Chapter 1

Probability

Contents

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Probability theory, like many other branches of mathematics, evolved out of practical considerations. It had its origin in the 16th century when an Italian physician and Mathematician Jerome Cardan (1501-1576) *wrote the first book on the subject* "Book on Games of Chance (Liber de Ludo Aleae)" *It was published in 1663 after his death.*

In 1654, a gambler Chevalier de Mere approached the well known French philosopher and mathematician Blaise Pascal (1623-1662) *for certain dice problems. Pascal became interested in these problems and discussed with famous* French mathematician Pierre de Fermat (1601-1665) *Both Pascal and Fermat solved the problem independently.*

Probability

1.1 Introduction

Numerical study of chances of occurrence of events is dealt in probability theory.

The theory of probability is applied in many diverse fields and the flexibility of the theory provides approximate tools for so great a variety of needs.

There are two approaches to probability viz. (i) Classical approach and (ii) Axiomatic approach.

In both the approaches we use the term 'experiment', which means an operation which can produce some well-defined outcome(*s*). There are two types of experiments:

(1) **Deterministic experiment :** Those experiments which when repeated under identical conditions produce the same result or outcome are known as deterministic experiments. When experiments in science or engineering are repeated under identical conditions, we get almost the same result everytime.

(2) **Random experiment :** If an experiment, when repeated under identical conditions, do not produce the same outcome every time but the outcome in a trial is one of the several possible outcomes then such an experiment is known as a probabilistic experiment or a random experiment.

In a random experiment, all the outcomes are known in advance but the exact outcome is unpredictable.

For example, in tossing of a coin, it is known that either a head or a tail will occur but one is not sure if a head or a tail will be obtained. So it is a random experiment.

1.2 Definitions of Various Terms

(1) **Sample space :** The set of all possible outcomes of a trial (random experiment) is called its sample space. It is generally denoted by S and each outcome of the trial is said to be a sample point.

Example : (i) If a dice is thrown once, then its sample space is $S = \{1, 2, 3, 4, 5, 6\}$

(ii) If two coins are tossed together then its sample space is $S = \{HT, TH, HH, TT\}$.

(2) **Event :** An event is a subset of a sample space.

(i) **Simple event :** An event containing only a single sample point is called an elementary or simple event.

Example : In a single toss of coin, the event of getting a head is a simple event.

Here $S = \{H, T\}$ and $E = \{H\}$

(ii) **Compound events :** Events obtained by combining together two or more elementary events are known as the compound events or decomposable events.

For example, In a single throw of a pair of dice the event of getting a doublet, is a compound event because this event occurs if any one of the elementary events (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) occurs.

(iii) **Equally likely events :** Events are equally likely if there is no reason for an event to occur in preference to any other event.

Example : If an unbiased die is rolled, then each outcome is equally likely to happen *i.e.*, all elementary events are equally likely.

(iv) **Mutually exclusive or disjoint events :** Events are said to be mutually exclusive or disjoint or incompatible if the occurrence of any one of them prevents the occurrence of all the others.

Example : E = getting an even number, F = getting an odd number, these two events are mutually exclusive, because, if E occurs we say that the number obtained is even and so it cannot be odd *i.e.*, F does not occur.

A_1 and A_2 are mutually exclusive events if $A_1 \cap A_2 = \phi$.

(v) **Mutually non-exclusive events :** The events which are not mutually exclusive are known as compatible events or mutually non exclusive events.

(vi) **Independent events :** Events are said to be independent if the happening (or non-happening) of one event is not affected by the happening (or non-happening) of others.

Example : If two dice are thrown together, then getting an even number on first is independent to getting an odd number on the second.

(vii) **Dependent events :** Two or more events are said to be dependent if the happening of one event affects (partially or totally) other event.

Example : Suppose a bag contains 5 white and 4 black balls. Two balls are drawn one by one. Then two events that the first ball is white and second ball is black are independent if the first ball is replaced before drawing the second ball. If the first ball is not replaced then these two events will be dependent because second draw will have only 8 exhaustive cases.

(3) **Exhaustive number of cases :** The total number of possible outcomes of a random experiment in a trial is known as the exhaustive number of cases.

Example : In throwing a die the exhaustive number of cases is 6, since any one of the six faces marked with 1, 2, 3, 4, 5, 6 may come uppermost.

(4) **Favourable number of cases :** The number of cases favourable to an event in a trial is the total number of elementary events such that the occurrence of any one of them ensures the happening of the event.

Example : In drawing two cards from a pack of 52 cards, the number of cases favourable to drawing 2 queens is ${}^4C_2$.

(5) **Mutually exclusive and exhaustive system of events :** Let S be the sample space associated with a random experiment. Let A_1, A_2, ...A_n be subsets of S such that

(i) $A_i \cap A_j = \phi$ for $i \neq j$ and (ii) $A_1 \cup A_2 \cup .... \cup A_n = S$

Then the collection of events $A_1, A_2, ..., A_n$ is said to form a mutually exclusive and exhaustive system of events.

If $E_1, E_2, ..., E_n$ are elementary events associated with a random experiment, then

(i) $E_i \cap E_j = \phi$ for $i \neq j$ and (ii) $E_1 \cup E_2 \cup .... \cup E_n = S$

So, the collection of elementary events associated with a random experiment always form a system of mutually exclusive and exhaustive system of events.

In this system, $P(A_1 \cup A_2 .... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n) = 1$.

Important Tips

- *Independent events are always taken from different experiments, while mutually exclusive events are taken from a single experiment.*
- *Independent events can happen together while mutually exclusive events cannot happen together.*
- *Independent events are connected by the word "and" but mutually exclusive events are connected by the word "or".*

Example: 1 Two fair dice are tossed. Let *A* be the event that the first die shows an even number and *B* be the event that second die shows an odd number. The two events *A* and *B* are **[IIT 1979]**

(a) Mutually exclusive (b) Independent and mutually exclusive

(c) Dependent (d) None of these

Solution: (d) They are independent events but not mutually exclusive.

Example: 2 The probabilities of a student getting I, II and III division in an examination are respectively $\frac{1}{10}, \frac{3}{5}$ and $\frac{1}{4}$. The probability that the student fail in the examination is **[MP PET 1997]**

(a) $\frac{197}{200}$ (b) $\frac{27}{200}$ (c) $\frac{83}{100}$ (d) None of these

Solution: (d) *A* denote the event getting I; *B* denote the event getting II;

C denote the event getting III; and *D* denote the event getting fail.

Obviously, these four events are mutually exclusive and exhaustive, therefore

$P(A) + P(B) + P(C) + P(D) = 1 \Rightarrow P(D) = 1 - 0.95 = 0.05$.

1.3 Classical definition of Probability

If a random experiment results in *n* mutually exclusive, equally likely and exhaustive outcomes, out of which *m* are favourable to the occurrence of an event *A*, then the probability of occurrence of *A* is given by

$$P(A) = \frac{m}{n} = \frac{\text{Number of outcomes favourable to } A}{\text{Number of total outcomes}}$$

It is obvious that $0 \le m \le n$. If an event *A* is certain to happen, then $m = n$, thus $P(A) = 1$.

If *A* is impossible to happen, then $m = 0$ and so $P(A) = 0$. Hence we conclude that

$$0 \le P(A) \le 1.$$

Further, if $\bar{A}$ denotes negative of *A* *i.e.* event that *A* doesn't happen, then for above cases *m*, *n*; we shall have

$$P(\overline{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

$\therefore \quad P(A) + P(\overline{A}) = 1$.

Notations : For two events A and B,

(i) A' or $\overline{A}$ or A^C stands for the non-occurrence or negation of A.

(ii) $A \cup B$ stands for the occurrence of at least one of A and B.

(iii) $A \cap B$ stands for the simultaneous occurrence of A and B.

(iv) $A' \cap B'$ stands for the non-occurrence of both A and B.

(v) $A \subseteq B$ stands for "the occurrence of A implies occurrence of B".

1.4 Some important remarks about Coins, Dice , Playing cards and Envelopes

(1) **Coins :** A coin has a head side and a tail side. If an experiment consists of more than a coin, then coins are considered to be distinct if not otherwise stated.

Number of exhaustive cases of tossing n coins simultaneously (or of tossing a coin n times) = 2^n.

(2) **Dice :** A die (cubical) has six faces marked 1, 2, 3, 4, 5, 6. We may have tetrahedral (having four faces 1, 2, 3, 4) or pentagonal (having five faces 1, 2, 3, 4, 5) die. As in the case of coins, if we have more than one die, then all dice are considered to be distinct if not otherwise stated.

Number of exhaustive cases of throwing n dice simultaneously (or throwing one dice n times) = 6^n.

(3) **Playing cards :** A pack of playing cards usually has 52 cards. There are 4 suits (Spade, Heart, Diamond and Club) each having 13 cards. There are two colours red (Heart and Diamond) and black (Spade and Club) each having 26 cards.

In thirteen cards of each suit, there are 3 face cards or coart cards namely king, queen and jack. So there are in all 12 face cards (4 kings, 4 queens and 4 jacks). Also there are 16 honour cards, 4 of each suit namely ace, king, queen and jack.

(4) **Probability regarding *n* letters and their envelopes :** If n letters corresponding to n envelopes are placed in the envelopes at random, then

(i) Probability that all letters are in right envelopes $= \frac{1}{n!}$.

(ii) Probability that all letters are not in right envelopes $= 1 - \frac{1}{n!}$.

(iii) Probability that no letter is in right envelopes $= \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots + (-1)^n \frac{1}{n!}$.

(iv) Probability that exactly r letters are in right envelopes $= \frac{1}{r!}\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots\ldots + (-1)^{n-r}\frac{1}{(n-r)!}\right]$.

Example: 3 If $(1+3p)/3, (1-p)/4$ and $(1-2p)/2$ are the probabilities of three mutually exclusive events, then the set of all values of p is **[IIT 1986; AMU 2002; AIEEE 2003]**

(a) $\frac{1}{3} \le p \le \frac{1}{2}$ (b) $\frac{1}{3} < p < \frac{1}{2}$ (c) $\frac{1}{2} \le p \le \frac{2}{3}$ (d) $\frac{1}{2} < p < \frac{2}{3}$

Solution: (a) Since $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ and $\left(\frac{1-2p}{2}\right)$ are the probabilities of the three events, we must have

$$0 \le \frac{1+3p}{3} \le 1, 0 \le \frac{1-p}{4} \le 1 \text{ and } 0 \le \frac{1-2p}{2} \le 1 \Rightarrow -1 \le 3p \le 2, -3 \le p \le 1 \text{ and } -1 \le 2p \le 1$$

$$\Rightarrow \quad -\frac{1}{3} \le p \le \frac{2}{3}, -3 \le p \le 1 \text{ and } -\frac{1}{2} \le p \le \frac{1}{2}.$$

Also as $\frac{1+3p}{3}, \frac{1-p}{4}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events,

$$0 \le \frac{1+3p}{3} + \frac{1-p}{4} + \frac{1-2p}{2} \le 1 \Rightarrow 0 \le 4 + 12p + 3 - 3p + 6 - 12p \le 12 \Rightarrow \frac{1}{3} \le p \le \frac{13}{3}$$

Thus the required values of p are such that $\max\left\{-\frac{1}{3}, -3, -\frac{1}{2}, \frac{1}{3}\right\} \le p \le \min\left\{\frac{2}{3}, 1, \frac{1}{2}, \frac{13}{3}\right\} \Rightarrow \frac{1}{3} \le p \le \frac{1}{2}$.

Example: 4 The probability that a leap year selected randomly will have 53 Sundays is **[MP PET 1991, 93, 95]**

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{4}{53}$ (d) $\frac{4}{49}$

Solution: (b) A leap year contain 366 days *i.e.* 52 weeks and 2 days, clearly there are 52 Sundays in 52 weeks.
For the remaining two days, we may have any of the two days
(i) Sunday and Monday, (ii) Monday and Tuesday, (iii) Tuesday and Wednesday, (iv) Wednesday and Thursday,
(v) Thursday and Friday, (iv) Friday and Saturday and (vii) Saturday and Sunday.

Now for 53 Sundays, one of the two days must be Sundays, hence required probability $= \frac{2}{7}$.

Example: 5 Three identical dice are rolled. The probability that same number will appear on each of them will be
[SCRA 1991; MP PET 1989; IIT 1984; Rajasthan PET 2000, 02; DCE 2001]

(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$

Solution: (b) If three identical dice are rolled then total number of sample points $= 6 \times 6 \times 6 = 216$.
Favourable events (same number appear on each dice) are

(1, 1, 1) (2, 2, 2) ....(6, 6, 6). $\therefore$ Required probability $= \frac{6}{216} = \frac{1}{36}$.

1.5 Problems based on Combination and Permutation

(1) **Problems based on combination or selection :** To solve such kind of problems, we use $^nC_r = \frac{n!}{r!(n-r)!}$.

Example: 6 Three of the six vertices of a regular hexagon are chosen at random. The probability that the triangle with these three vertices is equilateral, is equal to **[IIT 1995; MP PET 2002]**

(a) $\frac{1}{2}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{20}$

Solution: (c) Total number of triangles which can be formed = $^6C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$

Number of equilateral triangles = 2. $\therefore$ Required probability $= \frac{2}{20} = \frac{1}{10}$.

Example: 7 Three distinct numbers are selected from 100 natural number. The probability that all the three numbers are divisible by 2 and 3 is **[IIT Screening 2004]**

(a) $\frac{4}{25}$ (b) $\frac{4}{35}$ (c) $\frac{4}{55}$ (d) $\frac{4}{1155}$

Solution: (d) The numbers should be divisible by 6. Thus the number of favourable ways is ${}^{16}C_3$ (as there are 16 numbers in first 100 natural numbers, divisible by 6). Required probability is $\frac{{}^{16}C_3}{{}^{100}C_3} = \frac{16 \times 15 \times 14}{100 \times 99 \times 98} = \frac{4}{1155}$.

Example: 8 Out of 21 tickets marked with numbers from 1 to 21, three are drawn at random. The chance that the numbers on them are in A.P., is **[Roorkee 1988; DCE 1999]**

(a) $\frac{10}{133}$ (b) $\frac{9}{133}$ (c) $\frac{9}{1330}$ (d) None of these

Solution: (a) Total number of ways $= {}^{21}C_3 = 1330$. If common difference of the A.P. is to be 1, then the possible groups are 1, 2, 3; 2, 3, 4; ……19, 20, 21.

If the common difference is 2, then possible groups are 1, 3, 5; 2, 4, 6; ….. 17, 19, 21.

Proceeding in the same way, if the common difference is 10, then the possible group is 1, 10, 21.

Thus if the common difference of the A.P. is to be ≥ 11, obviously there is no favourable case.

Hence, total number of favourable cases = 19 +17 + 15 + …+ 3 + 1 =100

Hence, required probability $= \frac{100}{1330} = \frac{10}{133}$.

(2) **Problems based on permutation or arrangement :** To solve such kind of problems, we use ${}^nP_r = \frac{n!}{(n-r)!}$.

Example: 9 There are four letters and four addressed envelopes. The chance that all letters are not dispatched in the right envelope is

[Rajasthan PET 1997; MP PET 1999; DCE 1999]

(a) $\frac{19}{24}$ (b) $\frac{21}{23}$ (c) $\frac{23}{24}$ (d) $\frac{1}{24}$

Solution: (c) Required probability is 1 – P (they go in concerned envelopes) $= 1 - \frac{1}{4!} = \frac{23}{24}$.

Example: 10 The letters of the word 'ASSASSIN' are written down at random in a row. The probability that no two S occur together is

[BIT Ranchi 1990; IIT 1983]

(a) $\frac{1}{35}$ (b) $\frac{1}{14}$ (c) $\frac{1}{15}$ (d) None of these

Solution: (b) Total ways of arrangements $= \frac{8!}{2!.4!}$. $\bullet w \bullet x \bullet y \bullet z \bullet$

Now 'S' can have places at dot's and in places of w, x, y, z we have to put 2A's, one I and one N.

Therefore, favourable ways $= {}^5C_4\left(\frac{4!}{2!}\right)$. Hence, required probability $= \frac{5.4!2!4!}{2!8!} = \frac{1}{14}$.

1.6 Odds In favour and Odds against an Event

As a result of an experiment if "a" of the outcomes are favourable to an event E and "b" of the outcomes are against it, then we say that odds are a to b in favour of E or odds are b to a against E.

Thus odds in favour of an event $E = \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}} = \frac{a}{b} = \frac{a/(a+b)}{b/(a+b)} = \frac{P(E)}{P(\overline{E})}$.

Similarly, odds against an event $E = \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}} = \frac{b}{a} = \frac{P(\overline{E})}{P(E)}$.

Important Tips

☞ *If odds in favour of an event are a : b, then the probability of the occurrence of that event is* $\frac{a}{a+b}$ *and the probability of non-occurrence of that event is* $\frac{b}{a+b}$.

☞ *If odds against an event are a : b, then the probability of the occurrence of that event is* $\frac{b}{a+b}$ *and the probability of non-occurrence of that event is* $\frac{a}{a+b}$.

Example: 11 Two dice are tossed together. The odds in favour of the sum of the numbers on them as 2 are **[Rajasthan PET 1**

(a) 1 : 36 (b) 1 : 35 (c) 35 : 1 (d) None of these

Solution: (b) If two dice are tossed, total number of events = 6 × 6 = 36.

Favourable event is (1, 1). Number of favourable events = 1

$\therefore$ odds in favour $=\frac{1}{36-1}=\frac{1}{35}$.

Example: 12 A party of 23 persons take their seats at a round table. The odds against two persons sitting together are **[Rajasthan PET 1999]**

(a) 10 : 1 (b) 1 : 11 (c) 9 : 10 (d) None of these

Solution: (a) $P=\frac{(21)!\,2!}{(22)!}=\frac{1}{11}=\frac{1}{1+10}$. $\therefore$ odd against = 10 : 1.

