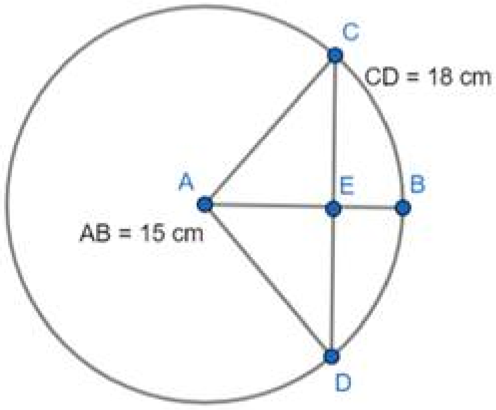
3. Geometry

Exercise 3.1

1. Question

The radius of a circle is 15 cm and the length of one of its chord is 18 cm. Find the distance of the chord from the centre.

Answer



The figure is attached above

A is the centre of the circle.

AB is the radius = 15 cm

CD is the chord = 18 cm.

We need to find the distance of the chord from the centre i.e. AE

In this circle, we draw the perpendicular

We know that perpendicular drawn from the centre to the chord, will bisect the chord, such that CE = ED = $\frac{18\,\text{cm}}{2}$ = 9 cm

Now,

In ΔAEC,

Applying Pythagoras theorem,

$AC^2 = AE^2 + EC^2$

$\Rightarrow AE^2 = AC^2 - EC^2$

$\Rightarrow AE^2 = (15cm)^2 – (9cm)^2$

$\Rightarrow AE^2 = 225 – 81 = 144$

$\Rightarrow AE = \sqrt{144}$

$\Rightarrow AE = 12$ cm

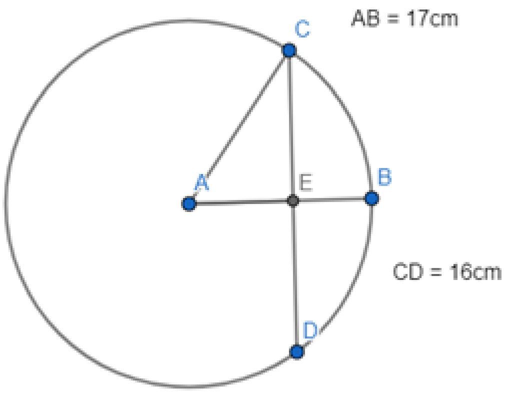
∴The chord is 12cm away from the center of the circle.

2. Question

The radius of a circles 17 cm and the length of one of its chord is 16 cm. Find the distance of the chord from the centre.

Answer

In the given figure:



Radius AB = 17cm

Chord CD = 17cm

Now we draw a perpendicular bisector of CD from A which is AB

Hence CE = DE = 8cm

We have to find the distance of chord from centre i.e. AE

In the triangle ACE

$AC^2 = AE^2 + EC^2$

Therefore $AE^2 = 17^2 – 8^2$

$AE^2 = 225$

AE = 15cm

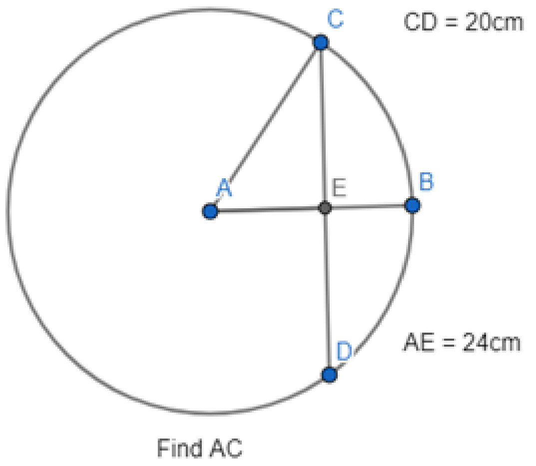
Hence the distance of chord from the centre is 15cm

3. Question

A chord of length 20 cm is drawn at a distance of 24 cm from the centre of a circle. Find the radius of the circle.

