Chapter 2

Time Response Analysis

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Time response
- Steady state response
- Standard test signals
- Type of the system
- Order of the system
- Unit step response
- Delay time
- Rise time
- Settling time
- Steady state error analysis
- Parabolic input

Time response

The response given by the system which is function of time for the applied excitation is called 'time response'.

Time response can be obtained by solving the differential equations governing the system (or) from the transfer function of the system and input given to the system.

$$C(t) = C_t(t) + C_{ss}(t)$$

The time response of a control system consists of two parts:

1. Transient response $C_t(t)$
2. Steady-state response $C_{ss}(t)$

Transient Response

The output variation during the time it takes to achieve its final value is called as 'transient response'.

The time required to achieve the final values is called 'transient period'.

For a stable operating system, transient response $C_t(t)$ can be written as

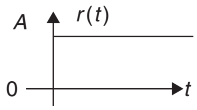
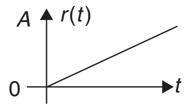
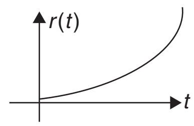
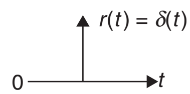
$$\underset{t \to \infty}{\text{Lt}}\ C_t(t) = 0$$

Steady-state Response

The response which remains after complete transient response vanishes (transient period). Steady-state response indicates the accuracy of a system.

Standard Test Signals

The information of input signals is required to pre-estimate the response of a system. The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and a constant acceleration. So the equivalent test signals are used as input signals to predict the performance of the system.

Name of the signal	Waveform	Physical interpretation	Mathematical representation	Laplace transform
Step		A sudden change/ constant position	$r(t) = A\ t \geq 0 = 0\ t < 0$	$\frac{A}{s}$
Ramp		A constant velocity	$r(t) = At\ t \geq 0 = 0\ t < 0$	$\frac{A}{s^2}$
Parabolic		A constant acceleration	$r(t) = \frac{At^2}{2}\ t \geq 0 = 0\ t < 0$	$\frac{A}{s^3}$
Impulse		A sudden shock	$r(t) = \infty;\ t = 0 = 0\ t \neq 0$	1

Type and Order or of the System

Type of the System

Type of a system is defined as the number of open loop poles at origin.

Steady-state behaviour of the system depends on the type of the system.

Example:

$$G(s)\,H(s) = \frac{K(1+T_{z_1}s)(1+T_{z_2}s)(1+T_{z_3}s)\ldots}{s^n(1+T_{p_1}s)(1+T_{p_2}s)(1+T_{p_3}s)\ldots}$$

If $n = 0 \Rightarrow$ type '0' system
If $n = 1 \Rightarrow$ type '1' system
$\therefore$ n = type of the system

Order of the System

The highest power of 's' in the denominator of the closed loop transfer function is defined as the order of the system.

Example:

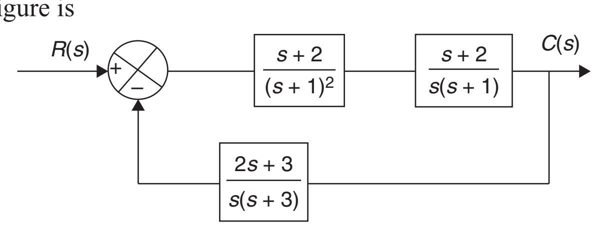
$$\text{Transfer function} = \frac{C(S)}{R(S)} = \frac{a_0s^n + a_1s^{n-1} + a_2s^{n-2} + \ldots + a_n}{b_0s^m + b_1s^{m-1} + b_2s^{m-2} + \ldots + b_m}$$

In the above behaviour of the system depends on the order of the system.

Transient behaviour of the system depends on the order of the system.

Solved Examples

Example 1: The feedback control system shown in the figure is



(A) Type '0' system (B) Type '1' system
(C) Type '2' system (D) Type '3' system

Solution: (C)
Number of integrators in the open loop transfer function are '2'. So the type of the system is '2'.

Example 2: A system has the following transfer function

$$G(s) = \frac{10(s+5)(s+50)}{s^4(s+1)(s^2+3s+10)}$$

The type and order of the system are, respectively,
(A) 4 and 9 (B) 4 and 7
(C) 5 and 7 (D) 7 and 5

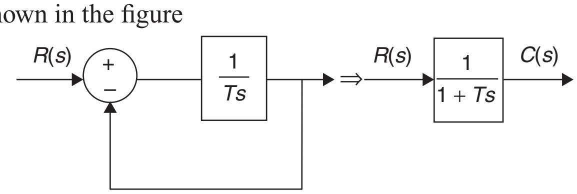
Solution: (B)
Number of integrators = 4 = type of system
Highest power of 's' in denominator = order of the system = 7

Example 3: Type of a system depends on the
(A) No. of poles
(B) Difference between no. of poles and zeros
(C) No. of real poles
(D) No. of poles it has at the origin
Solution: (D)

Time Response of a First-order System

A system with highest power of 's' in the denominator of its transfer function equal to 1 is known as 'first-order' system

Consider a first order unity feedback control system shown in the figure



$$\text{Transfer function of the system} = \frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$

Unit Step Response

Unit step input $r(t) = u(t)$

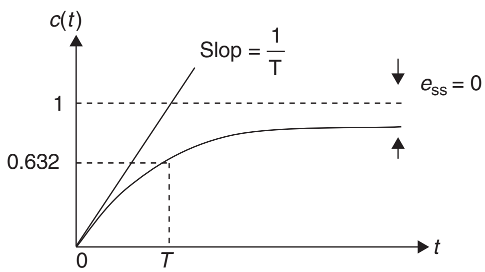
$$R(t) = \frac{1}{s}$$

Unit step response in 's' domain $[C(s)] = \dfrac{1}{1+Ts} * \dfrac{1}{s}$

Unit step response in time domain $[C(t)] = 1 - e^{-t/T}$

Steady-state value of the response $= \underset{t\to\infty}{\text{Lt}}\ C(t) = 1$

Transient response of the system $= e^{-t/\tau}$



Time Response for Unit Step Input

Time Constant

Time takes for the step response to raise to 63% of its final value.