1.7 Addition Theorems on Probability

Notations : (i) $P(A+B)$ or $P(A\cup B)$ = Probability of happening of A or B

= Probability of happening of the events A or B or both

= Probability of occurrence of at least one event A or B

(ii) $P(AB)$ or $P(A\cap B)$ = Probability of happening of events A and B together.

(1) **When events are not mutually exclusive :** If A and B are two events which are not mutually exclusive, then $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ or $P(A+B)=P(A)+P(B)-P(AB)$.

For any three events A, B, C

$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

or $P(A+B+C)=P(A)+P(B)+P(C)-P(AB)-P(BC)-P(CA)+P(ABC)$.

(2) **When events are mutually exclusive :** If A and B are mutually exclusive events, then

$n(A\cap B)=0 \Rightarrow P(A\cap B)=0$

$\therefore$ $P(A\cup B)=P(A)+P(B)$.

For any three events A, B, C which are mutually exclusive,

$P(A\cap B)=P(B\cap C)=P(C\cap A)=P(A\cap B\cap C)=0$ $\therefore$ $P(A\cup B\cup C)=P(A)+P(B)+P(C)$.

The probability of happening of any one of several mutually exclusive events is equal to the sum of their probabilities, *i.e.* if $A_1, A_2 .....A_n$ are mutually exclusive events, then

$P(A_1+A_2+...+A_n)=P(A_1)+P(A_2)+.....+P(A_n)$ *i.e.* $P(\sum A_i)=\sum P(A_i)$.

(3) **When events are independent :** If A and B are independent events, then $P(A\cap B)=P(A).P(B)$

$\therefore$ $P(A\cup B)=P(A)+P(B)-P(A).P(B)$.

(4) **Some other theorems**

(i) Let A and B be two events associated with a random experiment, then

(a) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$ (b) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

If $B \subset A$, then

(a) $P(A \cap \overline{B}) = P(A) - P(B)$ (b) $P(B) \le P(A)$

Similarly if $A \subset B$, then

(a) $(\overline{A} \cap B) = P(B) - P(A)$ (b) $P(A) \le P(B)$.

Note : ❑ *Probability of occurrence of neither A nor B is* $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$.

(ii) **Generalization of the addition theorem :** If $A_1, A_2, \ldots, A_n$ are n events associated with a random experiment, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{\substack{i,j=1 \\ i \neq j}}^{n} P(A_i \cap A_j) + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} P(A_i \cap A_j \cap A_k) + \ldots + (-1)^{n-1} P(A_1 \cap A_2 \cap \ldots \cap A_n).$$

If all the events $A_i (i = 1, 2 \ldots, n)$ are mutually exclusive, then $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$

i.e. $P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$.

(iii) **Booley's inequality :** If $A_1, A_2, \ldots A_n$ are n events associated with a random experiment, then

(a) $P\left(\bigcap_{i=1}^{n} A_i\right) \ge \sum_{i=1}^{n} P(A_i) - (n-1)$ (b) $P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$

These results can be easily established by using the Principle of Mathematical Induction.

Important Tips

Let *A, B,* and *C* are three arbitrary events. Then

Verbal description of event	***Equivalent Set Theoretic Notation***
(i) Only A occurs	*(i)* $A \cap \overline{B} \cap \overline{C}$
(ii) Both A and B, but not C occur	*(ii)* $A \cap B \cap \overline{C}$
(iii) All the three events occur	*(iii)* $A \cap B \cap C$
(iv) At least one occurs	*(iv)* $A \cup B \cup C$
(v) At least two occur	*(v)* $(A \cap B) \cup (B \cap C) \cup (A \cap C)$
(vi) One and no more occurs	*(vi)* $(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C)$
(vii) Exactly two of A, B and C occur	*(vii)* $(A \cap B \cap \overline{C}) \cup (\overline{A} \cap B \cap C) \cup (A \cap \overline{B} \cap C)$
(viii) None occurs	*(viii)* $\overline{A} \cap \overline{B} \cap \overline{C} = \overline{A \cup B \cup C}$
(ix) Not more than two occur	*(ix)* $(A \cap B) \cup (B \cap C) \cup (A \cap C) - (A \cap B \cap C)$
(x) Exactly one of A and B occurs	*(x)* $(A \cap \overline{B}) \cup (\overline{A} \cap B)$

Example: 13 A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, what is the probability that it is rusted or is a nail **[MP PET 1992, 2000]**

(a) 3/16 (b) 5/16 (c) 11/16 (d) 14/16

Solution: (c) Let *A* be the event that the item chosen is rusted and *B* be the event that the item chosen is a nail.

$\therefore$ $P(A) = \frac{8}{16}, P(B) = \frac{6}{16}$ and $P(A \cap B) = 3/16$

Required probability $= P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{8}{16}+\frac{6}{16}-\frac{3}{16}=\frac{11}{16}$.

Example: 14 The probability that a man will be alive in 20 years is $\frac{3}{5}$ and the probability that his wife will be alive in 20 years is $\frac{2}{3}$. Then the probability that at least one will be alive in 20 years is **[Bihar CEE 1994]**

(a) $\frac{13}{15}$ (b) $\frac{7}{15}$ (c) $\frac{4}{15}$ (d) None of these

Solution: (a) Let A be the event that the husband will be alive 20 years. B be the event that the wife will be alive 20 years. Clearly A and B are independent events. $\therefore$ $P(A\cap B)=P(A)P(B)$.

Given $P(A)=\frac{3}{5}, P(B)=\frac{2}{3}$.

The probability that at least one of them will be alive 20 years is

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=P(A)+P(B)-P(A).P(B)=\frac{3}{5}+\frac{2}{3}-\frac{3}{5}\cdot\frac{2}{3}=\frac{9+10-6}{15}=\frac{13}{15}$.

Example: 15 Let A and B be two events such that $P(A)=0.3$ and $P(A\cup B)=0.8$. If A and B are independent events, then $P(B)=$

[IIT 1990; UPSEAT 2001, 02]

(a) $\frac{5}{6}$ (b) $\frac{5}{7}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

Solution: (b) Here $P(A\cup B)=0.8$, $P(A)=0.3$ and A and B are independent events.

Let $P(B)=x$. $\therefore$ $P(A\cup B)=P(A)+P(B)-P(A\cap B) \Rightarrow P(A\cup B)=P(A)+P(B)-P(A).P(B)$

$\Rightarrow 0.8=0.3+x-0.3x \Rightarrow x=\frac{5}{7}$.

Example: 16 A card is chosen randomly from a pack of playing cards. The probability that it is a black king or queen of heart or jack is

[Rajasthan PET 1998]

(a) 1/52 (b) 6/52 (c) 7/52 (d) None of these

Solution: (c) Let A, B, C are the events of choosing a black king, a queen of heart and a jack respectively.

$\therefore$ $P(A)=\frac{2}{52}, P(B)=\frac{1}{52}, P(C)=\frac{4}{52}$

These are mutually exclusive events, $\therefore$ $P(A\cup B\cup C)=\frac{2}{52}+\frac{1}{52}+\frac{4}{52}=\frac{7}{52}$.

Example: 17 If A and B are events such that $P(A\cup B)=3/4, P(A\cap B)=1/4, P(\overline{A})=2/3$, then $P(\overline{A}\cap B)$ is **[AIEEE 2002]**

(a) 5/12 (b) 3/8 (c) 5/8 (d) 1/4

Solution: (a) $P(A\cup B)=\frac{3}{4}, P(A\cap B)=\frac{1}{4}$, $P(\overline{A})=\frac{2}{3} \Rightarrow P(A)=\frac{1}{3}$.

$\therefore$ $P(A\cap B)=P(A)+P(B)-P(A\cup B) \Rightarrow \frac{1}{4}=\frac{1}{3}+P(B)-\frac{3}{4} \Rightarrow P(B)=\frac{2}{3}$.

$P(\overline{A}\cap B)=P(B)-P(A\cap B)=\frac{2}{3}-\frac{1}{4}=\frac{8-3}{12}=\frac{5}{12}$.

Example: 18 The probability that A speaks truth is $\frac{4}{5}$, while this probability for B is $\frac{3}{4}$. The probability that they contradict each other when asked to speak on a fact is **[AIEEE 2004]**

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$ (c) $\frac{7}{20}$ (d) $\frac{3}{20}$

Solution: (c) Let E be the event that B speaks truth and F be the event that A speaks truth.

Now $P(E)=\frac{75}{100}=\frac{3}{4}$ and $P(F)=\frac{80}{100}=\frac{4}{5}$.

$\therefore$ P (A and B contradict each other)

= P [(B tells truth and A tells lie) or (B tells lie and A tells truth)]

$=P[(E\cap \overline{F})\cup(\overline{E}\cap F)]=P(E).P(\overline{F})+P(\overline{E}).P(F)=\frac{3}{4}\times\frac{1}{5}+\frac{1}{4}\times\frac{4}{5}=\frac{7}{20}$.

Example: 19 A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and $\frac{1}{2}$ respectively. If the probability that the student is successful is $\frac{1}{2}$, then **[IIT 1986]**

(a) $p=1, q=0$ (b) $p=\frac{2}{3}, q=\frac{1}{2}$

(c) There are infinitely many values of p and q (d) All of the above

Solution: (c) Let A, B and C be the events that the student is successful in test I, II and III respectively, then P (the student is successful)

$=P[(A\cap B\cap C')\cup(A\cap B'\cap C)\cup(A\cap B\cap C)]=P(A\cap B\cap C')+P(A\cap B'\cap C)+P(A\cap B\cap C)$

$=P(A).P(B).P(C')+P(A).P(B').P(C)+P(A).P(B).P(C)$ [$\because$ A, B, C are independent]

$=pq\left(1-\frac{1}{2}\right)+p(1-q)\left(\frac{1}{2}\right)+pq\left(\frac{1}{2}\right)=\frac{1}{2}p(1+q) \Rightarrow \frac{1}{2}=\frac{1}{2}p(1+q) \Rightarrow p(1+q)=1$.

This equation has infinitely many values of p and q.

Example: 20 A man and his wife appear for an interview for two posts. The probability of the husband's selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability that only one of them will be selected.**[AISSE 1**

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) None of these

Solution: (b) The probability of husband is not selected $=1-\frac{1}{7}=\frac{6}{7}$; The probability that wife is not selected $=1-\frac{1}{5}=\frac{4}{5}$

The probability that only husband is selected $=\frac{1}{7}\times\frac{4}{5}=\frac{4}{35}$; The probability that only wife is selected $=\frac{1}{5}\times\frac{6}{7}=\frac{6}{35}$

Hence, required probability $=\frac{6}{35}+\frac{4}{35}=\frac{10}{35}=\frac{2}{7}$.

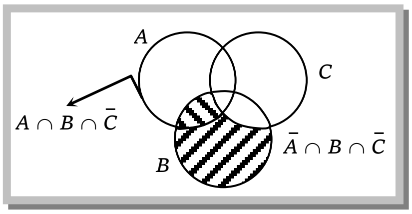
Example: 21 If $P(B)=\frac{3}{4}, P(A\cap B\cap \overline{C})=\frac{1}{3}$ and $P(\overline{A}\cap B\cap \overline{C})=\frac{1}{3}$, then $P(B\cap C)$ is **[IIT Screening 2003]**

(a) 1/12 (b) 1/6 (c) 1/15 (d) 1/9

Solution: (a) From Venn diagram, we can see that

$P(B\cap C)=P(B)-P(A\cap B\cap \overline{C})-P(\overline{A}\cap B\cap \overline{C})$

$=\frac{3}{4}-\frac{1}{3}-\frac{1}{3}=\frac{1}{12}$.



Example: 22 A purse contains 4 copper coins and 3 silver coins, the second purse contains 6 copper coins and 2 silver coins. If a coin is drawn out of any purse, then the probability that it is a copper coin is**[Ranchi BIT 199**

(a) 4/7 (b) 3/4 (c) 37/56 (d) None of these

Solution: (c) Required probability $=\frac{1}{2}\cdot\frac{4}{7}+\frac{1}{2}\cdot\frac{6}{8}=\frac{37}{56}$.

Example: 23 The probability of happening an event *A* is 0.5 and that of *B* is 0.3. If *A* and *B* are mutually exclusive events, then the probability of happening neither *A* nor *B* is **[IIT 1980; DCE 2000]**

(a) 0.6 (b) 0.2 (c) 0.21 (d) None of these

Solution: (b) $P(\bar{A}\cap\bar{B})=P(\overline{A\cup B})=1-P(A\cup B)$

Since *A* and *B* are mutually exclusive, so $P(A\cup B)=P(A)+P(B)$

Hence, required probability $=1-(0.5+0.3)=0.2$.

1.8 Conditional Probability

Let *A* and *B* be two events associated with a random experiment. Then, the probability of occurrence of *A* under the condition that *B* has already occurred and $P(B)\neq 0$, is called the conditional probability and it is denoted by $P(A/B)$.

Thus, $P(A/B)$ = Probability of occurrence of *A*, given that *B* has already happened.

$$=\frac{P(A\cap B)}{P(B)}=\frac{n(A\cap B)}{n(B)}.$$

Similarly, $P(B/A)$ = Probability of occurrence of *B*, given that *A* has already happened.

$$=\frac{P(A\cap B)}{P(A)}=\frac{n(A\cap B)}{n(A)}.$$

Note : ❑ Sometimes, $P(A/B)$ is also used to denote the probability of occurrence of *A* when *B* occurs. Similarly, $P(B/A)$ is used to denote the probability of occurrence of *B* when *A* occurs.

(1) **Multiplication theorems on probability**

(i) If *A* and *B* are two events associated with a random experiment, then $P(A\cap B)=P(A).P(B/A)$, if $P(A)\neq 0$ or $P(A\cap B)=P(B).P(A/B)$, if $P(B)\neq 0$.

(ii) **Extension of multiplication theorem :** If $A_1,A_2,....,A_n$ are *n* events related to a random experiment, then $P(A_1\cap A_2\cap A_3\cap....\cap A_n)=P(A_1)P(A_2/A_1)P(A_3/A_1\cap A_2)....P(A_n/A_1\cap A_2\cap...\cap A_{n-1})$,

where $P(A_i/A_1\cap A_2\cap...\cap A_{i-1})$ represents the conditional probability of the event A_i, given that the events $A_1,A_2,.....,A_{i-1}$ have already happened.

(iii) **Multiplication theorems for independent events :** If *A* and *B* are independent events associated with a random experiment, then $P(A\cap B)=P(A).P(B)$ *i.e.,* the probability of simultaneous occurrence of two independent events is equal to the product of their probabilities.

By multiplication theorem, we have $P(A\cap B)=P(A).P(B/A)$.

Since *A* and *B* are independent events, therefore $P(B/A)=P(B)$. Hence, $P(A\cap B)=P(A).P(B)$.

(iv) **Extension of multiplication theorem for independent events :** If $A_1,A_2,....,A_n$ are independent events associated with a random experiment, then $P(A_1\cap A_2\cap A_3\cap...\cap A_n)=P(A_1)P(A_2)...P(A_n)$.

By multiplication theorem, we have

$P(A_1 \cap A_2 \cap A_3 \cap ... \cap A_n) = P(A_1)P(A_2 / A_1)P(A_3 / A_1 \cap A_2)...P(A_n / A_1 \cap A_2 \cap ... \cap A_{n-1})$

Since $A_1, A_2, ..., A_{n-1}, A_n$ are independent events, therefore

$P(A_2 / A_1) = P(A_2), P(A_3 / A_1 \cap A_2) = P(A_3), ..., P(A_n / A_1 \cap A_2 \cap ... \cap A_{n-1}) = P(A_n)$

Hence, $P(A_1 \cap A_2 \cap ... \cap A_n) = P(A_1)P(A_2)....P(A_n)$.

(2) **Probability of at least one of the *n* independent events :** If $p_1, p_2, p_3, ..., p_n$ be the probabilities of happening of *n* independent events $A_1, A_2, A_3, ..., A_n$ respectively, then

(i) Probability of happening none of them

$= P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 ...... \cap \overline{A}_n) = P(\overline{A}_1).P(\overline{A}_2).P(\overline{A}_3).....P(\overline{A}_n) = (1 - p_1)(1 - p_2)(1 - p_3)....(1 - p_n)$.

(ii) Probability of happening at least one of them

$= P(A_1 \cup A_2 \cup A_3 .... \cup A_n) = 1 - P(\overline{A}_1)P(\overline{A}_2)P(\overline{A}_3)....P(\overline{A}_n) = 1 - (1 - p_1)(1 - p_2)(1 - p_3)...(1 - p_n)$.

(iii) Probability of happening of first event and not happening of the remaining

$= P(A_1)P(\overline{A}_2)P(\overline{A}_3).....P(\overline{A}_n) = p_1(1 - p_2)(1 - p_3).......(1 - p_n)$

Example: 24 If $4P(A) = 6, P(B) = 10, P(A \cap B) = 1$, then $P\left(\frac{B}{A}\right) =$ **[MP PET 2003]**

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{7}{10}$ (d) $\frac{19}{60}$

Solution: (a) $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{(1/10)}{(1/4)} = \frac{2}{5}$.

Example: 25 A coin is tossed three times in succession. If E is the event that there are at least two heads and F is the event in which first throw is a head, then $P\left(\frac{E}{F}\right) =$ **[MP PET 1996]**

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Solution: (a) $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$n(E) = 4, n(F) = 4$ and $n(E \cap F) = 3$

$\therefore P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{4/8} = \frac{3}{4}$.

Example: 26 Two cards are drawn one by one from a pack of cards. The probability of getting first card an ace and second an honour card is (before drawing second card first card is not placed again in the pack)**[UPSEAQT 19**

(a) 1/26 (b) 5/52 (c) 5/221 (d) 4/13

Solution: (c) $P(E_1) = \frac{4}{52} = \frac{1}{13}, P\left(\frac{E_2}{E_1}\right) = \frac{15}{51} = \frac{5}{17}$

$P(E_1 \cap E_2) = P(E_1).P\left(\frac{E_2}{E_1}\right) = \frac{1}{13}.\frac{5}{17} = \frac{5}{221}$.