Answer

The figure is given below:



CD = 20cm = length of chord

AE = 24cm = distance of chord from centre

From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

From here we get,

$r^2 = 100 + 576 = 676$

r = 26cm

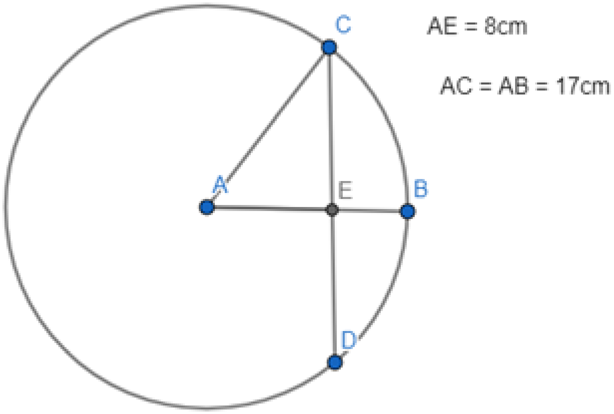
Hence radius is 26cm.

4. Question

A chord is 8 cm away from the centre of a circle of radius 17 cm. Find the length of the chord.

Answer

The figure is given below:



AE = 8cm

AB = AC = 17cm

From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

d = 8cm

r = 17cm

$$\left(\frac{l}{2}\right)^2 = 289 - 64 = 225$$

$$\frac{l}{2} = 15$$

$$l = 30\text{cm}$$

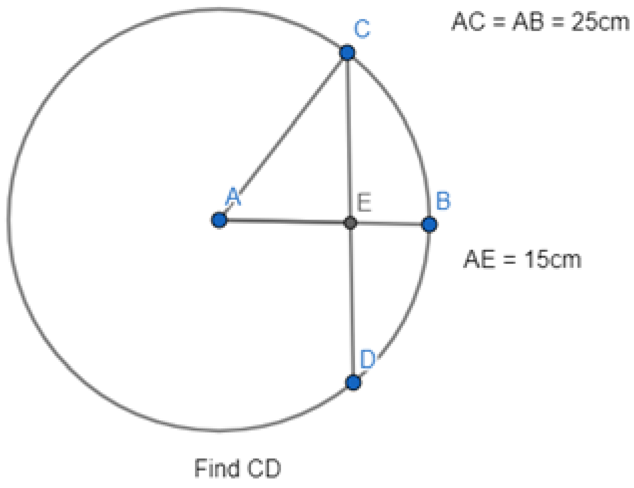
Hence the length of the chord is 30cm.

5. Question

Find the length of a chord which is at a distance of 15 cm from the centre of a circle of radius 25 cm.

Answer

The figure is given below:



AE = 15cm

AB = AC = 25cm

From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

Here d = 15cm and r = 25cm

$$\left(\frac{l}{2}\right)^2 = 625 - 225 = 400$$

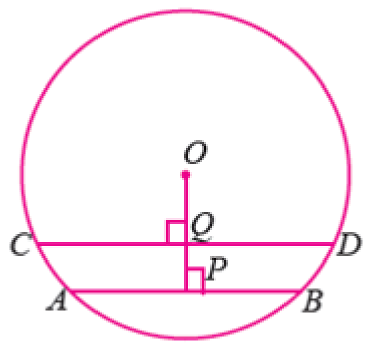
$$\frac{l}{2} = 20$$

l = 40cm

Hence the length of the chord is 40cm.

6. Question

In the figure at right, AB and CD are two parallel chords of a circle with centre O and radius 5 cm such that AB = 6 cm and CD = 8 cm. If OP ⊥ AB and CD ⊥ OQ determine the length of PQ.



