Electromagnetic Induction

basic concepts

1. Electromagnetic Induction

The phenomenon of generation of induced emf and induced current due to change in magnetic field lines associated with a closed circuit is called electromagnetic induction.

2. Magnetic Flux

Magnetic flux through a surface of area A placed in a uniform magnetic field is $\phi_m = \vec{B}.\vec{A} = BA\cos\theta$, θ being angle between $\vec{B}$ and normal to $\vec{A}$. If magnetic field is not uniform, then $\phi_m = \int_A \vec{B}.d\vec{A}$, where integral extends for whole area A.

The SI unit of magnetic flux is weber. Magnetic flux is a scalar quantity; because of being scalar product of two vectors $\vec{B}$ and $\vec{A}$.

3. Faraday's Laws of Electromagnetic Induction

(*i*) Whenever there is a change in magnetic flux linked with a coil, an emf is induced in the coil. The induced emf is proportional to the rate of change of magnetic flux linked with the coil.

i.e., $\varepsilon \propto \frac{\Delta\phi}{\Delta t}$

(*ii*) emf induced in the coil opposes the change in flux, *i.e.*,

$$\varepsilon \propto -\frac{\Delta\phi}{\Delta t} \Rightarrow \varepsilon = -k\frac{\Delta\phi}{\Delta t}$$

where k is a constant of proportionality.

Negative sign represents opposition to change in flux.

In SI system ϕ is in weber, t in second, ε in volt, when $k = 1, \varepsilon = -\frac{\Delta\phi}{\Delta t}$

If the coil has N-turns, then $\varepsilon = -N\frac{\Delta\phi}{\Delta t}$

4. Induced Current and Induced Charge

If a coil is closed and has resistance R, then current induced in the coil,

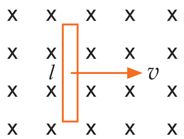
$$I = \frac{\varepsilon}{R} = -\frac{N}{R}\frac{\Delta\phi}{\Delta t}$$

Induced charge, $q = I\,\Delta t = -\frac{N\Delta\phi}{R} = \frac{\text{Total flux linkage}}{\text{Resistance}}$

5. Lenz's Law

It states that the direction of induced emf is such that it tends to produce a current which opposes the change in magnetic flux producing it.

6. EMF Induced in a Moving Conducting Rod



EMF induced in a conducting rod of length l moving with velocity v in a magnetic field of induction B, such that B, l and v are mutually perpendicular, is given by

$$\varepsilon = Bvl$$

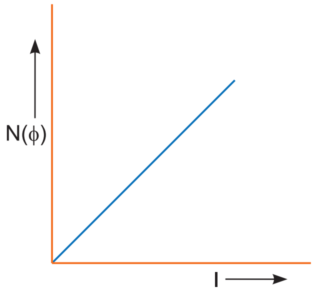
force required to keep the rod in constant motion is $F = BIL = \frac{B^2 l^2 v}{r}$

7. Self Induction

When the current in a coil is changed, an induced emf is produced in the same coil. This phenomenon is called self-induction. If L is self-inductance of coil, then

$$N\phi \propto I \quad \text{or} \quad N\phi = LI \Rightarrow L = \frac{N\phi}{I}$$

L is also called coefficient of self induction.



The graph between effective magnetic flux ($N\phi$) and current I is straight line of slope self inductance L.

Also induced emf $\varepsilon = -L\frac{\Delta I}{\Delta t}$

The unit of self inductance is henry (H). The self induction acts as inertia in electrical circuits; so it is also called electrical inertia.

The self inductance of a solenoid consisting core of relative permeability μ_r is $L = \mu_r \mu_0 n^2 A l$

where $n = \frac{N}{l}$ is the number of turns per metre length.

8. Mutual Induction

When two coils are placed nearby and the current in one coil (often called primary coil) is changed, the magnetic flux linked with the neighbouring coil (often called secondary coil) changes; due to which an emf is induced in the neighbouring coil. This effect is called the mutual induction. If M is mutual inductance of two coils, then $\phi_2 \propto I_1$ or $\phi_2 = MI_1$

Definition of mutual inductance: $M = \frac{\phi_2}{I_1}$.

The mutual inductance of two coils is defined as the magnetic flux linked with the secondary coil when the current in primary coil is 1 ampere.

Also induced emf in secondary coil $\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t} \Rightarrow M = \frac{\varepsilon_2}{\Delta I_1/\Delta t}$.

The mutual inductance of two coils is defined as the emf induced in the secondary coil when the rate of change of current in the primary coil is 1 A/s.

The SI unit of mutual inductance is also henry (H). The mutual inductance of two coils does not depend on the fact which coil carries the current and in which coil emf is induced *i.e.*, $M_{12} = M_{21} = M$

This is also called reciprocity theorem of mutual inductance.

If L_1 and L_2 are self-inductances of two coils with 100% flux linkage between them, then

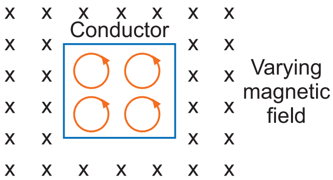
$M = \sqrt{L_1 L_2}$, otherwise $M = k\sqrt{L_1 L_2}$, where k is coefficient of flux linkage between the coils.

Mutual Inductance of solenoid-coil system

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

where A is area of coil, l is length of solenoid, N_1 is number of turns in solenoid and N_2 is number of turns in coil.

9. Eddy Currents



When a thick piece of a conductor is placed in a varying magnetic field the magnetic flux linked with the conductor changes, so currents are induced in the body of conductor, which causes heating of conductor.

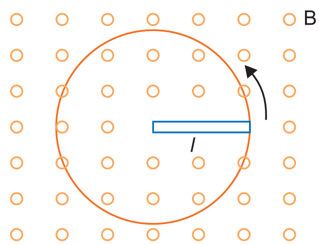
The currents induced in the conductor are called the eddy currents. In varying magnetic field, the free electrons of conductor experience Lorentz force and traverse closed paths; which are equivalent to small current loops. These currents are the eddy currents; they cause heating effect and sometimes the conductor becomes red-hot.

Eddy current losses may be reduced by using laminated soft iron cores in galvanometers, transformers, etc., and making holes in the core. Few of the application of eddy currents is in induction furnace, induction motor and many more.

Selected NCERT Textbook Questions

Induced emf

Q. 1. A 1.0 m metallic rod is rotated with an angular velocity of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.



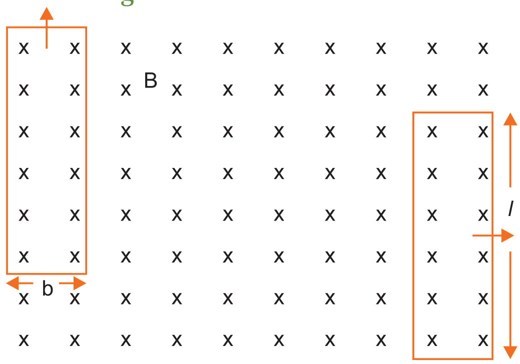
Ans. EMF developed between the centre of ring and the point on the ring.

$$\varepsilon = \frac{1}{2}B\omega l^2$$

Given $B = 0.5$ T, $\omega = 400$ rad/s, $l = 1.0$ m.

$$\therefore \quad \varepsilon = \frac{1}{2} \times 0.5 \times 400 \times (1.0)^2 = \textbf{100 volt}$$

Q. 2. A rectangular wire loop of sides 8 cm × 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed if the velocity of the loop is 1 cms^{-1} in a direction normal to the (*i*) longer side (*ii*) shorter side of the loop? For how long does the induced voltage last in each case?



Ans. Given $l = 8 \text{ cm} = 8 \times 10^{-2}$ m,

$b = 2 \text{ cm} = 2 \times 10^{-2}$ m

$v = 1 \text{ cm s}^{-1} = 1 \times 10^{-2}$ m/s, $B = 0.3$ T

(*i*) When velocity is normal to the longer side

Induced emf, $\varepsilon = Bvl$

$$= 0.3 \times 1 \times 10^{-2} \times 8 \times 10^{-2}$$
$$= \mathbf{24 \times 10^{-5}\ V}$$

emf will last only so long as the loop is in the magnetic field.

$$\text{Time taken} = \frac{\text{distance}}{\text{velocity}} = \frac{b}{v} = \frac{2 \times 10^{-2}}{1 \times 10^{-2}} = \mathbf{2\ s}$$

(*ii*) When velocity is normal to the shorter side

$$\varepsilon_2 = Bvb$$
$$= 0.3 \times 1 \times 10^{-2} \times 2 \times 10^{-2} = \mathbf{6 \times 10^{-5}\ V}$$

$$\text{Time taken} = \frac{l}{v} = \frac{8 \times 10^{-2}}{1 \times 10^{-2}} = \mathbf{8\ s}$$

Q. 3. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of earth's magnetic field equal to 0.30 ×10^{-4} Wbm^{-2}.

(*a*) What is the instantaneous value of the emf induced in the wire?

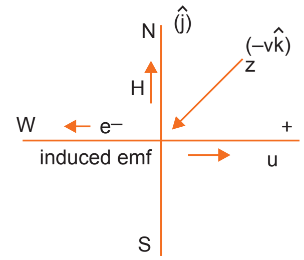
(*b*) What is the direction of emf?

(*c*) Which emf of the wire is at the higher electrical potential ?

Ans. (*a*) Instantaneous emf, $\varepsilon = B_n vl = Hvl$

Given, $H = 0.30 \times 10^{-4}$ T, $v = 5.0$ ms^{-1}, $l = 10$ m

$\therefore \quad \varepsilon = 0.30 \times 10^{-4} \times 5.0 \times 10 = 1.5 \times 10^{-3}$ V = **1.5 mV**



(*b*) By Fleming's right hand rule, the direction of induced current in wire is from west to east, therefore, direction of emf is from west to east.

(*c*) The direction of electron flow according to relation $\vec{F}_m = q\vec{v} \times \vec{B} = -e(-v\hat{k}) \times (B\hat{j}) = -evB\hat{i}$

i.e., along negative *x*-axis, *i.e.*, from east to west.

The induced emf will oppose the flow of electrons from east to west, so eastern end will be at higher potential.

Q. 4. A jet plane is travelling westward at a speed of 1800 km/h. What is the potential difference developed between the ends of a wing 25 m long? Its earth's magnetic field at the location has a magnitude of 5.0 $\times 10^{-4}$ T and the dip angle is 30°. **[*CBSE (AI) 2009*]**

Ans. The wing of horizontal travelling plane will cut the vertical component of earth's magnetic field, so emf is induced across the wing. The vertical component of earth's field is given by

$V = B_e \sin\theta$; where B_e is earth's magnetic field and θ is angle of dip

Induced emf of wing $\varepsilon = V\,v\,l = (B_e \sin\theta)\,v\,l$

Given $B_e = 5.0 \times 10^{-4}$ T, $l = 25$ m, $\theta = 30°$,

$$v = 1800\text{ km/h} = 1800 \times \frac{5}{18}\text{ m/s} = 500\text{ m/s}$$

$$\therefore \quad \varepsilon = (5.0 \times 10^{-4} \times \sin 30°) \times 500 \times 25$$
$$= (5.0 \times 10^{-4} \times 0.5) \times 500 \times 25 = \mathbf{3.1\ V}$$

Induced emf and Power

Q. 5. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad/s in a uniform horizontal magnetic field of magnitude 3.0 × 10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to joule heating. Where does the power come from?

Ans. Magnetic flux linked with the coil, $\phi = \vec{B} \cdot \vec{A}$

$$= NBA\cos\theta = NBA\cos\omega t \text{ (where } \theta = \omega t)$$

EMF induced in the coil $\varepsilon = -N\dfrac{d\phi}{dt}$

$$= -N\frac{d}{dt}(BA\cos\omega t) = NBA\,\omega\sin\omega t$$

Maximum emf induced $\varepsilon_{max} = NBA\omega = NB(\pi r^2)\,\omega$

Given $N = 20$, $r = 8.0$ cm $= 8.0 \times 10^{-2}$ m, $B = 3.0 \times 10^{-2}$ T, $\omega = 50$ rad/s

$$\therefore \quad \varepsilon_{max} = 20 \times 3.0 \times 10^{-2} \times 3.14 \times (8.0 \times 10^{-2})^2 \times 50$$
$$= \mathbf{0.603\ volt}$$

Average emf $= NBA\,\omega\,(\sin\omega t)_{av} = \mathbf{0}$

(Since average value of sin ωt over a complete cycle is zero.)

Maximum current induced,

$$I_{max} = \frac{\varepsilon_{max}}{R} = \frac{0.603}{10} = \mathbf{0.0603\ A}$$

Average power loss due to joule heating

$$P_{max} = (I^2)_{av} \; R = \frac{(\varepsilon^2)_{av}}{R}$$

Since average value of $\sin^2 \omega t$ for a complete cycle is $\frac{1}{2}$, *i.e.*, $(\sin^2 \omega t)_{av} = \frac{1}{2}$

$$\therefore \quad P_{max} = \frac{1}{2}\frac{N^2B^2A^2\omega^2}{R}$$

$$= \frac{1}{2}(NBA\omega)\left(\frac{NBA\omega}{R}\right) = \frac{1}{2}\varepsilon_{max} I_{max}$$

$$= \frac{1}{2} \times 0.603 \times 0.0603 = \mathbf{0.018\ W}$$

The current induced causes a torque which opposes the rotation of the coil. An external agency (rotor) must supply torque to counter this torque in order to keep the coil rotating uniformly. The source of power dissipated as heat is the rotor.

