integrating the work done from the initial position x_i to final position x_f.

$$K_f - K_i = \int_{x_i}^{x_f} F dx = W$$

  where, K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f.

Potential Energy

- It is the stored energy by virtue of the position and configuration of a body.
- **Gravitational potential energy** of an object is the negative of work done by the gravitational force in raising the object to that height.

$$U(h) = mgh$$

- If h is taken as a variable, the gravitational force F equals to the negative of the derivative of $U(h)$ with respect to h.

  Thus, $F = -\frac{d}{dh}U(h) = -mg$, negative sign indicates that the gravitational force is downward.
- Equation $\frac{1}{2}mv^2 = mgh$, shows that gravitational potential energy of the object at height h when the object is released, manifests itself as kinetic energy of the object on reaching the ground.
- Mathematically, the potential energy $U(x)$ is defined for the force $F(x)$ as

$$F(x) = -\frac{dU}{dx}$$

$$\Rightarrow \int_{x_i}^{x_f} F(x)\,dx = \int_{U_i}^{U_f} dU = U_i - U_f$$

- **Work done by conservative and non-conservative force**
  (i) The work done by a **conservative force** such as gravity depends upon initial and final positions only not upon the path taken.
  (ii) If the work done or the kinetic energy depend on other factors such as the velocity or the particular path taken by the object, then the force is known as **non-conservative force.**
- The change in potential energy for a conservative force ΔU is equal to the negative of work done by the force,

$$\Delta U = -F(x)dx$$

- **The principle of conservation of total mechanical energy** states that, ''the total mechanical energy of a system is conserved, if the forces doing work on it, are conservative.''

  Thus, over a whole path from x_i to x_f,

$$K_i + U(x_i) = K_f + U(x_f)$$

  The quantity $K + U(x)$ is called the **total mechanical energy** of the system.

Potential Energy of Spring

- The work done in stretching or compressing a spring by the spring force is called potential energy of spring and can be given as

$$W_s = -\frac{kx_m^2}{2}$$

- For compression of spring, the potential energy is negative while for expansion of spring, the potential energy is positive.
- So, for an extension or compression of x, the potential energy of spring is,

$$U(x) = \frac{1}{2}kx^2$$

- The potential energy $U(x)$ of the spring is zero in the equilibrium position.
- The maximum speed of the spring is given by

$$v_m = \sqrt{\frac{k}{m}}\ x_m.$$

  where, k = spring constant.

Various Forms of Energy

Energy comes in many forms which transform into one another as

- When a block slides on a rough horizontal surface, the work done by friction is not lost, but is transferred as **heat energy**. This raises the internal energy of the block.
- **Chemical energy** is the total binding energy of different particles in a molecule. It may be released or absorbed during a chemical reaction, often in the form of heat.
- **Electric energy** is the energy associated with the flow of electric charge and current.
- **Nuclear energy** is the energy in the nucleus or core of an atom that holds the nucleons together.

  Mass-Energy Equivalence Einstein showed that, mass and energy are equivalent and related by the relation

  $$E = mc^2$$

  where, c = speed of light in vacuum.

Conservation of Energy

- According to this principle "energy may be transformed from one form to another, but the total energy of an isolated system remains constant. Energy can neither be created nor destroyed".
- At a height H, the energy is purely potential mgH. It is partially converted to kinetic at height $h < H$ $\left(mgh + \frac{1}{2}mv_h^2\right)$ and is fully kinetic at ground level $\left(\frac{1}{2}mv_f^2\right)$.

Power

- It is defined as the time rate at which work is done or energy is transferred.
- The **average power** of a force is defined as the ratio of the work W, to the total time t taken, $P_{av} = \frac{W}{t}$.
- The **instantaneous power** is defined as the limiting value of the average power as time interval approaches zero.

  $$P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$$

  where, $\mathbf{v}$ is the instantaneous velocity when the force is $\mathbf{F}$.

  It is a scalar quantity and its dimensions are $[ML^2T^{-3}]$.

  In SI system, its unit is **watt (W) or Js^{-1}**.

  Another unit of power is **1 hp (horse power)** = 746 W

Collision

- It is an isolated event, in which two or more colliding bodies exert strong forces on each other for a short duration of time.
- In all collisions, the total linear momentum is conserved, i.e. the initial momentum of the system is equal to the final momentum of the system.

  This implies, $\Delta \mathbf{p}_1 + \Delta \mathbf{p}_2 = 0$

  where, $\mathbf{p} = mv$

- Collision is of two types, elastic and inelastic collisions.
- Kinetic energy of the colliding body and the system is conserved in elastic collision only.

Collision in One-Dimension

- If the initial and final velocities of both the bodies are along the same straight line, then it is called a **one-dimensional collision** or **head-on-collision.**
- In 1-D completely inelastic collision, the loss in kinetic energy on collision is

  $$\Delta K = \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)} v_{1i}^2$$

  where, v_{1i} = initial velocity of mass m_1.

- After collision, the velocity of two masses are

  $$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i}$$

  $$v_{2f} = \frac{2m_1 v_{1i}}{m_1 + m_2}$$

  There are two cases as given below

  ***Case* I** If the two masses are equal,

  $$v_{1f} = 0 \text{ and } v_{2f} = v_{1i}.$$

  ***Case* II** If one mass dominates, i.e. $m_2 > m_1$

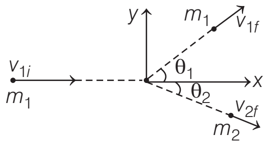
  $$v_{1f} \cong -v_{1i} \text{ and } v_{2f} \cong 0$$

Collision in Two-Dimensions

- For collision in two-dimensions or a plane as shown in the figure below the x and y-components equations are

  $$m_1 v_{1f} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$

  $$0 = m_1 v_{1f} \sin\theta_1 - m_2 v_{2f} \sin\theta_2$$



- If the **collision is elastic**, then $\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$
- In a **perfectly elastic collision**, total energy and total linear momentum of colliding particles remain conserved.
- **Coefficient of restitution** (e) is the ratio of the relative velocity of separation after the collision to the relative velocity of approach before collision,

  i.e. $$e = \frac{v_2 - v_1}{u_1 - u_2}$$

  For an elastic collision, $e = 1$.

  For an inelastic collision, $e < 1$.

- **Rebounding of a ball on collision with the floor**
  (i) Speed of the ball after the nth rebound,
  $v_n = e^n v_0 = e^n \sqrt{2gh_0}$
  (ii) Height covered by the ball after the nth rebound,
  $h_n = e^{2n} h_0$

KEY NOTES

Mastering NCERT

MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ Scalar Product and Work Done by Constant & Variable Forces

1 Scalar product of vectors $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ is

(a) 6 (b) 8
(c) 5 (d) 9

2 Find the angle between force $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$ unit and displacement $\mathbf{d} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ unit.

(a) $\cos^{-1}(0.49)$ (b) $\cos^{-1}(0.32)$
(c) $\cos^{-1}(0.60)$ (d) $\cos^{-1}(0.90)$

3 A force $\mathbf{F} = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ acting on a body produces a displacement $\mathbf{d} = 6\hat{\mathbf{i}} + 5\hat{\mathbf{k}}$. The work done by the force is

(a) 18 units (b) 15 units (c) 12 units (d) 10 units

4 A uniform force of $(3\hat{\mathbf{i}} + \hat{\mathbf{j}})$N acts on a particle of mass 2 kg. Hence, the particle is displaced from position $(2\hat{\mathbf{i}} + \hat{\mathbf{k}})$ m to position $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$ m. The work done by the force on the particle is ***CBSE AIPMT 2013***

(a) 9 J (b) 6 J
(c) 13 J (d) 15 J

5 The earth is moving around the sun in a circular orbit, is acted upon by a force and hence work done on the earth by the force is

(a) zero (b) + ve
(c) – v (d) None of these

6 Work done by gravitational force in one revolution of earth around the sun on its elliptical path is zero because

(a) force is always perpendicular to displacement
(b) displacement is zero
(c) displacement is non-zero
(d) Both (a) and (c)

7 In which case, work done will be zero

(a) a weight-lifter while holding a weight of 100 kg on his shoulders for 1 min
(b) a locomotive against gravity is running on a level plane with a speed of 60 kmh^{-1}
(c) a person holding a suitcase on his head and standing at a bus terminal
(d) All of the above

8 A ball is released from the top of a tower. The ratio of work done by force of gravity in 1st second, 2nd second and 3rd second of the motion of ball is

(a) 1 : 2 : 3 (b) 1 : 4 : 16
(c) 1 : 3 : 5 (d) 1 : 9 : 25

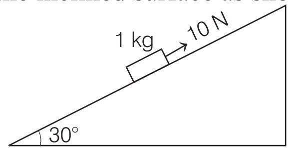
9 A force of 10 N is applied on an object of mass 2 kg placed on a rough surface having coefficient of friction equal to 0.2. Work done by applied force in 4 s is

(a) 120 J (b) 240 J
(c) 250 J (d) 100 J

10 A uniform chain of length L and mass M is lying on a smooth table and one-third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, work required to pull the hanging part on to the table is

(a) MgL (b) $\frac{MgL}{3}$
(c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

11 A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 30° by a force of 10 N parallel to the inclined surface as shown in the figure.



If the block is pushed up by 10 m along the incline, then the work against gravity is
(take, $g = 10\ \text{ms}^{-2}$)

(a) 10 J (b) 50 J
(c) 100 J (d) 150 J

12 A force $F = 20 + 10y$ acts on a particle in y-direction, where F is in newton and y in metre.
Work done by this force to move the particle from $y = 0$ to $y = 1$ m is ***NEET 2019***

(a) 5 J (b) 25 J
(c) 20 J (d) 30 J

13 A position-dependent force $F = 3x^2 - 2x + 7$ acts on a body of mass 7 kg and displaces it from $x = 0$ m to $x = 5$ m. The work done on the body is y joule. If both F and x are measured in SI units, the value of y is

(a) 135 (b) 235
(c) 335 (d) 935

14 A force $F = -k/x^2$ $(x \neq 0)$ acts on a particle in x-direction. Find the work done by the force in displacing the particle from $x = -a$ to $x = 2a$.

(a) $3k/2a$ (b) $4k/a^2$
(c) $-3k/2a^2$ (d) $\dfrac{-9k}{a^2}$

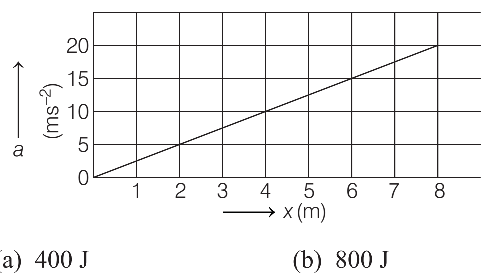
15 When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is

(a) $aL^2 + bL^3$ (b) $\dfrac{1}{2}(aL^2 + bL^3)$
(c) $\dfrac{aL^2}{2} + \dfrac{bL^3}{3}$ (d) $\dfrac{1}{2}\left(\dfrac{aL^2}{2} + \dfrac{bL^3}{3}\right)$

16 A body of mass 3 kg is under a force which causes a displacement in it given by $s = t^2/3$ (in metre). Work done by force in 2 s is

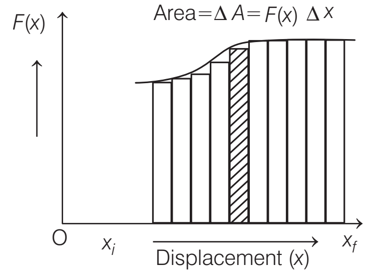
(a) 2 J (b) 3.8 J
(c) 5.2 J (d) 2.6 J

17 A 10 kg brick moves along X-axis. Its acceleration as a function of its position is shown in figure. What is the net work performed on the brick by the force causing the acceleration as the brick moves from $x = 0$ to $x = 8.0$ m?



(a) 400 J (b) 800 J
(c) 200 J (d) 100 J

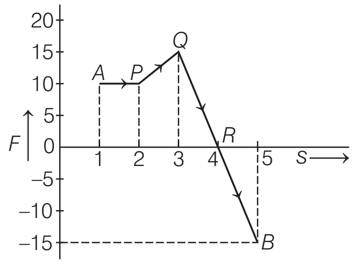
18 If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit, but the sum approaches a definite value equal to the area under the curve in given figure.



What is the net work done by varying force $F(x)$ from position x_i to x_f?

(a) $\int_{x_f}^{x_i} F(x)\,dx$ (b) $\int_0^{x_f} F(x)\,dx$
(c) $\int_{x_i}^{x_f} F(x)\,dx$ (d) $\int_{x_i}^{0} F(x)\,dx$

19 A body moves from point A to B under the action of a force varying in magnitude as shown in figure, then the work done is (force is expressed in newton and displacement in metre)



(a) 30 J (b) 22.5 J
(c) 25 J (d) 27 J

20 A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of 100 N over a distance of 10 m. Thereafter, she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance through which the trunk has been moved is 20 m. If frictional force is 50 N, then total work done by the two forces over 20 m is

(a) – 2000 J (b) 500 J
(c) 750 J (d) – 200 J

TOPIC 2 ~ *Kinetic Energy : Work-Energy Theorem*

21 When velocity of a moving car decreases by applying sudden brake, then its kinetic energy,

(a) increases (b) decreases
(c) remains same (d) none of these

22 A car of mass 1000 kg is moving with a speed of 80 m/s. The kinetic energy of the car is

(a) 6.4×10^6 J (b) 3.2×10^6 J
(c) 3.2×10^5 J (d) 4×10^5 J

23 The kinetic energy of an air molecule (10^{-21} J) in eV is

(a) 6.2 meV (b) 4.2 meV
(c) 10.4 meVeV (d) 9.7 meVeV

24 When a man increases his speed by 2 ms^{-1}, he finds that his kinetic energy is doubled, the original speed of the man is

(a) $2(\sqrt{2}-1)\,\text{ms}^{-1}$ (b) $2(\sqrt{2}+1)\,\text{ms}^{-1}$
(c) 4.5 ms^{-1} (d) None of these

25 Two vehicles X and Y have masses 40 kg and 10 kg respectively. Each vehicle is acted upon by a force of 80 N. If both vehicles acquire same kinetic energy in times t_X and t_Y respectively, then $\frac{t_X}{t_Y}$ is

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{2}{1}$ (d) $\frac{1}{4}$

26 A running man has half kinetic energy to that of a boy of half of his mass. The man speeds up by 1 ms^{-1}, so as to have same kinetic energy as that of the boy. The original speed of the man is

(a) $\sqrt{2}$ ms^{-1} (b) $\sqrt{2}-1$ms^{-1}
(c) $\frac{1}{\sqrt{2}-1}$ ms^{-1} (d) $\frac{1}{\sqrt{2}}$ ms^{-1}

27 An object of mass 10 kg is moving with velocity of 10 ms^{-1}. A force of 50 N acted upon it for 2 s. Percentage increase in its kinetic energy is

(a) 25% (b) 50% (c) 75% (d) 300%

28 A particle of mass 10 g moves along a circle of radius 6.4 cm with a constant tangential acceleration. What is the magnitude of this acceleration, if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after the beginning of the motion? ***NEET 2016***

(a) 0.15 ms^{-2} (b) 0.18 ms^{-2}
(c) 0.2 ms^{-2} (d) 0.1 ms^{-2}

29 A force which is inversely proportional to the speed, is acting on a body. The kinetic energy of the body starting from rest is

(a) a constant
(b) inversely proportional to time
(c) directly proportional to time
(d) directly proportional to square of time

30 Kinetic energy of a particle is increased by 4 times. What will be the relation between initial and final momentum? ***JIPMER 2018***

(a) $p_2 = 2p_1$ (b) $p_2 = \frac{p_1}{2}$
(c) $p_2 = p_1$ (d) $p_2 = 4p_1$

31 Two masses of 1 g and 4 g are moving with equal kinetic energy. The ratio of the magnitudes of their momentum is

(a) 4 : 1 (b) $\sqrt{2}:1$
(c) 1 : 2 (d) 1 : 16

32 If momentum of a moving body is increased to 50% of its initial value, then percentage increase in its kinetic energy will be

(a) 50% (b) 125%
(c) 100% (d) 75%

33 Two moving objects having same kinetic energy are stopped by application of equal retarding force. Which object will come to rest at short distance?

(a) Bigger (b) Smaller
(c) Both at same distance (d) Cannot say

34 A car weighing 1400 kg is moving at a speed of 54 kmh^{-1} up a hill. When the motor stops, it is just able to reach the destination which is at a height of 10 m above the point. Then, the work done against friction (negative of the work done by the friction) is (take, $g = 10\,\text{ms}^{-2}$)

(a) 10 kJ (b) 15 kJ
(c) 17.5 kJ (d) 25 kJ

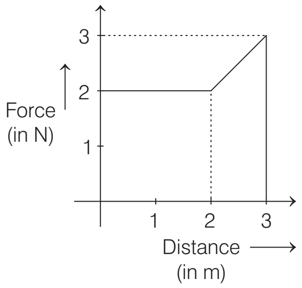
35 A bullet of mass 20 g is moving with a speed of 150 ms^{-1}. It strikes a target and is brought to rest after piercing 10 cm into it. Calculate the average force of resistance offered by the target.

(a) 2500 N (b) 2000 J
(c) 2250 N (d) 2100 J

36 A block of mass 10 kg moving in x-direction with a constant speed of 10 ms^{-1} is subjected to a retarding force $F = 0.1\,x$ J/m during its travelling from $x = 20$ m to 30 m. Its final kinetic energy will be ***CBSE AIPMT 2015***

(a) 475 J (b) 450 J
(c) 275 J (d) 250 J

37 A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is ***JEE Main 2019***



(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

38 Consider a drop of rain water having mass 1 g falling from a height of 1 km. It hits the ground with a speed of 50 m/s. Take g constant with a value of 10 m/s^2. The work done by the (i) gravitational force and the (ii) resistive force of air is ***NEET 2017***

(a) (i) -10 J and (ii) -8.25 J
(b) (i) 1.25 J and (ii) -8.25 J
(c) (i) 100 J and (ii) 8.75 J
(d) (i) 10 J and (ii) -8.75 J

39 A body which is moving with 10 ms^{-1} is sliding up on a rough inclined plane having inclination of 30°. Find the height upto which it can go, if coefficient of friction of the inclined surface is 0.1.

