ALL INDIA TEST SERIES

TEST – 18

JEE (Advanced)

Time Allotted: 3 Hours **Maximum Marks: 186**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-C.**

1. ***Section-A (01 – 08, 19 – 26, 37 – 44)*** *contains 24 multiple choice questions which have* ***one or more than one correct*** *answer. Each question carries* ***+4 marks*** *for all correct answer.*
   *Full Marks : **+4** If only (all) the four option(s) is (are) chosen.*
   *Partial Marks : **+3** If all the four options are correct but ONLY three options are chosen.*
   *Partial Marks : **+2** If three or more options are correct but ONLY two options are chosen and both of which are correct options.*
   *Partial Marks : **+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.*
   *Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).*
   *Negative Marks : **–1** In all other cases.*

   Section-A (09 – 12, 27 – 30, 45 – 48) *contains 6 List-Match sets with 12 questions (each set has 2 questions). Each question has 4 statements in LIST-I & 4, 5 or 6 statements in LIST-II. The codes for Lists have choices (A), (B), (C), (D) out of which only one is correct.*
   Each question has ***only one correct*** *answer and carries* ***+3 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

2. ***Section-C (13 – 18, 31 – 36, 49 – 54)*** *contains 18 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+3 marks*** *for correct answer. There is no negative marking.*

Physics

PART – I

SECTION – A
(One or More than one correct type)

This section contains **08** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

1. A soap bubble of radius 'r' and wall thickness 'h' is made in vacuum by blowing an ideal diatomic gas in it. The surface tension of the soap solution is 'σ' and density of the soap solution is 'ρ'. Assume that the heat capacity of the soap film is much greater than that of the gas in the bubble. Then choose the correct option(s). (R = ideal gas constant)

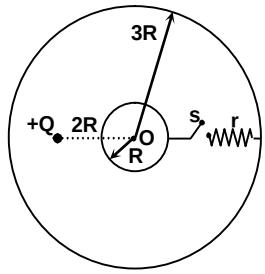
(A) The molar heat capacity of the gas in the bubble is 4R.

(B) The molar heat capacity of the gas in the bubble is 3R.

(C) The time period of small radial oscillations of the soap film is $2\pi\sqrt{\frac{\rho hr^2}{8\sigma}}$

(D) The time period of small radial oscillations of the soap film is $2\pi\sqrt{\frac{\rho hr^2}{4\sigma}}$

2. Two uncharged thin concentric spherical conducting shells 'A' and 'B' of radii R and 3R are connected through a resistance 'r' and switch 'S' initially opened as shown in the figure. A positive point charge 'Q' is fixed at a distance 2R from the centre of the shells. At t = 0, the switch 'S' is closed till the steady state is reached. Then choose the correct option(s).



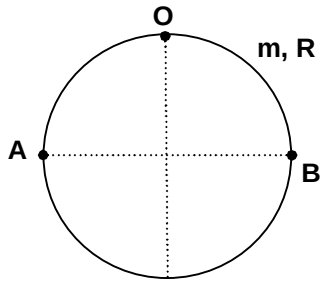
(A) The total heat dissipated in the resistor 'r' is $\frac{Q^2}{96\pi\varepsilon_0 R}$

(B) The total heat dissipated in the resistor 'r' is $\frac{Q^2}{192\pi\varepsilon_0 R}$

(C) The charge on the outer surface of shell 'A' after steady state reached is $-\frac{Q}{4}$.

(D) The charge on the outer surface of shell 'B' after steady state reached is +Q.

3. A uniform circular ring of mass 'm' and radius 'R' is hinged at its top point 'O' and suspended in a vertical plane so that it can freely rotate about a horizontal axis passing through the hinge 'O' as shown in the figure. The bead 'A' is rigidly attached to the ring and the bead 'B' can slide without friction on the ring. The mass of each bead is 'm'. The system is released from rest from the initial position shown in the figure. Then choose the correct option(s), immediately after the system is released from rest. (Take g = 10 m/s^2)



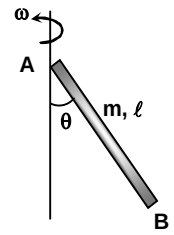
(A) The acceleration of bead 'A' is $2\sqrt{2}$ m/s^2.

(B) The acceleration of bead 'A' is 2 m/s^2.

(C) The acceleration of bead 'B' is 2 m/s^2.

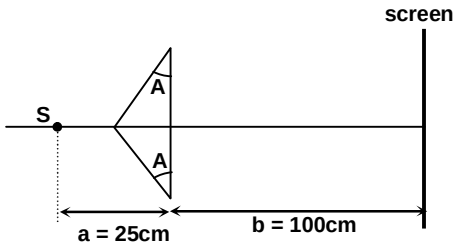
(D) The acceleration of bead 'B' is $2\sqrt{26}$ m/s^2.

4. A thin uniform rod AB of mass m = 1kg and length ℓ = 1m is hinged at end 'A'. The rod makes an angle θ = 53° with the vertical axis and it is rotating with a constant angular velocity 'ω' about the vertical axis passing through end 'A' as shown in the figure. Then choose the correct option(s).



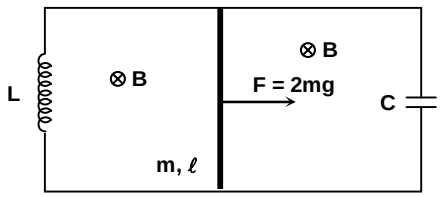
(A) The angular velocity 'ω' must be 5 rad/s.
(B) The angular velocity 'ω' must be 10 rad/s.
(C) The net hinge reaction on the rod at end 'A' is $10\sqrt{2}$ N.
(D) The net hinge reaction on the rod at end 'A' is 10 N.

5. Consider an arrangement of a thin biprism and screen as shown in the figure. The refraction angle of the biprism is A = 4° and the refractive index of its material is μ = 1.5. The wavelength of the monochromatic light emitted by the source 'S' is λ = 6000 Å. Then choose the correct option(s).



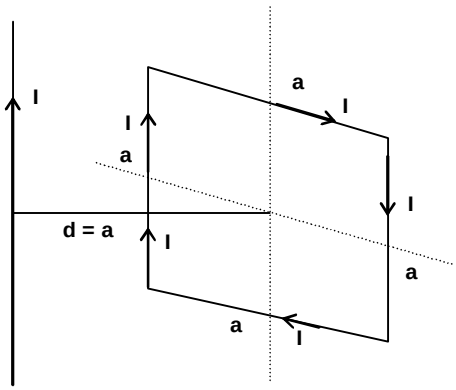
(A) The fringe width of the interference pattern obtained on the screen is 21.5 μm.
(B) The fringe width of the interference pattern obtained on the screen is 43 μm.
(C) If the source 'S' is located at large distance from the biprism then the fringe width obtained on the screen will be 8.6 μm.
(D) If the source 'S' is located at large distance from the biprism then the fringe width obtained on the screen will be 17.2 μm.

6. A conducting rod of mass 'm' and length 'ℓ' is placed on a pair of smooth horizontal conducting parallel rails connected at one end by an inductor of inductance L and by a capacitor of capacitance 'C' at the other end as shown in the figure. A uniform vertical magnetic field 'B' exists in the region perpendicular to the plane of rails. The conductor starts from rest at t = 0 from the left end of the rails due to constant external horizontal force F = 2mg applied to it. Then choose the correct option(s). (Take $CB^2\ell^2 = m$)



(A) The maximum horizontal displacement of the rod is $\frac{8mgL}{B^2\ell^2}$
(B) The maximum horizontal displacement of the rod is $\frac{4mgL}{B^2\ell^2}$
(C) The angular frequency of the oscillations of the rod is $\frac{B\ell}{\sqrt{mL}}$
(D) The angular frequency of the oscillations of the rod is $\frac{B\ell}{\sqrt{2mL}}$

7. A square loop of side 'a' and carrying current 'I' is placed at a distance d = a from an infinitely long straight wire carrying the same current 'I' as shown in the figure. The perpendicular bisector plane of the square loop contains the long straight current carrying wire. Then choose the correct options(s).



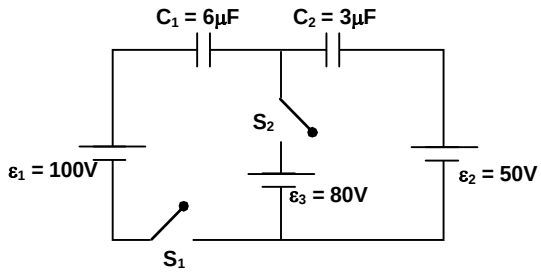
(A) The magnitude of force acting on the square loop due to long current carrying wire is $\frac{2\mu_0 I^2}{5\pi}$

(B) The magnitude of force acting on the square loop due to long current carrying wire is $\frac{\mu_0 I^2}{5\pi}$

(C) The torque experienced by the square loop due to long current carrying wire is $\frac{2\mu_0 I^2 a}{5\pi}$

(D) The torque experienced by the square loop due to long current carrying wire is $\frac{\mu_0 I^2 a}{5\pi}$

8. In the circuit as shown in the adjoining figure the switch S_1 is closed at t = 0. After steady state is reached, the switch S_2 is also closed for a long time. Then choose the correct option(s).



(A) The heat dissipated in the circuit after the switch 'S_2' is closed will be 100 μJ.

(B) The heat dissipated in the circuit after the switch 'S_2' is closed will be 50 μJ.

(C) The charge flown through the battery ε_1 after closing the switch 'S_2' is 20 μC.

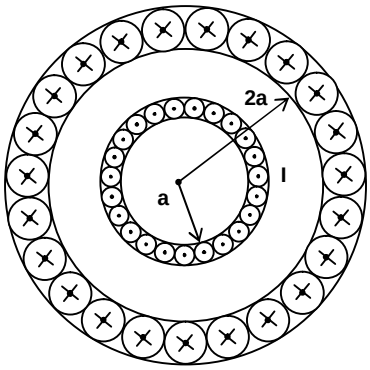
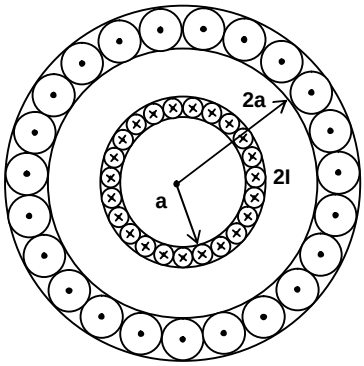
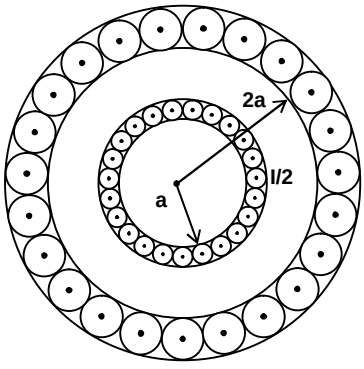
(D) The charge flown through the battery ε_2 after closing the switch 'S_2' is 10 μC.