Q. 6. A rectangular loop of sides 8 cm × 2 cm with a small cut is stationary in a uniform magnetic field produced by an electromagnet. If the current feeding the electromagnet is gradually reduced so that the magnetic field decreases from its initial value of 0·3 T at the rate of 0·02 Ts^{-1}. If the cut is joined and the loop has a resistance of 1·6 Ω, how much power is dissipated by the loop as heat ? What is the source of this power?

Ans. Area of loop, $A = 8\text{ cm} \times 2\text{ cm} = 16\text{ cm}^2 = 16 \times 10^{-4}\text{ m}^2$

Induced emf, $\varepsilon = -\frac{\Delta\phi}{\Delta t} = -\frac{\Delta}{\Delta t}(BA) = -A\frac{\Delta B}{\Delta t}$

Here, $\frac{\Delta B}{\Delta t} = -0.02\text{ Ts}^{-1}$

$\therefore$ Induced emf, $\varepsilon = -(16 \times 10^{-4}) \times (-0.02) = 3.2 \times 10^{-5}\text{ V}$

Induced current, $I = \frac{\varepsilon}{R} = \frac{3.2 \times 10^{-5}}{1.6} = 2 \times 10^{-5}\text{ A}$

Power dissipated, $P = I^2R = (2 \times 10^{-5})^2 \times 1.6 = \mathbf{6.4 \times 10^{-10}\ W}$

The source of the power is the external source feeding the electromagnet

Self Inductance and Mutual Inductance

Q. 7. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, calculate the self-induction of the circuit. [*CBSE (F) 2011*]

Ans. Induced emf $\quad E = -L\frac{\Delta I}{\Delta t}$ ...(*i*)

Here, $E = 200$ V,

$$\frac{\Delta I}{\Delta t} = \frac{I_2 - I_1}{\Delta t} = \frac{0.0 - 5.0}{0.1} = -50\text{ A/s}$$

$\therefore$ Substituting these values in (*i*), we get

$$L = \frac{E}{(-\Delta I/\Delta t)} = \frac{200}{50} = \mathbf{4H}$$

Q. 8. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside normal to the axis of the solenoid. The current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced emf in the loop while the current is changing? [*CBSE (F) 2016*]

Ans. Mutual inductance of solenoid coil system

$$M = \frac{\mu_0 N_1 N_2 A_2}{l}$$

Here $\quad N_1 = 15, N_2 = 1, l = 1\text{ cm} = 10^{-2}\text{ m}, A_2 = 2.0\text{ cm}^2 = 2.0 \times 10^{-4}\text{m}^2$

$$\therefore \quad M = \frac{4\pi \times 10^{-7} \times 15 \times 1 \times 2.0 \times 10^{-4}}{10^{-2}}$$

$$= 120\,\pi \times 10^{-9}\text{ H}$$

Induced emf, in the loop

$$\varepsilon_2 = M\frac{\Delta I_1}{\Delta t} \qquad \text{(numerically)}$$

$$= 120\,\pi \times 10^{-9}\frac{(4-2)}{0.1}$$

$$= 120 \times 3.14 \times 10^{-9} \times \frac{2}{0.1} = 7.5 \times 10^{-6}\text{ V} = \mathbf{7.5\ \mu V}$$

Q. 9. An air cored solenoid with length 30 cm, area of cross-section 25 cm^2 and number of turns 500 carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s. How much is the average back emf induced across the ends of the open switch in the circuit ? Ignore the variation in magnetic field near the ends of the solenoid.

Ans. Induced emf in a solenoid, $\varepsilon = -L\frac{\Delta I}{\Delta t}$ ...(*i*)

Inductance of solenoid $L = \frac{\mu_0 N^2 A}{l}$ ...(*ii*)

∴ Induced emf $\varepsilon = -\left(\frac{\mu_0 N^2 A}{l}\right)\frac{\Delta I}{\Delta t}$

Here $N = 500$, A = 25 cm^2, = 25 × 10^{-4} m^2, l=30 cm =0.30 m and

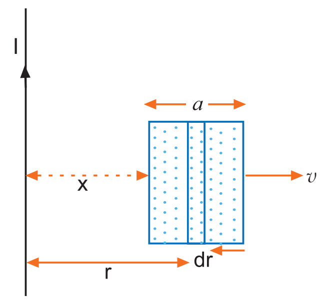
$$\frac{\Delta I}{\Delta t} = \frac{I_2 - I_1}{t} = \frac{0-2.5}{10^{-3}} = -2.5 \times 10^3\text{ A/s}$$

$$\therefore \quad \varepsilon = -\frac{4\pi \times 10^{-7} \times (500)^2 \times 25 \times 10^{-4}}{0.30} \times (-2.5 \times 10^3)$$

$$= \frac{3.14 \times 25 \times 2.5}{3} \times 10^{-1} = \mathbf{6.5\ V}$$

Q. 10. (*a*) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side '*a*' as shown in fig.

(*b*) Evaluate the induced emf in the loop if the wire carries a current of 50 A and the loop has an instantaneous velocity v =10 ms^{-1} at the location x = 0.2 m as shown. Take a = 0.1 m and assume that the loop has a large resistance.



Ans. (*a*) Suppose the loop is formed of a number of small elements parallel to the length of wire. Consider an element of width dr at a distance r from the wire. The magnetic field at the vicinity of wire, $B = \frac{\mu_0 I}{2\pi r}$ downward perpendicular to the plane of paper.

The magnetic flux linked with this element $\phi_2 = \left|\vec{B}\,.\,d\,\vec{A}_2\right|$

$$= \left|B\,dA_2 \cos\pi\right| = \frac{\mu_0 I}{2\pi r}(a\,dr)$$

$$= \frac{\mu_0 Ia}{2\pi}\frac{dr}{r}$$

Total magnetic flux linked with the loop, $\phi_2 = \frac{\mu_0 Ia}{2\pi}\int_x^{x+a}\frac{dr}{r}$

$$= \frac{\mu_0 Ia}{2\pi}\left[\log_e r\right]_x^{x+a} = \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{x+a}{x}\right)$$

$$\therefore \text{ Mutual inductance, } M = \frac{\phi_2}{I} = \frac{\mu_0 a}{2\pi}\log_e\left(1+\frac{a}{x}\right)$$

(*b*) The square loop is moving in non-uniform magnetic field. The magnetic flux linked with the loop at any instant is

$$\phi = \frac{\mu_0 I_a}{2\pi} \log_e\left(1 + \frac{a}{x}\right)$$

Induced emf set up in the loop,

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d\phi}{dx}\cdot\frac{dx}{dt} = -v\frac{d\phi}{dx}$$

$$= -v\frac{d}{dx}\left[\frac{\mu_0 I_a}{2\pi}\log_e\left(1+\frac{a}{x}\right)\right]$$

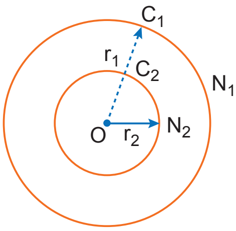
$$= -v.\frac{\mu_0 I_a}{2\pi}.\log_e\frac{1}{\left(1+\frac{a}{x}\right)}.\left(-\frac{a}{x^2}\right) = \frac{\mu_0}{2\pi}.\frac{a^2 v}{x(x+a)}.I$$

$$= \frac{4\pi\times10^{-7}}{2\pi}\times\frac{(0.1)^2\times10}{0.2(0.2+0.1)}\times50$$

$$= \mathbf{1.67 \times 10^{-5}\ V \simeq 1.7 \times 10^{-5}\ V.}$$

Q. 11. Two concentric circular coils, one of small radius r_2 and the other of large radius r_1, such that $r_2 << r_1$ are placed co-axially with centres coinciding. Obtain the mutual inductance of the arrangement. ***[CBSE Chennai 2015]***

Ans. Mutual Inductance of two plane coils: Consider two concentric circular plane coils C_1 and C_2 placed very near to each other. The number of turns in the primary coil is N_1 and radius is r_1 while the number of turns in the secondary coil is N_2 and its radius is r_2. If I_1 is the current in the primary coil, then magnetic field produced at its centre,



$$B_1 = \frac{\mu_0 N_1 I_1}{2r_1} \qquad \ldots (i)$$

If we suppose this magnetic field to be uniform over the entire plane of secondary coil, then total *effective magnetic flux linkage* with secondary coil

$$\phi_2 = N_2 B_1 A_2 = N_2\left(\frac{\mu_0 N_1 I_1}{2r_1}\right)A_2 = \frac{\mu_0 N_1 N_2 A_2}{2r_1}I_1$$

By definition, Mutual Inductance, $M = \dfrac{\phi_2}{I_1} = \dfrac{\mu_0 N_1 N_2 A_2}{2r_1}$

But $A_2 = \pi r_2^2 \quad \therefore\ M = \dfrac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$

Special case: If both coils have one turn each; then $N_1 = N_2 = 1$, so mutual inductance $M = \dfrac{\mu_0 \pi r_2^2}{2r_1}$

Multiple Choice Questions [1 mark]

Choose and write the correct option(s) in the following questions.

1. Whenever the flux linked with a circuit changes, there is an induced emf in the circuit. This emf in the circuit lasts

(*a*) for a very short duration

(*b*) for a long duration

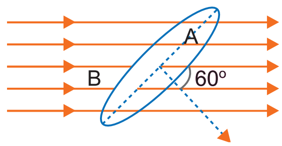
(*c*) forever

(*d*) as long as the magnetic flux in the circuit changes.

2. The area of a square shaped coil is 10^{-2} m^2. Its plane is perpendicular to a magnetic field of strength 10^{-3} T. The magnetic flux linked with the coil is

(*a*) 10 Wb (*b*) 10^{-5} Wb (*c*) 10^{5} Wb (*d*) 100 Wb

3. **An area $A = 0.5\ m^2$ shown in the figure is situated in a uniform magnetic field $B = 4.0\ Wb/m^2$ and its normal makes an angle of 60° with the field. The magnetic flux passing through the area A would be equal to**



(*a*) 2.0 weber (*b*) 1.0 weber (*c*) $\sqrt{3}$ weber (*d*) 0.5 weber

4. **A square of side L meters lies in the *X-Y* plane in a region, where the magnetic field is given by $B = B_o(2\hat{i} + 3\hat{j} + 4\hat{k})\,T$, where B_o is constant. The magnitude of flux passing through the square is** *[NCERT Exemplar]*

(*a*) $2\,B_o\,L^2$ Wb (*b*) $3\,B_o\,L^2$ Wb (*c*) $4\,B_o\,L^2$ Wb (*d*) $\sqrt{29}\,B_o L^2$ Wb

5. **A loop, made of straight edges has six corners at $A(0, 0, 0)$, $B(L, O, 0)$, $C(L, L, 0)$, $D(0, L, 0)$ $E(0, L, L)$ and $F(0, 0, L)$. A magnetic field $B = B_o(\hat{i} + \hat{k})\,T$ is present in the region. The flux passing through the loop *ABCDEFA* (in that order) is** *[NCERT Exemplar]*

(*a*) $B_o\,L^2$ Wb (*b*) $2\,B_o\,L^2$ Wb (*c*) $\sqrt{2}\,B_o L^2$ Wb (*d*) $4\,B_o\,L^2$ Wb

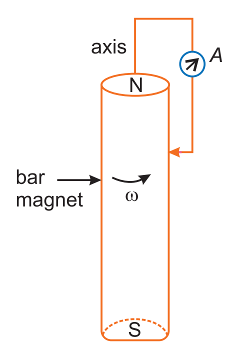
6. **An emf is produced in a coil, which is not connected to an external voltage source. This can be due to** *[NCERT Exemplar]*

(*a*) the coil being in a time varying magnetic field.
(*b*) the coil moving in a time varying magnetic field.
(*c*) the coil moving in a constant magnetic field.
(*d*) the coil is stationary in external spatially varying magnetic field, which does not change with time.

7. **A magnet is dropped with its north pole towards a closed circular coil placed on a table then**

(*a*) looking from above, the induced current in the coil will be anti-clockwise.
(*b*) the magnet will fall with uniform acceleration.
(*c*) as the magnet falls, its acceleration will be reduced.
(*d*) no current will be induced in the coil.

8. **A cylindrical bar magnet is rotated about its axis (Figure given alongside). A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then** *[NCERT Exemplar]*

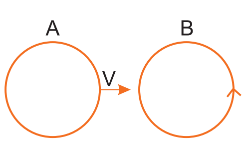


(*a*) a direct current flows in the ammeter A.
(*b*) no current flows through the ammeter A.
(*c*) an alternating sinusoidal current flows through the ammeter A with a time period $T=2\pi/\omega$.
(*d*) a time varying non-sinusoidal current flows through the ammeter A.

