Chapter 5

CIRCULAR & ROTATIONAL DYNAMICS

A SINGLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

1. When a bucket containing water is rotated fast in a vertical circle of radius R, the water in the bucket doesn't spill provided the bucket is

(a) whirled with a maximum speed of $\sqrt{2gR}$

(b) whirled around with a minimum speed of $\sqrt{(1/2)gR}$

(c) having an r.p.m of $n = \sqrt{900g/(\pi^2 R)}$

(d) having an r.p.m of $n = \sqrt{3600g/(\pi^2 R)}$

2. A disc of uniform thickness and radius 50.0 cm is made of two zones. The central zone of radius 20.0 cm is made of metal and has a mass of 4.00 kg. The outer zone is of wood and has a mass of 3.00 kg. The moment of inertia of the disc about a transverse axis through its centre is

(a) 0.510 kg-m^2 (b) 0.515 kg-m^2

(c) 0.500 kg-m^2 (d) 0.525 kg-m^2

3. A particle is confined to rotate in a circular path decreasing linear speed, then which of the following is correct?

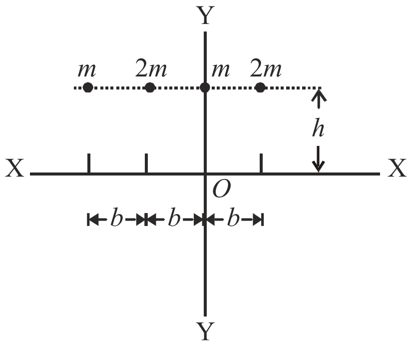
(a) $\vec{L}$ (angular momentum) is conserved about the centre

(b) only direction of angular momentum $\vec{L}$ is conserved

(c) It spirals towards the centre

(d) its acceleration is towards the centre.

4. With O as the origin of the coordinate axis, the X and Y-coordinates of the centre of mass of the system of particles shown in the figure may be given as :

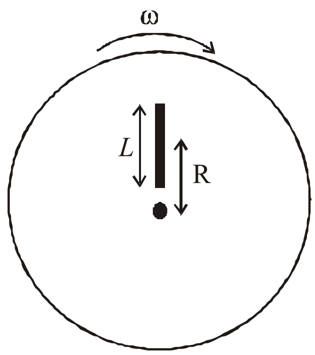


(a) $\left(-\frac{b}{2}, 0\right)$ (b) $\left(-\frac{b}{2}, b\right)$

(c) $\left(-\frac{b}{3}, b\right)$ (d) $\left(-\frac{2}{5}b, b\right)$

Here m and $2m$ represent the masses of the particles.

5. A uniform rod of mass m and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Then the kinetic energy of the rod is

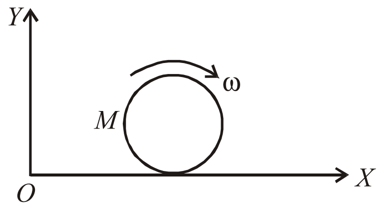


(a) $\frac{1}{2}m\omega^2\left(R^2 + \frac{L^2}{12}\right)$ (b) $\frac{1}{2}m\omega^2 R^2$

(c) $\frac{1}{24}m\omega^2 L^2$ (d) $\frac{1}{12}m\omega^2 L^2$

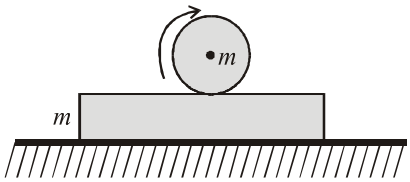
MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ

6. A disc of mass M and radius R is rolling with angular speed ω on a horizontal plane as shown in Figure. The magnitude of angular momentum of the disc about the origin O is



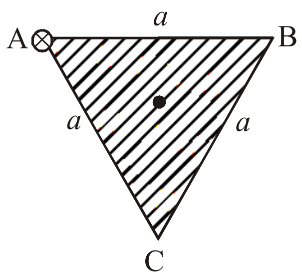
(a) $(1/2)\,MR^2\omega$ (b) $MR^2\omega$
(c) $(3/2)\,MR^2\omega$ (d) $2\,MR^2\omega$

7. A sphere of mass 'm' is given some angular velocity about a horizontal axis through its centre and gently placed on a plank of mass 'm'. The coefficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the plank is



(a) zero (b) $(7/5)\,\mu g$
(c) μg (d) $2\,\mu g$

8. A uniform triangular plate ABC of moment of inertia I (about an axis passing through A and perpendicular to plane of the plate) can rotate freely in the vertical plane about point 'A' as shown in figure. The plate is released from the position shown in the figure. Line AB is horizontal. The acceleration of centre of mass just after the release of plate is



(a) $\dfrac{mga^2}{\sqrt{3}I}$ (b) $\dfrac{mga^2}{4I}$

(c) $\dfrac{mga^2}{2\sqrt{3}I}$ (d) $\dfrac{mga^2}{3I}$

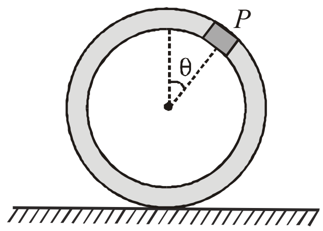
9. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are

(a) up the incline while ascending and down the incline descending
(b) up the incline while ascending as well as descending
(c) down the incline while ascending and up the incline while descending
(d) down the incline while ascending as well as descending.

10. A train of mass M is moving on a circular track of radius R with constant speed V. The length of the train is half of the perimeter of the track. The linear momentum of the train will be

(a) 0 (b) $2MV/\pi$
(c) MVR (d) MV

11. A small block of mass 'm' is rigidly attached at 'P' to a ring of mass '$3m$' and radius 'r'. The system is released from rest at $\theta = 90°$ and rolls without sliding.



The angular acceleration of hoop just after release is

(a) $g/4r$ (b) $g/8r$
(c) $g/3r$ (d) $g/2r$

12. A running man has half the K.E. that a body of half his mass. whenthe man speeds up by 1 ms^{-1} then he has the same K.E. as that of the body. The original speeds of the man and the boy in ms^{-1} are

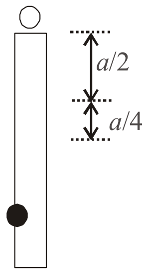
(a) 1.41 each (b) 2.42, 4.84
(c) 4.84, 0.8 (d) 2.41, 0.41

13. A horizontal circular plate is rotating about a vertical axis passing through its centre with an angular velocity ω_o. A man sitting at the centre having two blocks in his hands stretches out his hands so that the moment of inertia of the system doubles. If the kinetic energy of the system is K initially, its final kinetic energy will be

(a) $2\,K$ (b) $K/2$
(c) K (d) $K/4$

MARK YOUR RESPONSE					
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ		

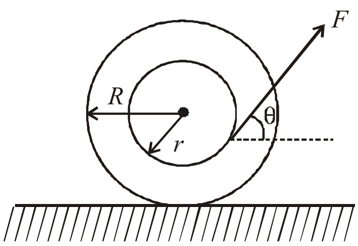
14. A particle of mass m is attached to a thin uniform rod of length a and mass 4 m. The distance of the particle from the centre of mass of the rod is $a/4$. The moment of inertia of the combination about an axis passing through O normal to the rod is



(a) $\frac{64}{48}ma^2$ (b) $\frac{91}{48}ma^2$

(c) $\frac{27}{48}ma^2$ (d) $\frac{51}{48}ma^2$

15. The spool shown in figure is placed on a rough horizontal surface has inner radius r and outer radius R.



The angle θ between the applied force and the horizontal can be varied. The critical angle (θ) for which the spool does not roll and remains stationary is given by

(a) $\theta = \cos^{-1}\left(\frac{r}{R}\right)$ (b) $\theta = \cos^{-1}\left(\frac{2r}{R}\right)$

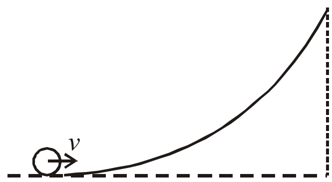
(c) $\theta = \cos^{-1}\sqrt{\frac{r}{R}}$ (d) $\theta = \sin^{-1}\left(\frac{r}{R}\right)$

16. The free end of a thread wound on a bobbin is passed round a nail A hammered into the wall. The thread is pulled at a constant velocity. Assuming pure rolling of bobbin, find the velocity v_0 of the centre of the bobbin at the instant when the thread forms an angle α with the vertical.

(a) $\frac{vR}{R\sin\alpha - r}$ (b) $\frac{vR}{R\sin\alpha + r}$

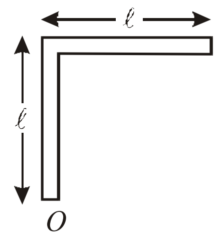
(c) $\frac{2vR}{R\sin\alpha + r}$ (d) $\frac{v}{R\sin\alpha + r}$

17. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is



(a) ring (b) solid sphere
(c) hollow sphere (d) disc

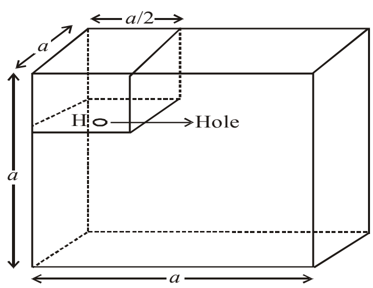
18. Two thin rods of mass m and length ℓ each are joined to form L shape as shown. The moment of inertia of rods about an axis passing through free end (O) of a rod and perpendicular to both the rods is



(a) $\frac{2}{7}m\ell^2$ (b) $\frac{m\ell^2}{6}$

(c) $m\ell^2$ (d) $\frac{5m\ell^2}{3}$

19. The figure shows a hollow cube of side 'a' of volume V. There is a small chamber of volume $V/4$ in the cube as shown. This chamber is completely filled by m kg of water. Water leaks through a hole H and spreads in the whole cube. Then the work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is



(a) (1/2) mga (b) (3/8) mga
(c) (5/8) mga (d) (1/8) mga

MARK YOUR RESPONSE	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ				

20. A solid sphere of mass M and radius R having moment of inertia I about its diameter is recast into a solid disc of radius r and thickness t. The moment of inertia of the disc about an axis passing the edge and perpendicular to the plane remains I. Then R and r are related as

(a) $r = \sqrt{\frac{2}{15}}R$ (b) $r = \frac{2}{\sqrt{15}}R$

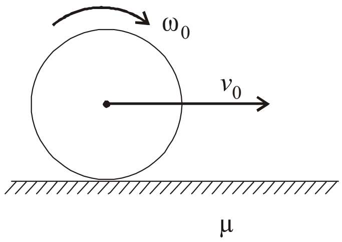
(c) $r = \frac{2}{15}R$ (d) $r = \frac{\sqrt{2}}{15}R$

21. A heavy particle of weight W, attached to a fixed point by a light inextensible string describes a circle in a vertical plane. The tension in the string has the values mW and nW respectively, when the particle is at the highest and the lowest points in the path. The value of $(n - m)$ is

(a) 5 (b) 4

(c) 6 (d) 7

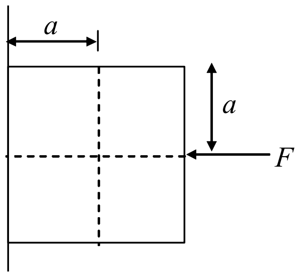
22. A sphere of mass M and radius R is moving on a rough fixed surface, having coefficient of friction μ as shown in figure. It will attain a minimum linear velocity after a time



(a) $v_0/\mu g$ (b) $\omega_0 R/\mu g$

(c) $(v_0 - \omega_0 R)/\mu g$ (d) $2(v_0 - \omega_0 R)/7\mu g$

23. A block of mass m is at rest under the action of force F against a wall as shown in figure. Which of the following statement is incorrect?



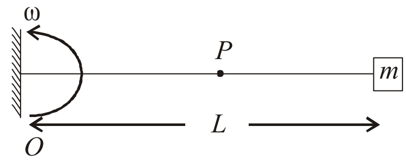
(a) $f = $ mg [where f is the friction force]

(b) $F = N$ [where N is the normal force]

(c) F will not produce torque

(d) N will not produce torque

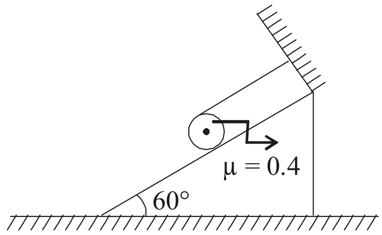
24. A particle of mass m is attached to a rod of length L and it rotates in a circle with a constant angular velocity ω. An observer P is rigidly fixed on the rod at a distance $L/2$ from the centre. The acceleration of m and the pseudo force on m from the frame of reference of P must be respectively.



(a) zero, zero (b) zero, $m\omega^2 \frac{L}{2}$

(c) $\omega^2 \frac{L}{2}, m\omega^2 \frac{L}{2}$ (d) zero, $m\omega^2 L$

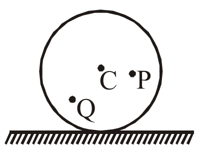
25. A solid cylinder is wrapped with a string and placed on an inclined plane as shown in the figure. Then the frictional force acting between cylinder and plane is



(a) zero (b) 5 mg

(c) $\frac{7mg}{2}$ (d) $\frac{mg}{5}$

26. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is

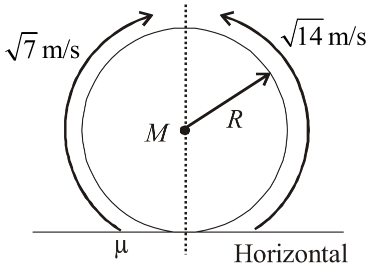


(a) $v_Q > v_C > v_P$ (b) $v_P > v_C > v_Q$

(c) $v_P = v_C, v_Q = v_C/2$ (d) $v_P < v_C > v_Q$

MARK YOUR RESPONSE					
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ			

27. A solid sphere of mass M and radius R is kept on a rough surface. The velocities of air (density ρ) around the sphere are as shown in the figure. Assuming R to be small and $M = \dfrac{4\pi\rho R^2}{g}$ kg, what is the minimum value of coefficient of friction so that the sphere starts pure rolling? (Assume force due to pressure difference is acting on centre of mass of the sphere)

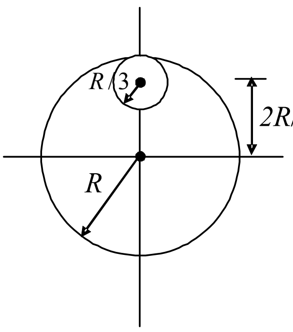


(a) 0.25 (b) 0.50
(c) 0.75 (d) 1.0

28. A racing car driver drives his car on a flat circular track of radius 25/3 m and a coefficient of friction 0.5. He drives the car in such a manner that he may attain the maximum possible velocity on the track in a minimum possible time. The magnitude of his tangential acceleration at an instant when his speed is 5m/s is

(a) 2 m/s^2 (b) 3 m/s^2
(c) 4 m/s^2 (d) 1 m/s^2

29. From a circular disc of radius R and mass $9M$, a small disc of radius $R/3$ is removed from the disc. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through O is



(a) $4MR^2$ (b) $\dfrac{40}{9}MR^2$

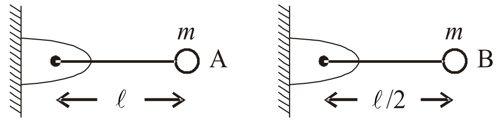
(c) $10\,MR^2$ (d) $\dfrac{37}{9}MR^2$

30. A circular disc of mass m and radius R is rotating on a rough surface having a coefficient of friction μ with an initial angular velocity ω. Assuming a uniform normal reaction on the entire contact surface, the time after which the disc comes to rest is

(a) $\dfrac{\omega R}{\mu g}$ (b) $\dfrac{3\omega R}{4\mu g}$

(c) $\dfrac{1}{2}\dfrac{\omega R}{\mu g}$ (d) $\dfrac{\sqrt{3}}{2}\dfrac{\omega R}{\mu g}$

31. Consider the two bobs are shown in the figure. The bobs are pivoted to the hinges through massless rods. If t_A be the time taken by the bob A to reach the lowest position and t_B be the time taken by the bob B to reach the lowest position. (Both bobs are released from rest from a horizontal position) then ratio t_A / t_B is



(a) $\sqrt{3}$ (b) $\sqrt{5}$

(c) $\sqrt{2}$ (d) $\dfrac{1}{\sqrt{2}}$

32. A uniform solid cube of mass M has edge length a. The moment of inertia of the cube about its face diagonal will be

(a) $\dfrac{\sqrt{3}}{2}Ma^2$ (b) $\dfrac{1}{2}Ma^2$

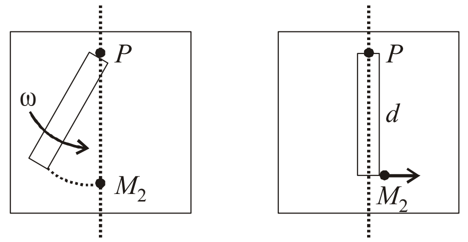
(c) $\dfrac{5}{12}Ma^2$ (d) $\dfrac{7}{12}Ma^2$

33. A particle undergoes uniform circular motion. About which point on the plane of the circle, will the angular momentum of the particle remain conserved?

(a) centre of the circle
(b) on the circumference of the circle.
(c) inside the circle
(d) outside the circle.

MARK YOUR RESPONSE					
	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ
	32. ⓐⓑⓒⓓ	33. ⓐⓑⓒⓓ			

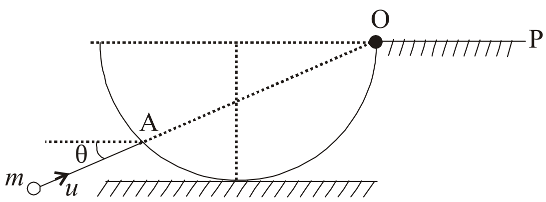
34. A system consists a ball of mass M_2 and a uniform thin rod of mass M_1 and length d. The rod is attached to a frictionless horizontal table by a pivot at point P and initially rotates at an angular speed ω as shown in figure. The rod strikes the ball, which is initially at rest. As a result just after collision, the rod stops and ball moves in the direction shown. If collision is elastic, the ratio M_1/M_2 is



Before collision After collision

(a) 3 (b) 2
(c) 1 : 2 (d) 1 : 3

35. A hemispherical shell of mass m and radius R is hinged at point O and placed on a horizontal surface. A ball of mass m moving with a velocity u inclined at an angle $\theta = \tan^{-1}(1/2)$ strikes the shell at point A (as shown in the figure) and stops. What is the minimum speed u if the given shell is to reach the horizontal surface OP ?



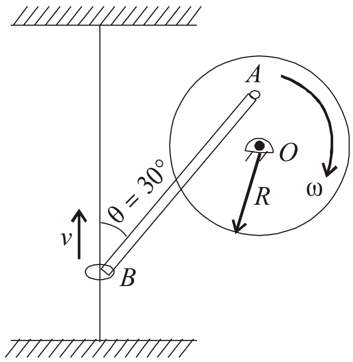
(a) Zero

(b) $\sqrt{\dfrac{2gR}{3}}$

(c) $\dfrac{gR}{\sqrt{5}}$

(d) it cannot come on the surface for any value of u.

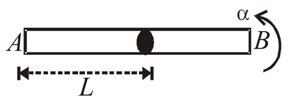
36. A disc is fixed at its centre O and rotating with constant angular velocity ω. There is a rod whose one end is connected at A on the disc and the other end is connected with a ring which can freely move along the fixed vertical smooth rod. At an instant when the rod is making an angle 30° with the vertical the ring is found to have a velocity v in the upward direction. Find ω of the disc. Given that the point A is R/2 distance above point O and length of the rod AB is l



(a) $\dfrac{v\sqrt{3}}{2l}$ (b) $\dfrac{v\sqrt{3}}{2R}$

(c) $\dfrac{2\sqrt{3}v}{R}$ (d) $\dfrac{2v}{l\sqrt{3}}$

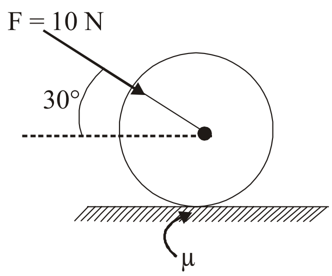
37. A long horizontal rod has a bead which can slide along its length and initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with constant angular acceleration α. If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is



(a) $\sqrt{\mu/\alpha}$ (b) $\mu/\sqrt{\alpha}$

(c) $\dfrac{1}{\sqrt{\mu\alpha}}$ (d) infinitesimal

38. A hollow sphere of mass 2 kg is kept on a rough horizontal surface. A force of 10 N is applied at the centre of the sphere as shown in the figure. Find the minimum value of μ so that the sphere starts pure rolling. (Take g = 10m/s^2)



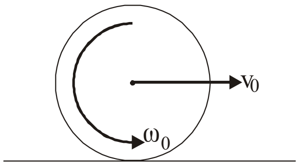
(a) $\sqrt{3} \times 0.16$ (b) $\sqrt{3} \times 0.08$

(c) $\sqrt{3} \times 0.1$ (d) Data insufficient

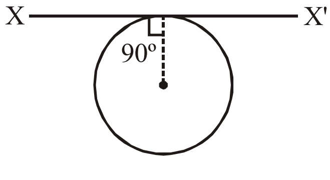
MARK YOUR RESPONSE	34. ⓐⓑⓒⓓ	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ

39. A uniform circular disc of radius r is placed on a rough horizontal surface and given a linear velocity v_0 and angular velocity ω_0 as shown. The disc comes to rest after moving some distance to right. It follows that

(a) $3v_0 = 2\omega_0 r$ (b) $2v_0 = \omega_0 r$
(c) $v_0 = \omega_0 r$ (d) $2v_0 = 3\omega_0 r$



40. A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



(a) $\dfrac{\rho L^3}{8\pi^2}$ (b) $\dfrac{\rho L^3}{16\pi^2}$

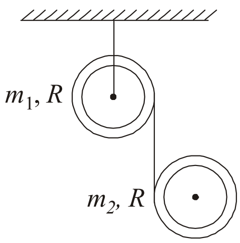
(c) $\dfrac{5\rho L^3}{16\pi^2}$ (d) $\dfrac{3\rho L^3}{8\pi^2}$

41. Two point masses A of mass M and B of mass $4M$ are fixed at the ends of a rod of length ℓ and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The work required for rotating the rod will be minimum when the distance of axis of rotation from the mass A is at

(a) $\dfrac{2}{5}\ell$ (b) $\dfrac{8}{5}\ell$

(c) $\dfrac{4}{5}\ell$ (d) $\dfrac{\ell}{5}$

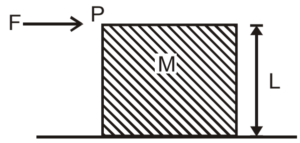
42. Linear acceleration of cylinder of mass m_2 is a_2. Then angular acceleration α_2 is (given that there is no slipping).



(a) $\dfrac{a_2}{R}$ (b) $\dfrac{(a_2 + g)}{R}$

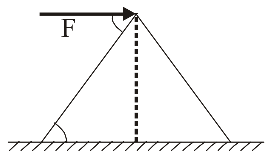
(c) $\dfrac{2(a_2 + g)}{R}$ (d) None of these

43. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is



(a) infinitesimal (b) mg/4
(c) mg/2 (d) $mg(1-\mu)$

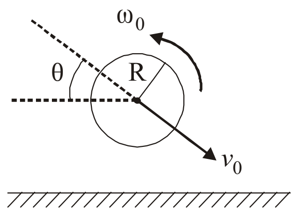
44. A horizontal force F is applied at the top of an equilateral triangular block having mass m and side a as shown in figure. The minimum value of the coefficient of friction required to topple the block before translation will be



(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{1}{2}$

(c) $\dfrac{1}{\sqrt{3}}$ (d) $\dfrac{1}{3}$

45. A solid spherical ball of radius R collides with a rough horizontal surface as shown in figure. At the time of collision its velocity is v_0 at an angle θ to the horizontal and angular velocity ω_0 as shown. After collision, the angular velocity of ball may



(a) decrease (b) increase
(c) remains constant (d) none of these.