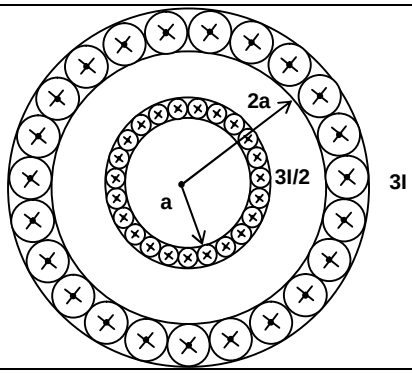
(Matching List Type)

This section contains **TWO (02)** List-Match Sets. Each List-Match set has **TWO(02)** Multiple Choice Questions. Each List-Match set has two lists: **List-I** and **List-II**. **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U). **FOUR** options are given in each Multiple Choice Question based On **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

Answer the following by appropriately matching the list based on the information given in the paragraph.

Two long current carrying thin walled coaxial cylinders are depicted in List-I and List-II gives the magnitude of magnetic pressure exerted on the wall of the cylinders.

List– I		List- II	
(I)		(P)	$\frac{\mu_0 I^2}{32\pi^2 a^2}$
(II)		(Q)	$\frac{\mu_0 I^2}{8\pi^2 a^2}$
(III)		(R)	$\frac{3\mu_0 I^2}{16\pi^2 a^2}$

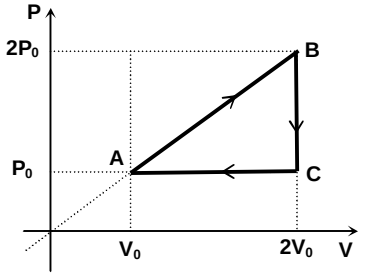
(IV)		(S)	$\frac{9\mu_0 I^2}{32\pi^2 a^2}$
		(T)	$\frac{\mu_0 I^2}{2\pi^2 a^2}$
		(U)	$\frac{9\mu_0 I^2}{16\pi^2 a^2}$

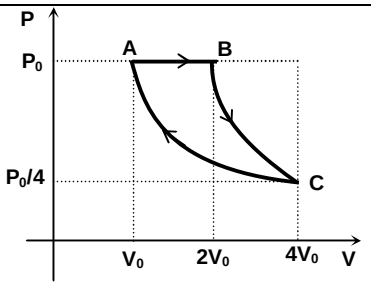
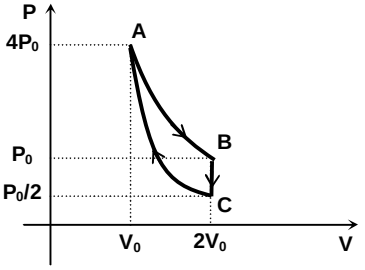
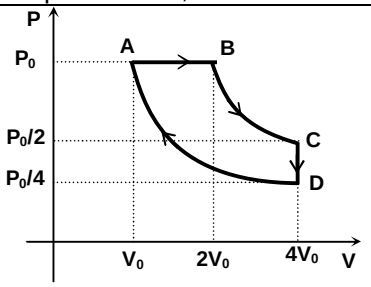
9. Find the correct match for the magnetic pressure exerted on the wall of each inner cylinder.
   (A) (I) → (Q), (II) → (T), (III) → (P), (IV) → (S)
   (B) (I) → (P), (II) → (T), (III) → (S), (IV) → (R)
   (C) (I) → (Q), (II) → (T), (III) → (R), (IV) → (S)
   (D) (I) → (P), (II) → (S), (III) → (R), (IV) → (Q)

10. Find the correct match for the magnetic pressure exerted on the wall of each outer cylinder.
   (A) (I) → (P), (II) → (Q), (III) → (T), (IV) → (S)
   (B) (I) → (Q), (II) → (S), (III) → (R), (IV) → (U)
   (C) (I) → (P), (II) → (Q), (III) → (R), (IV) → (U)
   (D) (I) → (S), (II) → (T), (III) → (R), (IV) → (P)

Answer the following by appropriately matching the list based on the information given in the paragraph.

One mole of an ideal gas is taken through four different cyclic processes as depicted in the List-I and List-II gives the values of total work done and total heat absorbed by the gas during four different cyclic processes.

List– I		List- II	
(I)	 **monoatomic gas is taken**	(P)	$\frac{P_0V_0}{2}$

(II)	 **monoatomic gas is taken** (For the process BC, PV^2 = constant and process CA is an isothermal process)	(Q)	P_0V_0
(III)	 **Diatomic gas is taken** (For the process AB, PV^2 = constant and for the process CA, PV^3 = constant)	(R)	$2P_0V_0(1-\ell n2)$
(IV)	 **Diatomic gas is taken** (Processes BC and DA are isothermal)	(S)	$\frac{5P_0V_0}{2}$
		(T)	$6P_0V_0$
		(U)	$\frac{P_0V_0}{2}(7+4\ell n2)$

11. Find the correct match for the total work done by the gas during each cyclic process.
(A) (I) → (S), (II) → (R), (III) → (P), (IV) → (T)
(B) (I) → (P), (II) → (R), (III) → (P), (IV) → (Q)
(C) (I) → (P), (II) → (Q), (III) → (R), (IV) → (T)
(D) (I) → (Q), (II) → (P), (III) → (S), (IV) → (R)

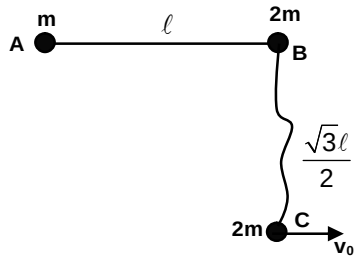
12. Find the correct match for the total heat absorbed by the gas during each cyclic process.
(A) (I) → (Q), (II) → (S), (III) → (T), (IV) → (P)
(B) (I) → (T), (II) → (S), (III) → (Q), (IV) → (R)
(C) (I) → (T), (II) → (S), (III) → (T), (IV) → (U)
(D) (I) → (P), (II) → (S), (III) → (R), (IV) → (Q)

SECTION – C
(Numerical Answer Type)

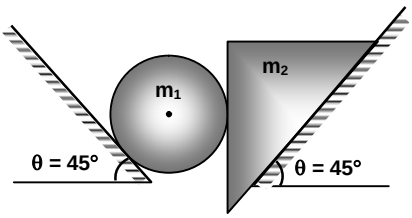
This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

---

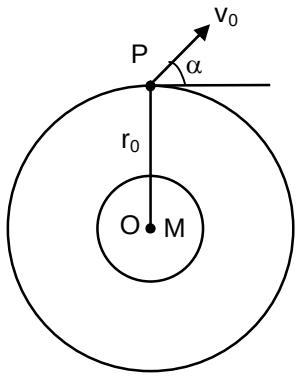
13. Three small balls A, B and C of masses m, 2m and 2m respectively are placed on a smooth horizontal surface. Balls A and B as well as B and C are connected by light inextensible strings each of equal length ℓ. The string connecting balls A and B is taut. The initial distance between the balls B and C is $\frac{\ell\sqrt{3}}{2}$ and BC is perpendicular to AB. The ball 'C' is given a horizontal velocity v_0 = 20.7 m/s parallel to AB as shown in the figure. Find the initial velocity (in m/s) of the ball 'A'.



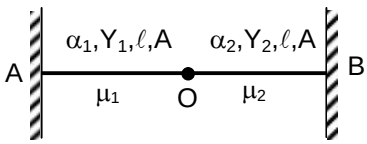
14. A cylinder and a wedge with a vertical face touching each other, move along two smooth inclined planes making the same angle θ = 45° with the horizontal as shown in the figure. The masses of the cylinder and the wedge are m_1 = 5 kg and m_2 = 3 kg respectively. There is no friction between the cylinder and the wedge. Find the normal force (in Newton) acting between the cylinder and the wedge. (Take g = 10 m/s^2)



15. A satellite is projected into space from a point 'P' at a distance 'r_0' from the centre of the earth with an initial velocity $v_0 = \sqrt{\frac{GM}{r_0}}$ at an angle α = 37° with the horizontal at point 'P' as shown in the figure. It is found that the satellite is propelled into an elliptical orbit. Find the eccentricity of the elliptical orbit.



16. Two wires of same length ℓ = 1 m and same cross-sectional area A = 10 mm^2 are welded together and their ends are rigidly clamped between two walls as shown in the figure. Given



$\alpha_1 = 2 \times 10^{-5}$ C^{-1}, $Y_1 = 1 \times 10^{10}$ N/m^2, μ_1 = 0.4 kg/m
$\alpha_2 = 1 \times 10^{-5}$ C^{-1}, $Y_2 = 2 \times 10^{10}$ N/m^2, μ_2 = 0.1 kg/m
The tension in the wires is produced by reducing the temperature of system by 20°C. Find the first overtone frequency (in Hz) of transverse vibration of the system with joint 'O' as a node.

17. A metallic surface is illuminated alternatively with electromagnetic radiations of wavelengths 4500 A^0 and 6000 A^0. It is observed that the maximum speeds of the photoelectrons emitted under these illuminations are in the ratio 2 :1. Find the work function (in eV) of the metal. (Take hC = 1242 eV-nm)

18. The pitch of a screw gauge is 1 mm and its circular scale is divided into 100 divisions. When nothing is placed between its studs, the zero of the circular scale lies 6 divisions below the reference line. When a wire is placed between its studs, the main scale reading is 2 mm and 56^{th} division of circular scale coincides with the reference line of the main scale. The length of wire is 8.68 cm. Find the curved surface area (in cm^2) of the wire in true significant figures.

Chemistry

PART – II

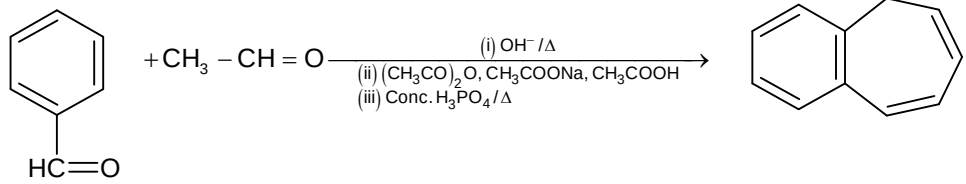
SECTION – A
(One or More than one correct type)

This section contains **08** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

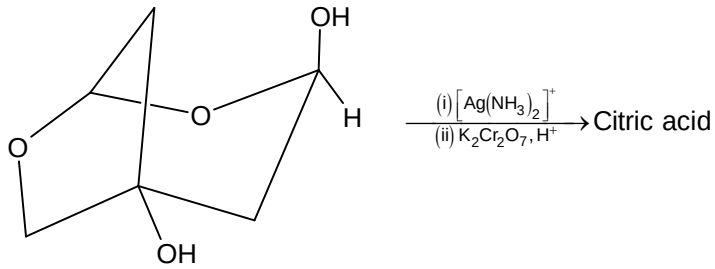
19. Identify the complex(es) in which C – O bond length is less than the C – O bond length in $Fe(CO)_5$:
    (A) $[Cr(CO)_6]$
    (B) $[Cr(NH_3)(CO)_5]$
    (C) $[Cr(NH_3)_2(CO)_4]$
    (D) $[Cr(NH_3)_3(CO)_3]$

20. Identify the reaction(s) with their correct major product?

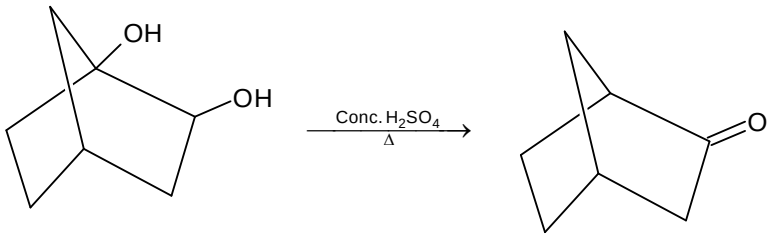
(A)



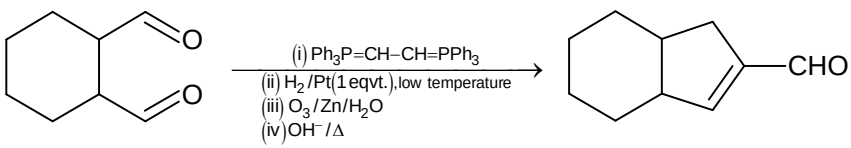
(B)



(C)



(D)



21. ACl_3 is an unstable compound and gradually changing into A_2O_5 by its reaction with atmospheric oxygen, according to the following partial reaction:

$2ACl_3 \longrightarrow A_2O_5$

The above reaction follows second order kinetics. The reaction is started by taking 1 mole of ACl_3 in a container of volume 1 L and allowed to stand for some time. After 10 min, the reaction mixture requires 250 mL of 0.8 M acidified $KMnO_4$ solution; where ACl_3 is oxidized into AO_3^- and assume A_2O_5 do not react with $KMnO_4$. Now, choose the correct statement(s) of the following:

(A) Half-life time of the reaction is 10 minutes.