Answer

From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

For Triangle COQ

OC = 5cm ,

CQ = half of chord length = 4cm

From Pythagoras theorem

$$OC^2 = CQ^2 + OQ^2$$

$$OQ^2 = 25 - 16 = 9$$

OQ = 3cm

For triangle POA

OA = 5cm, AP = half of chord length = 3cm

From Pythagoras theorem

$$OA^2 = AP^2 + OP^2$$

$$OP^2 = 25 - 9 = 16$$

OP = 4cm

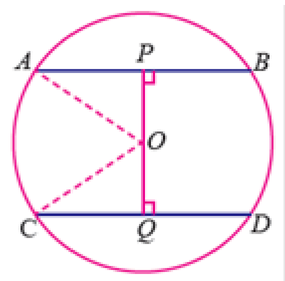
From figure PQ = OP – OQ

Therefore PQ = 1cm

7. Question

AB and CD are two parallel chords of a circle which are on either sides of the centre. Such that AB = 10 cm and CD = 24 cm. Find the radius if the distance between AB and CD is 17 cm.

Answer



From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

In the figure, AB = 10cm = l

$r^2 = 25 + OP^2$ .....(1)

And CD = 24cm

$r^2 = 144 + OQ^2$ .....(2)

As Distance between the two chords = PQ = 17cm

We have OP + OQ = 17....(3)

Equating equations 1 and 2 we get,

$$OP^2 - OQ^2 = 144 - 25 = 119$$

From identity $(a + b)(a - b) = a^2 - b^2$ we get

(OP + OQ)(OP–OQ) = 119

Using equation 3 we get

OP – OQ = 7 ....(4)

Solving equation 3 and 4 we get

OP = 12cm

OQ = 5cm

From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

Using this in Triangle APO we get

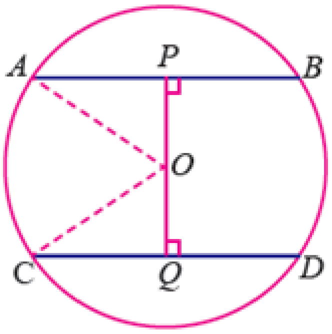
$$r^2 = 25 + 144 = 169$$

$$r = 14\text{cm}$$

Hence the radius of the triangle is 14cm

8. Question

In the figure at right, AB and CD are two parallel chords of a circle with centre O and radius 5 cm. Such that AB = 8 cm and CD = 6 cm. If OP ⊥ AB and OQ ⊥ CD determine the length PQ.



Answer

From Theorem: A circle with radius r and a chord of length l which is at distance d from the centre, follows the equation,

$$r^2 = \left(\frac{l}{2}\right)^2 + d^2$$

In triangle APO

$$OP^2 = 25 - 16 = 9$$

OP = 3cm.

In triangle COQ

$$OQ^2 = 25 - 9 = 16$$

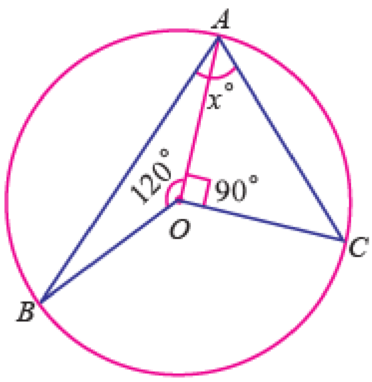
OQ = 4cm

From figure PQ = OP + OQ = 7cm.

Hence PQ = 7cm.

9 A. Question

Find the value of x in the following figures.



Answer

Triangle AOC is a right angled triangle at O and sides AO = OC.