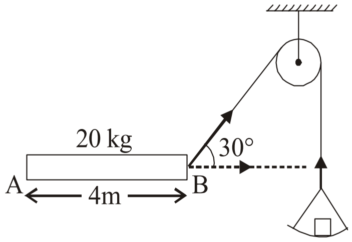
MARK YOUR RESPONSE					
	39. ⓐⓑⓒⓓ	40. ⓐⓑⓒⓓ	41. ⓐⓑⓒⓓ	42. ⓐⓑⓒⓓ	43. ⓐⓑⓒⓓ
	44. ⓐⓑⓒⓓ	45. ⓐⓑⓒⓓ			

46. A particle of mass m is moving in a circle of radius r. The centripetal acceleration (a_c) of the particle varies with the time according to the relation, $a_c = Kt^2$, where K is a positive constant and t is the time. The magnitude of the time rate of change of angular momentum of the particle about the centre of the circle is

(a) mKr (b) $\sqrt{m^2Kr^3}$

(c) $\sqrt{mKr}$ (d) mKr^2

47. A horizontal uniform beam AB of length 4m and a mass of 20 kg is supported at the end B by means of a string which passes over a fixed, smooth pulley supporting a counterbalancing weight of 8kg on the other side. What force, F, when applied at the point A in a suitable direction, will hold the beam in static equilibrium?



(a) F = 160 N (b) $F = 40\sqrt{19}$ N

(c) $F = (200 - 80\sqrt{3})$ N

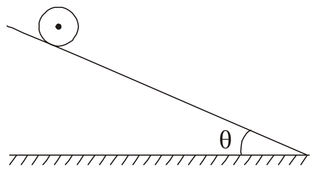
(d) no force F, as specified, can hold the beam in static equilibrium.

48. x, y, z are the cartesian axes of an inertial frame of reference. A particle of mass 1 kg moves with a uniform velocity of 1m/s from A (0, 3 m, 0) to B (4m, 0, 0). The motion of the particle is observed from a frame K which rotates with constant angular velocity ω about the z-axis by an observer O located at (0, 0, 7m) in xyz system. What is the magnitude of average pseudo force that observer O should consider as acting on the particle during its motion from A to B, if $\vec{\omega} = \frac{3\pi}{10}\hat{k}$ rad/s; $\hat{k}$ being a unit vector in the direction of positive z - axis?

(a) zero (b) $\frac{3\pi}{10}$ N

(c) $\frac{3\pi}{50}$ N (d) $\frac{5}{2}\left(\frac{3\pi}{10}\right)^2$ N

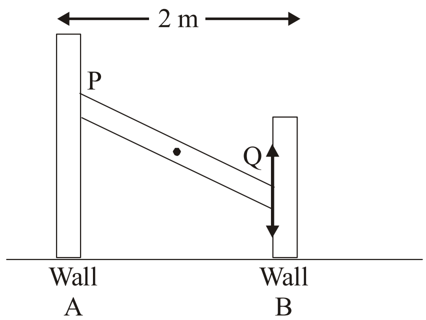
49. A thin uniform spherical shell and a uniform solid cylinder of the same mass and radius are allowed to roll down a fixed incline, without slipping, starting from rest. The times taken by them to roll down the same distance are in the ratio, $t_{sph} : t_{cyl}$



(a) $\sqrt{\frac{14}{15}}$ (b) $\sqrt{\frac{15}{14}}$

(c) $\frac{3}{\sqrt{10}}$ (d) $\sqrt{\frac{10}{3}}$

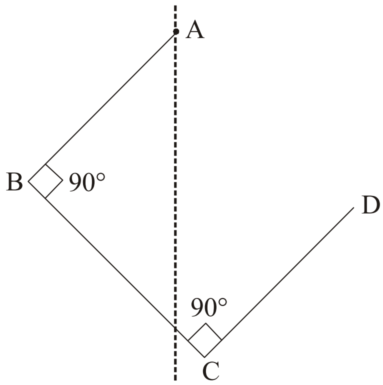
50. Two vertical walls are separated by a distance of 2 metres. Wall 'A' is smooth while wall B is rough with a coefficient of friction $\mu = 0.5$. A uniform rod is probed between them. The length of the longest rod that can be probed between the walls is equal to



(a) 2 metres (b) $2\sqrt{2}$ metres

(c) $\sqrt{2}$ metres (d) $\frac{\sqrt{17}}{2}$ metres

51. Three identical rods are hinged at point A as shown. The angle made by rod AB with vertical is



(a) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$

(c) $\tan^{-1}(1)$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

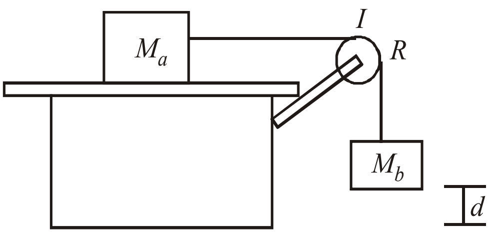
MARK YOUR RESPONSE	46. ⓐⓑⓒⓓ	47. ⓐⓑⓒⓓ	48. ⓐⓑⓒⓓ	49. ⓐⓑⓒⓓ	50. ⓐⓑⓒⓓ
	51. ⓐⓑⓒⓓ				

52. What is the average angular speed of the second hand on a clock (in rad/s) ?

(a) 6.28 (b) 0.105

(c) 0.0167 (d) 1.745×10^{-3}

53. The pulley in the figure has radius $R = 3$ cm and moment of inertia $I = 36 \times 10^{-4}$ kg m^2 about its axis of rotation. The high cord that connects the masses ($M_a = 2$ kg and $M_b = 3$ kg) goes over the pulley and does not slip over the pulley. If coefficient of friction between M_a and the tabletop is $\mu_k = 0.15$ and if the system starts from rest, determine the speed (in m/s) of the masses when M_b has descended a distance $d = 1.5$ from its original position. ($g = 10$ m/s^2).

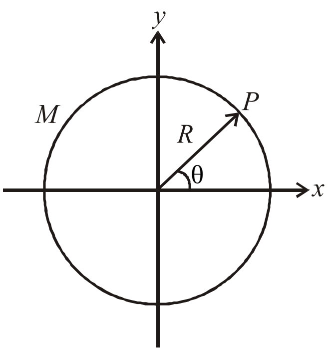


(a) 6 (b) 4 (c) 3 (d) 5

54. A uniform rigid rod hinged at one end is released from rest in the position shown in the vertical plane. Find the magnitude of reaction force (in N) at hinge just after its release. Express your answer after rounding it to nearest integer. (Use $M = 0.8$ kg, $\theta = 45°$, $g = 10$ m/s^2)

(a) 6 (b) 4 (c) 3 (d) 5

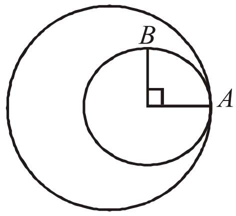
55. A ring of mass M and radius R lies in x-y plane with its centre at origin as shown. The mass distribution of ring is non-uniform such that at any point P on the ring, the mass per unit length is given by $\lambda = \lambda_0 \cos^2\theta$ (where λ_0 is a positive constant). Then the moment of inertia of the ring about z-axis is



(a) MR^2 (b) $\frac{1}{2}MR^2$

(c) $\frac{1}{2}\frac{M}{\lambda_0}R$ (d) $\frac{1}{\pi}\frac{M}{\lambda_0}R$

56. A and B are moving in 2 circular orbits with angular velocity 2ω and ω respectively. Their positions are as shown at $t = 0$. Find the time when they will meet for the first time.



(a) $\frac{\pi}{2\omega}$ (b) $\frac{3\pi}{2\omega}$

(c) $\frac{\pi}{\omega}$ (d) they will never meet

57. A homogeneous disc with a radius $0.2m$ and mass 5 kg rotates around an axis passing through its centre. The angular velocity of the rotation of the disc as a function of time is given by the formula $\omega = 2 + 6t$. The tangential force applied to the rim of the disc is

(a) 1 N (b) 2 N (c) 3 N (d) 4 N

58. A particle of mass m moves without friction along a semicubical parabolic curve, $y^2 = ax^3$, with constant speed v. Find the reaction force of the curve on the particle.

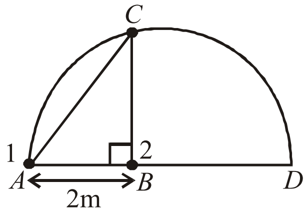
(a) $\frac{3}{4}a^{1/2}x^{-1/2}\left(1+\frac{9}{4}ax\right)^{-3/2} mv^2$

(b) $\frac{3}{4}a^{1/2}x^{-1/2}\left(1-\frac{9}{4}ax\right)^{-3/2} mv^2$

(c) $\frac{3}{4}a^{1/2}x^{-1/2}\left(1+\frac{9}{4}ax\right)^{-1/2} mv^2$

(d) None of these

59. A semicircle of radius $R = 5$m with diameter AD is shown in figure. Two particles 1 and 2 are at points A and B on diameter at $t = 0$ and move along segments AC and BC with constant speeds u_1 and u_2 respectively. Then the value of $\frac{u_1}{u_2}$ for both particles to reach point C simultaneously will be

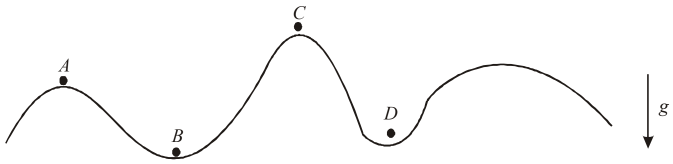


(a) $\frac{5\sqrt{2}}{4}$ (b) $\sqrt{\frac{5}{4}}$

(c) $\frac{2\sqrt{2}}{5}$ (d) $2\sqrt{2}$

Mark Your Response					
	52. ⓐⓑⓒⓓ	53. ⓐⓑⓒⓓ	54. ⓐⓑⓒⓓ	55. ⓐⓑⓒⓓ	56. ⓐⓑⓒⓓ
	57. ⓐⓑⓒⓓ	58. ⓐⓑⓒⓓ	59. ⓐⓑⓒⓓ		

60. Different locations of a cyclist moving with constant speed on road of hilly region are as shown in the figure below. At which of these locations would he feel heaviest

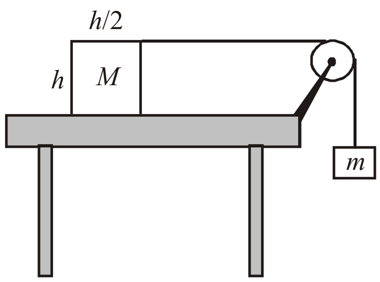


(a) A (b) B
(c) C (d) D

61. Consider the bowling ball in pure rolling motion and suppose that it is rotating with an angular velocity of magnitude ω. In applying the principles of classical mechanics to a rigid body, it is useful to regard the rigid body as being composed of an infinite number of point masses. The point masses that make up the sphere will have linear speeds (relative to the ground)

(a) that are all exactly $R\omega$
(b) that range from $-R\omega$ to $R\omega$
(c) that range from 0 to $R\omega$
(d) that range from 0 to $2R\omega$

62. A cylinder of height h, diameter $h/2$ and mass M and with a homogeneous mass distribution is placed on a horizontal table. One end of a string running over a pulley is fastened to the top of the cylinder, a body of mass m is hung from the other end and the system is released. Friction is negligible everywhere. At what minimum ratio m/M will the cylinder tilt?

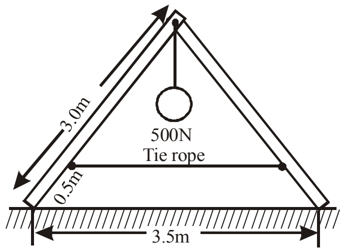


(a) 1 (b) 2
(c) 3 (d) 4

63. A ring of radius R rolls without sliding with a constant velocity with respect to ground. The radius of curvature of the path followed by any particle of the ring at the highest point of its path will be

(a) R (b) $2R$
(c) $4R$ (d) None of these

64. A truss is made by hinging two uniform 150 N rafters, as shown in figure. They rest on an essentially frictionless floor and are held together by a tie rope. A 500 N load is held at their apex. Find the tension in the tie rope.

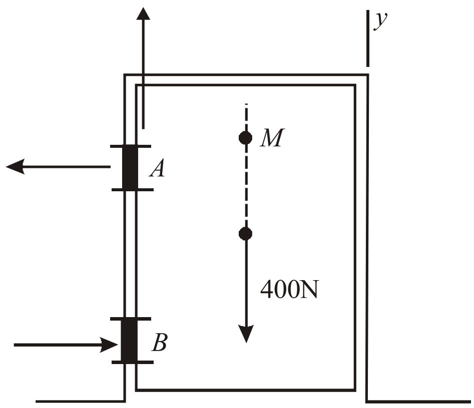


(a) 280 N (b) 460 N
(c) 320 N (d) 400 N

65. A boy of mass 30 kg starts running from rest along a circular path of radius 6 m with constant tangential acceleration of magnitude 2 m/s^2. After 2 sec from start he feels that his shoes started slipping on ground. The friction between his shoes and ground is (Take g = 10 m/s^2)

(a) 1/2 (b) 1/3
(c) 1/4 (d) 1/5

66. As shown in figure, the hinges A and B hold a uniform 400 N door in place. The upper hinge supports the entire weight of the door. Find the resultant force exerted on the door at the hinges. The width of the door is $h/2$, where h is the distance between the hinges.



(a) 312 N (b) 280 N
(c) 412 N (d) 480 N

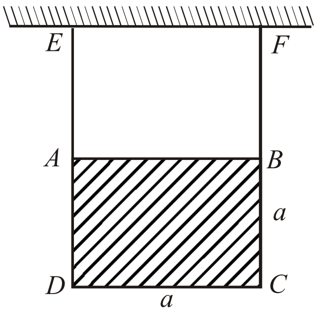
MARK YOUR RESPONSE	60. ⓐⓑⓒⓓ	61. ⓐⓑⓒⓓ	62. ⓐⓑⓒⓓ	63. ⓐⓑⓒⓓ	64. ⓐⓑⓒⓓ
	65. ⓐⓑⓒⓓ	66. ⓐⓑⓒⓓ			

67. A bowling ball of mass m, which can be treated as a uniform rigid sphere, is rolling without slipping on a horizontal surface. The coefficient of static friction between the ball and the surface is μ_s, the coefficient of kinetic friction is μ_k, and the acceleration of gravity is g. What is the force of friction acting on the ball ?

(a) zero (b) $\mu_s mg$

(c) $\mu_k mg$ (d) $\frac{2}{5}\mu_s mg$

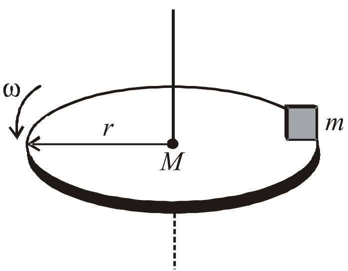
68. A thin, uniform square plate $ABCD$ of side 'a' and mass m = 1 kg is suspended in vertical plane as shown in the figure. AE and BF are two massless inextensible strings. The line AB is horizontal. Find the tension (in N) in the string AE just after BF is cut. (Take $g = 10$ m/s^2)



(a) $mg/5$ (b) $2mg/5$

(c) $3mg/5$ (d) $4mg/5$

69. A smooth disc is rotating with uniform angular speed ω about a fixed vertical axis passing through its centre and normal to its plane as shown. A small block of mass m is gently placed at the periphery of the disc. Then (pickup the correct alternative or alternatives).



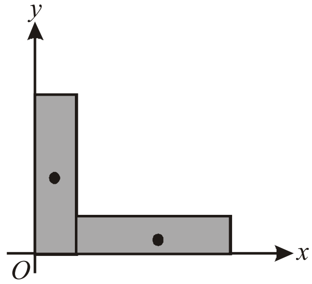
(a) In comparison to ω the angular speed of the disc now increases

(b) In comparison to ω the angular speed of the disc now decreases

(c) In comparison to ω the angular speed of the disc now remains same

(d) The block will move tangentially and fall off the disc

70. A metal sheet 14 cm × 2 cm of uniform thickness is cut into two pieces of width 2 cm. The two pieces are joined and laid along XY plane as shown. The centre of mass has the coordinates



(a) (1, 1) (b) (7/2, 7/2)

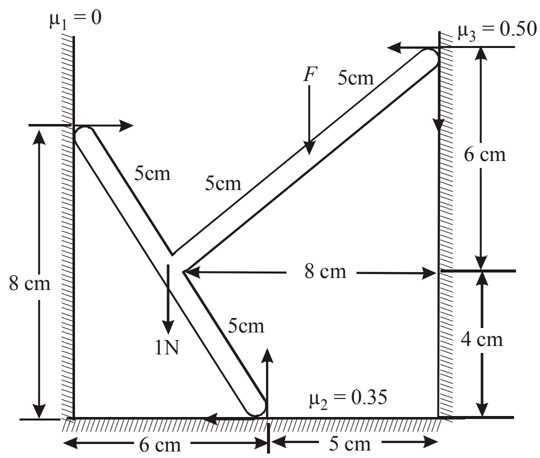
(c) (13/4, 9/4) (d) (12/7, 8/7)

71. A particle moves in a circle of radius $r = 4/3$cm at a speed given by $v = 2.0\, t^2$ where v is in cm/s and t in seconds. Find the magnitude of the acceleration (in cm/s^2) at t = 1s.

(a) 8 (b) 6

(c) 3 (d) 5

72. A light T bar, 10cm on each arm, rests between two vertical walls, as shown in figure. The left wall is smooth, the coefficients of static friction between the bar and floor, and between the bar and right wall, are 0.35 and 0.50, respectively. The bar is subjected to a vertical load of 1N, as shown. What is the smallest value of the vertical force F for which the bar will be in static equilibrium in the position shown ?



(a) 1/19 N (b) 2/19 N (c) 1/15 N (d) 20/19 N

MARK YOUR RESPONSE	67. ⓐⓑⓒⓓ	68. ⓐⓑⓒⓓ	69. ⓐⓑⓒⓓ	70. ⓐⓑⓒⓓ	71. ⓐⓑⓒⓓ
	72. ⓐⓑⓒⓓ				

73. A 2.0 kg block is attached to one end of a spring with a spring constant of 100 N/m and a 4.0 kg block is attached to the other end. The blocks are placed on a horizontal frictionless surface and set into motion. At one instant the 2.0kg block is observed to be travelling to the right with a speed of 0.5 m/s and the 4.0 kg block is observed to be travelling to the left with a speed of 0.30 m/s. Since the only force on the blocks is the force of gravity, the normal force of the spring, we conclude that

(a) the spring is compressed at the time of the observation
(b) the motion was started with the masses at rest
(c) the motion was started with at least one of the masses moving
(d) the motion was started by compressing the spring

74. An automobile of mass m is going around a curve in an arc of a circle of radius R at a speed v. The curve is banked at an angle θ to the horizontal and the coefficient of static friction between the tires and the road is μ_s. If θ is not very big, the maximum speed the car can be moving without skidding is

(a) $\sqrt{\dfrac{gR\,(\sin\theta+\mu_s\cos\theta)}{\cos\theta-\mu_s\sin\theta}}$ (b) $\sqrt{\dfrac{gR\,(\cos\theta+\mu_s\sin\theta)}{\cos\theta-\mu_s\sin\theta}}$

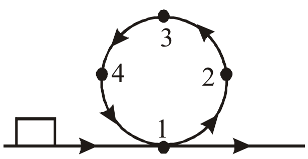
(c) $\sqrt{\dfrac{mgR\,(\sin\theta+\mu_s\cos\theta)}{\sin\theta-\mu_s\cos\theta}}$ (d) $\sqrt{\dfrac{gR\,(\cos\theta+\mu_s\sin\theta)}{\sin\theta-\mu_s\cos\theta}}$

75. A thin circular ring of mass 'M and radius r is rotating about its axis with a constant angular velocity ω, Two objects, each of mass m, are attached gently to the opposite ends of a diameter of the ring. The wheel now rotates with an angular velocity

(a) $\dfrac{\omega M}{(M+m)}$ (b) $\dfrac{\omega\,(M-2m)}{(M+2m)}$

(c) $\dfrac{\omega M}{(M+2m)}$ (d) $\dfrac{\omega\,(M+2m)}{M}$

76. A rectangular block is moving along a frictionless path when it encounters the circular loop as shown. The block passes points 1, 2, 3, 4, 1 before returning to the horizontal track. At point 3 :



(a) its mechanical energy is a minimum
(b) it is not accelerating
(c) its speed is a minimum
(d) it experiences a net upward force

77. Consider the following statements :

S_1 : The locations of centre of mass and centre of gravity may be different for an object.

S_2 : Internal forces can change the momentum of a non-rigid body.

S_3 : If the resultant force on a system of particles is non-zero, then the distance of the centre of mass of system may remain constant from a fixed point.

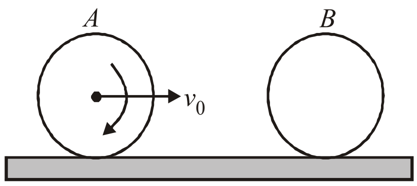
State, in order, whether S_1, S_2 and S_3 are true or false

(a) FTT (b) TFT (c) FFT (d) FTF

78. Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S_1 is closest to S_2, the angular speed of S_2 as observed by an astronaut in S_1 is

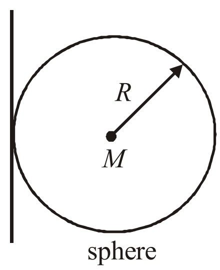
(a) π rad/hr (b) π/3 rad/hr
(c) 2π rad/hr (d) π/2 rad/hr

79. A hollow smooth uniform sphere A of mass m rolls without sliding on a smooth horizontal surface. It collides head on elastically with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of B to that of A just after the collision is



(a) 1 : 1 (b) 2 : 3
(c) 3 : 2 (d) None of these

80. What is the moment of inertia I of a uniform solid sphere of mass M and radius R, pivoted about an axis that is tangent to the surface of the sphere

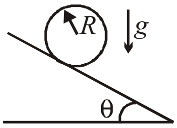


(a) $\dfrac{2}{5}MR^2$ (b) $\dfrac{3}{5}MR^2$

(c) $\dfrac{6}{5}MR^2$ (d) $\dfrac{7}{5}MR^2$

MARK YOUR RESPONSE					
	73. ⓐⓑⓒⓓ	74. ⓐⓑⓒⓓ	75. ⓐⓑⓒⓓ	76. ⓐⓑⓒⓓ	77. ⓐⓑⓒⓓ
	78. ⓐⓑⓒⓓ	79. ⓐⓑⓒⓓ	80. ⓐⓑⓒⓓ		

81. A bowling ball rolls without slipping down an inclined plane inclined at an angle θ to the horizontal, as shown. The coefficient of static friction between the ball and the surface is μ_s, and the coefficient of kinetic friction is μ_k. What is the magnitude of the force of friction acting on the ball ?



(a) zero (b) $\mu_s mg \cos\theta$

(c) $\mu_s \frac{7}{5} mg \sin\theta$ (d) $\frac{2}{7} mg \sin\theta$

82. Two students were given a physics problem for finding maximum extension of spring if blocks are imparted velocities v_1 and v_2 when spring is unstretched.

By student A : $\frac{1}{2} m (v_1 + v_2)^2 = \frac{1}{2} kx^2$

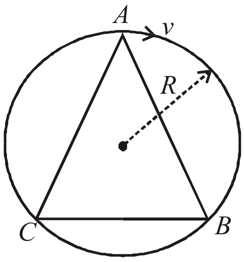
By student B : $\frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 = \frac{1}{2} kx^2$

(a) Student A is correct, Student B is wrong
(b) Student B is correct, Student A is wrong
(c) Both are correct
(d) Both are wrong

83. A particle is moving along a circular path of radius $r = \frac{5}{\pi}$ m with a uniform speed 5 m/s. What is the magnitude of average acceleration (in m/s^2) during the interval in which particle completes half revolution ?

(a) 2 m/s^2 (b) 10 m/s^2
(c) 20 m/s^2 (d) 5 m/s^2

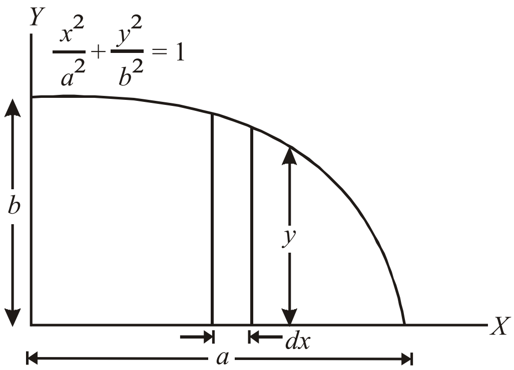
84. A particle is moving with constant speed v m/s along the circumference of a circle of radius R meters as shown. A, B and C are three points on periphery of the circle and ΔABC is equilateral. The magnitude of average velocity of particle, as it moves from A to C clockwise sense, will be



(a) $\frac{3v}{2\pi}$ (b) $\frac{3v}{4\pi}$

(c) $\frac{3\sqrt{3}v}{4\pi}$ (d) $\frac{3\sqrt{3}v}{2\pi}$

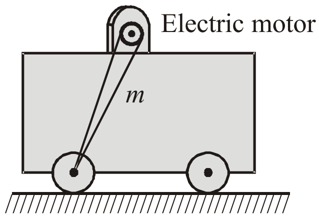
85. The center of mass of a quadrant of thin elliptical section made of material of mass per unit area σ (figure) is



(a) $4a/\pi$, $4b/3\pi$ (b) $4a/3\pi$, $4b/\pi$

(c) $2a/3\pi$, $4b/3\pi$ (d) $4a/3\pi$, $4b/3\pi$

86. A box of mass 1 kg is mounted with two cylinders each of mass 1 kg, moment of inertia 0.5 kg m^2 and radius 1m as shown in figure. Cylinders are mounted on their control axis of rotation and this system is placed on a rough horizontal surface. The rear cylinder is connected to battery operated motor which provides a torque of 100N-m to this cylinder via a belt as shown. If sufficient friction is present between cylinder and horizontal surface for pure rolling, find acceleration of the vehicle in m/s^2. (Neglect mass of motor, belt and other accessories of vehicle).