(a) 4.25 m (b) 5.25 m (c) 6.25 m (d) 7.25 m

40 A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds? ***JEE Main 2019***

(a) 850 J (b) 900 J (c) 950 J (d) 875 J

41 A block of mass $m = 1$ kg, moving on a horizontal surface with speed $v_i = 2$ ms^{-1} enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely proportional to x over this range,

$$F_r = \frac{-k}{x} \text{ for } 0.1 < x < 2.01 \text{ m}$$
$$= 0 \text{ for } x < 0.1 \text{ m and } x > 2.01 \text{ m, where } k = 0.5 \text{ J.}$$

What is the speed v_f of the block as it crosses this patch?

(a) 2 ms^{-1} (b) 40 ms^{-1}
(c) 1 ms^{-1} (d) 4 ms^{-1}

TOPIC 3 ~ *Conservative Forces and Potential Energy*

42 When a body is lifted above the surface of the earth, then its potential energy

(a) increases (b) decreases
(c) remains same (d) None of these

43 A body of mass 2 kg lifted at a height of 16 m from the surface of earth. The potential energy of the body at given height, is [take, $g = 10$ m/s^2]

(a) 640 J (b) 320 J (c) 80 J (d) 160 J

44 A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At highest point of their journey, the ratio of their potential energies will be

(a) 1 : 1 (b) 2 : 1 (c) 3 : 2 (d) 4 : 1

45 A ball bounces to 75% of its original height. The percentage loss of potential energy in each bounce is

(a) 100% (b) 75% (c) 50% (d) 25%

46 The potential energy $U(x)$ can be assumed zero when

(a) $x = 0$
(b) gravitational force is constant
(c) infinite distance from the gravitational source
(d) All of the above

47 The potential energy of a spring increases by 15 J when stretched by 3 cm. If it is stretched by 4 cm, the increase in potential energy is

(a) 27 J (b) 30 J (c) 33 J (d) 36 J

48 The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy, if they are stretched by the same force?

(a) 2 : 3 (b) 3 : 2
(c) 4 : 9 (d) 9 : 4

49 The potential energy of a body is increased in which of the following cases?

(a) If work is done by conservative force
(b) If work is done against conservative force
(c) If work is done by non-conservative force
(d) If work is done against non-conservative force

50 A body of mass $4m$ is lying in xy-plane at rest. It suddenly explodes into three pieces. Two pieces each of mass m move perpendicular to each other with equal speeds v. The total kinetic energy generated due to explosion is ***CBSE AIPMT 2014***

(a) mv^2 (b) $(3/2)mv^2$
(c) $2mv^2$ (d) $4mv^2$

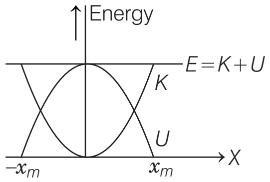
51 The potential energy of a 1 kg particle free to move along the X-axis is given by $U(x) = \left[\frac{x^4}{4} - \frac{x^2}{2}\right]$ J

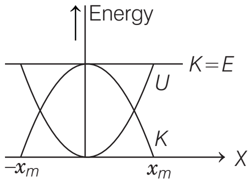
The total mechanical energy of the particle is 2 J. Then, maximum speed (in ms^{-1}) is

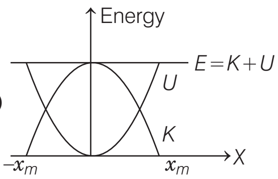
(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$

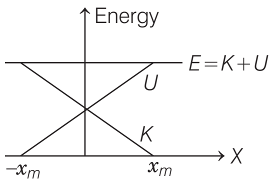
(c) $\frac{1}{\sqrt{2}}$ (d) 2

52 Which graph represents conservation of total mechanical energy?

(a)


(b)


(c)


(d)


53 A spring is compressed by 10 cm, if a block of mass is dropped on it from a height of 40 cm. If the force constant of the spring is 980 Nm^{-1}, then which of the following is the mass of the block?

(a) 1 kg (b) 2 kg (c) 3kg (d) 4 kg

54 A car of mass 1000 kg moving with a speed 18 km/h on a smooth road, collide with a horizontally mounted spring of spring constant 6.25×10^3 Nm^{-1}. What is the maximum compression of the spring?

(a) 1 m (b) 2 m (c) 3 m (d) 5 m

55 A spring gun of spring constant 90 N/cm is compressed 12 cm by a ball of mass 16 g. If the trigger is pulled, the velocity of the ball is

(a) 50 ms^{-1} (b) 40 ms^{-1} (c) 90 ms^{-1} (d) 60 ms^{-1}

56 A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)$th part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be ***JEE Main 2019***

(a) $\frac{2MgL}{n^2}$ (b) $nMgL$ (c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

TOPIC 4 ~ *The Law of Conservation of Energy*

57 When a body is falling from a certain height from the surface of earth, then

(a) its kinetic energy decreases continuously
(b) its potential energy increases continuously
(c) its total mechanical energy remains constant at each point
(d) kinetic energy and potential energy are equal at each point

58 An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Then, as it comes closer and closer to the earth its speed

(a) increases (b) equal to
(c) decreases (d) less than equal to

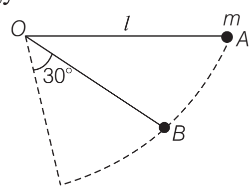
59 A 2 kg block slides on a horizontal floor with a speed of 4 ms^{-1}. It strikes an uncompressed spring and compresses it, till the block is motionless. The friction of kinetic force is 15 N and spring constant is 10000 Nm^{-1}. The spring is compressed by

(a) 5.5 cm (b) 2.5 cm (c) 11.0 cm (d) 8.5 cm

60 A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce?

(a) 20% (b) 25%
(c) 26% (d) 30%

61 A simple pendulum is released from A as shown in the figure. If m and l represent, the mass of the bob and length of the pendulum respectively, the gain in kinetic energy at B is



(a) $\frac{mgl}{2}$ (b) $\frac{\sqrt{3}}{2}mgl$ (c) $\frac{mgl}{\sqrt{2}}$ (d) $\frac{2}{\sqrt{3}}mgl$

62 U is the potential energy, K is the kinetic energy and E is the mechanical energy. Which of the following is not possible for a stable system?

(a) $U > E$ (b) $U < E$ (c) $E > K$ (d) $K > E$

63 A body of mass 5 kg is thrown vertically up with a kinetic energy of 490 J. The height at which the kinetic energy of the body becomes half of the original value is

(a) 12.5 m (b) 10 m
(c) 2.5 m (d) 5 m

64 What is the ratio of kinetic energy of a particle at the bottom to the kinetic energy at the top, when it just loops a vertical loop of radius r?

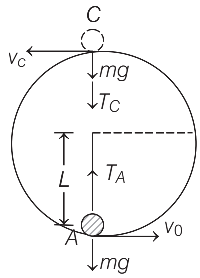
(a) 5 : 1 (b) 2 : 3 (c) 5 : 2 (d) 7 : 2

65 k/r^2 represents the force under which a particle is moving in a circle of radius r. The total energy of the particle is

(a) $\frac{k}{2r}$ (b) $\frac{2k}{r}$
(c) $\frac{-k}{r}$ (d) $-\frac{k}{2r}$

66 A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane. The string becomes slack only on reaching the topmost point C as shown in figure.

Then, the speed of bob (v_0) at point A is



(a) $\sqrt{5gL}$ (b) $\sqrt{2gL}$ (c) $\sqrt{3gL}$ (d) $\sqrt{7gL}$

67 When we rub two flint stones together; we got them to heat up and to ignite a heap of dry leaves in the form of

(a) chemical energy (b) sound energy
(c) heat energy (d) electrical energy

68 How much amount of energy is liberated in converting 1 kg of coal into energy?

(a) 9×10^{16} J (b) 9×10^{15} J (c) 3×10^{14} J (d) 4×10^{6} J

69 In daily life, intake of a human adult is 10^7 J, then average human consumption in a day is

(a) 2400 kcal (b) 1000 kcal
(c) 1200 kcal (d) 700 kcal

TOPIC 5 ~ *Power*

70 Which of the following represents the unit of power?

(a) Pascal (b) Kilowatt-hour
(c) Erg (d) Watt

71 A man can do work of 600 J in 2 min, then man's power is

(a) 7.5 W (b) 10 W (c) 5 W (d) 15 W

72 A one kilowatt motor is used to pump water from a well 10 m deep. The quantity of water pumped out per second is nearly

(a) 1kg (b) 10 kg (c) 100 kg (d) 1000 kg

73 The power of a windmill having blade area equal to A and wind velocity equal to v is (where, ρ is density of air)

(a) $\frac{A\rho v^3}{2}$ (b) $\frac{A\rho v^2}{2}$ (c) $\frac{A\rho v}{2}$ (d) $A\rho v^3$

74 In a hydroelectric power station, the water is flowing at 2 ms^{-1} in the river, which is 100 m wide and 5 m deep. The maximum power output from the river is

(a) 1.5 MW (b) 2 MW (c) 2.5 MW (d) 3 MW

75 A machine gun fires 360 bullets per minute, with a velocity of 600 ms^{-1}. If the power of the gun is 5.4 kW, then mass of each bullet is ***JIPMER 2018***

(a) 5 kg (b) 0.5 kg (c) 5 g (d) 0.5 g

76 An elevator in a building can carry a maximum of 10 persons with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed of 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $(g = 10 \text{ m/s}^2)$ must be at least ***JEE Main 2020***

(a) 62360 W (b) 48000 W
(c) 56300 W (d) 66000 W

77 At time $t = 0$ s, particle starts moving along X-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to

(a) $\frac{1}{\sqrt{t}}$ (b) $t^{3/2}$
(c) $t^{1/3}$ (d) $t^{4/3}$

78 A body of mass 1 kg begins to move under the action of a time dependent force $\mathbf{F} = (2t\,\hat{\mathbf{i}} + 3t^2\,\hat{\mathbf{j}})$ N, where $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along X and Y-axes.
What power will be developed by the force at the time t? ***NEET 2016***

(a) $(2t^2 + 4t^4)$ W (b) $(2t^3 + 3t^4)$ W
(c) $(2t^3 + 3t^5)$ W (d) $(2t + 3t^3)$ W

TOPIC 6 ~ Collisions

79 When two bodies collide to each other such that their kinetic energy remains conserved. Their collision belong to

(a) elastic collision (b) inelastic collision
(c) Both (a) and (b) (d) Neither (a) nor (b)

80 A particle of mass 1g moving with a velocity $\mathbf{v}_1 = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\ \text{ms}^{-1}$ experiences a perfectly elastic collision with another particle of mass 2 g and velocity $\mathbf{v}_2 = (4\hat{\mathbf{j}} - 6\hat{\mathbf{k}})\ \text{ms}^{-1}$. The velocity of the particle is

(a) $2.3\,\text{ms}^{-1}$ (b) $4.6\ \text{ms}^{-1}$ (c) $9.2\ \text{ms}^{-1}$ (d) $6\ \text{ms}^{-1}$

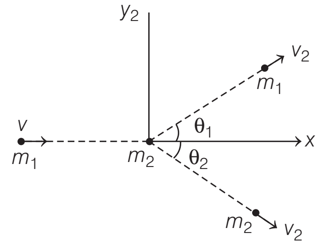
81 A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to

(a) 44% (b) 50% (c) 56% (d) 62%

82 In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is **JEE Main 2018**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}\,v_0$ (c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

83 Consider the collision depicted in figure to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 37°$.



Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1.

(a) 58° (b) 54° (c) 53° (d) 90°

84 Body of mass M is much heavier than the other body of mass m. The heavier body with speed v collides with the lighter body, which was at rest initially, elastically. The speed of lighter body after collision is **AIIMS 2018**

(a) $2v$ (b) $3v$ (c) v (d) $v/2$

85 A moving block having mass m collides with another stationary block having mass $4m$. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be **NEET 2018**

(a) 0.8 (b) 0.25
(c) 0.5 (d) 0.4

86 A ball of 0.5 kg collided with wall at 30° and bounced back elastically. The speed of ball was 12m/s. The contact remained for 1s. What is the force applied by wall on ball? **JIPMER 2018**

(a) $12\sqrt{3}$ N (b) $\sqrt{3}$ N
(c) $6\sqrt{3}$ N (d) $3\sqrt{3}$ N

87 Body A of mass $4m$ moving with speed u collides with another body B of mass $2m$ at rest. The collision is head-on and elastic in nature. After the collision, the fraction of energy lost by the colliding body A is **NEET 2019**

(a) $\frac{8}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{1}{9}$

88 A toy truck of mass $2m$ elastically collides with a toy car of mass m, speed of truck is v and car is at rest. Find the velocity of car after collision. **JIPMER 2019**

(a) $\frac{4v}{3}$ (b) $\frac{v}{3}$ (c) v (d) $\frac{2v}{3}$

89 Two objects of mass m each moving with speed u m/s collide at 90°, then final momentum is (assume collision is inelastic) **JIPMER 2019**

(a) mu (b) $2\,mu$ (c) $\sqrt{2}\,mu$ (d) $2\sqrt{2}\,mu$

90 A body of mass 5×10^3 kg moving with speed 2 m/s collides with a body of mass 15×10^3 kg inelastically and sticks to it. Then, loss in kinetic energy of the system will be **AIIMS 2019**

(a) 7.5 kJ (b) 15 kJ
(c) 10 kJ (d) 5 kJ

91 A ball is thrown vertically downwards from a height of 20 m with an initial velocity v_0. It collides with the ground, loses 50% of its energy in collision and rebounds to the same height. The initial velocity v_0 is (take, $g = 10\ \text{ms}^{-2}$) **CBSE AIPMT 2015**

(a) $14\ \text{ms}^{-1}$ (b) $20\ \text{ms}^{-1}$ (c) $28\ \text{ms}^{-1}$ (d) $10\ \text{ms}^{-1}$

92 The height attained by a ball after 3 rebounds on falling from a height of h on floor, having coefficient of restitution e is

(a) e^3h (b) e^4h
(c) e^5h (d) e^6h

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

■ **Direction** (Q. Nos. 93-101) *In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one.*

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

93 **Assertion** Stopping distance $= \dfrac{\text{Kinetic energy}}{\text{Stopping force}}$

Reason Work done in stopping a body is equal to change in kinetic energy of the body.

94 **Assertion** Friction is a non-conservative force.

Reason This is because work done against friction, in moving a body over a closed path is never zero.

95 **Assertion** Decrease in mechanical energy is more in case of an object sliding up a relatively less inclined plane due to friction.

Reason The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

96 **Assertion** If momentum of a body increases by 50%, its kinetic energy will increase by 125%.

Reason Kinetic energy is proportional to square of velocity.

97 **Assertion** Two springs of force constants k_1 and k_2 are stretched by the same force. If $k_1 > k_2$, then work done in stretching the first (W_1) is less than work done in stretching the second (W_2).

Reason Spring force, $F = k_1 x_1 = k_2 x_2$

98 **Assertion** Kilowatt hour is the unit of energy.

Reason One kilowatt hour is equal to 3.6×10^8 J.

99 **Assertion** Mass and energy are not conserved separately but are conserved as a single entity called 'mass-energy'.

Reason This is because one can be obtained at the cost of the other as per Einstein's equation

$$E = mc^2$$

100 **Assertion** Force applied on a block moving in one-dimension is producing a constant power, then the motion should be uniformly accelerated.

Reason This constant power multiplied with time is equal to the change in kinetic energy.

101 **Assertion** Two particles are moving in the same direction do not lose all their energy in completely inelastic collision.

Reason Principle of conservation of momentum holds true for all kinds of collisions. ***AIIMS 2018***

II. Statement Based Questions

102 Which of the following statement(s) is/are correct for work done to be zero?

I. If the displacement is zero.
II. If force applied is zero.
III. If force and displacement are mutually perpendicular to each other.

(a) Only I (b) Both I and II
(c) Only II (d) I, II and III

103 A force $F(x)$ is conservative, if

I. it can be derived from a scalar quantity $V(x)$.
II. it depends only on the end points.
III. work done by $F(x)$ in a closed path is zero.

Which of the above statement(s) is/are correct?

(a) Only I (b) Both I and III
(c) Only II (d) I, II and III

104 In which of the following cases, there is no loss of mechanical energy?

I. When a ball is moving on a rough surface under perfect rolling.
II. When a ball in sliding on a rough surface.
III. When a ball is falling under gravity.

Which of the above statement(s) is/are correct?

(a) Both I and II (b) Both I and III
(c) Both II and III (d) I, II and III

105 I. Total energy of an isolated system of constant mass remains constant.
II. Energy may be transformed from one form to another.
III. Energy can neither be created nor destroyed.

Which of the above statement(s) is/are correct?

(a) Only I (b) Both I and II
(c) Only III (d) I, II and III

106 In elastic collision,

I. initial kinetic energy is equal to the final kinetic energy.
II. kinetic energy during the collision time Δt is constant.
III. total momentum is conserved.

Which of the above statement(s) is/are correct?

(a) Only I (b) Both I and III
(c) Only III (d) Only II

107 Read the following statements and choose the correct statements in the codes given below.

I. If the total energy of the reactants is more than the products of the reaction, then heat is absorbed.
II. Chemical energy is associated with the forces that give rise to the stability of substance.
III. The mass of an isolated system is covertible into energy.

(a) Both I and II
(b) Both II and III
(c) Both III and I
(d) I, II, III

108 Which of the following statement is correct about non-conservative force?

(a) It depends on velocity of the object.
(b) It depends on the particular path taken by the object.
(c) It depend on the initial and final positions of the object.
(d) Both (a) and (b)

109 Which of the following statement(s) is/are correct?

(a) Absolute value of potential energy cannot be determined.
(b) Absolute value of kinetic energy can be determined because velocity is not measured in relative terms.
(c) Absolute value of force cannot be determined because measurement of acceleration is not possible.
(d) None of the above

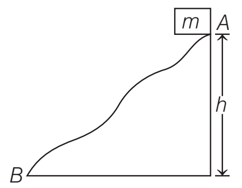
110 Which of the following statement(s) is/are correct?