9. **A copper ring is held horizontally and a magnet is dropped through the ring with its length along the axis of the ring. The acceleration of the falling magnet is**

(*a*) equal to that due to gravity
(*b*) less than that due to gravity
(*c*) more than that due to gravity
(*d*) depends on the diameter of the ring and the length of the magnet

10. **There are two coils A and B as shown in the figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter clockwise. B is kept stationary when A moves. We can infer that** *[NCERT Exemplar]*

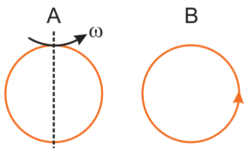


(*a*) there is a constant current in the clockwise direction in A.
(*b*) there is a varying current in A.

(*c*) there is no current in A.

(*d*) there is a constant current in the counterclockwise direction in A.

11. Same as the above problem except the coil A is made to rotate about a vertical axis refer to the figure. No current flows in B if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counterclockwise and the coil A is as shown at this instant, $t = 0$, is [*NCERT Exemplar*]



(*a*) constant current clockwise.

(*b*) varying current clockwise.

(*c*) varying current counterclockwise.

(*d*) constant current counterclockwise.

12. Lenz's law is essential for

(*a*) conservation of energy (*b*) conservation of mass

(*c*) conservation of momentum (*d*) conservation of charge

13. The self inductance L of a solenoid of length l and area of crosssection A, with a fixed number of turns N increases as [*NCERT Exemplar*]

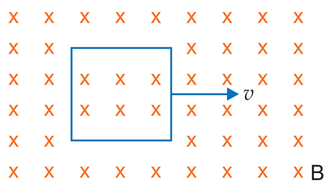
(*a*) l and A increase. (*b*) l decreases and A increases.

(*c*) l increases and A decreases. (*d*) both l and A decrease.

14. A thin circular ring of area A is held perpendicular to a uniform magnetic field of induction B. A small cut is made in the ring and a galvanometer is connected across its ends in such a way that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is

(*a*) $\frac{BR}{A}$ (*b*) $\frac{AB}{R}$ (*c*) ABR (*d*) $\frac{B^2A}{R^2}$

15. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane of the loop exists everywhere as in given figure. The current induced in the loop is



(*a*) Blv/R clockwise (*b*) Blv/R anticlockwise

(*c*) $2\ Blv/R$ anticlockwise (*d*) zero.

16. Inductance plays the role of

(*a*) inertia (*b*) friction (*c*) source of emf (*d*) force

17. A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because [*NCERT Exemplar*]

(*a*) the magnetic field is constant.

(*b*) the magnetic field is in the same plane as the circular coil and it may or may not vary.

(*c*) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably.

(*d*) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction.

18. When the current in a coil changes from 8A to 2A in 3×10^{-2} second, the emf induced in the coil is 2 volt. The self-inductance of the coil, in millihenry, is

(*a*) 1 (*b*) 5 (c) 20 (*d*) 10

19. A thin diamagnetic rod is placed vertically between the poles of an electromagnet. When the current in the electromagnet is switched on, then the diamagnetic rod is pushed up, out of the horizontal magnetic field. Hence the rod gains gravitational potential energy. The work required to do this comes from

(*a*) the current source

(*b*) the magnetic field

(*c*) the lattice structure of the material of the rod
(*d*) the induced electric field due to the changing magnetic field

20. The mutual inductance of two coils depends upon
(*a*) medium between coils
(*b*) separation between coils
(*c*) both on (*a*) and (*b*)
(*d*) none of (*a*) and (*b*)

Answers

1. (*d*)	**2.** (*b*)	**3.** (*b*)	**4.** (*c*)	**5.** (*b*)	**6.** (*a*), (*b*), (*c*)	**7.** (*a*)
8. (*b*)	**9.** (*b*)	**10.** (*d*)	**11.** (*a*)	**12.** (*a*)	**13.** (*b*)	**14.** (*b*)
15. (*d*)	**16.** (*a*)	**17.** (*b*), (*c*)	**18.** (*d*)	**19.** (*a*)	**20.** (*c*)	

Fill in the Blanks [1 mark]

1. The phenomenon in which electric current is generated by varying magnetic fields is appropriately called ______________.
2. The magnitude of the induced emf in a circuit is equal to the time rate of change of ______________ through the circuit.
3. The induced emf Blv is called ______________.
4. Lenz's law is consistent with the law of ______________.
5. The self-induced emf is also called the ______________ as it opposes any change in the current in a circuit.
6. Physically, the self-inductance plays the role of ______________.
7. The retarding force due to the eddy current inhibits the motion of a magnet. This phenomenon is known as ______________.

Answers

1. electromagnetic induction **2.** magnetic flux **3.** motional emf
4. conservation of energy **5.** back emf **6.** inertia **7.** electromagnetic damping

Very Short Answer Questions [1 mark]

Q. 1. Two spherical bobs, one metallic and the other of glass, of the same size are allowed to fall freely from the same height above the ground. Which of the two would reach earlier and why? [*CBSE Delhi 2014*]

Ans. Glass would reach earlier. This is because there is no effect of electromagnetic induction in glass, due to presence of earth's magnetic field, unlike in the case of metallic ball.

Q. 2. When current in a coil changes with time, how is the back emf induced in the coil related to it? [*CBSE (AI) 2008*]

Ans. The back emf induced in the coil opposes the change in current.

Q. 3. State the law that gives the polarity of the induced emf. [*CBSE (AI) 2009*]

Ans. Lenz's Law: The polarity of induced emf is such that it tends to produce a current which opposes the change in magnetic flux that produces it.

Q. 4. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify. [*CBSE Delhi 2017*]

Ans. No.

Justification: As the magnetic field due to current carrying wire will be in the plane of the circular loop, so magnetic flux will remain zero. Also, magnetic flux does not change with the change in current.

Q. 5. A light metal disc on the top of an electromagnet is thrown up as the current is switched on. Why? Give reason. [CBSE (AI) 2013]

Ans. A metal disc is placed on the top of a magnet, as the electric current flows through the coil, an induced current in the form of Eddies flows through the metal plate, the lower face attains the same polarity, and hence the metal disc is thrown up.

Q. 6. On what factors does the magnitude of the emf induced in the circuit due to magnetic flux depend? [CBSE (F) 2013]

Ans. It depends on the rate of change in magnetic flux (or simply change in magnetic flux).

$$|\varepsilon| = \frac{\Delta\phi}{\Delta t}$$

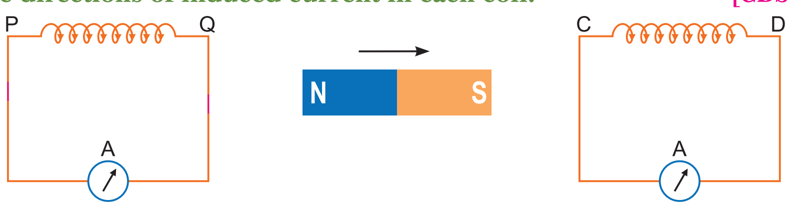
Q. 7. Give one example of use of eddy currents. [CBSE (F) 2016]

Ans. (*i*) Electromagnetic damping in certain galvanometers.

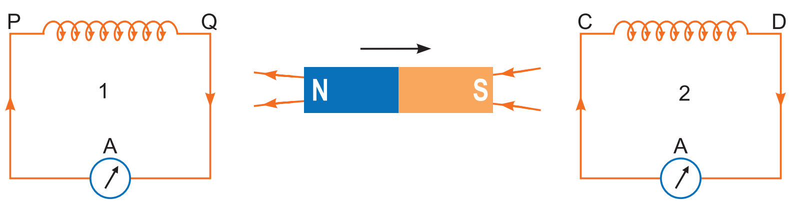
(*ii*) Magnetic braking in trains.

(*iii*) Induction furnace to produce high temperature. (*Any one*)

Q. 8. A bar magnet is moved in the direction indicated by the arrow between two coils *PQ* and *CD*. Predict the directions of induced current in each coil. [CBSE (AI) 2012, 2017]

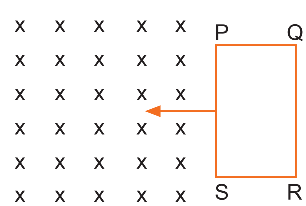


Ans.



In figure, N-pole is receding away coil (PQ), so in coil (PQ), the nearer faces will act as S-pole and in coil (CD) the nearer face will also act as S-pole to oppose the approach of magnet towards coil (CD), so currents in coils will flow clockwise as seen from the side of magnet. The direction of current will be from *P* to *Q* in coil (PQ) and from *C* to *D* in coil (CD).

Q. 9. The closed loop *PQRS* is moving into a uniform magnetic field acting at right angles to the plane of the paper as shown. State the direction of the induced current in the loop. [CBSE (F) 2012]



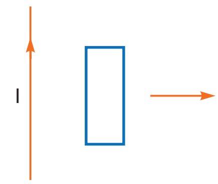
Ans. Due to the motion of coil, the magnetic flux linked with the coil increases. So by Lenz's law, the current induced in the coil will oppose this increase, hence tend to produce a field upward, so current induced in the coil will flow anticlockwise.

i.e., along *PSRQP*

Q. 10. A planar loop of rectangular shape is moved within the region of a uniform magnetic field acting perpendicular to its plane. What is the direction and magnitude of the current induced in it? [CBSE Ajmer 2015]

Ans. If planar loop moves within the region of uniform magnetic field, there is no magnetic flux changes by loop so, no current will be induced in the loop. Hence no direction.

Q. 11. A rectangular loop of wire is pulled to the right, away from the long straight wire through which a steady current *I* flows upwards. What is the direction of induced current in the loop? [CBSE (F) 2010]



Ans. Direction of induced current in loop is clockwise.

Reason: Induced current opposes the motion of loop away from wire; as similar currents attract, so in nearer side of loop the current will be upward, *i.e.*, in loop, current is clockwise.

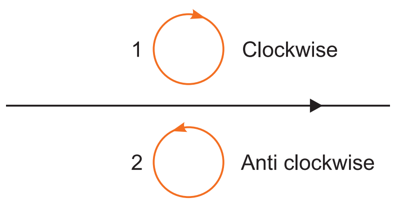
Q. 12. The motion of copper plate is damped when it is allowed to oscillate between the two poles of a magnet. What is the cause of this damping? ***[CBSE (AI) 2013]***

Ans. As the plate oscillate, the changing magnetic flux through the plate produces a strong eddy current in the direction, which opposes the cause.

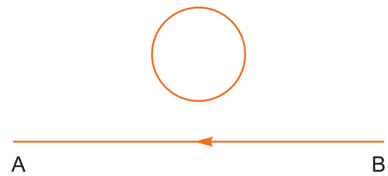
Also, copper being diamagnetic substance, it gets magnetised in the opposite direction, so the plate motion gets damped.

Q. 13. Predict the directions of induced currents in metal rings 1 and 2 lying in the same plane where current *I* in the wire is increasing steadily. ***[CBSE Delhi 2012, (AI) 2017] [HOTS]***

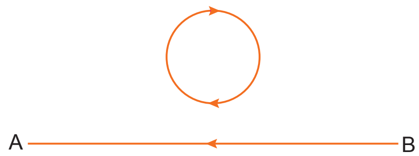
Ans.



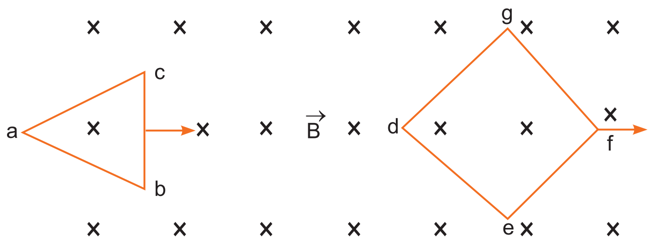
Q. 14. The electric current flowing in a wire in the direction from *B* to *A* is decreasing. Find out the direction of the induced current in the metallic loop kept above the wire as shown. ***[CBSE (AI) 2014]***



Ans. The current in the wire produces a magnetic field vertically downward in the vicinity of the coil. When the current in wire BA decreases, according to Lenz's law, the current induced in the coil opposes this decrease; so the current in the coil will be in clockwise direction.



Q. 15. Two loops of different shapes are moved in the region of a uniform magnetic field pointing downward. The loops are moved in the directions shown by arrows. What is the direction of induced current in each loop? ***[CBSE (F) 2010] [HOTS]***

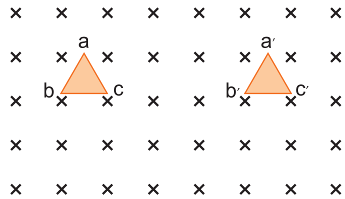


Ans. Loop *abc* is entering the magnetic field, so magnetic flux linked with it begins to increase. According to Lenz's law, the current induced opposes the increases in magnetic flux, so current induced will be **anticlockwise** which tends to decrease the magnetic field.