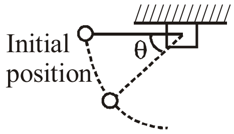


(a) 20 m/s^2 (b) 10 m/s^2

(c) 25 m/s^2 (d) 30 m/s^2

MARK YOUR RESPONSE	81. ⓐⓑⓒⓓ	82. ⓐⓑⓒⓓ	83. ⓐⓑⓒⓓ	84. ⓐⓑⓒⓓ	85. ⓐⓑⓒⓓ
	86. ⓐⓑⓒⓓ				

87. The given figure shows a small mass connected to a string, which is attached to a fixed vertical post. If the mass is released from rest when the string is horizontal as shown, the magnitude of the total acceleration of the mass as a function of the angle θ is



(a) $2g \sin\theta$ (b) $2g \cos\theta$

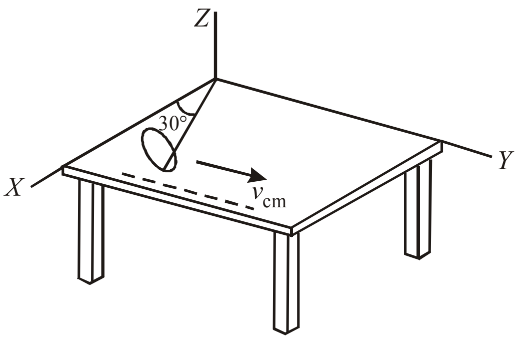
(c) $g\sqrt{3\cos^2\theta+1}$ (d) $g\sqrt{3\sin^2\theta+1}$

88. A particle of mass $m = 1$ kg moves in a circle of radius $R = 2m$ with uniform speed $v = 3\pi$ m/s. The magnitude of impulse given by centripetal force to the particle in one second is

(a) $\sqrt{2}\pi$ Ns (b) $\sqrt{3}\pi$ Ns

(c) $2\sqrt{3}\pi$ Ns (d) $3\sqrt{2}\pi$ Ns

89. A hoop of radius 0.10m and mass 0.50 kg rolls across a table parallel to one edge with a speed of 0.50 m/s. Refer its motion to a rectangular coordinate system with the origin at the left rear corner of the table. At a certain time t, a line drawn from the origin to the point of contact of the hoop with the table has length 1m and makes an angle of 30° with the X-axis (figure). What is the spin angular momentum of the hoop with respect to the origin at this time t ?

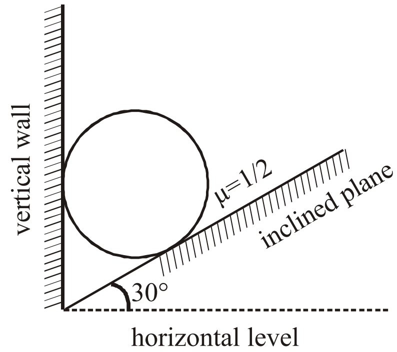


(a) $-0.25\,\hat{i}$ kg m^2/s (b) $-0.005\,\hat{i}$ kg m^2/s

(c) $-0.025\,\hat{i}$ kg m^2/s (d) $-0.5\,\hat{i}$ kg m^2/s

90. A uniform solid sphere of mass m is lying at rest between a vertical wall and a fixed inclined plane as shown. There is no friction between sphere and the vertical wall but coefficient of friction between the sphere and the fixed inclined plane is $\mu = 1/2$. Then the magnitude of frictional force exerted by fixed inclined plane on sphere is

(g is acceleration due to gravity)



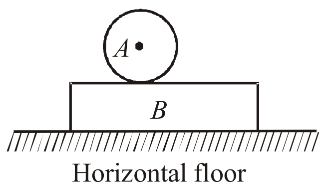
(a) $\frac{mg}{2}$ (d) $\frac{\sqrt{3}}{4}mg$

(c) mg (d) 0

91. Two point masses of 0.3 kg and 0.7 kg are fixed at the ends of a rod of length 1.4 m and of negligible mass. The rod is set rotating about an axis perpendicular to its length with a uniform angular speed. The point on the rod through which the axis should pass in order that the work required for rotation of the rod is minimum, is located at a distance of

(a) 0.42 m from mass of 0.3 kg
(b) 0.70 m from mass of 0.7 kg
(c) 0.98 m from mass of 0.3 kg
(d) 0.98 m from mass of 0.7 kg

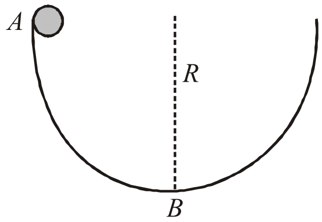
92. A cylinder A rolls without slipping on a plank B. The velocities of center of the cylinder and that of the plank are 4m/s and 2m/s respectively in same direction, with respect to the ground. Find the angular velocity of the cylinder (in rad/s) if its radius is 1m.



(a) 2 rad/sec (b) 4 rad/sec
(c) 6 rad/sec (d) 10 rad/sec

MARK YOUR RESPONSE	87. ⓐⓑⓒⓓ	88. ⓐⓑⓒⓓ	89. ⓐⓑⓒⓓ	90. ⓐⓑⓒⓓ	91. ⓐⓑⓒⓓ
	92. ⓐⓑⓒⓓ				

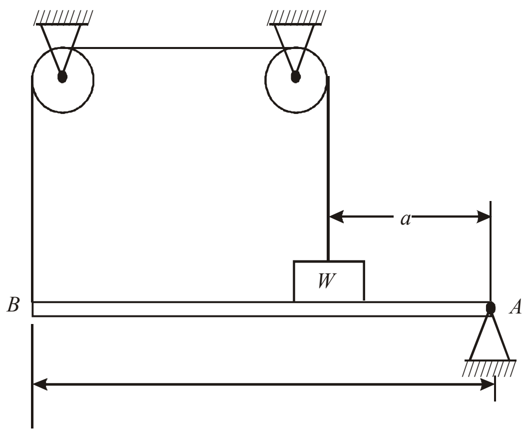
93. A small sphere A of mass m and radius r rolls without slipping inside a large fixed hemispherical bowl of radius R ($>> r$) as shown in figure. If the sphere starts from rest at the top point of the hemisphere find the normal force exerted by the small sphere on the hemisphere when it is at the bottom B of the hemisphere.



(a) $\frac{10}{7}mg$ (b) $\frac{17}{7}mg$

(c) $\frac{5}{7}mg$ (d) $\frac{7}{5}mg$

94. A weight W rests on the bar AB as shown in figure. The cable connecting W and B passes over frictionless pulleys. If bar AB has negligible weight, the vertical component of reaction at A is



(a) $W\frac{L-a}{L+a}$ (b) $W\frac{L+a}{L-a}$

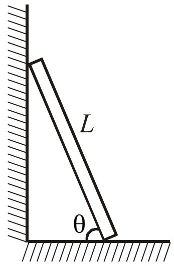
(c) $W\frac{L-2a}{L+a}$ (d) $W\frac{L-a}{L+2a}$

95. A constant force acting at the centre of a uniform disc of radius r is always perpendicular to the plane of the disc. The disc can rotate about a chord at a distance x from the centre of the disc. For what value of x will the angular acceleration of the disc be maximum ?

(a) $\frac{r}{\sqrt{3}}$ (b) $\frac{r}{3}$

(c) $\frac{r}{2}$ (d) $\frac{r}{\sqrt{2}}$

96. A uniform ladder of length L rests against a smooth frictionless wall. The floor is rough and the coefficient of static friction between the floor and ladder is μ. When the ladder is positioned at angle θ, as shown in the accompanying diagram, it is just about to slip. What is θ ?



(a) $\cos\theta = \mu$

(b) $\tan\theta = 2\mu$

(c) $\tan\theta = \frac{1}{2\mu}$

(d) $\sin\theta = \frac{1}{\mu}$

97. Two identical rings A and B are acted upon by torques τ_A and τ_B respectively. A is rotating about an axis passing through the centre of mass and perpendicular to the plane of the ring. B is rotating about a chord at a distance $\frac{1}{\sqrt{2}}$ times the radius from the centre of the ring. If the angular acceleration of the rings is the same, then

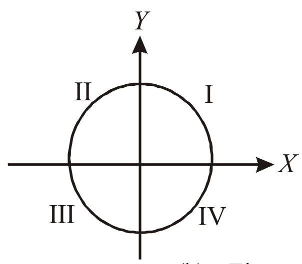
(a) $\tau_A = \tau_B$

(b) $\tau_A > \tau_B$

(c) $\tau_A < \tau_B$

(d) Nothing can be said about τ_A and τ_B as data are insufficient

98. A particle is moving along a circular path as shown in the figure. The instantaneous velocity of the particle is $\vec{v} = (4\text{m/s})\,\hat{i} - (3\text{m/s})\,\hat{j}$. The particle is moving through ...... quadrant if it is travelling clockwise and through .... quadrant if it is travelling anticlockwise, respectively around the circle

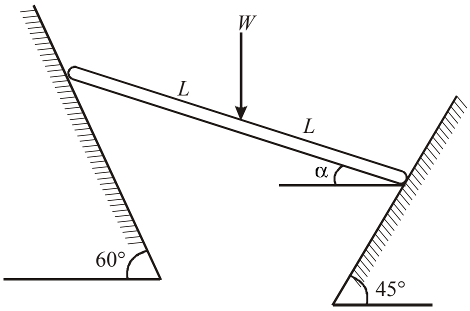


(a) First, first (b) First, second

(c) First, third (d) Third, first

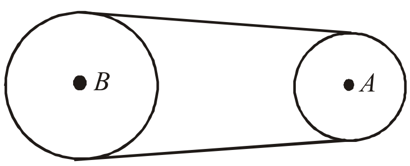
MARK YOUR RESPONSE	93. ⓐⓑⓒⓓ	94. ⓐⓑⓒⓓ	95. ⓐⓑⓒⓓ	96. ⓐⓑⓒⓓ	97. ⓐⓑⓒⓓ
	98. ⓐⓑⓒⓓ				

99. A uniform plank of weight W and total length $2L$ is placed as shown in figure with its ends in contact with the inclined planes. The angle of friction is 15°. Determine the maximum value of the angle α at which slipping impends.



(a) 18.1° (b) 48.4°
(c) 36.2° (d) 88.8°

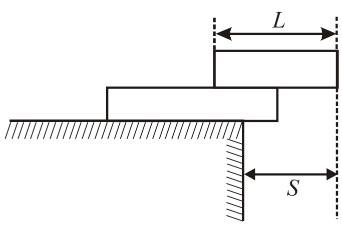
100. A wheel of radius 0.1m (wheel A) is attached by a non-stretching belt to a wheel of radius 0.2m (wheel B). The belt does not slip. By the time wheel B turns through 1 revolution, wheel A will rotate through



(a) $\frac{1}{2}$ revolution (b) 1 revolution
(c) 2 revolution (d) 4 revolution

101. A right circular cone of base diameter 3 cm. and height 6cm. is cut from a solid cylinder of diameter 5cm. and height 12cm. Find the position of CG of rest of the body.
(a) 2.1 cm. (b) 6.3cm (c) 8.2 cm. (d) 5.3cm.

102. Two identical bricks of length L are piled one on top of the other on a table as shown in the figure. The maximum distance S the top brick can overhang the table with the system still balanced is

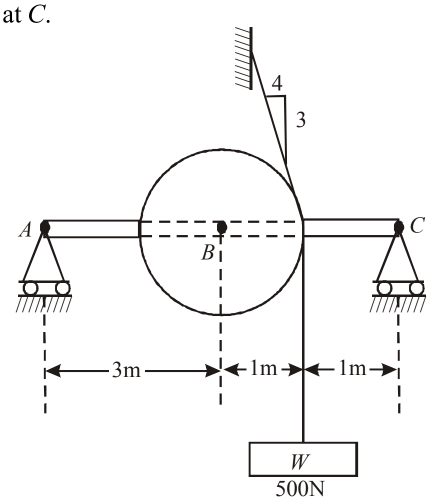


(a) $\frac{1}{2}L$ (b) $\frac{2}{3}L$
(c) $\frac{3}{4}L$ (d) $\frac{7}{8}L$

103. A particle collides with a uniform rod at rest lying on a smooth horizontal plane. Initially the velocity of the particle is not along the length of the rod. Then the ratio of the velocity of the centre of the rod to its angular velocity will

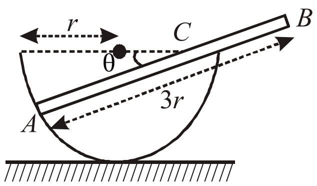
(a) depend on the coefficient of restitution
(b) depend on the masses of the particle and that of the rod
(c) depend on the initial velocity of the particle
(d) depend on the length of rod and the position of the point where the particle strikes

104. A pulley of 1m radius, supporting a load of 500N is mounted at B on a horizontal beam as shown in figure. The beam weighs 200N and the pulley weighs 50N, find the hinge force at C.



(a) 271.69 N (b) 671.69 N
(c) 371.69 N (d) 471.69 N

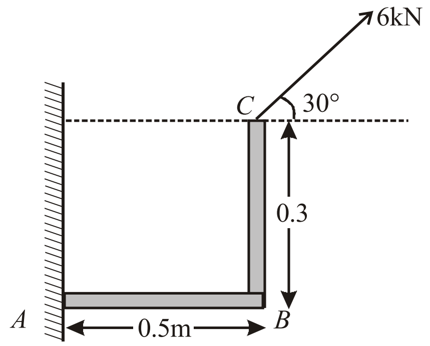
105. A uniform rod AB of length three times the radius of a hemisphered bowl remains in equilibrium in the bowl as shown. Neglecting friction find the inclination of the rod with the horizontal.



(a) $\sin^{-1}(0.92)$ (b) $\cos^{-1}(0.92)$
(c) $\cos^{-1}(0.49)$ (d) $\tan^{-1}(0.92)$

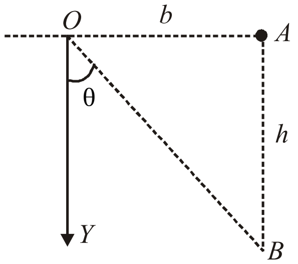
MARK YOUR RESPONSE					
	99. ⓐⓑⓒⓓ	100. ⓐⓑⓒⓓ	101. ⓐⓑⓒⓓ	102. ⓐⓑⓒⓓ	103. ⓐⓑⓒⓓ
	104. ⓐⓑⓒⓓ	105. ⓐⓑⓒⓓ			

106. Find the moment of force about point A as shown in the figure.



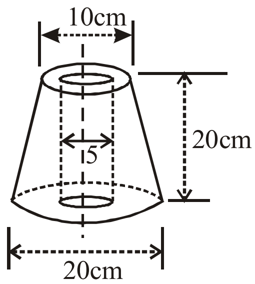
(a) 0.06 kNm clockwise
(b) 0.06 kNm anticlockwise
(c) 0.03 kNm clockwise
(d) 0.03 kNm anticlockwise

107. A particle of mass m is released from rest at point A in the figure falling freely under gravity parallel to the vertical Y-axis. The magnitude of angular momentum of particle about point O when it reaches B is
(where $OA = b$ and $AB = h$)



(a) mh/bg (b) $mb\sqrt{2gh}$
(c) $mh\sqrt{2gh}$ (d) None of these

108. A frustum of a solid right circular cone has a base diameter of 20cm, top diameter of 10cm. and height 20cm. It has an axial cylindrical hole of diameter 5cm. Determine the position of centre of gravity of this body



(a) 7.6cm. (b) 4.3cm
(c) 12.6cm. (d) 15.3 cm.

109. Consider the following statements :

S_1 : Zero net torque on a body means absence of rotational motion of the body.
S_2 : A particle may have angular momentum even though the particle is not moving in a circle.
S_3 : A ring of rolling without sliding on a fixed surface. The centripetal acceleration of each particle with respect to the centre of the ring is same.

State in order, whether S_1, S_2, S_3 are true or false.

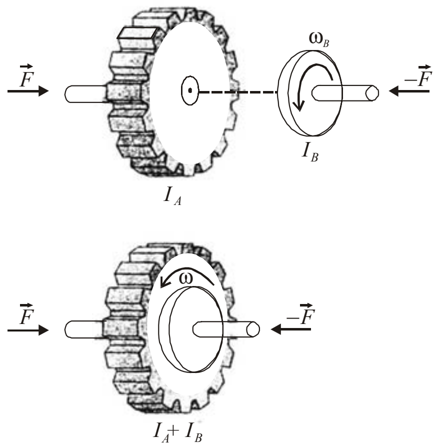
(a) FTT (b) FFT
(c) TTF (d) FTF

Mark Your Response	106.ⓐⓑⓒⓓ	107.ⓐⓑⓒⓓ	108.ⓐⓑⓒⓓ	109.ⓐⓑⓒⓓ	

B COMPREHENSION TYPE

This section contains groups of questions. Each group is followed by some multiple choice questions based on a paragraph. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct.

PASSAGE-1



The figure has two disks : one an engine flywheel, and the other a clutch plate attached to a transmission shaft. Their moments of inertial are I_A and I_B; initially, they are rotating with constant angular speeds ω_A and ω_B, respectively. We then push the disks together with forces acting along the axis, so as not to apply any torque on either disk. The disks rub against each other and eventually reach a common final angular speed ω. Suppose flywheel A has a mass of 2.0 kg, a radius of 0.20 m and an initial angular speed of 50 rad/sec. (about 500 rpm) and that clutch plate B has a mass of 4.0 kg, a radius of 0.10 m, and an initial angular speed of 200 rad/sec.

1. Find an expression for ω?

(a) $\dfrac{I_A\omega_A + I_B\omega_B}{I_A - I_B}$ (b) $\dfrac{I_A\omega_A - I_B\omega_B}{I_A - I_B}$

(c) $\dfrac{I_A\omega_A + I_B\omega_B}{I_A \quad I_B}$ (d) $\dfrac{I_A\omega_A - I_B\omega_B}{I_A \quad I_B}$

2. Find the common final angular speed ω after the disks are pushed into contact?

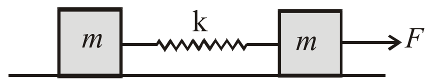
(a) 10 rad/sec (b) 100 rad/sec
(c) 1000 rad/sec (d) .010 rad/sec

3. What happens to the final kinetic energy during this process?

(a) 300 J (b) 3 J
(c) 30 J (d) 3000 J

PASSAGE-2

Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on the first block pulling it away from the other as shown in figure.



4 The displacement of the centre of mass at time t is

(a) $\dfrac{Ft^2}{2m}$ (b) $\dfrac{Ft^2}{3m}$

(c) $\dfrac{Ft^2}{4m}$ (d) $\dfrac{Ft^2}{m}$

5. If the extension of the spring is x_0 at time t, then the displacement of the first block at this instant is

(a) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m} + x_0\right)$ (b) $-\dfrac{1}{2}\left(\dfrac{Ft^2}{2m} + x_0\right)$

(c) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m} - x_0\right)$ (d) $\left(\dfrac{Ft^2}{2m} + x_0\right)$

6. If the extension of the spring is x_0 at time t, then the displacement of the second block at this instant is

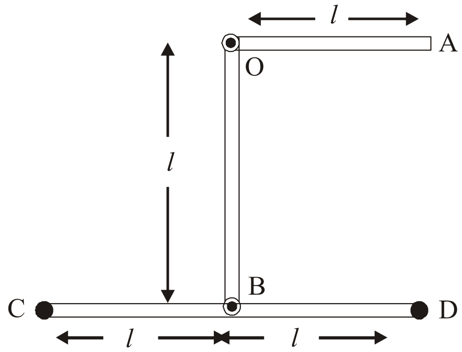
(a) $\left(\dfrac{Ft^2}{2m} - x_0\right)$ (b) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m} + x_0\right)$

(c) $\dfrac{1}{2}\left(\dfrac{2Ft^2}{m} - x_0\right)$ (d) $\dfrac{1}{2}\left(\dfrac{Ft^2}{2m} - x_0\right)$

PASSAGE-3

Rigid uniform L-shaped rod AOB has mass $2m$ and is free to rotate about a fixed point O on a horizontal frictionless plane. Now massless rigid rod CD is connected at end B of L-shaped rod such that it can freely rotate about B, as shown in the figure. Two masses, of mass m each, are connected to ends C and D. Now an impulse is given at point A (perpendicular to OA) such that the total assembly gets an initial angular speed ω.

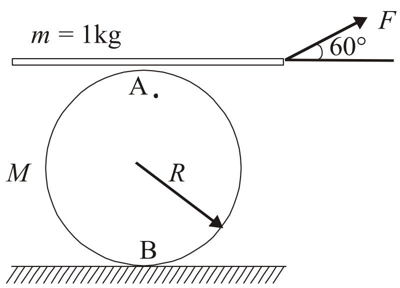
MARK YOUR RESPONSE	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ				



7. Angular velocity of AO will
   (a) be constant at ω.
   (b) be varying, with least value ω
   (c) be varying, with maximum value ω.
   (d) linearly drop to zero with time
8. Distance between A and C will
   (a) be constant at $\sqrt{5}\ell$
   (b) be varying, with max value 3ℓ
   (c) be varying, with max value $\sqrt{5}\ell$
   (d) be varying, with max value $\ell\left(\sqrt{2}+1\right)$
9. Magnitude of relative velocity of A w.r.t. C
   (a) be constant at $\sqrt{2}\ell\omega$
   (b) keeps varying, with a maximum value $\ell\omega$
   (c) keeps varying, with a max value $\sqrt{2}\ell\omega$
   (d) keeps varying, with a maximum value $\left(\sqrt{2}+1\right)\ell\omega$

PASSAGE-4

Consider a cylinder of mass $M = 1$kg and radius $R = 1$ m lying on a rought horizontal plane. It has a plank lying on its stop as shown in the figure.



A force F = 55 N is applied on the plank such that the plank moves and causes the cylinder to roll. The plank always remains horizontal. There is no slipping at any point of contact.

10. Calculate the acceleration of cylinder.
    (a) 20 m/s^2 (b) 10 m/s^2
    (c) 5 m/s^2 (d) None of these
11. Find the value of frictional force at *A*
    (a) 7.5 N (b) 5.0 N
    (c) 2.5 N (d) None of these
12. Find the value of frictional force at *B*
    (a) 7.5 N (b) 5.0 N
    (c) 2.5 N (d) None of these

PASSAGE-5

n identical rods each of mass *m* are welded at their ends to form a regular polygon and the corners are then welded to a metal ring of radius *R* and mass *M*, such that the plane of polygon and plane of ring are in same plane and centres of polygon and ring coincide.