(a) Conservation of mechanical energy does not consider only conservative force.
(b) Conservation of energy consider both conservative and non-conservative forces.
(c) Conservation of energy consider only conservative force.
(d) Mass converted into energy in nuclear reaction is called mass-defect.

111 According to the equivalence of mass and energy, which of the following statement(s) is/are incorrect?

(a) The mass of an isolated system is conserved.
(b) Matter is neither created nor destroyed.
(c) Matter might change its phase.
(d) All of the above

112 In the given curved road, if particle is released from A, then which of the following statement(s) is/are correct?



(a) Kinetic energy at B must be mgh.
(b) Kinetic energy at B must be zero.
(c) Kinetic energy at B must be less than mgh.
(d) Kinetic energy at B must not be equal to zero.

113 A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal and rebounds with the same speed. Which of the following statement(s) is/are correct?

(a) Momentum is not conserved.
(b) Elastic collision occurs here.
(c) Inelastic collision occurs here.
(d) Both (a) and (b)

114 A bullet of mass m fired at an angle 30° to the horizontal leaves the barrel of the gun with a velocity v. The bullet hits a soft target at a height h above the ground while it is moving downward and emerge out with half the kinetic energy it had before hitting the target. Which of the following statement(s) is/are correct in respect of bullet after it emerges out of the target?

(a) The velocity of the bullet will be reduced to half its initial value.
(b) The velocity of the bullet will be more than half of its earlier velocity.
(c) The bullet will continue to move along the same parabolic path.
(d) The bullet will move in a straight line.

III. Matching Type

115 Match the Column I (work done) with Column II (value) and select the correct answer from the codes given below.

	Column I		Column II
A.	Work done in pulling out a bucket from well by a person and by gravitational force	1.	Positive and negative
B.	Work done by friction on a body sliding down an inclined plane	2.	Negative
C.	Work done by a person in pulling a luggage on a rough surface	3.	Positive
D.	Work done by air in bringing a vibrating pendulum to rest and by gravitational force	4.	Negative and positive

	A	B	C	D		A	B	C	D
(a)	4	1	2	3	(b)	3	2	1	4
(c)	4	2	1	3	(d)	1	2	3	4

116 Match the Column I (angle) with Column II (work done) and select the correct answer from the codes given below.

Column I	Column II
A. $\theta < 90°$	1. Friction
B. $\theta = 90°$	2. Satellite rotating around the earth
C. $\theta > 90°$	3. Coolie is lifting a luggage

	A	B	C
(a)	1	2	3
(b)	3	2	1
(c)	1	3	2
(d)	3	1	2

117 Match the Column I (force) with Column II (feature or result) and select the correct answer from the codes given below.

Column I	Column II
A. Conservative force	1. Work done is zero
B. Non-conservative force	2. Potential energy
C. Centripetal force	3. Heat energy

	A	B	C		A	B	C
(a)	1	2	3	(b)	2	1	3
(c)	3	2	1	(d)	2	3	1

118 Match the Column I (collision) with Column II (feature) and select the correct answer from the codes given below.

Column I	Column II
A. Elastic collision	1. Deformation
B. Non-elastic collision	2. Conservation of KE
C. Scattering	3. Conservation of momentum

	A	B	C		A	B	C
(a)	1	2	3	(b)	2	1	3
(c)	3	2	1	(d)	2	3	1

NCERT & NCERT Exemplar

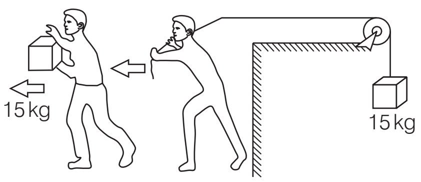
MULTIPLE CHOICE QUESTIONS

NCERT

119 A body constrained to move along Z-axis of a coordinate system is subjected to a constant force F is given by $F = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ N, where $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ are the unit vectors along the X, Y and Z-axes of the system, respectively. What is the work done by this force in moving the body a distance of 4 m along the Z-axis?

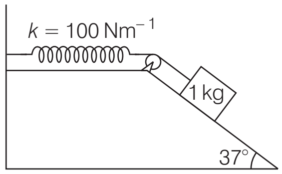
(a) 10 J (b) 12 J (c) 24 J (d) 28 J

120 Given below in Fig. (i), the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. (ii), he walks the same distance pulling the rope towards him. The rope goes over a pulley and a mass of 15 kg hangs at its other end. How much the work done is more in case II as compared to case I?



(a) 310 J (b) 294 J (c) 400 J (d) 500 J

121 A 1 kg block situated on a rough incline is connected to a spring of spring constant $100\ \text{N m}^{-1}$ as shown in figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley in frictionless.

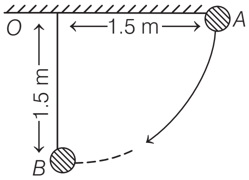


(a) 2.0 (b) 1.0
(c) 0.5 (d) 0.125

122 A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of $7\ \text{ms}^{-1}$. It hits the floor of the elevator (length of the elevator $= 3$ m) and does not rebound. What is the heat produced by the impact?

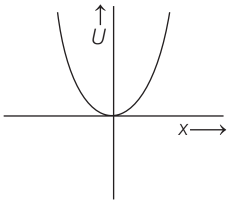
(a) 9.2 J (b) 8.82 J
(c) 10 J (d) 12 J

123 The bob of a pendulum is released from a horizontal position A as shown in the figure. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowest point B, given that it dissipated 5% of its initial energy against air resistance?



(a) $4\ ms^{-1}$ (b) $7\ ms^{-1}$
(c) $5.28\ ms^{-1}$ (d) $10\ ms^{-1}$

124 The potential energy function for a particle executing linear simple harmonic motion is given by $U(x) = \frac{1}{2}kx^2$, where k is the force constant of the oscillator. For $k = 0.5\ Nm^{-1}$, the graph $U(x)$ *versus* x is shown in the figure given below.



Find out position of a particle carrying total energy 1 J moving under this potential at which it must turn back to its original position.

(a) $\pm$ 0.5 m (b) $\pm$ 1 m
(c) $\pm$ 2 m (d) $\pm$ 3 m

125 A body is initially at rest. It undergoes one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(a) $t^{1/2}$ (b) t (c) $t^{3/2}$ (d) t^2

126 A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(a) $t^{1/2}$ (b) t (c) $t^{3/2}$ (d) t^2

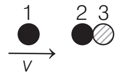
127 A pump on the ground floor of a building can pump up water to fill a tank of value $30\ m^3$ in 15 min. If the tank is 40 m above the ground and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

(a) 50 kW (b) 60 kW (c) 43.6 kW (d) 55 kW

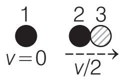
128 A large family uses 8 kW of power. Direct solar energy is incident on the horizontal surface at an average rate of $200\ Wm^{-2}$. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW?

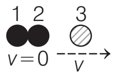
(a) $1000\ m^2$ (b) $20\ m^2$ (c) $200\ m^2$ (d) $2000\ m^2$

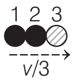
129 Two identical balls bearing in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v.



If the collision is elastic, which of the following is a possible result after collision?

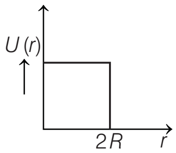
(a) $\Rightarrow$ 

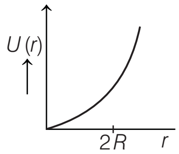
(b) $\Rightarrow$ 

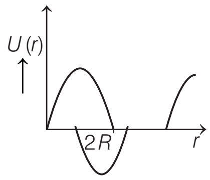
(c) $\Rightarrow$ 

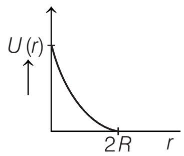
(d) None of these

130 Which of the following potential energy curves in figure given below can possibly describe the elastic collision of two billiard balls? Here, r is the distance between centre of the balls and R is the radius of each ball.

(a) 

(b) 

(c) 

(d) 

131 A person trying to lose weight (dieter) lifts a 10 kg mass to 0.5 m 1000 times. Assume that, the potential energy lost each time she lowers the mass is dissipated.

How much work does she do against the gravitational force?

(a) 50000 J (b) 20000 J
(c) 49000 J (d) 30000 J

132 Fat supplies 3.8×10^7 J of energy per kilogram, which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up by doing work of 49000 J?

(a) 6.45×10^{-3} kg (b) 9×10^{-4} kg
(c) 7×10^{-2} kg (d) 10^{-3} kg

133 An electron and proton have kinetic energy equal to 10 keV and 100 keV, respectively. The ratio of their speeds is

(a) 13.5 (b) 15.5
(c) 16.5 (d) 17.5

NCERT Exemplar

134 An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because

(a) the two magnetic forces are equal and opposite, so they produce no net effect
(b) the magnetic forces do not work on each particle
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle
(d) the magnetic forces are necessarily negligible

135 A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments; one in which the charged particle is also a proton and in another, a positron. In same time t, the work done on the two moving charged particles is

(a) same as the same force law is involved in the two experiments
(b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens
(c) more for the case of a positron, as the positron moves away a larger distance
(d) same as the work is done by charged particle on the stationary proton

136 A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200 N and is directly opposed to the motion. The work done by the cycle on the road is

(a) + 2000 J (b) – 200 J
(c) zero (d) – 20000 J

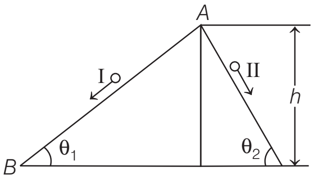
137 A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?

(a) Kinetic energy (b) Potential energy
(c) Total mechanical energy (d) Total linear momentum

138 During inelastic collision between two bodies, which of the following quantities always remain conserved?

(a) Total kinetic energy (b) Total mechanical energy
(c) Total linear momentum (d) Speed of each body

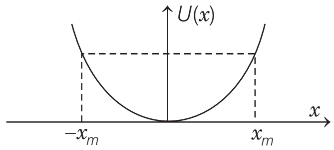
139 Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in figure.



Which of the following statement is correct?

(a) Both the stones reach the bottom at the same time but not with the same speed.
(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.
(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.
(d) Both the stones reach the bottom at different times and with different speeds.

140 The potential energy function for a particle executing linear SHM is given by $U(x) = \frac{1}{2}kx^2$, where k is the force constant of the oscillator. For $k = 0.5\ \text{Nm}^{-1}$, the graph of $U(x)$ *versus* x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If U and K indicate the PE and KE, respectively of the particle at $x = +x_m$, then which of the following is correct?



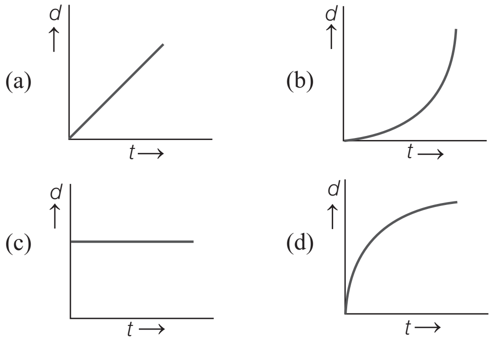
(a) $U = 0, K = E$
(b) $U = E, K = 0$
(c) $U < E, K = 0$
(d) $U = 0, K < E$

141 A body of mass 0.5 kg travels in a straight line with velocity $v = a\,x^{3/2}$, where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is

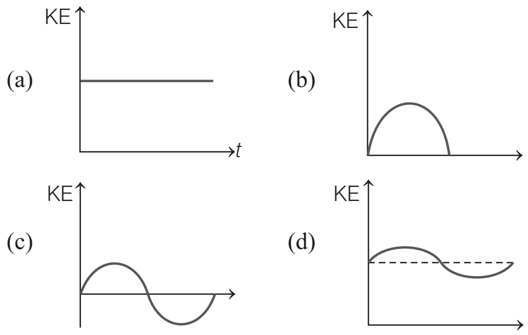
(a) 1.5 J (b) 50 J
(c) 10 J (d) 100 J

142 A body is moving unidirectionally under the influence of a source of constant power supplying energy.

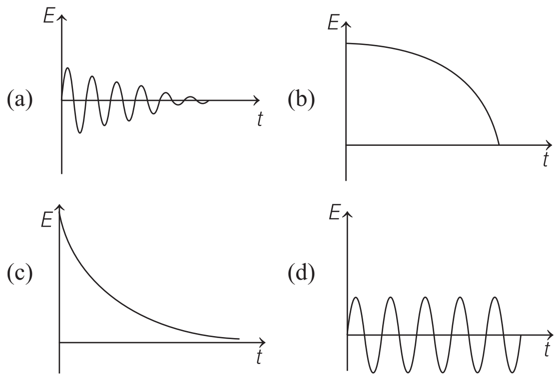
Which of the diagrams shown in figure correctly shows the displacement-time curve for its motion?



143 Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit?



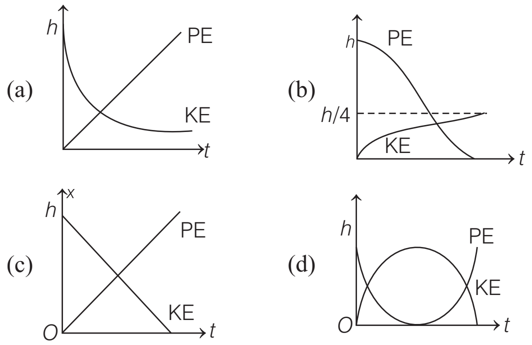
144 Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as a function of time?



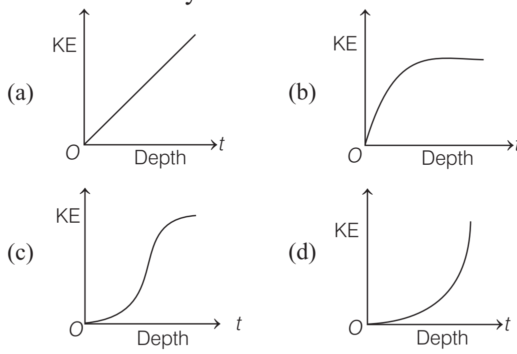
145 A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 rev min^{-1}, then its kinetic energy (in J) would be

(a) $250\pi^2$ (b) $100\pi^2$ (c) $5\pi^2$ (d) 0

146 A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the following diagrams shown in figure correctly shows the change in kinetic and potential energy of the drop during its fall on to the ground?



147 Which of the diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity?



148 In a shotput event, an athlete throws the shotput of mass 10 kg with an initial speed of 1 ms^{-1} at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 ms^{-2}, the kinetic energy of the shotput when it just reaches the ground will be

(a) 2.5 J (b) 5.0 J (c) 52.5 J (d) 155.0 J

149 A cricket ball of mass 150 g moving with a speed of 126 kmh^{-1} hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat.

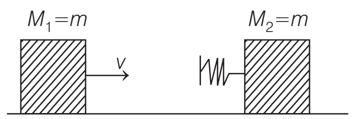
Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001s, the force that the batsman had to apply to hold the bat firmly at its place would be

(a) 10.5 N (b) 21 N

(c) 1.05×10^4 N (d) 2.1×10^4 N

150 Two blocks M_1 and M_2 having equal masses are free to move on a horizontal frictionless surface and M_2 is attached to a massless spring as shown in figure. Initially, M_2 is at rest and M_1 is moving towards M_2 with speed v and collides head-on with M_2.

Then, which of the following statement is correct?



(a) While spring is fully compressed the system, all the KE of M_1 is stored as PE of spring.

(b) While spring is fully compressed the system, momentum is not conserved, though final momentum is equal to initial momentum.

(c) If spring is massless, the final state of the M_1 is state of rest.

(d) If the surface on which blocks are moving has friction, then collision cannot be elastic.

Answers

> Mastering NCERT with MCQs

1 (d)	2 (b)	3 (d)	4 (a)	5 (a)	6 (b)	7 (d)	8 (c)	9 (b)	10 (d)
11 (b)	12 (b)	13 (a)	14 (a)	15 (c)	16 (d)	17 (b)	18 (c)	19 (b)	20 (c)
21 (b)	22 (b)	23 (a)	24 (b)	25 (c)	26 (c)	27 (d)	28 (d)	29 (c)	30 (a)
31 (c)	32 (b)	33 (c)	34 (c)	35 (c)	36 (a)	37 (c)	38 (d)	39 (a)	40 (b)
41 (c)	42 (a)	43 (b)	44 (d)	45 (d)	46 (d)	47 (a)	48 (b)	49 (b)	50 (b)
51 (a)	52 (c)	53 (a)	54 (b)	55 (c)	56 (d)	57 (c)	58 (a)	59 (a)	60 (a)
61 (b)	62 (a)	63 (d)	64 (a)	65 (c)	66 (a)	67 (a)	68 (a)	69 (a)	70 (d)
71 (c)	72 (b)	73 (a)	74 (b)	75 (c)	76 (d)	77 (a)	78 (c)	79 (a)	80 (b)
81 (c)	82 (b)	83 (c)	84 (a)	85 (b)	86 (c)	87 (a)	88 (a)	89 (c)	90 (a)
91 (b)	92 (d)								

> Special Types Questions

93 (a)	94 (a)	95 (c)	96 (a)	97 (a)	98 (c)	99 (a)	100 (d)	101 (b)	102 (d)
103 (d)	104 (b)	105 (d)	106 (b)	107 (b)	108 (d)	109 (a)	110 (b)	111 (b)	112 (a)
113 (b)	114 (b)	115 (d)	116 (b)	117 (d)	118 (b)				

> NCERT & NCERT Exemplar MCQs

119 (b)	120 (b)	121 (d)	122 (b)	123 (c)	124 (c)	125 (b)	126 (c)	127 (c)	128 (c)
129 (b)	130 (d)	131 (c)	132 (a)	133 (a)	134 (b)	135 (c)	136 (c)	137 (c)	138 (c)
139 (c)	140 (b)	141 (b)	142 (b)	143 (d)	144 (c)	145 (a)	146 (b)	147 (b)	148 (d)
149 (c)	150 (c)								

Hints & Explanations

1 (*d*) Given, $\mathbf{A} = 2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\mathbf{B} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$

$\therefore$ Scalar product of **A** and **B** is

$$\mathbf{A}\cdot\mathbf{B} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})\cdot(\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 2 + 3 + 4 = 9$$

2 (*b*) Given, $\mathbf{F} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}})$ unit

and $\mathbf{d} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ unit

$$\therefore \quad \mathbf{F}\cdot\mathbf{d} = F_x d_x + F_y d_y + F_z d_z = 3(5) + 4(4) + (-5)(3) = 16 \text{ units}$$

Now, $\mathbf{F}\cdot\mathbf{F} = F^2 = F_x^2 + F_y^2 + F_z^2$

$$= 9 + 16 + 25 = 50 \text{ units}$$

$$\Rightarrow \quad F = \sqrt{50} \text{ units}$$

and $\mathbf{d}\cdot\mathbf{d} = d^2 = d_x^2 + d_y^2 + d_z^2$

$$= 25 + 16 + 9 = 50 \text{ units}$$

$$\Rightarrow \quad d = \sqrt{50} \text{ units}$$

$$\therefore \quad \cos\theta = \frac{16}{\sqrt{50}\sqrt{50}} = \frac{16}{50} = 0.32 \quad \left(\because \cos\theta = \frac{\mathbf{F}\cdot\mathbf{d}}{Fd}\right)$$

$$\Rightarrow \quad \theta = \cos^{-1}(0.32)$$

3 (*d*) Given, force, $\mathbf{F} = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

and displacement, $\mathbf{d} = 6\hat{\mathbf{i}} + 5\hat{\mathbf{k}}$

$\therefore$ Work done, $W = \mathbf{F}\cdot\mathbf{d}$

$$= (5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}})\cdot(6\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 30 + 0 - 20 = 10 \text{ units}$$

Therefore, the work done by the force is 10 units.