Hence ∠OAC = ∠OCA = y

As the sum of all angles = 180°

90 + 2y = 180

y = 45°

∠OAC = 45°

Similarly in triangle AOB

Sides AO = OB and hence

∠OBA = ∠OAB = a

As sum of all angles of a triangle = 180°

120 + 2a = 180

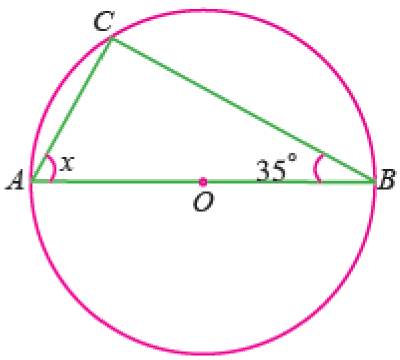
a = 30°

In the figure,

∠BAC = ∠CAO + ∠BAO = 45 + 30 = 75°

9 B. Question

Find the value of x in the following figures.



Answer

According to theorem, angle subtended by the diameter on the circle is 90°.

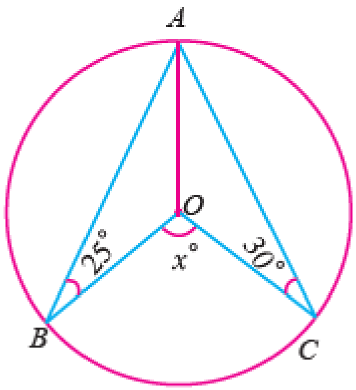
∠ACB = 90° ; ∠ABC = 35°

As the sum of all angles = 180°

∠CAB = 180– 35 = 145°

9 C. Question

Find the value of x in the following figures.



Answer

According to theorem that the angle which is subtended by an arc at the centre of a circle is double the size of the angle subtended at any point on the circumference.

∠AOB = 2 × 25 = 50°

∠AOC = 2 × 30 = 60°

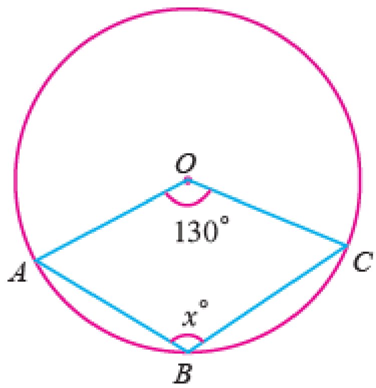
At centre O

∠AOB + ∠AOC + x° = 360°

x = 360–60–50 = 250°

9 D. Question

Find the value of x in the following figures.



Answer

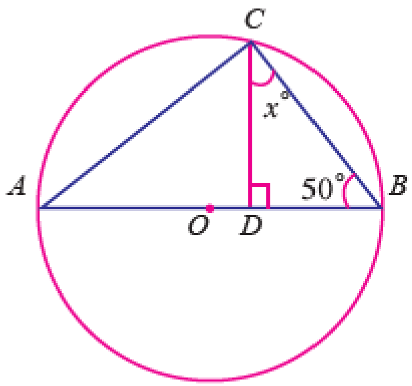
According to theorem that the angle which is subtended by an arc at the centre of a circle is double the size of the angle subtended at any point on the circumference.

Hence Angle by chord AC on the circumference = $\frac{1}{2}$(angle subtended at centre)

∠ABC = $\frac{1}{2}(130)$ = 65°

9 E. Question

Find the value of x in the following figures.



Answer

As the sum of all angles of a triangle is 180°

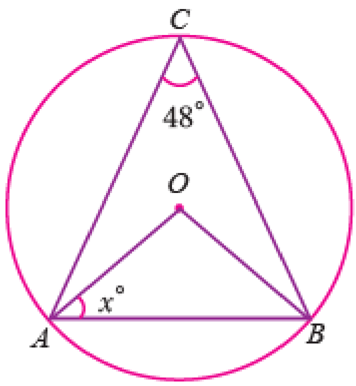
Angles of triangle DBC are 90°, 50° and x°

$90 + 50 + x = 180$

$x = 180 - 90 - 50 = 40°$

9 F. Question

Find the value of x in the following figures.



Answer

According to theorem that the angle which is subtended by an arc at the centre of a circle is double the size of the angle subtended at any point on the circumference.