Example: 27 If A and B are two events such that $P(A) \neq 0$ and $P(B) \neq 1$, then $P\left(\frac{\overline{A}}{\overline{B}}\right) =$

[IIT 1982; RPET 1995, 2000; DCE 2000; UPSEAT 2001]

(a) $1 - P\left(\frac{A}{B}\right)$ (b) $1 - P\left(\frac{\overline{A}}{B}\right)$ (c) $\frac{1 - P(A \cup B)}{P(\overline{B})}$ (d) $\frac{P(\overline{A})}{P(\overline{B})}$

Solution: (c) $P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})}$.

Example: 28 If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the true relation is **[IIT 1985]**

(a) $P(A) + P(B) = 0$ (b) $P(A) + P(B) = P(A)P\left(\frac{B}{A}\right)$

(c) $P(A) + P(B) = 2P(A)P\left(\frac{B}{A}\right)$ (d) None of these

Solution: (c) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cap B)$ $\{\because P(A \cap B) = P(A \cup B)\}$

$\Rightarrow 2P(A \cap B) = P(A) + P(B) \Rightarrow 2P(A).\frac{P(A \cap B)}{P(A)} = P(A) + P(B) \Rightarrow 2P(A)P\left(\frac{B}{A}\right) = P(A) + P(B).$

Example: 29 Let E and F be two independent events. The probability that both E and F happens is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$, then **[IIT 1993]**

(a) $P(E) = \frac{1}{3}, P(F) = \frac{1}{4}$ (b) $P(E) = \frac{1}{2}, P(F) = \frac{1}{6}$ (c) $P(E) = \frac{1}{6}, P(F) = \frac{1}{2}$ (d) None of these

Solution: (a) We are given $P(E \cap F) = \frac{1}{12}$ and $P(\overline{E} \cap \overline{F}) = \frac{1}{2}$

$\Rightarrow P(E).P(F) = \frac{1}{12}$ .....(i) and $P(\overline{E}).P(\overline{F}) = \frac{1}{2}$ .....(ii)

$\Rightarrow \{1 - P(E)\}\{(1 - P(F)\} = \frac{1}{2} \Rightarrow 1 + P(E)P(F) - P(E) - P(F) = \frac{1}{2} \Rightarrow 1 + \frac{1}{12} - [P(E) + P(F)] = \frac{1}{2}$

$\Rightarrow P(E) + P(F) = \frac{7}{12}$ .....(iii)

On solving (i) and (iii), we get $P(E) = \frac{1}{3}, \frac{1}{4}$ and $P(F) = \frac{1}{4}, \frac{1}{3}$.

Example: 30 Let p denotes the probability that a man aged x years will die in a year. The probability that out of n men $A_1, A_2, A_3, ... A_n$ each aged x, A_1 will die in a year and will be the first to die, is **[MNR 1987; UPSEAT 2000]**

(a) $\frac{1}{n}[1 - (1-p)^n]$ (b) $[1 - (1-p)^n]$ (c) $\frac{1}{n-1}[1 - (1-p)^n]$ (d) None of these

Solution: (a) Let E_i denotes the event that A_i dies in a year.

Then $P(E_i) = p$ and $P(E_i') = 1 - p$ for i = 1, 2, ...n

P (none of $A_1, A_2, ... A_3$ dies in a year) $= P(E_1' \cap E_2' \cap ..... E_n') = P(E_1')P(E_2')....P(E_n') = (1-p)^n$,

because $E_1, E_2, ... E_n$ are independent.

Let E denote the event that at least one of $A_1, A_2, ... A_n$ dies in a year.

Then $P(E) = 1 - P(E_1' \cap E_2' \cap ..... \cap E_n') = 1 - (1-p)^n$

Let F denote the event that A_1 is the first to die.

Then $P(F/E) = \frac{1}{n}$. Also, $P(F) = P(E).P(F/E) = \frac{1}{n}[1 - (1-p)^n]$.

Example: 31 A problem of mathematics is given to three students whose chances of solving the problem are 1/3, 1/4 and 1/5 respectively. The probability that the question will be solved is **[BIT Ranchi 1991; MP PET 1990]**

(a) 2/3 (b) 3/4 (c) 4/5 (d) 3/5

Solution: (d) The probabilities of students not solving the problem are $1 - \frac{1}{3} = \frac{2}{3}, 1 - \frac{1}{4} = \frac{3}{4}$ and $1 - \frac{1}{5} = \frac{4}{5}$.

Therefore the probability that the problem is not solved by any one of them $= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} = \frac{2}{5}$.

Hence, the probability that problem is solved $= 1 - \frac{2}{5} = \frac{3}{5}$.

Example: 32 The probability of happening an event *A* in one trial is 0.4. The probability that the event *A* happens at least once in three independent trials is **[IIT 1980; Kurukshetra CEE 1998; DCE 2001]**

(a) 0.936 (b) 0.784 (c) 0.904 (d) 0.216

Solution: (b) Here $P(A) = 0.4$ and $P(\bar{A}) = 0.6$

Probability that *A* does not happen at all $= (0.6)^3$. Thus required probability $= 1 - (0.6)^3 = 0.784$.

1.9 Total Probability and Baye's rule

(1) **The law of total probability :** Let *S* be the sample space and let $E_1, E_2, ...E_n$ be *n* mutually exclusive and exhaustive events associated with a random experiment. If *A* is any event which occurs with E_1 or E_2 or .... or E_n, then $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + ... + P(E_n)P(A/E_n)$.

(2) **Baye's rule :** Let *S* be a sample space and $E_1, E_2, ...E_n$ be *n* mutually exclusive events such that $\bigcup_{i=1}^{n} E_i = S$ and $P(E_i) > 0$ for *i* = 1, 2, ..., *n*. We can think of (E_i's as the causes that lead to the outcome of an experiment. The probabilities $P(E_i)$, *i* = 1, 2, ..., *n* are called prior probabilities. Suppose the experiment results in an outcome of event *A*, where *P*(*A*) > 0. We have to find the probability that the observed event *A* was due to cause E_i, that is, we seek the conditional probability $P(E_i / A)$. These probabilities are called posterior probabilities, given by Baye's rule as $P(E_i / A) = \frac{P(E_i).P(A/E_i)}{\sum_{k=1}^{n} P(E_k)P(A/E_k)}$.

Example: 33 In a bolt factory, machines *A*, *B* and *C* manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. Then the probability that the bolt drawn is defective is

(a) 0.0345 (b) 0.345 (c) 3.45 (d) 0.0034

Solution: (a) Let E_1, E_2, E_3 and *A* be the events defined as follows:

E_1 = the bolts is manufactured by machine *A*; E_2 = the bolts is manufactured by machine *B*; E_3 = the bolts is manufactured by machine *C*, and *A* = the bolt is defective.

Then $P(E_1) = \frac{25}{100} = \frac{1}{4}, P(E_2) = \frac{35}{100}, P(E_3) = \frac{40}{100}$.

$P(A/E_1)$ = Probability that the bolt drawn is defective given the condition that it is manufactured by machine *A* = 5/100.

Similarly $P(A/E_2) = \frac{4}{100}$ and $P(A/E_3) = \frac{2}{100}$.

Using the law of total probability, we have $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$

$$= \frac{25}{100} \times \frac{5}{100} + \frac{35}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100} = 0.0345.$$

Example: 34 A lot contains 20 articles. The probability that the lot contains 2 defective articles is 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn at random one by one without replacement and tested till all the defective articles are found. The probability that the testing procedure ends at the twelfth testing is

(a) $\frac{9}{1900}$ (b) $\frac{19}{1000}$ (c) $\frac{99}{1900}$ (d) $\frac{19}{900}$

Solution: (c) The testing procedure may terminate at the twelfth testing in two mutually exclusive ways.

(I) When lot contains 2 defective articles, (II) When lot contains 3 defective articles.

Consider the following events.

A = Testing procedure ends at the twelfth testing.

A_1 = Lot contains 2 defective articles.

A_2 = Lot contains 3 defective articles.

Required probability

$= P(A) = P(A\cap A_1)\cup(A\cap A_2) = P(A\cap A_1) + P(A\cap A_2) = P(A_1)P(A/A_1) + P(A_2)P(A/A_2)$

Now, $P(A/A_1) =$ Probability that first 11 draws contain 10 non-defective and one defective and 12th draw contains a defective article.

$= \frac{^{18}C_{10}\times {}^2C_1}{^{20}C_{11}}\times\frac{1}{9}$.

And $P(A/A_2)$ = Probability that first 11 draws contain 9 non defective and 2 defective articles and 12th draw contains a defective article = $\frac{^{17}C_9\times {}^3C_2}{^{20}C_{11}}\times\frac{1}{9}$

Hence, required probability $= 0.4\times\frac{^{18}C_{10}\times {}^2C_1}{^{20}C_{11}}\times\frac{1}{9} + 0.6\times\frac{^{17}C_9\times {}^3C_2}{^{20}C_{11}}\times\frac{1}{9} = \frac{99}{1900}$.

Example: 35 A bag A contains 2 white and 3 red balls and bag B contains 4 white and 5 red balls. One ball is drawn at random from a randomly chosen bag and is found to be red. The probability that it was drawn from B is **[BIT Ranchi 1988; IIT 1976]**

(a) $\frac{5}{14}$ (b) $\frac{5}{16}$ (c) $\frac{5}{18}$ (d) $\frac{25}{52}$

Solution: (d) Let E_1 be the event that the ball is drawn from bag A, E_2 the event that it is drawn from bag B and E that the ball is red.

We have to find $P(E_2/E)$.

Since both the bags are equally likely to be selected,

we have $P(E_1) = P(E_2) = \frac{1}{2}$. Also $P(E/E_1) = 3/5$ and $P(E/E_2) = 5/9$.

Hence by Baye's theorem, we have $P(E_2/E) = \frac{P(E_2)P(E/E_2)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)} = \frac{\frac{1}{2}\cdot\frac{5}{9}}{\frac{1}{2}\cdot\frac{3}{5}+\frac{1}{2}\cdot\frac{5}{9}} = \frac{25}{52}$.

Example: 36 A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is

(a) $\frac{3}{8}$ (b) $\frac{1}{5}$ (c) $\frac{3}{4}$ (d) None of these

Solution: (a) Let E denote the event that a six occurs and A the event that the man reports that it is a '6', we have

$P(E) = \frac{1}{6}, P(E') = \frac{5}{6}, P(A/E) = \frac{3}{4}$ and $P(A/E') = \frac{1}{4}$.

By Baye's theorem, $P(E/A)=\dfrac{P(E).P(A/E)}{P(E).P(A/E)+P(E')P(A/E')}=\dfrac{\frac{1}{6}\times\frac{3}{4}}{\frac{1}{6}\times\frac{3}{4}+\frac{5}{6}\times\frac{1}{4}}=\dfrac{3}{8}$.

Example: 37 A pack of playing cards was found to contain only 51 cards. If the first 13 cards which are examined are all red, then the probability that the missing cards is black, is

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{{}^{25}C_{13}}{{}^{51}C_{13}}$

Solution: (b) Let A_1 be the event that the black card is lost, A_2 be the event that the red card is lost and let E be the event that first 13 cards examined are red.

Then the required probability $=P\left(\frac{A_1}{E}\right)$. We have $P(A_1)=P(A_2)=\frac{1}{2}$; as black and red cards were initially equal in number.

Also $P\left(\frac{E}{A_1}\right)=\frac{{}^{26}C_{13}}{{}^{51}C_{13}}$ and $P\left(\frac{E}{A_2}\right)=\frac{{}^{25}C_{13}}{{}^{51}C_{13}}$.

The required probability $=P\left(\frac{A_1}{E}\right)=\dfrac{P(E/A_1)P(A_1)}{P(E/A_1)P(A_1)+P(E/A_2)P(A_2)}=\dfrac{\frac{1}{2}\cdot\frac{{}^{26}C_{13}}{{}^{51}C_{13}}}{\frac{1}{2}\cdot\frac{{}^{26}C_{13}}{{}^{51}C_{13}}+\frac{1}{2}\cdot\frac{{}^{25}C_{13}}{{}^{51}C_{13}}}=\dfrac{2}{3}$.

1.10 Binomial Distribution

(1) **Geometrical method for probability :** When the number of points in the sample space is infinite, it becomes difficult to apply classical definition of probability. For instance, if we are interested to find the probability that a point selected at random from the interval [1, 6] lies either in the interval [1, 2] or [5, 6], we cannot apply the classical definition of probability. In this case we define the probability as follows:

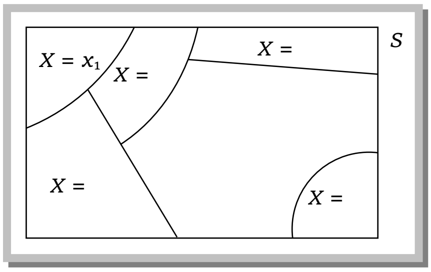
$$P\{x \in A\}=\frac{\text{Measure of region } A}{\text{Measure of the sample space } S},$$

where measure stands for length, area or volume depending upon whether S is a one-dimensional, two-dimensional or three-dimensional region.

(2) **Probability distribution :** Let S be a sample space. A random variable X is a function from the set S to R, the set of real numbers.

For example, the sample space for a throw of a pair of dice is $S=\begin{matrix}\{11, & 12, & \cdots, & 16 \\ 21, & 22, & \cdots, & 26 \\ \vdots & \vdots & \ddots & \vdots \\ 61, & 62, & \cdots, & 66\}\end{matrix}$

Let X be the sum of numbers on the dice. Then $X(12)=3, X(43)=7$, etc. Also, $\{X = 7\}$ is the event {61, 52, 43, 34, 25, 16}. In general, if X is a random variable defined on the sample space S and r is a real number, then $\{X = r\}$ is an event. If the random variable X takes n distinct values $x_1, x_2, ..., x_n$, then $\{X=x_1\}$, $\{X=x_2\}$,....,$\{X=x_n\}$ are mutually exclusive and exhaustive events.



Now, since $(X = x_i)$ is an event, we can talk of $P(X = x_i)$. If $P(X = x_i) = P_i (1 \le i \le n)$, then the system of numbers.

$$\begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ p_1 & p_2 & \cdots & p_n \end{pmatrix}$$

is said to be the probability distribution of the random variable *X*. The expectation (mean) of the random variable *X* is defined as $E(X) = \sum_{i=1}^{n} p_i x_i$

and the variance of *X* is defined as $\text{var}(X) = \sum_{i=1}^{n} p_i (x_i - E(X))^2 = \sum_{i=1}^{n} p_i x_i^2 - (E(X))^2$.

(3) **Binomial probability distribution :** A random variable *X* which takes values 0, 1, 2, ..., *n* is said to follow binomial distribution if its probability distribution function is given by $P(X = r) = {}^nC_r p^r q^{n-r}, r = 0, 1, 2, ..., n$

where *p*, *q* > 0 such that *p* + *q* = 1.

The notation *X* ~ *B*(*n*, *p*) is generally used to denote that the random variable *X* follows binomial distribution with parameters *n* and *p*.

We have $P(X = 0) + P(X = 1) + ... + P(X = n) = {}^nC_0 p^0 q^{n-0} + {}^nC_1 p^1 q^{n-1} + ... + {}^nC_n p^n q^{n-n} = (q + p)^n = 1^n = 1$.

Now probability of

(a) Occurrence of the event exactly *r* times

$P(X = r) = {}^nC_r q^{n-r} p^r$.

(b) Occurrence of the event at least *r* times

$$P(X \ge r) = {}^nC_r q^{n-r} p^r + ... + p^n = \sum_{X=r}^{n} {}^nC_X p^X q^{n-X} .$$

(c) Occurrence of the event at the most *r* times

$$P(0 \le X \le r) = q^n + {}^nC_1 q^{n-1} p + ... + {}^nC_r q^{n-r} p^r = \sum_{X=0}^{r} p^X q^{n-X} .$$

(iv) If the probability of happening of an event in one trial be *p*, then the probability of successive happening of that event in *r* trials is p^r.

Note : ❑ If n trials constitute an experiment and the experiment is repeated *N* times, then the frequencies of 0, 1, 2, ..., *n* successes are given by $N.P(X = 0), N.P(X = 1), N.P(X = 2), ..., N.P(X = n)$.

(i) **Mean and variance of the binomial distribution :** The binomial probability distribution is

X	0	1	2	n
$P(X)$	${}^nC_0 q^n p^0$	${}^nC_1 q^{n-1} p$	${}^nC_2 q^{n-2} p^2$.....	${}^nC_n q^0 p^n$

The mean of this distribution is $\sum_{i=1}^{n} X_i p_i = \sum_{X=1}^{n} X.\,{}^nC_X q^{n-X} p^X = np$,

the variance of the Binomial distribution is $\sigma^2 = npq$ and the standard deviation is $\sigma = \sqrt{(npq)}$.

(ii) **Use of multinomial expansion :** If a die has m faces marked with the numbers 1, 2, 3, ...m and if such n dice are thrown, then the probability that the sum of the numbers exhibited on the upper faces equal to p is given by the coefficient of x^p in the expansion of $\frac{(x + x^2 + x^3 + ... + x^m)^n}{m^n}$.

Example: 38 A random variable X has the probability distribution :

***X* :**	1	2	3	4	5	6	7	8
***P*(*X*) :**	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the events E = {X is a prime number} and F = {X < 4}, the probability $P(E \cup F)$ is **[AIEEE 2004]**

(a) 0.50 (b) 0.77 (c) 0.35 (d) 0.87

Solution: (b) E = {X is a prime number}

$P(E) = P(2) + P(3) + P(5) + P(7) = 0.62$, $F = \{x < 4\}$

$P(F) = P(1) + P(2) + P(3) = 0.50$ and $P(E \cap F) = P(2) + P(3) = 0.35$

$\therefore$ $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.62 + 0.50 - 0.35 = 0.77$.