4 (*a*) Given, force, $\mathbf{F} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})$ N

and positions, $\mathbf{r}_1 = (2\hat{\mathbf{i}} + \hat{\mathbf{k}})$ m and $\mathbf{r}_2 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})$m

$\therefore$ Displacement, $\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1 = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}) - (2\hat{\mathbf{i}} + \hat{\mathbf{k}})$

$$= (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})\text{m}$$

$\therefore$ Work done, $\mathbf{W} = \mathbf{F}\cdot\mathbf{s} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

$$= 3\times 2 + 3\times 1 + 0 = 6 + 3 = 9\text{J}$$

5 (*a*) When earth is moving around the sun in a circular orbit, then gravitational attraction on earth due to the sun provides required centripetal force, which is in radially inward direction, i.e. in a direction perpendicular to the direction of motion of the earth in its circular orbit around the sun.

As a result, the work done on the earth by the force will be zero, i.e. $W = Fd\cos 90° = 0$.

6 (*b*) Gravitational force is conservative and hence it is independent of path, i.e. displacement is zero.

Due to this reason, work done by gravitational force in one revolution around the sun on its elliptical path is zero.

7 (*d*) Work done by weight-lifter is zero because there is no displacement.

In a locomotive, work done is zero because force due to gravity and displacement are mutually perpendicular to each other.

In case of a person holding a suitcase on his head and standing at a bus terminal, work done is zero because there is no displacement.

Hence, options (a), (b) and (c) are correct.

8 (*c*) Initial velocity of ball is zero, i.e. $u = 0$

$\therefore$ Displacement of ball in tth second,

$$s = gt - \frac{1}{2}g \qquad \left[\because s_n = u + \frac{1}{2}g(2t-1)\right]$$

$$= g\left(t - \frac{1}{2}\right) \Rightarrow \quad s \propto \left(t - \frac{1}{2}\right)$$

or $s_1 : s_2 : s_3 = \left(1 - \frac{1}{2}\right) : \left(2 - \frac{1}{2}\right) : \left(3 - \frac{1}{2}\right) = 1 : 3 : 5$

Now, $\quad W = mgs \qquad (\because W = Fs)$

$\quad W \propto s$

$\therefore \quad W_1 : W_2 : W_3 = 1 : 3 : 5$

9 (*b*) Force of friction acting in opposite direction $= \mu\, mg$

$$= 0.2 \times 2 \times 10 = 4\,\text{N}$$

Net force on the body, $F = 10\ \text{N} - 4\,\text{N} = 6\,\text{N}$

Acceleration, $a = \dfrac{F}{m} = \dfrac{6}{2} = 3\ \text{ms}^{-2}$

As initial velocity, $u = 0$

$\therefore$ Distance travelled in 4 s, $s = \dfrac{1}{2}at^2$

$$= \frac{1}{2} \times 3 \times 16 = 24\ \text{m}$$

Work done by applied force, i.e.

$$W = F \cdot s = 10 \times 24 = 240\ \text{J}$$

10 (*d*) The weight of hanging part $\left(\dfrac{L}{3}\right)$ of chain is $\left(\dfrac{1}{3}Mg\right)$.

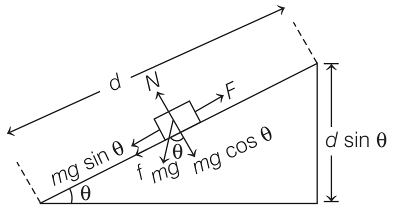
This weight acts at the centre of gravity of the hanging part, which is at a distance of $\left(\dfrac{L}{6}\right)$ from the table.

Hence, work required to pull hanging part,

$$W = \text{force} \times \text{displacement}$$

$$\therefore \quad W = \frac{Mg}{3} \times \frac{L}{6} = \frac{MgL}{18}$$

11 (*b*) The various forces acting on the block are as shown in the figure.



Given, $m = 1\,\text{kg}, \theta = 30°, F = 10\,\text{N}$ and $d = 10\ \text{m}$

$\therefore$ Work done against gravity is

$$W_g = mgd\sin\theta = (1\,\text{kg}) \times (10\ \text{ms}^{-2}) \times 10\ \text{m}$$

$$\times \sin 30° = 100 \times \frac{1}{2} = 50\ \text{J}$$

12 (*b*) Work done by a force F, which is variable in nature, in moving a particle from y_1 to y_2 is given by

$$W = \int_{y_1}^{y_2} F \cdot dy \qquad \ldots \text{(i)}$$

Given, force, $F = 20 + 10y$, $y_1 = 0$ and $y_2 = 1\,\text{m}$

Substituting the given values in Eq. (i), we get

$$\Rightarrow \quad W = \int_0^1 (20 + 10y)dy = \left[20y + \frac{10y^2}{2}\right]_0^1$$

$$= 20(1-0) + 5(1-0)^2 = 25\ \text{J}$$

$\therefore$ Work done will be 25 J.

13 (*a*) This is the case of work done by a variable force,

$$\therefore \quad W = \int_{x_1}^{x_2} F \cdot dx$$

$$W = \int_0^5 (3x^2 - 2x + 7)\,dx = (x^3 - x^2 + 7x)_0^5$$

$$W = 5 \times 5 \times 5 - 5 \times 5 + 7 \times 5$$

$$\Rightarrow \quad W = 125 - 25 + 35 = 135\ \text{J}$$

14 (*a*) Work done by the force in displacing the particle from $x = -a$ to $x = 2a$ will be

$$W = \int F dx = \int_{x=-a}^{x=2a} \left(-\frac{k}{x^2}\right) dx = \left[\frac{k}{x}\right]_{-a}^{2a}$$

$$= \frac{k}{2a} - \frac{k}{(-a)} = \frac{3k}{2a}$$

15 (*c*) Given, $F = ax + bx^2$

We know that, work done in stretching the rubber band by L is $|dW| = |Fdx|$

$$|W| = \int_0^L (ax + bx^2)\,dx = \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$$

$$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$$

$$|W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

16 (*d*) Given, $s = \dfrac{t^2}{3}$

So, $\quad v = \dfrac{ds}{dt} = \dfrac{2t}{3}; \ a = \dfrac{d^2s}{dt^2} = \dfrac{2}{3}$

Force is constant, because acceleration is constant.

Work done, $W = \displaystyle\int_0^2 F ds = \int_0^2 m\frac{d^2s}{dt^2}\,ds$

$$= \int_0^2 m\frac{d^2s}{dt^2}\frac{ds}{dt} \cdot dt$$

$$= \int_0^2 3 \times \frac{2}{3} \times \frac{2t}{3}\,dt = \frac{4}{3}\left(\frac{t^2}{2}\right)_0^2$$

$$= \frac{2}{3}[t^2]_0^2 = \frac{2}{3}[4-0] = \frac{8}{3} = 2.6\ \text{J}$$

17 (*b*) According to the graph, the acceleration a varies linearly with the coordinate x. We may write $a = \alpha x$, where α is the slope of the graph.

$$\Rightarrow \qquad \alpha = \frac{a}{x} = \frac{20}{8} = 2.5 \text{ s}^{-2}$$

The force on the brick is in the positive x-direction and according to Newton's second law, its magnitude is given by

$$F = ma = m\alpha x$$

If x_f is the final coordinate, the work done by the force is

$$W = \int_0^{x_f} F dx = m\alpha \times \int_0^{x_f} x dx$$

$$= m\alpha \times \left(\frac{x^2}{2}\right)_0^{x_f} = \frac{m\alpha \times x_f^2}{2}$$

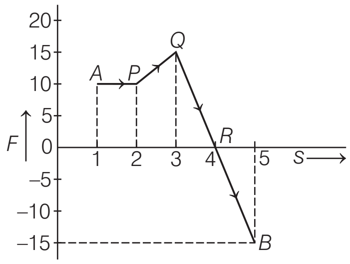
$$= \frac{10 \times 2.5 \times 64}{2} = 800 \text{ J} \quad (\text{given, } m = 10 \text{ kg})$$

18 (*c*) As we know that, total work done by varying force $F(x)$,

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\, \Delta x = \int_{x_i}^{x_f} F(x)\, dx$$

where, lim stands for the limit of the sum when Δx tends to zero.

19 (*b*) Work done = Area under F-s curve



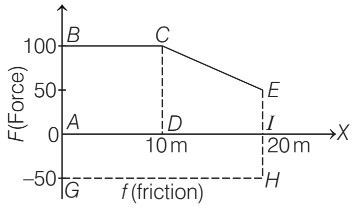
$$W_{AB} = W_{12} + W_{23} + W_{34} + W_{45}$$

$$= \text{Area under } AP + \text{Area under } PQ + \text{Area under } QR - \text{Area above } RB$$

$$= 10 \times 1 + \frac{1}{2}(10 + 15) \times 1 + \frac{1}{2} \times 1 \times 15 - \frac{1}{2} \times 1 \times 15$$

$$= 10 + 12.5 = 22.5 \text{ J}$$

20 (*c*) The plot of the applied force is shown in figure. At $x = 20$ m, $F = 50$ N ($\neq 0$). We are given that, the frictional force f is $|\mathbf{f}| = 50$ N. It opposes motion and acts in a direction opposite to $\mathbf{F}$. It is therefore shown on the negative side of the force axis. The work done by the woman is $W_F \rightarrow$ area of the rectangle $ABCD$ + area of the trapezium $CEID$.



$$\therefore \qquad W_F = 100 \times 10 + \frac{1}{2}(100 + 50) \times 10$$

$$= 1000 + 750 = 1750 \text{ J}$$

The work done by the frictional force is

$$W_f = \text{area of the rectangle } AGHI$$

$$W_f = (-50) \times 20 = -1000 \text{ J}$$

The area on the negative side of the force axis has a negative sign.

$\therefore$ Total work done by the two forces,

$$W = W_F + W_f$$

$$= 1750 - 1000 = 750 \text{ J}$$

21 (*b*) When velocity of car decreases, then its kinetic energy decreases because kinetic energy of car is directly proportional to the square of its velocity.

i.e. $\qquad K = \frac{1}{2}mv^2$ or $K \propto v^2$

22 (*b*) Given, mass of car, $m = 1000$ kg

Speed of car, $v = 80$ m/s

$\therefore$ Kinetic energy of car,

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1000 \times (80)^2$$

$$= 3.2 \times 10^6 \text{ J}$$

23 (*a*) The kinetic energy of an air molecule is

$$K = \frac{10^{-21} \text{ J}}{1.6 \times 10^{-19} \text{ J/eV}} \simeq 0.0062 \text{ eV}$$

This is the same as 6.2 meV.

24 (*b*) $\therefore$ Kinetic energy, $K = \frac{1}{2}mv^2$

Given, $\quad v_2 = (v_1 + 2)\,\text{ms}^{-1}$

$$\therefore \qquad \frac{K_1}{K_2} = \left(\frac{v_1}{v_2}\right)^2 \Rightarrow \frac{1}{2} = \frac{v_1^2}{(v_1 + 2)^2} \qquad (\because K_2 = 2K_1)$$

$$\Rightarrow \qquad v_1^2 + 4v_1 + 4 = 2v_1^2$$

$$\Rightarrow \qquad v_1^2 - 4v_1 - 4 = 0$$

$\because$ The above equation is a quadratic equation, so the roots of the equation will be

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

Then, $\qquad v_1 = \frac{4 \pm \sqrt{16 + 16}}{2} = \frac{4 \pm \sqrt{32}}{2}$

$$\Rightarrow \qquad v_1 = \frac{4 \pm 4\sqrt{2}}{2} = 2(1 \pm \sqrt{2})\,\text{ms}^{-1}$$

$$v_1 = 2(\sqrt{2} + 1) \text{ or } 2(1 - \sqrt{2})$$

Hence, option (b) is correct.

25 (*c*) Given, $m_X = 40$ kg, $m_Y = 10$ kg, $F = 80$ N

Acceleration, $a_Y = \frac{F}{m_X} = \frac{80}{40} = 2 \text{ m/s}^2$

and $$a_Y = \frac{F}{m_Y} = \frac{80}{10} = 8\text{ m/s}^2$$

As, $$K_X = K_Y$$

$$\frac{1}{2}m_X v_X^2 = \frac{1}{2}m_Y v_Y^2$$

$$\Rightarrow \quad \frac{1}{2}m_X(a_X t_X)^2 = \frac{1}{2}m_Y(a_Y t_Y)^2$$

$$\Rightarrow \quad \frac{1}{2}40\,(2\cdot t_X)^2 = \frac{1}{2}10\,(8\,t_Y)^2$$

$$\Rightarrow \quad 16t_X^2 = 64\,t_Y^2$$

$$\Rightarrow \quad \frac{t_X^2}{t_Y^2} = 4 \Rightarrow \frac{t_X}{t_Y} = \frac{2}{1}$$

Hence, the ratio of $t_X : t_Y$ is 2 : 1.

26 (*c*) Let m = mass of boy, M = mass of man,

v = velocity of boy and v' = velocity of man

According to first condition,

$$\frac{1}{2}Mv'^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) \quad \ldots\text{(i)}$$

When man speed up by 1 m/s, then

$$\Rightarrow \quad \frac{1}{2}M\,(v'+1)^2 = 1\left(\frac{1}{2}mv^2\right) \quad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\frac{1}{2}M(v'+1)^2}{\frac{1}{2}M(v')^2} = \frac{\frac{1}{2}mv^2}{\frac{1}{4}mv^2}$$

$$\Rightarrow \quad (v'+1)^2 = 2v'^2$$

$$\Rightarrow \quad v'+1 = \sqrt{2}v'$$

$$\therefore \quad v' = \frac{1}{\sqrt{2}-1}\text{ ms}^{-1}$$

27 (*d*) Given, initial velocity, $u = 10\text{ ms}^{-1}$, $m = 10\text{ kg}$, $F = 50\text{ N}$ and $t = 2\text{s}$

Acceleration of object, $a = \frac{F}{m} = \frac{50}{10} = 5\text{ m/s}^2$

If v be the final velocity, then

$v = u + at = 10 + 5\times 2 = 20\text{ m/s}$

Initial kinetic energy $= \frac{1}{2}mu^2 = \frac{1}{2}\times 10\times 10\times 10$

$= 5\times 10^2$ J

Final kinetic energy $= \frac{1}{2}mv^2 = \frac{1}{2}\times 10\times 20\times 20$

$= 20\times 10^2$ J

% increase in kinetic energy

$$= \frac{\text{Initial kinetic energy} - \text{Final kinetic energy}}{\text{Initial kinetic energy}}\times 100$$

$$= \frac{(20-5)\times 10^2}{5\times 10^2}\times 100 = 300\%$$

28 (*d*) Given, mass of particle, $m = 10\text{g} = 0.01$ kg

Radius of circle along which particle is moving, $r = 6.4\text{cm}$

Kinetic energy of particle, KE $= 8\times 10^{-4}$ J

$$\therefore \text{ KE} = \frac{1}{2}mv^2 = 8\times 10^{-4}\text{ J}$$

$$\Rightarrow v^2 = \frac{8\times 2\times 10^{-4}}{m} = \frac{16\times 10^{-4}}{0.01} = 16\times 10^{-2} \quad \ldots\text{(i)}$$

As it is given that, kinetic energy of particle is equal to 8×10^{-4} J by the end of second revolution after the beginning of motion of particle. It means, its initial velocity u is 0 m/s at this moment.

By Newton's third equation of motion,

$$v^2 = u^2 + 2a_t\,s \Rightarrow v^2 = 2a_t\,s \qquad [\because u = 0]$$

or $$v^2 = 2a_t\,(4\pi r)$$

($\because$ particle covers 2 revolutions)

$$\Rightarrow \quad a_t = \frac{v^2}{8\pi r} = \frac{16\times 10^{-2}}{8\times 3.14\times 6.4\times 10^{-2}}$$

[from Eq. (i)]

$$\therefore \quad a_t = 0.1\text{ ms}^{-2}$$

29 (*c*) It is given that, force acting on a body is inversely proportional to its velocity.

i.e. $$F \propto \frac{1}{v}$$

$$\Rightarrow \quad F = \frac{k}{v} \Rightarrow ma = \frac{k}{v}$$

$$\Rightarrow \quad m\frac{dv}{dt} = \frac{k}{v} \Rightarrow \int mv\,dv = \int k\,dt$$

$$\Rightarrow \quad m\frac{v^2}{2} = kt \Rightarrow \text{KE} \propto t \qquad \left(\because \text{KE} = \frac{1}{2}mv^2\right)$$

$\therefore$ Kinetic energy of body starting from rest is directly proportional to time.

