JEE 2023

Definite Integration

DPP-03

1. The value of $\int_0^{\sqrt{2}}[x^2]\,dx$, where [.] is the greatest integer function, is:

(1) $2-\sqrt{2}$ (2) $2+\sqrt{2}$

(3) $\sqrt{2}-1$ (4) $\sqrt{2}-2$

2. $\int_0^{3\pi/2}\sin\left[\frac{2x}{\pi}\right]dx$, where [.] denotes the greatest integer function, is equal to:

(1) $\frac{\pi}{2}(\sin 1+\cos 1)$ (2) $\frac{\pi}{2}(\sin 1+\sin 2)$

(3) $\frac{\pi}{2}(\sin 1-\cos 1)$ (4) $\frac{\pi}{2}(\sin\pi+\sin 2)$

3. The value of $\int_{-\pi/2}^{\pi/2}\log\left(\frac{2-\sin\theta}{2+\sin\theta}\right)d\theta$ is

(1) 0 (2) 1

(3) 2 (4) None of these

4. The value of $\int_{-\pi/2}^{\pi/2}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ is

(1) 0 (2) 2

(3) π (4) None of these

5. $\int_{\pi/6}^{\pi/3}\tan x\,dx+\int_{1/\sqrt{3}}^{1}\tan^{-1}x\,dx=a-b+\ln\sqrt{2}$

then a/b is

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{3}{2}$

(3) $3\sqrt{3}$ (4) $\frac{3\sqrt{3}}{2}$

6. $\int_0^{\pi}xf\left(\sin x\right)dx=$

(1) $\int_0^{\pi}f\left(\sin x\right)dx$ (2) $\pi\int_0^{\pi}f\left(\sin x\right)dx$

(3) $\frac{\pi}{2}\int_0^{\pi}f\left(\sin x\right)dx$ (4) None of these

7. If [x] stands for the greatest integer function, then $\int_4^{10}\frac{\left[x^2\right]dx}{\left[x^2-28x+196\right]+\left[x^2\right]}$ is

(1) 1 (2) 2

(3) 3 (4) 4

8. $\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx=$

(1) $\sqrt{2}\log\left(\sqrt{2}+1\right)$ (2) $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

(3) $\log\left(\sqrt{2}+1\right)$ (4) $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}-1\right)$

9. $\int_2^7\frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}dx$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$

(3) $\frac{5}{2}$ (4) 0

10. $\int_0^{1000}e^{x-[x]}dx=$ (where [ ] is G.I.F.)

(1) $\frac{e^{1000}-1}{1000}$ (2) $\frac{e^{1000}-1}{e-1}$

(3) $1000(e-1)$ (4) $\frac{e-1}{1000}$

Answer Key

1.	**(3)**	**6.**	**(3)**
2.	**(2)**	**7.**	**(3)**
3.	**(1)**	**8.**	**(2)**
4.	**(3)**	**9.**	**(3)**
5.	**(4)**	**10.**	**(3)**