(B) Rate constant of the reaction is 0.05 $\ell mol^{-1} min^{-1}$.

(C) Rate constant of the reaction is 0.10 $\ell mol^{-1} min^{-1}$.

(D) After 20 mins from the start, 33.33% of ACl_3 is left unreacted.

22. 100 ml of 0.1 M aqueous solution of sodium phosphate is titrated using 0.1 M HCl. Now, choose the correct statement(s) of the following:

(For H_3PO_4 : $K_{a_1} = 10^{-3}; K_{a_2} = 10^{-8}$ and $K_{a_3} = 10^{-13}$ at 25^oC)

(A) The pH of the solution is 13 at 25^oC, when the titre value is 50 mL.

(B) pH of the solution is 10.5 at 25^oC, when the titre value is 100 mL.

(C) pH of the solution is 8 at 25^oC, when the titre value is 150 mL.

(D) pH of the solution is 3 at 25^oC, when the titre value is 200 mL.

23. Choose the correct statement(s) among the following:

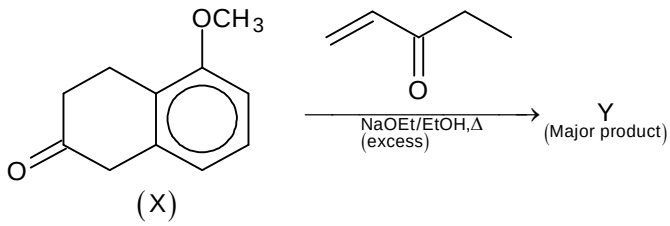
(A) When ideal gases are mixed isothermally, the entropy of mixing is always positive.

(B) At equilibrium state, chemical potential of a species is the same in all the phases of the system.

(C) A reversible adiabatic process is always isoentropic.

(D) When a gas is allowed to expand, the maximum work is obtained when the process is carried out irreversibly.

24.



(X)

Identify the correct statement (s) about (X) and (Y)

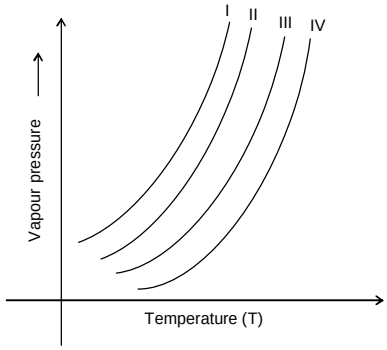
(A) Both (X) and (Y) gives +ve Bromine water test.

(B) Both (X) and (Y) gives positive 2,4-dinitro phenyl hydrazine (D.N.P.) test.

(C) Degree of unsaturation of (Y) is 8.

(D) (Y) on heating with triphenyl phosphine ethylidene gives a product which also gives Bromine water test.

25. The following diagram shows the vapour pressure curves for CH_3F, CH_3OH, CH_3COOH and CH_2O.



Now choose the correct match from the options given below:
(A) III = CH_3OH
(B) II = CH_3F
(C) I = CH_2O
(D) IV = CH_3COOH

26. Identify the correct statement(s) of the following:
(A) The mineral kionite ($Ca_2Cu_2Si_3O_{10}.2H_2O$) is a chain silicate.
(B) HF is used as etching agent in glass industry.
(C) In the titration of Fe^{3+} with Sn^{2+}, KCNS can be used as a suitable indicator.
(D) On reaction of red lead with conc. HCl, a golden yellow gas is evolved.

(Matching List Type)

This section contains **TWO (02)** List-Match Sets. Each List-Match set has **TWO(02)** Multiple Choice Questions. Each List-Match set has two lists: **List-I** and **List-II**. **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U). **FOUR** options are given in each Multiple Choice Question based On **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

Answer the following by appropriately matching the list based on the information given in the paragraph.

Some statement(s) are given in List-I and some thermodynamic properties in List-II:

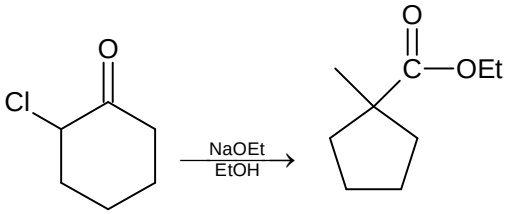
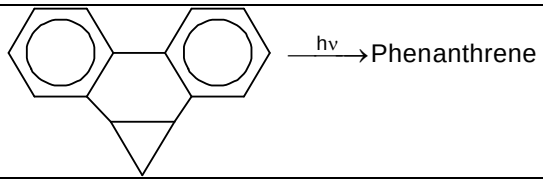
LIST–I		**LIST–II**	
(I)	A process in which heat change is positive at constant volume (V) and constant pressure (P)	(P)	$W = 0$
(II)	An isobaric process in which ΔU is negative and $\Delta S = 0$	(Q)	$\Delta U = 0$
(III)	An adiabatic process in which ΔS is positive and work done is zero.	(R)	Process must be reversible
(IV)	An isobaric, iso-entropic process in which ΔH is negative.	(S)	ΔV must be equal to 0 for spontaneity
		(T)	$\Delta H = 0$
		(U)	The process must be spontaneous

27. Choose the INCORRECT match from the options given below:
(A) (I), (U)
(B) (II), (S)
(C) (III), (R)
(D) All are incorrect

28. Identify the correct matching from the following options:
(A) (II), (U)
(B) (III), (U)
(C) (I), (U)
(D) (IV), (T)

Answer the following by appropriately matching the list based on the information given in the paragraph.

Some reactions/processes are given in List-I and some reaction intermediates (which can be formed in the reaction given in List - I) are given in List-II:

LIST–I		**LIST–II**	
(I)	To determine the sequence of amino acids in a polypeptide chain by using 2,4-Dinitrofluorobenzene	(P)	Carbocation
(II)		(Q)	Carbanion
(III)	Cyclohexanone oxime $\xrightarrow{AlCl_3}$ Caprolactam	(R)	Free radical
(IV)		(S)	Benzyne
		(T)	Nitrene
		(U)	Carbene

29. Choose the correct match from the options given below:
(A) (I), (S)
(B) (IV), (U)
(C) (III), (Q)
(D) (II), (P)

30. Choose the INCORRECT match from the options given below:
(A) (IV), (R)
(B) (II), (Q)
(C) (III), (P)
(D) All are correct

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

---

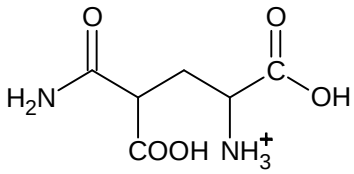
31. Half the number of compounds (given below) which reacts with PCl_5 to give $POCl_3$; is:
Sulphuric acid, orthophosphoric acid, sulphurous acid, ethanoic anhydride, benzene sulphonic acid, H_2O, P_4O_6, P_4O_{10}, SO_2, SO_3, ethoxy ethane, CO_2 Palmitic acid, N_2.

32. To an aqueous solution of weak acid (HA), 20 mL NaOH solution is added and connected to a reference $Ag|AgCl|Cl^-$ (0.1 M) electrode. EMF of the cell was found to be 0.56 V. To this solution of HA, 30 mL of NaOH from same stock solution was added further and new EMF was found to be 0.62 V. Determine the dissociation constant (pK_a) of the weak acid (HA) [Take partial pressure of H_2 = 1 bar and temperature of reaction is 298 K, $\log_{10} 2 = 0.3, \frac{2.303RT}{F} = 0.06$]

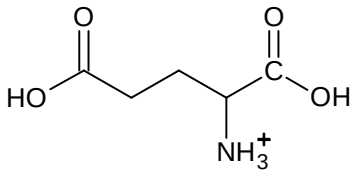
$\left(E^{o}_{Cl^-|AgCl/Ag} = 0.20\ V\right)$

33. Some amino acids are given below:

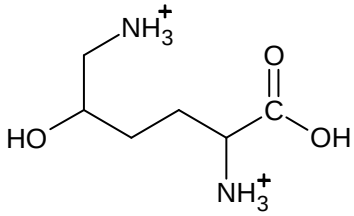
(I)



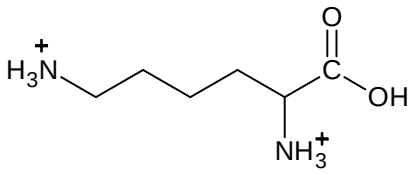
(II)



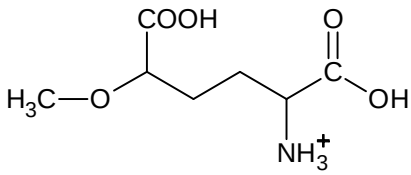
(III)



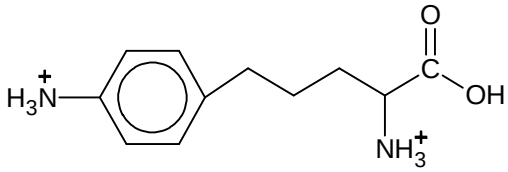
(IV)



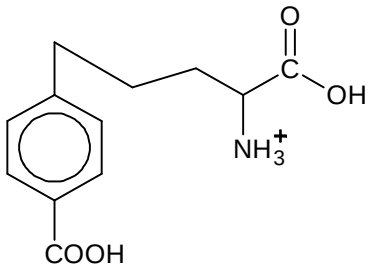
(V)



(VI)



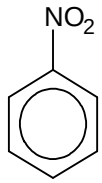
(VII)



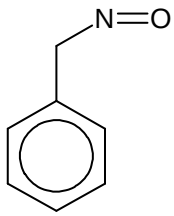
Let, the number of amino acids which exists as cation at pH = 7 be 'x' and the number of amino acids which exists as anion at pH = 7 be 'y', then find the value of $\frac{x}{y}$.