30 (*a*) Relation between kinetic energy and momentum is

$$p_1 = \sqrt{2\,mK_1} \quad \ldots\text{(i)}$$

$\because$ Kinetic energy is increased by 4 times, so

$$K_2 = 4K_1$$

Hence, $$p_2 = \sqrt{2mK_2} = \sqrt{2m(4K_1)} = 2\sqrt{2mK_1}$$

or $$p_2 = 2\,p_1 \qquad \text{[from Eq. (i)]}$$

31 (*c*) As we know that, linear momentum,

$$p = \sqrt{2mK} \qquad \left(\because K = \frac{p^2}{2m}\right)$$

For same kinetic energy, $p \propto \sqrt{m}$

$$\Rightarrow \quad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 1:2$$

The ratio of the magnitudes of their momentum is 1 : 2.

32 (*b*) Given, initial momentum of body $= p_i$

Final momentum, $p_f = p_i + 50\%$ of p_i

$$= p_i + \frac{50}{100} \times p_i$$

$$p_f = \frac{3p_i}{2} \quad \text{...(i)}$$

Kinetic energy in terms of momentum is given as

$$K = \frac{p^2}{2m}$$

$$\therefore \quad K_i = \frac{p_i^2}{2m}$$

$$K_f = \frac{p_f^2}{2m} = \frac{\left(\frac{3p_i}{2}\right)^2}{2m} \quad \text{[from Eq. (i)]}$$

$$= \frac{9p_i^2}{8m}$$

$\therefore$ % increase in kinetic energy

$$= \frac{K_f - K_i}{K_i} \times 100$$

$$= \frac{\frac{9p_i^2}{8m} - \frac{p_i^2}{2m}}{\frac{p_i^2}{2m}} \times 100 = \frac{5}{4} \times 100 = 125\%$$

33 (*c*) Applying work-energy theorem on both moving objects,

$$\frac{1}{2} m_1 v_1^2 = Fx_1$$

and $$\frac{1}{2} m_2 v_2^2 = Fx_2$$

Since, both moving objects have same kinetic energy,

i.e. $$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 \Rightarrow Fx_1 = Fx_2$$

$$\Rightarrow \quad x_1 = x_2$$

Therefore, both the objects will come to rest at the same distance.

34 (*c*) Here, work done by a car,

i.e. $$W_g = mgh$$

$$= 1400 \times 10 \times 10 = 140000 \text{J}$$

Speed of car $= 54 \text{ kmh}^{-1} = 54 \times \frac{5}{18} = 15 \text{ms}^{-1}$

According to work-energy theorem, we get

$$W_g + W_r = \frac{1}{2} mv^2$$

$$1400 \times 10 \times 10 + W = \frac{1}{2} \times 1400 \times 15 \times 15$$

$$\Rightarrow \quad W = (700 \times 15 \times 15 - 1400 \times 10 \times 10)$$

$$= 700\,(225 - 200)$$

$$= 700 \times 25 \text{ J} = 17.5 \text{ kJ}$$

35 (*c*) Given, $m = 20\text{ g} = 0.02$ kg, $u = 150 \text{ ms}^{-1}$, $v = 0$ and $s = 10 \text{ cm} = 0.1 \text{ m}$

According to the work-energy theorem, we have

$$K - K' = W = Fs$$

$$\therefore \quad \frac{1}{2} mu^2 - 0 = Fs$$

$$\Rightarrow \quad F = \frac{mu^2}{2s} = \frac{0.02 \times (150)^2}{2 \times 0.1} = 2250 \text{ N}$$

36 (*a*) From work-energy theorem,

Work done = Change in kinetic energy

$$\Rightarrow \quad W = K_f - K_i$$

$$\Rightarrow \quad K_f = W + K_i = \int_{x_1}^{x_2} Fx dx + \frac{1}{2} mv^2$$

$$= \int_{20}^{30} -0.1x dx + \frac{1}{2} \times 10 \times (10)^2$$

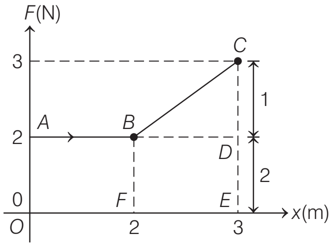
$$= -0.1 \left[\frac{x^2}{2}\right]_{20}^{30} + 500$$

$$= -0.05\,[30^2 - 20^2] + 500$$

$$= -0.05\,[900 - 400] + 500$$

$$\Rightarrow \quad K_f = -25 + 500 = 475 \text{ J}$$

37 (*c*)



$\therefore$ Work done on the particle

$$= \text{Area under the curve } ABC$$

$$W = \text{Area of square } ABFO + \text{Area of } \Delta BCD + \text{Area of rectangle } BDEF$$

$$= 2 \times 2 + \frac{1}{2} \times 1 \times 1 + 2 \times 1 = 6.5 \text{ J}$$

Now, from work-energy theorem,

$$\Delta W = K_f - K_i$$

$$\Rightarrow \quad K_f = \Delta W = 6.5 \text{ J} \quad [\because K_i = 0]$$

38 (*d*) Given, $m = 1\text{g} = 10^{-3}$ kg, $h = 1 \text{ km} = 10^3$ m

$v = 50$ m/s and $g = 10 \text{ m/s}^2$.

(i) Work done by gravitational force,

$$W_g = mgh = 10^{-3} \times 10 \times 1 \times 10^3 = 10 \text{ J}$$

(ii) Now, from work-kinetic energy theorem, we have

Change in kinetic energy = Work done by all of the forces

$$\Delta K = W_{\text{gravity}} + W_{\text{air resistance}}$$

$$\Rightarrow \quad \frac{1}{2} mv^2 = mgh + W_{\text{air resistance}}$$

$$\Rightarrow \quad W_{\text{air resistance}} = \frac{1}{2}mv^2 - mgh = m\left(\frac{1}{2}v^2 - gh\right)$$

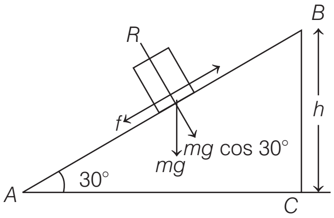
$$= 10^{-3}\left(\frac{1}{2}\times 50\times 50 - 10\times 10^3\right)$$

$$= -8.75\text{ J}$$

39 (a) According to work-energy theorem

Kinetic energy = Work against gravity + Work against friction

Thus, using the figure given below, we have



$$\frac{1}{2}mv^2 = mgh + \mu R\cdot AB$$

$$\Rightarrow \frac{1}{2}mv^2 = mgh + \mu\, mg\cos 30°\cdot\frac{h}{\sin 30°}\left[\because AB = \frac{h}{\sin 30°}\right]$$

$$\Rightarrow \frac{1}{2}\times m\times(10)^2 = mgh + \mu mg\cot 30°\times h$$

$$\Rightarrow 50 = h[10 + 0.1\times 10\times\sqrt{3}] = h\,[10 + 1.732]$$

$$\Rightarrow \quad h = \frac{50}{11.732} = 4.25\text{ m}$$

40 (b) Here, the displacement of an object is given by

$$x = (3t^2 + 5)\text{ m}$$

Therefore, velocity $(v) = \dfrac{dx}{dt} = \dfrac{d(3t^2+5)}{dt}$

or $\qquad v = 6t$ m/s $\qquad$ ...(i)

The work done in moving the object from $t = 0$ to $t = 5$ s

$$W = \int_{x_0}^{x_5} F\cdot dx \qquad \text{...(ii)}$$

The force acting on this object is given by

$$F = ma = m\times\frac{dv}{dt}$$

$$= m\times\frac{d(6t)}{dt} \qquad [\therefore \text{ using (i)}]$$

$$F = m\times 6 = 6\text{m} = 12\text{ N}$$

Also, $\quad x_0 = 3t^2 + 5 = 3\times(0)^2 + 5 = 5$ m

and at $t = 5$ s,

$$x_5 = 3\times(5)^2 + 5 = 80\text{ m}$$

Put the values in Eq. (ii),

$$W = 12\times\int_{x_0}^{x_5} dx = 12\,[80 - 5]$$

$$W = 12\times 75 = 900\text{ J}$$

41 (c) According to the work-energy theorem,

$$K_f = K_i + W = K_i + \int_{x_1}^{x_2} F dx$$

$$K_f = K_i + \int_{0.10}^{2.01}\frac{-k}{x}\,dx$$

$$= \frac{1}{2}mv_i^2 - |k\ \ln(x)|_{0.10}^{2.01}$$

$$= \frac{1}{2}\times 1\times(2)^2 - k\ \ln(2.01/0.10)$$

$$[\because m = 1\text{ kg},\ v_i = 2\text{ ms}^{-1}]$$

$$\Rightarrow \quad K_f = 2 - 0.5\times 3 \qquad [\because k = 0.5\text{ J}]$$

$$= 2 - 1.5 = 0.5\text{ J}$$

$$\Rightarrow \quad v_f = \sqrt{2K_f/m} = \sqrt{2\times 0.5/(1)} = 1\text{ ms}^{-1}$$

42 (a) Potential energy of a body above the earth's surface is given by

$$U = mgh$$

i.e. $\qquad U \propto h$

Hence, when a body is lifted above the surface of the earth, then its potential energy increases.

43 (b) Given, mass of body, $m = 2$ kg,

height, $h = 16$ m and $g = 10\text{ m/s}^2$

$\therefore$ Potential energy, $U = mgh = 2\times 10\times 16 = 320$ J

44 (d) For first ball at maximum height of vertical motion potential energy is equal to kinetic energy

i.e. $\qquad mgh_1 = \dfrac{1}{2}mu^2$

i.e. $\qquad h_1 = \dfrac{u^2}{2g}$

For second ball, thrown at an angle θ,

$$mgh_2 = mg\,\frac{u^2\cos^2\theta}{2g} \qquad \left[\because h_2 = \frac{u^2\cos^2\theta}{2g}\right]$$

Ratio of potential energy $\Rightarrow \dfrac{mgh_1}{mgh_2} = \dfrac{1}{\cos^2\theta}$

$$= \frac{1}{\cos^2 60} = 4:1$$

The ratio of their potential energies will be 4 : 1.

45 (d) Let ball is dropped from height h.

Therefore, its initial potential energy at height h,

$$U_i = mgh$$

Potential energy of ball after first bounce,

$$U_f = mg\times(75\%\text{ of }h) = mg(0.75\,h)$$

$$U_f = 0.75\,mgh$$

Loss in potential energy of ball in each bounce,

$$\Delta U = U_i - U_f = mgh - 0.75\,mgh$$

$$= 0.25\,mgh$$

$\therefore$ Percentage loss in potential energy in each bounce

$$= \frac{\Delta U}{U_i}\times 100 = \frac{0.25\,mgh}{mgh}\times 100 = 25\%$$

46 (*d*) The zero of the potential energy is arbitrary. It is set according to convenience. For the spring force, we took $U(x) = 0$, at $x = 0$, i.e. the unstretched spring has zero potential energy. For the constant gravitational force mg, we took $U = 0$ on the earth's surface.

From universal law of gravitation, the force on a body at infinite distance from the gravitational source is zero. Hence, potential energy is also zero.

Hence, options (a), (b) and (c) are correct.

47 (*a*) Potential energy of spring, $\text{PE} = \frac{1}{2}kx^2$

$\Rightarrow \quad \text{PE} \propto x^2$

$\because \quad \frac{(\text{PE})_2}{(\text{PE})_1} = \left(\frac{x_2}{x_1}\right)^2$

$\Rightarrow \quad (\text{PE})_2 = (\text{PE})_1 \times \left(\frac{x_2}{x_1}\right)^2 = 15 \times \left(\frac{4}{3}\right)^2 = 27\,\text{J}$

48 (*b*) The spring forces are

$\because \quad F = k_1x_1$ and $F = k_2x_2$

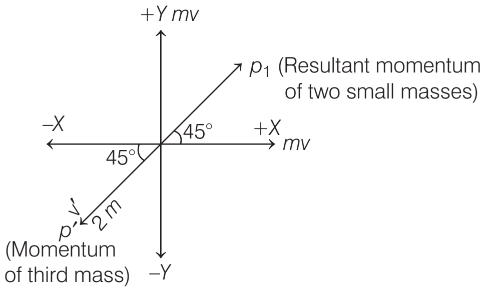
$\therefore \quad k_1x_1 = k_2x_2$

$\Rightarrow \quad \frac{k_1}{k_2} = \frac{x_2}{x_1} \quad \Rightarrow \quad \frac{(\text{PE})_1}{(\text{PE})_2} = \frac{k_1\,x_1^2}{k_2\,x_2^2}$

$$= \frac{k_1}{k_2} \times \left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1} = \frac{3}{2} \qquad \left[\because \frac{k_1}{k_2} = \frac{2}{3}\right]$$

49 (*b*) Potential energy of a body increases, when work is done against a conservative force, e.g. if we raise the height of an object, its potential energy increases. It is because work is done against gravitational force which is a conservative force.

50 (*b*) According to question, a body of mass 4 m is lying in xy-plane at rest suddenly explodes into three pieces. Two pieces of mass m which are moving perpendicular to each other with equal speeds v. So, the third part of mass $2m$ will move as shown in the figure below,



The total momentum of the system after explosion must remains zero.

Let the velocity of third part be v'. From the conservation of momentum,

$$p_1 = p' = \sqrt{p^2 + p^2 + p'\cos 90^\circ} = 2mv'$$

where, $p = mv$ = momentum due to each small mass

$\Rightarrow \quad \sqrt{2p^2 + p^2(0)} = 2mv'$

$$\sqrt{2}(mv) = (2m)v' \quad \Rightarrow v' = \frac{v}{\sqrt{2}}$$

So, total kinetic energy generated by the explosion

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(2m)v'^2$$

$$= mv^2 + m \times \left(\frac{v}{\sqrt{2}}\right)^2$$

$$= mv^2 + \frac{mv^2}{2} = \frac{3}{2}mv^2$$

51 (*a*) As we know that, potential energy,

$$U(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$$

For minimum value of U, $\frac{dU}{dx} = 0$

$\Rightarrow \quad \frac{4x^3}{4} - \frac{2x}{2} = 0 \quad \Rightarrow x^3 - x = 0$

$\Rightarrow \quad x(x^2 - 1) = 0$

$\Rightarrow \quad x = 0, x = \pm 1$

Minimum potential $= \frac{1^4}{4} - \frac{1^2}{2} = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$

According to law of conservation of energy,

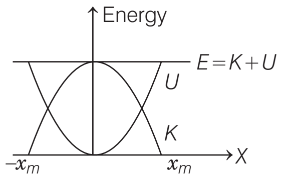
maximum kinetic energy = total mechanical energy − minimum potential energy

Maximum kinetic energy $= 2 - \left(-\frac{1}{4}\right) = \frac{9}{4}$

$\therefore$ Maximum speed $= \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2 \times 9}{1 \times 4}} = \frac{3}{\sqrt{2}}$

52 (*c*) When a spring is compressed to extreme distance $-x_m$, the kinetic energy K decreases due to resistive internal forces, while the potential energy U increases. The same is true for expansion of spring.

This variation is shown below, where parabolic plots of the potential energy U and kinetic energy K of a block attached to a spring obey Hooke's law. The two plots are complementary, i.e. one decreases as the other increases. The total mechanical energy, $E = K + U$ remains constant. This is shown correctly in option (c).



53 (*a*) Given, $k = 980\,\text{Nm}^{-1}$, $h = 40\,\text{cm}$

Stored energy of compressed spring

$$= \frac{1}{2}kx^2 = \frac{1}{2} \times 980 \times \frac{10 \times 10}{100 \times 100} = 4.9\,\text{J}$$

$$\left[\because x = 10\,\text{cm} = \frac{10}{100}\,\text{m}\right]$$

Loss of potential energy of mass m

$$= mgh = m \times g \times \frac{(40+10)}{100} = m \times 9.8 \times \frac{1}{2} = 4.9\, m$$

According to conservation of energy,

$$4.9\, m = 4.9 \Rightarrow m = \frac{4.9}{4.9} = 1\,\text{kg}$$

54 (b) Given, $m = 10^3$ kg and $k = 6.25 \times 10^3\ \text{Nm}^{-1}$

At maximum compression x_m, the potential energy U of the spring is equal to the kinetic energy K of the moving car, i.e. $\frac{1}{2}mv^2 = \frac{1}{2}kx_m^2$

$$\Rightarrow \quad 10^3 \times 5 \times 5 = 6.25 \times 10^3 \times x_m^2$$

$$\left[\because v = 18\ \text{km/h} = 18 \times \frac{5}{18} = 5\ \text{ms}^{-1}\right]$$

$$\Rightarrow x_m^2 = \frac{25}{6.25} \Rightarrow x_m = 2\,\text{m}$$

55 (c) Given, $k = 90$ N/cm $= 90 \times 10^2$ N/m,

$$x = 12\,\text{cm} = 12 \times 10^{-2}\ \text{m and } m = 16\,\text{g} = 16 \times 10^{-3}\ \text{kg}$$

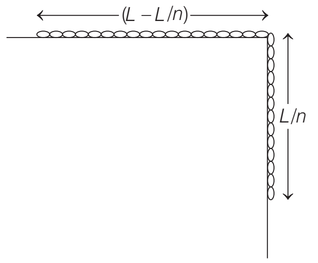
Loss in potential energy of spring = Gain in kinetic energy of ball

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$90 \times 10^2 \times (12 \times 10^{-2})^2 = 16 \times 10^{-3} \times v^2$$

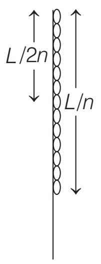
$$\Rightarrow v = \sqrt{\frac{90 \times 144 \times 10^{-2}}{16 \times 10^{-3}}} = 90\ \text{ms}^{-1}$$

56 (d)



Given, mass of the cable is M.

So, mass of $\frac{1}{n}$th part of the cable, i.e. hanged part of the cable is $= M/n$ …(i)



Now, centre of mass of the hanged part will be its middle point.

So, its distance from the top of the table will be $L/2n$.