Rise Time (T_r)

Time taken by the step response to go from 10% to 90% of its final value.

$$T_r = 2.31T - 0.11T = 2.22T$$

Setting Time (T_s)

Time taken by the step response to reach and stay within 2% or 5% of its final value.

$$T_s = 4T \text{ for } \pm 2\% \text{ tolerance}$$
$$= 3T \text{ for } \pm 5\% \text{ tolerance}$$

Unit Ramp Response

Unit ramp input $r(t) = tu(t)$

$$R(s) = \frac{1}{s^2}$$

Unit ramp response in 's' domain $= \dfrac{1}{1+T_s} \times \dfrac{1}{s^2}$

Unit ramp response in time domain $c(t) = L^{-1}\left\{\dfrac{1}{s^2(1+T_s)}\right\}$

$$e(t) = (t - \tau + \tau e^{-t/\tau})$$

Error for unit ramp input $e(t) = r(t) - c(t) = (\tau - \tau e^{-t/\tau})$

The steady-state error for unit ramp input $= \underset{t\to\infty}{\text{Lt}}\ e(t)$

$$= \underset{t\to\infty}{\text{Lt}}\left(\tau - \tau e^{-t/\tau}\right)$$

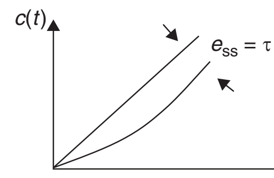


Figure 1 Unit ramp response of first-order system

Unit Impulse Response

Unit impulse input $r(t) = \delta(t)$

$$R(s) = 1$$

Unit impulse response in 's' domain $c(s) = \dfrac{1}{1+T_s}$

Unit impulse response in time domain $c(t) = \dfrac{1}{T} e^{-t/T}$ for $t \geq 0$

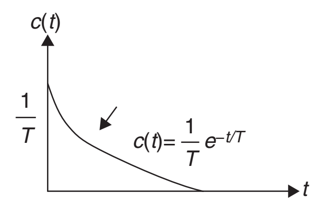


Figure 2 Unit impulse response of first-order system

Example 4: Transfer function of a system is given by

$$G(s) = \frac{20}{(s+20)}$$

rise time of the system is

(A) 0.33 s (B) 44 s
(C) 0.11 s (D) 0.2 s

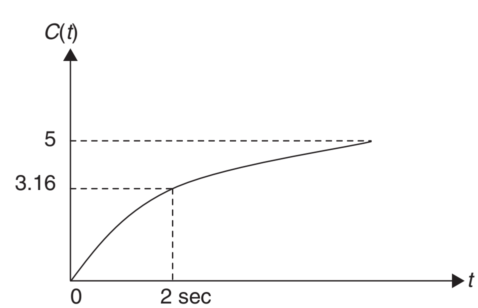
Solution: (C)

Transfer function $= \dfrac{20}{s+20} = \dfrac{1}{(0.05s+1)}$

Time constant = 0.05 s

Rise time = 2.2 × Time constant = 0.11 s

Example 5: Time response of the system $G(s) = \dfrac{K}{s+a}$ is given by



Then the values of 'K' and 'a' are

(A) $K = 5, a = 2$ (B) $K = 5, a = 0.5$
(C) $K = 2.5, a = 0.5$ (D) $K = 2.5, a = 2$

Solution: (C)
Time constant = 2 s
Steady-state value = 5

$$\text{Transfer function} = \frac{5}{2s+1} = \frac{2.5}{(s+0.5)}$$

$$\therefore \qquad K = 2.5$$
$$a = 0.5$$

Example 6: Transfer function of the system is given by

$$G(s) = \frac{2}{(s+2)}$$

Time required for the system unit step response to reach 95% of its final value is
(A) 0.5 s (B) 1 s (C) 1.5 s (D) 2 s

Solution: (C)

$$\text{Transfer function} = \frac{2}{s+2} = \frac{1}{1+0.5\,s}$$

Time constant = 0.5

Time required for unit step response to reach 95% of its final value = 3 × Time constant = 1.5 s

Time Response of a Second-order System

A system with highest power of 's' in the denominator of its transfer function equal to '2' is known as 'second-order' system.

Closed loop transfer function for a standard second-order system is given by

$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{N(s)}{D(s)}$$

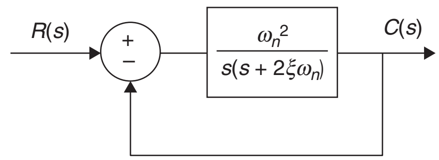


Figure 3 Block diagram of standard second-order system

where ω_n = Undamped natural frequency
ξ = Damping ratio constant

Tendency to oppose the oscillatory behaviour of the system is called as 'damping', it is denoted by 'ξ'. As 'ξ' is increased, the response becomes progressively less oscillatory till it becomes critically damped ($\xi = 1$) and becomes over damped for $\xi > 1$.

The denominator polynomial of the transfer function $D(s)$ is called as 'characteristic equation'. The characteristic equation of the second-order system is

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

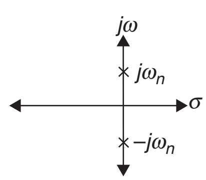
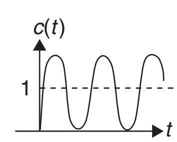
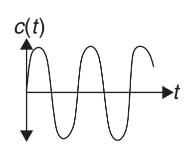
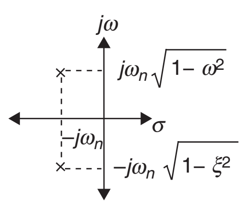
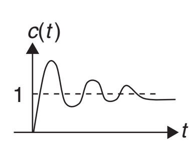
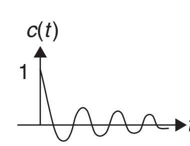
The roots of the characteristic equation are given by

$$s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$$

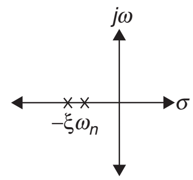
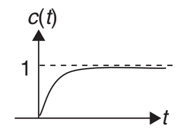
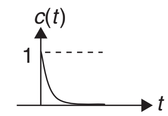
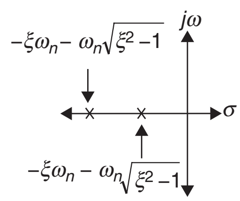
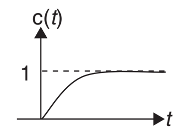
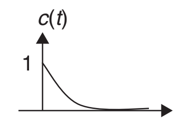
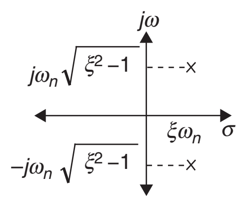
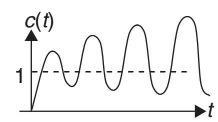
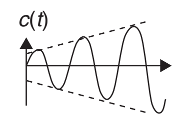
where $\omega_d = \omega_n\sqrt{1-\xi^2}$ ⇒ Damping natural frequency.

The time response of any system is characterized by the roots of the denominator polynomial (poles of the system), which depends on the damping ratio (ξ) of the system. Detailed summary of the system response characterization with variation is ξ/poles is given in the table.

Summary of Second-order System Response

System characterization	Range of ξ	Poles of second-order system	Poles on the *S*-plane	Nature of the response	Unit step response	Impulse response
Undamped	$\xi = 0$	$\pm j\omega_n$		Constant frequency and magnitude oscillations		
Under damped	$0 < \xi < 1$	$-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$		Damped oscillations		

(*Continued*)

System characterization	Range of ξ	Poles of second-order system	Poles on the S-plane	Nature of the response	Unit step response	Impulse response
Critically damped	$\xi = 1$	$-\xi\omega_n$		Pure exponential		
Over damped	$\xi > 1$	$-\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$		Slow exponential		
Negative damped	$\xi < 0$	$\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$		Increasing oscillations		

Transient Response Specifications of Second-order System

The common model for physical problems is under damped second-order system due to its speed in reaching steady state. The specifications of the under damped second-order system are shown in figure.