13. The moment of inertia of the system about an axis passing through the centre of mass of system and perpendicular to the plane of system will be
    (a) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{3}+\cos^2\frac{\pi}{n}\right]+MR^2$
    (b) $nmR^2\left[\dfrac{\tan^2\frac{\pi}{n}}{4}+\sin^2\frac{\pi}{n}\right]+MR^2$
    (c) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{4}+\cos^2\frac{\pi}{n}\right]+MR^2$
    (d) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{4}+\dfrac{\cos^2\frac{\pi}{n}}{3}\right]+MR^2.$

MARK YOUR RESPONSE	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ
	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ			

14. The moment of inertia of system about any one of the corner of polygon and perpendicular to its plane will be

(a) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{3}+\cos^2\dfrac{\pi}{n}+1\right]+MR^2.$

(b) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{3}+\cos^2\dfrac{\pi}{n}+1\right]+2MR^2$

(c) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{3}+\cos^2\dfrac{\pi}{n}+2\right]+2MR^2$

(d) $nmR^2\left[\dfrac{\sin^2\frac{\pi}{n}}{3}+\cos^2\dfrac{\pi}{n}+4\right]+MR^2.$

15. If the rigid assembly of rods and hoop is allowed to roll down the incline of inclination θ, the minimum value of the coefficient of static friction that will prevent slipping will be

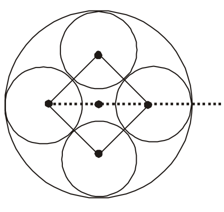
(a) $\dfrac{\text{I}\tan\theta}{2I+(M+nm)R^2}$ (b) $\dfrac{\text{I}\sin\theta}{\text{I}+(M+nm)R^2}$

(c) $\dfrac{\text{I}\cos\theta}{2\text{I}+(M+nm)R^2}$ (d) $\dfrac{\text{I}\tan\theta}{\text{I}+(M+nm)R^2}.$

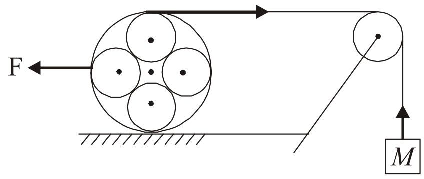
(where I is moment of inertia about centre of mass)

PASSAGE-6

Four identical spheres having mass M and radius R are fixed tightly within a massless ring such that the centres of all spheres lie in the plane of ring. The ring is kept on a rough horizontal table as shown. The string is wrapped around the ring which can roll without slipping. The other end of the string is passed over a massless frictionless pulley to a block of mass M. A force F is applied horizontally on the ring, at the same level as the centre, so that the system is in equilibrium.



16. The moment of inertia of the combined ring system about the centre of ring will be



(a) $\dfrac{12}{5}MR^2$ (b) $\dfrac{48}{15}MR^2$

(c) $\dfrac{24}{5}MR^2$ (d) $\dfrac{48}{5}MR^2$

17. The magnitude of F equals

(a) Mg (b) $2Mg$

(c) $\dfrac{Mg}{2}$ (d) none of these

18. If the masses of the spheres were doubled keeping their dimensions same, the force of friction between the ring and the horizontal surface would

(a) be doubled

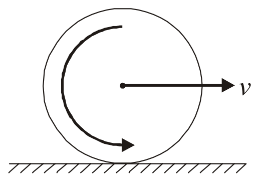
(b) increase but be less than double

(c) remain the same

(d) decrease

PASSAGE-7

A disc of radius R is spun to an angular speed ω_0 about its axis and then imparted a horizontal velocity $v_0=\dfrac{\omega_0 R}{4}$ (at $t=0$) with its plane remaining vertical. The coefficient of friction between the disc and plane is μ. The direction of v_0 and ω_0 are shown in the figure.



19. The disc will return to starting point at time

(a) $\left(\dfrac{25}{48}\right)\dfrac{R\omega_0}{\mu g}$ (b) $\dfrac{5}{12}\dfrac{R\omega_0}{\mu g}$

(c) $\dfrac{5}{48}\dfrac{R\omega_0}{\mu g}$ (d) $\dfrac{R\omega_0}{6\mu g}$

MARK YOUR RESPONSE	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ				

20. The disc will start rolling without slipping at time

(a) $\frac{R\omega_0}{6\mu g}$ (b) $\frac{5R\omega_0}{12\mu g}$

(c) $\frac{R\omega_0}{\mu g}$ (d) $\frac{R\omega_0}{4\mu g}$

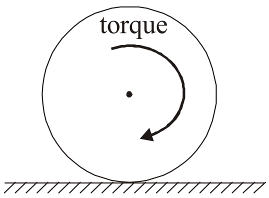
21. The angular momentum of the disc about the point of contact, when slipping ceases is equal to

(a) $MR^2\frac{\omega_0}{2}$ (b) $MR^2\frac{\omega_0}{12}$

(c) $MR^2\frac{\omega_0}{4}$ (d) $MR^2\frac{\omega_0}{3}$

PASSAGE-8

A uniform disc rolls on a rough horizontal surface without slipping. It starts rolling with a small initial velocity v_0, and is continuously acted upon by a torque (provided through the axle), that delivers constant power. The initial kinetic energy of the disc can be ignored.



Rough horizontal surface

22. The velocity v, of the centre of the wheel varies with time (t) according to (assume that t is large enough)

(a) $v \propto t$ (b) $v \propto \frac{1}{t}$

(c) $v \propto \sqrt{t}$ (d) $v \propto \frac{1}{t^2}$.

23. If the magnitude of the force of friction acting on the wheel be f when it travelled a distance x (which is large), then

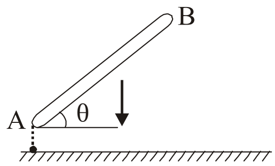
(a) f = constant
(b) fx = constant
(c) f^2x = constant
(d) f^3x = constant.

24. If the coefficient of static friction between the wheel and the horizontal surface be μ, the time for which the wheel rolls without slipping is proportional to

(a) $\mu^{1/2}$
(b) μ^{-2}
(c) μ^2
(d) the wheel will continue to roll without slipping forever, independent of μ.

PASSAGE-9

A uniform rod AB of length 2ℓ falls without rotation on a smooth horizontal surface at an angle θ to the horizontal. The speed of rod just before collision is v_0 and the collision is elastic. The magnitude of the angular velocity and magnitude of the velocity of centre of mass after collision are ω and v' respectively.



25. The direction of force of impact on the rod is

(a) along the surface
(b) vertically upward
(c) along the rod
(d) any direction is possible

26. The relation between v_0, ω and v' is

(a) $v_0 = v'$ (b) $v_0 = v' + \ell\omega$

(c) $v_0 = v' - \ell\omega$ (d) $v_0 = v' + \ell\omega\cos\theta$.

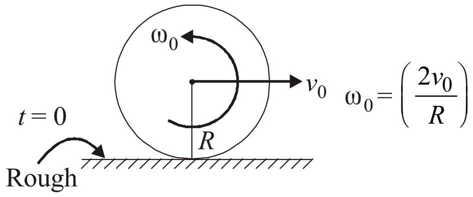
27. The angular momentum before collision about the point on the ground at which the rod strikes has the magnitude (m : mass of rod)

(a) $\frac{mv_0\ell}{2}$ (b) $mv_0\ell\cos\theta$

(c) $\frac{mv_0\ell}{2\cos\theta}$ (d) $\left(\frac{m\ell}{3}\right)\frac{v_0}{\ell}$.

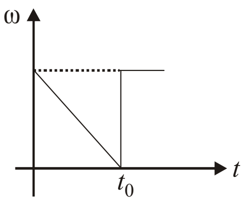
PASSAGE-10

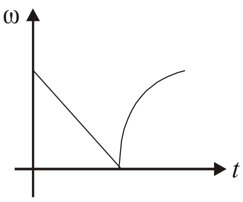
Rolling is the combination of translation and rotation. The point of contact plays a crucial role as it decides the direction of friction. A uniform disc of mass M and radius R is given a linear velocity v_0 and angular speed ω_0 as shown in the diagram on a rough horizontal surface.

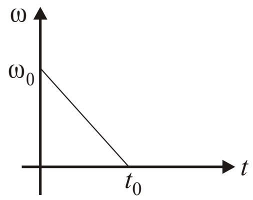


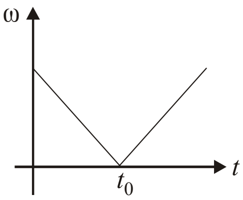
MARK YOUR RESPONSE					
	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ
	25. ⓐⓑⓒⓓ	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ		

28. The graph of angular speed of disc about its centre is best represented by

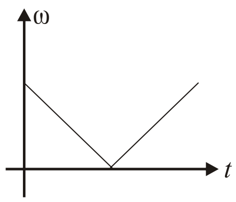
(a) 

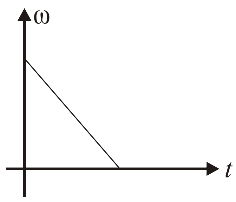
(b) 

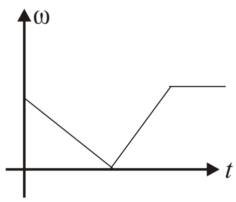
(c) 

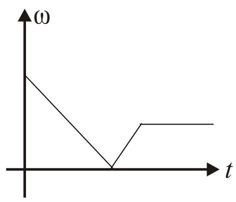
(d) 

29. The disc is now replaced by a uniform solid sphere of mass M and radius R. Again sphere is given same velocity and angular speed under same situation. Then group of angular speed about centre vs time 't' is given by

(a) 

(b) 

(c) 

(d) 

30. The velocity of centre of mass of sphere in previous question when pure rolling starts will be equal to

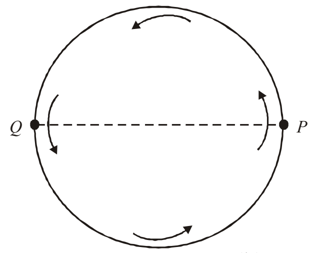
(a) $\frac{2v_0}{5}$ (b) $\frac{5v_0}{6}$

(c) $\frac{v_0}{7}$ (d) zero

PASSAGE-11

A particle continuously moves in a circular path at constant speed in a counter clockwise direction. Consider a time interval during which the particle moves along this circular path from point P to point Q. Point Q is exactly half-way around the circle from point P.

31. What is the direction of the average velocity during this time interval ?



(a) $\rightarrow$ (b) $\leftarrow$

(c) $\uparrow$ (d) $\downarrow$

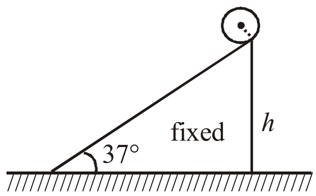
32. What is the direction of the average acceleration during this time interval ?

(a) $\rightarrow$ (b) $\leftarrow$

(c) $\uparrow$ (d) $\downarrow$

PASSAGE-12

In the figure shown a uniform solid sphere is released on the top of a fixed inclined plane of inclination of 37° and height h. It rolls without sliding. (Take $\sin 37° = 3/5$ and g is acceleration due to gravity)



33. The acceleration of the centre of the sphere is

(a) $3g/5$ (b) $4g/5$ (c) $4g/7$ (d) $3g/7$

34. The speed of the point of contact of the sphere with the inclined plane when the sphere reaches the bottom of the incline is

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{10gh}{7}}$

(c) zero (d) $2\sqrt{2gh}$

MARK YOUR RESPONSE					
	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ
	33. ⓐⓑⓒⓓ	34. ⓐⓑⓒⓓ			

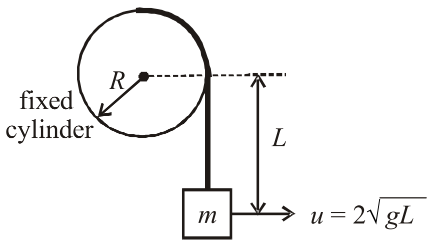
35. The time taken by the sphere to reach the bottom is

(a) $\sqrt{\dfrac{2h}{g}}$ (b) $\sqrt{\dfrac{70h}{9g}}$

(c) $\sqrt{\dfrac{25h}{18g}}$ (d) $\sqrt{\dfrac{25h}{6g}}$

PASSAGE-13

The cross-section of a fixed cylinder (not allowed to rotate and translate) with horizontal axis is as shown. One end of a light inelastic string is fixed at top of cylinder of radius R and a small block of mass m is tied to the other end of string. Initially the block is at rest with the portion of string not in contact with cylinder being vertical and having length L as shown. At the lowest position the block is given initial horizontal velocity $u \quad \sqrt{2gL}$ and the block moves in vertical plane. When the block reaches the highest point of its trajectory, the length of string not in contact with cylinder is $L \quad \dfrac{R\pi}{3}$ (where g is acceleration due to gravity).



36. The distance between block and centre of cylinder when block is at highest position will be

(a) $2R$ (b) $\sqrt{5}R$

(c) $3R$ (d) $2R/\sqrt{3}$

37. The least tension in string is

(a) $\dfrac{mg}{\sqrt{6}}$ (b) $\dfrac{mg}{\sqrt{5}}$

(c) $\dfrac{\sqrt{3}}{2}mg$ (d) $\dfrac{mg}{2}$

38. Tangential acceleration of block at highest position is

(a) $\dfrac{g}{2}$ (b) $\dfrac{\sqrt{3}}{2}g$

(c) $\dfrac{g}{3}$ (d) $\dfrac{g}{\sqrt{6}}$

PASSAGE-14

An 80 kg cyclist riding a 16 kg bicycle is travelling along a city street at 18 km / hour. A taxi pulls in front of him and stops suddenly to pick up a passenger. By the time the cyclist assesses the situation and begins to apply his brakes, he is 5 m away from the taxi. The cyclist knows from experience that if he jams on the brakes and his wheels skid, he will have less braking force, so he avoids this. The brake pads have a surface area of 8 mm by 70mm and a coefficient of friction of 0.5, the bicycle has 700 mm diameter wheels and the diameter of the middle of the rim (where the brake pads are applied) is 650 mm.

39. If the braking force is distributed equally between the front and back wheels, calculate the force that must be applied to each pad in order to stop in time.

(a) 129 N (b) 25.8 N

(c) 240 N (d) 258 N

40. If the brake lever (on the handlebar) moves 2 cm for each brake pad to move 1 mm, calculate the force which must be applied to the brake lever.

(a) 129 N (b) 25.8 N

(c) 240 N (d) 258 N

MARK YOUR RESPONSE	35. ⓐⓑⓒⓓ	36. ⓐⓑⓒⓓ	37. ⓐⓑⓒⓓ	38. ⓐⓑⓒⓓ	39. ⓐⓑⓒⓓ
	40. ⓐⓑⓒⓓ				

C

REASONING TYPE

In the following questions two Statements (1 and 2) are provided. Each question has 4 choices (a), (b), (c) and (d) for its answer, out of which ONLY ONE is correct. Mark your responses from the following options :

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.
(b) Both Statement-1 and Statement-2 are true and Statement-2 is not the correct explanation of Statement-1.
(c) Statement-1 is true but Statement-2 is false.
(d) Statement-1 is false but Statement-2 is true.

1. **Statement - 1** : A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.

Statement - 2 : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

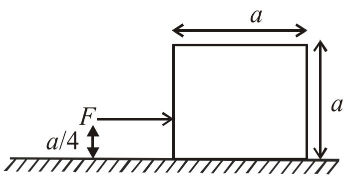
2. **Statement - 1** : A uniform thin rod of length L is hinged about one of its end and is free to rotate about the hinge without friction. Neglect the effect of gravity. A force F is applied at a distance x from the hinge on the rod such that force always is perpendicular to the rod. As the value of x is increased from zero to L, the component of reaction by hinge on the rod perpendicular to length of rod increases.

Statement - 2 : Under the conditions given in statement 1 as x is increased from zero to L, the angular acceleration of rod increases.

3. **Statement - 1** : If two different axes are at same distance from centre of mass of a rigid body, then moment of inertia of the given rigid body about both axis will always be same.

Statement - 2 : From parallel axis theorem $I = I_{cm} + md^2$, where all terms have usual meaning.

4. **Statement - 1** : A uniform cubical block (of side a) undergoes translational motion on a smooth horizontal surface under action of horizontal force F as shown.



Under the given condition, the horizontal surface exerts normal reaction non-uniformly on lower surface of the block.

Statement – 2 : For the cubical block given in statement-1, the horizontal force F has tendency to rotate the cube about its centre in clockwise sense. Hence, the lower right edge of cube presses the horizontal surface harder in comparison to the force exerted by lower left edge of cube on horizontal surface.

5. **Statement - 1** : Radius of gyration of a body is not a constant quantity.

Statement - 2 : The radius of gyration of a body about an axis of rotation may be defined as the root mean square distance of the particle from the axis of rotation.

6. **Statement - 1** : Two bodies A and B are attracted towards each other due to gravitation. If A is much heavier than B then the center of mass of the bodies moves towards A.

Statement - 2 : The centre of mass depends upon mass distribution of a body or a system of bodies.

7. **Statement - 1** : A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling motion).

Statement - 2 : For perfect rolling motion, work done against friction is zero.

8. **Statement - 1** : Centrifugal force is reaction force of the centripetal force.

Statement - 2 : A rotating frame of reference is a non-inertial frame.

9. **Statement - 1** : In non-uniform circular motion, velocity vector and acceleration vector are not perpendicular to each other.

Statement - 2 : In non-uniform circular motion, particle has normal as well as tangential acceleration.

10. **Statement - 1** : When a body rolls on a rough surface, friction force is always zero.

Statement - 2 : A particle cannot roll on a surface.

11. **Statement - 1** : A particle is moving in circular path. The net work done on the particle is zero.

Statement - 2 : For a particle undergoing uniform circular motion, net force acting on the particle and velocity of the particle are always perpendicular.

12. **Statement - 1** : If two different axes are at same distance from centre of mass of a rigid body, then moment of inertia of the given rigid body about both axes will always be same.

Statement - 2 : From parallel axis theorem $I = I_{cm} + md^2$, where all terms have usual meaning.

13. **Statement - 1** : Kinetic energy of a system is minimum in centre of mass frame of reference.

Statement - 2 : In centre of mass frame kinetic energy of all particles is smaller than their respective kinetic energy in ground frame.

MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ
	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ	8. ⓐⓑⓒⓓ	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ
	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ		

14. **Statement - 1 :** A cyclist is cycling on rough horizontal circular track with increasing speed. Then the frictional force on cycle is always directed towards centre of the circular track.

Statement - 2 : For a particle moving in a circle, radial component of net force should be directed towards centre.

15. **Statement - 1 :** If two different axes are at same distance from centre of mass of a rigid body, then moment of inertia of the given rigid body about both axis will always be same.

Statement - 2 : From parallel axis theorem $I = I_{cm} + md^2$, where all terms have usual meaning.

16. **Statement - 1 :** KE of rotating rigid body in *CM* frame is $\frac{1}{2}I_{cm}\omega^2$, where symbols have usual meaning.

Statement - 2 : In *CM* frame rigid body has pure rotational motion.

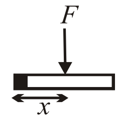
17. **Statement - 1 :** If net force $\vec{F}$ acting on a system is changing in direction only, the linear momentum $(\vec{p})$ of system changes in direction.

Statement - 2 : In case of uniform circular motion, magnitude of linear momentum is constant but direction of centripetal force changes at every instant.

18. **Statement - 1 :** A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.

Statement - 2 : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.

19. A uniform thin rod of length *L* is hinged about one of its end and is free to rotate about the hinge without friction. Neglect the effect of gravity. A force *F* is applied at a distance *x* from the hinge on the rod such that force is always perpendicular to the rod. As the value of *x* is increased from zero to *L*,



Statement - 1 : The component of reaction force by hinge on the rod perpendicular to length of rod increases.

Statement - 2 : The angular acceleration of rod increases.

20. **Statement - 1 :** Net external torque (τ_{ext}) on a system of particles is equal to rate of change of angular momentum $\frac{d\vec{L}}{dt}$, if τ_{ext} and $\vec{L}$ are measured with respect to any fixed point in an inertial frame.

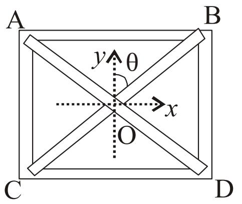
Statement-2 : If a body is in rotational equilibrium, then net torque on a body about any fixed point is zero.

MARK YOUR RESPONSE					
	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ			

D MULTIPLE CORRECT CHOICE TYPE

Each of these questions has 4 choices (a), (b), (c) and (d) for its answer, out of which ONE OR MORE is/are correct.

1. As shown in figure, a planner assembly, having six rods, each of mass *m* is lying in *x*-*y* plane with O at origin, lengths of AD and BC are ℓ . If I_z denotes the moment of inertia of the assembly about *z*-axis and I_y denotes moment of inertia about *y*-axis.



(a) I_z will have its highest value for $\theta = 45°$
(b) I_z will have its highest value for $\theta = 90°$
(c) I_z will have its highest value for $\theta = 0°$
(d) I_y will have its highest value for $\theta = 90°$

2. A particle is moving along an expanding spiral in such a manner that magnitude of normal acceleration of particle remains constant. Choose the correct options
(a) Linear speed of particle is increasing
(b) Linear speed of particle is decreasing
(c) Angular speed of paricle is increasing
(d) Angular speed of particle is decreasing

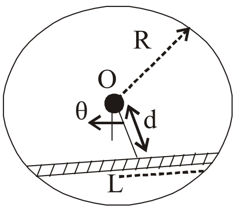
MARK YOUR RESPONSE					
	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ			

3. The moment of inertia of a hollow cylinder of mass M and inner radius R_1 and outer radius R_2 about its central axis is

(a) $\frac{1}{2}M(R_2^2 - R_1^2)$ (b) $\frac{1}{2}M(R_1^2 + R_2^2)$

(c) $M(R_1^2 - R_2^2)$ (d) $\frac{1}{2}M(R_2 - R_1)^2$

4. A uniform rod moves in a vertical circle. Its ends are constrained to move on the track without friction. The angular frequency of small oscillation is given by



(a) $\sqrt{\frac{\left(R^2+\frac{L^2}{4}\right)}{\left(R^2-\frac{L^2}{4}\right)}g}$ (b) $\sqrt{\frac{\left(R^2-\frac{L^2}{4}\right)^{1/2}}{\left(R^2-\frac{L^2}{6}\right)}g}$

(c) $\sqrt{\frac{(4R^2+L^2)}{(R^2-L^2)}g}$ (d) $\sqrt{\frac{\left(R^2+\frac{L^2}{6}\right)}{\left(R^2-\frac{L^2}{6}\right)}g}$

5. A torque τ on a body about a given point is found to be equal to $\vec{C}\times\vec{L}$, where $\vec{C}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this, it follows that

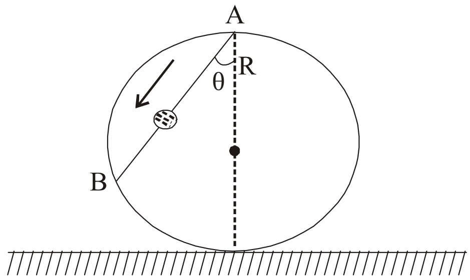
(a) $\vec{L}$ does not change with time.

(b) $\frac{d\vec{L}}{dt}$ is perpendicular to $\vec{L}$ at all instants of time.

(c) the magnitude of $\vec{L}$ does not change with time.

(d) all the above

6. A bead is free to slide down a smooth wire tightly stretched between the points A and B on a fixed vertical circle. If the bead starts from rest at A, the highest point on the circle.



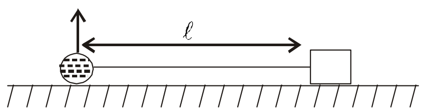
(a) its velocity v on arriving at B is proportional to cos θ.

(b) its velocity v on arriving at B is proportional to tan θ.

(c) time to arrive at B is proportional to cos θ.

(d) time to arrive at B is independent of θ.

7. A small ball is connected to a block by a light string of length ℓ. Both are initially on the ground. There is sufficeint friction om the ground to prevent the block from slipping. The ball is projected vertically up with a velocity u, where $2g\ell < u^2 < 3g\ell$. The centre of mass of the block + ball system is C.



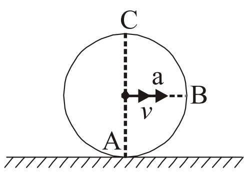
(a) C will move along a circle.

(b) C will move along a parabola

(c) C will move along a straight line

(d) The horizontal component of the velocity of the ball will be maximum when the string makes an angle q = $\sin^{-1}(u^2/3g\ell)$ with the horizontal

8. A wheel is rolling on a horizontal plane without slipping. At a certain instant, it has velocity 'v' and acceleration 'a' of c.m. as shown in the figure. Acceleration of



(a) A is vertically upwards

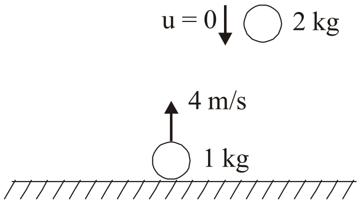
(b) B may be vertically downwards

(c) C cannot be horizontal

(d) Some point on the rim may be horizontal leftwards.

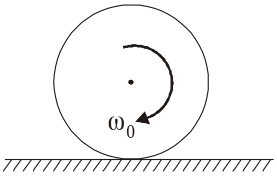
MARK YOUR RESPONSE	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ	5. ⓐⓑⓒⓓ	6. ⓐⓑⓒⓓ	7. ⓐⓑⓒⓓ
	8. ⓐⓑⓒⓓ				

9. A ball of mass 1 kg is thrown up with an initial speed of 4 m/s. A second ball of mass 2 kg is released from rest from some height as shown in the figure.