$\therefore$ Initial potential energy of the hanged part of cable,

$$U_i = \left(\frac{M}{n}\right)(-g)\left(\frac{L}{2n}\right)$$

$$\Rightarrow \qquad U_i = -\frac{MgL}{2n^2} \qquad \text{…(ii)}$$

When whole cable is on the table, its potential energy will be zero.

$$\therefore \qquad U_f = 0 \qquad \text{…(iii)}$$

Now, using work-energy theorem,

$$W_{\text{net}} = \Delta U = U_f - U_i$$

$$\Rightarrow \qquad W_{\text{net}} = 0 - \left(-\frac{MgL}{2n^2}\right)$$

[using Eqs. (ii) and (iii)]

$$\Rightarrow \qquad W_{\text{net}} = \frac{MgL}{2n^2}$$

58 (a) When an artificial satellite comes closer to the earth, its gravitational potential energy decreases (as its height from the earth surface decreases). Since, according to the law of conservation of energy, the sum of kinetic and potential energies remain constant.

Therefore, to keep the total energy constant, kinetic energy increases and hence velocity of the satellite increases $\{\because \text{KE} = (1/2)\, mv^2\}$.

However, total energy of the satellite continuously decreases at a very small rate due to atmospheric resistance.

Therefore, speed of satellite increases progressively as it comes closer and closer to the earth.

59 (a) Given, $m = 2\,\text{kg}$, $v = 4\ \text{ms}^{-1}$, $k = 10000\,\text{Nm}^{-1}$

and $f_k = 15\,\text{N}$

Suppose the spring gets compressed by length x. Then, initial kinetic energy of the block = potential energy stored in the spring + work done against friction

$$\frac{1}{2} \times 2 \times 4^2 = \frac{1}{2} \times 10000 \times x^2 + 15x$$

$$\text{or} \qquad 5000x^2 + 15x - 16 = 0$$

On solving the above quadratic equation, we get

$$x = \frac{-15 \pm \sqrt{15^2 - 4(5000)(-16)}}{2 \times 5000} = \frac{-15 \pm 565}{10000}$$

As distance cannot be negative, so

$$x = \frac{550}{10000}\ \text{m}$$

$$\therefore \qquad x = 0.055\ \text{m} = 5.5\ \text{cm}$$

60 (a) Let a ball falls from a height h, then kinetic energy of ball at the time of just striking the ground = potential energy of ball at height h

$$\Rightarrow \qquad K = mgh$$

Similarly on rebounding, the ball moves to a maximum height h', then kinetic energy of ball on rebounding K' = potential energy of ball at a height $h'(mgh')$

$\therefore$ Loss of kinetic energy due to the rebounce,

$$K - K' = mgh - mgh' = mg\,(h - h')$$

$$= mg\left(h - \frac{80}{100}h\right) = mgh \times 0.2$$

$\therefore$ Fractional loss in kinetic energy of ball in each rebounce

$$= \frac{K - K'}{K}$$

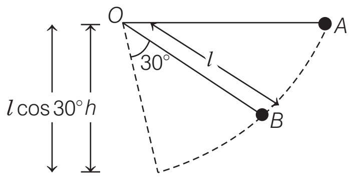
$$= \frac{mgh \times 0.2}{mgh} = 0.2$$

% Fractional loss $= 0.2 \times 100\% = 20\%$

61 (*b*) From figure given below, vertical height $= h = l\cos\theta = l\cos 30°$

Loss of potential energy $= mgh = mgl\cos 30°$

$$= \frac{\sqrt{3}}{2}\, mgl$$



According to law of conservation of energy, loss of potential energy = gain in kinetic energy

$\therefore$ Kinetic energy gained $= \dfrac{\sqrt{3}}{2}\, mgl$

62 (*a*) According to the law of conservation of energy, kinetic energy + potential energy = total mechanical energy

$$\Rightarrow \quad K + U = E$$

$$\Rightarrow \quad K = E - U \qquad \ldots\text{(i)}$$

$$\text{or} \quad U = E - K \qquad \ldots\text{(ii)}$$

Since, kinetic energy K is always positive, hence

$$K > 0$$

$$E - U > 0 \qquad \text{[from Eq. (i)]}$$

$$E > U$$

Since, potential energy may be negative, hence

$$U < 0$$

$$E - K < 0 \qquad \text{[from Eq. (ii)]}$$

$$E < K$$

If kinetic energy and potential energy both have some small positive value, then

$$U < E \text{ and } K < E$$

Hence, $U > E$ is not possible.

63 (*d*) According to the law of conservation of energy,

Initial kinetic energy = Total energy at height h

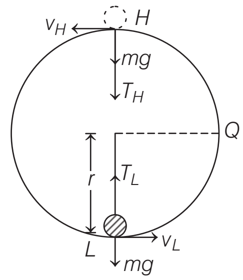
$$\frac{1}{2}mu^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) + mgh$$

$$\Rightarrow \quad 490 = 245 + 5 \times 9.8 \times h$$

$$h = \frac{245}{49} = 5\text{m}$$

Therefore, at $h = 5$m, the kinetic energy of the body becomes half of the original value.

64 (*a*) Consider the circular motion of a particle as shown below, where v_L and v_H be the velocities at bottom (L) and top (H) of the circular loop of radius r.



At bottom point, the total energy is only kinetic energy as potential energy is zero.

i.e. $$E = \frac{1}{2}mv_L^2 \qquad \ldots\text{(i)}$$

The tension on the string is

$$T_L = \frac{mv_L^2}{r} + mg$$

At top point, the tension (T_H) in string becomes zero, so

$$mg = \frac{mv_H^2}{r} \Rightarrow v_H = \sqrt{gr}$$

and total energy, $E = \dfrac{1}{2}mv_H^2 + 2mgr$

$$E = \frac{1}{2}mgr + 2mgr = \frac{5}{2}mgr \qquad \ldots\text{(ii)}$$

From law of conservation of energy, equating Eqs. (i) and (ii), we get

$$\frac{5}{2}mgr = \frac{1}{2}mv_L^2 \Rightarrow v_L = \sqrt{5gr}$$

Therefore, the ratio of kinetic energies at bottom and top is

$$\frac{K_L}{K_H} = \frac{\frac{1}{2}mv_L^2}{\frac{1}{2}mv_H^2} = \left(\frac{v_L}{v_H}\right)^2$$

$$= \frac{5gr}{gr} = \frac{5}{1} = 5:1$$

Hence, the ratio of kinetic energies is 5 : 1.

65 (*d*) For circular motion of the particle, centripetal force is required

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{k}{r^2} \Rightarrow mv^2 = \frac{k}{r}$$

$$\therefore \quad \text{KE} = \frac{1}{2}mv^2 = \frac{k}{2r}$$

As, $$\frac{dU}{dr} = -F,\ U = \int_r^0 -F\,dr = \int_r^0 -\frac{k}{r^2}\,dr$$

$$U \text{ or PE} = \left[\frac{k}{r}\right]_r^0 = -\frac{k}{r}$$

$$\text{Total energy} = \text{KE} + \text{PE} = \frac{k}{2r} + \frac{-k}{r} = \frac{-k}{2r}$$

66 (a) The total mechanical energy E of the system is conserved. We take the potential energy of the system to be zero at the lowest point A. Thus, at A,

$$E = \frac{1}{2} mv_0^2 \qquad \text{... (i)}$$

The resultant force at A provides the necessary centripetal force.

i.e. $\quad T_A - mg = \dfrac{mv_0^2}{L}$

where, T_A is the tension in the string at A.

At the highest point C, the string becomes slack, as the tension in the string (T_C) becomes zero.

Thus, at C, $mg = \dfrac{mv_C^2}{L}$ ... (ii)

and total energy, $E = \dfrac{1}{2} mv_C^2 + 2mgL$

where, v_C is the speed at C.

$\Rightarrow \qquad v_C^2 = gL$ ...(iii)

From Eqs. (ii) and (iii), we get

$$E = \frac{1}{2} mgL + 2mgL = \frac{5}{2} mgL \qquad \text{...(iv)}$$

Equating Eqs. (i) and (iv), we get

$$\frac{5}{2} mgL = \frac{m}{2} v_0^2 \quad \text{or} \quad v_0 = \sqrt{5gL}$$

67 (a) One of the greatest technical achievements of human kind occurred, when we discovered how to ignite and control fire .

We learnt to rub two flint stones together (mechanical energy), got them to heat up and to ignite a heap of dry leaves (chemical energy), which then provided sustained warmth.

68 (a) According to mass-energy equivalence,

$$E = mc^2 = 1 \times (3 \times 10^8)^2 = 9 \times 10^{16} \text{ J}$$

So, 9×10^{16} J of energy is liberated in converting 1 kg of coal into energy.

69 (a) The average human consumption in a day is

$$= \frac{10^7 \text{ J}}{4.2 \times 10^3 \text{ J / kcal}} \simeq 2400 \text{ kcal}$$

$$\left(\begin{array}{l} \because 1 \text{ cal} = 4.2 \text{ J} \\ \text{and } 1 \text{ kcal} = 4.2 \times 10^3 \text{ J} \end{array} \right)$$

71 (c) Given, $W = 600$ J and $t = 2 \text{ min} = 2 \times 60 = 120$ s

$\therefore$ Power, $P = \dfrac{W}{t} = \dfrac{600}{120} = 5$ W

72 (b) Given, $P = 1 \text{kW} = 1 \times 10^3$ W and $h = 10$ m

As, power, $P = \dfrac{\text{work done}}{\text{time taken}}$

$\Rightarrow \qquad P = \dfrac{W}{t} = \dfrac{mgh}{t} \qquad [\because W = mgh]$

$\Rightarrow \qquad \dfrac{P}{gh} = \dfrac{m}{t} \Rightarrow \dfrac{1000}{10 \times 10} = \dfrac{m}{t}$

$\Rightarrow \qquad \dfrac{1000}{100} \times 1 = m \qquad (\because t = 1 \text{ s})$

$\Rightarrow \qquad m = 10$ kg

So, quantity of water pumped out per second is 10 kg.

73 (a) Mass of the air flowing out of windmill per second $= Av\rho$ (where, ρ is density of air)

Kinetic energy per second $= \dfrac{1}{2} \times Av\rho \times v^2 = \dfrac{1}{2} A\rho v^3$

This will be the power of the windmill.

74 (b) Given, area of river, $A = 100 \text{ m} \times 5 \text{ m} = 500 \text{ m}^2$

Density of water, $\rho = 10^3$ kg/m^3 and $v = 2 \text{ ms}^{-1}$

$\therefore$ Mass of water flowing per second, $m = A\rho v$

$= 500 \times 10^3 \times 2 = 10^6$ kg/s

Power of power station, P = Kinetic energy of water flowing per second

$$= \frac{1}{2} mv^2 = \frac{1}{2} \times 10^6 \times 2^2$$

$$= 2 \times 10^6 \text{ W} = 2 \text{MW}$$

75 (c) Here, $n = \dfrac{360}{60} = 6$ bullets s^{-1}

Velocity, $v = 600 \text{ ms}^{-1}$, $m = ?$

Power of gun = Power of bullets

$$5.4 \times 10^3 = \frac{1}{2} (nm) v^2$$

$$2 \times 5400 = 6 \times m (600)^2$$

$$\Rightarrow \qquad m = \frac{2 \times 5400}{6 \times 600 \times 600} \text{ kg}$$

$$= \frac{1}{200} \text{ kg} = \frac{1000}{200} \text{ g} = 5 \text{g}$$

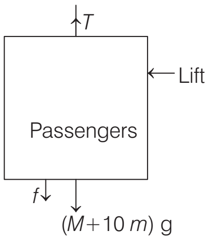
76 (d) Mass of elevator, $M = 920$ kg

Mass of all '10' passengers carried by elevator $= 10 \times m$

$= 10 \times 68 = 680$ kg

Total weight of elevator and passengers

$= (M + 10m) g = (920 + 680) \times 10 = 16000$ N



Force of friction = 6000 N

Total force (T) applied by the motor of elevator

$= 16000 + 6000 = 22000$ N

Power delivered by elevator's motor,

$P = F \cdot v = 22000 \times 3 \qquad [\because v = 3 \text{ms}^{-1}]$

$= 66000$ W

77 (*a*) It is given that, $\frac{dK}{dt}$ = constant

where, K= kinetic energy.

$$\Rightarrow \qquad K \propto t \Rightarrow v \propto \sqrt{t} \qquad \left(\because K = \frac{1}{2}mv^2\right)$$

Also, power $(P) = Fv = \frac{dK}{dt}$ = constant

$$\Rightarrow \qquad F \propto \frac{1}{v} \Rightarrow F \propto \frac{1}{\sqrt{t}}$$

78 (*c*) According to question, a body of mass 1 kg begins to move under the action of time dependent force,

$\mathbf{F} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})$ N

where, $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ are unit vectors along X and Y-axes.

$$\therefore \qquad \mathbf{F} = m\mathbf{a} \Rightarrow \mathbf{a} = \mathbf{F}/m$$

$$\Rightarrow \qquad \mathbf{a} = \frac{(2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})}{1} \qquad \text{(given, } m = 1\,\text{kg})$$

$$\Rightarrow \qquad \mathbf{a} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\ \text{ms}^{-2}$$

Acceleration, $a = \frac{dv}{dt} \Rightarrow dv = a\ dt$ ...(i)

On integrating both sides of Eq. (i), we get

$$\int dv = \int a\ dt = \int (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\ dt$$

$$\mathbf{v} = t^2\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}$$

$\therefore$ Power developed by the force at time t will be given as

$$P = \mathbf{F}\cdot\mathbf{v} = (2t\hat{\mathbf{i}} + 3t^2\hat{\mathbf{j}})\cdot(t^2\hat{\mathbf{i}} + t^3\hat{\mathbf{j}})$$

$$= (2t\cdot t^2 + 3t^2\cdot t^3)$$

$$P = (2t^3 + 3t^5)\ \text{W}$$

80 (*b*) From conservation of momentum,

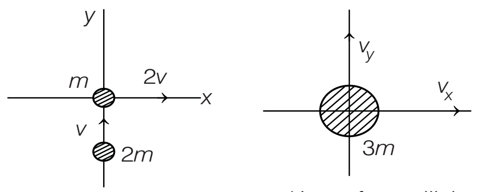
$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}$$

$$1\times(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2\times(4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}) = (1+2)\ \mathbf{v}$$

$$\Rightarrow 3\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}} = 3\mathbf{v} \Rightarrow \mathbf{v} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

$$\therefore \qquad v = |\mathbf{v}| = \sqrt{1+4+16} = 4.6\ \text{ms}^{-1}$$

81 (*c*) Consider the movement of two particles as shown below



(Just before collision) (Just after collision)

According to conservation of linear momentum in x-direction, we have

$$(p_i)x = (p_f)x \text{ or } 2mv = (2m+m)\,v_x \text{ or } v_x = \frac{2}{3}v$$

By conserving linear momentum in y-direction, we get

$$(p_i)y = (p_f)y \text{ or } 2mv = (2m+m)\,v_y \text{ or } v_y = \frac{2}{3}v$$

Initial kinetic energy of the two particles system is

$$E_i = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(v)^2$$

$$= \frac{1}{2}\times 4mv^2 + \frac{1}{2}\times 2mv^2$$

$$= 2mv^2 + mv^2 = 3mv^2$$

Final kinetic energy of the combined two particles system is

$$E_f = \frac{1}{2}(3m)(v_x^2 + v_y^2) = \frac{1}{2}(3m)\left(\frac{4v^2}{9} + \frac{4v^2}{9}\right)$$

$$= \frac{3m}{2}\left(\frac{8\,v^2}{9}\right) = \frac{4mv^2}{3}$$

Loss in the energy, $\Delta E = E_i - E_f$

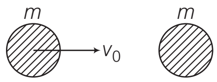
$$= mv^2\left(3 - \frac{4}{3}\right) = \frac{5}{3}mv^2$$

Percentage loss in the energy during the collision,

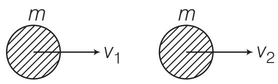
$$\frac{\Delta E}{E_i}\times 100 = \frac{5/3\ mv^2}{3mv^2}\times 100 = \frac{5}{9}\times 100 \simeq 56\%$$

82 (*b*) Final kinetic energy is 50% more than initial kinetic energy

$$\Rightarrow \qquad \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100}\times\frac{1}{2}mv_0^2 \qquad \text{...(i)}$$



Before collision



After collision

Conservation of momentum gives,

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_2 + v_1 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

$$\Rightarrow \qquad 2v_1v_2 = \frac{-v_0^2}{2}$$

$$\therefore \qquad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = 2v_0^2$$

$$\text{or} \qquad v_{\text{rel}} = \sqrt{2}\,v_0$$

83 (*c*) When two equal masses undergo a glancing elastic collision with one of them at rest, then after the collision, they will move at right angles to each other.

So, according to question, the first ball on hitting the second ball, makes an angle of 90° with the other one.

So, $\theta_1 = 90° - \theta_2 = 90° - 37° = 53°$.

84 (*a*) From conservation of momentum,

$$Mv + m\times 0 = Mv_1 + mv_2$$

where, v_1 and v_2 be the velocities of M and m after collision.

$\Rightarrow \qquad M(v - v_1) = mv_2 \qquad \ldots\text{(i)}$

Again, from the conservation of kinetic energy (as collision is of elastic nature),

$$\frac{1}{2}Mv^2 + \frac{1}{2}m \times 0 = \frac{1}{2}Mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \qquad M(v^2 - v_1^2) = mv_2^2 \qquad \ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M(v - v_1)}{M(v + v_1)(v - v_1)} = \frac{mv_2}{mv_2^2}$$

$$v_2 = v + v_1 \qquad \ldots\text{(iii)}$$

Now, solving Eqs. (i) and (iii), we get

$$M(v - v_1) = m(v + v_1)$$

$$v_1 = \frac{(M - m)v}{(M + m)} \quad \text{and} \quad v_2 = \frac{2Mv}{(M + m)}$$

As $\quad M >> m$

So, $\quad v_1 = v \Rightarrow v_2 = v + v = 2v$

85 (*b*) Since, the collision mentioned is an elastic head-on-collision. Thus, according to the law of conservation of linear momentum, we get

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

where, m_1 and m_2 are the masses of the two blocks respectively, u_1 & u_2 are their initial velocities and v_1 & v_2 are their final velocities, respectively.