34. Consider the following compounds:

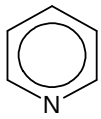
(I)



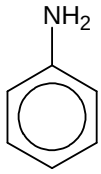
(II)



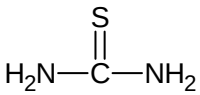
(III)

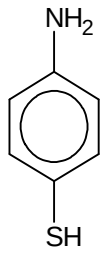


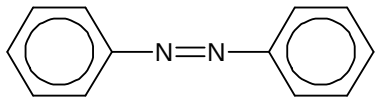
(IV)



(V)

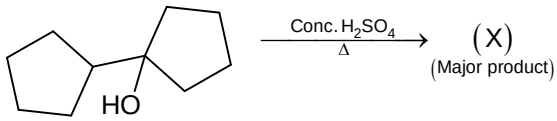


(VI) 

(VII) 

Let, the compound(s) which cannot be Kjeldahlised be 'x'
the compound(s) which gives blue colour in Lassaigne's test of nitrogen be y and
the compound(s) which gives red colour in Lassaigne's test of nitrogen be z.
So, the value of $\left(\frac{x+y}{z}\right)$ will be:

35. The chromium in a 1.0 g sample of chromite ($FeCr_2O_4$) was oxidized to Cr^{+6} state by fusion with Na_2O_2. The fused mixture was treated with excess of water and boiled to destroy the excess of peroxide. After acidification, the sample was treated with 50 mL of 0.16 M $FeSO_4$ solution. 3.67 mL of 0.05 M $Cr_2O_7^{2-}$ solution was required to oxidize the Fe^{2+} ion left unreacted. Determine the mass % of chromite in the original sample. [Cr = 52, Fe = 56]

36. 

Let, the degree of unsaturation in 'X' be 'x' and the number of peaks in the energy profile for the whole course of the above reaction be 'y', then find the value of $\frac{2x}{y}$.

Mathematics PART – III

SECTION – A
(One or More than one correct type)

This section contains **08** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.

37. The roots α, β of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ lie in the interval [0, 1], then?

(A) the maximum value of $\frac{(a-b)(2a-b)}{a(a-b+c)}$ is 3

(B) the minimum value of $\frac{(a-b)(2a-b)}{a(a-b+c)}$ is 2

(C) if $\frac{(a-b)(2a-b)}{a(a-b+c)}$ is at its maximum value, then $\alpha^2 + \beta^2$ can be 2

(D) if $\frac{(a-b)(2a-b)}{a(a-b+c)}$ is at its maximum value, then $\alpha^2 + \beta^2$ can be 1

38. Let $a_1, a_2, a_3, \ldots, a_n$ be a sequence of real numbers which satisfies the relation $a_{n+1} = \sqrt{a_n^2 + 1}$ $\forall$ $n \in N$. Suppose that there exists a positive integer n_0 such that $a_{2n_0} = 3a_{n_0}$, then?

(A) $\lim\limits_{n\to\infty} \frac{2a_n}{\sqrt{n+2020}} = 2$

(B) if $T_r = \sum\limits_{n=1}^{n=r} a_n^2$, the min$(T_1, T_2, \ldots, T_r) = -1$

(C) $\lim\limits_{n\to\infty} (a_{n+1} + a_n) n^{-\frac{1}{2}} = 2$

(D) $\left[\frac{1}{2}\sum\limits_{n=1}^{49}\sqrt{\frac{8}{8a_n^2+7}}\right] = 6$, (where [.] denotes the greatest integer function)

39. If exactly one number from the set {1, 2, 3, ….., 8} is assigned to the each vertex of a cube $A_1A_2A_3$ ….. A_8, then?

(A) number of ways of assigning the numbers such that the sum of any two numbers with a common edge is always odd is $2 \times (4!)^2$

(B) number of ways of assigning the numbers such that the sum of any two numbers with a common body diagonal is always even is $6 \times (4!)^2$

(C) number of ways of assigning the numbers such that the sum of any two numbers with a common edge is always even in $(4!)^2$

(D) the number of equilateral triangles formed by joining vertices of the given cube is equal to 8

40. The lines L_1, L_2, L_3 are parallel to each other and L_2 lies between L_1 and L_3, the distance between L_1 and L_2 is 'a' unit and that between L_2 and L_3 is 'b' unit. If the area of an equilateral triangle having one vertex on each of the three given parallel lines is a function g(a, b), then?

(A) $g(1,1) = \sqrt{3}$ unit^2

(B) minimum value of g(1, b) is $\frac{\sqrt{3}}{4}$ unit^2

(C) eccentricity of the curve g(x, y) = $\sqrt{3}$ is $\frac{2\sqrt{2}}{3}$

(D) $g(2,1) = \frac{7}{\sqrt{3}}$ unit^2

41. If all the pairs (x, y) such that x, y $\in$ I and x, y satisfying the following system of inequalities, $x^2 + x + y \leq 3$ and $y^2 + y + x \leq 3$, then?
(A) total number of such pair of (x, y), x, y $\in$ I are 9
(B) total number of squares formed by joining such point are 14
(C) total number of such pairs of (x, y) are 17
(D) total number of squares formed by joining such point are 20

42. A bug starts from the origin O(0, 0) in the co-ordinate plane jumping from one point to another at the rate of one jump per second. It moves according to the rule that from (m, n), it jumps to either (m, n + 1) or (m + 1, n), (m, n $\in$ W), either been equally likely. Five second from the start the bug reaches (α, β), then?
(A) the number of different values that $|\alpha - \beta|$ can take is 3
(B) probability that bug reaches (α, β) if $|\alpha - \beta| = 1$, is $\frac{10}{16}$
(C) probability that bug reaches (α, β) if $|\alpha - \beta| > 1$ is $\frac{1}{8}$
(D) probability that bug reaches (α, β) if $|\alpha - \beta| > 4$ is $\frac{1}{3}$

43. Let ABC be a triangle and r, r_1, r_2, r_3 denote its in-radius and the exradii opposite to vertices A, B C respectively. If $a > r_1$, $b > r_2$ and $c > r_3$, then?
(A) $\angle$B is obtuse angle
(B) $\angle$A is acute angle
(C) 3r > s, where s is semi-perimeter
(D) 3r < s, where s is semi-perimeter

44. Let S be the curve $y = x^2$ and S′ be the mirror image of S with respect to the line passing through the points A(0, –10) and B(2, 0), then?
(A) if C is a point on S′, then the minimum value of area of triangle ABC is $\frac{15}{2}$ unit^2
(B) shortest distance between S and S′ is $\frac{15}{2\sqrt{26}}$ unit
(C) if D is a point on S′, then the minimum area of triangle ABD is 0 unit^2
(D) if C (α, β) is point on S′ such that the area of the ΔABC is minimum then the value of $(\alpha - 2)^2 + \beta^2$ is $\frac{629}{16}$ unit^2

(Matching List Type)

This section contains **TWO (02)** List-Match Sets. Each List-Match set has **TWO(02)** Multiple Choice Questions. Each List-Match set has two lists: **List-I** and **List-II**. **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U). **FOUR** options are given in each Multiple Choice Question based On **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.

Answer the following by appropriately matching the list based on the information given in the paragraph.

MM-I. Let $S_1 : x^2 - y^2 - 4 = 0$, $S_2 : y^2 - 4x = 0$, $S_3 : x^2 + y^2 - 4 = 0$ and $S_4 : \frac{x^2}{4} + \frac{y^2}{1} - 1 = 0$ are four given conics and A, B, C, D are set of all the tangents of S_1, S_2, S_3 and S_4 respectively.
List-I contains the sets A, B, C and D. List-II contains information regarding the elements in these sets

LIST-I		**LIST-II**	
(I)	A	(P)	$4x - 2y + 1 = 0$
(II)	B	(Q)	$2x - y + 2\sqrt{3} = 0$
(III)	C	(R)	$x - y - \sqrt{5} = 0$
(IV)	D	(S)	$2x - y + 2\sqrt{5} = 0$
		(T)	$x - 2 = 0$
		(U)	$x + 2 = 0$

45. Which of the following is the only CORRECT combination?
(A) (I), (P), (T)
(B) (III), (R), (S)
(C) (I), (Q), (U)
(D) (II), (P), (T)

46. Which of the following is the only CORRECT combination?
(A) (IV), (P), (T), (U)
(B) (I), (Q), (T), (U)
(C) (II), (P), (S)
(D) (IV), (R), (S)

Answer the following by appropriately matching the list based on the information given in the paragraph.

MM-II. Let $f(x)=\frac{x}{2}+\frac{\sin 2x}{4}$, $g(x)=\left(\frac{x}{2}+\frac{\sin 2x}{4}\right)e^{\sin x}$ and

$h(x)=\begin{cases}\left(x\sin\frac{1}{x^4}\right)e^{-\frac{1}{x^2}} & ;\ x\neq 0\\ 0 & ;\ x=0\end{cases}$, $t(x)=\begin{cases}e^{-\frac{1}{x^2}} & ;\ x\neq 0\\ 0 & ;\ x=0\end{cases}$ are four real functions.

List-I contains some functions related to functions f, g, h and t and List-II contains some information regarding functions given in List-I

LIST-I		LIST-II	
(I)	$\frac{f(x)}{g(x)}$	(P)	Limit x tends to 0 exist
(II)	$\frac{f'(x)}{g'(x)}$	(Q)	Limit x tends to 0 does not exist
(III)	$\frac{h(x)}{t(x)}$	(R)	Limit x tends to ∞ exist
(IV)	$\frac{h'(x)}{t'(x)}$	(S)	Limit x tends to ∞ does not exist
		(T)	Limit x tends to ∞ is equal 0
		(U)	Limit x tends to 0 is equal 1

47. Which of the following is the only CORRECT combination?
(A) (I), (P), (R)
(B) (II), (Q), (T)
(C) (III), (P), (T)
(D) (II), (P), (S)

48. Which of the following is the only INCORRECT combination?
(A) (IV), (Q), (T)
(B) (I), (P), (S)
(C) (II), (P), (T), (U)
(D) (IV), (P), (S)

SECTION – C
(Numerical Answer Type)

This section contains **06** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

49. A red ball and a green ball are randomly and independently tossed into bins numbered 1, 2, 3 ….. such that for each ball the probability that it gets tossed into bin k is $2^{-k}\ \forall\ k \in N$. If P is the probability that the red ball is tossed into a higher numbered bin than the green ball, then 7P is?

50. Let $(a^2 + 1)(b^2 + 1)(c^2 + 1) = 9$, where $a, b, c \in R$, then the number of integers in the range of $ab + bc + ca$ is?

51. If $\int_a^b e^{\cos x}(380 - x - x^2)\,dx$ attains its maximum value for some a, b such that $a \le b$, then the maximum value of b – a is?

52. The number of continuous functions f : R $\to$ R, such that $(2f(x) + x)(6f^2(x) + 5xf(x) + x^2) = 0$, $\forall\, x \in$ R is equal to ?

53. If [p, q) is the exhaustive interval of values of x, satisfying the equation $\left[\frac{x^2 + x - 1}{x^2 - x + 1}\right] = \left[\frac{x - 1}{x + 1}\right]$, (where [.] denotes the greatest integer function), then $\left(q + p + \frac{1}{2}\right)^2$ is equal to?

54. If f(x) is a function continuous at all points except at x = 0, such that f: R – {0} $\to$ R, f(2020) = 1, f(–3) = –1, and $f(x)f(y) = 2f(xy) - f\left(\frac{2020}{x}\right)f\left(\frac{2020}{y}\right)$, $\forall$ x, y $\in$ R – {0}, then the value of $\int_{-1}^{2019} f(x)\,dx$ is equal to?