Here angle subtended by a chord at circumference = 48°

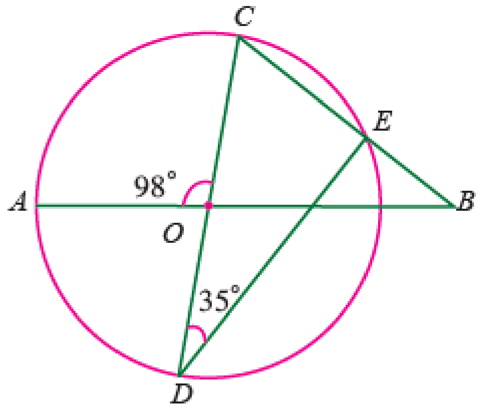
Hence angle subtended at centre = 2x 48°

Therefore x = 96°

10. Question

In the figure at right, AB and CD are straight lines through the centre O of a circle. If ∠AOC = 98° and ∠CDE = 35°

find (i) ∠DCE (ii) ∠ABC



Answer

According to theorem the angle subtended by a diameter on the circumference of a circle is 90°

Hence ∠CED = 90°

As the sum of all angles of a triangle CDE = 180°

∠DCE = 180 – 90 – 35 = 55°

As the sum of all angles of triangle OCB = 180°

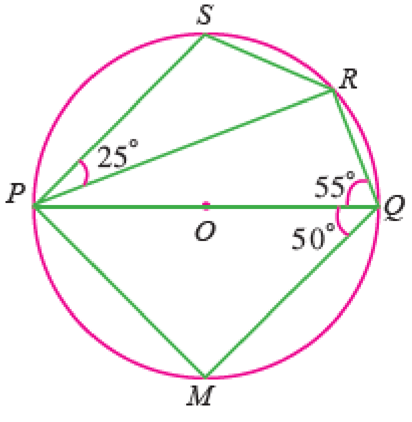
As AB is a straight line ∠COB = 180–98 = 82°

Hence ∠ABC = 180 – 82 – 55 = 43°

11. Question

In the figure at left, PQ is a diameter of a circle with centre O. If ∠PQR = 55°, ∠SPR = 25°and ∠PQM = 50°.

Find (i) ∠QPR, (ii) ∠QPM and (iii) ∠PRS.



Answer

According to theorem the angle subtended by a diameter on the circumference of a circle is 90°

$\angle PRQ = 90°$

As the sum of all angles of a triangle is 180°

In triangle PRQ,

$\angle QPR = 180 - 55 - 90 = 35°$

$\angle PMQ = 90°$

As sum of all angles of triangle = 180°

In triangle PQM,

$\angle QPM = 180 - 90 - 50 = 40°$

Triangle formed by POS is an isosceles triangle as two of sides are radius OP and OS.

Hence $\angle OPS = \angle OSP = 35 + 25 = 60°$

As sum of all angles of a triangle is 180°.

$\angle POS = 180 - 60 - 60 = 60°$

According to theorem that the angle which is subtended by an arc at the centre of a circle is double the size of the angle subtended at any point on the circumference.

Angle subtended by chord SP on centre = 60°

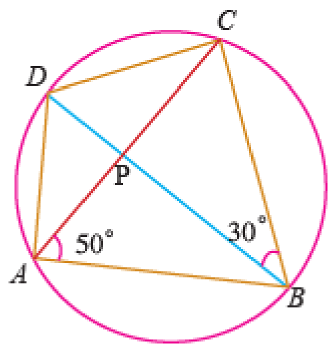
Hence angle subtended by chord SP on circumference is $\frac{60}{2}$

Hence $\angle PRS = 30°$

12. Question

In the figure at right, ABCD is a cyclic quadrilateral whose diagonals intersect at P such that $\angle DBC = 30°$ and $\angle BAC = 50°$.