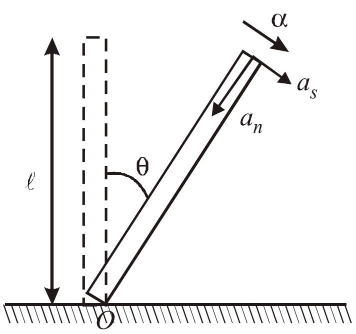
(a) The centre of mass of the two balls comes down with acceleration $g/3$.
(b) The centre of mass first moves up and then comes down
(c) The acceleration of the centre of mass is g downwards
(d) The centre of mass of the two balls remains stationary.

10. A disc is given an initial angular velocity ω_0 and placed on rough horizontal surface as shown. The quantities which will not depend on the coefficient of friction is/are



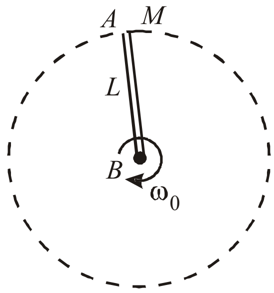
(a) The time until rolling begins.
(b) The displacement of the disc until rolling begins.
(c) The velocity when rolling begins.
(d) The work done by the force of friction.

11. The angular acceleration of the toppling pole shown in figure is given by $\alpha = k \sin\theta$, where θ is the angle between the axis of the pole and the vertical, and k is a constant. The pole starts from rest at $\theta = 0$. Choose the correct options



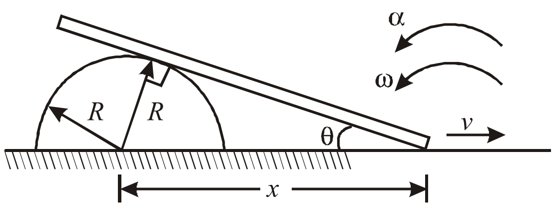
(a) The tangential acceleration of the upper end of the pole is $\ell k \sin\theta$
(b) The centripetal acceleration of the upper end of the pole is $2k\ell(1 - \cos\theta)$
(c) The tangential acceleration of the upper end of the pole is $2k\ell(1 - \cos\theta)$
(d) The centripetal acceleration of the upper end of the pole is $\ell k \sin\theta$

12. A thin rod AB of mass M and length L is rotating with angular speed ω_0 about vertical axis passing through its end B on a horizontal smooth table as shown. If at some instant the hinge at end B of rod is opened then which of the following statement is/are correct about motion of rod ?



(a) The angular speed of rod after opening the hinge will remain ω_0.
(b) The angular speed of rod after opening the hinge will be less than ω_0.
(c) In the process of opening the hinge the kinetic energy of rod will remain conserved.
(d) Angular momentum of rod will remain conserved about centre of mass of rod in the process of opening the hinge.

13. A rod leans against a stationary cylindrical body as shown in figure, and its right end slides to the right on the floor with a constant speed v. Choose the correct option(s).



(a) the angular speed ω is $\dfrac{-Rv^2(2x^2 - R^2)}{x^2(x^2 - R^2)^{3/2}}$

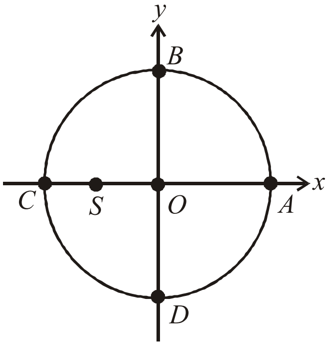
(b) the angular acceleration α is $\dfrac{Rv}{x\sqrt{x^2 - R^2}}$

(c) the angular speed ω is $\dfrac{Rv}{x\sqrt{x^2 - R^2}}$

(d) the angular acceleration α is $\dfrac{-Rv^2(2x^2 - R^2)}{x^2(x^2 - R^2)^{3/2}}$

MARK YOUR RESPONSE	9. ⓐⓑⓒⓓ	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ

14. A small object moves counter clockwise along the circular path whose centre is at origin as shown in figure. As it moves along the path, its acceleration vector continuously points towards point *S*. Then the object



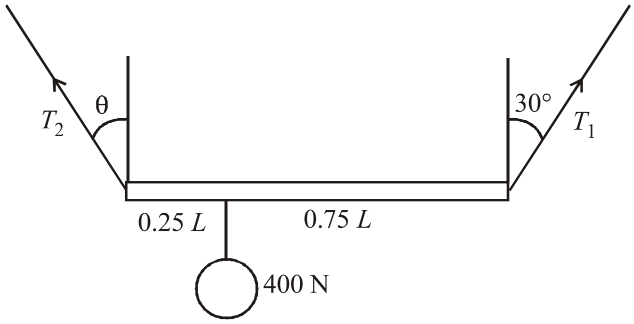
(a) Speeds up as it moves from *A* to *C* via *B*
(b) Slows down up as it moves from *A* to *C* via *B*
(c) Slows down as it moves from *C* to *A* via *D*
(d) Speeds up as it moves from *C* to *A* via *D*

15. Which of the following are not correct about centre of mass?
(a) Centre of mass of a system of four particles in a plane must lie within the quadrilateral formed by the four particles.
(b) In centre of mass frame momentum of a system is always zero.
(c) Internal force may affect the motion of centre of mass.
(d) Centre of mass and centre of gravity are synonymous in all situations.

16. A ball tied to the end of a string swings in a vertical circle under the influence of gravity.
(a) When the string makes an angle 90° with the vertical the tangential acceleration is zero and radial acceleration is somewhere between maximum and minimum.
(b) When the string makes an angle 90° with the vertical the tangential acceleration is maximum and radial acceleration is somewhere between maximum and minimum.
(c) At no place in the circular motion, tangential acceleration is equal to radial acceleration.
(d) Throughout the path whenever radial acceleration has its extreme value, the tangential acceleration is zero.

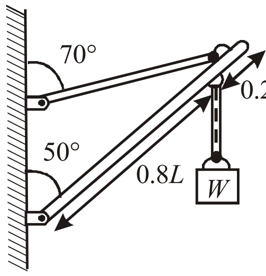
17. A cylinder rolls without slipping on a rough floor, moving with a speed *v*. It makes an elastic collision with smooth vertical wall. After impact
(a) it will move with a speed v initially
(b) its motion will be rolling without slipping
(c) its motion will be rolling with slipping initially and its rotational motion will stop momentarily at some instant.
(d) its motion will be rolling without slipping only after some time

18. The uniform 120 N board shown in figure is supported by two ropes. A 400 N weight is suspended one-fourth of the way from the left end. Choose the correct options



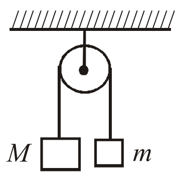
(a) $T_1 = 185$ N
(b) $T_2 = 371$ N
(c) $T_2 = 185$ N
(d) $\tan\theta = 0.257$

19. Neglecting the weight of the beam in figure, choose the correct options



(a) Tension in the tied rope is 1.80 *W*
(b) The force components at the hinge are 1.69 *W*, 1.62 *W*
(c) If the uniform beam weighs *W*/2 then *T* = 2.35 *W*
(d) Force component at the hinge are 2.21 *W* and 2.30 *W* if uniform beam weighs *W*/2.

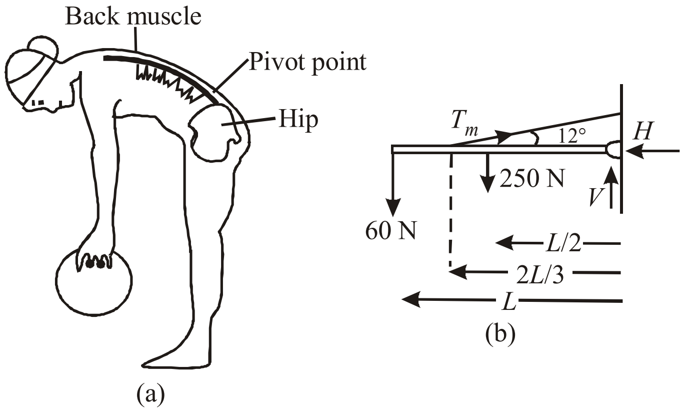
20. A light rope passes over a light frictionless pulley attached to the ceiling. An object with a large mass is tied to one end and an object with a smaller mass is tied to the other end. Both masses are released from rest. Which of the following statement (s) is/are false for the system consisting of the two moving masses while string remains taut?



(a) the centre of mass remains at rest
(b) the net external force is zero
(c) the velocity of the center of mass is a constant
(d) the acceleration of the center of mass is g downward

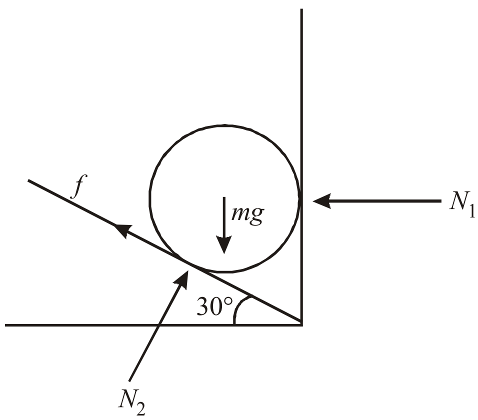
MARK YOUR RESPONSE	14. ⓐⓑⓒⓓ	15. ⓐⓑⓒⓓ	16. ⓐⓑⓒⓓ	17. ⓐⓑⓒⓓ	18. ⓐⓑⓒⓓ
	19. ⓐⓑⓒⓓ	20. ⓐⓑⓒⓓ			

21. Consider a woman lifting a 60N bowling ball as shown in figure (a). Approximate the situation as shown in figure (b) and assume the upper part of her body to weigh 250N with center of gravity as indicated. Choose the correct options



(a) The tension in her back muscle is 1335 N
(b) The compressional force in her spine when her back is horizontal is 1305 N
(c) The tension in her back muscle is 1305 N
(d) The compressional force in her spine when her back is horizontal is 1335 N

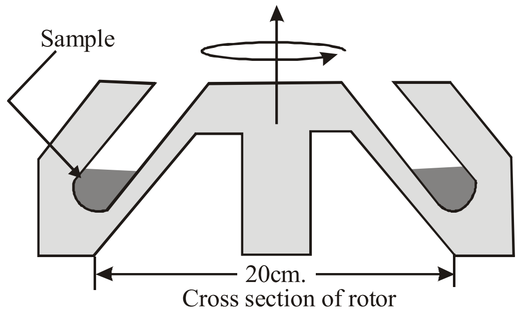
22. A sphere of radius 0.10m and mass 10 kg rests in the corner formed by a 30° inclined plane and a smooth vertical wall. Choose the correct options



(a) $N_1 = 56.5$ N
(b) $N_2 = 113$ N
(c) $f = 0$
(d) $f \neq 0$

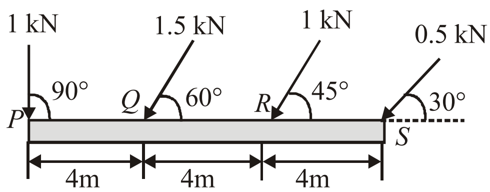
23. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground. Then pickup the correct alternative or alternatives.
(a) The magnitude of angular momentum of the particle about O is increasing
(b) The magnitude of torque of the gravitational force on the particle about O is decreasing
(c) The moment of inertia of the particle about O is decreasing
(d) The magnitude of angular velocity of the particle about O is increasing

24. Four samples of a colloidal aqueous mixture each weighing 12.0 g are placed in the rotor of a high speed centrifuge, equally spaced around the circumference of the rotor. The samples are located at 10 cm from the axis of rotation of the rotor. The centrifuge motor delivers a constant torque of 0.25 Nm and the empty rotor has a moment of inertia of 0.06 kg m^2. Choose the correct options



(a) In 456.2 sec. rotor accelerate to its operating state of 18,000 rpm (rotations per minute).
(b) When the centrifuge is up to speed 18,000 rpm the force exerted on the sample by the rotor is 2264 N
(c) In 756.2 sec. rotor accelerate to its operating state of 18,000 rpm (rotations per minute).
(d) When the centrifuge is up to speed 18,000 rpm the force exerted on the sample by the rotor is 4264 N

25. A horizontal beam $PQRS$ is 12m long. Forces of 1kN, 1.5kN, 1 kN and 0.5 kN act at P, Q, R, S respectively in downward direction. The line of action of these forces make angle of 90°, 60°, 45°, 30° respectively with line PS. If $PQ = QR = RS$ = 4m, choose the correct options related to resultant force.



(a) the magnitude of resultant force is 3.77 kN
(b) position of the resultant force is 3.67m from P
(c) the magnitude of resultant force is 1.77 kN
(d) position of the resultant force is 1.67m from P

MARK YOUR RESPONSE	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ	25. ⓐⓑⓒⓓ

26. A tube of length L is filled completely with an incomressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is

(a) $\dfrac{M\omega^2 L}{2}$ (b) $M\omega^2 L$

(c) $\dfrac{M\omega^2 L}{4}$ (d) $\dfrac{M\omega^2 L^2}{2}$

27. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A pendulum bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with track is

(a) zero (b) 30°

(c) 45° (d) 60°

28. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(a) I (b) $I\sin^2\theta$

(c) $I\cos^2\theta$ (d) $I\cos^2(\theta/2)$

29. The torque τ on a body about a given point is found to be equal to $\boldsymbol{A} \times \boldsymbol{L}$ where $\boldsymbol{A}$ is a constant vector, and $\boldsymbol{L}$ is the angular momentum of the body about that point. From this it follows that

(a) $\dfrac{d\boldsymbol{L}}{dt}$ is perpendicular to $\boldsymbol{L}$ at all instants of time.

(b) the component of $\boldsymbol{L}$ in the direction of $\boldsymbol{A}$ does not change with time.

(c) the magnitude of $\boldsymbol{L}$ does not change with time.

(d) $\boldsymbol{L}$ does not change with time

30. A solid cylinder is rolling down a rough inclined plane of inclination θ. Then

(a) The friction force is dissipative

(b) The friction force is necessarily changing

(c) The friction force will aid rotation but hinder translation

(d) The friction force is reduced if θ is reduced

31. A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}, \vec{v}, \vec{\alpha}$ and $\vec{a}_c$. Which of the following relations is/are correct ?

(a) $\vec{\omega}.\vec{v} = 0$ (b) $\vec{\omega}.\vec{\alpha} = 0$

(c) $\vec{\omega}.\vec{a}_c = 0$ (d) $\vec{v}.\vec{a}_c = 0$

32. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

(a) linear momentum of the system does not change in time

(b) kinetic energy of the system does not change in time

(c) angular momentum of the system does not change in time

(d) potential energy of the system does not change in time

MARK YOUR RESPONSE					
	26. ⓐⓑⓒⓓ	27. ⓐⓑⓒⓓ	28. ⓐⓑⓒⓓ	29. ⓐⓑⓒⓓ	30. ⓐⓑⓒⓓ
	31. ⓐⓑⓒⓓ	32. ⓐⓑⓒⓓ			

E MATRIX-MATCH TYPE

Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labeled A, B, C and D, while the statements in Column-II are labelled p, q, r, s and t. Any given statement in Column -I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example:
If the correct matches are A–p, s and t; B–q and r; C–p and q; and D–s and t; then the correct darkening of bubbles will look like the given.

	p	q	r	s	t
A	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
B	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
C	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ
D	ⓟ	ⓠ	ⓡ	ⓢ	ⓣ

1. A particle of 500 gm mass moves along a horizontal circle of radius 16 m such that normal acceleration of particle varies with time as $a_n = 9t^2$

	Column I		**Column II**
(A)	Tangential force on particle at $t = 1$ second (in newton)	(p)	72
(B)	Total force on particle at $t = 1$ second (in newton)	(q)	36
(C)	Power delivered by total force at $t = 1$ sec (in watt)	(r)	7.5
(D)	Average power developed by total force over first one second (in watt)	(s)	6

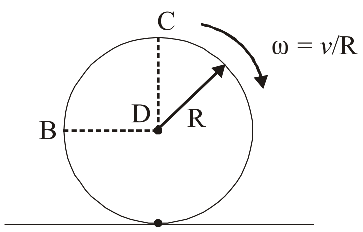
2. A rigid body of mass M and radius R rolls without slipping on an inclined plane of inclination θ, under gravity. Match the type of body with magnitude of the force of friction.

	Column I		**Column II**
(A)	For ring	(p)	$\dfrac{Mg\sin\theta}{2.5}$
(B)	For solid sphere	(q)	$\dfrac{Mg\sin\theta}{3}$
(C)	For solid cylinder	(r)	$\dfrac{Mg\sin\theta}{3.5}$
(D)	For hollow spherical shell	(s)	$\dfrac{Mg\sin\theta}{2}$

3.

	Column I		**Column II**
(A)	Axial vector	(p)	Rotational K.E.
(B)	Scalar quantities	(q)	Translational K.E.
(C)	Turning ability of force	(r)	Angular momentum
(D)	A rolling body can have	(s)	Torque

4. A rigid body is rolling without slipping on horizontal surface. At a given instant BD is perfectly horizontal and CD is perfectly vertical.



	Column I		**Column II**
(A)	Velocity at point A; v_A	(p)	$v\sqrt{2}$
(B)	Velocity at point B; v_B	(q)	Zero
(C)	Velocity at point C; v_C	(r)	v
(D)	Velocity at point D; v_D	(s)	$2v$

MARK YOUR RESPONSE

1.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

2.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

3.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

4.

	p	q	r	s
A	(p)	(q)	(r)	(s)
B	(p)	(q)	(r)	(s)
C	(p)	(q)	(r)	(s)
D	(p)	(q)	(r)	(s)

5. A uniform disc is acted upon by some forces and it rolls on a horizontal plank without slipping from north to south. The plank, in turn lies on a smooth horizontal surface. Match the following regarding this situation

Column I	Column II
(A) Frictional force on the disc by the surface	(p) May be directed towards north
(B) Velocity of the lowermost point of the disc	(q) May be directed towards south
(C) Acceleration of centre of mass of the disc	(r) May be zero
(D) Vertical component of the acceleration of centre of mass	(s) Must be zero

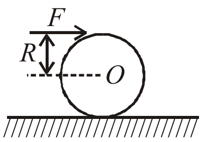
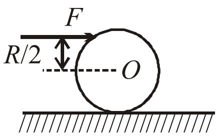
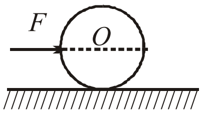
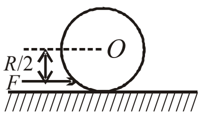
6. Match the following :

Column I	Column II
(A) Work done by friction force may be	(p) Dependent on the property of metal
(B) Work done by pseudo force may be	(q) Positive
(C) Work done by torque may be	(r) Negative
(D) Work function of a metal surface is	(s) Zero

7. A motorcycle moves around a vertical circle with a constant speed under the influence of the force of gravity $\vec{W}$, friction between wheel and track $\vec{f}$ and normal reaction between wheel and track $\vec{N}$.

Column I	Column II
(A) Constant magnitude	(p) $\vec{N}$
(B) Directed towards centre when value is non-zero	(q) $\vec{N}+\vec{f}$
(C) Total reaction force by track	(r) $\vec{f}+\vec{W}$
(D) When motion is along vertical the value is zero	(s) $\vec{N}+\vec{W}+\vec{f}$

8. A uniform solid cylinder of mass m and radius *R* is placed on a rough horizontal surface where friction is sufficient to provide pure rolling. A horizontal force of magnitude *F* is applied on cylinder at different positions with respect to its centre *O* in each of four situations of column-I, due to which magnitude of acceleration of centre of mass of cylinder is 'a'. Match the appropriate results in column-II for conditions of column-I.

Column I	Column II
(A) 	(p) Friction force on cylinder will not be zero
(B) 	(q) $a=\dfrac{F}{m}$
(C) 	(r) $a\neq\dfrac{F}{m}$
(D) 	(s) the direction of friction force acting on cylinder is towards left

MARK YOUR RESPONSE

5. A ⓟⓠⓡⓢ B ⓟⓠⓡⓢ C ⓟⓠⓡⓢ D ⓟⓠⓡⓢ

6. A ⓟⓠⓡⓢ B ⓟⓠⓡⓢ C ⓟⓠⓡⓢ D ⓟⓠⓡⓢ

7. A ⓟⓠⓡⓢ B ⓟⓠⓡⓢ C ⓟⓠⓡⓢ D ⓟⓠⓡⓢ

8. A ⓟⓠⓡⓢ B ⓟⓠⓡⓢ C ⓟⓠⓡⓢ D ⓟⓠⓡⓢ

9. A small object of mass 0.5 kg is attached to an end of a massless 2 meter long inextensible string with the other end of the string being fixed. Initially, the string is vertical and the object is at its lowest position having initial horizontal velocity of magnitude u. The tension in string is T when the object is at its lowest position. The object subsequently moves in vertical plane. The forces acting on object are tension exerted by string and gravitational pull by earth. Match the statements in column I with corresponding results in column II (Take $g = 10$ m/s^2).

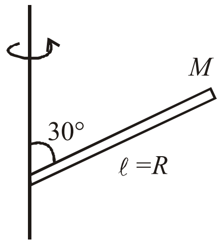
Column I	Column II
(A) $u = 3.5$ m/s	(p) There will be some point on the trajectory of object at which speed of the object is zero but tension in the string is not zero.
(B) $u = 9.5$ m/s	(q) There will be some point on the trajectory of object for which tension in the string is zero but speed of the object is not zero.
(C) $T = 15$N	(r) There will be some point on the trajectory of object for which tension in the string as well as speed of object are both zero.
(D) $T = 35$N	(s) The acceleration of the object will be in vertical direction

10. Match the columns

Column I
(Object)

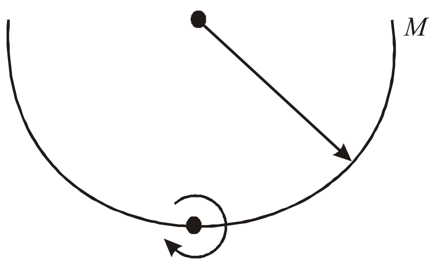
Column II
(Moment of Inertia)

(A) Uniform rod



(p) $\dfrac{8MR^2}{11}$

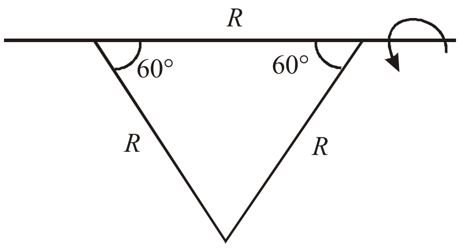
(B) Uniform semicircular ring. Axis is perpendicular to plane of ring



[$\pi = 22/7$]

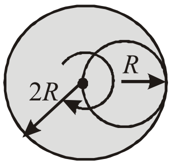
(q) $\dfrac{MR^2}{12}$

(C) Uniform triangular plate of mass M



(r) $\dfrac{13MR^2}{8}$

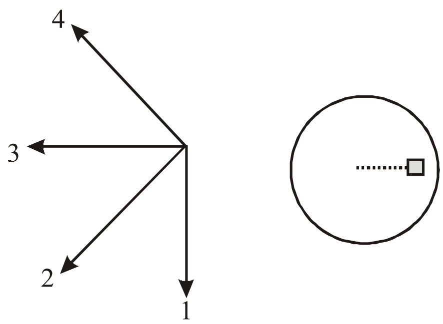
(D) Uniform disk of initial mass M from which circular portion of radius R is then removed. M.I. of remaining mass about axis which is perpendicular to plane of plate. (s) $\frac{MR^2}{8}$



11. If net external force on a system of particles is zero, then match the following

Column I	**Column II**
(A) Acceleration of centre of mass	(p) must be constant
(B) Kinetic energy of the system	(q) must be zero
(C) Velocity of centre of mass	(r) may not be zero
(D) Velocity of an individual particle of the system	(s) may not be constant

12. A block is placed on a horizontal table which can rotate about its axis. A block is placed at a certain distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure. Then match the column.



Column I	**Column II**
(A) When rotation is clockwise with constant ω	(p) 1
(B) When rotation is clockwise with decreasing ω	(q) 2
(C) When rotation is clockwise with increasing ω	(r) 3
(D) Just after clockwise rotation begins from rest	(s) 4

MARK YOUR RESPONSE

	9. p q r s	10. p q r s	11. p q r s	12. p q r s
A	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ
B	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ
C	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ
D	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ	ⓟⓠⓡⓢ

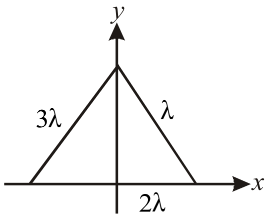
13. A particle moves with position given by $r = 3t\hat{i} + 4\hat{j}$. Where r is measured in meters and t (> 0) in seconds

Column I	**Column II**
(A) Rate of change of distance from origin	(p) Increasing with time
(B) Magnitude of linear acceleration of particle	(q) decreasing with time
(C) Magnitude of angular velocity of particle about origin	(r) constant
(D) Magnitude of angular momentum of particle about origin	(s) zero

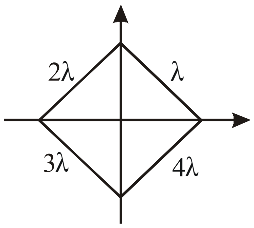
14. In each situation of column I a mass distribution is given and information regarding x and y coordinates of centre of mass is given in column II. Match the figures in column I with corresponding information of centre of mass in column II.