Example: 39 8 coins are tossed simultaneously. The probability of getting at least 6 heads is**[AISSE 1985; MNR 1985; MP PE**

(a) $\frac{57}{64}$ (b) $\frac{229}{256}$ (c) $\frac{7}{64}$ (d) $\frac{37}{256}$

Solution: (d) The required probability $= {}^8C_6\left(\frac{1}{2}\right)^6.\left(\frac{1}{2}\right)^2 + {}^8C_7\left(\frac{1}{2}\right)^7.\left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8 = \frac{37}{256}$.

Example: 40 An unbiased die with faces marked 1, 2, 3, 4, 5 and 6 is rolled four times. Out of four face values obtained the probability that the minimum face value is not less than 2 and the maximum face value is not greater than 5, is

[IIT 1993; DCE 2000; Roorkee 2000]

(a) $\frac{16}{81}$ (b) $\frac{1}{81}$ (c) $\frac{80}{81}$ (d) $\frac{65}{81}$

Solution: (a) P(minimum face value is not less than 2 and maximum face value is not greater than 5)

= P(2 or 3 or 4 or 5) $= \frac{4}{6} = \frac{2}{3}$.

Hence required probability $= {}^4C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 = \frac{16}{81}$.

Example: 41 One hundred identical coins each with probability p of showing up heads are tossed once. If $0 < p < 1$ and the probability of heads showing on 50 coins is equal to that of heads showing on 51 coins, then the value of p is

[IIT 1988; CEE 1993; MP PET 2001]

(a) $\frac{1}{2}$ (b) $\frac{49}{101}$ (c) $\frac{50}{101}$ (d) $\frac{51}{101}$

Solution: (d) We have ${}^{100}C_{50}p^{50}(1-p)^{50} = {}^{100}C_{51}p^{51}(1-p)^{49}$ or $\frac{1-p}{p} = \frac{100\,!}{51\,!.\,49\,!} \times \frac{50\,!.\,50\,!}{100\,!} = \frac{50}{51}$ or $51 - 51p = 50p \Rightarrow p = \frac{51}{101}$.

Example: 42 The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

[AIEEE 2004]

(a) $\frac{28}{256}$ (b) $\frac{219}{256}$ (c) $\frac{128}{256}$ (d) $\frac{37}{256}$

Solution: (a) $\left.\begin{matrix} np = 4 \\ npq = 2 \end{matrix}\right\} \Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$p(X = 2) = {}^{8}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^6 = 28.\frac{1}{2^8} = \frac{28}{256}$.

Example: 43 A man takes a step forward with probability 0.4 and backward with probability 0.6. The probability that at the end of eleven steps he is one step away from the starting point is

(a) ${}^{11}C_6(0.24)^5$ (b) ${}^{11}C_6(0.4)^6(0.6)^5$ (c) ${}^{11}C_6(0.6)^6(0.4)^5$ (d) None of these

Solution: (a) The man will be one step away from the starting point if (i) either he is one step ahead or (ii) one step behind the starting point.

$\therefore$ The required probability = $P(i) + P(ii)$

The man will be one step ahead at the end of eleven steps if he moves six step forward and five steps backward.

The probability of this event is ${}^{11}C_6(0.4)^6(0.6)^5$.

The man will be one step behind at the end of eleven steps if he moves six steps backward and five steps forward.

The probability of this event is ${}^{11}C_6(0.6)^6(0.4)^5$.

Hence the required probability $= {}^{11}C_6(0.4)^6(0.6)^5 + {}^{11}C_6(0.6)^6(0.4)^5 = {}^{11}C_6(0.4)^5(0.6)^5(0.4 + 0.6) = {}^{11}C_6(0.24)^5$.

Example: 44 A person can kill a bird with probability 3/4. He tries 5 times. What is the probability that he may not kill the bird

[Rajasthan PET 1997]

(a) 243/1024 (b) 781/1024 (c) 1/1024 (d) 1023/1024

Solution: (c) Probability to kill a bird $p = \frac{3}{4}$, $p + q = 1$

$\Rightarrow q = 1 - p = 1 - \frac{3}{4} = \frac{1}{4}$ and $n = 5$.

Probability that he may not kill the bird,

$P(X = 0) = {}^{5}C_0\left(\frac{3}{4}\right)^0 \cdot \left(\frac{1}{4}\right)^{5-0} = \frac{1}{1024}$.

Example: 45 If X follows a binomial distribution with parameters n = 8 and $p = \frac{1}{2}$, then $P(|X - 4| \le 2)$ equals

(a) $\frac{118}{128}$ (b) $\frac{119}{128}$ (c) $\frac{117}{128}$ (d) None of these

Solution: (b) We have, $P(|X-4| \le 2) = P(-2 \le X - 4 \le 2) = P(2 \le X \le 6) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

$= {}^{8}C_2\left(\frac{1}{2}\right)^8 + {}^{8}C_3\left(\frac{1}{2}\right)^8 + {}^{8}C_4\left(\frac{1}{2}\right)^8 + {}^{8}C_5\left(\frac{1}{2}\right)^8 + {}^{8}C_6\left(\frac{1}{2}\right)^8 = \frac{1}{2^8}[28 + 56 + 70 + 56 + 28] = \frac{238}{2^8} = \frac{119}{128}$.

Example: 46 Three six faced fair dice are thrown together. The probability that the sum of the numbers appearing on the dice is $k(3 \le k \le 8)$, is

(a) $\frac{(k-1)(k-2)}{432}$ (b) $\frac{k(k-1)}{432}$ (c) $\frac{k^2}{432}$ (d) None of these

Solution: (a) The total number of cases $= 6 \times 6 \times 6 = 216$

The number of favourable ways

= Coefficient of x^k in $(x + x^2 + ... + x^6)^3$

= Coefficient of x^{k-3} in $(1-x^6)^3(1-x)^{-3}$

= Coefficient of x^{k-3} in $(1-x)^{-3}$ $\{0 \le k-3 \le 5\}$

= Coefficient of x^{k-3} in $(1 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + ...) = {}^{k-1}C_2 = \frac{(k-1)(k-2)}{2}$

Thus the probability of the required event is $\frac{(k-1)(k-2)}{432}$.

Example: 47 If three dice are thrown simultaneously, then the probability of getting a score of 7 is **[Kurukshetra CEE 1998]**

(a) 5/216 (b) 1/6 (c) 5/72 (d) None of these

Solution: (c) $n(S) = 6 \times 6 \times 6$

$n(E)$ = The number of solutions of $x + y + z = 7$,

where $1 \le x \le 5, 1 \le y \le 5, 1 \le z \le 5$

= Coefficient of x^7 in $(x + x^2 + ... + x^5)^3$

= Coefficient of x^4 in $(1 + x + ... + x^4)^3$ = Coefficient of x^4 in $\left(\frac{1-x^5}{1-x}\right)^3$

= Coefficient of x^4 in $(1 - 3x^5 + 3x^{10} - x^{15})(1-x)^{-3}$

= Coefficient of x^4 in $(1 - 3x^5 + 3x^{10} - x^{15})({}^2C_0 + {}^3C_1 x + {}^4C_2 x^2 + {}^5C_3 x^3 + {}^6C_4 x^4 + ...)$

$= {}^6C_4 = \frac{6!}{4!2!} = \frac{6 \times 5}{2} = 15$.

$\therefore\ p(E) = \frac{n(E)}{n(S)} = \frac{15}{6 \times 6 \times 6} = \frac{5}{72}$.

Assignment

Definition of various terms

Basic Level

1. Two coins are tossed. Let A be the event that the first coin shows head and B be the event that the second coin shows a tail. Two events A and B are

(a) Mutually exclusive (b) Dependent
(c) Independent and mutually exclusive (d) None of these

2. A card is drawn from a pack of 52 cards. If A = card is of diamond, B = card is an ace and $A \cap B$ = card is ace of diamond, then events A and B are

(a) Independent (b) Mutually exclusive (c) Dependent (d) Equally likely

3. The probabilities of three mutually exclusive events are $2/3$, 1/4 and 1/6. The statement is

(a) True (b) False (c) Could be either (d) Do not know

4. If $P(A_1 \cup A_2) = 1 - P(A_1^c) P(A_2^c)$, where c stands for complement, then the events A_1 and A_2 are

(a) Mutually exclusive (b) Independent (c) Equally likely (d) None of these

5. If $\frac{1-3p}{2}, \frac{1+4p}{3}$ and $\frac{1+p}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of p is

[MNR 1992; Rajasthan PET 2000; UPSEAT 2000]

(a) [0, 1] (b) $\left[-\frac{1}{4}, \frac{1}{3}\right]$ (c) $\left[0, \frac{1}{3}\right]$ (d) $(0, \infty)$

6. The event A is independent of itself if and only if $P(A) =$

(a) 0 (b) 1 (c) 0, 1 (d) None of these

7. If A and B are independent events and $P(C) = 0$, then

(a) A and C are independent (b) B and C are independent
(c) A, B and C are independent (d) All of these

Definition of Probability

Basic Level

8. The probability that an ordinary or a non-leap year has 53 Sundays, is

(a) 2/7 (b) 1/7 (c) 3/7 (d) None of these

9. Three letters are to be sent to different persons and addresses on the three envelopes are also written. Without looking at the addresses, the probability that the letters go into the right envelope is equal to

(a) 1/27 (b) 1/9 (c) 4/27 (d) 1/6

10. The probability of getting head and tail alternately in three throws of a coin (or a throw of three coins), is

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{3}{8}$

11. In a lottery there were 90 tickets numbered 1 to 90. Five tickets were drawn at random. The probability that two of the tickets drawn numbers 15 and 89 is

(a) 2/801 (b) 2/623 (c) 1/267 (d) 1/623

12. Two numbers are selected randomly from the set $S = \{1,2,3,4,5,6\}$ without replacement one by one. The probability that minimum of the two numbers is less than 4 is

(a) 1/15 (b) 14/15 (c) 1/5 (d) 4/5

13. Among 15 players, 8 are batsmen and 7 are bowlers. Find the probability that a team is chosen of 6 batsmen and 5 bowlers

[UPSEAT 2002]

(a) $\frac{{}^8C_6 \times {}^7C_5}{{}^{15}C_{11}}$ (b) $\frac{{}^8C_6 + {}^7C_5}{{}^{15}C_{11}}$ (c) $\frac{15}{28}$ (d) None of these

14. The probability of obtaining sum '8' in a single throw of two dice

(a) $\frac{1}{36}$ (b) $\frac{5}{36}$ (c) $\frac{4}{36}$ (d) $\frac{6}{36}$

15. Three mangoes and three apples are in a box. If two fruits are chosen at random, the probability that one is a mango and the other is an apple is

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{1}{3}$ (d) None of these

16. A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square is

[EAMCET 1989]

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{10}$ (d) None of these

17. A bag contains 5 white, 7 black and 4 red balls. Three balls are drawn from the bag at random. The probability that all the three balls are white, is

(a) $\frac{3}{16}$ (b) $\frac{3}{5}$ (c) $\frac{1}{60}$ (d) $\frac{1}{56}$

18. Two dice are thrown together. The probability that at least one will show its digit 6 is

(a) $\frac{11}{36}$ (b) $\frac{36}{11}$ (c) $\frac{5}{11}$ (d) $\frac{1}{6}$

19. The sum of two positive numbers is 100. The probability that their product is greater than 1000 is

(a) 7/9 (b) 7/10 (c) 2/5 (d) None of these

20. Two integers are chosen at random and multiplied. The probability that the product is an even integer is

(a) 1/2 (b) 2/3 (c) 3/4 (d) 4/5

21. A pair of a dice thrown, if 5 appears on at least one of the dice, then the probability that the sum is 10 or greater is **[MP PET 2001]**

(a) $\frac{11}{36}$ (b) $\frac{2}{9}$ (c) $\frac{3}{11}$ (d) $\frac{1}{12}$

22. Four boys and three girls stand in a queue for an interview, probability that they will in alternate position is**[UPSEAT 2**

(a) 1/34 (b) 1/35 (c) 1/17 (d) 1/68

23. If two dice are thrown simultaneously then probability that 1 comes on first dice is **[Rajasthan PET 2002]**

(a) 1/36 (b) 5/36 (c) 1/6 (d) None of these

24. Out of 30 consecutive numbers, 2 are chosen at random. The probability that their sum is odd, is

(a) $\frac{14}{29}$ (b) $\frac{16}{29}$ (c) $\frac{15}{29}$ (d) $\frac{10}{29}$

25. Three integers are chosen at random from the first 20 integers. The probability that their product is even, is**[Kurukshe**

(a) $\frac{2}{19}$ (b) $\frac{3}{29}$ (c) $\frac{17}{19}$ (d) $\frac{4}{19}$

26. Two dice are thrown. The probability that the sum of the points on two dice will be 7, is **[IIT 1974; MNR 1981, 91; Rajasthan PET 1995, 97, 2002; UPSEAT 2000]**

(a) $\frac{5}{36}$ (b) $\frac{6}{36}$ (c) $\frac{7}{36}$ (d) $\frac{8}{36}$

27. A bag contains tickets numbered from 1 to 20. Two tickets are drawn. The probability that both the numbers are prime, is **[AISSE 1981]**

(a) $\frac{14}{95}$ (b) $\frac{7}{95}$ (c) $\frac{1}{95}$ (d) None of these

28. In a single throw of two dice, the probability of getting more than 7 is

(a) $\frac{7}{36}$ (b) $\frac{7}{12}$ (c) $\frac{5}{12}$ (d) $\frac{5}{36}$

29. If two balanced dice are tossed once, the probability of the event that the sum of the integers coming on the upper sides of the two dice is 9 is

(a) 7/18 (b) 5/36 (c) 1/9 (d) 1/6

30. The probability of getting number 5 in throwing a die is **[MP PET 1988]**

(a) 1 (b) 1/3 (c) 1/6 (d) 5/6

31. The probability of getting a number greater than 2 in throwing a die is

(a) 1/3 (b) 2/3 (c) 1/2 (d) 1/6

32. The chance of throwing at least 9 in a single throw with two dice, is

(a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{7}{18}$ (d) $\frac{11}{18}$

33. The probability that the three cards drawn from a pack of 52 cards are all red is **[MP PET 1999]**

(a) $\frac{1}{17}$ (b) $\frac{3}{19}$ (c) $\frac{2}{19}$ (d) $\frac{2}{17}$

34. The probability of getting a total of 5 or 6 in a single throw of 2 dice is

(a) 1/2 (b) 1/4 (c) 1/3 (d) 1/6

35. If a committee of 3 is to be chosen from a group of 38 people of which you are a member. What is the probability that you will be on the committee

(a) $\binom{38}{3}$ (b) $\binom{37}{2}$ (c) $\binom{37}{2}\Big/\binom{38}{3}$ (d) 666/8436

36. The chance of getting a doublet with 2 dice is **[Kurukshetra CEE 2002]**

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{5}{6}$ (d) $\frac{5}{36}$

37. A bag contains 3 white and 5 black balls. If one ball is drawn, then the probability that it is black, is

(a) $\frac{3}{8}$ (b) $\frac{5}{8}$ (c) $\frac{6}{8}$ (d) $\frac{10}{20}$

38. Two dice are thrown together. The probability that sum of the two numbers will be a multiple of 4 is

(a) 1/9 (b) 1/3 (c) 1/4 (d) 5/9

39. The probability of happening of an impossible event *i.e.,* $P(\phi)$ is

(a) 1 (b) 0 (c) 2 (d) – 1

40. For any event A **[Rajasthan PET 1995]**

(a) $P(A)+P(\overline{A})=0$ (b) $P(A)+P(\overline{A})=1$ (c) $P(A)>1$ (d) $P(\overline{A})<1$

41. A bag contains 3 red, 4 white and 5 black balls. Three balls are drawn at random. The probability of being their different colours is

[Rajasthan PET 1999]

(a) 3/11 (b) 2/11 (c) 8/11 (d) None of these

42. Find the probability that the two digit number formed by digits 1, 2, 3, 4, 5 is divisible by 4 (while repetition of digit is allowed)

[UPSEAT 2002]

(a) $\frac{1}{30}$ (b) $\frac{1}{20}$ (c) $\frac{1}{40}$ (d) None of these

43. If $P(A)=0.65, P(B)=0.15,$ then $P(\bar{A})+P(\bar{B})=$ **[Pb. CET 1989; EAMCET 1988]**

(a) 1.5 (b) 1.2 (c) 0.8 (d) None of these

44. If four persons are chosen at random from a group of 3 men, 2 women and 4 children. Then the probability that exactly two of them are children, is
[Kurukshetra CEE 1996; DCE 1999]

(a) 10/21 (b) 8/63 (c) 5/21 (d) 9/21

45. A single letter is selected at random from the word "PROBABILITY". The probability that the selected letter is a vowel is

[MNR 1986; UPSEAT 2000]

(a) 2/11 (b) 3/11 (c) 4/11 (d) 0

46. The probability of three persons having the same date and month for the birthday is

(a) 1/365 (b) $1/(365)^2$ (c) $1/(365)^3$ (d) None of these

47. Out of 20 consecutive positive integers, two are chosen at random, the probability that their sum is odd is

(a) 1/20 (b) 10/19 (c) 19/20 (d) 9/19

48. A and B play a game where each is asked to select a number from 1 to 25. If the two numbers match, both of them win a prize. The probability that they will not win a prize in a single trial is

(a) 1/25 (b) 24/25 (c) 2/25 (d) None of these

49. If E and F are events with $P(E) \le P(F)$ and $P(E \cap F) > 0$, then

(a) Occurrence of $E \Rightarrow$ occurrence of F (b) Occurrence of $F \Rightarrow$ occurrence of E

(c) Non-occurrence of $E \Rightarrow$ non-occurrence of F (d) None of the above implications holds

50. A single letter is selected form the word 'KURUKSHETRA UNIVERSITY' the probability that it is a vowel is **[Kurukshetr**

(a) 4/5 (b) 3/7 (c) 8/21 (d) 2/5

51. From the word 'POSSESSIVE', a letter is chosen at random. The probability of it to be S is

(a) $\frac{3}{10}$ (b) $\frac{4}{10}$ (c) $\frac{3}{6}$ (d) $\frac{4}{6}$

52. Out of 40 consecutive natural numbers, two are chosen at random. Probability that the sum of the numbers is odd, is

[MP PET 2002]