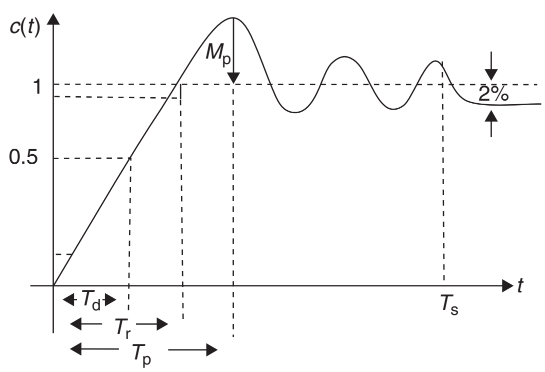


Figure 4 Unit step response of the under damped second-order system

Delay Time, t_d

The delay time is the time required for the response to reach half the final value for the very first time.

$$t_d = \frac{1 + 0.7\xi}{\omega_n}\, s$$

Rise Time, t_r

It is the time required for the response to rise from 0 to 100% of its final value for undamped case. For underdamped case and overdamped case, it is 10% to 90% of its final value.

i.e.,
$$t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{-\sigma}\right) = \frac{\pi - \beta}{\omega_d}$$

where $\beta = \cos^{-1}\xi$

Peak Time, t_p

The peak time is the time required for the response to reach the first peak of the overshoot.

i.e.,
$$t_p = \frac{\pi}{\omega_d}$$

The peak time t_p corresponds to one half-cycle of the frequency of damped oscillation.

Peak Overshoot, M_p

The maximum overshoot is the maximum peak value of the response curve measured from unity.

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\%\text{ peak overshoot} = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100$$

Settling Time (t_s)

This is the time required for the response to reach and remain within a specified tolerance band of its final value.

The settling time corresponding to a ±2% or ±5% tolerance band may be measured in terms of the time constant $T = \dfrac{1}{\xi\omega_n}$

$$t_s = 4T = \frac{4}{\xi\omega_n} \quad \text{(2\% band)}$$

$$t_s = 3T = \frac{3}{\xi\omega_n} \quad \text{(5\% band)}$$

Note: Time period of oscillations $T_{osci} = \dfrac{2\pi}{\omega_d}$

Number of oscillations $(N) = \dfrac{t_s}{T_{osci}} = \dfrac{t_s\,\omega_d}{2\pi}$

Example 7: A unity feedback system has open loop transfer function as $G(s) = \dfrac{25}{s(s+25)}$

The natural frequency of the system is

(A) 10 (B) 5 (C) 25 (D) 2.5

Solution: (B)

Given open loop transfer function $= \dfrac{25}{s(s+25)}$

Standard open loop transfer function $= \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

$\therefore \quad \omega_n^2 = 25$

Natural frequency $\omega_n = 5$ rad/s

Example 8: Given a unity feedback with

$$G(s) = \frac{K}{s(s+5)}$$

The value of 'K' for damping ratio of 0.5 is

(A) 1 (B) 25 (C) 5 (D) 2.5

Solution: (B)

Standard second-order OLTF $= \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

Comparing with given OLTE $2\xi\omega_n = 5$

$$2 \times 0.5 \times \omega_n = 5$$

$\Rightarrow \quad \omega_n = 5$

$\therefore \quad K = \omega_n^2 = 25$

Example 9: The unit impulse response of a second-order system is $\dfrac{1}{6}e^{-0.8t}\sin(0.6t)$. Then the natural frequency and damping ratio of the system are, respectively,

(A) 2 and 0.3 (B) 1 and 0.6
(C) 1 and 0.8 (D) 2 and 0.8

Solution: (C)

Transfer function = L\{Impulse response\}

$$= \frac{1}{6}\cdot\frac{0.6}{(s+0.8)^2 + (0.6)^2}$$

Transfer function $= \dfrac{0.1}{s^2 + 1.6s + 1}$

After comparing with standard second-order transfer function

$$\omega_n = 1,\ 2\xi\omega_n = 1.6$$

$$\xi = 0.8$$

Example 10: A second-order system has

$$M(j\omega) = \frac{100}{100 - \omega^2 + j20\omega}$$

If M_p (peak magnitude) is

(A) 100 (B) 1 (C) 0.5 (D) 2

Solution: (B)

Comparing with standard second-order system,

$$\xi = 1,\ \omega_n = 10$$

System is critically damped ⇒ Peak magnitude = 1

Example 11: A unity feedback control system has a forward path transfer function equal to $G(s) = \dfrac{30.25}{s(s+5.5)}$

The unit step response of the system starting form rest will have its maximum value at a time equal to

(A) 0.5 s (B) 0.66 s
(C) 0 s (D) Infinity

Solution: (B)

Standard form of second-order OLTF $= \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$

$\therefore \quad \omega_n = 5.5,\ \xi = 0.5$

Peak time $(t_p) = \dfrac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.66$ s

Example 12: A plant has the following transfer function

$$G(s) = \frac{1}{s^2 + 0.4s + 1}$$

For a step input, it is required that the response value settles to within 2% of its final value. The plant setting time is

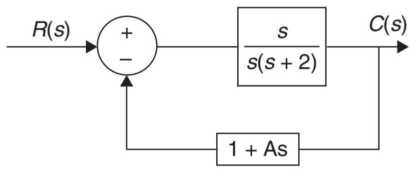
(A) 10 s (B) 20 s
(C) 30 s (D) Infinity

Solution: (B)

For the given system transfer function $\omega_n = 1,\ \xi = 0.2$

Setting time of 2% tolerance $(t_s) = \dfrac{4}{\xi\omega_n} = \dfrac{4}{0.2} = 20$ s

Example 13: The block diagram of a closed loop control system is given in figure. The value of 'K' and 'A' such that the system has a damping ratio of 0.8 and undamped natural frequency ω_n of 5 rad/s, are respectively equal to



(A) 4 and 0.8 (B) 5 and 0.24
(C) 25 and 0.24 (D) 20 and 0.8

Solution: (C)
Transfer function of the given system

$$= \frac{G(s)}{1+G(s)H(s)} = \frac{K}{s^2+(2+AK)s+(K)} \quad (1)$$

$$\text{Required transfer function} = \frac{25}{s^2+8s+25} \quad (2)$$

Comparing equations (1) and (2),we have

$$K = 25$$

$$2 + AK = 8$$

$$25A = 6$$

$$\Rightarrow \quad A = \frac{6}{25} = 0.24$$

Steady-state Error Analysis

Steady-state error is the difference between the input and the output for a prescribed test input as $t \to \infty$.

A generalized expression for steady-state error is obtained by using feedback system shown in the following figure.