Column I **Column II**

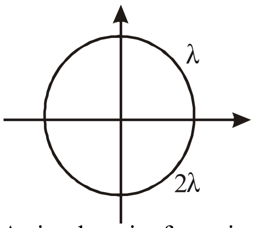
(A) An equilateral wire frame is made using three thin uniform rods of mass per unit lengths λ, 2λ and 3λ as shown. (p) $x_{cm} \geq 0$



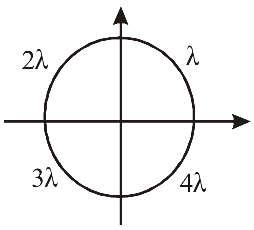
(B) An square frame is made using four thin uniform rods of mass per unit lengths λ, 2λ, 3λ and 4λ as shown. (q) $y_{cm} \geq 0$



(C) A circular wire frame is made of two uniform semicircular wires of same radius and of mass per unit length λ and 2λ as shown. (r) $x_{cm} < 0$



(D) A circular wire frame is made of four uniform quarter circular of same radius and of mass per unit length λ, 2λ, 3λ and 4λ as shown. (s) $y_{cm} < 0$



MARK YOUR RESPONSE

13.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

14.

	p	q	r	s
A	p	q	r	s
B	p	q	r	s
C	p	q	r	s
D	p	q	r	s

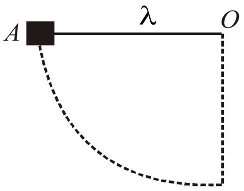
15. A solid sphere, hollow sphere, solid cylinder, hollow cylinder and ring each of mass M and radius R are simultaneously released at rest from top of incline and roll (pure rolling) down the incline then match column I and column II.

Column I	**Column II**
(A) Time taken to reach bottom is maximum for	(p) Solid sphere
(B) Angular acceleration is maximum for	(q) Hollow cylinder
(C) Kinetic energy at bottom is same for	(r) Hollow sphere
(D) Rotational kinetic energy is maximum for	(s) Ring

16. An object is allowed to roll down the incline starting from rest. All are uniform and have same mass and radius.

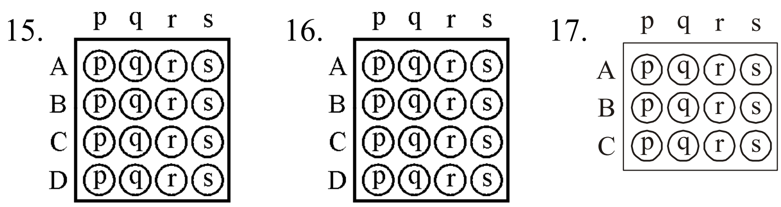
Column I	**Column II**
(A) The object which has largest rotational inertia about its axes of symmetry	(p) Solid sphere
(B) The object which will experience the largest net torque	(q) Spherical shell
(C) The object which will have the largest speed at the bottom of the incline	(r) Solid disc
(D) The object which will reach the bottom of incline in the shortest time	(s) Thin hollow cylinder

17. A block of mass m is tied with an inextensible light string of length ℓ. One end of the string is fixed at point O. Block is released (from rest) at A. Find acceleration of particle during its motion in vertical plane at positions specified in column I and match them with column II. Given that A and O are at same horizontal level.



Column I	**Column II**
(A) At highest point	(p) Acceleration is horizontal
(B) At lowest point	(q) Acceleration is vertically upwards
(C) At $\theta = \tan^{-1}(\sqrt{3})$ with vertical	(r) Acceleration is vertically downwards
	(s) Acceleration has both horizontal and vertical components

MARK YOUR RESPONSE



F

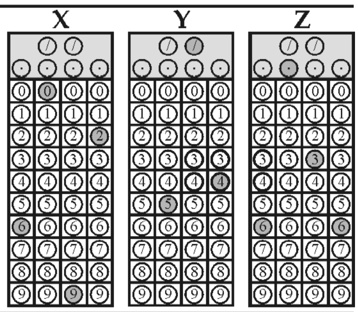
NUMERIC/INTEGER ANSWER TYPE

The answer to each of the questions is either numeric (eg. 304, 40, 3010, 3 etc.) or a fraction (2/3, 23/7) or a decimal (2.35, 0.546).

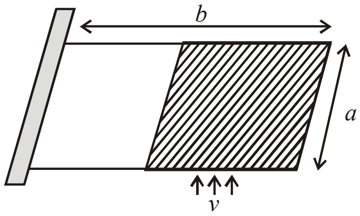
The appropriate bubbles below the respective question numbers in the response grid have to be darkened.

For example, if the correct answers to question X, Y & Z are 6092, 5/4 & 6.36 respectively then the correct darkening of bubbles will look like the following.

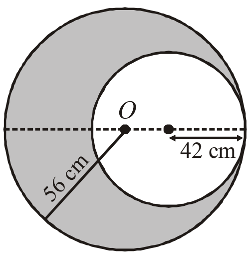
For single digit integer answer darken the extreme right bubble only.



1. A rectangular plate of mass M and dimension $(a \times b)$ is held in horizontal position by striking n small balls (each of mass m) per unit area per second by a velocity v. The balls are striking in the shaded half region of the plate. The collision of the balls with the plate is elastic. What is the value of $(0.1\ v)$ in m/s ?
(Given $n = 100$, $M = 3$ kg, $m = 0.01$ kg; $b = 2$ m; $a = 1$ m; $g = 10\ m/s^2$).

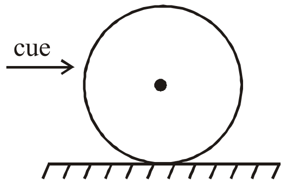


2. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown. At what distance (in cm) to the left from the centre of the disc is the centre of mass of the remaining portion ?

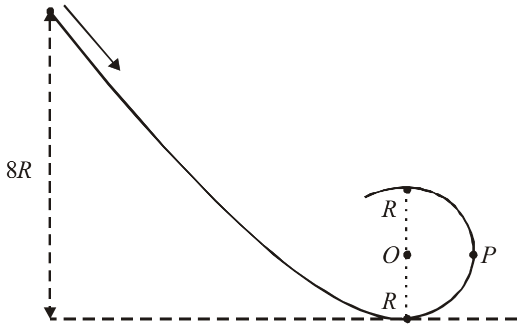


3. Two blocks of masses m_1 (=1kg) and m_2 (= 2kg) connected by a weightless spring of stiffness k (= 0.2 N/m)rest on a smooth horizontal plane as shown in fig. Block 2 is shifted a small distance x (= 0.1m) to the left and then released. Find the velocity (in m/s) of centre of mass of the system after block 1 breaks off the wall.

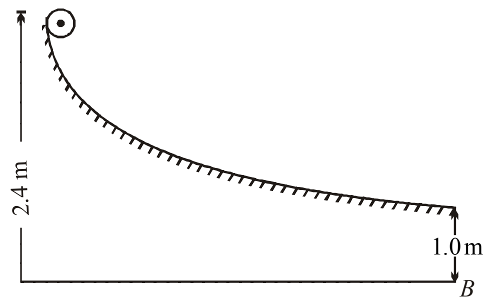
4. A sphere at rest on a horizontal rough surface is hit by a cue. At what height (in cm) above the centre of the sphere should it be hit so that it starts pure rolling, just after hitting ? Assume the radius of the sphere to be 0.5 cm.



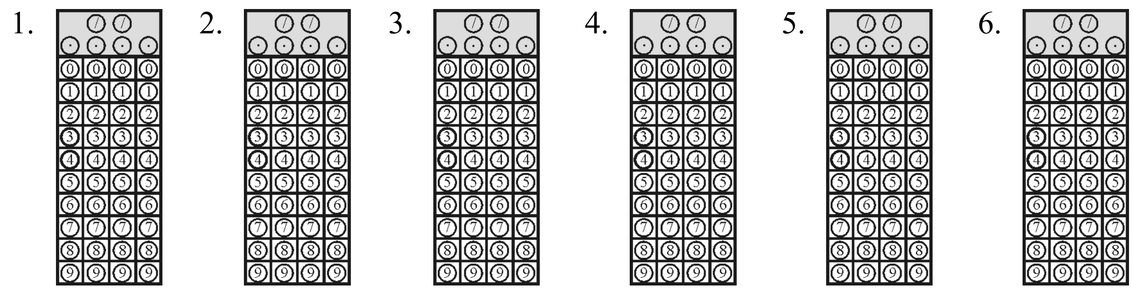
5. A small solid ball (mass = 0.1kg) rolls without slipping along the track shown in Fig. The radius of the circular part of track is R. If the ball starts from rest at a height $8R$ above the bottom, what is the horizontal forced (in N) acting on it at P?



6. A small sphere rolls down without slipping from the top of a track in a vertical plane. The track has an elevated section and a horizontal part. The horizontal part is 1.0 metre above the ground level and the top of the track is 2.4 metres above the ground. Find the distance (in m) on the ground with respect to the point B (which is vertically below the end of the track as shown in fig.) where the sphere lands.



MARK YOUR RESPONSE



Answerkey

A SINGLE CORRECT CHOICE TYPE

1	(c)	**12**	(b)	**23**	(d)	**34**	(a)	**45**	(a)	**56**	(d)	**67**	(a)	**78**	(b)	**89**	(c)	**100**	(c)
2	(b)	**13**	(b)	**24**	(d)	**35**	(d)	**46**	(b)	**57**	(c)	**68**	(b)	**79**	(c)	**90**	(d)	**101**	(b)
3	(b)	**14**	(b)	**25**	(d)	**36**	(c)	**47**	(d)	**58**	(a)	**69**	(c)	**80**	(d)	**91**	(c)	**102**	(c)
4	(c)	**15**	(a)	**26**	(b)	**37**	(a)	**48**	(b)	**59**	(b)	**70**	(c)	**81**	(d)	**92**	(a)	**103**	(d)
5	(a)	**16**	(a)	**27**	(a)	**38**	(b)	**49**	(d)	**60**	(d)	**71**	(d)	**82**	(d)	**93**	(b)	**104**	(d)
6	(c)	**17**	(d)	**28**	(c)	**39**	(b)	**50**	(d)	**61**	(d)	**72**	(a)	**83**	(b)	**94**	(a)	**105**	(b)
7	(c)	**18**	(d)	**29**	(a)	**40**	(d)	**51**	(b)	**62**	(a)	**73**	(c)	**84**	(c)	**95**	(c)	**106**	(a)
8	(c)	**19**	(c)	**30**	(b)	**41**	(c)	**52**	(b)	**63**	(c)	**74**	(a)	**85**	(d)	**96**	(c)	**107**	(b)
9	(b)	**20**	(b)	**31**	(c)	**42**	(c)	**53**	(c)	**64**	(a)	**75**	(c)	**86**	(a)	**97**	(a)	**108**	(a)
10	(b)	**21**	(c)	**32**	(c)	**43**	(c)	**54**	(a)	**65**	(b)	**76**	(c)	**87**	(d)	**98**	(c)	**109**	(d)
11	(b)	**22**	(d)	**33**	(a)	**44**	(c)	**55**	(a)	**66**	(c)	**77**	(c).	**88**	(d)	**99**	(c)		

B COMPREHENSION TYPE

1	(c)	**5**	(a)	**9**	(a)	**13**	(a)	**17**	(b)	**21**	(c)	**25**	(b)	**29**	(d)	**33**	(d)	**37**	(d)
2	(b)	**6**	(b)	**10**	(b)	**14**	(b)	**18**	(c)	**22**	(c)	**26**	(d)	**30**	(c)	**34**	(c)	**38**	(b)
3	(a)	**7**	(a)	**11**	(a)	**15**	(d)	**19**	(a)	**23**	(d)	**27**	(b)	**31**	(b)	**35**	(b)	**39**	(a)
4	(c)	**8**	(d)	**12**	(c)	**16**	(d)	**20**	(b)	**24**	(d)	**28**	(c).	**32**	(d)	**36**	(a)	**40**	(b)

C REASONING TYPE

1	(d)	**3**	(d)	**5**	(a)	**7**	(b)	**9**	(a)	**11**	(d)	**13**	(c)	**15**	(d)	**17**	(b)	**19**	(d)
2	(d)	**4**	(c)	**6**	(c)	**8**	(d)	**10**	(d)	**12**	(d)	**14**	(d)	**16**	(a)	**18**	(d)	**20**	(c)

D MULTIPLE CORRECT CHOICE TYPE

1	(a, b, c, d)	**5**	(b, c)	**9**	(b, c)	**13**	(c, d)	**17**	(a, c, d)	**21**	(a, b)	**25**	(a, b)	**29**	(a, b, c)
2	(a, d)	**6**	(a, d)	**10**	(c, d)	**14**	(a, c)	**18**	(a, b, d)	**22**	(a, b, c)	**26**	(a)	**30**	(c, d)
3	(b)	**7**	(a, d)	**11**	(a, b)	**15**	(a, c, d)	**19**	(a, b, c, d)	**23**	(a, c, d)	**27**	(c)	**31**	(a,c)
4	(b)	**8**	(a, b, c, d)	**12**	(a, c, d)	**16**	(b, d)	**20**	(a, b, c, d)	**24**	(a, d)	**28**	(a)	**32**	(a)

E MATRIX-MATCH TYPE

1. A-s; B-r; C-p; D-q
2. A-s; B-r; C-q; D-p
3. A-r, s; B-p, q; C-s; D-p, q, r, s
4. A-q; B-p; C-s; D-r
5. A-p, q, r; B-p, q, r; C-p, q, r; D-s
6. A-q, r, s; B-q, r, s; C-q, r, s; D-p, q
7. A-s; B-p, s; C-q; D-r
8. A- p, r; B-q, r; C-p, r, s; D-p, r, s
9. A-p, s; B-q, s; C-r, s; D-s
10. A-q; B-p; C-s; D-r
11. A-p, q; B-r, s; C-p, r; D-r, s
12. A-r; B-s; C-q; D-p
13. A-p; B-r, s; C-q; D-r
14. A-q, r; B-p, s; C-p, s; D-p, s
15. A-q, s; B-p; C-p, q, r, s; D-q, s
16. A-s; B-s; C-p; D-p
17. A-r; B-q; C-s

F NUMERIC/INTEGER ANSWER TYPE

1	1	**2**	9	**3**	0.02	**4**	0.2	**5**	10	**6**	2

Solutions

A SINGLE CORRECT CHOICE TYPE

1. **(c)** At the highest point,
weight = centrifugal force gives $mg = mR\omega^2$

$$\therefore \omega = \sqrt{\frac{g}{R}} = 2\pi n \quad \therefore n = \frac{1}{2\pi}\sqrt{\frac{g}{R}} = \sqrt{\frac{g}{4\pi^2 R}}$$

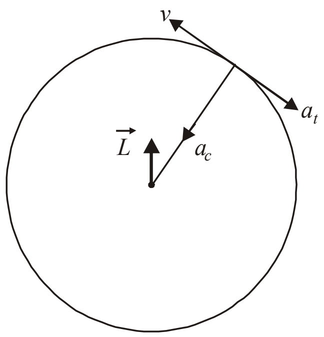
$$\text{r.p.m} = 60\,n = 60\sqrt{\frac{g}{4\pi^2 R}} = \sqrt{\frac{900g}{\pi^2 R}}$$

2. **(b)** $I = I_{metal} + I_{wood}$

$$= \frac{m_1 R_1^2}{2} + m_2\left(\frac{R_1^2 + R_2^2}{2}\right)$$

$$= 4\left(\frac{0.2}{2}\right)^2 + 3\left(\frac{0.2^2 + 0.3^2}{2}\right) = 0.515$$

3. **(b)** Since v is changing (decreasing), L is not conserved in magnitude. Since it is given that a particle is confined to rotate in a circular path, it cannot have spiral path. Since the particle has two accelerations a_c and a_t therefore the net acceleration is not towards the centre.



The direction of $\vec{L}$ remains same even when the speed decreases.

4. **(c)** $\bar{x} = \dfrac{m(-2b) + 2m(-b) + m \times 0 + 2m(b)}{m + 2m + m + 2m} = \dfrac{-b}{3}$ and

$\bar{y} = +b.$

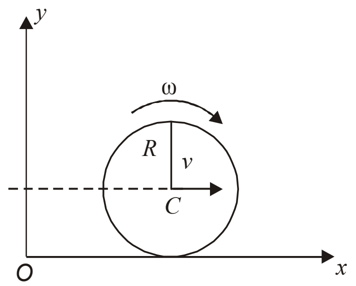
5. **(a)** Moment of inertia of the rod w.r.t. the axis through centre of the disc is (by parallel axis theorem)

$$I = \frac{mL^2}{12} + m\mathrm{R}^2 \text{ and K.E. of rod w.r.t. disc}$$

$$= \frac{1}{2}I\omega^2 = \frac{1}{2}m\omega^2\left[\mathrm{R}^2 + \frac{L^2}{12}\right]$$

6. **(c)** The disc has two types of motion namely translational and rotational. Therefore there are two types of angular momentum and the total angular momentum is the vector sum of these two.

In this case both the angular momentum have the same direction (perpendicular to the plane of paper and away from the reader).



$$\vec{L} = \vec{L}_T + \vec{L}_R$$

L_T = angular momentum due to translational motion.

L_R = angular momentum due to rotational motion about $C.M.$

$L = MV \times R + I_{\text{cm}}\omega$

$I_{\text{cm}} = M.I.$ about centre of mass C.

$$= M(R\omega)R + \frac{1}{2}MR^2\omega$$

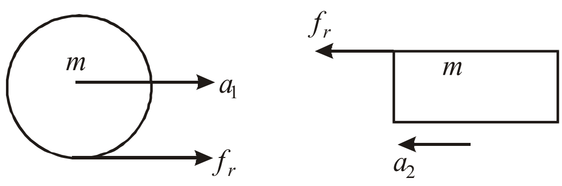
($v = R\omega$ in case of rolling motion and surface at rest)

$$= \frac{3}{2}MR^2\omega$$

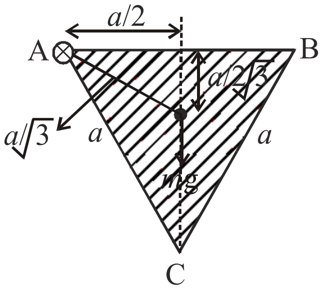
7. **(c)** FBD for sphere and block

$$a_1 = \frac{f_r}{m} = \frac{\mu mg}{m}, \quad a_2 = \frac{f_r}{m} = \frac{\mu mg}{m}$$

$$\vec{a}_1 = \mu g\hat{i}, \quad \vec{a}_2 = -\mu g\hat{i}$$



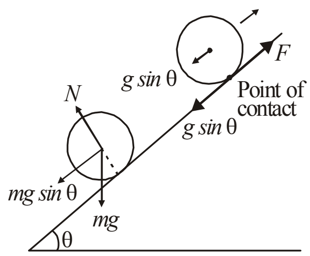
8. **(c)** Torque about A : $mg\frac{a}{2} = I\alpha$



$$\Rightarrow \alpha = \frac{mga}{2I}$$

$$\text{acceleration} = \frac{a}{\sqrt{3}}\alpha = \frac{mga^2}{2\sqrt{3}\,I}$$

9. **(b)** Imagine the cylinder to be moving on a frictionless surface. In both the cases the acceleration of the centre of mass of the cylinder is g sin θ. This is also the acceleration of the point of contact of the cylinder with the inclined surface. Also no torque (about the centre of cylinder) is acting on the cylinder since we assumed the surface to be frictionless and the forces acting on the cylinder is mg and N which pass through the centre of cylinder. Therefore the net movement of the point of contact in both the cases is in the downward direction as shown. Therefore the frictional force will act in the upward direction in both the cases.



In general we find the acceleration of the point of contact due to translational and rotational motion and then find the net acceleration of the point of contact. The frictional force acts in the opposite direction to that of net acceleration of point of contact.

10. **(b)** If we treat the train as a ring of mass M then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle.

Velocity of centre of mass is :

$$V_{CM} = R_{CM}.\omega$$

$$= \frac{2R}{\pi}\omega = \frac{2R}{\pi}\left(\frac{V}{R}\right) \quad \left(\because \omega = \frac{V}{R}\right)$$

$$\Rightarrow V_{CM} = \frac{2V}{\pi} \Rightarrow MV_{CM} = \frac{2MV}{\pi}$$

As the linear momentum of any system = MV_{CM}.

$\therefore$ The linear momentum of the train = $\frac{2MV}{\pi}$

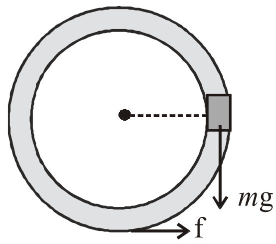
11. **(b)** $f = 4\,ma$ .... (1)

$(mg - f)\,r = (3mr^2 + mr^2)$ a

$mg - f = 4ma$ .... (2)

From (1) and (2)

$8ma = mg$



$$\Rightarrow a = \frac{9}{8} \Rightarrow \alpha = \frac{g}{8r}$$

12. **(b)** $\frac{1}{2}mv_m^2 = \frac{1}{2}\left(\frac{1}{2}\frac{m}{2}v_b^2\right)$

$$\frac{1}{2}m(v_m + 1)^2 = \frac{1}{2}\frac{m}{2}v_b^2$$

Divide both equation,

$$\frac{v_m + 1}{v_m} = \sqrt{2}$$

$$v_m = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1 = 2.414 \text{ m/s}$$

Put in any above equation $v_b = 4.84$ m/s

13. **(b)** The angular momentum (L) is conserved, since τ_{ext} is zero.

Here, $I_i = I_0$ $\quad$ $I_f = 2I_0$

$\omega_i = \omega_0$ $\quad$ $\omega_f = \omega$ (say)

Then $I_0\omega_0 = 2I_0\omega \Rightarrow \omega = \frac{\omega_0}{2}$

$$K_i = \frac{1}{2}I_0{\omega^2}_0 = K$$

$$K_f = \frac{1}{2}(2I_0)\times\left(\frac{\omega_0}{2}\right)^2 = \frac{1}{2}\times 2I_0\times\frac{\omega_0^2}{4}$$

$$= \left\{\frac{1}{2}\left(I_0\omega_0^2\right)\right\}\times\frac{1}{2} = \frac{K}{2}$$

Alternatively,

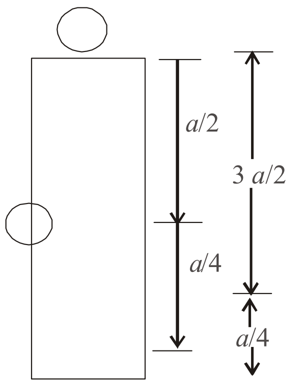
$$(K.E.)_{\text{rotation}} = \frac{L^2}{2I}.$$

Here , L = constant

$\therefore$ $(K.E.)_{\text{rotational}} \times I$ = constant.

When I is doubled, $K.E._{\text{rotational}}$ becomes half.

14. **(b)** Moment of inertia



$$= m\left(\frac{3a}{4}\right)^2 + m_1\frac{a^2}{3}$$

For the centre of rod

$$\left(\frac{m_1a^2}{12}+\frac{m_1a^2}{4}\right)=\frac{m_1a^2}{3}$$

$\therefore \quad m_1 = 4m$

$$\text{Total } I = m\left(\frac{3a}{4}\right)^2 + \frac{4ma^2}{3} = \frac{9ma^2}{16}+\frac{4ma^2}{3}$$

$$= \frac{(27+64)}{48}ma^2 = \frac{91}{48}ma^2$$

15. **(a)** If spool is not to translate

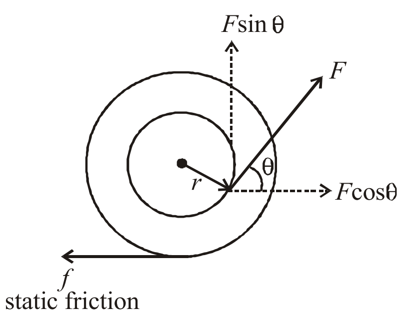
$F\cos\theta = f$ .... (1)

If spool is not to rotate

$Fr = fR$ .... (2)

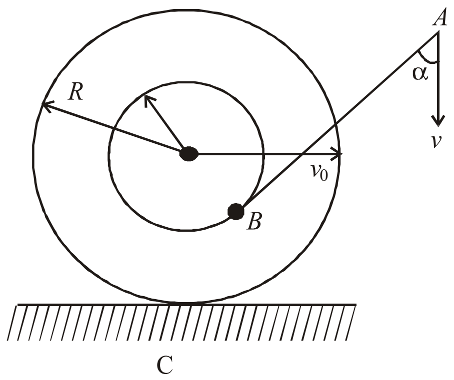
From eq. (1) and (2) we get static friction

$$\frac{fR}{r}\cos\theta = f$$



or $\cos\theta = \dfrac{r}{R}$ or $\theta = \cos^{-1}\left(\dfrac{r}{R}\right)$

16. **(a)** When the thread is pulled, the bobbin rolls to the right.