(a) $\frac{14}{29}$ (b) $\frac{20}{39}$ (c) $\frac{1}{2}$ (d) None of these

53. Two dice are tossed. The probability that the total score is a prime number is

(a) $\frac{1}{6}$ (b) $\frac{5}{12}$ (c) $\frac{1}{2}$ (d) None of these

54. A lot consists of 12 good pencils, 6 with minor defects and 2 with major defects. A pencil is choosen at random. The probability that this pencil is not defective is

(a) 3/5 (b) 3/10 (c) 4/5 (d) 1/2

55. 7 white balls and 3 black balls are placed in a row at random. The probability that no two black balls are adjacent is

(a) $\frac{1}{2}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$

Advance Level

56. Twenty children are standing in a line outside a ticket window at Appu Ghar in New Delhi. Ten of these children have a one-rupee coin each and the remaining 10 have a two-rupee coin each. The entry ticket is priced Re. 1. If all the arrangements of the 20 children are equally likely, the probability that the 10^{th} will be the first to wait for change is (Assume that the cashier has no change to begin with)

(a) $\frac{2^{10}}{^{20}C_{10}}$ (b) $\frac{^{20}C_{10}}{2^{10}}$ (c) 0 (d) None of these

57. 4 five-rupee coins, 3 two-rupee coins and 2 one-rupee coins are stacked together in a column at random. The probability that the coins of the same denomination are consecutive is

(a) $\frac{13}{9!}$ (b) $\frac{1}{210}$ (c) $\frac{1}{35}$ (d) None of these

58. Two small squares on a chess board are chosen at random. Probability that they have a common side is

(a) 1/3 (b) 1/9 (c) 1/18 (d) None of these

59. There are n persons ($n \ge 3$), among whom are A and B, who are made to stand in a row in random order. Probability that there is exactly one person between A and B is

(a) $\frac{n-2}{n(n-1)}$ (b) $\frac{2(n-2)}{n(n-1)}$ (c) $2/n$ (d) None of these

60. If m rupee coins and n ten paise coins are placed in a line, then the probability that the extreme coins are ten paise coins is

(a) $^{m+n}C_m$ (b) $\frac{n(n-1)}{(m+n)(m+n-1)}$ (c) $^{m+n}P_m$ (d) $^{m+n}P_n$

61. Twelve balls are distributed among three boxes. The probability that the first box contains 3 balls is

(a) $\frac{110}{9}\left(\frac{2}{3}\right)^{10}$ (b) $\frac{9}{110}\left(\frac{2}{3}\right)^{10}$ (c) $\frac{^{12}C_3}{12^3}.2^9$ (d) $\frac{^{12}C_3}{3^{12}}$

62. Six boys and six girls sit in a row. What is the probability that the boys and girls sit alternately

(a) 1/462 (b) 1/924 (c) 1/2 (d) None of these

63. Word 'UNIVERSITY' is arranged randomly. Then the probability that both 'I' does not come together, is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

64. A fair coin is tossed repeatedly. If tail appears on first four tosses, then the probability of head appearing on fifth toss equals **[IIT 1998]**

(a) 1/2 (b) 1/32 (c) 31/32 (d) 1/5

65. A determinant is chosen at random. The set of all determinants of order 2 with elements 0 or 1 only. The probability that value of the determinant chosen is positive, is

(a) $\frac{3}{16}$ (b) $\frac{3}{8}$ (c) $\frac{1}{4}$ (d) None of these

66. Out of 13 applicants for a job, there are 5 women and 8 men. It is desired to select 2 persons for the job. The probability that at least one of the selected persons will be a woman is

(a) 25/39 (b) 14/39 (c) 5/13 (d) 10/13

67. Two numbers are selected at random from 1, 2, 3......100 and are multiplied, then the probability correct to two places of decimals that the product thus obtained is divisible by 3, is

(a) 0.55 (b) 0.44 (c) 0.22 (d) 0.33

68. Five digit numbers are formed using the digits 1, 2, 3, 4, 5, 6, and 8. What is the probability that they have even digits at both the ends **[Rajasthan PET 1999]**

(a) 2/7 (b) 3/7 (c) 4/7 (d) None of these

69. The corners of regular tetrahedrons are numbered 1, 2, 3, 4. Three tetrahedrons are tossed. The probability that the sum of upward corners will be 5 is

(a) 5/24 (b) 5/64 (c) 3/32 (d) 3/16

70. If four vertices of a regular octagon are chosen at random, then the probability that the quadrilateral formed by them is a rectangle is

[AMU 1999]

(a) 1/8 (b) 2/21 (c) 1/32 (d) 1/35

71. In a college, 25% of the boys and 10% of the girls offer Mathematics. The girls constitute 60% of the total number of students. If a student is selected at random and is found to be studying Mathematics, the probability that the student is a girl, is **[MP PET 2001]**

(a) $\frac{1}{6}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{5}{6}$

72. There are m persons sitting in a row. Two of them are selected at random. The probability that the two selected persons are not together, is

(a) $\frac{2}{m}$ (b) $1-\frac{2}{m}$ (c) $\frac{m(m-1)}{(m+1)(m+2)}$ (d) None of these

73. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{49}$

74. Cards are drawn one by one at random from a well shuffled full pack of 52 cards until two aces are obtained for the first time. If N is the number of cards required to be drawn, then $P_r[N=n]$, where $2 \le n \le 50$, is

(a) $\frac{(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$ (b) $\frac{2(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$ (c) $\frac{3(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$ (d) $\frac{4(n-1)(52-n)(51-n)}{50\times 49\times 17\times 13}$

75. A locker can be opened by dialing a fixed three digit code (between 000 and 999). A stranger who does not know the code tries to open the locker by dialing three digits at random. The probability that the stranger succeeds at the k^{th} trial is

(a) $\frac{k}{999}$ (b) $\frac{k}{1000}$ (c) $\frac{k-1}{1000}$ (d) None of these

76. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently equals

(a) 1/2 (b) 7/15 (c) 2/15 (d) 1/3

77. A committee consists of 9 experts taken from three institutions *A, B* and *C*, of which 2 are from *A*, 3 from *B* and 4 from *C*. If three experts resign, then the probability that they belong to different institutions is

(a) $\frac{1}{729}$ (b) $\frac{1}{24}$ (c) $\frac{1}{21}$ (d) $\frac{2}{7}$

78. There are four machines and it is known that exactly two of them are faulty. They are tested, one by one, in a random order till both the faulty machines are identified. The probability that only two tests are needed is

(a) 1/3 (b) 1/6 (c) 1/2 (d) 1/4

79. A five digit number is formed by writing the digits 1, 2, 3, 4, 5 in a random order without repetitions. Then the probability that the number is divisible by 4 is

(a) 3/5 (b) 18/5 (c) 1/5 (d) 6/5

80. Five persons entered the lift cabin on the ground floor of an 8-floor house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first. The probability of all five persons leaving at different floors is

(a) $\frac{7^5}{{}^7P_5}$ (b) $\frac{{}^7P_5}{7^5}$ (c) $\frac{5!}{7^5}$ (d) 1

81. If A and B are two events than the value of the determinant choosen at random from all the determinants of order 2 with entries 0 or 1 only is positive or negative respectively. Then

(a) $P(A) \ge P(B)$ (b) $P(A) \le P(B)$ (c) $P(A) = P(B) = 1/2$ (d) None of these

82. $x_1, x_2, x_3, ... x_{50}$ are fifty real numbers such that $x_r < x_{r+1}$ for $r = 1,2,3,....., 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is

(a) $\frac{{}^{20}C_2 \times {}^{30}C_2}{{}^{50}C_5}$ (b) $\frac{{}^{30}C_2 \times {}^{19}C_2}{{}^{50}C_5}$ (c) $\frac{{}^{19}C_2 \times {}^{31}C_3}{{}^{50}C_5}$ (d) None of these

83. A card is drawn from a pack. The card is replaced and the pack is reshuffled. If this is done six times, the probability that 2 hearts, 2 diamonds and 2 black cards are drawn is

(a) $90.\left(\frac{1}{4}\right)^6$ (b) $\frac{45}{2}.\left(\frac{3}{4}\right)^4$ (c) $\frac{90}{2^{10}}$ (d) None of these

84. An even number of cards is drawn from a pack of 52 cards. The probability that half of these cards will be red and the other half black is

(a) $\frac{{}^{52}C_2}{2^{51}-1}$ (b) $\frac{{}^{52}C_{26}-1}{2^{51}-1}$ (c) $\frac{{}^{52}C_2-1}{2^{51}-1}$ (d) $\frac{{}^{52}C_2}{2^{51}+1}$

85. Two numbers *a* and *b* are chosen at random from the set {1, 2, 3,.....,3*n*} the probability that $a^2 - b^2$ is divisible by 3 is

(a) $\frac{5(n-3)}{3n-1}$ (b) $\frac{5(n+3)}{3n-1}$ (c) $\frac{5n-3}{3(3n-1)}$ (d) None of these

86. The probability that the birth days of six different persons will fall in exactly two calendar months is

(a) $\frac{1}{6}$ (b) ${}^{12}C_2 \times \frac{2^6}{12^6}$ (c) ${}^{12}C_2 \times \frac{2^6-1}{12^6}$ (d) $\frac{341}{12^5}$

87. A bag contains *n* white and *n* red balls. Pairs of balls are drawn without replacement until the bag is empty. The probability of each pair consisting of balls of different colours is

(a) $\frac{2^n}{{}^{2n}C_n}$ (b) $\frac{2^{n-1}}{{}^{2n}C_n}$ (c) $\frac{2^n}{{}^{2n-1}C_n}$ (d) 1

88. To avoid detection at customs, a traveller has placed six narcotic tablets in a bottle containing nine vitamin pills that are similar in appearance. If the customs official selects three of the tablets at random for analysis, the probability that traveller will be arrested for illegal possession of narcotics is

(a) $\frac{53}{63}$ (b) $\frac{53}{65}$ (c) $\frac{51}{65}$ (d) $\frac{13}{63}$

89. Six different balls are put in three different boxes, no box being empty. The probability of putting balls in the boxes in equal numbers is

(a) 3/10 (b) 1/6 (c) 1/5 (d) None of these

90. A man and a woman appear in an interview for two vacancies in the same post. The probability of man's selection is 1/4 and that of woman's selection is 1/3. What is the probability that none of them will be selected

(a) 1/2 (b) 1/12 (c) 1/4 (d) None of these

91. Three six faced unbiased dice are thrown together. The probability that exactly two of the three numbers are equal is

(a) 117/216 (b) 5/12 (c) 165/216 (d) None of these

92. If the papers of 4 students can be checked by any one of the seven teachers, then the probability that all the four papers are checked by exactly two teachers is

(a) 2/7 (b) 12/49 (c) 32/343 (d) None of these

93. *m* boys and *m* girls take their seats randomly around a circle. The probability of their sitting is $({}^{2m-1}C_m)^{-1}$ when

(a) No two boys sit together (b) No two girls sit together

(c) Boys and girls sit alternatively (d) All the boys sit together

94. *m* men and *w* women seat themselves at random on $m+w$ seats arranged in row (circle). If $p_1(p_2)$ denote the probability of all women sitting together when they are arranged in row (circle), then

(a) $p_1 = \frac{m+1}{^{m+w}C_m}$ (b) $p_1 + p_2 = \frac{2m+w+1}{^{m+w}C_m}$ (c) $p_1 = p_2$ if and only if $w = 1$ (d) $p_2 < p_1$ if $w > 1$

95. Three player *A, B* and *C*, toss a coin cyclically in that order (that is *A, B, C, A, B, C, A, B*,....) till a head shows. Let *p* be the probability that the coin shows a head. Let α, β and γ be, respectively, the probabilities that A, B and *C* gets the first head. Then

(a) $\beta = (1-p)\alpha$ (b) $\gamma + 2p\alpha = (1+p^2)\alpha$ (c) $\alpha + \beta + \gamma = 1$ (d) $\alpha = 1/(3 - 3p + p^2)$

96. Two players *A* and *B* toss a fair coin cyclically in the following order $A, A, B, A, A, B....$ till a head shows (that is, *A* will be allowed first two tosses, followed by a single toss of *B*). Let $\alpha(\beta)$ denote the probability that $A(B)$ gets the head first. Then

(a) $\alpha = 6/7$ (b) $\alpha = 5/7$ (c) $\beta = 1/7$ (d) $\beta = 2/7$

97. Three political parties are contesting election for $(2n+1)$ Lok Sabha seats. the probability that there will be a coalition government after the election is

(a) $\frac{4n+6}{n}$ (b) $\frac{n}{4n+6}$ (c) $\frac{n}{2n+3}$ (d) 1

98. *A* and *B* each throw a dice. The probability that *A*'s throw is not greater than *B*'s is

(a) 1/6 (b) 5/12 (c) 1/2 (d) 7/12

99. A binary operation is chosen at random from the set of all binary operations on a set *A* containing *n* elements. The probability that the binary operation is commutative is

(a) $\frac{n^n}{n^{n^2}}$ (b) $\frac{n^{n/2}}{n^{n^2}}$ (c) $\frac{n^{n/2}}{n^{n^2/2}}$ (d) None of these

100. Let a die is loaded in such a way that even faces are twice as likely to occur as the odd faces. The probability that a prime number will show up when the die is tossed is

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{9}$ (d) $\frac{2}{3}$

101. A special die with numbers $1, -1, 2, -2, 0$ and 3 is thrown thrice. The probability that the total is zero is

(a) $\frac{25}{216}$ (b) $\frac{214}{217}$ (c) $\frac{11}{216}$ (d) None of these

102. If four small squares are chosen at random on a chess board, the probability that they lie on a diagonal line is

(a) $\frac{13}{22692}$ (b) $\frac{11}{22692}$ (c) $\frac{7}{22692}$ (d) $\frac{2}{7}$

103. A letter is taken at random out of each of the words CHOICE and CHANCE. The probability that they should be the same letter is

(a) 1/6 (b) 1/9 (c) 5/36 (d) 1/324

104. Let *X* be a set containing *n* elements. If two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is

(a) $\frac{^{2n}C_n}{2^{2n}}$ (b) $\frac{1}{^{2n}C_n}$ (c) $\frac{1.3.5....(2n-1)}{2^n(n!)}$ (d) $\frac{3^n}{4^n}$

105. A four figure number is formed of the figures 1, 2, 3, 5 with no repetitions. The probability that the number is divisible by 5 is

(a) 3/4 (b) 1/4 (c) 1/8 (d) None of these

106. An elevator starts with *m* passengers and stops at *n* floors $(m \le n)$. The probability that no two passengers alight at the same floor is

(a) $\frac{^nP_m}{m^n}$ (b) $\frac{^nP_m}{n^m}$ (c) $\frac{^nC_m}{m^n}$ (d) $\frac{^nC_m}{n^m}$

107. If ten objects are distributed at random among ten persons, the probability that at least one of them will not get any thing is

(a) $\frac{10^{10}-10}{10^{10}}$ (b) $\frac{10^{10}-10!}{10^{10}}$ (c) $\frac{10^{10}-1}{10^{10}}$ (d) None of these

108. Cards are drawn one by one without replacement from a pack of 52 cards. The probability that 10 cards will precede the first ace is

(a) $\frac{241}{1456}$ (b) $\frac{164}{4165}$ (c) $\frac{451}{884}$ (d) None of these

109. Five different objects A_1, A_2, A_3, A_4, A_5 are distributed randomly in 5 places marked 1, 2, 3, 4, 5. One arrangement is picked at random. The probability that in the selected arrangement, none of the object occupies the place corresponding to its number, is

(a) 119/120 (b) 1/15 (c) 11/30 (d) None of these

110. 4 gentlemen and 4 ladies take seats at random round a table. The probability that they are sitting alternately is

(a) $\frac{4}{35}$ (b) $\frac{1}{70}$ (c) $\frac{2}{35}$ (d) $\frac{1}{35}$

111. Let $x = 33^n$. The index n is given a positive integral value at random. The probability that the value of x will have 3 in the units place is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

112. There are 7 seats in a row. Three persons take seats at random. The probability that the middle seat is always occupied and no two persons are consecutive is

(a) $\frac{9}{70}$ (b) $\frac{9}{35}$ (c) $\frac{4}{35}$ (d) None of these

113. 10 different books and 2 different pens are given to 3 boys so that each gets equal number of things. The probability that the same boy does not receive both the pens is

(a) $\frac{5}{11}$ (b) $\frac{7}{11}$ (c) $\frac{2}{3}$ (d) $\frac{6}{11}$

114. The probability that out of 10 persons, all born in April, at least two have the same birthday is

(a) $\frac{^{30}P_{10}}{(30)^{10}}$ (b) $1-\frac{^{30}C_{10}}{30!}$ (c) $\frac{(30)^{10}-{^{30}P_{10}}}{(30)^{10}}$ (d) None of these

115. A and B draw two cards each, one after another, from a pack of well-shuffled pack of 52 cards. The probability that all the four cards drawn are of the same suit is

(a) $\frac{44}{85\times 49}$ (b) $\frac{11}{85\times 49}$ (c) $\frac{13\times 24}{17\times 25\times 49}$ (d) None of these

116. Three different numbers are selected at random from the set $A = \{1, 2, 3, ..., 10\}$. The probability that the product of two of the numbers is equal to the third is

(a) $\frac{3}{4}$ (b) $\frac{1}{40}$ (c) $\frac{1}{8}$ (d) None of these

117. A point is selected at random from the interior of a circle. The probability that the point is closer to the centre than the boundary of the circle is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) None of these

118. Let A and B are two independent events. The probability that both A and B occur together is 1/6 and the probability that neither of them occurs is 1/3. The probability of occurrence of A is
[Rajasthan PET 2000; Roorkee 1989]