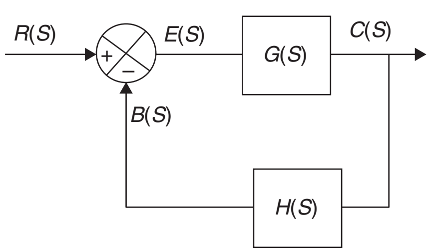


Figure 5 Feedback control system

From the above block diagram

$$E(s) = R(s) - B(s) = R(s) - C(s)\,H(s)$$

$$E(s) = R(s) - E(s)\,G(s)\,H(s)$$

$$E(s) = \frac{R(s)}{1+G(s)\,H(s)}$$

The steady-state error using final value theorem can be written as

$$e_{ss} = \underset{t\to\infty}{\mathrm{Lt}}\, e_{ss}(t) = \underset{s\to 0}{\mathrm{Lt}}\, s\,E(S) = \underset{s\to 0}{\mathrm{Lt}}\, s.\frac{R(s)}{1+G(s)H(s)}$$

From the above expression is evident that steady-state error depends on input $R(s)$ and the open loop transfer function $G(s)H(s)$.

The expressions for steady-state errors for various types of standard test signals are given below.

(i) **Unit step input:**

$$\text{Steady-state error} = e_{ss} = \underset{s\to 0}{\mathrm{Lt}} \frac{sR(s)}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{\mathrm{Lt}} \frac{s\,.\,1/s}{1+G(s)H(s)}$$

$$e_{ss} = \frac{1}{1+\underset{s\to 0}{\mathrm{Lt}}\, G(s)H(s)} = \frac{1}{1+K_P}$$

$K_P = \underset{s\to 0}{\mathrm{Lt}}\; G(s)H(s)$ = Position error constant

(ii) **Unit ramp input:**

$$\text{Steady-state error} = e_{ss} = \underset{s\to 0}{\mathrm{Lt}} \frac{s\,.\,1/s^2}{1+G(s)H(s)}$$

$$= \underset{s\to 0}{\mathrm{Lt}} \frac{1}{s+sG(s)H(s)}$$

$$= \frac{1}{\underset{s\to 0}{\mathrm{Lt}}\, G(s)H(s)} = \frac{1}{K_V}$$

$K_V = \underset{s\to 0}{\mathrm{Lt}}\; sG(s)H(s)$ = Velocity error constant

(iii) **Unit parabolic input:**

$$\text{Steady-state error } e_{ss} = \underset{s\to 0}{\mathrm{Lt}} \frac{s\,.\,1/s^3}{1+G(s)H(s)}$$

$$= \frac{1}{\underset{s\to 0}{\mathrm{Lt}}\, s^2G(s)H(s)} = \frac{1}{K_a}$$

$K_a = \underset{s\to 0}{\mathrm{Lt}}\; s^2G(s)H(s)$ = Acceleration error constant.

Steady-state Errors for Different Types of System

Type 0 System

Open loop transfer function $G(S)H(s)$

$$= \frac{K(T_{z1}S+1)(T_{z2}S+1)..........}{(T_{p1}S+1)(T_{p2}S+1)......}$$

For step input: $e_{ss} = \dfrac{1}{1+K_P}$

Position error constant $(K_p) = \underset{s\to 0}{\mathrm{Lt}}\; G(S)\,H(s) = K$

Steady-state error $(e_{ss}) = \dfrac{1}{1+K}$

For Ramp Input

Velocity error constant $(K_V) = \underset{s\to 0}{\text{Lt}}\ s\ G(S)\ H(s) = 0$

Steady-state error $(e_{ss}) = \infty$

For Parabolic Input

Acceleration error constant $(K_a) = \underset{s\to 0}{\text{Lt}}\ s^2\ G(S)H(s) = 0$

Steady-state error $(e_{ss}) = \infty$

Type I System

Open loop transfer function

$$G(S)H(s) = \frac{K(T_{z1}s+1)(T_{z2}s+1)(T_{z3}s+1)\ldots}{S(T_{p1}s+1)(T_{p2}s+1)(T_{p3}s+1)\ldots}$$

Step Input

Position error constant $(K_p) = \underset{s\to 0}{\text{Lt}}\ G(S)\ H(s) = \infty$

$$\text{Steady-state error } (e_{ss}) = \frac{1}{1+K_p} = 0$$

Parabolic Input

Acceleration constant $(K_a) = \underset{s\to 0}{\text{Lt}}\ S^2\ G(S)\ H(s) = 0$

$$\text{Steady-state error } (e_{ss}) = \frac{1}{K_a} = \infty$$

Type 2 System

Open loop transfer function

$$G(S)H(s) = \frac{K(T_{z1}s+1)(T_{z2}s+1)\ldots\ldots}{s^2(T_{p1}s+1)(T_{p2}s+1)\ldots\ldots}$$

Step Input

Position error constant $(K_p) = \underset{s\to 0}{\text{Lt}}\ G(S)\ H(s) = \infty$

$$\text{Steady-state error } (e_{ss}) = \frac{1}{1+K_p} = 0$$

Ramp Input

Velocity error constant $(K_v) = \underset{s\to 0}{\text{Lt}}\ s\ G(S)\ H(s) = \infty$

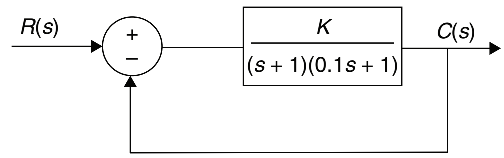
$$\text{Steady-state error } (e_{ss}) = \frac{1}{K_v} = 0$$

Parabolic Input

Acceleration error constant $(K_a) = \underset{s\to 0}{\text{Lt}}\ s\ G(S)H(s) = K$

$$\text{Steady-state error } (e_{ss}) = \frac{1}{K_a} = \frac{1}{K}$$

Example 14: The system shown in the figure has a unit step input. In order that the steady-state error is 0.1, the value of 'K' required is



(A) 0.1 (B) 9
(C) 0.9 (D) 1.0

Solution: (B)

Steady-state error $= \underset{s\to 0}{\text{Lt}}\ s.\dfrac{R(s)}{1+G(s)H(s)}$

Input is unit step

$$e_{ss} = \frac{1}{1+\underset{s\to 0}{\text{Lt}}\ G(s)H(s)} = \frac{1}{1+K_p}$$

$$K_p = \underset{s\to 0}{\text{Lt}}\ G(s)H(s) = \underset{s\to 0}{\text{Lt}}\ \frac{K}{(s+1)(0.1s+1)} = K$$

From the given problem,

$$e_{ss} = \frac{1}{1+K} = 0.1$$

$$K = 9$$

Example 15: If the time response of the system is given by thefollowing expression

$$y(t) = 5 + 4\sin(\omega t + \delta_1) + e^{-8t}\sin(\omega t + \delta_2) + e^{-4t},$$

then the steady-state part of the above response is given by

(A) $5 + e^{-4t}$
(B) $5 + 4(\sin(\omega t + \delta_1)$
(C) $5 + 4\sin(\omega t + \delta_1) + e^{-8t}(\omega t + \delta_2)$
(D) 5

Solution: (B)

Steady-state part $= \underset{t\to \infty}{\text{Lt}}\ y(t)$

$$= 5 + 4\sin(\omega t + \delta_1)$$

Example 16: Which of the following equation gives the steady-state error for a unity feedback system excited by?