ALL INDIA TEST SERIES

TEST – 18

JEE (Advanced)

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. A, C

Sol. $P = P_0 + \frac{4\sigma}{r}$

Since outside pressure, $P_0 = 0$

$P = \frac{4\sigma}{r} \Rightarrow PV^{1/3}$ = constant

Hence molar heat capacity of the gas, $C = C_V + \frac{R}{(1-x)} = \frac{5R}{2} + \frac{3R}{2}$

$\Rightarrow C = 4R$

Now, when the radius of the soap bubble is increased by dr = x(say) from equilibrium, then the net restoring force acting on the soap bubble

$F = 2 \times \frac{4\sigma}{r^2} dr 4\pi r^2 = 32\sigma\pi dr = 32\sigma\pi x$

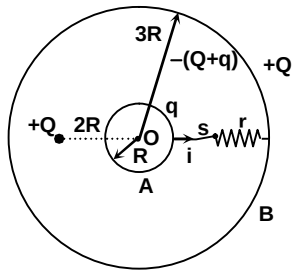
Now, $\rho 4\pi r^2 h \frac{dv}{dt} = -32\sigma\pi x$

$\frac{d^2x}{dt^2} = -\left(\frac{8\sigma}{\rho hr^2}\right)x$

Time period of small radial oscillations of the soap film, $T = 2\pi\sqrt{\frac{\rho hr^2}{8\sigma}}$

2. B, C, D

Sol. In steady state,



$V_A = V_B$

$$\frac{kq}{R}+\frac{kQ}{2R}-\frac{kq}{3R}=\frac{kQ}{3R}$$

$$\frac{2kq}{3R}=-\frac{kQ}{6R}$$

$$\Rightarrow q=-\frac{Q}{4}$$

At any time 't' the current through the resistor 'r' is

$$ir=\left(\frac{kq}{R}-\frac{kq}{3R}+\frac{kQ}{2R}\right)-\frac{kQ}{3R}$$

$$ir=\frac{2kq}{3R}+\frac{kQ}{6R}=\frac{k}{6R}(4q+Q)$$

$$i=\frac{k}{6Rr}(4q+Q)=\alpha(4q+Q) \qquad \left[\text{where } \alpha=\frac{k}{6Rr}\right]$$

$$-\frac{dq}{dt}=\alpha(4q+Q) \Rightarrow -\int_0^q\frac{dq}{(4q+Q)}=\alpha\int_0^t dt$$

$$\Rightarrow \ell n\left(\frac{4q+Q}{Q}\right)=-4\alpha t$$

$$q=\frac{Q}{4}\left[e^{-4\alpha t}-1\right]$$

$$i=-\frac{dq}{dt}=Q\alpha e^{-4\alpha t}$$

Now the total heat dissipated in the resistance 'r' is

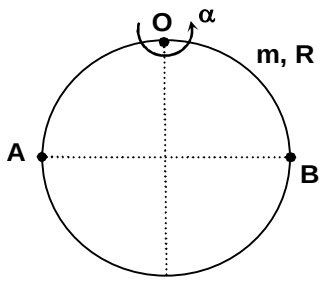
$$H=\int_0^\infty i^2 r dt=Q^2\alpha^2 r\int_0^\infty e^{-8\alpha t}dt=\frac{Q^2\alpha^2 r}{8\alpha}$$

$$H=\frac{Q^2\alpha r}{8}=\frac{Q^2 r}{8}\left(\frac{k}{6Rr}\right)=\frac{kQ^2}{48R}$$

$$H=\frac{Q^2}{192\pi\varepsilon_0 R}$$

3. A, D

Sol. $\tau = I\alpha$



$mgR - NR = 4mR^2\alpha$

$mg - N = 4m\alpha R$ …(i)

$N = m\alpha R$ …(ii)

From (i) and (ii)

$mg = 5m\alpha R$

$\alpha = g/5R$

$\therefore$ acceleration of bead 'A' is

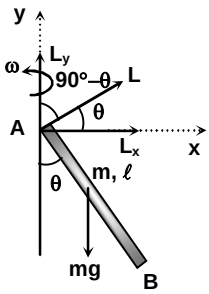
$$a_A=\alpha R\sqrt{2}=\frac{g\sqrt{2}}{5}=2\sqrt{2}\ m/s^2$$

$\therefore$ acceleration of bead 'B' is $a_B=\frac{g}{5}\sqrt{26}=2\sqrt{26}\ m/s^2$

Then normal force on the bead 'B' due to ring $N = m\alpha R = mg/5$

4. A, C

Sol. Total angular momentum of the rod about end 'A' is $L = \frac{m\ell^2}{3}\omega\sin\theta$



$$L_x = L\cos\theta = \frac{m\ell^2}{3}\omega\sin\theta\cos\theta$$

$$L_y = L\sin\theta = \frac{m\ell^2}{3}\omega\sin^2\theta$$

$$\vec{L} = \vec{L}_x + \vec{L}_y$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}_x}{dt} + \frac{d\vec{L}_y}{dt}$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}_x}{dt}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d\vec{L}_x}{dt}$$

$$mg\frac{\ell}{2}\sin\theta = L_x\omega$$

$$mg\frac{\ell}{2}\sin\theta = \frac{m\ell^2}{3}\omega^2\sin\theta\cos\theta$$

$$\omega = \sqrt{\frac{3g}{2\ell\cos\theta}} = \sqrt{\frac{3\times10\times5}{2\times1\times3}} = 5 \text{ rad/s}$$

Now let F_h and F_v are the horizontal and vertical components of hinge reaction acting on the rod at end 'A'

$$F_h = \frac{m\omega^2\ell}{2}\sin\theta = 1\times25\times\frac{1}{2}\times\frac{4}{5} = 10 \text{ N}$$

$F_V = mg = 10$ N

The net hinge reaction

$$F = \sqrt{F_h^2 + F_v^2} = 10\sqrt{2} \text{ N}$$

5. B, C

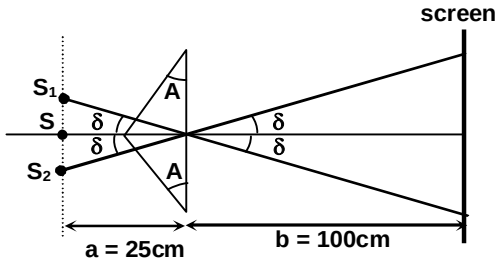
Sol. $\delta = (\mu - 1)A = (1.5 - 1)4 = 2 = 2\times\frac{\pi}{180} = \frac{\pi}{90}$ radian

The separation between two images formed is $S_1S_2 = 2a\delta$

$d = 2a\delta$

and $D = (a + b)$

$$\therefore \text{ fringe width, } \omega = \frac{\lambda D}{d} = \frac{\lambda(a+b)}{2a\delta} = \frac{6\times10^{-7}}{2\times\frac{\pi}{90}}\left(\frac{125}{25}\right) = 43\,\mu\text{m}$$



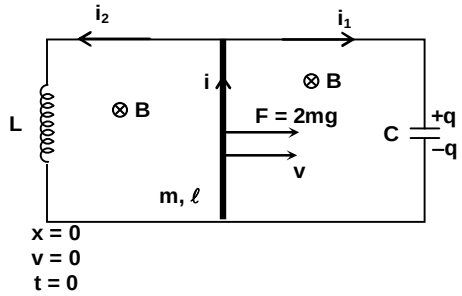
If the source is located at large distance form the biprism ($a \to \infty$)

$\therefore$ fringe width, $\omega = \frac{\lambda}{2\delta}\left(\frac{a+b}{a}\right) = \frac{\lambda}{2\delta}\left(1+\frac{b}{a}\right) = \frac{\lambda}{2\delta}$

$$\omega = \frac{\lambda}{2\delta} = \frac{6\times10^{-7}}{2\times\frac{\pi}{90}} = 8.6\ \mu m$$

6. B, D

Sol. $\varepsilon = Bv\ell$



$q = C\varepsilon = CBv\ell$

$$i_1 = \frac{dq}{dt} = CB\ell\frac{dv}{dt} \qquad \ldots(i)$$

$$\frac{Ldi_2}{dt} = Bv\ell$$

$$\int di_2 = \left(\frac{B\ell}{L}\right)\int v dt$$

$$\int_0^{i_2} di_2 = \frac{B\ell}{L}\int_0^x dx$$

$$i_2 = \left(\frac{B\ell}{L}\right)x \qquad \ldots(ii)$$

$$m\frac{dv}{dt} = F - Bi\ell$$

$$\frac{mdv}{dt} = F - B\ell(i_1 + i_2)$$

$$\frac{mdv}{dt} = F - B\ell i_1 - B\ell i_2$$

$$m\frac{dv}{dt} = 2mg - B\ell\left(CB\ell\frac{dv}{dt}\right) - B\ell\left(\frac{B\ell}{L}\right)x$$

$$(m + CB^2\ell^2)\frac{d^2x}{dt^2} = -\frac{B^2\ell^2}{L}\left(x - \frac{2mgL}{B^2\ell^2}\right)$$

$$\frac{d^2x}{dt^2} = -\frac{B^2\ell^2}{2mL}\left(x - \frac{2mgL}{B^2\ell^2}\right) \quad [\because CB^2\ell^2 = m)$$

$\therefore$ angular frequency of SHM executed by the rod is

$$\omega = \frac{B\ell}{\sqrt{2mL}}$$

The amplitude of SHM executed by the rod is

$$A = \frac{2mgL}{B^2\ell^2}$$

The maximum horizontal displacement of the rod

$$x_{max} = 2A = \frac{4mgL}{B^2\ell^2}$$

7. A, C

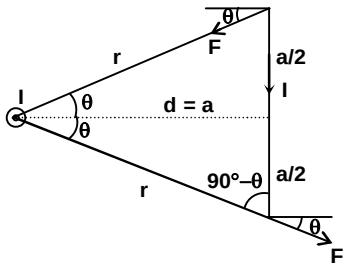
Sol. The net force acting on the square loop,

$F_0 = 2F \sin\theta$

$= 2BIa \sin\theta$

$= \frac{2\mu_0 I^2 a}{2\pi r}\frac{a}{2r}$

$F_0 = \frac{2\mu_0 I^2}{5\pi}$



The torque experienced by the square loop,

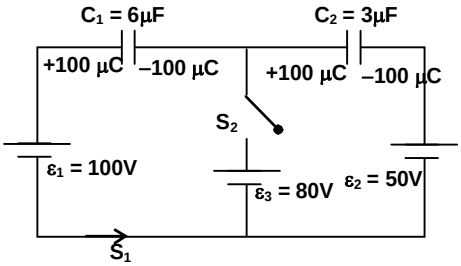
$\tau = BIa\cos\theta\, a = BIa^2\cos\theta$

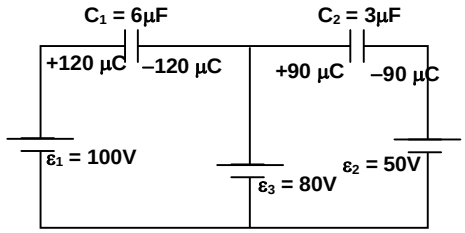
$\tau = \frac{\mu_0 I^2 a^2}{2\pi r}\frac{a}{r} = \frac{2\mu_0 I^2 a}{5\pi}$

$\tau = \frac{2\mu_0 I^2 a}{5\pi}$

8. B, C, D

Sol.