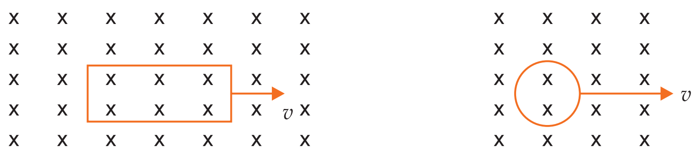
Loop *defg* is leaving the magnetic field; so flux linked with it tends to decrease, the induced current will be **clockwise** to produce magnetic field downward to oppose the decrease in magnetic flux.

Q. 16. A triangular loop of wire placed at *abc* is moved completely inside a magnetic field which is directed normal to the place of the loop away from the reader to a new position *a′ b′ c′*. What is the direction of the current induced in the loop? Give reason. [*CBSE (F) 2014*] [*HOTS*]

Ans. As there is no change in magnetic flux, so no current is induced in the loop.



Q. 17. A rectangular loop and a circular loop are moving out of a uniform magnetic field region to a field free region with a constant velocity. In which loop do you expect the induced emf to be a constant during the passage out of the field region? The field is normal to the loop.

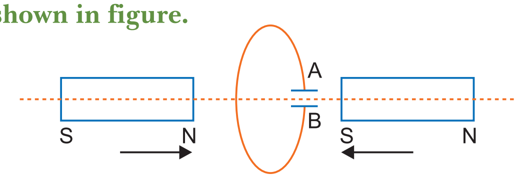


(a) (b)

[*CBSE (AI) 2010*]

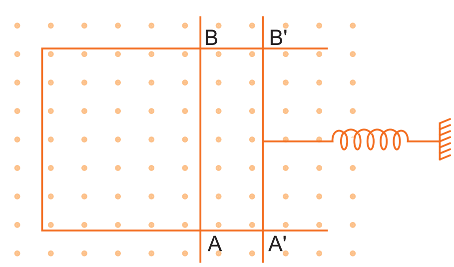
Ans. In rectangular coil the induced emf will remain constant because in this the case rate of change of area in the magnetic field region remains constant, while in circular coil the rate of change of area in the magnetic field region is not constant.

Q. 18. Predict the polarity of the capacitor *C* connected to coil, which is situated between two bar magnets moving as shown in figure. [*CBSE Delhi 2011, (AI) 2017*]

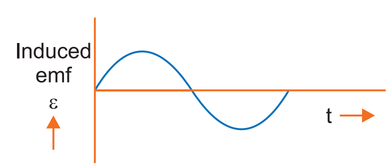


Ans. Current induced in coil will oppose the approach of magnet; therefore, left face of coil will act as *N*-pole and right face as *S*-pole. For this the current in coil will be anticlockwise as seen from left, therefore, the plate *A* of capacitor will be positive and plate *B* will be negative.

Q. 19. A rectangular wire frame, shown below, is placed in a uniform magnetic field directed upward and normal to the plane of the paper. The part *AB* is connected to a spring. The spring is stretched and released when the wire *AB* has come to the position *A′ B′* (t = 0) How would the induced emf vary with time? Neglect damping. [*HOTS*]



Ans. When the spring is stretched and released, the wire AB will execute simple harmonic (sinusoidal) motion, so induced emf will vary periodically. At $t = 0$, wire is at the extreme position $v = 0$.



Sinusoidal variation of induced emf

$$v = A\omega \sin \omega t$$

$$\text{Induced emf } \varepsilon = Bvl$$

$$= BA\,\omega l \sin \omega t$$

where $A = BB' = AA'$ is the amplitude of motion and ω is angular frequency.

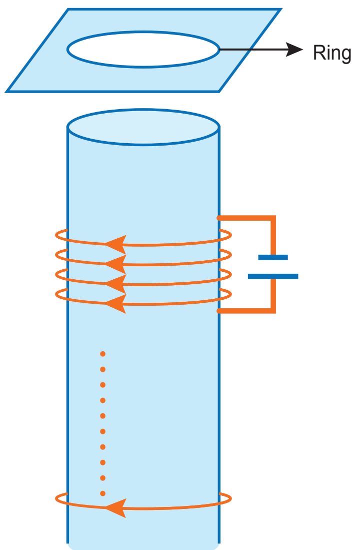
Q. 20. A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain. *[NCERT Exemplar]*

Ans. The current will increase. As the wires are pulled apart the flux will leak through the gaps. Lenz's law demands that induced emf resist this decrease, which can be done by an increase in current.

Q. 21. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain. *[NCERT Exemplar]*

Ans. The current will decrease. As the iron core is inserted in the solenoid, the magnetic field increases and the flux increases. Lenz's law implies that induced emf should resist this increase, which can be achieved by a decrease in current. However, this change will be momentarily.

Q. 22. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (in figure). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring? *[NCERT Exemplar]*



Ans. When the current in the solenoid decreases a current flows in the same direction in the metal ring as in the solenoid. Thus there will be a downward force. This means the ring will remain on the cardboard. The upward reaction of the cardboard on the ring will increase.

Q. 23. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

[NCERT Exemplar]

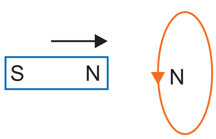
Ans. For the magnet, eddy currents are produced in the metallic pipe. These currents will oppose the motion of the magnet. Therefore magnet's downward acceleration will be less than the acceleration due to gravity g. On the other hand, an unmagnetised iron bar will not produce eddy currents and will fall an acceleration g. Thus the magnet will take more time.

Short Answer Questions–I [2 marks]

Q. 1. State Lenz's Law.

A metallic rod held horizontally along east-west direction, is allowed to fall under gravity. Will there be an emf induced at its ends? Justify your answer. *[CBSE Delhi 2013]*

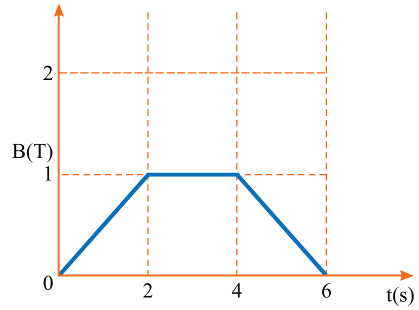
Ans. Lenz's law: According to this law "***the direction of induced current in a closed circuit is always such as to oppose the cause that produces it***."



The direction of induced current in a circuit is such that it opposes the very cause which generates it. Yes, an emf will be induced at its ends. Justification:

When a metallic rod held horizontally along east-west direction is allowed to fall freely under gravity *i.e.*, fall from north to south, the intensity of earth magnetic field changes through it *i.e.*, the magnetic flux changes and hence the emf is induced at it ends.

Q. 2. The magnetic field through a circular loop of wire 12 cm in radius and 8.5 Ω resistance, changes with time as shown in the figure. The magnetic field is perpendicular to the plane of the loop. Calculate the induced current in the loop and plot it as a function of time. ***[CBSE (F) 2017]***



Ans. We know,

$$\varepsilon = \frac{-d\phi}{dt} = \frac{-d(BA)}{dt} = -A\frac{dB}{dt}$$

$$I = \frac{\varepsilon}{R} = \frac{-A\left(\frac{dB}{dt}\right)}{R} = \frac{-\pi r^2\left(\frac{dB}{dt}\right)}{R}$$

For $0 < t < 2$

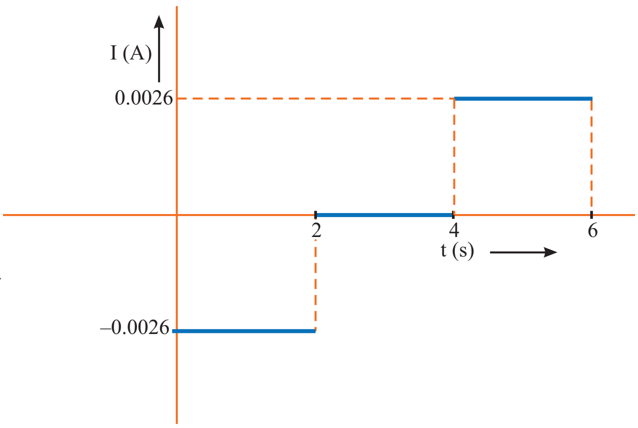
$$I = \frac{-3.14(0.12)^2 \times 1}{2 \times 8.5} = \mathbf{-0.0026\ A}$$

For, $2 < t < 4$

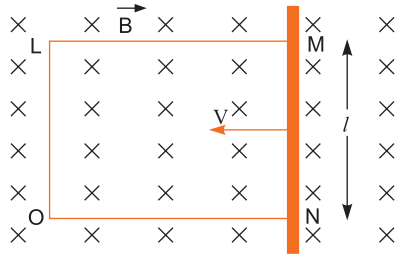
$$\frac{dB}{dt} = 0 \quad \Rightarrow \quad I = \mathbf{0}$$

For, $4 < t < 6$

$$I = \mathbf{+0.0026\ A}$$



Q. 3. A rectangular conductor LMNO is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor. When the arm MN of length of 20 cm is moved towards left with a velocity of 10 ms^{-1}. Calculate the emf induced in the arm. Given the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance), find the value of the current in the arm. ***[CBSE (AI) 2013]***



Ans. Induced emf in a moving rod in a magnetic field is given by

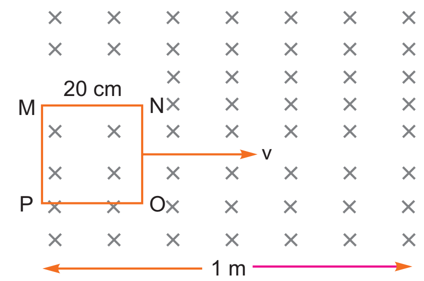
$$\varepsilon = Blv$$

Since the rod is moving to the left so

$$\varepsilon = Blv = 0.5 \times 0.2 \times 10 = \mathbf{1\ V}$$

Current in the rod $I = \frac{\varepsilon}{R} = \frac{1}{5} = \mathbf{0.2\ A}$

Q. 4. A square loop *MNOP* of side 20 cm is placed horizontally in a uniform magnetic field acting vertically downwards as shown in the figure. The loop is pulled with a constant velocity of 20 cms^{-1} till it goes out of the field.



(*i*) Depict the direction of the induced current in the loop as it goes out of the field. For how long would the current in the loop persist?

(*ii*) Plot a graph showing the variation of magnetic flux and induced emf as a function of time.

[*CBSE Panchkula 2015*]

Ans. (*i*) Clockwise MNOP.

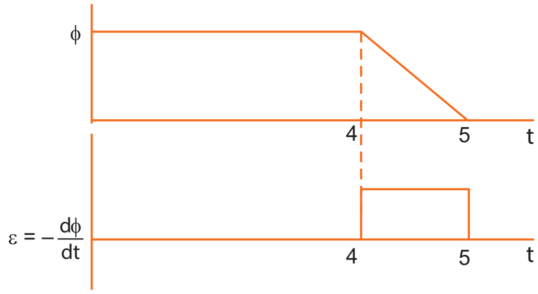
$$v = 20 \text{ cm/s}; d = 20 \text{ cm}$$

Time taken by the loop to move out of magnetic field

$$t = \frac{d}{v} = \frac{20}{20} = \mathbf{1s}$$

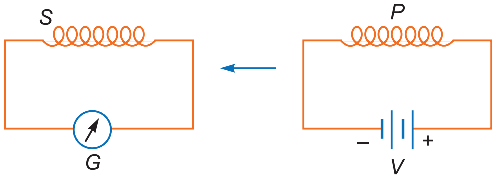
Induced current will last for 1 second till the length 20 cm moves out of the field.

(*ii*)



Q. 5. (*i*) When primary coil *P* is moved towards secondary coil *S* (as shown in the figure below) the galvanometer shows momentary deflection. What can be done to have larger deflection in the galvanometer with the same battery?

(*ii*) State the related law. [*CBSE Delhi 2010*]



Ans. (*i*) For larger deflection, coil P should be moved at a **faster rate**.

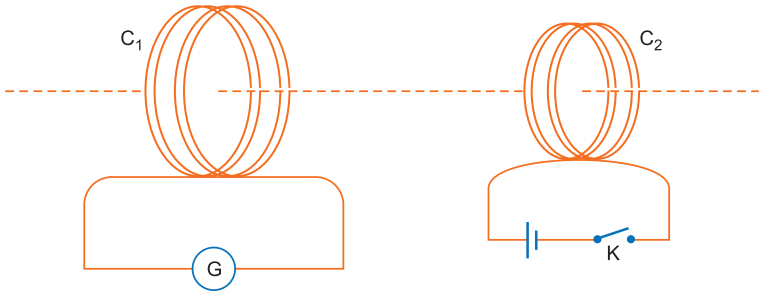
(*ii*) **Faraday law:** The induced emf is directly proportional to rate of change of magnetic flux linked with the circuit.

Q. 6. A current is induced in coil C_1 due to the motion of current carrying coil C_2.

(*a*) Write any two ways by which a large deflection can be obtained in the galvanometer G.

(*b*) Suggest an alternative device to demonstrate the induced current in place of a galvanometer.

[*CBSE Delhi 2011*]



Ans. (*a*) The deflection in galvanometer may be made large by

(*i*) moving coil C_2 towards C_1 with high speed.

(*ii*) by placing a soft iron laminated core at the centre of coil C_1.