Given, $\quad m_1 = m, m_2 = 4m$

$\qquad u_1 = v, u_2 = 0$ and $v_1 = 0$

$$\Rightarrow \qquad mv + 4m \times 0 = 0 + 4mv_2$$

$$\Rightarrow \qquad mv = 4mv_2 \text{ or } v_2 = \frac{v}{4} \qquad \ldots\text{(i)}$$

As, the coefficient of restitution is given as

$$e = \frac{\text{relative velocity of separation after collision}}{\text{relative velocity of approach}}$$

$$= \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{v}{4} - 0}{v - 0} \qquad \text{[from Eq. (i)]}$$

$$= \frac{1}{4}$$

$$\therefore \qquad e = 0.25$$

86 (*c*) Given, $m = 0.5\text{ kg}, v = 12\text{ m/s}, \Delta t = 1\text{s}, \theta = 30°$

Force applied by wall on ball, $F = \frac{\Delta p}{\Delta t}$

or $\qquad F = \frac{(p_f)_H - (p_i)_H}{\Delta t}$

In this elastic collision, final and initial velocities will be same but direction will be changed.

$\therefore$ Horizontal component, $(p_f)_H = mv\cos\theta$

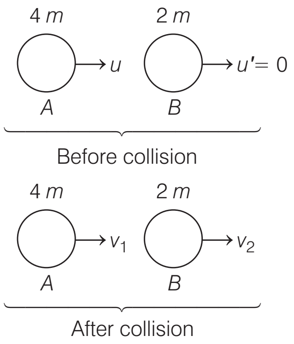
and $(p_i)_H = -mv\cos\theta$

$$\therefore \qquad F = \frac{mv\cos\theta - (-mv\cos\theta)}{\Delta t} = \frac{2mv\cos\theta}{\Delta t}$$

$$= \frac{2 \times 0.5 \times 12 \times \cos 30°}{1} = 6\sqrt{3}\text{ N}$$

87 (*a*) In head-on-elastic collision, momentum and kinetic energy before and after the collision are conserved.

The given situation of collision can be drawn as



Applying conservation of linear momentum,

Initial momentum of system = Final momentum of system

$$\Rightarrow \qquad (4m)u + (2m)u' = (4m)v_1 + (2m)v_2$$

$$4mu = 4mv_1 + 2mv_2 \qquad [\because u = 0]$$

or $\qquad 2u = 2v_1 + v_2 \qquad \ldots\text{(i)}$

The kinetic energy of A before collision is

$$(\text{KE})_A = \frac{1}{2}(4m)u^2 = 2mu^2$$

Kinetic energy of B before collision, $(\text{KE})_B = 0$

The kinetic energy of A after collision is

$$(\text{KE}')_A = \frac{1}{2}(4m)v_1^2 = 2mv_1^2$$

Kinetic energy of B after collision,

$$(\text{KE}')_B = \frac{1}{2}(2m)v_2^2 = mv_2^2$$

As, initial kinetic energy of the system = final kinetic energy of the system

$$\Rightarrow (\text{KE})_A + (\text{KE})_B = (\text{KE}')_A + (\text{KE}')_B$$

$$2mu^2 + 0 = 2mv_1^2 + mv_2^2$$

$$2mu^2 = 2mv_1^2 + mv_2^2 \text{ or } 2u^2 = 2v_1^2 + v_2^2 \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$4u^2 = 4v_1^2 + v_2^2 + 4v_1v_2$$

$$\Rightarrow 4v_1^2 + 2v_2^2 = 4v_1^2 + v_2^2 + 4v_1v_2 \Rightarrow v_2 = 4v_1$$

From Eq. (i), we get

$$2u = 2v_1 + 4v_1$$

$$v_1 = \frac{1}{3}u \quad \text{and } v_2 = \frac{4}{3}u$$

or the final velocity of A can be directly calculated by using the formula.

The velocity after collision is given by

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$= \left(\frac{4m - 2m}{4m + 2m}\right)u + \frac{2(2m) \times 0}{(4m + 2m)} [\because u_2 = u' = 0]$$

$$= \frac{2m}{6m}u = \frac{1}{3}u$$

$\therefore$ Net decreases in kinetic energy of A,

$$\Delta\text{KE} = (\text{KE})_A - (\text{KE}')_A$$
$$= 2mu^2 - 2mv_1^2 = 2m(u^2 - v_1^2)$$

Substituting the value of v_1, we get

$$\Delta\text{KE} = 2m\left(u^2 - \frac{u^2}{9}\right) = \frac{16\,mu^2}{9}$$

$\therefore$ The fractional decreases in kinetic energy is

$$\frac{\Delta\text{KE}}{(\text{KE})_A} = \frac{16mu^2}{9} \times \frac{1}{2mu^2} = \frac{8}{9}$$

88 (a) Mass of toy truck, $m_t = 2m$

Mass of toy car, $m_c = m$

Initial speed of truck, $v_t = v$

and initial speed of car, $u_c = 0$

If v_1 and v_2 be the final velocity of truck and car after collision, then by law of conservation of momentum,

Total initial momentum = Total final momentum

$$m_t u_t + m_c u_c = m_t v_1 + m_c v_2$$
$$2mv + 0 = 2mv_1 + mv_2$$
$$2v = 2v_1 + v_2 \quad \ldots\text{(i)}$$

For elastic collision, coefficient of restitution, $e = 1$

i.e. $$\frac{v_2 - v_1}{u_t - u_c} = 1$$

$$\frac{v_2 - v_1}{v - 0} = 1$$

$\Rightarrow$ $$v = v_2 - v_1 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$2v_2 - 2v_1 = 2v_1 + v_2$$
$$v_2 = 4v_1$$

$\Rightarrow$ $$2v = \frac{2v_2}{4} + v_2 \quad \text{[from Eq. (i)]}$$

$\Rightarrow$ $$v_2 = \frac{4v}{3}$$

89 (c) Given, speed of objects = u m/s

Since, both objects collide at 90°.

Hence, by the law of conservation of momentum,

Total momentum before collision

= Total momentum after collision

$$|mu\hat{\mathbf{i}} + mu\hat{\mathbf{j}}| = p_f$$

$$\sqrt{m^2u^2 + m^2u^2} = p_f \Rightarrow p_f = \sqrt{2}\,mu$$

90 (a) Given, mass of body, $m_1 = 5 \times 10^3$ kg

Velocity, $v_1 = 2$ m/s

Mass of another body, $m_2 = 15 \times 10^3$ kg

For perfectly inelastic collision ($e = 0$),

Loss in kinetic energy of system,

$$\Delta E_K = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2} \times v_1^2$$

$$= \frac{1}{2} \times \frac{5 \times 10^3 \times 15 \times 10^3}{5 \times 10^3 + 15 \times 10^3} \times 2^2$$

$$= 7.5 \times 10^3 \text{ J} = 7.5 \text{ kJ}$$

91 (b) Suppose a ball rebounds with speed v. As at collision, the speed become zero. So, from equation of motion,

$$v^2 - u^2 = 2gh$$
$$v^2 = 2gh \quad [\because u = 0]$$

$\Rightarrow$ $$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}$$

Energy of a ball just after rebound, $E = \frac{1}{2}mv^2 = 200\,\text{m}$

As, 50% of energy loses in collision means just before collision energy is 400 m.

According to the law of conservation of energy, we have

$$\frac{1}{2}mv_0^2 + mgh = 400\,\text{m}$$

$\Rightarrow$ $$\frac{1}{2}mv_0^2 + m \times 10 \times 20 = 400\,\text{m}$$

$$\frac{v_0^2}{2} = 200 \Rightarrow v_0 = 20\,\text{ms}^{-1}$$

92 (d) In collision with the ground, the velocity of ball before collision is

$$v_i = \sqrt{2gh_i} \quad [\text{using } v^2 = u^2 + 2gh]$$

and after collision is

$$v_f = ev_i \quad \left[\because e = \frac{\text{velocity after collision}}{\text{velocity before collision}}\right]$$
$$= e\sqrt{2gh_i}$$

$\therefore$ Height attained after first rebound,

$$h_f = \frac{v_f^2}{2g} = e^2 h_i$$

Similarly, after nth rebound, velocity is

$$v_n = e^n v_i = e^n\sqrt{2gh_i}$$

and height attained is $h_n = \frac{v_n^2}{2g} = e^{2n}h_i$

For third rebound, $n = 3$ and $h_i = h$

$\therefore$ $$h_3 = e^{2 \times 3}h = e^6 h$$

93 (a) According to work-energy theorem, work done by a body is equal to change in kinetic energy of the body.

$\Rightarrow$ $$W = \Delta\text{KE} = \frac{1}{2}mv^2 \quad \ldots\text{(i)}$$

But, W = Stopping force $\times$ Stopping distance

$$W = F \cdot d \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we have

$$\text{Stopping distance } (d) = \frac{\text{Kinetic energy}\left(\frac{1}{2}mv^2\right)}{\text{Stopping force}\,(F)}$$

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

94 (*a*) In case of friction, work done in moving a body over a closed path is never zero. It is because some work is converted into heat energy.

So, friction is a non-conservative force.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

95 (*c*) Mechanical energy consists of both PE and KE. In the given cases, some of the mechanical energy is converted into heat energy and it is more in the case when inclination is less due to increased (as θ decreases, value of $\cos\theta$ will increases) friction force on an inclined plane.

$$f_r = \mu mg\cos\theta$$

The coefficient of friction does not depend on the angle of inclination of the plane. It depends only on the nature of surfaces in contact.

Therefore, Assertion is correct but Reason is incorrect.

96 (*a*) As momentum of a body increases by 50% of its initial momentum, $p_2 = p_1 + 50\%$ of $p_1 = \frac{3}{2}p_1$

$$\therefore \quad v_2 = \frac{3}{2}v_1$$

As, $\quad K \propto v^2$; so $K_2 = \frac{9}{4}K_1$

$$\text{Increase in KE} = \frac{K_2 - K_1}{K_1} \times 100$$

$$= \frac{\frac{9}{4}K_1 - K_1}{K_1} \times 100$$

$$= 125\%$$

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

97 (*a*) As we know that, spring force, $F_s = kx$

Given, $\quad F = k_1x_1 = k_2x_2 ; \frac{k_1}{k_2} = \frac{x_2}{x_1}$

The work done in stretching is equal to elastic potential energy,

i.e. $\quad W = E = kx^2$

$$\Rightarrow \frac{W_1}{W_2} = \frac{E_1}{E_2} = \frac{k_1x_1^2}{k_2x_2^2} = \frac{k_1}{k_2}\cdot\left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1} \quad \left(\because \frac{x_1}{x_2} = \frac{k_2}{k_1}\right)$$

$\Rightarrow$ Since, $k_1 > k_2$, so $W_2 > W_1$.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

98 (*c*) $\text{Power} = \frac{\text{Work done (or energy)}}{\text{Time}}$

$\Rightarrow$ Work done $=$ Power $\times$ Time

$$W = P \times t$$

If $\quad P = 1\,\text{kilowatt}, t = 1\,\text{hour}$, then

$$W = 1\,\text{kilowatt} \times 1\,\text{hour} = 1\,\text{kilowatt-hour}$$
$$= 10^3\,\text{watt} \times 60 \times 60\,\text{s}$$
$$= 3.6 \times 10^6\,\text{J}$$

Therefore, Assertion is correct but Reason is incorrect.

99 (*a*) The mass may be converted into energy and given by using Einstein's equation,

$$\therefore \quad E = mc^2$$

So, it is conserved as a single entity called as mass-energy relation.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

100 (*d*) From work-energy theorem,

$$W = \Delta\text{KE} = \frac{1}{2}mv^2 \quad \ldots\text{(i)}$$

The power, $P = \frac{\text{work done}}{\text{Time}} = \frac{W}{t}$

$$\Rightarrow \quad W = P \times t \quad \ldots\text{(ii)}$$

From Eqs (i) and (ii), we get

$$P \times t = \Delta\text{KE} = \frac{1}{2}mv^2 \quad \ldots\text{(iii)}$$

$\therefore$ Power multiplied with time is equal to the change in kinetic energy.

Also, $\quad P = F \cdot v$

From Eq. (iii),

$$v^2 \propto t \quad \text{or} \quad v \propto t^{1/2}$$

Differentiating Eq. (iv), we get

$$\frac{dv}{dt} \propto t^{-1/2} \quad \text{or} \quad a \propto t^{-1/2}$$

Thus, the motion is not uniformly accelerating.

Therefore, Assertion is incorrect but Reason is correct.

101 (*b*) In completely inelastic collision of two particles, they stick together and move as a single particle with a common velocity.

In perfectly inelastic collision of two particles maximum loss of kinetic energy occurs but they do not lose all their energy.

Principle of conservation of momentum holds true for all kinds of collisions.

Therefore, Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

102 (*d*) The work done in displacing an object by applying force F is given by

$$W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta$$

So, work done will be zero, when

(i) either applied force F or displacement s is zero.

(ii) the force and displacement are mutually perpendicular to each other, i.e. $\theta = 90°$.

So, all statements are correct.

103 (d) A force $F(x)$ is conservative, if it can be derived from a scalar quantity $V(x)$ by the relation given by equation, $\Delta V = -F(x)\Delta x$.

The work done by the conservative force depends only on the end points. This can be seen from the relation,

$$W = K_f - K_i = U(x_i) - U(x_f)$$

which depends on the end points.

The work done by this force in a closed path is zero. This is once again apparent from equation, $K_i + U(x_i) = K_f + U(x_f)$, since $x_i = x_f$.

So, all statements are correct.

104 (b) The statements I and III are correct but rest is incorrect and it can be corrected as,

In case of sliding on a rough surface, mechanical energy is not conserved and a fraction of it is converted into heat. Sliding causes displacement of point of contact.

106 (b) The statements I and III are correct but rest is incorrect and it can be corrected as,

During collision time, some KE is stored as PE in the form of deformation.

107 (b) The statements II and III are correct but rest is incorrect and it can be corrected as,

If the total energy of the reactants is more than the products of reaction, then heat is released and reaction is said to be an exothermic.

108 (d) If the work done or the kinetic energy depend on other factors such as the velocity or the particular path taken by the object, the force would be called non-conservative.

Thus, the statements given in options (a) and (b) are correct, rest is incorrect.

109 (a) The statement given in option (a) is correct but rest are incorrect and these can be corrected as,

Like velocity, potential energy is also measured only in relative terms. Their absolute value cannot be determined.

Difference of velocity and difference of potential energy can be measured. So, acceleration can also be measured.

110 (b) In elastic collision, the conservation of mechanical energy consider only conservative force while conservation of energy consider both conservative and non-conservative force.

Mass converted into energy in nuclear reaction is called nuclear energy.

Thus, the statement given in option (b) is correct, rest are incorrect.

111 (b) Till the end of the nineteenth century, physicists believed that in every physical and chemical process, the mass of an isolated system is conserved.

Matter might change its phase, e.g. glacial ice could melt into a gushing stream, but matter is neither created nor destroyed.

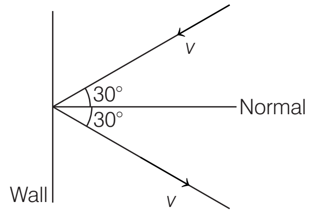
But Einstein showed that matter can be destroyed into energy and *vice-versa*.

Thus, the statement given in option (b) is incorrect, rest are correct.

112 (a) In a conservative field, loss of PE or gain of KE depends only on the initial and final points and not on path covered. So, kinetic energy at point B will be equal to PE, i.e. mgh.

Thus, the statement given in option (a) is correct, rest are incorrect.

113 (b) Linear momentum remains conserved in elastic collision as well as in inelastic collision but kinetic energy remains conserved only in an elastic collision.

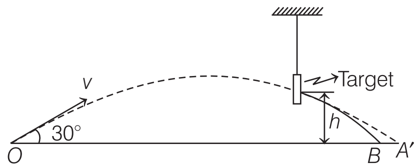


Given that the speed of the molecule of gas is same before and after the collision, therefore its kinetic energy is conserved. Hence, the collision is elastic.

A molecule in a gas container hits a horizontal wall with speed of 200 ms^{-1} and angle 30° with the normal and rebounds with the same speed. During this process, momentum of the system remains conserved.

Thus, the statement given in option (b) is correct, rest are incorrect.

114 (b) Consider the below diagram for the given situation in the question.



Conserving energy between O and target,

$$U_i + K_i = U_f + K_f$$

$$\Rightarrow \quad 0 + \frac{1}{2}mv^2 = mgh + \frac{1}{2}mv'^2$$

$$\Rightarrow \quad \frac{(v')^2}{2} = \frac{v^2}{2} - gh$$

$$\Rightarrow \quad (v')^2 = v^2 - 2gh$$

$$\Rightarrow \quad v' = \sqrt{v^2 - 2gh} \qquad \text{...(i)}$$

where, v' is the speed of the bullet just before hitting the target.

Let speed after emerging from the target is v'', then

By question, $\frac{1}{2}(mv'')^2 = \frac{1}{2}\left[\frac{1}{2}m(v')^2\right]$

$$\frac{1}{2}m(v'')^2 = \frac{1}{4}m(v')^2 = \frac{1}{4}m[v^2 - 2gh]$$

[from Eq. (i)]

$$\Rightarrow \quad (v'')^2 = \frac{v^2 - 2gh}{2} = \frac{v^2}{2} - gh$$

$$\Rightarrow \quad v'' = \sqrt{\frac{v^2}{2} - gh} \qquad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{v'}{v''} = \frac{\sqrt{v^2 - 2gh}}{\frac{\sqrt{v^2 - 2gh}}{\sqrt{2}}} = \sqrt{2}$$

$$\Rightarrow \quad v'' = \frac{v'}{\sqrt{2}} = \sqrt{2}\left(\frac{v'}{2}\right)$$

$$\Rightarrow \quad \frac{v''}{v'/2} = \sqrt{2} = 1.414 > 1$$

$$\Rightarrow \quad v'' > \frac{v'}{2}$$

Hence, after emerging from the target velocity of the bullet (v'') is more than half of its earlier velocity v' (velocity before emerging into the target).

As the velocity of the bullet changes to v'' which is less than v', hence path followed will change and the bullet reaches at point B instead of A' as shown in the figure but motion is still parabolic.