Find (i) $\angle BCD$ (ii) $\angle CAD$



Answer

As ABCD is a cyclic quadrilaterals, sum of all angles = 360°

As the angle subtended by chord BC on circumference = 50°

Therefore $\angle$BDC = 50°

Sum of all angles of triangle BCD = 180°

Hence $\angle$BCD = 180– 30–50 = 100°

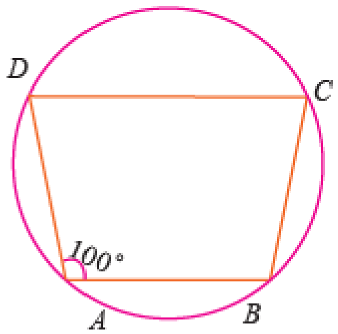
As the angle subtended by chord DC on circumference = 30°

Therefore $\angle$CAD = 50°

13. Question

In the figure at left ,ABCD is a cyclic quadrilateral in which AB || DC. If ∠BAD = 100°

find (i) ∠BCD (ii) ∠ADC (iii) ∠ABC.



Answer

As ABCD is a cyclic quadrilaterals, sum of all angles = 360°

As AB is parallel to CD

∠ADC = 180 – ∠DAB = 80° as they are interior angles on the same side.

As angle subtended by chord BD on circumference = 100°

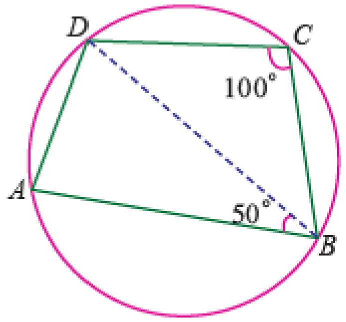
Therefore ∠BCD = 100°

As ∠BCD and ∠ABC are interior angles

∠ABC = 180 – ∠BCD = 80°

14. Question

In the figure at right, ABCD is a cyclic quadrilateral in which ∠BCD = 100°and ∠ABD = 50°. Find ∠ADB



Answer

As ABCD is a cyclic quadrilaterals, sum of all angles = 360°

As angle subtended by chord BD on circumference = 100°

Therefore ∠DAB = 100°

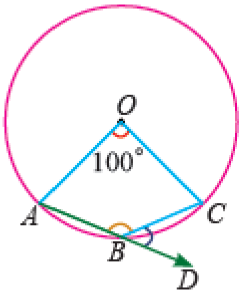
As sum of all angles of triangle ADB is 180°

Hence ∠ADB = 180–50–100 = 30°

15. Question

In the figure at left, O is the centre of the circle, ∠AOC = 100° and side AB is produced to D.

Find (i) ∠CBD (ii) ∠ABC



Answer

As the angle subtended by chord AC on center O is 100°.

Angle subtended by chord AC on major segment = 50°

As the sum of angle subtended by chord AC on major and minor segments is 180°

Hence ∠ABC = 180–50 = 130°

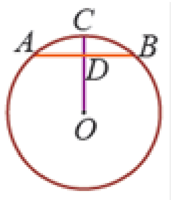
As line AD is straight

Hence ∠CBD = 180 – 130 = 50°

Exercise 3.2

1. Question

O is the centre of the circle. AB is the chord and D is mid-point of AB. If the length of CD is 2cm and the length of chord is12 cm, what is the radius of the circle



A. 10cm

B. 12cm

C. 15cm

D. 18cm

Answer

In triangle AOD

AD = 6cm

AO = r

OD = OC–CD = r–2

Applying Pythagoras theorem we get

$r^2 = 36 + (r-2)^2$

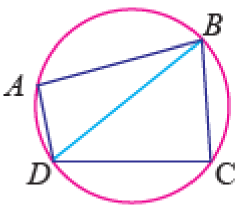
$r^2 = 36 + r^2 + 4 - 4r$

$4r = 40$

r = 10cm

2. Question

ABCD is a cyclic quadrilateral. Given that ∠ADB + ∠DAB = 120°and ∠ABC + ∠BDA = 145°. Find the value of ∠CDB