(*b*) The induced current can be demonstrated by connecting a torch bulb (in place of galvanometer) in coil C_1. Due to induced current the bulb begins to glow.

Q. 7. (*i*) Define mutual inductance.

(*ii*) A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil? [*CBSE Delhi 2016*]

Ans. (*i*) Mutual inductance of two coils is the magnetic flux linked with the secondary coil when a unit current flows through the primary coil,

i.e., $\phi_2 = MI_1$ or $M = \dfrac{\phi_2}{I_1}$

(*ii*) Change of flux for small change in current

$$d\phi = MdI = 1.5\,(20 - 0)\text{ weber} = \mathbf{30\ weber}$$

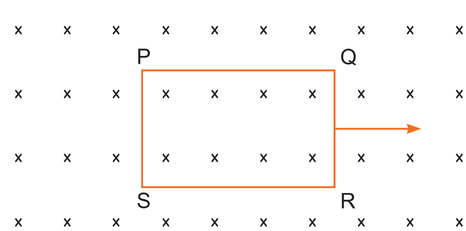
Q. 8. A toroidal solenoid with air core has an average radius of 15 cm, area of cross-section 12 cm^2 and has 1200 turns. Calculate the self-inductance of the toroid. Assume the field to be uniform across the cross-section of the toroid. [*CBSE (F) 2014*]

Ans. Here, $r = 15\text{ cm} = 0.15\text{ m}$, $A = 12\text{ cm}^2 = 12\times10^{-4}\text{ m}^2$ and $N = 1200$

Self inductance, $L = \dfrac{\mu_0 N^2 A}{l} = \dfrac{\mu_0 N^2 A}{2\pi r}$

$$= \frac{4\pi\times10^{-7}\times(1200)^2\times12\times10^{-4}}{2\pi\times0.15} = \mathbf{2.3\times10^{-3}\ H.}$$

Q. 9. The closed loop (*PQRS*) of wire is moved out of a uniform magnetic field at right angles to the plane of the paper as shown in the figure. Predict the direction of the induced current in the loop. [*CBSE (F) 2012*]



Ans. So far the loop remains in the magnetic field, there is no change in magnetic flux linked with the loop and so no current will be induced in it, but when the loop comes out of the magnetic field, the flux linked with it will decrease and so the current will be induced so as to oppose the decrease in magnetic flux, *i.e.,* it will cause magnetic field downwards; so the *direction of current will be clockwise*.

Q. 10. A small flat search coil of area 5 cm^2 with 140 closely wound turns is placed between the poles of a powerful magnet producing magnetic field 0.09 T and then quickly removed out of the field region. Calculate [*CBSE 2019, 55/3/1*]

(*a*) change of magnetic flux through the coil, and

(*b*) emf induced in the coil.

Ans. (*a*) M flux $\phi_1 = N\,\vec{B}.\vec{A} = NBA\cos\theta$

$= NBA\cos 0° = NBA$

$= 140\times 0.09\times 5\times 10^{-4} = 63\times 10^{-4}$ Wb

$\phi_2 = NBA \qquad [\because B = 0]$

$= 0$

Change in magnetic flux $= \phi_2 - \phi_1$

$$= \mathbf{63 \times 10^{-2}\ Wb}$$

(*b*) εmf induced $= -\dfrac{d\phi}{dt} = \dfrac{-63\times 10^{-4}}{\Delta t}$.

[Here time is not given. Question is incomplete.]

Q. 11. A 0.5 m long solenoid of 10 turns/cm has area of cross-section 1 cm^2. Calculate the voltage induced across its ends if the current in the solenoid is changed from 1A to 2A in 0.1s.

[*CBSE 2019, 55/3/1*]

Ans. Here $l = 0.5$ m

$n = 10$ tuns/cm $= 1000$/m

$A = 1\text{ cm}^2 = 1\times 10^{-4}$ m

Change in current $dI = (2 - 1) = 1$ A, $dt = 0.1$s

The induced voltage

$$|V| = L\frac{dI}{dt}$$

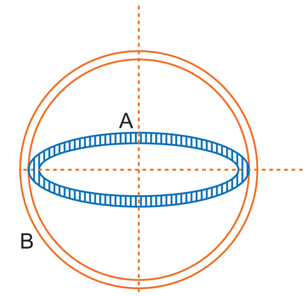
$$= \mu_0 n^2 A l \frac{dI}{dt}$$

$$= 4\pi\times 10^{-7}\times (1000)^2\times 10^{-4}\times 0.5\times \frac{1\text{ A}}{0.1\text{ s}}$$

$$= 4\pi\times 5\times 10^{-5}$$

$$= 20\pi\times 10^{-5} = \mathbf{0.628\ mV}$$

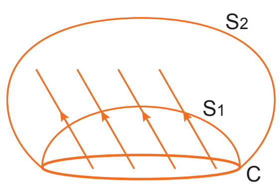
Q. 12. Two coils of wire *A* and *B* are placed mutually perpendicular as shown in figure. When current is changed in any one coil, will the current induce in another coil?



Ans. No; this is because the magnetic field due to current in coil (*A* or *B*) will be parallel to the plane of the other coil (*A* or *B*) Hence, the magnetic flux linked with the other coil will be zero and so no current will be induced in it.

Q. 13. Consider a closed loop C in a magnetic field (see figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = \vec{B_1}.d\,\vec{A_1} + \vec{B_2}.d\,\vec{A_2}$ ... Now if we chose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.

[*NCERT Exemplar*]



Ans. One gets the same answer for flux. Flux can be thought of as the number of magnetic field lines passing through the surface (we draw $dN = BA$ lines in a area ΔA perpendicular to B). As field lines of B cannot end or start in space (they form closed loops), number of lines passing through surface S_1 must be the same as the number of lines passing through the surface S_2.

Short Answer Questions–II [3 marks]

Q. 1. In an experimental arrangement of two coils C_1 and C_2 placed coaxially parallel to each other, find out the expression for the emf induced in the coil C_1 (of N_1 turns) corresponding to the change of current I_2 in the coil C_2 (of N_2 turns). [*CBSE Chennai 2015*]

Ans. Let ϕ_1 be the flux through coil C_1 (of N_1 turns) when current in coil C_2 is I_2. Then, we have

$$N_1\phi_1 = MI_2 \quad ...(i)$$

For current varying with time,

$$\frac{d(N_1\phi_1)}{dt} = \frac{d(MI_2)}{dt} \quad ...(ii)$$

Since induced *emf* in coil C_1 is given by

$$\varepsilon_1 = -\frac{d(N_1\phi_1)}{dt}$$

From (*ii*), $$-\varepsilon_1 = M\left(\frac{dI_2}{dt}\right)$$

$$\varepsilon_1 = -M\frac{dI_2}{dt} \quad \text{[from } (i)]$$

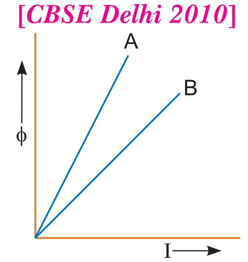
It shows that varying current in a coil induces emf in the neighbouring coil.

Q. 2. (*a*) How does the mutual inductance of a pair of coils change when

(*i*) distance between the coils is increased and

(*ii*) number of turns in the coils is increased? [*CBSE (AI) 2013*]

(*b*) A plot of magnetic flux (ϕ) versus current (*I*), is shown in the figure for two inductors *A* and *B*. Which of the two has large value of self-inductance? [*CBSE Delhi 2010*]



(*c*) How is the mutual inductance of a pair of coils affected when

(*i*) separation between the coils is increased?

(*ii*) the number of turns in each coil is increased?

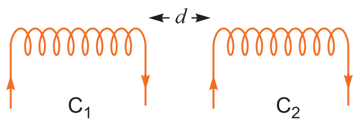
(*iii*) a thin iron sheet is placed between the two coils, other factors remaining the same?

Justify your answer in each case. [*CBSE (AI) 2013*]

Ans. (*a*) (*i*) Mutual inductance decreases.

(*ii*) Mutual inductance increases.

Concept: (*i*) If distance between two coils is increased as shown in figure.



It causes decrease in magnetic flux linked with the coil C_2. Hence induced emf in coil C_2 decreases by relation $\varepsilon_2 = \frac{-d\varphi_2}{dt}$. Hence mutual inductance decreases.

(*ii*) From relation $M_{21} = \mu_0 n_1 n_2 Al$, if number of turns in one of the coils or both increases, means mutual inductance will increase.

(*b*) $\phi = LI \Rightarrow \frac{\phi}{I} = L$

The slope of $\frac{\phi}{I}$ of straight line is equal to self-inductance *L*. It is larger for inductor *A*; therefore inductor *A* has larger value of self inductance '*L*'.

(*c*) (*i*) When the relative distance between the coil is increased, the leakage of flux increases which reduces the magnetic coupling of the coils. So magnetic flux linked with all the turns decreases. Therefore, mutual inductance will be decreased.

(*ii*) Mutual inductance for a pair of coil is given by

$$M = K\sqrt{L_1 L_2}$$

where $L = \dfrac{\mu N^2 A}{l}$ and L is called self inductance. Therefore, when the number of turns in each coil increases, the mutual inductance also increases.

(*iii*) When a thin iron sheet is placed between the two coils, the mutual inductance increases because M$\propto$ permeability. The permeability of the medium between coils increases.

Q. 3. Define self-inductance of a coil. Show that magnetic energy required to build up the current *I* in a coil of self inductance *L* is given by $\frac{1}{2}LI^2$. **[*CBSE Delhi 2012*]**

OR

Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance *L* to build up a current *I* through it. **[*CBSE (AI) 2014*]**

Ans. Self inductance – Using formula $\phi = LI$, if I = 1 Ampere then $L = \phi$

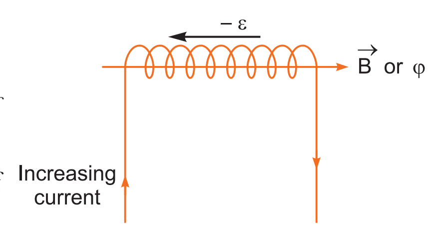
Self inductance of the coil is equal to the magnitude of the magnetic flux linked with the coil, when a unit current flows through it.

Alternatively

Using formula $|-\varepsilon| = L\dfrac{dI}{dt}$

If $\dfrac{dI}{dt}$ = 1 A/s then $L = |-\varepsilon|$

Self inductance of the coil is equal to the magnitude of induced emf produced in the coil itself, when the current varies at rate 1 A/s.



Expression for magnetic energy

When a time varying current flows through the coil, back *emf* (–ε) produces, which opposes the growth of the current flow. It means some work needs to be done against induced emf in establishing a current *I*. This work done will be stored as magnetic potential energy.

For the current *I* at any instant, the rate of work done is

$$\frac{dW}{dt} = (-\varepsilon)I$$

Only for inductive effect of the coil $|-\varepsilon| = L\dfrac{dI}{dt}$

$$\therefore \quad \frac{dW}{dt} = L\left(\frac{dI}{dt}\right)I \quad \Rightarrow \quad dW = LI\,dI$$

From work-energy theorem

$$dU = LI\,dI$$

$$\therefore \quad U = \int_0^I LI dI = \frac{1}{2}LI^2$$

Q. 4. Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare (*i*) the induced emf and (*ii*) the current produced in the two coils. Justify your answer. **[*CBSE (AI) 2010*]**

Ans. (*i*) Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t)$

$$= BA\,\omega\sin\omega t$$

As B, A, ω are same for both loops, so induced emf is same in both loops.

(*ii*) Current induced, $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\rho l/A} = \frac{\varepsilon A}{\rho l}$

As area A, length l and emf ε are same for both loops but resistivity ρ is less for copper, therefore current I induced is larger in copper loop.

Q. 5. A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the plane is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel. How will the value of emf be affected if the number of spokes were increased? [*CBSE (AI) 2013*]

Ans. If a rod of length 'l' rotates with angular speed ω in uniform magnetic field 'B'

$$\varepsilon = \frac{1}{2}Bl^2\omega$$

In case of earth's magnetic field $B_H = |B_e|\cos\delta$

and $B_V = |B_e|\sin\delta$

$$\therefore \quad \varepsilon = \frac{1}{2}|B_e|\cos\delta . l^2\omega$$

$$= \frac{1}{2}\times 0.4\times 10^{-4}\cos 60^o \times (0.5)^2 \times 2\pi\nu$$

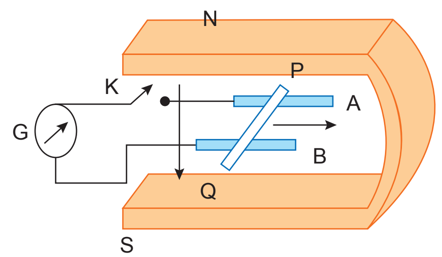
$$= \frac{1}{2}\times 0.4\times 10^{-4}\times\frac{1}{2}\times (0.5)^2\times 2\pi\times\left(\frac{120\text{ rev}}{60\text{ s}}\right)$$

$$= 10^{-5}\times 0.25\times 2\times 3.14\times 2$$

$$\mathbf{= 3.14\times 10^{-5}\ volt}$$

Induced emf is independent of the number of spokes *i.e.*, it remain same.