So, only statement given in option (b) is correct.

116 (*b*) Work done by an agent is given by

$$W = \mathbf{F}\cdot\mathbf{s} = Fs\cos\theta$$

where, F is the applied force, s is the displacement and θ is the smaller angle between F & s.

(A) If $\theta < 90°$, i.e. acute angle, then work done is positive, as in case of coolie lifting luggage.

(B) If $\theta = 90°$, i.e. right angle, then work done is zero, as in case of satellite rotation around the earth.

(C) If $\theta > 90°$, i.e. obtuse angle, work done is negative, as in case of friction.

Hence, A $\rightarrow$ 3, B $\rightarrow$ 2 and C $\rightarrow$ 1.

118 (*b*) Elastic collision keeps kinetic energy constant.

In non-elastic collision, there is loss of kinetic energy due to deformation at the point of contact.

Scattering is also a type of collision without bringing bodies in contact. So, momentum remains conserved.

Hence, A $\rightarrow$ 2, B $\rightarrow$ 1 and C $\rightarrow$ 3.

119 (*b*) Work done, $W = \mathbf{F}\cdot\mathbf{d}$

Force, $\mathbf{F} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$N

Displacement, $\mathbf{d} = (4\hat{\mathbf{k}})$ m $= (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$m

$\therefore$ Work done by the force is given by

$$W = \mathbf{F}\cdot\mathbf{d} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})\cdot(0\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$$

$$= (-1\times 0) + (2\times 0) + (3\times 4) = 0 + 12$$

$[\because \hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1 \text{ and } \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{i}} = 0]$

$$= 12 \text{ J}$$

120 (*b*) In first case, the man applies a force on the mass 15 kg in vertically upward direction against its weight and walks 2 m in horizontal direction. So, the angle between the applied force and displacement is 90°.

$\therefore$ Work done, $W = Fd\cos 90° = 0 \quad (\because \cos 90° = 0)$

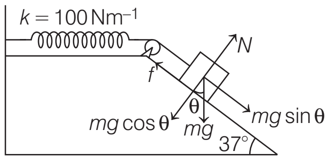
In second case, the man applies a force in horizontal direction and moves also in horizontal direction. So, angle between the applied force and displacement is 0°.

$\therefore$ Work done $= Fd\cos 0° = Fd = mg \times d$

$$= 15 \times 9.8 \times 2 = 294 \text{ J}$$

Difference in work done = 294 − 0 = 294 J

121 (*d*) Given, spring constant, $k = 100\,\text{Nm}^{-1}$, mass of block, $m = 1\,\text{kg}$, $\theta = 37°$ and distance moved by block, $x = 10\,\text{cm} = 0.1$ m.



As shown in figure, the net accelerating force acting on block is

$$F = mg\sin\theta - f = mg\sin\theta - \mu N$$

$$= mg\sin\theta - \mu\, mg\cos\theta$$

$\therefore$ Work done by the force F for motion of block,

$$W = Fx = mg(\sin\theta - \mu\cos\theta)x$$

When the block stops, the work done is stored in the spring in the form of its potential energy, $U = \frac{1}{2}kx^2$

$$\therefore \quad mg(\sin\theta - \mu\cos\theta)x = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \mu = \frac{1}{\cos\theta}\left(\sin\theta - \frac{kx}{2mg}\right)$$

Substituting the values, we get

$$\mu = \frac{1}{\cos 37°}\left(\sin 37° - \frac{100\times 0.1}{2\times 1\times 10}\right)$$

$$\Rightarrow \quad \mu = \frac{1}{0.8}[0.6 - 0.5] = 0.125$$

122 (*b*) Mass of the bolt, $m = 0.3$ kg

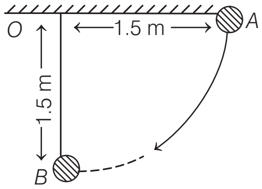
Length of the elevator, $h = 3$ m

As the bolt does not rebound, therefore its total PE is converted into heat.

$\therefore$ Heat produced = PE of the bolt

$$= mgh = 0.3 \times 9.8 \times 3 = 8.82 \text{ J}$$

123 (*c*) Length of the pendulum = 1.5 m



Potential energy of the bob at position $A = mgh$

As bob moves from position A towards position B, its potential energy converted into kinetic energy. 5% of its potential energy is dissipated against air resistance.

KE at position B = 95% of its PE at position A

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

or $$v = \sqrt{\frac{2 \times 95 \times gh}{100}} = \sqrt{\frac{19}{10} \times 9.8 \times 1.5}$$

$$= 5.28\ \text{ms}^{-1}$$

124 (*c*) Total energy of a particle executing linear simple harmonic motion at any instant is given by

$$E = \text{PE} + \text{KE} \qquad \ldots\text{(i)}$$

Given, potential energy, $V(x) = \frac{1}{2}kx^2$

Total energy, $E = 1\,\text{J}$ and $k = 0.5\,\text{Nm}^{-1}$

When particle is at extreme position (the position from which the particle starts to come back to its mean position), then speed of the particle is zero and hence

$$\text{KE} = \frac{1}{2}mv^2 = 0 \qquad (\because v = 0)$$

Substituting values in Eq. (i), we get

$$1 = \frac{1}{2}kx^2 + 0 \Rightarrow 1 = \frac{1}{2} \times 0.5 \times x^2$$

$$\Rightarrow \quad x^2 = \frac{2}{0.5} = 4 \text{ or } x = \pm 2\,\text{m}$$

125 (*b*) Let a body of mass m which is initially at rest undergoes one-dimensional motion under a constant force F with a constant acceleration a.

Acceleration, $$a = \frac{F}{m} \qquad \ldots\text{(i)}$$

Using equation of motion, $v = u + at$

$$\Rightarrow \quad v = 0 + \frac{F}{m}\cdot t \qquad (\because u = 0)$$

$$\Rightarrow \quad v = \frac{F}{m}t \qquad \ldots\text{(ii)}$$

Power delivered, $P = Fv$

Substituting the value from Eq. (ii), we get

$$\Rightarrow \quad P = F \times \frac{F}{m} \times t \Rightarrow P = \frac{F^2}{m}t$$

Dividing and multiplying by m in RHS,

$$P = \frac{F^2}{m^2} \times mt = a^2\,mt \qquad \text{[using Eq. (i)]}$$

As, mass m and acceleration a are constants.

$$\therefore \quad P \propto t$$

126 (*c*) Velocity attained by the body in time t can be obtained using equation of motion, $v = u + at$

$$v = 0 + at \qquad [\because u = 0]$$

or $$v = at \qquad \ldots\text{(i)}$$

Power delivered, $P = Fv$

But, $F = ma$

$$\therefore \quad P = ma \times at \qquad \text{[using Eq. (i)]}$$

$$P = ma^2t$$

or $$a = \sqrt{\frac{P}{mt}} \qquad \ldots\text{(ii)}$$

Using equation of motion, we get

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = \frac{1}{2}\left(\sqrt{\frac{P}{mt}}\right) \times t^2$$

$$s \propto t^{3/2}$$

127 (*c*) Time taken to fill the tank = 15 min

$$= 15 \times 60 = 900\,\text{s}$$

Work done $= mgh = (V \times d)gh$

[$\because$ mass = volume × density]

$$= (30 \times 1000) \times 9.8 \times 40$$

$$= 1.176 \times 10^7\ \text{J}$$

$\therefore$ Power required, $P = \frac{\text{Work done}}{\text{Time taken}}$

$$\Rightarrow \quad P = \frac{1.176 \times 10^7}{900} = 13.07 \times 10^3\ \text{W}$$

$$= 13.07\ \text{kW}$$

Efficiency of the pump, $\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$

$\therefore$ Input power $= \frac{\text{Output power}}{\eta} \times 100 = \frac{13.07}{30} \times 100$

$$= \frac{130.7}{3}\ \text{kW} = 43.56\ \text{kW}$$

The pump consumes, 43.6 kW electric power.

128 (*c*) Power used by a family, $P = 8\,\text{kW}$

Solar energy incident on horizontal surface per square metre = 200 W

Electrical energy obtained from solar energy per unit area $= 200 \times \frac{20}{100}\ \text{W} = 40\ \text{W}$

$\therefore$ Area needed to supply 8 kW $= \frac{8\ \text{kW}}{40\ \text{W}} = \frac{8000}{40}$

$$= 200\ \text{m}^2$$

129 (*b*) When two bodies of equal masses collides elastically, their velocities are interchanged.

When ball 1 collides with ball 2, then velocity of ball 1, v_1 becomes zero and velocity of ball 2 becomes v.

Similarly when ball 2 collides with ball 3, then velocity of ball 2 becomes zero and velocity of ball 3 becomes v. Thus, option (b) denotes this condition.

130 (*d*) When two billiard balls collide, then distance between their centres is $2R$. Due to impact of collision, there is small temporary deformation of balls. In this process, KE of ball is gradually reduced to zero and converted into elastic potential energy $U(r)$ of balls.

This phenomenon can be successfully explained only by potential energy curve in option (d), because here as $r < 2R$, the potential energy function $U(r)$ is increasing gradually on decreasing value of r and become maximum at $r = 0$.

131 (*c*) Mass, $m = 10$ kg, height, $h = 0.5$ m

Number of times the mass lifted, $n = 1000$

Work done against gravitational force

$$= n \times mgh = 1000 \times 10 \times 9.8 \times 0.5$$
$$= 49000 \text{ J}$$

132 (*a*) Energy supplied by fat per kilogram $= 3.8 \times 10^7$ J

Mechanical energy supplied by fat per kilogram

$$= 20\% \text{ of total energy supplied by fat}$$
$$= \frac{20}{100} \times 3.8 \times 10^7 = 0.76 \times 10^7 \text{ Jkg}^{-1}$$

Fat used up by the dieter $= \dfrac{1}{0.76 \times 10^7} \times 49000$

$$= 6.45 \times 10^{-3} \text{ kg}$$

133 (*a*) The ratio of kinetic energy of electron and proton is

$$\frac{K_e}{K_p} = \frac{\frac{1}{2} m_e v_e^2}{\frac{1}{2} m_p v_p^2} = \frac{10}{100} \Rightarrow \left(\frac{v_e}{v_p}\right)^2 = \frac{10 \times m_p}{100\, m_e}$$

$$\Rightarrow \left(\frac{v_e}{v_p}\right)^2 = \frac{1}{10} \times \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = \frac{183.5}{1}$$

$$\frac{v_e}{v_p} = \frac{13.5}{1}$$

134 (*b*) When an electron and a proton are moving under influence of their mutual forces, the magnetic forces will be perpendicular to their motion, hence no work is done by these forces.

135 (*c*) Force between two protons is same as that of between proton and a positron.

As positron is much lighter than proton, it moves away through much larger distance compared to proton.

We know that, work done = force × distance. As forces are same in case of proton and positron but distance moved by positron is larger, hence work done will be more in case of positron.

136 (*c*) Here, work is done by the frictional force on the cycle and is equal to $-200 \times 10 = -2000$ J.

As the road is not moving, hence work done by the cycle on the road = zero.

137 (*c*) As the body is falling freely under gravity, the potential energy decreases and kinetic energy increases but total mechanical energy (PE + KE) of the body and earth system will be constant as external force on the system is zero.

138 (*c*) When we are considering the two bodies as system, the total external force on the system will be zero.

Hence, total linear momentum of the system remains conserved.

139 (*c*) As the given tracks are frictionless, hence mechanical energy will be conserved. As both the tracks having common height h.

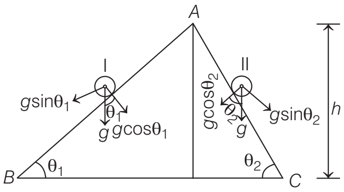
From conservation of mechanical energy,

$$\frac{1}{2} mv^2 = mgh \qquad \text{(for both tracks I and II)}$$

$$v = \sqrt{2gh}$$

Hence, speed is same for both stones.

For stone as shown below, a_1 = acceleration along inclined plane $= g \sin\theta_1$



Similarly, for stone II, $a_2 = g \sin\theta_2$ as $\theta_2 > \theta_1$, hence $a_2 > a_1$.

Also, lengths for track II are also less, hence stone II reaches earlier than stone I.

Thus, the statement given in option (c) is correct, rest are incorrect.

140 (*b*) Total energy, $E = \text{PE} + \text{KE}$ ...(i)

When particle is at $x = x_m$, i.e. at extreme position, it returns back. Hence, at $x = x_m$, $v = 0 \Rightarrow \text{KE} = 0$

From Eq. (i),

$$E = \text{PE} + 0 = \text{PE} = U(x_m) = \frac{1}{2} kx_m^2$$

$$\Rightarrow \qquad E = U$$

Thus, option (b) is correct.

141 (*b*) Given, $v = ax^{3/2}$, $m = 0.5$ kg, $a = 5 \text{ m}^{-1/2}\text{s}^{-1}$

Work done, $W = ?$

We know that, acceleration,

$$a_0 = \frac{dv}{dt} = v\frac{dv}{dx} = ax^{3/2} \frac{d}{dx}(ax^{3/2})$$
$$= ax^{3/2} \times a \times \frac{3}{2} \times x^{1/2}$$
$$= \frac{3}{2} a^2 x^2$$

Now, force $= ma_0 = \dfrac{3}{2} ma^2 x^2$

$$\text{Work done} = \int_{x=0}^{x=2} F dx = \int_0^2 \frac{3}{2} ma^2 x^2 dx$$
$$= \frac{3}{2} ma^2 \times (x^3/3)_0^2$$
$$= \frac{1}{2} ma^2 \times 8 = \frac{1}{2} \times (0.5) \times (5)^2 \times 8 = 50\text{J}$$

142 (*b*) Given, power = constant

We know that, power, $P = \frac{dW}{dt} = \frac{\mathbf{F}\cdot d\mathbf{s}}{dt}$

As, body is moving unidirectionally.

Hence, $\mathbf{F}\cdot ds = Fds\cos 0°$

$$P = \frac{Fds}{dt} = Fv = \text{constant}$$

($\because P$ = constant by question)

Now, writing dimensions, [F] [v] = constant

$\Rightarrow \quad [MLT^{-2}][LT^{-1}] = \text{constant}$

$\Rightarrow \quad L^2 T^{-3} = \text{constant} \qquad (\because \text{mass is constant})$

$\Rightarrow \quad L \propto T^{3/2} \quad \text{or displacement } (d) \propto t^{3/2}$

Thus, only graph (b) shows this condition.

143 (*d*) When the earth is closest to the sun, speed of the earth is maximum, hence KE is maximum. When the earth is farthest from the sun, speed is minimum, hence KE is minimum but never zero and negative.

Therefore, option (d) is correct.

144 (*c*) When a pendulum oscillates in air, it loses energy continuously in overcoming resistance due to air. Therefore, total mechanical energy of the pendulum decreases continuously with time which is shown in graph (c).

145 (*a*) Given, mass, $m = 5$ kg

Radius, $R = 1$ m

Revolutions per minute, $\omega = 300 \text{ rev min}^{-1}$

$$= (300 \times 2\pi) \text{ rad min}^{-1}$$
$$= (300 \times 2 \times \pi) \text{ rad/60 s}$$
$$= \frac{300 \times 2 \times \pi}{60} \text{ rads}^{-1} = 10\pi \text{ rads}^{-1}$$

Linear speed, $v = \omega R = (10\pi \times 1) = 10\pi \text{ ms}^{-1}$

$$\text{KE} = \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times (10\pi)^2$$
$$= 250\pi^2 \text{ J}$$

146 (*b*) When raindrop falls first velocity increases, hence first KE also increases. After sometime speed (velocity) becomes constant, this is called terminal velocity, hence KE also becomes constant. PE decreases continuously as the drop is falling continuously.

Hence, only graph (b) shows this condition correctly.

147 (*b*) First velocity of the iron sphere increases and after sometime becomes constant. Hence, accordingly first KE increases and then becomes constant which is best represented by (b).

148 (*d*) Given, $h = 1.5$ m, $v = 1 \text{ ms}^{-1}$,

$$m = 10 \text{ kg}, g = 10 \text{ ms}^{-2}$$

From conservation of mechanical energy,

$(PE)_i + (KE)_i = (PE)_f + (KE)_f$

$\Rightarrow \quad mgh + \frac{1}{2} mv^2 = 0 + (KE)_f$

$\Rightarrow \quad (KE)_f = mgh + \frac{1}{2} mv^2$

$\Rightarrow \quad (KE)_f = 10 \times 10 \times 1.5 + \frac{1}{2} \times 10 \times (1)^2$

$$= 150 + 5 = 155 \text{ J}$$

149 (*c*) Given, $m = 150 \text{ g} = \frac{150}{1000} \text{ kg} = \frac{3}{20} \text{ kg}$

$$\Delta t = \text{time of contact} = 0.001 \text{ s}$$
$$u = 126 \text{ kmh}^{-1} = \frac{126 \times 1000}{60 \times 60} \text{ ms}^{-1}$$
$$= 35 \text{ ms}^{-1}$$

As collision is completely elastic, so

$$v = -126 \text{ kmh}^{-1} = -35 \text{ ms}^{-1}$$

Change in momentum of the ball,

$$\Delta p = m(v - u)$$
$$= \frac{3}{20}(-35 - 35)$$
$$= \frac{3}{20}(-70)$$
$$= -\frac{21}{2} \text{ kg-ms}^{-1}$$

We know that, force, $F = \frac{\Delta p}{\Delta t} = \frac{-21/2}{0.001}$ N

$$= -1.05 \times 10^4 \text{ N}$$

Here, negative sign shows that force will be opposite to the direction of motion of the ball before hitting.

150 (*c*) Consider the following diagram below when M_1 comes in contact with the spring, M_1 is retarded by the spring force and M_2 is accelerated by the spring force.

Then,

(a) The spring will continue to compress until the two blocks acquire common velocity.

(b) As surfaces are frictionless, so momentum of the system will be conserved.

(c) If spring is massless, whole energy of M_1 will be imparted to M_2 and M_1 will be at rest.

(d) Collision is inelastic, even if friction is not involved. This is because energy is stored as PE during collision.

Hence, only statement given in option (c) is correct.