(a) 0 or 1 (b) 1/2 or 1/3 (c) 1/2 or 1/4 (d) 1/3 or 1/4

Odds in favour and Odds against

Basic Level

119. For an event, odds against is 6 : 5. The probability that event does not occur, is

(a) $\frac{5}{6}$ (b) $\frac{6}{11}$ (c) $\frac{5}{11}$ (d) $\frac{1}{6}$

120. An event has odds in favour 4 : 5, then the probability that event occurs, is

(a) $\frac{1}{5}$ (b) $\frac{4}{5}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

121. A card is drawn from a pack of 52 cards. A gambler bets that it is a spade or an ace. What are the odds against his winning this bet

(a) 17 : 52 (b) 52 : 17 (c) 9 : 4 (d) 4 : 9

122. The odds in favour of a certain event are 2 : 5 and odds against of another event are 5 : 6. If the events are independent, then the probability of happening of at least one of them is

(a) 50/77 (b) 51/77 (c) 52/77 (d) 53/77

123. In a horse race the odds in favour of three horses are 1 : 2, 1 : 3 and 1 : 4. The probability that one of the horse will win the race is

(a) $\frac{37}{60}$ (b) $\frac{47}{60}$ (c) $\frac{27}{60}$ (d) $\frac{17}{60}$

Advance Level

124. Odds 8 to 5 against a person who is 40 years old living till he is 70 and 4 to 3 against another person now 50 till he will be living 80. Probability that one of them will be alive next 30 years is

(a) 59/91 (b) 44/91 (c) 51/91 (d) 32/91

125. One of the two events must occur. If the chance of one is 2/3 of the other, then odds in favour of the other are

(a) 2 : 3 (b) 1 : 3 (c) 3 : 1 (d) 3 : 2

126. If a party of n persons sit at a round table, then the odds against two specified individuals sitting next to each other are **[MP PET 2002]**

(a) $2:(n-3)$ (b) $(n-3):2$ (c) $(n-2):2$ (d) $2:(n-2)$

127. If odds against solving a question by three students are 2 : 1, 5 : 2 and 5 : 3 respectively, then probability that the question is solved only by one student is

(a) 31/56 (b) 24/56 (c) 25/56 (d) None of these

128. Odds in favour of an event A are 2 to 1 and odds in favour of $A \cup B$ are 3 to 1. Consistent with this information the smallest and largest values for the probability of event B are given by

(a) $\frac{1}{6} \le P(B) \le \frac{1}{3}$ (b) $\frac{1}{3} \le P(B) \le \frac{1}{2}$ (c) $\frac{1}{12} \le P(B) \le \frac{3}{4}$ (d) None of these

129. The chance of an event happening is the square of the chance of a second event but the odds against the first are the cube of the odds against the second. The chances of the events are

(a) $\frac{1}{9}, \frac{1}{3}$ (b) $\frac{1}{16}, \frac{1}{4}$ (c) $\frac{1}{4}, \frac{1}{2}$ (d) None of these

Addition Theorem on Probability

Basic Level

130. If A and B are two mutually exclusive events, then $P(A+B)=$
[MNR 1978; MP PET 1991, 1992]

(a) $P(A)+P(B)-P(AB)$ (b) $P(A)-P(B)$ (c) $P(A)+P(B)$ (d) $P(A)+P(B)+P(AB)$

131. If A and B are two events such that $P(A \cup B)+P(A \cap B)=\frac{7}{8}$ and $P(A)=2P(B)$, then $P(A)=$

(a) 7/12 (b) 7/24 (c) 5/12 (d) 17/24

132. A bag contains 5 brown and 4 white socks. A man pulls out two socks. The probability that these are of the same colour is

[UPSEAT 1999; MP PET 2000]

(a) 5/108 (b) 18/108 (c) 30/108 (d) 48/108

133. The probability that a leap year will have 53 Fridays or 53 Saturdays is **[MP PET 2002; Roorkee 1999]**

(a) 2/7 (b) 3/7 (c) 4/7 (d) 1/7

134. A box contains 10 good articles and 6 with defects. One article is chosen at random. What is the probability that it is either good or has a defect

(a) 24/64 (b) 40/64 (c) 49/64 (d) 64/64

135. The probabilities of occurrence of two events are respectively 0.21 and 0.49. The probability that both occurs simultaneously is 0.16. Then the probability that none of the two occurs is

(a) 0.30 (b) 0.46 (c) 0.14 (d) None of these

136. A bag contains 30 balls numbered from 1 to 30, one ball is drawn randomly. The probability that number on the ball is multiple of 5 or 7 is

(a) 1/2 (b) 1/3 (c) 2/3 (d) 1/4

137. If $P(A)=P(B)=x$ and $P(A\cap B)=P(A'\cap B')=\frac{1}{3}$, then x = **[UPSEAT 2003]**

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

138. If the probability of X to fail in the examination is 0.3 and that for Y is 0.2, then the probability that either X or Y fail in the examination is

(a) 0.5 (b) 0.44 (c) 0.6 (d) None of these

139. A card is drawn from a well shuffled pack of cards. The probability of getting a queen of club or king of heart is **[MP PI**

(a) 1/52 (b) 1/26 (c) 1/18 (d) None of these

140. If A and B are two independent events, then $P(A+B)=$ **[MP PET 1992]**

(a) $P(A)+P(B)-P(A)P(B)$ (b) $P(A)-P(B)$ (c) $P(A)+P(B)$ (d) $P(A)+P(B)+P(A)P(B)$

141. In two events $P(A\cup B)=5/6$, $P(A^c)=5/6$, $P(B)=2/3$, then A and B are **[UPSEAT 2001]**

(a) Independent (b) Mutually exclusive (c) Mutually exhaustive (d) Dependent

142. The probability that at least one of the events A and B occurs is 3/5. If A and B occur simultaneously with probability 1/5, then $P(A')+P(B')$ is

(a) 2/5 (b) 4/5 (c) 6/5 (d) 7/5

143. If A and B are arbitrary events, then **[DCE 2002]**

(a) $P(A\cap B)\geq P(A)+P(B)$ (b) $P(A\cup B)\leq P(A)+P(B)$ (c) $P(A\cap B)=P(A)+P(B)$ (d) None of these

144. If $P(A)=2/3$, $P(B)=1/2$ and $P(A\cup B)=5/6$ then events A and B are **[Kerala (Engg.) 2002]**

(a) Mutually exclusive (b) Independent as well as mutually exhaustive

(c) Independent (d) Dependent only on A

145. A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected randomwise, the probability that it is a black or red ball is

[EAMCET 2002]

(a) 1/3 (b) 1/4 (c) 5/12 (d) 2/3

146. A card is drawn from a pack of cards. Find the probability that the card will be a queen or a heart

(a) $\frac{4}{3}$ (b) $\frac{16}{3}$ (c) $\frac{4}{13}$ (d) $\frac{5}{3}$

147. The chance of India winning toss is 3/4. If it wins the toss, then its chance of victory is 4/5 otherwise it is only 1/2. Then chance of India's victory is

(a) 1/5 (b) 3/5 (c) 3/40 (d) 29/40

148. Let A and B be events for which $P(A) = x$, $P(B) = y$, $P(A \cap B) = z$, then $P(\overline{A} \cap B)$ equals **[AMU 1999]**

(a) $(1-x)y$ (b) $1-x+y$ (c) $y - z$ (d) $1-x+y-z$

149. A and B are two events such that $P(A) = 0.4$, $P(A+B) = 0.7$ and $P(AB) = 0.2$, then $P(B) =$

(a) 0.1 (b) 0.3 (c) 0.5 (d) None of these

150. A card is drawn at random from a pack of cards. The probability of this card being a red or a queen is

(a) 1/13 (b) 1/26 (c) 1/2 (d) 7/13

151. If $P(A) = 0.4, P(B) = x, P(A \cup B) = 0.7$ and the events A and B are mutually exclusive, then x =

(a) 3/10 (b) 1/2 (c) 2/5 (d) 1/5

152. One card is drawn randomly from a pack of 52 cards, then the probability that it is a king or spade is **[Rajasthan PET 2001, 1996; MP PET 1990, 94]**

(a) 1/26 (b) 3/26 (c) 4/13 (d) 3/13

153. The chance of throwing a total of 7 or 12 with 2 dice, is **[Kurukshetra CEE 2002]**

(a) $\frac{2}{9}$ (b) $\frac{5}{9}$ (c) $\frac{5}{36}$ (d) $\frac{7}{36}$

154. The probability of three mutually exclusive events A, B and C are given by 2/3, 1/4 and 1/6 respectively. The statement **[MNR 1987]**

(a) Is true (b) False (c) Nothing can be said (d) Could be either

155. If $A_1, A_2, ..., A_n$ are any n events, then

(a) $P(A_1 \cup A_2 \cup .... \cup A_n) = P(A_1) + P(A_2) + ... + P(A_n)$ (b) $P(A_1 \cup A_2 \cup .... \cup A_n) > P(A_1) + P(A_2) + ... + P(A_n)$

(c) $P(A_1 \cup A_2 \cup .... \cup A_n) \le P(A_1) + P(A_2) + ... + P(A_n)$ (d) None of these

156. In a class of 125 students 70 passed in Mathematics, 55 in Statistics and 30 in both. The probability that a student selected at random from the class, has passed in only one subject is

(a) 13/25 (b) 3/25 (c) 17/25 (d) 8/25

157. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing single event is

(a) 0.56 (b) 0.54 (c) 0.38 (d) 0.94

158. The chances of throwing a total of 3 or 5 or 11 with two dice is

(a) 5/36 (b) 1/9 (c) 2/9 (d) 19/36

159. In a box there are 2 red, 3 black and 4 white balls. Out of these three balls are drawn together. The probability of these being of same colour is

(a) $\frac{1}{84}$ (b) $\frac{1}{21}$ (c) $\frac{5}{84}$ (d) None of these

160. A card is drawn at random from a well shuffled pack of 52 cards. The probability of getting a two of heart or diamond is **[DSSE 1979]**

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$ (c) $\frac{1}{13}$ (d) None of these

161. A committee of five is to be chosen from a group of 9 people. The probability that a certain married couple will either serve together or not at all is

(a) $\frac{1}{2}$ (b) $\frac{5}{9}$ (c) $\frac{4}{9}$ (d) $\frac{2}{3}$

162. A and B toss a coin alternately, the first to show a head being the winner. If A starts the game, the chance of his winning is **[MP PET 1987]**

(a) 5/8 (b) 1/2 (c) 1/3 (d) 2/3

163. If A and B are two events, then the probability of the event that at most one of A, B occurs, is

(a) $P(A' \cap B) + P(A \cap B') + P(A' \cap B')$ (b) $1 - P(A \cap B)$

(c) $P(A') + P(B') + P(A \cup B) - 1$ (d) All of these

164. Three persons work independently on a problem. If the respective probabilities that they will solve it are 1/3, 1/4 and 1/5, then the probability that none can solve it

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{1}{3}$ (d) None of these

165. The probability of hitting a target by three marksmen are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that one and only one of them will hit the target when they fire simultaneously, is

(a) $\frac{11}{24}$ (b) $\frac{1}{12}$ (c) $\frac{1}{8}$ (d) None of these

166. If A speaks truth in 75% cases and B in 80% cases, then the probability that they contradict each other in stating the same statement, is

[MP PET 1997, 2002]

(a) $\frac{7}{20}$ (b) $\frac{13}{20}$ (c) $\frac{12}{20}$ (d) $\frac{2}{5}$

167. The probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is

(a) $p+q$ (b) $p+q-2qp$ (c) $p+q-pq$ (d) $p+q+pq$

168. If from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls. One ball is drawn at random. Then the probability that 2 white and 1 black ball will be drawn

(a) 13/32 (b) 1/4 (c) 1/32 (d) 3/16

169. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2, then

(a) $p = q = 1$ (b) $p = q = 1/2$

(c) $p = 1, q = 0$ (d) There are infinite values of p, q

170. A bag contains 3 white, 3 black and 2 red balls. One by one three balls are drawn without replacing them. The probability that the third ball is red, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

171. The probability of A, B, C solving a problem are $\frac{1}{3}, \frac{2}{7}, \frac{3}{8}$ respectively. If all the three try to solve the problem simultaneously, the probability that exactly one of them will solve it, is

(a) $\frac{25}{168}$ (b) $\frac{25}{56}$ (c) $\frac{20}{168}$ (d) $\frac{30}{168}$

172. The two events A and B have probabilities 0.25 and 0.50 respectively. The probability that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is

(a) 0.39 (b) 0.25 (c) 0.904 (d) None of these

173. A basket contains 5 apples and 7 oranges and another basket contains 4 apples and 8 oranges. One fruit is picked out from each basket. Find the probability that the fruits are both apples or both oranges

(a) 24/144 (b) 56/144 (c) 68/144 (d) 76/144

174. A, B, C are any three events. If $P(S)$ denotes the probability of S happening then $P(A \cap (B \cup C)) =$

(a) $P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$ (b) $P(A) + P(B) + P(C) - P(B)P(C)$

(c) $P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$ (d) None of these

175. If A and B are any two events, then the probability that exactly one of them occur is **[Ranchi BIT 1990; IIT 1984; Rajasthan PET 2002]**

(a) $P(A) + P(B) - P(A \cap B)$ (b) $P(A) + P(B) - 2P(A \cap B)$ (c) $P(A) + P(B) - P(A \cup B)$ (d) $P(A) + P(B) - 2P(A \cup B)$

176. If A and B are any two events, then $P(\overline{A} \cap B) =$ **[MP PET 2001]**

(a) $P(\overline{A})P(\overline{B})$ (b) $1 - P(A) - P(B)$ (c) $P(A) + P(B) - P(A \cap B)$ (d) $P(B) - P(A \cap B)$

177. If A and B are any two events, then the true relation is **[IIT 1988]**

(a) $P(A \cap B)$ is not less than $P(A) + P(B) - 1$ (b) $P(A \cap B)$ is not greater than $P(A) + P(B)$

(c) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ (d) $P(A \cap B) = P(A) + P(B) + P(A \cup B)$

178. A bag contains 3 black and 4 white balls. Two balls are drawn one by one at random without replacement. The probability that the second drawn ball is white, is

(a) $\frac{4}{49}$ (b) $\frac{1}{7}$ (c) $\frac{4}{7}$ (d) $\frac{12}{49}$

179. If $P(A) = 0.25, P(B) = 0.50$ and $P(A \cap B) = 0.14$, then $P(A \cap \overline{B})$ is equal to **[Rajasthan PET 2001]**

(a) 0.61 (b) 0.39 (c) 0.48 (d) None of these

180. Suppose that A, B, C are events such that $P(A) = P(B) = P(C) = \frac{1}{4}, P(AB) = P(CB) = 0, P(AC) = \frac{1}{8}$, then $P(A + B) =$ **[MP PET 1992]**

(a) 0.125 (b) 0.25 (c) 0.375 (d) 0.5

181. For any two independent events E_1 and E_2 $P\{(E_1 \cup E_2) \cap (\overline{E}_1 \cap \overline{E}_2)\}$ is **[IIT 1991]**

(a) $\leq \frac{1}{4}$ (b) $> \frac{1}{4}$ (c) $\geq \frac{1}{2}$ (d) None of these

182. Two cards are drawn without replacement from a well-shuffled pack. Find the probability that one of them is an ace of heart

[UPSEAT 2002]

(a) $\frac{1}{25}$ (b) $\frac{1}{26}$ (c) $\frac{1}{52}$ (d) None of these

183. If $P(A \cup B) = 0.8$ and $P(A \cap B) = 0.3$, then $P(\overline{A}) + P(\overline{B}) =$ **[EAMCET 2003]**

(a) 0.3 (b) 0.5 (c) 0.7 (d) 0.9

184. If A and B are two independent events such that $P(A \cap B') = \frac{3}{25}$ and $P(A' \cap B) = \frac{8}{25}$, then $P(A) =$

(a) $\frac{1}{5}$ (b) $\frac{3}{8}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

185. If A and B are two independent events such that $P(A) = 0.40$, $P(B) = 0.50$, then P (neither A nor B) is equal to

(a) 0.90 (b) 0.10 (c) 0.2 (d) 0.3

Advance Level

186. The probability of India winning a test match against West Indies is $\frac{1}{2}$. Assuming independence from match to match, the probability that in a 5 match series India's second win occurs at the third test is

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

187. A box contains 3 white and 2 red balls. A ball is drawn and another ball is drawn without replacing first ball, then the probability of second ball to be red is

(a) $\frac{8}{25}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{21}{25}$

188. The probability of solving a question by three students are $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ respectively. Probability of question is being solved will be

[UPSEAT 1999]

(a) $\frac{33}{48}$ (b) $\frac{35}{48}$ (c) $\frac{31}{48}$ (d) $\frac{37}{48}$

189. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, one girl and 3 boys. One child is selected at random from each group. The chance that three selected consisting of 1 girl and 2 boys, is]

(a) $\frac{9}{32}$ (b) $\frac{3}{32}$ (c) $\frac{13}{32}$ (d) None of these

190. *A, B, C* are three events for which $P(A)=0.6, P(B)=0.4, P(C)=0.5, P(A\cup B)=0.8, P(A\cap C)=0.3$ and $P(A\cap B\cap C)=0.2$. If $P(A\cup B\cup C)\geq 0.85$ then the interval of values of $P(B\cap C)$ is

(a) [0.2, 0.35] (b) [0.55, 0.7]

(c) [0.2, 0.55] (d) None of these

191. A student has to match three historical events-Dandi March, Quit India Movement and Mahatma Gandhi's assassination with the years 1948, 1930 and 1942. The student has no knowledge of the correct answers and decides to match the events and years randomly. Let $E_i (0\leq i\leq 3)$ denote the event that the student gets exactly *i* correct answers. Then

(a) $P(E_0)+P(E_3)=P(E_1)$ (b) $P(E_0)P(E_1)=P(E_3)$ (c) $P(E_0\cap E_1)=P(E_2)$ (d) $P(E_0)+P(E_1)+P(E_3)=1$

192. Given that *A*, *B* and *C* are events such that $P(A)=P(B)=P(C)=1/5, P(A\cap B)=P(B\cap C)=0$ and $P(A\cap C)=1/10$.