Q. 6. Figure shows a metal rod *PQ* of length *l*, resting on the smooth horizontal rails *AB* positioned between the poles of a permanent magnet. The rails, rod and the magnetic field *B* are in three mutually perpendicular directions. A galvanometer *G* connects the rails through a key '*K*'. Assume the magnetic field to be uniform. Given the resistance of the closed loop containing the rod is *R*.



(*i*) Suppose *K* is open and the rod is moved with a speed *v* in the direction shown. Find the polarity and the magnitude of induced emf.

(*ii*) With *K* open and the rod moving uniformly, there is no net force on the electrons in the rod *PQ* even though they do experience magnetic force due to the motion of the rod. Explain.

(*iii*) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular? [*CBSE Sample Paper 2018*]

Ans. (*i*) The magnitude of the induced emf is given by

$$|\varepsilon| = Blv\sin\theta$$

As the conductor *PQ* moves in the direction shown, the free electrons in it experience magnetic Lorentz force. By Fleming's left hand rule, the electrons move from the end *P* towards the end *Q*. Deficiency of electrons makes the end *P* positive while the excess of electrons makes the end *Q* negative.

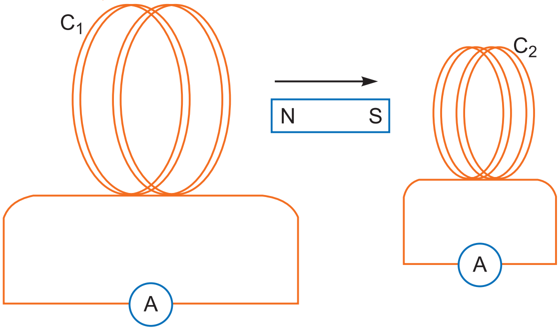
(*ii*) The magnetic Lorentz force $[\vec{F}_m = -e(\vec{v} \times \vec{B})]$ is cancelled by the electric force $[\vec{F}_m = e\vec{E}]$ exerted by the electric field set up by the opposite charges at its ends.

(*iii*) In this case, the angle θ made by the rod with the field $\vec{B}$ is zero.

$$\therefore \quad \varepsilon = Blv\sin 0° = 0$$

This is because the motion of the loop does not cut across the field lines. There is no change in magnetic flux. So the induced emf is zero.

Q. 7. A magnet is quickly moved in the direction indicated by an arrow between two coils C1 and C2 as shown in the figure. What will be the direction of induced current in each coil as seen from the magnet? Justify your answer. [*CBSE Delhi 2011*]



Ans. According to Lenz's law, the direction of induced current is such that it opposes the relative motion between coil and magnet.

The near face of coil C_1 will become S-pole, so the direction of current in coil C_1 will be **clockwise**.

The near face of coil C_2 will also become S-pole to oppose the approach of magnet, so the current in coil C_2 will also be **clockwise**.

Q. 8. The currents flowing in the two coils of self-inductance L_1=16 mH and L_2=12 mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of (*i*) induced voltages, (*ii*) the currents and (*iii*) the energies stored in the two coils at a given instant. [*CBSE (F) 2014*]

Ans. (*i*) Induced voltage (emf) in the coil,

$$\varepsilon = -L\frac{dI}{dt}$$

$$\therefore \quad \frac{\varepsilon_1}{\varepsilon_2} = \frac{-L_1\frac{dI}{dt}}{-L_2\frac{dI}{dt}} = \frac{L_1}{L_2} = \frac{16\text{ mH}}{12\text{ mH}} = \mathbf{\frac{4}{3}}$$

(*ii*) Power supplied, $P = \varepsilon I$

Since power is same for both the coils

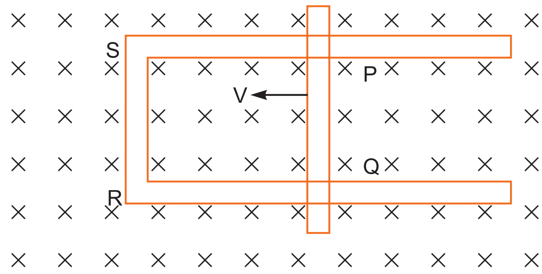
$$\therefore \quad \varepsilon_1 I_1 = \varepsilon_2 I_2 \quad = \frac{I_1}{I_2} = \frac{\varepsilon_2}{\varepsilon_1} = \mathbf{\frac{3}{4}}$$

(*iii*) Energy stored in the coil is given by

$$U = \frac{1}{2}LI^2$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2}L_1 I_1^2}{\frac{1}{2}L_2 I_2^2} = \frac{L_1}{L_2} \times \left(\frac{I_1}{I_2}\right)^2 = \frac{4}{3} \times \left(\frac{3}{4}\right)^2 = \mathbf{\frac{3}{4}}$$

Q. 9. Figure shows a rectangular loop conducting *PQRS* in which the arm *PQ* is free to move. A uniform magnetic field acts in the direction perpendicular to the plane of the loop. Arm *PQ* is moved with a velocity v towards the arm *RS*. Assuming that the arms *QR*, *RS* and *SP* have negligible resistances and the moving arm *PQ* has the resistance r, obtain the expression for (*i*) the current in the loop (*ii*) the force and (*iii*) the power required to move the arm *PQ*. **[*CBSE Delhi 2013*]**



Ans. (*i*) Current in the loop *PQRS*,

$$I = \frac{\varepsilon}{r}$$

Since $\varepsilon = \frac{d\phi}{dt} = Blv$ So, $I = \frac{Blv}{r}$

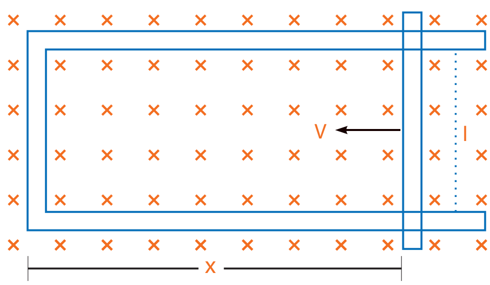
(*ii*) The force required to keep the arm *PQ* in constant motion

$$F = BI\,l = B\left(\frac{Blv}{r}\right)l = \frac{B^2l^2v}{r}$$

(*iii*) Power required to move the arm *PQ*

$$P = F\,|\,v\,| = \left(\frac{B^2l^2v}{r}\right)|\,v\,| = \left(\frac{B^2l^2v^2}{r}\right)$$

Q. 10. (*a*) A rod of length l is moved horizontally with a uniform velocity 'v' in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.



(*b*) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain. **[*CBSE (AI) 2014*]**

Ans. (*a*) Suppose a rod of length 'l' moves with velocity v inward in the region having uniform magnetic field B.

Initial magnetic flux enclosed in the rectangular space is $\phi = |B|lx$

As the rod moves with velocity $-v = \frac{dx}{dt}$

Using Lenz's law

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(Blx) = Bl\left(-\frac{dx}{dt}\right)$$

$\therefore$ $\varepsilon = Blv$

(*b*) Suppose any arbitrary charge 'q' in the conductor of length 'l' moving inward in the field as shown in figure, the charge q also moves with velocity v in the magnetic field B.

The Lorentz force on the charge 'q' is $F = qvB$ and its direction is downwards.

So, work done in moving the charge 'q' along the conductor of length l

$$W = F.l$$

$$W = qvBl$$

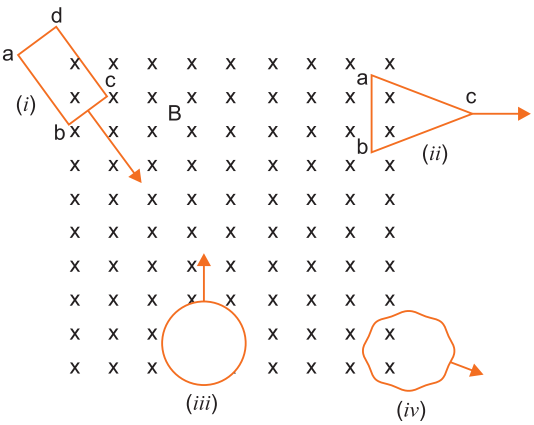
Since emf is the work done per unit charge

$$\therefore \qquad \varepsilon = \frac{W}{q} = Blv$$

This equation gives emf induced across the rod.

Q. 11. Figure shows planar loops of different shapes moving out of or into a region of magnetic field which is directed normal to the plane of loops downwards. Determine the direction of induced current in each loop using Lenz's law.

[CBSE (AI) 2010, (F) 2014]



Ans. (*a*) In Fig. (*i*) the rectangular loop *abcd* and in Fig. (*iii*) circular loop are entering the magnetic field, so the flux linked with them increases; The direction of induced currents in these coils, will be such as to oppose the increase of magnetic flux; hence the magnetic field due to current induced will be upward, *i.e.*, currents induced will flow *anticlockwise*.

(*b*) In Fig. (*ii*), the triangular loop *abc* and in fig. (*iv*) the zig-zag shaped loop are emerging from the magnetic field, therefore magnetic flux linked with these loops decreases. The currents induced in them will tend to increase the magnetic field in downward direction, so the currents will flow *clockwise*.

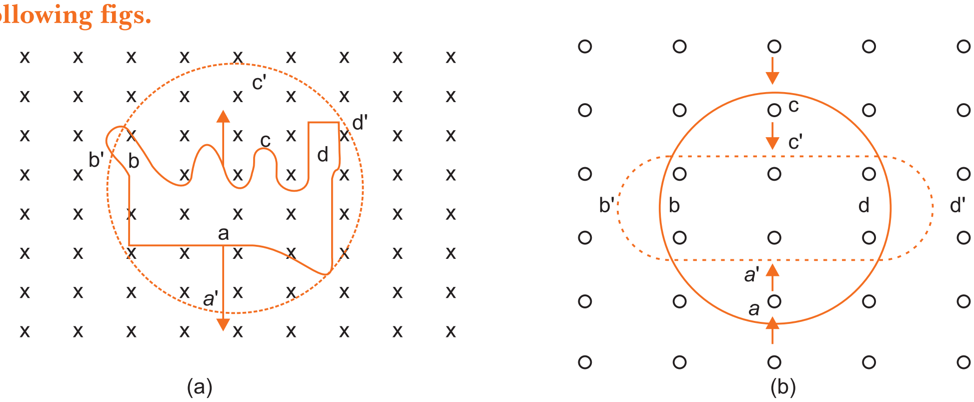
Thus in fig. (*i*) current flows anticlockwise,

in fig. (*ii*) current flows clockwise,

in fig. (*iii*) current flows anticlockwise,

in fig. (*iv*) current flows clockwise.

Q. 12. Use Lenz's law to determine the direction of induced current in the situation described by following figs.



(*a*) A wire of irregular shape turning into a circular shape. **[CBSE (F) 2014]**

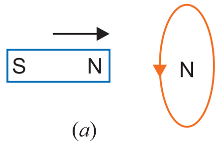
(*b*) A circular loop being deformed into a narrow straight wire.

Ans. (*a*) For the given periphery the area of a circle is maximum. When a coil takes a circular shape, the magnetic flux linked with coil increases, so current induced in the coil will tend to decrease the flux and so will produce a magnetic field upward. As a result the current induced in the coil will flow anticlockwise *i.e.*, along *a′d′c′b′*.

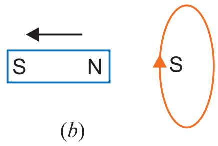
(*b*) For given periphery the area of circle is maximum. When circular coil takes the shape of narrow straight wire, the magnetic flux linked with the coil decreases, so current induced in the coil will tend to oppose the decrease in magnetic flux; hence it will produce upward magnetic field, so current induced in the coil will flow anticlockwise *i.e.*, along *a′ d′ c′ b′*.

Q. 13. Show that Lenz's law is in accordance with the law of conservation of energy. [*CBSE (F) 2017*]

Ans. Lenz's law: According to this law "*the direction of induced current in a closed circuit is always such as to oppose the cause that produces it*."



(*a*)



(*b*)

Example: When the north pole of a coil is brought near a closed coil, the direction of current induced in the coil is such as to oppose the approach of north pole. For this the nearer face of coil behaves as north pole. This necessitates an anticlockwise current in the coil, when seen from the magnet side [fig. (*a*)]

Similarly when north pole of the magnet is moved away from the coil, the direction of current in the coil will be such as to attract the magnet. For this the nearer face of coil behaves as south pole. This necessitates a clockwise current in the coil, when seen from the magnet side [fig. (*b*)].

Conservation of Energy in Lenz's Law: Thus, in each case whenever there is a relative motion between a coil and the magnet, a force begins to act which opposes the relative motion. Therefore to maintain the relative motion, a mechanical work must be done. This work appears in the form of electric energy of coil. Thus *Lenz's law is based on principle of conservation of energy*.

Long Answer Questions [5 marks]

Q. 1. (*a*) What is induced emf? Write Faraday's law of electromagnetic induction. Express it mathematically.

(*b*) A conducting rod of length '*l*', with one end pivoted, is rotated with a uniform angular speed 'ω' in a vertical plane, normal to a uniform magnetic field '*B*'. Deduce an expression for the emf induced in this rod. [*CBSE Delhi 2013, 2012*]

If resistance of rod is *R*, what is the current induced in it?