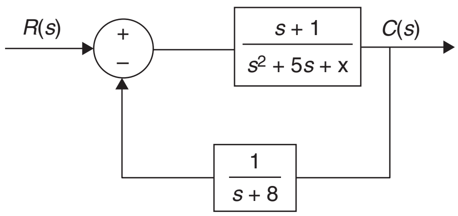
$$u_s(t) + tu_s(t) + t^2\frac{u_s(t)}{2}$$

(A) $\dfrac{1}{1+K_p} + \dfrac{1}{K_v} + \dfrac{1}{K_a}$ (B) $\dfrac{1}{1+K_p} + \dfrac{1}{K_v} + \dfrac{2}{K_a}$

(C) $\dfrac{1}{K_p} + \dfrac{1}{K_v} + \dfrac{1}{K_a}$ (D) $\dfrac{1}{K_p} + \dfrac{1}{K_v} + \dfrac{2}{K_a}$

Solution: (A)

Example 17: For what values of x, does the system shown in figure have a zero steady-state error (timed) for a step input?



(A) $x = 5$
(B) $x > 1$
(C) $x = 0$
(D) For no value of x

Solution: (C).
When step input is applied, the steady-state error is zero when type of the system is greater than zero (1).
For $X = 0 \Rightarrow$ type of the system = 1

Example 18: The system $G(S) = \dfrac{0.4}{s^2 + s - 2}$ is excited by a unit step input. The steady-state output is

(A) 0 (B) 0.83
(C) 1.25 (D) Unbounded

Solution: (D)
Given system is unstable, so the error is unbounded.

Example 19: The transfer function of a system is given by $\dfrac{(s+2)}{s(2s^3 + 3s^2 + s)}$

The steady-state error of the system with input $r(t) = (8t + 4t^2)\,u(t)$ is
(A) 4 (B) 12 (C) 8 (D) 2

Solution: (A)
Given system is type '2' system. Steady-state error for $8t(u(t))$ is zero.
Steady-state error for $4t^2$

$$= \frac{A}{K_a}\left[\because \frac{At^2}{2} = \frac{8}{2}t^2\right]$$

$$= \frac{8}{\underset{s\to 0}{\mathrm{Lt}}\, s^2 G(s)H(s)} = \frac{8}{2} = 4$$

Example 20: Which of the following is the steady-state error for a step input applied to a unity feedback system with open loop? transfer function $G(S) = \dfrac{5}{s^2 + 14s + 25}$

(A) $e_{ss} = 0.83$ (B) $e_{ss} = 0$
(C) $e_{ss} = 1$ (D) $e_{ss} = \infty$

Solution: (A)
Steady-state error for step input $= \dfrac{A}{1 + K_p}$

$$= \frac{1}{1 + \underset{s\to 0}{\mathrm{Lt}}\, G(s)H(s)}.$$

$$= \frac{1}{1 + 0.2}\, e^4 = 0.833$$

Exercises

Practice Problems I

Directions for questions 1 to 25: Select the correct alternative from the given choices.

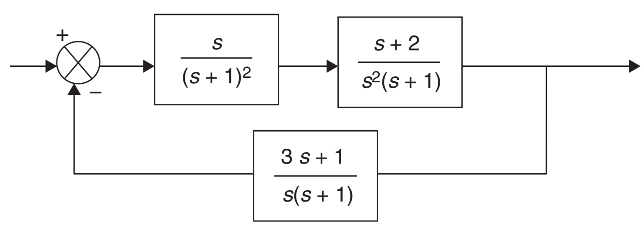
1. The closed loop transfer function of a second-order system is

$$\frac{C(s)}{R(s)} = \frac{100}{s^2 + 16s + 100}$$

The type of damping in the system is
(A) Underdamped.
(B) Overdamped.
(C) Critically damped.
(D) Undamped

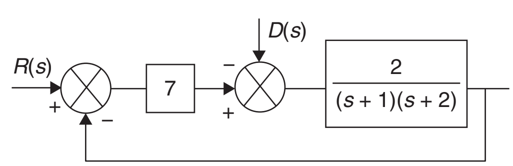
2. The unit impulse response of a second-order system is $\frac{1}{6}e^{-0.8t}\sin(0.6t)$. The natural frequency and damped frequency oscillations are, respectively,
(A) 1 and 0.8. (B) 1 and 0.6.
(C) 0.8 and 1. (D) 1.6 and 1.

3. The feedback control system shown in the figure represents a



(A) Type 1 system (B) Type 2 system
(C) Type 3 system (D) None

4. The steady-state error due to unit ramp disturbance input $D(s)$ is given by



(A) 0 (B) 0.012 (C) 0.021 (D) 0.025

5. A unity feedback control system is represented by the open loop transfer function $G(S) = \dfrac{K}{s(s+2)}$. The range values of K, so that the system remain under damped.

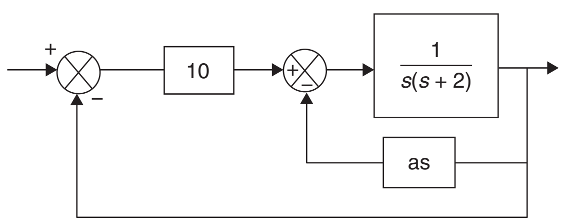
(A) $K < 1$ (B) $K > 1$
(C) $K = 1$ (D) None of these

6. For a proportional control action, which among the following is true?
(i) As gain K increases, ξ decreases.
(ii) The system tend to have large overshoots as gain increases.
(iii) As gain increases, the system will be over damped.
(iv) As gain increases, settling time will be more.
(A) All true (B) i, ii, iv are true
(C) i, ii true (D) ii, iii, iv true

7. The unit impulse response of a system is given by $C(t) = 4e^{-3t}$. Find the step response of the system for $t > 0$.
(A) $1 - e^{-3t}$ (B) $3e^{-3t}$
(C) $\frac{4}{3}(1 - e^{-3t})$ (D) e^{-3t}

8. If the system's transfer function is given by $T(s) = \frac{6}{s(s+2)(s+3)}$, then its time constant is
(A) $\frac{1}{6}$ (B) 6 (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

Common Data for Questions 9 and 10:
A feedback system employing output-rate damping is shown in the following figure.