Resultant velocity of point B along the thread is $v = v_0\sin\alpha - \omega r$, where $v_0\sin\alpha$ is the component of translational velocity along the thread and ωr linear velocity due to rotation. As the bobbin rolls without slipping, $v_0 = \omega R$. Solving the obtained equations, we get $v_0 = \dfrac{vR}{R\sin\alpha - r}$

17. **(d)** By the concept of energy conservation

$$\frac{1}{2}mv^2+\frac{1}{2}I\omega^2 = mg\left(\frac{3v^2}{4g}\right)$$

For rolling motion $v = R\omega$

$$\therefore \quad \frac{1}{2}mv^2+\frac{1}{2}I\frac{v^2}{R^2}=\frac{3}{4}mv^2$$

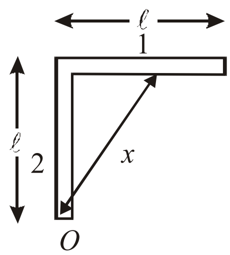
$$\therefore \quad \frac{1}{2}I\frac{v^2}{R^2}=\frac{3}{4}mv^2-\frac{1}{2}mv^2=\frac{1}{4}mv^2$$

$$\frac{1}{2}I\frac{v^2}{R^2}=\frac{1}{4}mv^2$$

$$\Rightarrow \quad I = \frac{1}{2}mR^2$$

This is the formula of the moment of inertia of the disc.

18. **(d)** Moment of inertia of rod 2 about an axis passing through O perpendicular to both roots, $I_2 = \dfrac{m\ell^2}{3}$



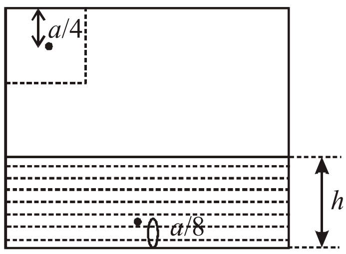
The distance from O to the parallel axis through the centre of mass of rod 2 is $\left[\ell^2 + \left(\frac{1}{2}\ell\right)^2\right]^{\frac{1}{2}}$

Moment of inertia of rod 1 about an axis passing through O perpendicular to both rods,

$$I_1 = \frac{m\,\ell^2}{12} + mx^2 = \frac{m\,\ell^2}{12} = \frac{5\,m\ell^2}{4} = \frac{4}{3}\;m\ell^2$$

$$I = I_1 + I_2 = \frac{4\,m\,\ell^2}{3} + \frac{m\,\ell^2}{3} = \frac{5\,m\,\ell^2}{3}$$

19. **(c)** Let h be the height of water surface, finally

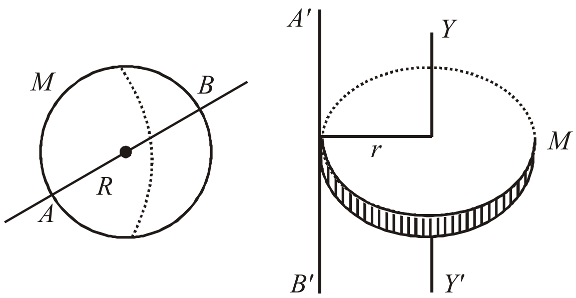


$$a^2h = a.\frac{a}{2}.\frac{a}{2} \Rightarrow h = \frac{a}{4}$$

$\therefore$ C.M. gets lowered by $a - \left(\frac{a}{4} + \frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$

$\therefore$ Work done by gravity $= mg\frac{5a}{8}$

20. **(b)**



For solid sphere

$I_{AB} = \frac{2}{5}MR^2 = I$ (given) ... (i)

For solid disc

$$I_{A'B'} = I_{YY'} + Mr^2 = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2$$

$I_{AB} = I_{A'B'}$ (given) ... (ii)

From (i) and (ii),

$$\frac{2}{5}MR^2 = \frac{3}{2}Mr^2$$

$$\Rightarrow r = \frac{2}{\sqrt{15}}R$$

21. **(c)** Let a be the length of the string.
Tension T at any point is given by

$$T = \frac{M}{a}[u^2 + 3ag\cos\theta - 2ag] \quad ...(i)$$

where M is the mass of the particle and as such $M = \frac{W}{g}$.

$\therefore$ From (i), we get :

$$T = \frac{W}{ag}[u^2 + 3ag\cos\theta - 2ag] \quad ...(ii)$$

At the highest point $T = mW$ (given) and $\theta = \pi$

$\therefore$ From (ii), $mW = \frac{W}{ag}[u^2 - 3ag - 2ag]$

$$mag = u^2 - 5ag \quad ...(iii)$$

At the lowest point $T = nW$ (given) and $\theta = 0$

$\therefore$ From (ii), $nW = \frac{W}{ag}[u^2 + 3ag - 2ag]$

$$nag = u^2 + ag \quad ...(iv)$$

Eq. (iii) – Eq. (iv)

$\Rightarrow$ $mag - nag = -5ag - ag$

$m + 6 = n \Rightarrow n - m = 6$

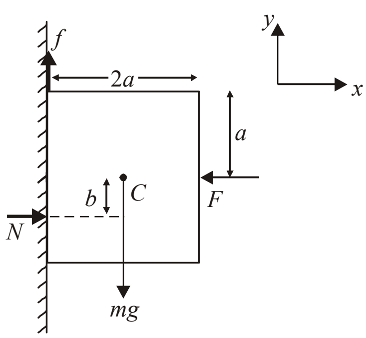
22. **(d)** $v_0 - \mu gt = R\left(\omega_0 + \frac{5\mu g}{2R}.t\right)$

$$t = \frac{2(v_0 - \omega_0 R)}{7\mu g}$$

23. **(d)** The cubical block is in equilibrium.
For translational equilibrium

(a) $\Sigma F_x = 0 \Rightarrow F = N$

(b) $\Sigma F_y = 0 \Rightarrow f = mg$



For Rotational Equilibrium

$\Sigma\tau_c = 0$

Where τ_c = torque about c.m.

Torque created by frictional force (f) about $C = f \times a$ in clockwise direction.

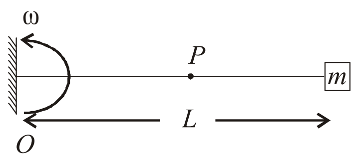
There should be another torque which should counter this torque. The normal reaction N on the block acts as shown. This will create a torque $N \times b$ in the anticlockwise direction.

Such that $f \times a = N \times b$

The normal force does not act through the centre of the body always. The point of application of normal force depends on all the forces acting on the body.

24. **(d)** In the shown frame the particle appears to be at rest.

$\therefore$ Net force on it must be zero. Therefore pseudo force must be equal and opposite to the tension.



25. **(d)** Let us assume cylinder is not moving then

$T + f_s = mg \sin\theta$

$T.R - f_s R = 0$

$$\Rightarrow f_s = \frac{mg\sqrt{3}}{4}$$

But $(f_s)_{max} = \mu N = \mu mg \cos\theta$

$$= 0.4 \times mg \times \frac{1}{2} = \frac{mg}{5}$$

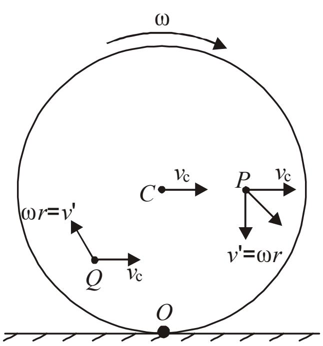
$\therefore (f_s) < (f_s)_{max}$, our assumption is wrong. So, friction existing must be kinetic

$$f_k = \mu\, m\, g \cos\theta = 0.4 \times mg \times \frac{1}{2} = \frac{mg}{5}$$

26. **(b)** Here $v_C = v_c$

$$v_P = \sqrt{v_c^2 + \omega^2 r^2}$$

$$v_Q = \sqrt{v_c^2 + \omega^2 r^2 + 2v_c \omega r \cos\theta}$$



Alternatively

In pure rolling, the point of contact is the instantaneous centre of rotation of all the particles of the disc.

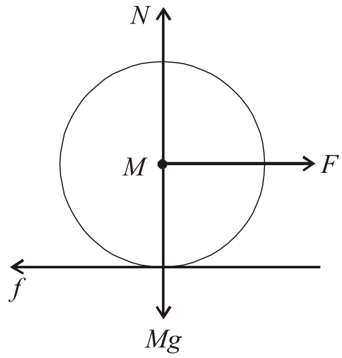
On applying

$v = r\omega$

We find ω is same for all the particles then $v \propto r$.

Farther the particles from O, higher is its velocity.

27. **(a)** Force due to pressure difference is



$$F = \frac{1}{2}\rho\pi R^2 (v_2^2 - v_1^2) = \frac{7\rho\pi R^2}{2}$$

Now, $F - f = Ma$

$$fR = \frac{2}{3}MR^2 \times \frac{a}{R} \Rightarrow f = \frac{2}{5}Ma$$

$$\Rightarrow f = \frac{2}{5}F = \rho\pi R^2 \Rightarrow \mu Mg = \rho\pi R^2$$

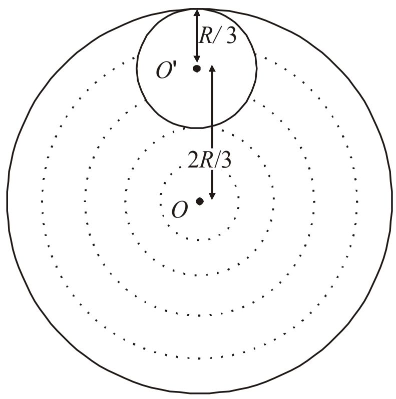
$$\Rightarrow \mu = \frac{\pi\rho R^2}{Mg} = \frac{1}{4} = 0.25$$

28. **(c)** $F = ma$

$$\Rightarrow \mu mg = m\sqrt{\left(\frac{v^2}{r}\right)^2 + a_t^2}$$

$$\Rightarrow a_t = \sqrt{\mu^2 g^2 - \left(\frac{v^2}{r}\right)^2} = 4\,\text{m/s}^2$$

29. **(a)** Let σ be the mass per unit area.



The total mass of the disc

$= \sigma \times \pi R^2 = 9M$

The mass of the circular disc cut

$$= \sigma \times \pi\left(\frac{R}{3}\right)^2$$

$$= \sigma \times \frac{\pi R^2}{9} = M$$

Let us consider the above system as a complete disc of mass $9M$ and a negative mass M super imposed on it. Moment of inertia (I_1) of the complete disc = $\frac{1}{2}9MR^2$ about an axis passing through O and perpendicular to the plane of the disc.

M.I. of the cut out portion about an axis passing through O' and perpendicular to the plane of disc

$$= \frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2$$

$\therefore$ *M.I.* (I_2) of the cut out portion about an axis passing through O and perpendicular to the plane of disc

$$= \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

[Using perpendicular axis theorem]

$\therefore$ The total *M.I.* of the system about an axis passing through O and perpendicular to the plane of the disc is $I = I_1 + I_2$

$$= \frac{1}{2}9MR^2 - \left[\frac{1}{2} \times M \times \left(\frac{R}{3}\right)^2 + M \times \left(\frac{2R}{3}\right)^2\right]$$

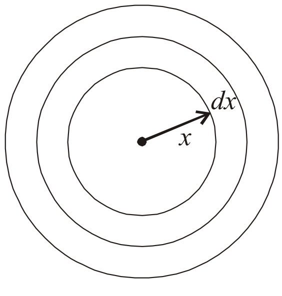
$$= \frac{1}{2}9MR^2 - MR^2\left[\frac{1}{18} + \frac{4}{9}\right]$$

$$= \frac{1}{2}9MR^2 - MR^2 \times \left[\frac{1+8}{18}\right]$$

$$= \frac{9MR^2}{2} - \frac{9MR^2}{18}$$

$$= \frac{(9-1)MR^2}{2} = 4MR^2$$

30. (b) Consider a small element of disc of thickness dx at a radius x.



Now $dN = \mu mg.\frac{2\pi x dx}{\pi R^2} = \frac{2\mu mgx}{R^2}dx$

$$\Rightarrow d\tau = xdN = \frac{2\mu mgx^2 dx}{R^2}$$

$$\Rightarrow \int d\tau = \frac{2\mu mg}{R^2}\int_0^R x^2 dx = \frac{2}{3}\mu mgR$$

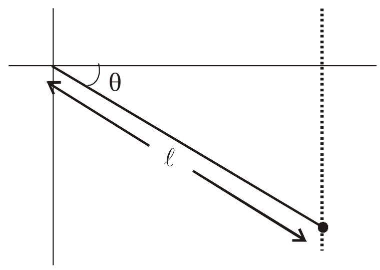
$$= I\alpha = \frac{1}{2}mR^2\alpha$$

$\Rightarrow \alpha = \frac{4}{3}\frac{\mu g}{R}$, If t be the time for complete stop.

$$\alpha t = \omega \Rightarrow t = \frac{\omega}{\alpha} = \frac{3\omega R}{4\mu g}$$

31. (c) Consider a situation when the bob A has fallen through an angle θ.

Loss in PE = Gain in KE



$$\Rightarrow \frac{1}{2}I\omega^2 = mg\ell\sin\theta$$

$$\Rightarrow \omega_A = \sqrt{\frac{2mg\ell\sin\theta}{I}} = \sqrt{\frac{2mg\ell\sin\theta}{m\ell^2}}$$

$$\omega_A = \sqrt{\frac{2g\sin\theta}{\ell}}$$

In the similar position

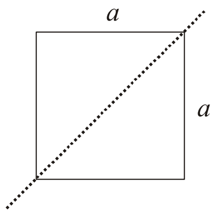
$$\omega_B = \sqrt{\frac{4g\sin\theta}{\ell}}$$

$$\Rightarrow \frac{\omega_A}{\omega_B} = \frac{t_B}{t_A} = \sqrt{\frac{1}{2}}$$

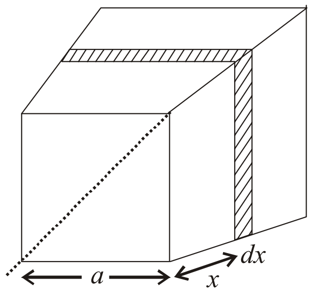
$$\Rightarrow \frac{t_A}{t_B} = \sqrt{2}$$

32. (c) Consider a square lamina of mass (dm).

Moment of inertia from this about the shown diagonal is $(dm)\frac{a^2}{12}$.



Now consider an element of the cube of thickness dx and a distance x from the face diagonal.



Now $dI = (dm)\dfrac{a^2}{12} + (dm)x^2$

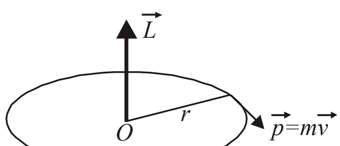
($\because$ Parallel axis theorem)

$$\Rightarrow dI = \frac{Mdx}{a}\left[\frac{a^2}{12} + x^2\right]$$

As $I = \dfrac{M}{a}\displaystyle\int_0^a \left(\frac{a^2}{12} + x^2\right)dx = \frac{M}{a}\left[\frac{a^2x}{12} + \frac{x^3}{3}\right]_0^a$

$$= Ma^2\left[\frac{1}{12} + \frac{1}{3}\right] = \frac{5}{12}Ma^2$$

33. (a) $|\vec{L}| = mvr$



The direction of $\vec{L}$ (about the center) is perpendicular to the plane containing the circular path. Both magnitude and direction of the angular momentum of the particle moving in a circular path about its center O is constant.

Alternatively, The net force acting on a particle undergoing uniform circular motion is centripetal force which always passes through the centre of the circle. The torque due to this force about the centre is zero, therefore, $\vec{L}$ is conserved about O.

34. (a) $\dfrac{M_1 d^2}{3}\omega = M_2 v d$

$v = d\,\omega \Rightarrow \dfrac{M_1}{M_2} = \dfrac{3}{1}$

35. (d) The angular momentum of the system about O = 0
$\Rightarrow \omega = 0.$

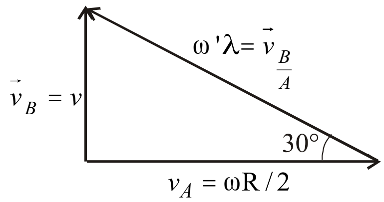
36. (c) $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$

$\vec{v}_B = v$

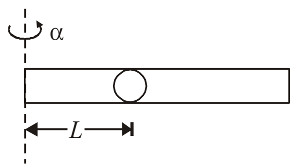
ω' is for rod AB

$$\tan 30° = \frac{v}{\omega\left(\frac{R}{2}\right)}$$

$$\omega = \frac{2v\sqrt{3}}{R} = \frac{2\sqrt{3}v}{R}$$



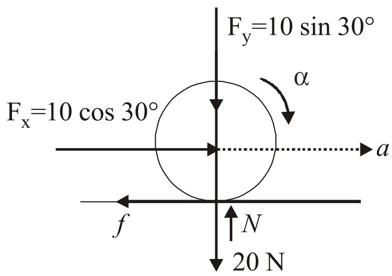
37. (a) When we are giving an angular acceleration to the rod, the bead is also having an instantaneous acceleration $a = L\alpha$. This will happen when a force is exerted on the bead by the rod. The bead has a tendency to move away from the centre. But due to the friction between the bead and the rod, this does not happen to the extent to which frictional force is capable of holding the bead. The frictional force here provides the necessary centripetal force. If instantaneous angular velocity is ω then



$mL\omega^2 = \mu(ma)$
$mL\omega^2 = \mu\, mL\alpha$
$\Rightarrow \quad \omega^2 = \mu\alpha$
By applying
$\Rightarrow \quad \omega = \omega_0 + \alpha t,$
We get $\omega = \alpha t$
$\therefore\ \alpha^2 t^2 = \mu\alpha$

$$\Rightarrow \quad t = \sqrt{\frac{\mu}{\alpha}}$$

38. (b) $10\cos 30° - f = 2a$ ...(1)



$\tau = I\alpha$

$\Rightarrow fr = \frac{2}{3}\times 2\times r^2 \times \alpha$ ...(2)

(where r is radius of sphere)

From (1) and (2), we get

$f = 2\sqrt{3}$ newton, $N = 20 + 10 \sin 30° = 25$

$f = \mu N \Rightarrow \mu = \frac{f}{N} = \frac{2\sqrt{3}}{25} = 0.08\times\sqrt{3}$

39. **(b)** $L_{initial} = L_{final}$

$mv_0 r = I_0\omega_0$

$mv_0 r = \frac{1}{2}mr^2\omega_0 \Rightarrow 2v_0 = \omega_0 r$

40. **(d)** Moment of inertia about the diameter of the circular loop (ring) $= \frac{1}{2}MR^2$

Using parallel axis theorem

The moment of inertia of the loop about XX' axis is

$I_{XX'} = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

Where M = mass of the loop and R = radius of the loop

Here $M = L\rho$ and $R = \frac{L}{2\pi}$;

$\therefore\ I_{XX'} = \frac{3}{2}(L\rho)\left(\frac{L}{2\pi}\right)^2 = \frac{3L^3\rho}{8\pi^2}$

41. **(c)** $W = \frac{1}{2}I\omega^2$

Let x is the distance of CM from A

$I = Mx^2 + 4M(\ell - x)^2$

and if I is minimum, W will be minimum

$\therefore\ \frac{dI}{dx} = 2Mx + 4M\times 2(\ell - x)\times(-1)$

$= 2Mx - 8M(\ell - x)$

$\frac{dI}{dx} = 10Mx - 8M\ell$

$\therefore\ 10Mx - 8M\ell = 0$

$x = \frac{4}{5}\ell$

42. **(c)** $m_2 g - T = m_2 a_2$ ...(1)

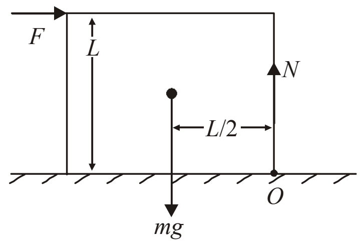
$TR = \frac{m_2R^2}{2}\alpha_2$ ...(2)

$\alpha_1 R = a_2 - \alpha_2 R$ ...(3)

$TR = \left(\frac{m_1R^2}{2}\right)\alpha_1$ ...(4)

$\alpha_2 = \frac{2T}{m_2R} = \frac{2}{m_2R}.(m_2a_2 + m_2g) = \frac{2(a_2 + g)}{R}$

43. **(c)** The applied force shifts the normal reaction to one corner as shown. At this situation, the cubical block starts topping about O. Taking torque about O

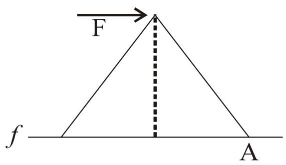


$F\times L = mg\times\frac{L}{2} \Rightarrow F = \frac{mg}{2}$

44. **(c)** Torque about point A

$F.\frac{a\sqrt{3}}{2} = mg.\frac{a}{2}$

$F = \frac{mg}{\sqrt{3}}$



For equilibrium of block

$F - f = 0$

$F = f$

μ should be greater than $\frac{1}{\sqrt{3}}$.

45. **(a)** During collision with the rough surface, a frictional torque will act in the clockwise direction.

46. **(b)** $v = \sqrt{Kr}\,.t$

$L = mvr = m\sqrt{Kr^3}\ t \Rightarrow \frac{dL}{dt} = m\sqrt{Kr^3}$

47. **(d)** Taking moments of all forces about A, the net torque due to tension (T) in the string and the weight of the beam (W) is non-zero. Hence, the beam can never be in equilibrium.

48. **(b)** Average pseudo force

$= \frac{\text{change in momentum in non-inertial frame}}{\text{time elapsed}}$

$= \frac{3\pi}{10}$ N

49. **(d)** From the F.B.D., it is clear that we get the equations,

$mg\sin\theta - f = ma$

$fr = I\alpha$

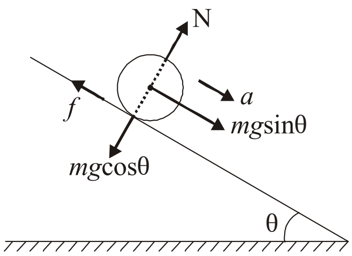
$= I\frac{\alpha}{r}\left(\text{since } \alpha = \frac{a}{r}\right)$

Solving,

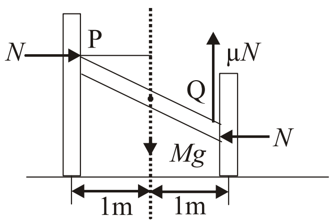
$a = \dfrac{mg\sin\theta}{m + \dfrac{I}{r^2}}$; the time taken by the bodies to roll down

a distance, ℓ is $\sqrt{\dfrac{2\ell}{a}}$.

Using $I = \dfrac{1}{2}mr^2$ for the solid cylinder and $I = \dfrac{2}{3}mr^2$ for the hollow spherical shell we get the required result.