The charge flown through the battery ε_1 and ε_2 after closing the switch 'S_2',

$\Delta Q_1 = 120 - 100 = 20\ \mu C$

$\Delta Q_2 = -90 - (-100) = 10\ \mu C$

Now, $\Delta W_b = \Delta U + \Delta H$

$20 \times 10^{-6} \times 100 + 10 \times 10^{-6} \times 50 - 30 \times 10^{-6} \times 80 =$

$$\frac{1}{2}\times 6\times 10^{-6}\times 400 + \frac{1}{2}\times 3\times 10^{-6}\times 900 - \frac{1}{2}\times 2\times 10^{-6}\times 2500 + \Delta H$$

$\Rightarrow 100 \times 10^{-6} = 50 \times 10^{-6} + \Delta H$

$\Rightarrow \Delta H = 50 \times 10^{-6} = 50\ \mu J$

9. A

Sol. The magnetic pressure exerted on the wall of inner cylinder.

(I) $P_i = \frac{\mu_0}{2}\left(\frac{I}{2\pi a}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 a^2}$

(II) $P_i = \frac{\mu_0}{2}\left(\frac{2I}{2\pi a}\right)^2 = \frac{\mu_0 I^2}{2\pi^2 a^2}$

(III) $P_i = \frac{\mu_0}{2}\left(\frac{I}{2\times 2\pi a}\right)^2 = \frac{\mu_0 I^2}{32\pi^2 a^2}$

(IV) $P_i = \frac{\mu_0}{2}\left(\frac{3I}{2\times 2\pi a}\right)^2 = \frac{9\mu_0 I^2}{32\pi^2 a^2}$

10. C

Sol. The magnetic pressure exerted on the wall of outer cylinder.

(I) $P_0 = \frac{I}{4\pi a}\left(\frac{\mu_0 I}{4\pi a} - \frac{\mu_0 I}{8\pi a}\right) = \frac{\mu_0 I^2}{32\pi^2 a^2}$

(II) $P_0 = \frac{2I}{4\pi a}\left(\frac{\mu_0 2I}{4\pi a} - \frac{\mu_0 2I}{8\pi a}\right) = \frac{\mu_0 I^2}{8\pi^2 a^2}$

(III) $P_0 = \frac{2I}{4\pi a}\left(\frac{\mu_0 I}{8\pi a} + \frac{\mu_0 2I}{8\pi a}\right) = \frac{3\mu_0 I^2}{16\pi^2 a^2}$

(IV) $P_0 = \frac{3I}{4\pi a}\left(\frac{\mu_0 3I}{8\pi a} + \frac{\mu_0 3I}{8\pi a}\right) = \frac{9\mu_0 I^2}{16\pi^2 a^2}$

11. B

12. C

Sol. **(For Q.11 to Q.12):**

(I) $\Delta W_{cycle} = \frac{1}{2} \times V_0 \times P_0 = \frac{P_0 V_0}{2}$

$\Delta Q_{AB} = nC\Delta T = n2R(4T_0 - T_0) = 6nRT_0 = 6P_0V_0$

$\Delta Q_{BC} = nC_V\Delta T = n\frac{3R}{2}(2T_0 - 4T_0) = -3nRT_0 = -3P_0V_0$

$\Delta Q_{CA} = nC_P\Delta T = \frac{5}{2}nR(T_0 - 2T_0) = -\frac{5}{2}nRT_0 = -\frac{5}{2}P_0V_0$

Total heat absorbed by the gas = $6P_0V_0$

(II) $\Delta Q_{AB} = nC_P\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}P_0V_0$

$\Delta Q_{BC} = nC\Delta T = n\frac{R}{2}(T_0 - 2T_0) = -\frac{nRT_0}{2} = -\frac{P_0V_0}{2}$

$\Delta Q_{CA} = nRT_0\ell n\left(\frac{V_0}{4V_0}\right) = -2nRT_0\ell n2 = -2P_0V_0\ln 2$

$\Delta W_{cycle} = \frac{5P_0V_0}{2} - \frac{P_0V_0}{2} - 2P_0V_0\ell n2 = 2P_0V_0(1 - \ell n2)$

Total heat absorbed by the gas = $\frac{5P_0V_0}{2}$

(III) $\Delta Q_{AB} = nC\Delta T = n\frac{3R}{2}\left(\frac{T_0}{2} - T_0\right) = -\frac{3}{4}nRT_0 = -\frac{3}{4} \times 4P_0V_0 = -3P_0V_0$

$\Delta Q_{BC} = nC_V\Delta T = n\frac{5R}{2}\left(\frac{T_0}{4} - \frac{T_0}{2}\right) = -\frac{5}{2}\frac{nRT_0}{4} = -\frac{5P_0V_0}{2}$

$\Delta Q_{CA} = nC\Delta T = n2R\Delta T = 2nR\left(T_0 - \frac{T_0}{4}\right) = \frac{6nRT_0}{4} = 6P_0V_0$

$\Delta W_{cycle} = \Delta Q_{cycle} = 6P_0V_0 - 3P_0V_0 - \frac{5P_0V_0}{2} = \frac{P_0V_0}{2}$

Total heat absorbed by the gas = $6P_0V_0$

(IV) $\Delta Q_{AB} = nC_P\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}P_0V_0$

$\Delta Q_{BC} = nR2T_0\ell n2 = 2P_0V_0\ell n2$

$\Delta Q_{CD} = nC_V\Delta T = n\frac{5R}{2}(T_0 - 2T_0) = -\frac{5}{2}nRT_0 = -\frac{5}{2}P_0V_0$

$\Delta Q_{DA} = nRT_0\ell n\left(\frac{1}{4}\right) = -2nRT_0\ell n2 = -2P_0V_0\ell n2$

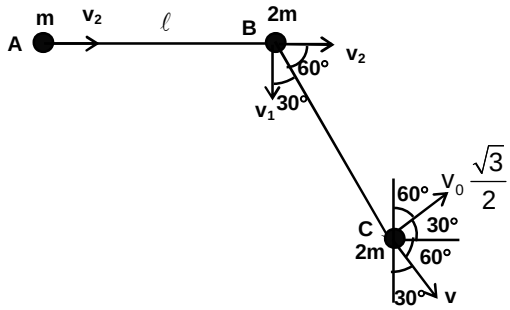
$\Delta W_{cycle} = \Delta Q_{cycle} = \frac{7P_0V_0}{2} - \frac{5P_0V_0}{2} = P_0V_0$

Total heat absorbed by the gas = $\frac{7P_0V_0}{2} + 2P_0V_0\ell n2 = \frac{P_0V_0}{2}(7 + 4\ell n2)$

SECTION – C

13. 00001.80

Sol. Using conservation of momentum of system.



$2mv_0 = 3mv_2 + 2m\left(\frac{3v_0}{4} + \frac{v}{2}\right)$

$2v_0 = 3v_2 + v + \frac{3v_0}{2}$

$3v_2 + v = \frac{v_0}{2}$ …(i)

$2mv_1 = 2m\left(\frac{\sqrt{3}v_0}{4} - \frac{v\sqrt{3}}{2}\right)$

$v_1 = \frac{\sqrt{3}v_0}{4} - \frac{\sqrt{3}v}{2}$

$\sqrt{3}v_1 = \frac{3v_0}{4} - \frac{3v}{2}$ …(ii)

Also, $v_1\cos30° + v_2\cos60° = v$

$\frac{\sqrt{3}v_1}{2} + \frac{v_2}{2} = v$

$\sqrt{3}v_1 = 2v - v_2$ …(iii)

From equation (ii) and (iii)

$2v - v_2 = \frac{3v_0}{4} - \frac{3v}{2}$

$7v - 2v_2 = \frac{3v_0}{2}$ ….(iv)

Solving equation (i) and (iv)

$23v_2 = 2v_0$

$v_2 = \frac{2v_0}{23} = \frac{2\times20.7}{23} = 1.8$ m/s

14. 00037.50

Sol. $a_1 \cos\theta = a_2 \cos\theta$

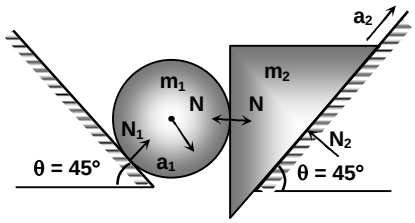
$a_1 = a_2 = a$ (say)

$m_1 g \sin\theta - N\cos\theta = m_1 a$ …(i)

$N\cos\theta - m_2 g \sin\theta = m_2 a$ …(ii)

Solving (i) and (ii) we get

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g\sin\theta \quad \text{…(iii)}$$



From equation (i) and (iii)

$N\cos\theta = m_1 g\sin\theta - m_1 g\sin\theta$

$$\left(\frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$N\cos\theta = \frac{2m_1 m_2 g\sin\theta}{(m_1 + m_2)}$$

$$N = \frac{2m_1 m_2 g\tan\theta}{(m_1 + m_2)} = \frac{2\times5\times3\times10\times1}{8} = 37.5\ N$$

N = 37.50 N

15. 00000.60

Sol. Total mechanical energy

$$\varepsilon = \frac{-GMm}{r_0} + \frac{1}{2}mv_0^2$$

$$= -\frac{GMm}{r_0} + \frac{1}{2}mv_0^2$$

$$= -\frac{GMm}{r_0} + \frac{GMm}{2r_0}$$

$$\varepsilon = -\frac{GMm}{2r_0}$$

$$\frac{-GMm}{2a} = -\frac{GMm}{2r_0}$$

$\therefore$ Length of semi-major axis, $a = r_0$

Hence point 'P' must be an extreme

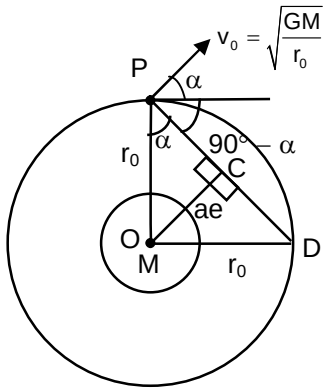
Point of the minor axis of the elliptical orbit

$\because$ $a\sin\alpha = ae$

$e = \sin\alpha = \sin 37° = 0.6$

Hence the eccentricity of the elliptical orbit

e = 0.60



16. 00020.00

Sol. The tension produced in each wire,

$$F = \frac{(\alpha_1 + \alpha_2)\ell\Delta\theta}{\frac{\ell}{A}\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)} = \frac{3\times10^{-5}\times20\times10^{-5}}{1.5\times10^{-10}} = 40N$$

Now, $\frac{n_1}{2\ell}\sqrt{\frac{F}{\mu_1}} = \frac{n_2}{2\ell}\sqrt{\frac{F}{\mu_2}}$

$$\Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{0.4}{0.1}} = 2$$

In first overtone, $n_1 = 4$, $n_2 = 2$

Hence the first overtone frequency of the transverse vibration of the system,

$$f = \frac{n_1}{2\ell}\sqrt{\frac{F}{\mu_1}}$$

$$f = \frac{4}{2\times 1}\sqrt{\frac{40}{0.4}} = \frac{4}{2}\times 10 = 20\ \text{Hz}$$