9. In the absence of derivative feedback, the damping factor and natural frequency of the system are
(A) 0.1 and 10 (B) 1 and 10
(C) $\frac{1}{\sqrt{10}}$ and $\sqrt{10}$ (D) $\frac{1}{\sqrt{5}}$ and $\sqrt{5}$

10. In the absence of derivative feedback the steady-state error resulting from unit ramp input is
(A) 5 (B) 0.2 (C) 2 (D) 0.02

11. A unity feedback system is characterized by an open loop transfer function $G(s) = \frac{K}{s(s+20)}$. The values of 'K' so that the system will have a damping ratio of 0.5 and the peak time are
(A) 200 and 0.157 s (B) 400 and 0.1813 s
(C) 100 and 0.157 s (D) $\sqrt{200}$ and 0.181 s

12. A unity feedback has a loop transfer function of $G(S) = \frac{20(s+5)}{s(s+1)(s+2)}$

The steady-state error for unit ramp input is
(A) 50 (B) Zero
(C) Infinite (D) 0.02

13. A second-order control system has

$$M(j\omega) = \frac{64}{64 - \omega^2 + 8\sqrt{2}j\omega}$$

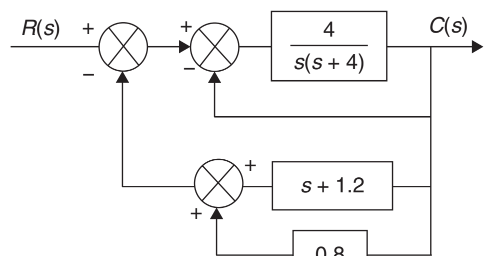
Its M_p (Peak magnitude) is
(A) 0.5 (B) 1.5 (C) $\sqrt{2}$ (D) 1

14. A servo system is given by the equation

$$C(t) = 1 - 2e^{-5t} + e^{-10t}$$

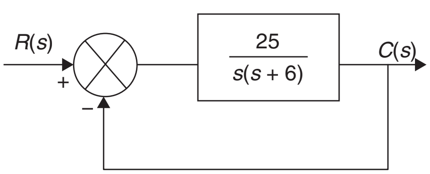
Find the natural frequency of the system, if step input is applied to the system.
(A) 17.52 (B) 50.01
(C) 7.07 (D) 2.04

15. Obtain the damping factor of the following system



(A) 0.8 (B) 4 (C) 4.3 (D) 3.46

16. The transfer function of second-order closed loop control system is shown below. The value of its response is maximum when $t = t_{max}$. Then t_{max} = ?



(A) $\pi/8$ (B) $\pi/4$ (C) $\pi/2$ (D) π

Common Data for Questions 17 and 18:
A unity feedback system is characterized by the open loop transfer function $G(S) = \frac{100}{s(s+10)}$

17. The static error constants (K_p, K_v, K_a) for the system are
(A) 0, 10 and ∞ (B) ∞, 10 and 0
(C) ∞, 0.1 and 0 (D) 0, 10 and 0

18. The steady-state error of the system when subjected to an input given by

$$r(t) = 3 + 4t + \frac{7}{2}t^2$$

(A) 0.4 (B) 0
(C) ∞ (D) 0.35

Common Data for Questions 19 and 20:
Open loop transfer function of a system is given by $G(s)H(s) = \dfrac{15}{5s^3 + 2s^2 + 3s}$

19. The steady-state error of the system when it is subjected to an input $r(t) = 5t$
(A) 1 (B) 3
(C) 5 (D) 15

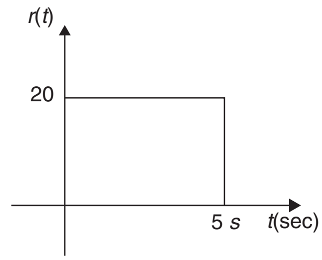
20. The steady-state error of the system when it is subjected to an input $r(t) = 5 + 8t + 3t^2$
(A) 0 (B) 1
(C) 3 (D) ∞

21. Match the following

Transfer function	Type of damping
1. $\dfrac{5}{s^2 + 10s + 5}$	P. Undamped
2. $\dfrac{25}{s^2 + 10s + 25}$	Q. Underdamped
3. $\dfrac{25}{s^2 + s + 25}$	R. Critically damped
4. $\dfrac{5}{s^2 + 5}$	S. Overdamped

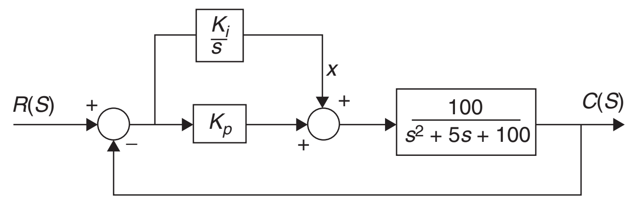
Code:	P	Q	R	S
(A)	4	3	2	1
(B)	3	2	4	1
(C)	4	2	3	1
(D)	3	4	1	2

22. The steady-state error of a unity feedback linear system for unit step input is 0.08. The steady-state error of the same system for a pulse input $r(t)$ having a magnitude of 20 and duration of 5 s as shown in the following figure.



(A) 0 (B) 08 (C) 1.6 (D) 0.16

23. Consider the feedback system shown below which is subjected to a unit input. The system is stable and has the controller parameters as $K_p = 5$, $K_i = 10$. The steady-state value of 'X' is



(A) 100 (B) 0 (C) 1 (D) 0.1

24. The roots of the characteristic equation are symmetric about origin and are on the real axis of the S-plane, then which one of the following is true?
(A) One row of zeros present in the RH table.
(B) System is unstable.
(C) One sign change present in the first column of the RH table.
(D) All the above

25. If the imaginary part of second-order underdamped closed loop control system is increased and real part remains same
(A) ξ increases and ω_n decreases.
(B) ξ decreases and ω_n increases.
(C) ξ and ω_n decrease
(D) ξ and ω_n increase

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. Consider the unity feedback control system with open loop transfer function

$$G(s) = \frac{k}{s(s+a)}$$

The steady-state error of the system due to a unit step input is
(A) Always zero.
(B) Depends on the value of 'k'.
(C) Depends on 'a'.
(D) Depends on both 'k' and 'a'.

2. Given that

$$e(t) = C_0 r(t) + C_1 \dot{r}(t) + \frac{C_2}{2!}\ddot{r}(t) + \frac{C_3}{3!}\dddot{r}(t) + \ldots\ldots + \frac{Cn}{n!} r^n(t)$$

where $e(t)$ and $r(t)$ represents error and input signal, respectively, and K_p, K_v and K_a represent static error constants then which of the following statements is true?

(i) Generalized error series gives error signal as a function of time.
(ii) Generalized error constants C_0, C_1, C_2 … C_n are functions of time.
(iii) Using generalized error constants steady-state error can be determined for any type of input.
(iv) Using static error constants the steady-state error can be determined for any type of input.

(A) i, ii and iv (B) i, ii and iii
(C) i, iii and iv (D) i and iii

3. If $y(t) = A + X\sin(\omega t + \theta_1) + e^{-bt}\sin(\omega t + \theta_1)$ represents the equation of a system, then the steady-state part of the above response is

(A) A (B) $A + X\sin(\omega t + \theta_1)$
(C) $A + e^{-bt}(\sin\omega t + \theta_1)$ (D) $X\sin(\omega t + \theta_1)$

4. The steady-state error of first-order system with ramp input is

(A) 0 (B) T(C) 1 (D) $1 - T$

5. Which among the following are true

(i) An overdamped system gives sluggish response.
(ii) An integral controller improves the steady-state response of the system.
(iii) The pole of an over damped system is in the real axis.
(iv) When $\xi = 0$, the system is an undamped one.