A. 75°

B. 115°

C. 35°

D. 45°

Answer

As $\angle$ADB + $\angle$DAB = 120°

In triangle ABD as sum of all angles = 180°

$\angle$ABD = 180–120 = 60°

As ABCD is cyclic, opposite angles have sum of 180°

Let $\angle$BAD = x°

Hence $\angle$BCD = 180–x

Let $\angle$BDC = a°

Let $\angle$DBC = z°; $\angle$ABC = 60 + z

Let $\angle$ADB = y°

As sum of angles of triangle BDC = 180°

180–x + a + z = 180

a = x–z ...(1)

According to question,

x + y = 120 ..(2)

and

60 + z + y = 145...(3)

Subtracting (3) from (2)

We get

x + y–60–z–y = –25

x–z = 35

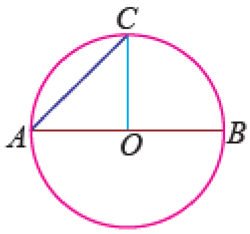
Equating from (1) we get

a = 35°

hence $\angle$CDB = 35°

3. Question

In the given figure, AB is one of the diameters of the circle and OC is perpendicular to it through the center O. If AC is $7\sqrt{2}$ cm, then what is the area of the circle in cm^2



A. 24.5

B. 49

C. 98

D. 154

Answer

In triangle AOC AO = OC = radius of circle = r

Using Pythagoras theorem,

$$(7\sqrt{2})^2 = r^2 + r^2$$

$$2r^2 = 98$$

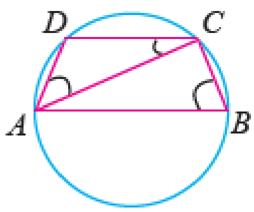
$$r^2 = 49$$

$$r = 7\text{cm}$$

As area of a circle is given by $\frac{22}{7}r^2 = \frac{22}{7}(49) = 154 \text{ cm}^2$

4. Question

In the given figure, AB is a diameter of the circle and points C and D are on the circumference such that ∠CAD = 30°and ∠CBA = 70° what is the measure of ACD?



A. 40°

B. 50°

C. 30°

D. 90°

Answer

According to theorem the angle subtended by a diameter on the circumference of a circle is 90°

$\angle ACB = 90°$

Angle subtended by chord AC on major segment = $\angle ABC = 70°$

Angle subtended on minor segment = 180–70 = 110° = $\angle ADC$

As sum of angles of triangle ADC = 180°

$\angle ACD = 180–30–110 = 40°$

5. Question

Angle in a semi circle is

A. obtuse angle

B. right angle

C. an acute angle

D. supplementary

Answer

As a semicircle is formed by a diameter and the angle subtended on the circumference by the diameter is 90°.

6. Question

Angle in a minor segment is

A. an acute angle

B. an obtuse angle

C. a right angle

D. a reflexive angle

Answer

The angle formed by a chord in the major segment is acute whereas in the minor segment is obtuse and the sum of both angles is 180°.

7. Question

In a cyclic quadrilateral ABCD, $\angle A = 5x$, $\angle C = 4x$ the value of x is

A. 12°

B. 20°

C. 48°

D. 36°

Answer

In a cyclic quadrilateral sum of opposite angles is 180°

$$5x + 4x = 180$$

$$9x = 180$$

$$x = 20$$

8. Question

Angle in a major segment is

A. an acute angle

B. an obtuse angle

C. a right triangle

D. a reflexive angle

Answer

The angle formed by a chord in the major segment is acute whereas in the minor segment is obtuse and the sum of both angles is 180°.

9. Question

If one angle of a cyclic quadrilateral is 70°, then the angle opposite to it is

A. 20°

B. 110°

C. 140°

D. 160°

Answer

As the sum of opposite angles in a cyclic quadrilateral is 180°

Let opposite angle be x°

$$70 + x = 180$$

$$x = 100°$$