17. 00001.84

Sol. $E_1 = k_1 + \phi$ …(i)

$E_2 = k_2 + \phi$ …(ii)

Also, $\frac{k_1}{k_2} = 4 \Rightarrow k_1 = 4k_2$ …(iii)

From (i) and (ii)

$$\frac{E_1}{E_2} = \frac{k_1+\phi}{k_2+\phi}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{4k_2+\phi}{k_2+\phi}$$

$$\frac{6000}{4500} = \frac{4k_2+\phi}{k_2+\phi}$$

$$\frac{4}{3} = \frac{4k_2+\phi}{k_2+\phi}$$

$\Rightarrow 4k_2 + 4\phi = 12k_2 + 3\phi$

$\Rightarrow \phi = 8k_2$ …(iv)

Now form (ii) and (iv)

$E_2 = k_2 + 8k_2$

$$\Rightarrow \frac{hC}{\lambda_2} = 9k_2$$

$$\Rightarrow \frac{1242}{600} = 9k_2$$

$\Rightarrow 2.07 = 9\ k_2$

$\Rightarrow k_2 = 0.23$ eV

$\therefore \phi = 8k_2 = 8 \times 0.23 = 1.84$ eV

18. 00006.82

Sol. The pitch of screw gauge, P = 1mm

$$\text{Least count, L.C.} = \frac{P}{N} = \frac{1}{100} = 0.01\,\text{mm}$$

Diameter of the wire,

$d = 2\text{mm} + 56 \times \text{L.C.} - 6 \times \text{L.C.}$

$= 2\text{mm} + 56 \times 0.01\ \text{mm} - 6 \times 0.01\ \text{mm}$

$d = 2.50\ \text{mm} = 0.250\ \text{cm}$

$\ell = 8.68$ cm

Curved surface area of the wire,

$S = \pi d\ell$

$$= \frac{22}{7}\times 0.250 \times 8.68 = 6.82\ \text{cm}^2$$

Chemistry

PART – II

SECTION – A

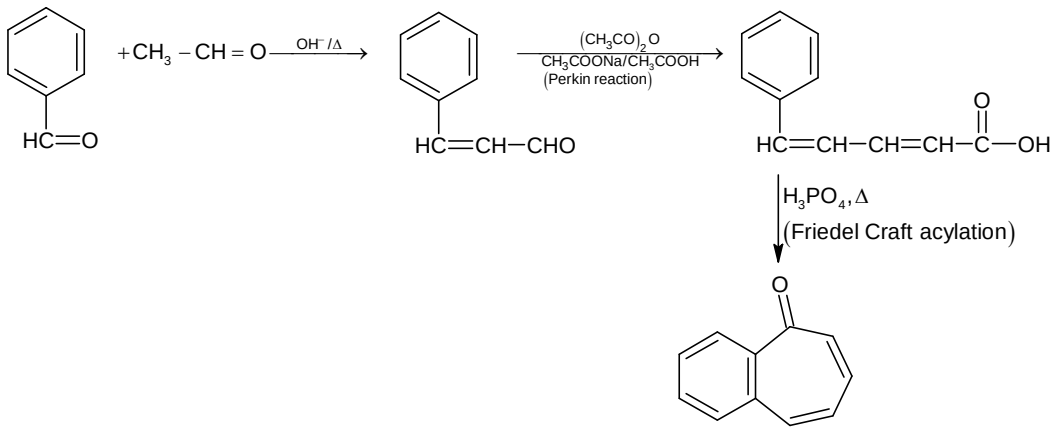
19. A, B, C

Sol. Average C – O bond length in metal compounds $\propto \frac{\text{No. of } d\pi - \text{electron pairs}}{\text{No. of CO groups}}$

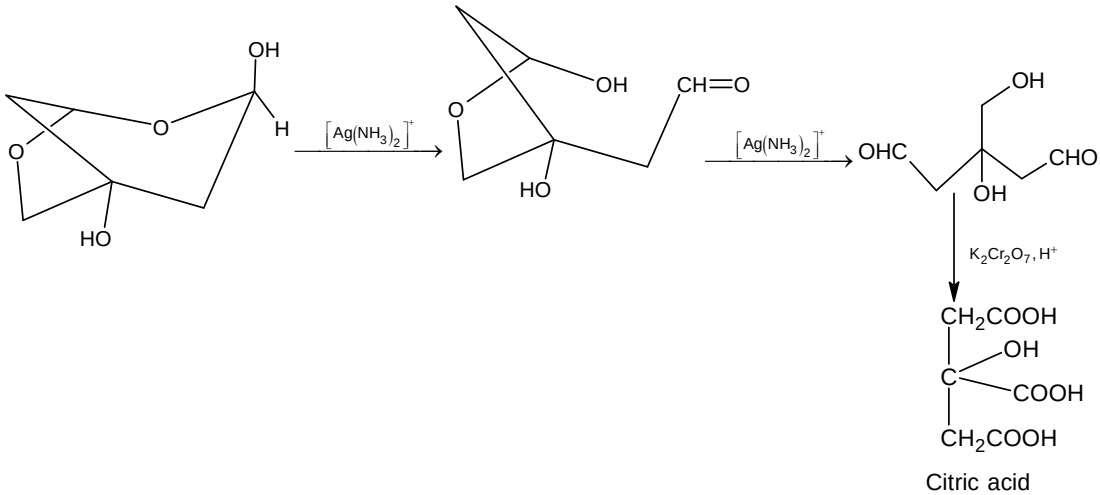
Complex	Ratio
$[Fe(CO)_5]$	0.8
$[Cr(CO)_6]$	0.5
$[Cr(NH_3)(CO)_5]$	0.6
$[Cr(NH_3)_2(CO)_4]$	0.75
$[Cr(NH_3)(CO)_3]$	1.0

20. A, B, C, D

Sol. (A)

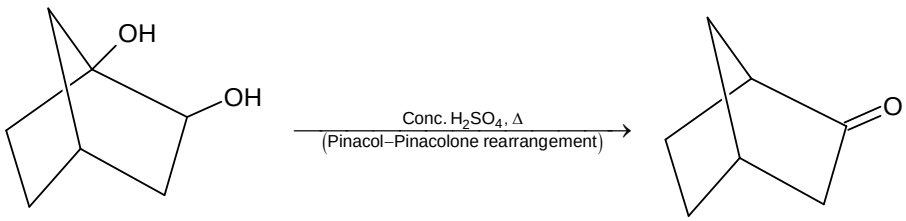


(B)

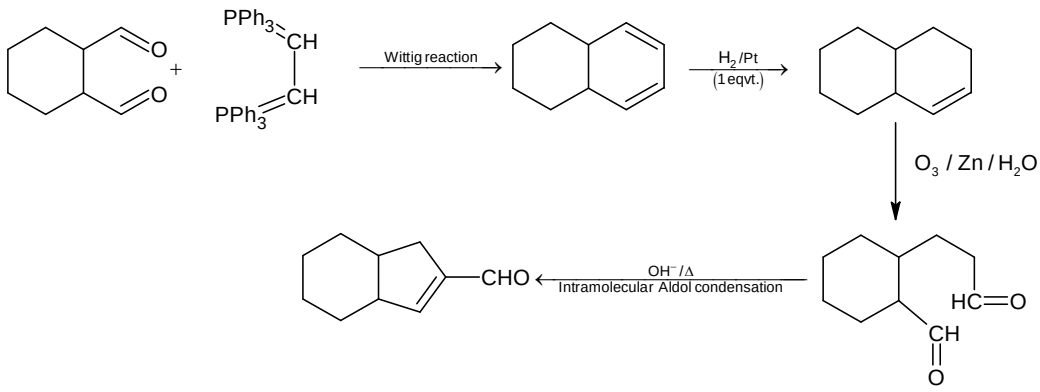


Citric acid

(C)



(D)



21. A, B, D

Sol.

$$2ACl_3 \longrightarrow A_2O_5$$

at $t = 0$ 1M 0

After time $= 10$ min $(1-x)$ $\frac{x}{2}$

So, $(1-x)\times 2 = \frac{1}{4}\times 0.8\times 5$

$\therefore x = 0.5$ mole

So, $t_{1/2} = 10\,\text{min}$

Now, $k = \frac{1}{2\times t_{1/2}\times [A]_o} = \frac{1}{2\times 10\times 1} = 0.05\ \ell\,\text{mol}^{-1}\,\text{min}^{-1}$

Also, after t = 20 min,

$$20 = \frac{1}{2\times 0.05}\left[\frac{1}{[A]_t} - \frac{1}{[A]_o}\right]$$

$$2 = \frac{1}{[A]_t} - 1$$

$$\therefore [A]_t = \frac{1}{3}$$

So, % of ACl_3 left unreacted after 20 min $= \frac{1}{3}\times 100 = 33.33\%$

22. A, B, C

Sol. (i)

$$PO_4^{3-} + H^+ \longrightarrow HPO_4^{2-}$$

m.mol.	10	5	0
after neutralization	5	0	5

So, $pH = pK_{a_3} = 13$ (Buffer)

(ii)

$$PO_4^{3-} + H^+ \longrightarrow HPO_4^{2-}$$

m.mol.	10	10	0
after neutralization	0	0	10

So, $pH = \frac{8+13}{2} = 10.5$ (Amphiprotic anion)

(iii)

$PO_4^{3-} + H^+ \longrightarrow HPO_4^{2-}$

m.mol.	10	15	0
after neutralization	0	5	10

$HPO_4^{2-} + H^+ \longrightarrow H_2PO_4^-$

10	5	0
5	0	5

So, $pH = pK_{a_2} = 8$ (Buffer)

(iv)

$PO_4^{3-} + H^+ \longrightarrow HPO_4^{2-}$

m.mol.	10	20	0
after neutralization	0	10	10

$HPO_4^{2-} + H^+ \longrightarrow H_2PO_4^-$

m.mol.	10	10	0
after neutralization	0	0	10

So, $pH = pK_{a_3} = 5.5$ (Amphiprotic anion)

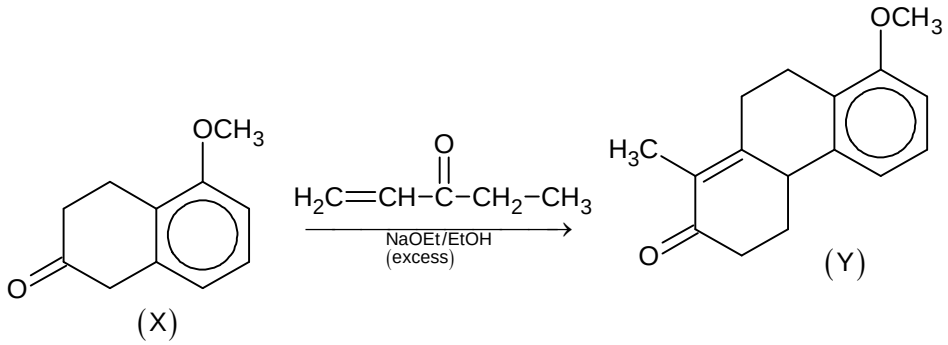
23. A, B, C

Sol. Mixing of gases is always accompanied with the increase in the entropy.