The probability that at least one of the events *A, B* or *C* occurs is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

193. Suppose that a die (with faces marked 1 to 6) is loaded in such a manner that for *K* = 1, 2, 3...., 6, the probability of the face marked *K* turning up when die is tossed is proportional to *K*. The probability of the event that the outcome of a toss of the die will be an even number is equal to

(a) $\frac{1}{2}$ (b) $\frac{4}{7}$ (c) $\frac{2}{5}$ (d) $\frac{1}{21}$

194. An unbiased die is tossed until a number greater than 4 appears. The probability that an even number of tosses is needed is

[IIT Screening 1994]

(a) 1/2 (b) 2/5 (c) 1/5 (d) 2/3

195. For the three events A, B and C ; *P*(exactly one of the events *A* or *B* occurs) = *P* (exactly one of the events *B* or *C* occurs) = *P*(exactly one of the events *C* or *A* occurs) = *p* and *P* (all the three events occur simultaneously) = p^2, where $0 < p < 1/2$. Then the probability of at least one of the three events *A*, *B* and *C* occurring is **[IIT 1996]**

(a) $\frac{3p+2p^2}{2}$ (b) $\frac{p+3p^2}{4}$ (c) $\frac{p+3p^2}{2}$ (d) $\frac{3p+2p^2}{4}$

196. A man alternately tosses a coin and throws a dice beginning with the coin. The probability that he gets a head in the coin before he gets a 5 or 6 in the dice is

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) None of these

Conditional Probability

Basic Level

197. Two cards are drawn successively with replacement from a pack of 52 cards. The probability of drawing two aces is

[MNR 1988; UPSEAT 2000]

(a) $\frac{1}{169}$ (b) $\frac{1}{221}$ (c) $\frac{1}{2652}$ (d) $\frac{4}{663}$

198. A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random. The probability that at least one of these in an ace, is

(a) $\frac{9}{20}$ (b) $\frac{3}{16}$ (c) $\frac{1}{6}$ (d) $\frac{1}{9}$

199. From a pack of 52 cards, two cards are drawn one by one without replacement. The probability that first drawn card is king and second is queen, is

(a) $\frac{2}{13}$ (b) $\frac{8}{663}$

(c) $\frac{4}{663}$ (d) $\frac{103}{663}$

200. From a pack of 52 cards two cards are drawn in succession one by one without replacement. The probability that both are aces **OR** the probability that both are kings is
[Rajasthan PET 2001; MP PET 1994]
(a) 2/13 (b) 1/51 (c) 1/221 (d) 2/21

201. A problem in Mathematics is given to three students *A, B, C* and their respective probability of solving the problem is 1/2, 1/3 and 1/4. Probability that the problem is solved is
[Rajasthan PET 2001; AIEEE 2002]
(a) 3/4 (b) 1/2 (c) 2/3 (d) 1/3

202. A coin is tossed and a dice is rolled. The probability that the coin shows the head and the dice shows 6 is
(a) 1/8 (b) 1/12 (c) 1/2 (d) 1

203. A coin is tossed until a head appears or until the coin has been tossed five times. If a head does not occur on the first two tosses, then the probability that the coin will be tossed 5 times is
(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

204. A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn one by one without replacement, what is the probability that all are white
(a) $\frac{1}{969}$ (b) $\frac{1}{380}$ (c) $\frac{5}{20}$ (d) None of these

205. A bag contains 19 tickets numbered from 1 to 19. A ticket is drawn and then another ticket is drawn without replacement. The probability that both the tickets will show even number, is
(a) $\frac{9}{19}$ (b) $\frac{8}{18}$ (c) $\frac{9}{18}$ (d) $\frac{4}{19}$

206. For two events *A* and *B*, if $P(A)=P\left(\frac{A}{B}\right)=\frac{1}{4}$ and $P\left(\frac{B}{A}\right)=\frac{1}{2}$, then
(a) *A* and *B* are independent (b) $P\left(\frac{A'}{B}\right)=\frac{3}{4}$ (c) $P\left(\frac{B'}{A'}\right)=\frac{1}{2}$ (d)

207. If $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{4}$, then $P\left(\frac{B}{A}\right)=$
(a) 1 (b) 0 (c) 1/2 (d) 1/3

208. From a pack of 52 cards two are drawn with replacement. The probability that the first is a diamond and the second is a king is
[MNR 1979]
(a) 1/26 (b) 17/2704 (c) 1/52 (d) None of these

209. The probability that a teacher will give an unannounced test during any class meeting is 1/5. If a student is absent twice, then the probability that the student will miss at least one test is
(a) 1/5 (b) 2/5 (c) 7/5 (d) 9/25

210. If *E* and *F* are independent events such that $0<P(E)<1$ and $0<P(F)<1$, then **[IIT 1989]**
(a) *E* and F^c (the complement of the event *F*) are independent (b) E^c and F^c are independent
(c) $P\left(\frac{E}{F}\right)+P\left(\frac{E^c}{F^c}\right)=1$ (d) All of these

211. The probability of getting at least one tail in 4 throws of a coin is
[MNR 1983; Kurukshetra CEE 1998]
(a) 15/16 (b) 1/16 (c) 1/4 (d) None of these

212. If any four numbers are selected and they are multiplied, then the probability that the last digit will be 1, 3, 5 or 7 is
[Rajasthan PET 2002]
(a) 4/625 (b) 18/625 (c) 16/625 (d) None of these

213. A bag contains 4 white balls and 2 black balls. Another contains 3 white balls and 5 black balls. If one ball is drawn from each bag, then the probability that both are white, is

(a) 0.25 (b) 0.2 (c) 0.3 (d) None of these

214. A binary number is made up of 16 bits. The probability of an incorrect bit appearing is *p* and the errors in different bits are independent of one another. The probability of forming an incorrect number is

(a) $p/16$ (b) p^{16} (c) $^{16}C_1 p^{16}$ (d) $1-(1-p)^{16}$

215. The probabilities of winning the race by two athletes *A* and *B* are $\frac{1}{5}$ and $\frac{1}{4}$. The probability of winning by neither of them, is

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{4}{5}$

216. Seven chits are numbered 1 to 7. Three are drawn one by one with replacements. The probability that the least number on any selected chit is 5, is

(a) $1-\left(\frac{2}{7}\right)^4$ (b) $4\left(\frac{2}{7}\right)^4$ (c) $\left(\frac{3}{7}\right)^3$ (d) None of these

217. A box contains 100 tickets numbered 1, 2.....100. Two tickets are chosen at random. It is given that the maximum number on the two chosen tickets is not more than 10. The minimum number on them is 5 with probability **[IIT 1985]**

(a) $\frac{1}{8}$ (b) $\frac{13}{15}$ (c) $\frac{1}{9}$ (d) None of these

218. There are 20 cards. 10 of these cards have the letter '*I*' printed on them and the other 10 have the letter '*T*' printed on them. If three cards are picked up at random and kept in the same order, the probability of making word *IIT* is

(a) $\frac{4}{27}$ (b) $\frac{5}{38}$ (c) $\frac{1}{8}$ (d) $\frac{9}{80}$

219. Let $A=\{2,3,4,......,20\}$. A number is chosen at random from the set *A* and it is found to be a prime number. The probability that it is more than 10 is

(a) $\frac{9}{10}$ (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{1}{2}$

220. A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the digits is 12. If he choose three numbers with replacement then the probability that he will laugh at least once is

(a) $1-\left(\frac{3}{5}\right)^3$ (b) $\left(\frac{43}{45}\right)^3$ (c) $1-\left(\frac{4}{25}\right)^3$ (d) $1-\left(\frac{43}{45}\right)^3$

221. The probability that a married man watches a certain T.V. show is 0.4 and the probability that a married woman watches the show is 0.5. The probability that a man watches the show, given that his wife does, is 0.7. Then the probability that a wife watches the shows given that her husband does is

(a) $\frac{7}{8}$ (b) $\frac{3}{5}$ (c) $\frac{2}{7}$ (d) 1

222. A pair of fair dice is rolled together till a sum of either 5 or 7 is obtained. Then the probability that 5 comes before 7 is **[IIT 1989]**

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) None of these

223. A bag contains 3 red and 5 black balls and a second bag contains 6 red and 4 black balls. A ball is drawn from each bag. The probability that one is red and other is black, is

(a) $\frac{3}{20}$ (b) $\frac{21}{40}$ (c) $\frac{3}{8}$ (d) All of these

224. Two persons *A* and *B* take turns in throwing a pair of dice. The first person to through 9 from both dice will be avoided the prize. If *A* throws first then the probability that *B* wins the game is

(a) $\frac{9}{17}$ (b) $\frac{8}{17}$ (c) $\frac{8}{9}$ (d) $\frac{1}{9}$

Advance Level

225. An anti-aircraft gun take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. The probability that the gun hits the plane is

(a) 0.25 (b) 0.21 (c) 0.16 (d) 0.6976

226. If *A* and *B* are two events such that $P(A/B) = P(A'/B') = p$ and $P(B) = 0.05$, then value of *p* so that $P(B/A) = 0.5$ is

(a) 0.75 (b) 0.85 (c) 0.95 (d) 1

227. Eight tickets numbered 000, 010, 011, 011, 100, 101, 101 and 110 are placed in a bag. One ticket is drawn from the bag at random. Let *A, B* and *C* denote the following events: *A* – "the first digit is 0" *B*– "the second digit is 0" and *C* – "the third digit is 0". then *A, B* and *C* are

(a) Independent (b) Mutually exclusive (c) Mutually non-exclusive (d) Not independent

228. A die is rolled three times. Let E_1 denote the event of getting a number larger than the previous number each time and E_2 denote the event that the numbers form an increasing A.P., then

(a) $P(E_2) \le P(E_1)$ (b) $P(E_2 \cap E_1) = 1/36$ (c) $P(E_2 | E_1) = 3/10$ (d) $P(E_1) = (10/3)P(E_2)$

229. A reputed coaching employed 8 professors in the staff. Their respective probabilities of remaining in employment for three years are $\frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}$. The probability that after 3 years at least six of these still work in the coaching is

(a) 0.15 (b) 0.19 (c) 0.3 (d) None of these

230. For a biased die the probabilities for different faces to turn up are given below

Face:	1	2	3	4	5	6
Probability:	.1	.32	.21	.15	.05	.17

The die is tossed and you are told that either face 1 or 2 has turned up. Then the probability that it is face 1, is **[IIT 1981**

(a) 5/21 (b) 5/22 (c) 4/21 (d) None of these

231. A biased die is tossed and the respective probabilities for various faces to turn up are given below

Face:	1	2	3	4	5	6
Probability:	.1	.24	.19	.18	.15	.14

If an even face has turned up, then the probability that it is face 2 or face 4, is **[MNR 1992]**

(a) 0.25 (b) 0.42 (c) 0.75 (d) 0.9

232. A bag *X* contains 2 white and 3 black balls and another bag *Y* contains 4 white and 2 black balls. One bag is selected at random and a ball is drawn from it. Then the probability for the ball chosen to be white is

(a) 2/15 (b) 7/15 (c) 8/15 (d) 14/15

233. A man draws a card from a pack of 52 playing cards, replaces it and shuffles the pack. He continues this processes until he gets a card of spade. The probability that he will fail the first two times is

(a) 9/16 (b) 1/16 (c) 9/64 (d) None of these

234. For any two events *A* and *B* in a sample space **[IIT 1991]**

(a) $P\left(\frac{A}{B}\right) \ge \frac{P(A)+P(B)-1}{P(B)}, P(B) \ne 0$ is always true (b) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$ does not hold

(c) $P(A \cup B) = 1 - P(\bar{A})P(\bar{B})$, if *A* and *B* are disjoint (d) None of these

235. Three groups *A*, *B*, *C* are competing for positions on the Board of Directors of a company. The probabilities of their winning are 0.5, 0.3, 0.2 respectively. If the group *A* wins, the probability of introducing a new product is 0.7 and the corresponding probabilities for group *B* and *C* are 0.6 and 0.5 respectively. The probability that the new product will be introduced, is **[Roorkee 1994]**

(a) 0.18 (b) 0.35 (c) 0.10 (d) 0.63

236. If $\bar{E}$ and $\bar{F}$ are the complementary events of events *E* and *F* respectively and if $0 < P(F) < 1$, then

(a) $P(E/F) + P(\bar{E}/F) = 1$ (b) $P(E/F) + P(E/\bar{F}) = 1$ (c) $P(\bar{E}/F) + P(E/\bar{F}) = 1$ (d) $P(E/\bar{F}) + P(\bar{E}/\bar{F}) = 1$

237. Let *A*, *B*, *C* be three mutually independent events. Consider the two statements S_1 and S_2

$S_1 : A$ and $B \cup C$ are independent; $S_2 : A$ and $B \cap C$ are independent

Then **[IIT Screening 1994]**

(a) Both S_1 and S_2 are true (b) Only S_1 is true (c) Only S_2 is true (d)

238. In a certain town, 40% of the people have brown hair, 25% have brown eyes and 15% have both brown hair and brown eyes. If a person selected at random from the town, has brown hair, the probability that he also has brown eyes, is **[MNR 1988]**

(a) 1/5 (b) 3/8 (c) 1/3 (d) 2/3

239. Let $0 < P(A) < 1, 0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)P(B)$. Then **[IIT 1995]**

(a) $P(B/A) = P(B) - P(A)$ (b) $P(A^c \cup B^c) = P(A^c) + (B^c)$ (c) $P(A \cup B)^c = P(A^c)P(B^c)$ (d) $P(A/B) = P(A)$

240. It has been found that if *A* and *B* play a game 12 times, A wins 6 times, B wins 4 times and they draw twice. A and B take part in a series of 3 games. The probability that they will win alternately is

(a) $\frac{5}{72}$ (b) $\frac{5}{36}$ (c) $\frac{19}{27}$ (d) None of these

241. Two dice are rolled one after the other. The probability that the number on the first is smaller than the number on the second is

(a) 1/2 (b) 7/18 (c) 3/4 (d) 5/12

242. The two friends *A* and *B* have equal number of daughters. There are three cinema tickets which are to be distributed among the daughters of *A* and *B* . The probability that all the tickets go to daughters of *A* is 1/20. The number of daughters each of them have is

(a) 4 (b) 5 (c) 6 (d) 3

243. A bag contains $(2n+1)$ coins. It is known that *n* of these have a head on both the sides, whereas the remaining $(n+1)$ coins are fair. A coin is picked up at random from the bag and tossed. If the probability that the toss results in a head is 31/42, then the value of *n* is

(a) 10 (b) 8 (c) 6 (d) 25

244. The letters of the word PROBABILITY are written down at random in a row. Let E_1 denote the event that two *I*'s are together and E_2 denote the event that two *B's* are together, then

(a) $P(E_1) = P(E_2)$ (b) $P(E_1 \cap E_2) = 2/55$ (c) $P(E_1 \cup E_2) = 18/55$ (d) $P(E_2 / E_1) = 1/5$

Baye's rule and Total probability

Basic Level

245. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing, is

(a) $\frac{37}{40}$ (b) $\frac{1}{37}$ (c) $\frac{36}{37}$ (d) $\frac{1}{9}$

246. Three urns contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black balls respectively. One of the urns is selected at random and a ball is drawn from it. If the ball drawn is red, the probability that it is drawn from the first urn is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

247. There are 3 bags, each containing 5 white balls and 3 black balls. Also there are 2 bags, each containing 2 white balls and 4 black balls. A white ball is drawn at random. The probability that this white ball is from a bag of the first group, is

(a) 2/63 (b) 45/61 (c) 2/49 (d) None of these

248. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be hearts. Find the probability of the missing card to be a heart

(a) $\frac{5}{9}$ (b) $\frac{6}{37}$ (c) $\frac{11}{50}$ (d) $\frac{13}{31}$

249. One bag contains four white balls and three black balls and a second bag contains three white balls and five black balls. One ball is drawn from the first bag and placed unseen in the second bag. The probability that a ball now drawn from second bag is black is

(a) $\frac{38}{65}$ (b) $\frac{38}{63}$ (c) $\frac{17}{65}$ (d) $\frac{1}{3}$

250. A real estate man has eight master keys to open several new homes. Only one master key will open any given house. If 40% of these homes are usually left unlocked, the probability that the real estate man can get into a specific home if he selects three master keys at random before leaving the office is

(a) $\frac{3}{8}$ (b) $\frac{7}{8}$ (c) $\frac{5}{8}$ (d) None of these

251. A coin is tossed 3 times by 2 persons. What is the probability that both get equal number of heads

(a) 3/8 (b) 1/9 (c) 5/16 (d) None of these

252. A bag x contains 3 white balls and 2 black balls and another bag y contains 2 white balls and 4 black balls. A bag and a ball out of it are picked at random. The probability that the ball is white is

(a) $\frac{3}{5}$ (b) $\frac{7}{15}$ (c) $\frac{1}{2}$ (d) None of these

253. The probability that in a year of the 22^{nd} century chosen at random there will be 53 Sundays is

(a) 3/28 (b) 2/28 (c) 7/28 (d) 5/28

254. If a coin be tossed n times then probability that the head comes odd times is **[Rajasthan PET 2002]**

(a) 1/2 (b) $1/2^n$ (c) $1/2^{n-1}$ (d) None of these

Advance Level

255. In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total bolts. Of their output 5, 4 and 2 percent respectively are defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, the probability that it is manufactured by the machine B is

(a) $\frac{28}{69}$ (b) $\frac{7}{69}$ (c) $\frac{32}{69}$ (d) $\frac{11}{69}$

256. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver and a truck driver is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver

(a) $\frac{1}{52}$ (b) $\frac{1}{62}$ (c) $\frac{2}{51}$ (d) 1

257. From an urn containing 3 white and 5 black balls, 4 balls are transferred into an empty urn. From this urn a ball is drawn and is found to be white. The probability that out of the four balls transferred, 3 are white and 1 black is

