DPP No. 8

SYLLABUS : FUNCTION

1. If $f(x)= \log\left(\frac{2+x}{2-x}\right)$, $0 < a < 2$ then $\frac{1}{2}f\left(\frac{8a}{4+a^2}\right)$ =

(A) f (a) (B) 2f (a) (C) $\frac{1}{2}$ f (a) (D) –f (a)

2. The range of $\cos^2 x + \sin^4 x$ is –

(A) $\left[\frac{3}{4},1\right]$ (B) [0 , 1] (C) $\left[0,\frac{3}{4}\right]$ (D) $\left[-1,\frac{-3}{4}\right]$

3. If a function f : [2 , ∞) $\to$ B defined by $f(x) = x^2 - 4x + 5$ is a bijection, then B is equal to

(A) R (B) [1, ∞) (C) [4, ∞) (D) [5, ∞)

4. $f(x)=\frac{\cos x}{\left[\frac{2x}{\pi}\right]+\frac{1}{2}}$, where x is not an integral multiple of π and [.] denotes the greatest integer function is -

(A) an odd function (B) even function
(C) neither odd nor even (D) none of these

5. If $f(x)=\frac{2^x+2^{-x}}{2}$, then f(x + y). f(x–y) is equal to-

(A) $\frac{1}{2}$[f(x + y) + f(x – y)] (B) $\frac{1}{2}$[f(2x) + f(2y)]

(C) $\frac{1}{2}$ [f(x + y). f(x – y)] (D) none of these

6. The domain of definition of the function f(x) = $\frac{\sqrt{\sin^{-1}x+\sqrt{x^2+1}}+\sqrt{x-[x]+\log x}}{e^{\sqrt{\sin x+\cos x}}+\log\left(\sin\left(\frac{1}{\sqrt{-x^2}}\right)\right)}$ is -

(A) (–1, 1) (B) (0, 1)
(C) (1, 0) (D) None of these

7. If $f(x) = \frac{x}{\sqrt{(1+x^2)}}$, then (fofof) (x) =

(A) $\frac{3x}{\sqrt{(1+x^2)}}$ (B) $\frac{x}{\sqrt{(1+3x^2)}}$ (C) $\frac{3x}{\sqrt{(1-x^2)}}$ (D) None of these

8. Which of the following statement is incorrect -

(A) $x \operatorname{sgn} x = |x|$ (B) $|x| \operatorname{sgn} x = x$

(C) $x (\operatorname{sgn} x) (\operatorname{sgn} x) = x$ (D) $|x| (\operatorname{sgn} x)^3 = |x|$

9. The number of solutions of the equation $\sin \pi x = |\log| x||$ is -

10. Let $f : R \to R$ be a function such that $f(x) = x^3 + x^2 + 3x + \sin x$. Then -

(A) f is one-one and into (B) f is one-one and onto

(C) f is many-one and into (D) f is many-one and onto

11. $f(x) = (\tan x^5)\ e^{x^3 \operatorname{sgn} x^7}$ is -

(A) an even function (B) an odd function

(C) neither even nor odd function (D) None of these

12. If the function $f : [1, +\infty) \to [1, +\infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ is -

(A) $\left(\frac{1}{2}\right)^{x(x-1)}$ (B) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(C) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (D) not defined

13. The graph of $f(x) = \dfrac{2x(\sin x + \tan x)}{2\left[\dfrac{x+21\pi}{\pi}\right] - 41}$ is symmetric about -

(A) x-axis (B) y-axis (C) origin (D) None of these

14. Which pair of functions is identical?

(A) $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1} x)$ (B) $\log_e e^x,\ e^{\log_e x}$

(C) $\log_e x^2$, $2 \log_e x$ (D) None of these

15. If f(x) is a periodic function having period 7 and g(x) is periodic having period 11 then the period of

$D(x) = \begin{vmatrix} f(x) & f\left(\frac{x}{3}\right) \\ g(x) & g\left(\frac{x}{5}\right) \end{vmatrix}$ is -

16. The range of the function $f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$ is

(A) $[0,\ \sqrt{3}\,]$ (B) $(0,\ \sqrt{3}\,)$ (C) $[0,\ \sqrt{3}\,)$ (D) $\left[0, \frac{1}{\sqrt{3}}\right]$

17. The range of the function $f(x) = \log(3x^2 - 4x + 5)$ is

(A) $(-\infty, \infty)$ (B) $\left[\log\left(\frac{11}{3}\right), \infty\right)$ (C) $\left(\log\left(\frac{11}{3}\right), \infty\right)$ (D) none of these

18. If $f(x) = \cos\left[\frac{\pi^2}{2}\right]x + \sin\left[-\frac{\pi^2}{2}\right]x$, where [.] denotes the greatest integer function, then which of the following is not correct

(A) $f(0) = 1$ (B) $f\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}+1}$ (C) $f\left(\frac{\pi}{2}\right) = 0$ (D) $f(\pi) = 0$

19. Let $f(n) = \left[\frac{1}{2} + \frac{n}{100}\right]$, where [.] denotes the greatest integer function, then the value of $\sum_{n=1}^{151} f(n)$ is

20. $f(x) \begin{cases} 1 & x \in Q \\ -1 & x \in R - Q \end{cases}$. If $f(1) + f(2) + f(\pi) + f(p) = 0$, then p cannot be

(A) –e (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 4

21. The number of solutions of equation $e^{\{x\}} = -x^2$ (where $\{\bullet\}$ is a fractional part function)

22. The number of solutions of equation $\sin \pi x = |x|$ are

23. Let g (x) = 1 + x – [x] and f (x) = $\begin{cases} -1 & ,x<0 \\ 0 & ,x=0 \\ 1 & ,x>0 \end{cases}$. Then for all x, f (g (x)) is equal to (where [.] denotes greatest integer function)

$\therefore$ f(g(x)) = sgn (1 + {x}) = 1

24. The graph of the function y = f(x) is symmetrical about the line x = 2, then :

(A) f(x + 2) = f(x – 2) (B) f(2 + x) = f(2 – x)

(C) f(x) = f(–x) (D) f(x) = –f(–x)

25. Period of the function f(x) = [5x + 7] + cosπx – 5x where $[\cdot]$ denotes greatest integer function is

ANSWER KEY OF DPP NO. : 08

1.	**(A)**	**2.**	**(A)**	**3.**	**(B)**	**4.**	**(A)**	**5.**	**(B)**	**6.**	**(D)**	**7.**	**(B)**
8.	**(D)**	**9.**	**(6)**	**10.**	**(B)**	**11.**	**(B)**	**12.**	**(B)**	**13.**	**(C)**	**14.**	**(D)**
15.	**(1155)**	**16.**	**(A)**	**17.**	**(B)**	**18.**	**(D)**	**19.**	**(104)**	**20.**	**(D)**	**21.**	**(0)**
22.	**(2)**	**23.**	**(1)**	**24.**	**(B)**	**25.**	**(2)**						