- At equilibrium, chemical potential is same in all phases of the system.
- Since in reversible adiabatic process, dS = 0, therefore these are isoentropic process.
- During expansion, work is maximum under reversible conditions.

24. B, C, D

Sol.



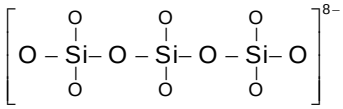
25. A, D

Sol. The order of volatility is IV < III < II < I

So, I = CH_3F, II = CH_2O, III = CH_3OH, IV = CH_3COOH

26. A, B, C

Sol. (A)



(B) HF reacts with glass, so it is used to make marking on the glass (etching).

(C) Since Fe^{3+} reacts with KCNS to produce red colour. So, it can be used as an indicator in the titration of Fe^{3+} with Sn^{2+}.

(D) $Pb_3O_4 + HCl \longrightarrow PbCl_2 + H_2O + \underset{\text{(Greenish yellow)}}{Cl_2} \uparrow$

27. D

Sol. (A) Spontaneity requires ΔG to be negative. Here ΔH is positive but nothing can be said about ΔS, so the process may or may not be spontaneous. Hence, option (A) is incorrect.

(B) Since, $\Delta S = 0$ so $\Delta G = \Delta H$ but $\Delta H = \Delta U + P\Delta V$, so this process can be spontaneous if $\Delta V$is negative or zero or even small positive. But nothing is mentioned about ΔV, so the process may or may not be spontaneous. So, (B) is incorrect.

(C) For a reversible adiabatic process,
$\Delta S_{Syst} = \Delta S_{surr} = 0$ and for an irreversible adiabatic process, $\Delta S_{syst} > 0$ and $\Delta S_{surr} = 0$
So, (C) is incorrect.

28. B

Sol. An adiabatic process implies q = 0, and with volume not changing we have $\Delta V = 0$; therefore, W = 0 and thus $\Delta U = 0$. The constant 'U' and 'V' allows us to apply the strict entropy spontaneity test; if $\Delta S > 0$ the process is spontaneous. Since, we are given that $\Delta S$is +ve, so the process must be spontaneous.

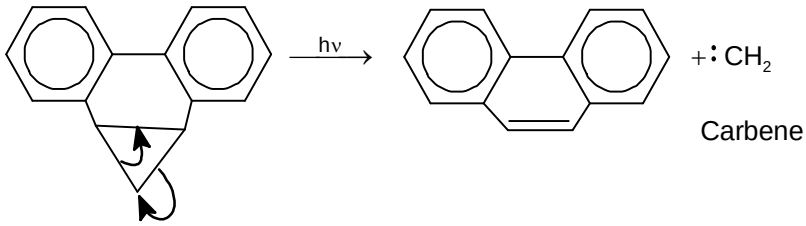
$\Delta G = \Delta H - T\Delta S$

Here, $\Delta H = \Delta U = 0$, $\Delta S = +ve$

So, ΔG must be negative.

29. B

Sol.



30. D

Sol. (A) Carbene is a diradical.
(B) It is Favorskii Rearrangement, where carbanion is formed as intermediate.
(C) It is Beckman's Rearrangement, where carbocation is the intermediate.

SECTION – C

31. 00006.50

Sol. All the above mentioned compounds except N_2 can react with PCl_5 to give $POCl_3$.

32. 00005.30

Sol. $\underset{(1\,bar)}{Pt;H_2(g)} \,|\, HA \,||\, AgCl(s), Cl^- \,|\, Ag(s)$

The cell reaction is

$$\frac{1}{2}H_2(g) + AgCl(s) \longrightarrow H^+ + Cl^- + Ag(s)$$

$E_{cell} = E^\circ_{cell} - 0.06\log_{10}\left[H^+\right]\left[Cl^-\right]$

$= 0.20 - 0.06 \times \log\left[Cl^-\right] + 0.06 \times pH$

$E_{cell} = 0.26 + 0.06 \times pH$ … (1)

$0.56 = 0.26 + 0.06 \times pH_1$

$pH_1 = 5$

Similarly: $0.62 = 0.26 + 0.06 \times pH_2$

$pH_2 = 6$

Now, let there be initially x mol HA present and 20 mL NaOH containing 'y' mol of NaOH.

So, $pH_1 = 5 = pK_a + \log\frac{y}{x-y}$ … (2)

$pH_2 = 6 = pK_a + \log\frac{2.5y}{x-2.5y}$ … (3)

On solving Eqn. (1) and (2), we get

$x = 3y$

Now, substituting x = 3y in Eqn. (2)

We get $pK_a = 5.30$

33. 00000.75

Sol. I, II, V and VII will exist as anion at pH = 7, So, y = 4 and rest III, IV and VI will exist as cation at pH = 7. So, x = 3.

So, $\frac{x}{y} = 0.75$

34. 00004.50

Sol. x = 4 (I, II, III, VII)
y = 5 (I, II, III, IV, VII)
z = 2 (V, VI)

35. 00025.76

Sol. Meqvt. of Fe^{2+} taken = 50 × 0.16 = 8

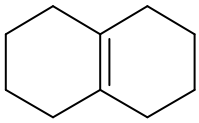
- Meqvts. of Fe^{2+} ion left = 3.67 × 0.05 × 6 = 1.10
- Meqvts. of Cr^{+6} in solution = 8 – 1.10 = 6.90
- m. mol. of Cr^{+6} in solution = 6.90/3 = 2.30
- m. mol. of Chromite = 2.30/2 = 1.15

Mass of chromite in original sample $= \frac{1.15 \times 224}{1000} = 0.2576$ g

% of chromite in the original sample = 25.76%

36. 00001.20

Sol.

X = 

So, x = 3, y = 5.

Mathematics PART – III

SECTION – A

37. A, B, C, D

Sol. $\frac{\left(1-\frac{b}{a}\right)\left(2-\frac{b}{a}\right)}{\left(1-\frac{b}{a}+\frac{c}{a}\right)}=\frac{\alpha}{\beta+1}+\frac{\beta}{\alpha+1}+2$. Let $0\le\alpha\le\beta\le1\Rightarrow\frac{\alpha}{\beta+1}+\frac{\beta}{\alpha+1}\le\frac{\alpha}{\alpha+1}+\frac{1}{\alpha+1}\le1$

Maximum value is 3, if $\alpha = 0$, $\beta = 1$ or $\alpha = 1$, $\beta = 1$ and minimum value = 2, if $\alpha = \beta = 0$

38. A, C, D

Sol. $a_{n+1}^2-a_n^2=1$. By Telescopic sum $a_n=\sqrt{n-1+a_1^2}$ now $a_{2n_0}=3a_{n_0}\Rightarrow 8a_1^2=8-7n_0\ge0\Rightarrow n_0=1$

$\Rightarrow a_1^2=\frac{1}{8}\Rightarrow a_n=\sqrt{n-\frac{7}{8}}$ and $\sqrt{\frac{8}{8a_n^2+7}}=\frac{1}{\sqrt{n}}$, $\sqrt{n+1}-\sqrt{n}<\frac{1}{2\sqrt{n}}<\sqrt{n}-\sqrt{n-1}$

$\Rightarrow \sqrt{50}-1<\frac{1}{2}\sum_{n=1}^{49}\sqrt{\frac{8}{8a_n^2+7}}<7$

39. A, B, D

Sol. (A) Number of ways so that the sum of a common edge is odd = $2\times4!\times4!$

(B) Number ways assigning numbers such that the sum of any two numbers with a common body diagonal even is $6\times(4!)^2$

40. A, D

Sol. $g(a,b)=\frac{a^2+b^2+ab}{\sqrt{3}}$

41. C, D

Sol. Total number of points will be 17 and number of squares joining these 16 point will be equal to $(3^2\times1+2^2\times2+1^2\times3)=20$

42. A, B

Sol. After 5 second bug can reach at (5, 0), (4, 1), (3, 2), (2, 3), (1, 4) and (0, 5) along 1, 5, 10, 10, 5, 1 different paths.

43. B, D

Sol. $r_1=s\tan\left(\frac{A}{2}\right)\Rightarrow\frac{r_1}{s}=\tan\left(\frac{A}{2}\right)$; $a>r_1\Rightarrow1>\frac{a}{s}>\tan\left(\frac{A}{2}\right)\Rightarrow$ A is acute and $r=(s-a)\tan\left(\frac{A}{2}\right)$

44. B, D

Sol. The point on S for which are of triangle with point B and C will be minimum, is $\left(\frac{5}{2},\frac{25}{4}\right)$

So, minimum area = $\frac{15}{4}$ unit^2 and $\sqrt{(\alpha-2)^2+\beta^2}=\sqrt{\left(\frac{5}{2}-2\right)^2+\left(\frac{25}{4}\right)^2}=\frac{\sqrt{629}}{4}$

45. C

46. B

Sol. **(for Q.45 to Q.46):**

$P \in B$, $Q \in A$, $R \in D$, $S \in C$ and $T \in A, C, D$; $U \in A, C, D$

47. C

48. D

Sol. **(for Q.47 to 48):**

$$\lim_{x\to 0}\frac{f(x)}{g(x)}=1;\ \lim_{x\to\infty}\frac{f(x)}{g(x)}=\text{D.N.E.}\ \Rightarrow\ \lim_{x\to 0}\frac{f'(x)}{g'(x)}=1;\ \lim_{x\to\infty}\frac{f'(x)}{g'(x)}=0$$

$$\lim_{x\to 0}\frac{h(x)}{t(x)}=0;\ \lim_{x\to\infty}\frac{h(x)}{t(x)}=0\ \Rightarrow\ \lim_{x\to 0}\frac{h'(x)}{t'(x)}=\text{D.N.E.};\ \lim_{x\to\infty}\frac{h'(x)}{t'(x)}=0$$

SECTION – C

49. 00002.33

Sol. If both go to same bin, then probability = $\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}.....\infty=\frac{1}{3}$

So, required probability $(P)=\frac{1}{2}\left(1-\frac{1}{3}\right)=\frac{1}{3}$

50. 00007.00

Sol. Let $z = (1 + ai)(1 + bi)(1 + ci) \Rightarrow |z| = 3$

$|Re(z)| \le |z| \Rightarrow -2 \le ab + bc + ca \le 4$

51. 00039.00

Sol. The integral will be at its maximum value if $380 - x - x^2 \ge 0$ $x \in [-20, 19]$

$\Rightarrow a = 20, b = 19$

52. 00004.00

Sol. $f(x)=-\frac{x}{2}$, $\forall\ x \in R$, $f(x)=-\frac{x}{3}$, $\forall\ x \in R$

$$f(x)=\begin{cases}-\frac{x}{2} & ;\ x\ge 0\\ -\frac{x}{3} & ;\ x<0\end{cases},\quad f(x)=\begin{cases}-\frac{x}{3} & ;\ x\ge 0\\ -\frac{x}{2} & ;\ x<0\end{cases}$$

53. 00001.25

Sol. The exhaustive solution is $\left[0,\frac{\sqrt{5}-1}{2}\right)$

54. 02018.00

Sol. $f(1) = 1 \Rightarrow f(x)+f\left(\frac{2020}{x}\right)=2f(x) \Rightarrow f^2(x) = 1$

$$f(x)=\begin{cases}1 & ;\ x>0\\ -1 & ;\ x<0\end{cases}$$