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$

258. In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct, given that he copied it, is 1/8. The probability that he knew the answer to the question, given that he correctly answered it, is

(a) $\frac{24}{27}$ (b) $\frac{24}{29}$ (c) $\frac{24}{31}$ (d) None of these

259. A company manufactures T.Vs at two different plants A and B. Plant 'A' produces 80% and B produces 20% of the total production. 85 out of 100 T.Vs produced at plant A meet the quality standards while 65 out of 100 T.Vs produced at plant B meet the quality standard. A T.V. produced by the company is selected at random and is not found of meeting the quality standard. The probability that selected T.V. was manufactured by the plant B is

(a) $\frac{7}{11}$ (b) $\frac{7}{19}$ (c) $\frac{2}{3}$ (d) None of these

Binomial distribution

Basic Level

260. A coin is tossed 3 times (**OR** Three coins are tossed all together). The probability of getting at least two heads is **[MP PET 1995]**

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

261. The probability of having at least one head in 3 throws with a coin is

(a) 7/8 (b) 3/8 (c) 1/8 (d) None of these

262. A fair coin is tossed n time. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then n is equal to
[Kurukshetra CEE 1998; AMU 2000]

(a) 15 (b) 14 (c) 12 (d) 7

263. The mean and variance of a binomial distribution are 4 and 3 respectively, then the probability of getting exactly six successes in this distribution is

(a) ${}^{16}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^6$ (b) ${}^{16}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^{10}$ (c) ${}^{12}C_6\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^6$ (d) ${}^{12}C_6\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^6$

264. In a binomial probability distribution, mean is 3 and standard deviation is $\frac{3}{2}$. Then the probability distribution is **[AISSE 1979]**

(a) $\left(\frac{3}{4}+\frac{1}{4}\right)^{12}$ (b) $\left(\frac{1}{4}+\frac{3}{4}\right)^{12}$ (c) $\left(\frac{1}{4}+\frac{3}{4}\right)^9$ (d) $\left(\frac{3}{4}+\frac{1}{4}\right)^9$

265. If X follows a binomial distribution with parameters $n=6$ and p and $4(P(X=4))=P(X=2)$, then $p=$

(a) 1/2 (b) 1/4 (c) 1/6 (d) 1/3

266. The mean and variance of a binomial distribution are 6 and 4. The parameter n is **[MP PET 2000]**

(a) 18 (b) 12 (c) 10 (d) 9

267. Suppose X follows a binomial distribution with parameters n and p, where $0<p<1$. If $\frac{P(X=r)}{P(X=n-r)}$ is independent of n and r, then

(a) $p=\frac{1}{2}$ (b) $p=\frac{1}{3}$ (c) $p=\frac{1}{4}$ (d) None of these

268. If x denotes the number of sixes in four consecutive throws of a dice, then $P(x=4)$ is

(a) 1/1296 (b) 4/6 (c) 1 (d) 1295/1296

269. The probability that an event will fail to happen is 0.05. The probability that the event will take place on 4 consecutive occasions is **[Roorkee 1990]**

(a) 0.00000625 (b) 0.18543125 (c) 0.00001875 (d) 0.81450625

270. A die is thrown three times. Getting a 3 or a 6 is considered success. Then the probability of at least two successes is **[DSSE 1981]**

(a) $\frac{2}{9}$ (b) $\frac{7}{27}$ (c) $\frac{1}{27}$ (d) None of these

271. Let p be the probability of happening an event and q its failure, then the total chance of r successes in n trials is **[MP PET 1999]**

(a) ${}^nC_{n+r}p^rq^{n-r}$ (b) ${}^nC_rp^{r-1}q^{r+1}$ (c) ${}^nC_rq^{n-r}p^r$ (d) ${}^nC_rp^{r+1}q^{r-1}$

272. In tossing 10 coins, the probability of getting exactly 5 heads is

(a) $\frac{9}{128}$ (b) $\frac{63}{256}$ (c) $\frac{1}{2}$ (d) $\frac{193}{256}$

273. Assuming that for a husband-wife couple the chances of their child being a boy or a girl are the same, the probability of their two children being a boy and a girl is

(a) $\frac{1}{4}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

274. The probability that a student is not a swimmer is 1/5. What is the probability that out of 5 students, 4 are swimmers **[DCE 1999]**

(a) ${}^5C_4\left(\frac{4}{5}\right)^4\frac{1}{5}$ (b) $\left(\frac{4}{5}\right)^4\frac{1}{5}$ (c) ${}^5C_1\frac{1}{5}\left(\frac{4}{5}\right)^4\times{}^5C_4$ (d) None of these

275. Three coins are tossed together, then the probability of getting at least one head is **[Rajasthan PET 2001]**
(a) 1/2 (b) 3/4 (c) 1/8 (d) 7/8

276. A bag contains 2 white and 4 black balls. A ball is drawn 5 times with replacement. The probability that at least 4 of the balls drawn are white is
(a) $\frac{8}{141}$ (b) $\frac{10}{243}$ (c) $\frac{11}{243}$ (d) $\frac{8}{41}$

277. A die is tossed 5 times. Getting an odd number is considered a success. Then the variance of distribution of success is **[AIEEE 2002]**
(a) 8/3 (b) 3/8 (c) 4/5 (d) 5/4

278. A coin is tossed 10 times. The probability of getting exactly six heads is
(a) 512/513 (b) 105/512 (c) 100/153 (d) ${}^{10}C_6$

279. An experiment succeeds twice as often as it fails. Find the probability that in 4 trials there will be at least three success **[AMU 1999]**
(a) 4/27 (b) 8/27 (c) 16/27 (d) 24/27

280. The records of a hospital show that 10% of the cases of a certain disease are fatal. If 6 patients are suffering from the disease, then the probability that only three will die is
(a) 1458×10^{-5} (b) 1458×10^{-6} (c) 41×10^{-6} (d) 8748×10^{-5}

281. If the probabilities of boy and girl to be born are same, then in a 4 children family the probability of being at least one girl, is
(a) $\frac{14}{16}$ (b) $\frac{15}{16}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

282. A committee has to be made of 5 members from 6 men and 4 women. The probability that at least one woman is present in committee, is
(a) $\frac{1}{42}$ (b) $\frac{41}{42}$ (c) $\frac{2}{63}$ (d) $\frac{1}{7}$

283. A die is tossed thrice. A success is getting 1 or 6 on a toss. The mean and the variance of number of successes **[AI CBSE**
(a) $\mu = 1, \sigma^2 = 2/3$ (b) $\mu = 2/3, \sigma^2 = 1$ (c) $\mu = 2, \sigma^2 = 2/3$ (d) None of these

284. A coin is tossed 4 times. The probability that at least one head turns up is **[MP PET 2000]**
(a) 1/16 (b) 2/16 (c) 14/16 (d) 15/16

285. If a dice is thrown twice, the probability of occurrence of 4 at least once is **[UPSEAT 2003]**
(a) 11/36 (b) 7/12 (c) 35/36 (d) None of these

286. In a binomial distribution the probability of getting a success is 1/4 and standard deviation is 3, then its mean is **[EAMCET 2002]**
(a) 6 (b) 8 (c) 12 (d) 10

287. If two coins are tossed 5 times, then the probability of getting 5 heads and 5 tails is **AMU 2002]**
(a) $\frac{63}{256}$ (b) $\frac{1}{1024}$ (c) $\frac{2}{205}$ (d) $\frac{9}{64}$

288. 6 ordinary dice are rolled. The probability that at least half of them will show at least 3 is
(a) $41 \times \frac{2^4}{3^6}$ (b) $\frac{2^4}{3^6}$ (c) $20 \times \frac{2^4}{3^6}$ (d) None of these

289. A fair die is tossed eight times. Probability that on the eighth throw a third six is observed is
(a) ${}^8C_3\frac{5^5}{6^8}$ (b) $\frac{{}^7C_2.5^5}{6^8}$ (c) $\frac{{}^7C_2.5^5}{6^7}$ (d) None of these

290. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, the probability of getting two heads is
(a) $15/2^8$ (b) $2/15$ (c) $15/2^{13}$ (d) None of these

291. The probability that a candidate secures a seat in Engineering through "EAMCET" is 1/10. 7 candidates are selected at random from a centre. The probability that exactly two will get seats is

(a) $15(0.1)^2(0.9)^5$ (b) $20(0.1)^2(0.9)^5$ (c) $21(0.1)^2(0.9)^5$ (d) $23(0.1)^2(0.9)^5$

292. The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target at least three times is

[MNR 1994]

(a) 291/364 (b) 371/464
(c) 471/502 (d) 459/512

293. A fair coin is tossed 100 times. The probability of getting tails an odd number of times is

(a) 1/2 (b) 1/8 (c) 3/8 (d) None of these

294. A coin is tossed 7 times. Each time a man calls head. The probability that he wins the toss on more occasions is

(a) $\frac{1}{4}$ (b) $\frac{5}{8}$ (c) $\frac{1}{2}$ (d) None of these

295. A man draws a card from a pack of 52 cards and then replaces it. After shuffling the pack, he again draws a card. This he repeats a number of times. The probability that he will draw a heart for the first time in the third draw is

(a) $\frac{9}{64}$ (b) $\frac{27}{64}$ (c) $\frac{1}{4}\times\frac{^{39}C_2}{^{52}C_2}$ (d) None of these

Advance Level

296. A fair coin is tossed n times. Let X be the number of times head is observed. If $P(X=4), P(X=5)$ and $P(X=6)$ are in H.P., then n is equal to

(a) 7 (b) 10 (c) 14 (d) None of these

297. Five coins whose faces are marked 2, 3 are tossed. The chance of obtaining a total of 12 is

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$ (c) $\frac{3}{16}$ (d) $\frac{5}{16}$

298. A coin is tossed $2n$ times. The chance that the number of times one gets head is not equal to the number of times one gets tail is

[DCE 2002]

(a) $\frac{(2n!)}{(n!)^2}\left(\frac{1}{2}\right)^{2n}$ (b) $1-\frac{(2n!)}{(n!)^2}$ (c) $1-\frac{(2n!)}{(n!)^2}.\frac{1}{4^n}$ (d) None of these

299. A coin is tossed n times. The probability of getting head at least once is greater than 0.8, then the least value of n is **[EAMCET 2003]**

(a) 2 (b) 3 (c) 4 (d) 5

300. A box contains 24 identical balls, of which 12 are white and 12 are black. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the 4^{th} time on the 7^{th} draw is **[IIT Screening 1994]**

(a) 5/64 (b) 27/32 (c) 5/32 (d) 1/2

301. A die is tossed twice. Getting a number greater than 4 is considered a success. Then the variance of the probability distribution of the number of successes is

(a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{1}{3}$ (d) None of these

302. In order to get at least once a head with probability ≥ 0.9, the number of times a coin needs to be tossed is **[Roorkee 19**

(a) 3 (b) 4 (c) 5 (d) None of these

303. India plays two matches each with West Indies and Australia. In any match the probabilities of India getting point 0, 1 and 2 are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is **[IIT 1992]**

(a) 0.8750 (b) 0.0875 (c) 0.0625 (d) 0.0250

304. In a box of 10 electric bulbs, two are defective. Two bulbs are selected at random one after the other from the box. The first bulb after selection being put back in the box before making the second selection. The probability that both the bulbs are without defect is

[MP PET 1987]

(a) 9/25 (b) 16/25 (c) 4/5 (d) 8/25

305. If the mean and variance of a binomial variate *X* are 2 and 1 respectively, then the probability that *X* takes a value greater than 1, is

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) $\frac{7}{8}$ (d) $\frac{15}{16}$

306. A die is tossed thrice. If getting a four is considered a success, then the mean and variance of the probability distribution of the number of successes are

(a) $\frac{1}{2}, \frac{1}{12}$ (b) $\frac{1}{6}, \frac{5}{12}$ (c) $\frac{5}{6}, \frac{1}{2}$ (d) None of these

307. Suppose *A* and *B* shoot independently until each hits his target. They have probabilities 3/5, 5/7 of hitting the targets at each shot. The probability that *B* will require more shots than *A* is

(a) 6/31 (b) 7/31 (c) 8/31 (d) None of these

308. A fair coin is tossed *n* times. Let *X* be the number of times head occurs. If $P(X=4), P(X=5)$ and $P(X=6)$ are in A.P., then value of *n* is

(a) 7 (b) 10 (c) 12 (d) 14

309. In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. The minimum number of bombs which should be dropped to give a 99% chance or better of completely destroying the target is

(a) 10 (b) 11 (c) 12 (d) None of these

310. If the mean of a binomial distribution is 25, then its standard deviation lies in the interval given below

(a) [0, 5) (b) (0, 5] (c) [0, 25) (d) (0, 25]

311. If *n* integers taken at random are multiplied together, then the probability that the last digit of the product is 1, 3, 7 or 9 is

(a) $\frac{2^n}{5^n}$ (b) $\frac{8^n - 2^n}{5^n}$ (c) $\frac{4^n - 2^n}{5^n}$ (d) None of these

312. A bag contains 14 balls of two colours, the number of balls of each colour being the same. 7 balls are drawn at random one by one. The ball in hand is returned to the bag before each new draw. If the probability that at least 3 balls of each colour are drawn is *p* then

(a) $p > \frac{1}{2}$ (b) $p = \frac{1}{2}$ (c) $p < 1$ (d) $p < \frac{1}{2}$

313. An ordinary dice is rolled a certain number of times. The probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times. Then the probability of getting an odd number an odd number of times is

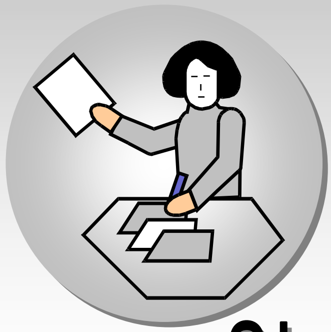
(a) $\frac{1}{32}$ (b) $\frac{5}{16}$ (c) $\frac{1}{2}$ (d) None of these

314. The probability of a bomb hitting a bridge is $\frac{1}{2}$ and two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9, is

(a) 8 (b) 7 (c) 6 (d) 9

315. All the spades are taken out from a pack of cards. From these cards, cards are drawn one by one without replacement till the ace of spade comes. The probability that the ace comes in the 4th draw is

(a) $\frac{1}{13}$ (b) $\frac{12}{13}$ (c) $\frac{4}{13}$ (d) None of these



Answer Sheet

Probability — ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
d	c	b	b	b	c	d	b	d	b	a	d	a	b	b	c	d	a	a	b
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
d	b	c	c	c	b	a	c	c	c	b	b	d	b	c	b	b	c	b	b
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
a	d	b	a	c	b	b	b	d	c	b	b	b	a	b	c	b	c	b	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
a	a	c	a	a	a	a	a	c	d	b	b	a	a	b	b	d	a	c	b
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a,b	b	c	b	c	d	a	b	b	a	b	d	a,b,c	a,b,c,d	a,b,c,d	a,c	b	d	c	b
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**	**116**	**117**	**118**	**119**	**120**
a	a	a	a,c	b	b	b	b	c	d	a	c	d	c	a	b	c	b	b	c
121	**122**	**123**	**124**	**125**	**126**	**127**	**128**	**129**	**130**	**131**	**132**	**133**	**134**	**135**	**136**	**137**	**138**	**139**	**140**
c	c	b	b	d	b	c	c	a	c	a	d	b	d	b	b	a	b	b	a
141	**142**	**143**	**144**	**145**	**146**	**147**	**148**	**149**	**150**	**151**	**152**	**153**	**154**	**155**	**156**	**157**	**158**	**159**	**160**
b	c	b	c	d	c	d	c	c	d	a	c	d	b	c	a	b	c	c	a
161	**162**	**163**	**164**	**165**	**166**	**167**	**168**	**169**	**170**	**171**	**172**	**173**	**174**	**175**	**176**	**177**	**178**	**179**	**180**
c	d	d	a	a	a	b	a	c,d	d	b	a	d	c	b	d	a,b,c	c	d	d
181	**182**	**183**	**184**	**185**	**186**	**187**	**188**	**189**	**190**	**191**	**192**	**193**	**194**	**195**	**196**	**197**	**198**	**199**	**200**
a	b	d	a	d	c	b	a	c	a	a,b,c,d	a	b	b	a	a	a	a	c	c
201	**202**	**203**	**204**	**205**	**206**	**207**	**208**	**209**	**210**	**211**	**212**	**213**	**214**	**215**	**216**	**217**	**218**	**219**	**220**
a	b	c	a	d	d	c	c	d	d	a	c	a	d	a	c	c	b	d	d
221	**222**	**223**	**224**	**225**	**226**	**227**	**228**	**229**	**230**	**231**	**232**	**233**	**234**	**235**	**236**	**237**	**238**	**239**	**240**
a	b	b	a	d	c	a	a,b,c,d	b	a	c	c	c	a	d	a,d	a	b	c,d	d
241	**242**	**243**	**244**	**245**	**246**	**247**	**248**	**249**	**250**	**251**	**252**	**253**	**254**	**255**	**256**	**257**	**258**	**259**	**260**
d	d	a	a,b,c,d	b	c	b	c	b	c	c	b	d	a	a	a	d	b	b	c
261	**262**	**263**	**264**	**265**	**266**	**267**	**268**	**269**	**270**	**271**	**272**	**273**	**274**	**275**	**276**	**277**	**278**	**279**	**280**
a	b	b	a	d	a	a	a	d	b	c	b	c	a	d	c	d	b	c	a
281	**282**	**283**	**284**	**285**	**286**	**287**	**288**	**289**	**290**	**291**	**292**	**293**	**294**	**295**	**296**	**297**	**298**	**299**	**300**
b	b	a	d	a	c	a	a	b	c	c	d	a	c	a	d	d	c	b	c
301	**302**	**303**	**304**	**305**	**306**	**307**	**308**	**309**	**310**	**311**	**312**	**313**	**314**	**315**					
b	b	b	b	d	d	a	a	b	a	a	a	c	a	a					