(A) i, iii are true (B) i, ii, iv are true
(C) i, ii are true (D) All true

6. The position and velocity error constants for the following system are

$$G(S)\,H(s) = \frac{100}{(s+2)\,(s+5)}$$

(A) 10, 0 (B) 10, infinity
(C) .1, 0 (D) 0, 10

7. $\dfrac{K}{s(\tau s+2)}$ represents the open loop transfer function of a unity feedback system. Keeping gain constant, what should be the change in time constant in order to reduce the damping factor from .8 to .4

(A) $T_2 = 4T_1$ (B) $T_2 = 2T_1$
(C) $T_2 = \dfrac{T_1}{4}$ (D) $T_2 = \dfrac{T_1}{2}$

Common Data for Questions 8 and 9:

The open loop transfer function of a unity feedback system is given by $G(s) = \dfrac{K}{s(1+sT)}$, where T and K are constants having positive values. By what factor should the amplifier gain be reduced so that

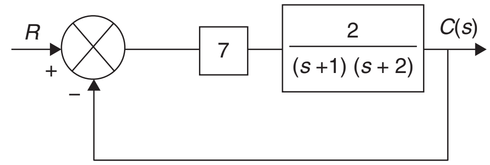
8. the peak overshoot of unit response of the systems are reduced from 75% to 50%?

(A) 5.6 (B) 10 (C) 2.5 (D) 20

9. the damping ratio increases from 0.1 to 0.5?

(A) 25 (B) 5 (C) $\sqrt{5}$ (D) $\dfrac{1}{\sqrt{5}}$

10. The maximum undershoot of the system given in the figure below will be



(A) 0.14 (B) 0.28
(C) 0.079 (D) 0.092

11. The open loop transfer function of a unity feedback system is given as $\dfrac{10}{s(s+5)}$. The system is subjected to an input $r(t) = 1 + 5t$. Find the steady-state error.

(A) 2 (B) α (C) 2.5 (D) 1

12. A unity feedback system is characterized by the open loop transfer function

$$G(S) = \frac{100}{s(5s+10)(2s+10)}.$$

The steady-state errors for unit-step, unit-ramp and unit-acceleration inputs are

(A) 1, 1, ∞ (B) 0, 1, 1
(C) 0, 1, ∞ (D) 1, 1, ∞

13. A servo system is given by the equation

$$C(t) = 1 - 2e^{-5t} + e^{-10t}$$

Find the natural frequency of the system, if step input is applied to the system.

(A) 17.52 (B) 50.01
(C) 7.07 (D) 2.04

14. A unity feedback control system has its open loop transfer function $G(S) = \dfrac{8s+1}{16s^2}$. Determine an expression for time response when it is subjected to a unit impulse input

(A) $\dfrac{e^{-0.5t}}{20}[1+t]$ (B) $\dfrac{e^{-0.25t}}{20}[1+t]$
(C) $\dfrac{e^{-0.25t} - 4te^{-0.25t}}{20}$ (D) $\dfrac{te^{-.25t}+1}{20}$

15. The steady-state error of the system with loop transfer function $\dfrac{K}{(s+1)\,(s+2)}$ is given as 0.1. Find the value of gain K, if system is subjected to step input

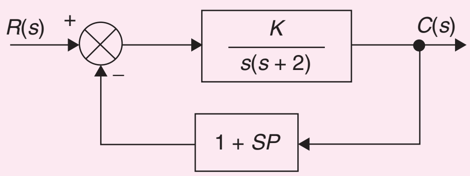
(A) 10 (B) 16
(C) 18 (D) 20

Previous Years' Questions

1. Consider the function $F(s) = \dfrac{5}{s(s^2+3s+2)}$, where $F(s)$ is the Laplace transform of the function $f(t)$. The initial value of $f(t)$ is equal to **[2004]**

(A) 5 (B) $\frac{5}{2}$ (C) $\frac{5}{3}$ (D) 0

2. The block diagram of a closed loop control system is given by figure. The values of K and P such that the system has a damping ratio of 0.7 and an undamped natural frequency ω_n of 5 rad/s are, respectively, equal to **[2004]**



(A) 20 and 0.3 (B) 20 and 0.2
(C) 25 and 0.3 (D) 25 and 0.2

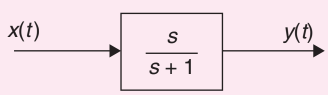
3. The unit impulse response of a second-order under-damped system starting from rest is given by

$$c(t) = 12.5e^{-6t} \sin 8t, \; t \geq 0$$

The steady-state value of the unit step response of the system is equal to **[2004]**

(A) 0 (B) 0.25 (C) 0.5 (D) 1.0

4. In the system shown in figure, the input $x(t) = \sin t$. In the steady state, the response $y(t)$ will be **[2004]**



(A) $\frac{1}{\sqrt{2}}\sin(t-45°)$ (B) $\frac{1}{\sqrt{2}}\sin(t+45°)$
(C) $\sin(t-45°)$ (D) $\sin(t+45°)$

5. The open loop transfer function of a unity feedback control system is given as $G(S) = \dfrac{as+1}{s^2}$

The value of 'a' to give a phase margin of 45° is equal to **[2004]**

(A) 0.141 (B) 0.441
(C) 0.841 (D) 1.141

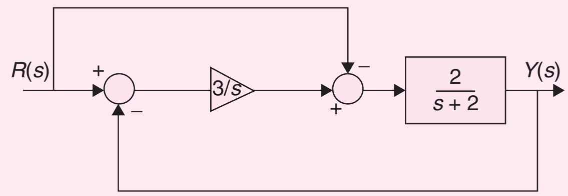
6. The Laplace transform of a function $f(t)$ is $F(s) = \dfrac{5s^2+23s+6}{s(s^2+2s+2)}$. As $t \to \infty$, $f(t)$ approaches **[2005]**

(A) 3 (B) 5 (C) $\frac{17}{2}$ (D) ∞

7. A system with zero initial conditions has the closed loop transfer function $T(s) = \dfrac{s^2+4}{(s+1)(s+4)}$. The system output is zero at the frequency **[2005]**

(A) 0.5 rad/s (B) 1 rad/s
(C) 2 rad/s (D) 4 rad/s

8. When subjected to a unit step input, the closed loop control system shown in the following figure will have a steady-state error of **[2005]**



(A) –1.0 (B) –0.5 (C) 0 (D) 0.5

9. If the loop gain K of a negative feedback system having a loop transfer function $\dfrac{K(s+3)}{(s+8)^2}$ is to be adjusted to induce a sustained oscillation then **[2007]**

(A) The frequency of this oscillation must be $\frac{4}{\sqrt{3}}$ rad/s.
(B) The frequency of this oscillation must be 4 rad/s.
(C) The frequency of this oscillation must be $\frac{4}{\sqrt{3}}$ rad/s.
(D) Such a K does not exist.