Ans. (*a*) **Induced emf:** The emf developed in a coil due to change in magnetic flux linked with the coil is called the induced emf.

Faraday's Law of Electromagnetic Induction: On the basis of experiments, Faraday gave two laws of electromagnetic induction:

1. *When the magnetic flux linked with a coil or circuit changes, an emf is induced in the coil.*

   If coil is closed, the current is also induced. The emf and current last so long as the change in magnetic flux lasts. *The magnitude of induced emf is proportional to the rate of change of magnetic flux linked with the circuit*. Thus if $\Delta\phi$ is the change in magnetic flux linked in time Δt then rate of change of flux is $\frac{\Delta\phi}{\Delta t}$,

   So emf induced $\qquad \varepsilon \propto \frac{\Delta\phi}{\Delta t}$

2. *The emf induced in the coil (or circuit) opposes the cause producing it.*

$$\varepsilon \propto -\frac{\Delta\phi}{\Delta t}$$

Here the negative sign shows that the induced emf 'ε' opposes the change in magnetic flux.

$$\varepsilon = -K\frac{\Delta\phi}{\Delta t}$$

where *K* is a constant of proportionality which depends on units chosen for ϕ, t and ε. In SI system the unit of flux ϕ is weber, unit of time t is second and unit of emf ε is volt and $K = 1$

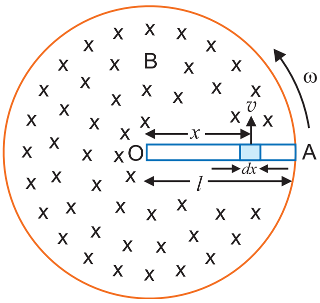
$$\therefore \qquad \varepsilon = -\frac{\Delta\phi}{\Delta t} \qquad ...(i)$$

If the coil contains N turns of insulated wire, then the flux linked with each turn will be same and the emf induced in each turn will be in the same direction, hence the emfs of all turns will be added. Therefore the emf induced in the whole coil,

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t} = -\frac{\Delta(N\phi)}{\Delta t} \quad \text{...(ii)}$$

$N\phi$ is called the ***effective magnetic flux*** or the ***number of flux linkages*** in ***the coil and may be denoted by*** ψ.

(*b*) Expression for Induced emf in a Rotating Rod



Consider a metallic rod OA of length l which is rotating with angular velocity ω in a uniform magnetic field B, the plane of rotation being perpendicular to the magnetic field. A rod may be supposed to be formed of a large number of small elements. Consider a small element of length dx at a distance x from centre. If v is the linear velocity of this element, then area swept by the element per second $= v\,dx$

The emf induced across the ends of element

$$d\varepsilon = B\frac{dA}{dt} = Bv\,dx$$

But $v = x\omega$

$\therefore \quad d\varepsilon = B\,x\,\omega\,dx$

$\therefore \quad$ The emf induced across the rod

$$\varepsilon = \int_0^l B\,x\omega\,dx = B\omega\int_0^l x\,dx$$

$$= B\omega\left[\frac{x^2}{2}\right]_0^l = B\omega\left[\frac{l^2}{2} - 0\right] = \frac{B\omega l^2}{2}$$

Current induced in rod $\quad I = \dfrac{\varepsilon}{R} = \dfrac{1}{2}\dfrac{B\omega l^2}{R}$.

If circuit is closed, power dissipated $= \dfrac{\varepsilon^2}{R} = \dfrac{B^2\omega^2 l^4}{4R}$

Q. 2. (*a*) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce an induced current which opposes the change of magnetic flux that produces it.

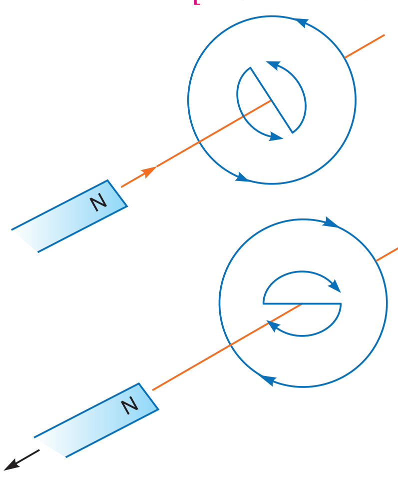
(*b*) The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of

(*i*) Magnetic flux versus the current

(*ii*) Induced emf versus *dI*/*dt*

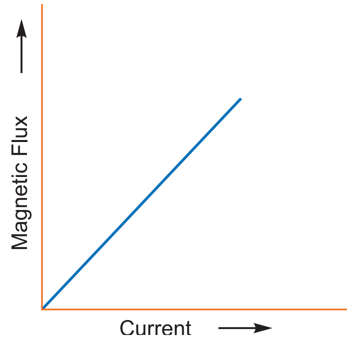
(*iii*) Magnetic potential energy stored versus the current. **[*CBSE Delhi 2014*]**

Ans. (*a*) When the North pole of a bar magnet moves towards the closed coil, the magnetic flux through the coil increases. This produces an induced emf which produces (or tend to produce if the coil is open) an induced current in the anti-clockwise sense. The anti-clockwise sense corresponds to the generation of North pole which opposes the motion of the approaching N pole of the magnet. The face of the coil, facing the approaching magnet, then has the same polarity as that of the approaching pole of the magnet. The induced current, therefore, is seen to oppose the change of magnetic flux that produces it.

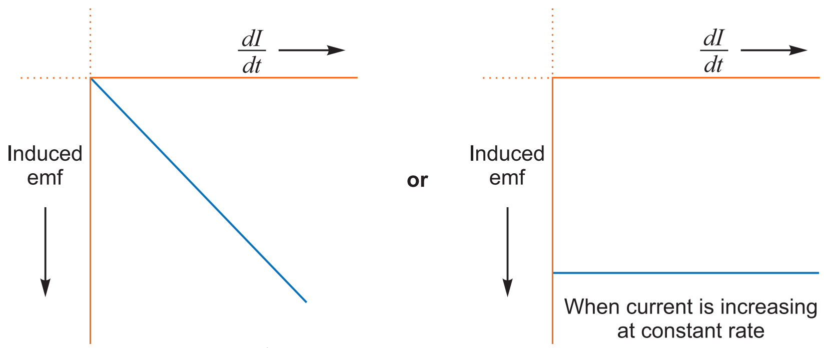


When a North pole of a magnet is moved away from the coil, the current (I) flows in the clock-wise sense which corresponds to the generation of South pole. The induced South pole opposes the motion of the receding North pole.

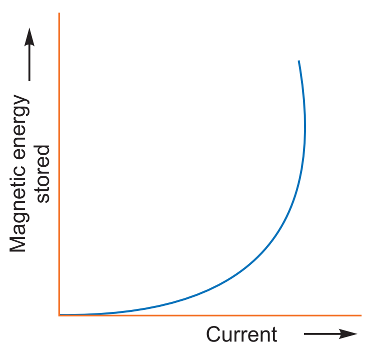
(*b*) (*i*) Magnetic flux versus the current



(*ii*) Induced emf versus dI/dt



(*iii*) Magnetic energy stored versus current



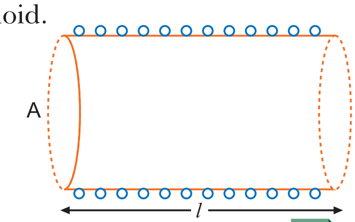
Q. 3. Derive expression for self inductance of a long air-cored solenoid of length *l*, cross-sectional area *A* and having number of turns *N*. **[*CBSE Delhi 2012, 2009*]**

Ans. Self Inductance of a long air-cored solenoid:

Consider a long air solenoid having 'n' number of turns per unit length. If current in solenoid is I, then magnetic field within the solenoid, $B = \mu_0 nI$ ...(*i*)

where $\mu_0 = 4\pi \times 10^{-7}$ henry/metre is the permeability of free space.

If A is cross-sectional area of solenoid, then effective flux linked with solenoid of length l is $\phi = NBA$ where $N = nl$ is the number of turns in length 'l' of solenoid.



$\therefore \quad \phi = (nl\, BA)$

Substituting the value of B from (*i*)

$\therefore \quad \phi = \mu_0 n^2 AlI$ ...(*ii*)

$\therefore$ Self-inductance of air solenoid

$$L = \frac{\phi}{I} = \mu_0 n^2 Al \qquad ...(iii)$$

If N is the total number of turns in length l then

$$n = \frac{N}{l}$$

$\therefore$ Self-inductance $L = \mu_0 \left(\frac{N}{l}\right)^2 Al$

$$= \frac{\mu_0 N^2 A}{l} \qquad ..(iv)$$

Remark: If solenoid contains a core of ferromagnetic substance of relative permeability μ_r, then self inductance, $L = \frac{\mu_r \mu_0 N^2 A}{l}$.

Q. 4. Obtain the expression for the mutual inductance of two long co-axial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2 and n_1 and n_2 be number of turns per unit length, when a current I is set up in the outer solenoid S_2. **[*CBSE Delhi 2017*]**

OR

(*a*) Define mutual inductance and write its SI units. **[*CBSE 2019*, (55/1/1)]**

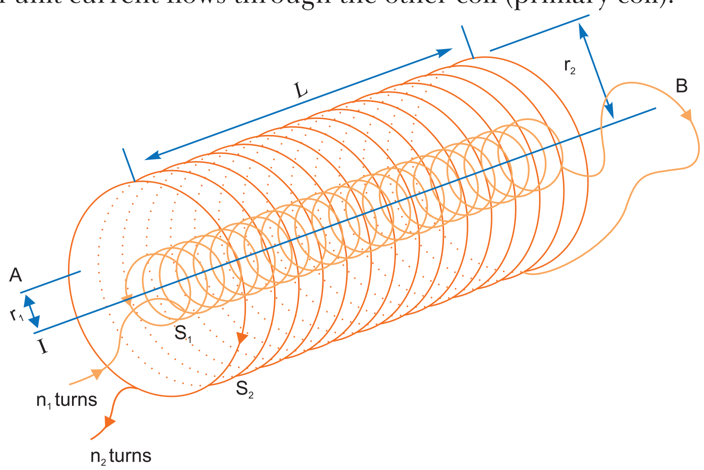
(*b*) Derive an expression for the mutual inductance of two long co-axial solenoids of same length wound one over the other.

(*c*) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for the emf induced in the coil C_1 due to a change in the current through the coil C_2. **[*CBSE Delhi 2015*]**

Ans. (*a*) When current flowing in one of two nearby coils is changed, the magnetic flux linked with the other coil changes; due to which an emf is induced in it (other coil). This phenomenon of electromagnetic induction is called the mutual induction. The coil, in which current is changed is called the primary coil and the coil in which emf is induced is called the secondary coil.

The SI unit of mutual inductance is henry.

(*b*) Mutual inductance is numerically equal to the magnetic flux linked with one coil (secondary coil) when unit current flows through the other coil (primary coil).



Consider two long co-axial solenoids, each of length L. Let n_1 be the number of turns per unit length of the inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of the outer solenoid S_2 of radius r_2.

Imagine a time varying current I_2 through S_2 which sets up a time varying magnetic flux ϕ_1 through S_1.

$$\therefore \qquad \phi_1 = M_{12}(I_2) \qquad ...(i)$$

where, M_{12} = Coefficient of mutual inductance of solenoid S_1 with respect to solenoid S_2

Magnetic field due to the current I_2 in S_2 is

$$B_2 = \mu_0 n_2 I_2$$

$\therefore$ Magnetic flux through S_1 is

$$\phi_1 = B_2 A_1 N_1$$

where, $N_1 = n_1 L$ and L = length of the solenoid

$$\phi_1 = (\mu_0 n_2 I_2)(\pi r_1^2)(n_1 L)$$

$$\phi_1 = \mu_0 n_1 n_2 \pi r_1^2 L I_2 \quad ...(ii)$$

From equations (*i*) and (*ii*), we get

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 L \quad ...(iii)$$

Let us consider the reverse case.

A time varying current I_1 through S_1 develops a flux ϕ_2 through S_2.

$$\therefore \quad \phi_2 = M_{21}(I_1) \quad ...(iv)$$

where, M_{21} = Coefficient of mutual inductance of solenoid S_2 with respect to solenoid S_1

Magnetic flux due to I_1 in S_1 is confined solely inside S_1 as the solenoids are assumed to be very long.

There is no magnetic field outside S_1 due to current I_1 in S_1.

The magnetic flux linked with S_2 is

$$\therefore \quad \phi_2 = B_1 A_1 N_2 = (\mu_0 n_1 I_1)(\pi r_1^2)(n_2 L)$$

$$\phi_2 = \mu_0 n_1 n_2 \pi r_1^2 L I_1 \quad ...(v)$$

From equations (*iv*) and (*v*), we get

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 \quad ...(vi)$$

From equations (*iii*) and (*vi*), we get

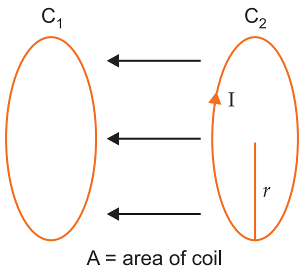
$$M_{12} = M_{21} = M = \mu_0 n_1 n_2 \pi r_1^2 L$$

We can write the above equation as

$$M = \mu_0 \left(\frac{N_1}{L}\right)\left(\frac{N_2}{L}\right)\pi r^2 \times L$$

$$M = \frac{\mu_0 N_1 N_2 \pi r^2}{L}$$

(*c*) When the current in coil C_2 changes, the flux linked with C_1 changes. This change in flux linked with C_1 induces emf in C_1.



Flux linked with C_1 = flux of C_2

$$\phi_{12} = B.A = \frac{\mu_0 I}{2r}.A$$

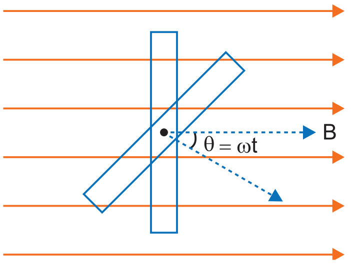
emf in $$C_1 = \frac{d\phi_{12}}{dt} = \frac{d}{dt}\frac{\mu_0 AI}{2r} = \frac{\mu_0 A}{2r} \times \frac{dI}{dt}$$

Q. 5. A coil of number of turns N, area A is rotated at a constant angular speed ω, in a uniform magnetic field B and connected to a resistor R. Deduce expression for

(*i*) maximum emf induced in the coil.

(*ii*) power dissipation in the coil.

Ans. (*i*) Suppose initially the plane of coil is perpendicular to the magnetic field B. When coil rotates with angular speed ω, then after time t, the angle between magnetic field $\vec{B}$ and normal to plane of coil is



$$\theta = \omega t$$

$\therefore$ At this instant magnetic flux linked with the coil $\phi = BA\cos\omega t$

If coil constants, N-turns, then emf induced in the coil

$$\varepsilon = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(BA\cos\omega t)$$

$$= +NBA\,\omega\sin\omega t \qquad \ldots(i)$$

$\therefore$ For maximum value of emf ε,

$$\sin\omega t = 1$$

$\therefore$ Maximum emf induced, $\varepsilon_{max} = NBA\,\omega$

(*ii*) If R is resistance of coil, the current induced, $I = \dfrac{\varepsilon}{R}$

$\therefore$ Instantaneous power dissipated, $P = \varepsilon I = \varepsilon\left(\dfrac{\varepsilon}{R}\right) = \dfrac{\varepsilon^2}{R} \qquad \ldots(ii)$

$$= \frac{N^2B^2A^2\omega^2\sin^2\omega t}{R} \text{ [using } (i)] \qquad \ldots(iii)$$

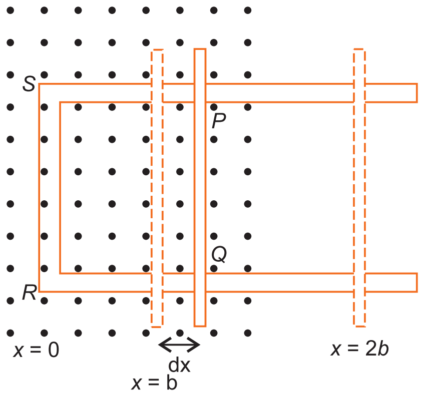
Average power dissipated in a complete cycle is obtained by taking average value of $\sin^2\omega t$ over a complete cycle which is $\dfrac{1}{2}$

i.e., $(\sin^2\omega t)_{av} = \dfrac{1}{2}$

$\therefore$ Average power dissipated $P_{av} = \dfrac{N^2B^2A^2\omega^2}{2R}$

Q. 6. State Faraday's law of electromagnetic induction.

Figure shows a rectangular conductor *PQRS* in which the conductor *PQ* is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from x = 0 to x = b and is zero for $x > b$. Assume that only the arm PQ possesses resistance r. When the arm *PQ* is pulled outward from x = 0 to x = 2b and is then moved backward to x =0 with constant speed v, obtain the expressions for the flux and the induced emf. Sketch the variations of these quantities with distance $0 \le x \le 2b$. **[*CBSE (AI) 2010, (North) 2016*]**



Ans. Refer to Point 3 of Basic Concepts.

Let length of conductor $PQ = l$

When PQ moves a small distance from x to $x + dx$ then magnetic flux linked $= BdA = Bldx$

The magnetic field is from $x = 0$ to $x = b$, to so final magnetic flux

$$= \sum Bldx = Bl \sum dx = Blx \text{ (increasing)}$$

We consider forward motion from $x = 0$ to $x = 2b$

$$\phi = Blx \ , \ 0 \le x < b$$

$$= Blb \ , \ b \le x < 2b$$

Mean magnetic flux from $x = 0$ to $x = b$ is $\frac{1}{2}Blb$

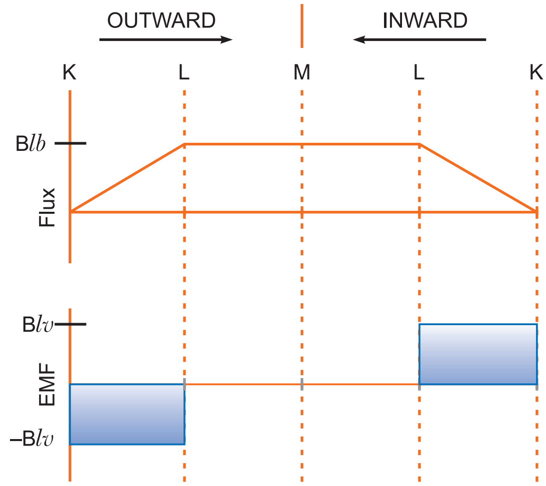
Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(Bldx) = -Bl\frac{dx}{dt} = -Blv$ for, $0 \le x < b$

where $v = \frac{dx}{dt}$ velocity of arm PQ from $x = 0$ to $x = b$.

$$\varepsilon = -\frac{d}{dt}(Blb) = 0 \ \text{for,} \ b \le x < 2b$$

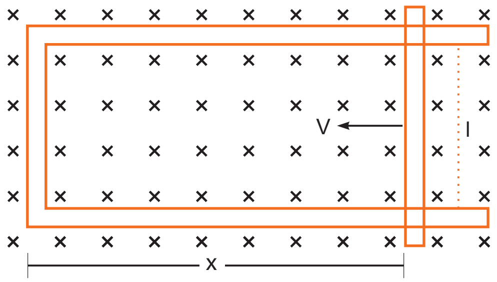
During return from $x = 2b$ to $x = b$ the induced emf is zero; but now area is decreasing so magnetic flux is decreasing, and induced emf will be in opposite direction.

$$\varepsilon = Blv$$



Q. 7. What are eddy currents? How are they produced? In what sense eddy currents are considered undesirable in a transformer? How can they be minimised? Give two applications of eddy currents. **[*CBSE (AI) 2011, (F) 2015*]**

Ans. **Eddy currents:** When a thick metallic piece is placed in a time varying magnetic field, the magnetic flux linked with the plate changes, the induced currents are set up in the conductor; these currents are called **eddy currents.** These currents are sometimes so strong, that the metallic plate becomes red hot.



Due to heavy eddy currents produced in the core of a transformer, large amount of energy is wasted in the form of undesirable heat.

Minimisation of Eddy Currents: Eddy currents may be minimised by using **laminated core** of soft iron. The resistance of the laminated core increases and the eddy currents are reduced and wastage of energy is also reduced.

Application of Eddy Currents:

1. **Induction Furnace:** In induction furnance, the metal to be heated is placed in a rapidly varying magnetic field produced by high frequency alternating current. Strong eddy currents are set up in the metal produce so much heat that the metal melts. This process is used in extracting a metal from its ore. The arrangement of heating the metal by means of strong induced currents is called the induction furnace.
2. **Induction Motor:** The eddy currents may be used to rotate the rotor. Its principle is: When a metallic cylinder (or rotor) is placed in a rotating magnetic field, eddy currents are produced in it. According to Lenz's law, these currents tend to opposes to relative motion between the cylinder and the field. The cylinder, therefore, begins to rotate in the direction of the field. This is the principle of induction motor.

Self-Assessment Test

Time allowed: 1 hour **Max. marks: 30**

1. Choose and write the correct option in the following questions. **(3 × 1 = 3)**

(*i*) If the number of turns in a coil is doubled, then its self-inductance becomes

(*a*) double (*b*) half

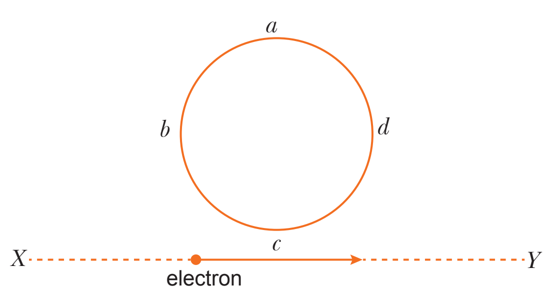
(*c*) four times (*d*) unchanged

(*ii*) The magnetic potential energy stored in a certain inductor is 25 mJ, when the current in the inductor is 60 mA. This inductor is of inductance

(*a*) 0.138 H (*b*) 138.88 H

(*c*) 1.389 H (*d*) 13.89 H

(*iii*) An electron moves on a straight line path *XY* as shown. The *abcd* is a coil adjacent to the path of electron. What will be the direction of current, if any, induced in the coil?



(*a*) The current will reverse its direction as the electron goes past the coil

(*b*) No current induced

(*c*) *abcd*

(*d*) *adcb*

2. Fill in the blanks. **(2 × 1 = 2)**

(*i*) ____________ of induced emf is such that it tends to produce a current which opposes the change in ____________ that produced it.

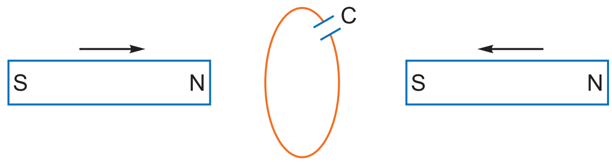
(*ii*) The magnitude of the induced emf depends upon the rate of change of current and ______________ of the two coils.

3. (*i*) Name the three elements of the earth's magnetic field.

(*ii*) Name the physical quantity which is the ratio of magnetic flux and induced current? Write its SI unit. **1**

4. Predict the direction of induced current in metal rings 1 and 2 when current I in the wire is steadily decreasing. **1**

5. Two bar magnets are quickly moved towards a metallic loop connected across a capacitor 'C' as shown in the figure. Predict the polarity of the capacitor. **1**



6. A rectangular coil rotates in a uniform magnetic field. Obtain an expression for induced emf and current at any instant. Also find their peak values. Show the variation of induced emf versus angle of rotation (ωt) on a graph.. **2**

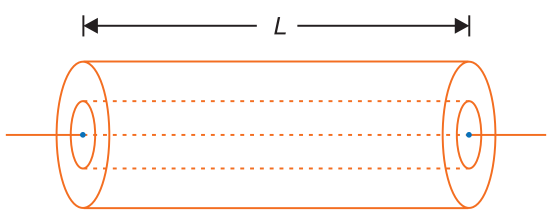
7. Obtain the expression for the mutual inductance of a pair of coaxial circular coils of radii r and R ($R > r$) placed with their centres coinciding. **2**

8. (*i*) How are eddy currents reduced in a metallic core?

(*ii*) Give two uses of eddy currents. **2**

9. An iron bar falling through the hollow region of a thick cylindrical shell made of copper experiences a retarding force. What can you conclude about the nature of the iron bar ? Explain. **2**

10. Figure shows two long coaxial solenoids, each of length 'L'. The outer solenoid has an area of cross-section A_1 and number of turns/length n_1 The corresponding values for the inner solenoid are A_2 and n_2 Write the expression for self inductance L_1, L_2 of the two coils and their mutual inductance M. Hence show that $M < \sqrt{L_1 L_2}$. **3**



11. (*a*) How are eddy currents generated in a conductor which is subjected to a magnetic field?

(*b*) Write two examples of their useful applications.

(*c*) How can the disadvantages of eddy currents be minimized? **3**

12. State Lenz's law. Illustrate, by giving an example, how this law helps in predicting the direction of the current in a loop in the presence of a changing magnetic flux.

In a given coil of self-inductance of 5 mH, current changes from 4 A to 1 A in 30 ms. Calculate the emf induced in the coil. **3**

13. (*a*) A metallic rod of length 'l' and resistance 'R' is rotated with a frequency 'ν' with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius 'l', about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field 'B' parallel to the axis is present everywhere.

(*i*) Derive the expression for the induced emf and the current in the rod.

(*ii*) Due to the presence of current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

(*iii*) Hence, obtain an expression for the power required to rotate the rod.

(*b*) A copper coil is taken out of a magnetic field with a fixed velocity. Will it be easy to remove it from the same field if its ohmic resistance is increased?

Answers

1. (*i*) (*c*) (*ii*) (*d*) (*iii*) (*a*)

2. (*i*) magnetic flux (*ii*) mutual inductance

12. 0.5 V