10. If $u(t)$, $r(t)$ denote the unit step and unit ramp functions respectively and $u(t) * r(t)$ their convolution, then the function $u(t+1) * r(t-2)$ is given by **[2007]**

(A) $(1/2)(t-1)(t-2)$ (B) $(1/2)(t-1)(t-2)$
(C) $(1/2)(t-1)^2 u(t-1)$ (D) None of these

11. A function $y(t)$ satisfies the following differential equation

$$\frac{dy(t)}{dt} + y(t) = \delta(t)$$

where $\delta(t)$ is the delta function. Assuming zero initial condition, and denoting the unit step function by $u(t)$, $y(t)$ can be of the form **[2008]**

(A) e^t (B) e^{-t}
(C) $e^t u(t)$ (D) $e^{-t} u(t)$

12. A system with input $x(t)$ and output $y(t)$ is defined by the input-output relation

$$y(t) = \int_{-\infty}^{-2t} x(t)\,dt$$

The system will be **[2008]**

(A) Causal, time-invariant and unstable
(B) Causal, time-invariant and stable
(C) Non-causal, time-invariant and unstable
(D) Non-causal, time-variant and unstable

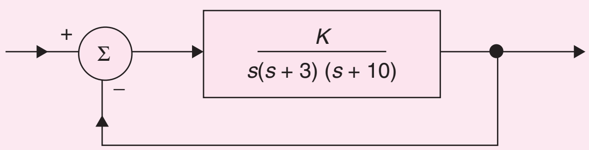
13. The transfer function of a linear time invariant system is given as

$$G(S) = \frac{1}{s^2 + 3s + 2}$$

The steady-state value of the output of this system for a unit impulse input applied at time instant $t = 1$ will be **[2008]**

(A) 0 (B) 0.5 (C) 1 (D) 2

14. Figure shows a feedback system where $K > 0$. The range of K for which the system is stable will be given by **[2008]**



(A) $0 < K < 30$ (B) $0 < K < 39$
(C) $0 < K < 390$ (D) $K > 390$

15. The transfer function of a system is given as $\dfrac{100}{s^2 + 20s + 100}$ This system is **[2008]**

(A) An overdamped system
(B) An underdamped system
(C) A critically damped system
(D) An unstable system

16. The open loop transfer function of a unity feedback system is given by $G(S) = (e^{-0.1s})/s$. The gain margin of this system is **[2009]**

(A) 11.95 dB (B) 17.67 dB
(C) 21.33 dB (D) 23.9 dB

17. Consider an LTI system with transfer function $H(s) = \dfrac{1}{S(S+4)}$ If the input to the system is $\cos(3t)$ and the steady-state output is $A \sin(3t + \alpha)$, then the value of A is **[2014]**

(A) 1/30 (B) 1/15 (C) 3/4 (D) 4/3

18. Consider an LTI system with impulse response $h(t) = e^{-5t}\,u(t)$. If the output of the system is $y(t) = e^{-3t}\,u(t) - e^{-5t}\,u(t)$ then the input, $x(t)$, is given by **[2014]**

(A) $e^{-3t}\,u(t)$ (B) $2\,e^{-3t}\,u(t)$
(C) $e^{-5t}\,u(t)$ (D) $2\,e^{-5t}\,u(t)$

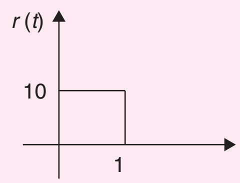
19. The characteristic equation of a closed-loop system is $s(s + 1)(s + 3) + k(s + 2) = 0$, $k > 0$. Which of the following statements is true? **[2010]**

(A) Its roots are always real.
(B) It cannot have a breakaway point in the range $-1 < \text{Re}[s] < 0$.
(C) Two of its roots tend to infinity along the asymptotes $\text{Re}[s] = -1$.
(D) It may have complex roots in the right half plane.

20. For the system $\dfrac{2}{(s+1)}$, the approximate time taken for a step response to reach 98% of its final value is **[2010]**

(A) 1s (B) 2s (C) 4s (D) 8s

21. The steady-state error of a unity feedback linear system for a unit step input is 0.1. The steady-state error of the same system, for a pulse input $r(t)$ having a magnitude of 10 and a duration of one second, as shown in the figure is **[2011]**



(A) 0 (B) 0.1 (C) 1 (D) 10

22. The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relax condition is $h(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is **[2011]**

(A) $u(t) + e^{-t} + e^{-2t}$ (B) $(e^{-t} + e^{-2t})\,u(t)$
(C) $(1.5 - e^{-t} - 0.5e^{-2t})$ (D) $e^{-t}\,\delta(t) + e^{-2t}\,u(t)$

23. A system with transfer function

$$G(S) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

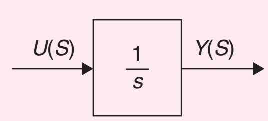
is excited by $\sin(\omega t)$. The steady-state output of the system is zero at **[2012]**

(A) $\omega = 1$ rad/s (B) $\omega = 2$ rad/s
(C) $\omega = 3$ rad/s (D) $\omega = 4$ rad/s

24. The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is **[2013]**

(A) 0 (B) 1 (C) 2 (D) 3

25. Assuming zero initial condition, the response $y(t)$ of the system given below to a unit step input $u(t)$ is **[2013]**



(A) $u(t)$ (B) $tu(t)$
(C) $\dfrac{t^2}{2}u(t)$ (D) $e^{-t}u(t)$

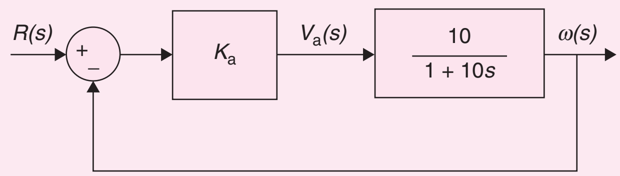
26. The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is **[2013]**

(A) $\dfrac{t^2}{2}u(t)$ (B) $\dfrac{t(t-1)}{2}u(t-1)$
(C) $\dfrac{(t-1)^2}{2}u(t-1)$ (D) $\dfrac{t^2-1}{2}u(t-1)$

27. Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system? **[2013]**

(A) All the poles of the system must lie on the left side of the $j\omega$ axis.
(B) Zeros of the system can lie anywhere in the S-plane.
(C) All the poles must lie within $|s| = 1$.
(D) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

28. The open-loop transfer function of a DC motor is given as $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$ When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is **[2013]**



(A) 1 (B) 5 (C) 10 (D) 100

Answer Keys

Exercises

Practice Problems 1

1. A	**2.** B	**3.** B	**4.** A	**5.** B	**6.** C	**7.** C	**8.** C	**9.** C	**10.** B
11. B	**12.** D	**13.** D	**14.** C	**15.** B	**16.** A	**17.** B	**18.** C	**19.** A	**20.** D
21. A	**22.** A	**23.** B	**24.** D	**25.** B					

Practice Problems 2

1. A	**2.** B	**3.** B	**4.** B	**5.** D	**6.** A	**7.** A	**8.** A	**9.** A	**10.** C
11. C	**12.** C	**13.** C	**14.** B	**15.** C					

Previous Years' Questions

1. D	**2.** D	**3.** D	**4.** B	**5.** C	**6.** A	**7.** C	**8.** C	**9.** B	**10.** C
11. D	**12.** D	**13.** A	**14.** C	**15.** C	**16.** D	**17.** B	**18.** B	**19.** C	**20.** C
21. A	**22.** C	**23.** C	**24.** B	**25.** B	**26.** C	**27.** C	**28.** C		