50. (d) Drawing F.B.D. of rod PQ



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Rightarrow \mu N = mg$ ... (1)

$\Sigma\tau$ about centre of mass = 0

$$\left[N \times \frac{\ell}{2}\sin\theta\right][2] = \mu N \times \frac{\ell}{2}\cos\theta$$

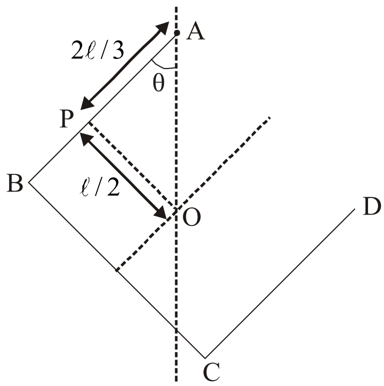
$$\Rightarrow \tan\theta = \frac{\mu}{2}$$

Hence $\sec\theta = \dfrac{(\sqrt{\mu^2+4})}{2}$

Thus, $\left(\dfrac{\ell/2}{1}\right) = \dfrac{\sqrt{\mu^2+4}}{2} \Rightarrow \ell = \sqrt{\mu^2+4}$

$\Rightarrow \quad \ell = \sqrt{0.25+4} = \dfrac{\sqrt{17}}{2}$ metres

51. (b)



Vertical line from hinge A must pass through C.M. of rod system.

$$\tan\theta = \frac{OP}{AP} = \frac{\ell/2}{2\ell/3}$$

$$\tan\theta = \frac{3}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

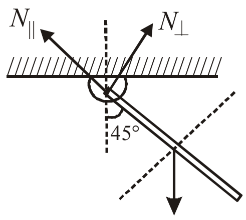
52. (b) $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{2\pi \text{ rad}}{60\text{ sec}} = \dfrac{3.14}{30}$ rad/sec = 0.105 rad/sec

53. (c) Applying work-energy theorem :

$$M_b gd - \mu M_a gd = \left[\frac{1}{2}(M_b + M_a)v^2 + \frac{1}{2}I_{pulley}\omega^2\right] - [0]$$

and $v = \omega R,\ v = 3.$

54. (a) $\tau = I\alpha$



$mg\sin 45° \times \dfrac{\ell}{2} = \dfrac{m\ell^2}{3}\alpha \Rightarrow \alpha = \dfrac{3g}{2\sqrt{2}\ell}$

Tangential acceleration of centre of mass

$(a_t) = \alpha \times \dfrac{\ell}{2} = \dfrac{3g}{4\sqrt{2}}$

Radial acceleration of centre of mass $(a_r) = \dfrac{m\omega^2\ell}{2} = 0$

(initially ω is 0)

$mg\sin 45° - N_{\perp} = ma_t \Rightarrow N_{\perp} = \dfrac{mg}{4\sqrt{2}}$

$mg\cos 45° - N_{||} = ma_r \Rightarrow N_{||} = \dfrac{mg}{\sqrt{2}}$

Net hinge force $\sqrt{N_{\perp}^2 + N_{||}^2} = mg\dfrac{\sqrt{17}}{4\sqrt{2}} \Rightarrow \sqrt{34} = 5.83 = 6$

(rounded to nearest integer)

55. **(a)** Divide the ring into infinitely small lengths of mass dm_1. Even though mass distribution is non-uniform, each mass dm_1 is at same distance R from origin.

$\therefore$ MI of ring about z-axis is

$$= dm_1 R^2 + dm_2 R^2 + ...+ dm_n R^2 = MR^2$$

56. **(d)** **Case 1 :** When they rotate in same sense

$2m\pi = 2\omega t$

$$\frac{3\pi}{2} + 2n\pi = \omega t \;; \qquad 2m\pi = 2\left(\frac{3\pi}{2} + 2n\pi\right)$$

$$2m = 3 + 4n \;;\; m = \frac{3}{2} + 2n \Rightarrow m - 2n = \frac{3}{2}$$

Not possible for m and n being integer.

Case 2 : When they rotate in opposite sense

$2m\pi = 2\omega t$

$$\frac{\pi}{2} + 2n\pi = \omega t \;; \qquad 2m\pi = 2\left(\frac{\pi}{2} + 2n\pi\right)$$

$2m\pi = \pi + 4n\pi$; $2m - 4n = 1$

Not possible for m and n integer.

57. **(c)** $FR = I\alpha$

where R is the radius of the disc.

$$F = \frac{1}{2}MR\alpha$$

$$= \frac{1}{2}MR\left(\frac{d\omega}{dt}\right) = \frac{1}{2}MR(6)$$

$$= \frac{1}{2} \times 5 \times 0.2 \times 6 = 3.0\,\text{N}$$

58. **(a)** The local radius of curvature of the curve is

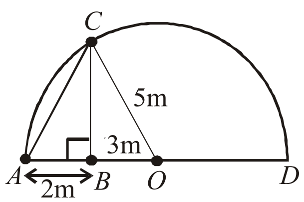
$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{d^2y/dx^2} = \frac{[1 + (9/4)ax]^{3/2}}{(3/4)\,a^{1/2}x^{-1/2}}$$

In this motion of the particle, the curve exerts a normal or centripetal force, causing the particle momentarily to move in an arc of a circle of radius ρ. Thus,

$$F = \frac{mv^2}{\rho} = \frac{3}{4}a^{1/2}x^{-1/2}\left(1 + \frac{9}{4}ax\right)^{-3/2} mv^2$$

59. **(b)** From geometry $BC = \sqrt{5^2 - 3^2} = 4\text{m}$

and $AC = \sqrt{AB^2 + BC^2} = \sqrt{2^2 + 4^2} = 2\sqrt{5}\,\text{m}$



For both particles to reach C simultaneously, we have

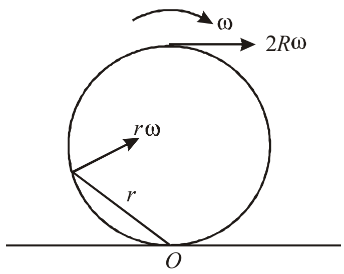
$t_{AC} = t_{BC}$

$$\Rightarrow \frac{AC}{u_1} = \frac{BC}{u_2} \quad \text{or} \quad \frac{u_1}{u_2} = \frac{AC}{BC} = \frac{2\sqrt{5}}{4}$$

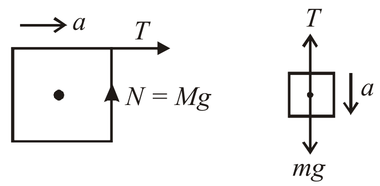
60. **(d)** At a point such as B or D (the centripetal force necessary for circular motion) = (the normal reaction given by the road) – (the weight of the cyclist). Therefore, the normal reaction = (the weight of the cyclist) + (the centripetal force). The radius of curvature is smaller at D than it is at point B, giving a larger normal reaction at D than at B. This is why the cyclist feels heaviest at D. Note that at points A or C, the necessary centripetal force is weight minus the normal reaction.

61. **(d)** Point O can be taken to be the instantaneous axis of rotation. Distance of this point from any other point is between 0 & $2R$.

$\therefore$ These linear speeds vary between 0 & $2R\omega$



62. **(a)**



$mg - T = ma$ .... (i) and $T = Ma$ .... (ii)

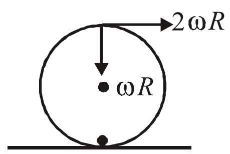
Solving (i) and (ii),

$$a = \frac{mg}{M+m} \qquad \Rightarrow \quad T = Ma = \frac{Mmg}{M+m}$$

$$T\frac{h}{2} = Mg\frac{h}{4} \qquad \Rightarrow \quad \frac{Mmg}{M+m}\frac{h}{2} = Mg\frac{h}{4}$$

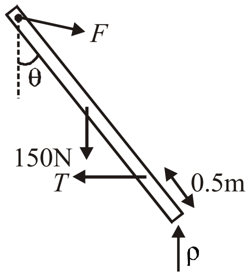
$$2 = 1 + \frac{M}{m} \Rightarrow \frac{M}{m} = 1 \quad \text{or} \quad \frac{m}{M} = 1$$

63. **(c)**



$$R = \frac{v^2}{a_\perp} = \frac{(2\omega R)^2}{\omega^2 R} = 4R$$

64. **(a)** There are many approaches to this problem, but the one to take should yield the desired answer as quickly as possible. We are interested in only one unknown, the tension in the tie rope. Considering one of the rafters to be our system, we show all forces acting on it in figure.



The force $\vec{F}$ includes the effect of the 500 N weight as well as the force due to the other rafter. If we take moments about an axis through the top of the rafter, we get

$\Sigma\tau = 0$

$\Rightarrow \quad (3.0\times\rho\times\sin\theta) - (1.5\times150\times\sin\theta) - 2.5\times T\times\sin(90° - \theta)$

$= 1.75\rho - (1.75/2)(150) - 2.5\cos\theta\, T = 0.$

Since $\sin\theta = (1.75/3.0)$, $\theta = 35.7°$, and $\cos\theta = 0.812$, therefore $2.03\,T = 1.75\rho - 131$.

To find ρ we return to the complete system in figure and appeal to symmetry. The normal force due to the floor on each rafter is the same. Furthermore, considering the entire system as one rigid body,

$\Sigma F_y = 0$ yields $2\rho = (500 + 150 + 150)$ N, and

$\rho = 400$ N.

Substituting into our moment equation and solving we get $T = 280$ N.

65. (b) After 2 sec speed of boy will be

$v = 2\times2 = 4$ m/s

At this moment centripetal force on boy is

$$F_r = \frac{mv^2}{R} = \frac{30\times16}{6} = 80N$$

Tangential force on boy is

$F_t = ma = 30\times2 = 60$N

Total friction acting on boy is

$$F = \sqrt{F_r^2 + F_t^2} = 100N$$

At the time of slipping $F = \mu mg$

or $100 = \mu\times30\times10 \Rightarrow \mu = 1/3$.

66. (c) Only a horizontal force acts at hinge B, because hinge A is assumed to support the door's weight. Let us take torques about A as axis.

$\Sigma\tau = 0$ becomes $(F_2)(h) - (400\text{ N})(h/4) = 0$,

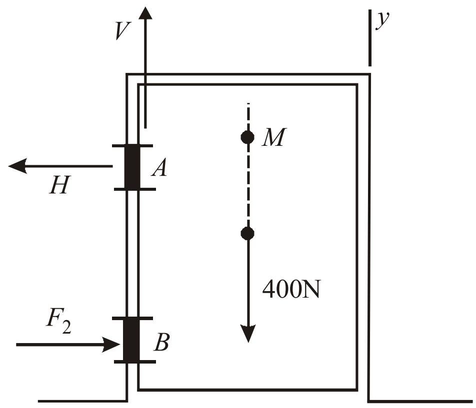
from which $F_2 = 100$ N.

We also have

$\Sigma F_x = 0$ or $F_2 - H = 0$

$\Sigma F_y = 0$ or $V - 400\text{ N} = 0$

We find from these that $H = 100$ N and $V = 400$ N

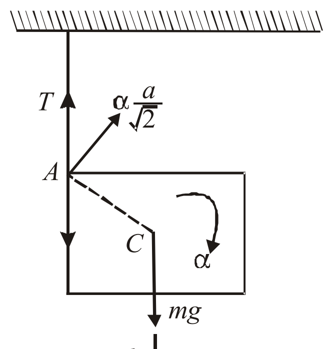


To find the resultant force $\vec{R}$ on the hinge at A, we have $R = \sqrt{(400)^2 + (100)^2} = 412$ N.

The tangent of the angle that $\vec{R}$ makes with the negative x-direction is V/H, so the angle is $\tan^{-1}(4) = 76°$

67. (a) If friction acts backwards its ω will increase & v_{cm} will decrease hence voiding pure rolling condition & if friction acts in forward direction the ω will decrease & v_{cm} will increase, again voiding pure rolling condition hence no friction acts.

68. (b) Let b and α are linear acceleration of centre of mass and angular acceleration of the plane, just after BF is cut.



$mg - T = mb$ .... (1)

Taking torques about COM

$$\frac{Ta}{2} = \frac{ma^2}{6}\alpha \qquad \text{......... (2)}$$

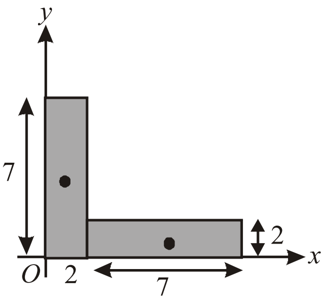
$$\Rightarrow \quad g = b + \frac{a\alpha}{3} \text{ and } b = \alpha\frac{a}{2}$$

$$g = b + \frac{2b}{3} = \frac{5b}{3} \Rightarrow b = \frac{3g}{5}$$

$$\Rightarrow \quad T = mg - \frac{m.3g}{5} = \frac{2mg}{5}$$

69. (c) The only force block exerts on disc is parallel to axis of rotation of disc. This additional force does not cause any torque on disc. Hence angular momentum of disc remains same. Since there is no friction between block and disc, the block remains in its position.

70. (c)



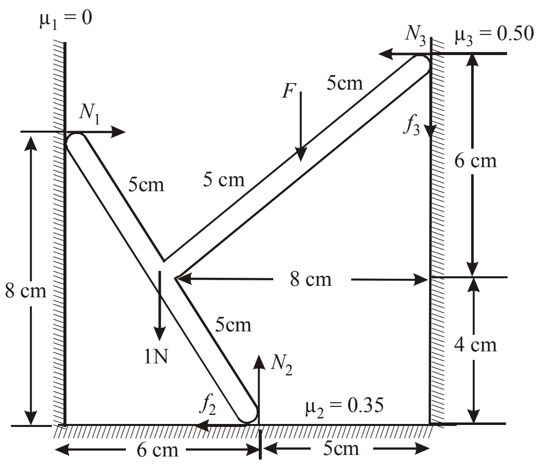
$$X_{cm} = \frac{m \times 5.5 + 1 \times m}{2m} = \frac{6.5}{2} = \frac{13}{4} cm.$$

$$Y_{cm} = \frac{m \times 3.5 + 1 \times m}{2m} = \frac{4.5}{2} = \frac{9}{4} cm.$$

71. (d) $a_t = \frac{du}{dt} = 4t = 4 m/s^2$

$$a_n = \frac{v^2}{R} = \frac{4}{4/3} = 3 m/s^2 \Rightarrow a = \sqrt{a_t^2 + a_c^2} = 5$$

72. (a) The solution of this problem is greatly simplified by an intuitive consideration of the situation when F is very small. The 1N force then sets up a counter-clockwise torque that because of the low frictional resistance offered by the left wall and the floor immediately causes the bar to lose contact with the right wall. Therefore, if a value of F can be found that puts the bar in equilibrium with its end just touching the right wall (i.e. $N_3 = f_3 = 0$), this value of F must represent the desired minimum.



With $N_3 = f_3 = 0$, the force conditions are

$N_2 - 1 - F = 0$ .... (1)

$N_1 - f_2 = 0$ .... (2)

and the torque condition (about the contact point with the floor) is

$-N_1(0.08) + 1(0.03) - F(0.01) = 0$ .... (3)

Elimination of F and N_1 between these three equation gives $f_2 = 0.5 - 0.125 N_2$.

But the largest possible value of f_2 is $\mu_2 N_2 = 0.35 N_2$.

Hence, $0.5 - 0.125 N_2 \le 0.35 N_2$

or $N_2 \ge \frac{0.5}{0.475} = \frac{20}{19}$ N

and, from (1)

$$F = N_2 - 1 \ge \frac{1}{19} \text{N}$$

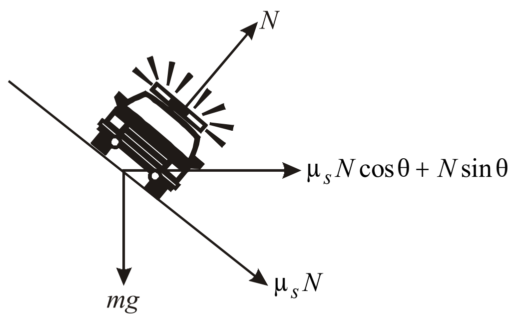
The minimum force is thus (1/19) N, corresponding to which

$$N_1 = \frac{7}{19} N = f_2, \quad N_2 = \frac{20}{19} N$$

The force may be increased above this value, still keeping $N_3 = f_3 = 0$, up to $F = 3$N, at which point N_1 and f_2 vanish. Thus there is a whole range of solutions such that the right wall might just as well not be there.

73. (c) Speed of COM $= \frac{0.5 \times 2 - 0.3 \times 4}{6} = 0.2$ m/s

74. (a)



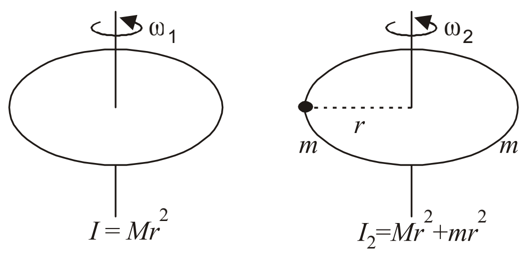
$$\mu_s N \cos\theta + N \sin\theta = \frac{mv^2}{R} \quad \text{............. (1)}$$

$$N \cos\theta - \mu_s N \sin\theta = mg \quad \text{............. (2)}$$

(1) and (2) give, $\frac{v^2}{Rg} = \frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}$

$$\Rightarrow v = \sqrt{gR\left(\frac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

75. (c) Since the objects are placed gently, therefore no external torque is acting on the system. Hence angular momentum is constant.



i.e., $I_1\omega_1 = I_2\,\omega_2$

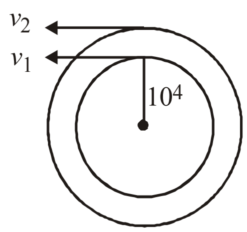
$$Mr^2 \times \omega_1 = (Mr^2 + 2mr^2)\,\omega_2 \quad (\because \omega_1 = \omega)$$

$$\therefore \quad \omega_2 = \frac{M\omega}{M + 2m}$$

76. (c) Its mechanical energy is conserved.
It has a centripetal acceleration, downward.
Its speed is minimum.

77. (c) S_1: If the object is large so that gravitational acceleration is not same at every point, both will have different locations.
S_2: Internal forces cannot change momentum of any kind of system.
S_3: If resultant force on a system of particles is non-zero, the centre of mass shall accelerate and in some condition it may move along a circular path. Thus the distance of centre of mass from centre of circle shall be constant. Hence the statement is true.

78. (b)



$$\omega_1 = \frac{2\pi}{1}\text{ rad/hr},\quad \omega_2 = \frac{2\pi}{8}\text{ rad/hr}$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \frac{R_2}{R_1} = 4 \Rightarrow R_2 = 4\times10^4\text{ km}$$

$$v_1 = \frac{2\pi R_1}{1h} = 2\pi\times10^4\text{ km/hr}$$

$$v_2 = \frac{2\pi R_2}{8h} = \pi\times10^4\text{ km/hr}$$

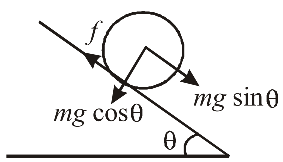
At closest separation

$$\omega = \frac{v_{rel} \perp \text{ to line joining}}{\textit{length}\text{ of line joining}} = \frac{\pi\times10^4\text{ km/hr}}{3\times10^4\text{ km}} = \frac{\pi}{3}\text{ rad/hr.}$$

79. (c) After collision velocity of COM of A becomes zero and that of B becomes equal to initial velocity of COM of A. But angular velocity of A remains unchanged as the two spheres are smooth.

80. (d) $I = I_{cm} + MR^2 = \frac{7}{5}MR^2$

81. (d)



$$mg\sin\theta - f = ma_{cm}$$

$$fR = \frac{2}{5}MR^2\alpha \qquad \text{(torque} = I\alpha \text{ about COM)}$$

$$a_{cm} = R\alpha \Rightarrow f = \frac{2mg\sin\theta}{7}$$

82. (d) Velocity of centre of mass is non zero.

83. (b) $a = \frac{\Delta v}{t} = \frac{v_f - v_i}{t}$

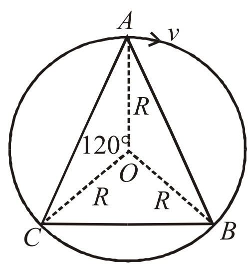
$$\Rightarrow t = \frac{\pi r}{v} = \frac{\pi\times5/\pi}{5} = 1$$

$$a = \frac{10}{1} = 10\text{ m/s}^2$$

84. (c) In ΔAOC, by sine rule,

$$\frac{AC}{\sin120^\circ} = \frac{AO}{\sin30^\circ}$$

$$\Rightarrow AC = \sqrt{3}R$$

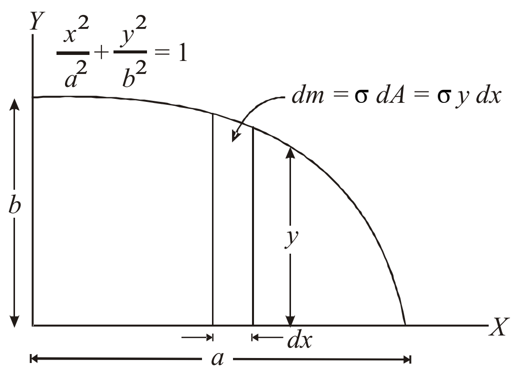


Time taken in moving from A to C will be

$$= \frac{A\text{ to }C\text{ distance}}{v} = \frac{4\pi R}{3v}$$

$$\therefore \quad v_{av} = \frac{\sqrt{3}R}{\frac{4\pi R}{3v}} = \frac{3\sqrt{3}v}{4\pi}$$

85. (d) The quadrant is bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the coordinate axes, its area is $A = \pi ab/4$



$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^a \sigma\,xy\,dx}{\sigma A} = \frac{1}{A}\int_0^a xy\,dx$$

But, along the ellipse,

$$\frac{2x\,dx}{a^2}+\frac{2y\,dy}{b^2}=0 \quad \text{or} \quad x\,dx=-\frac{a^2}{b^2}y\,dy$$

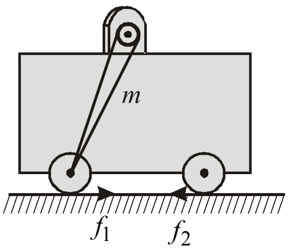
and so $x_{cm}=\frac{1}{A}\left(-\frac{a^2}{b^2}\right)\int_b^0 y^2dy$

$$=\frac{1}{A}\left(-\frac{a^2}{b^2}\right)\left(-\frac{b^3}{3}\right)=\frac{4a}{3\pi}$$

By symmetry, $y_{cm}=4b/3\pi$

86. **(a)** For whole system

$f_1-f_2=3\,(1)a$ .... (1)



For rear cylinder

$100-f_1=0.5\,(a)$ .... (2)

For front cylinder

$f_2=0.5\,(a)$ .... (3)

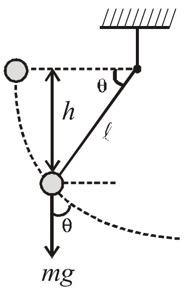
From (1), (2) and (3)

$100=4a$

$a=20\text{ m/s}^2$

87. **(d)** From conservation of energy

$$mgh=\frac{1}{2}mv^2$$



$$\Rightarrow mg\ell\sin\theta=\frac{1}{2}mv^2$$

$$\Rightarrow 2g\sin\theta=\frac{v^2}{\ell}=a_c$$

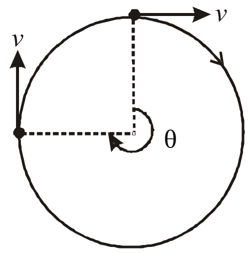
$$g\cos\theta=a_t$$

Total acceleration, $a=\sqrt{a_c^2+a_t^2}$

$$=g\sqrt{\cos^2\theta+(2\sin\theta)^2}=g\sqrt{3\sin^2\theta+1}$$

88. **(d)** The angular displacement of the particle in $t=1$ sec is

$$\theta=\omega t=\frac{v}{R}t=\frac{3\pi}{2}$$



$\therefore$ The magnitude of impulse by centripetal force in t = 1 second is equal to change in momentum

$=\sqrt{2}mv=3\sqrt{2}\pi\ Ns$

89. **(c)** The position vector of the center of mass at the time t is

$$\vec{r}_{cm}=\hat{i}\,(\cos 30°)+\hat{j}\,(\sin 30°)+\hat{k}\,(0.10)$$

$$=0.866\,\hat{i}+0.5\,\hat{j}+0.10\,\hat{k}$$

and the total momentum of the hoop is

$$\vec{p}=m\vec{v}_{cm}=(0.50)\,(0.50\hat{j})=0.25\hat{j}$$

Thus, $\vec{L}_{orb}=\vec{r}_{cm}\times\vec{p}$

$$=(0.866\,\hat{i}+0.5\,\hat{j}+0.10\,\hat{k})\times 0.25\hat{j}$$

$$=-0.025\,\hat{i}+0.216\,\hat{k}\ \text{kg m}^2/\text{s}$$

To find the spin angular momentum, note that every element of mass of the hoop is at the same distance from the centre of mass $r'=0.10$m, and every element rotates about the center of mass with a velocity $\vec{v}'$ (of magnitude 0.50m/s) perpendicular to $\vec{r}'$. Thus,

$$\vec{L}_{spin}=\int\vec{r}'\times\vec{v}'\,dm$$

$$=\int\vec{r}'\,\vec{v}'\,(-\hat{i})\,dm$$

$$=-mr'v'\,\hat{i}=-0.025\,\hat{i}\ \text{kg m}^2/\text{s}$$

90. **(d)** All forces on sphere pass through its centre except the force of friction exerted by inclined plane. Since net torque on sphere in equilibrium about its centre is zero, the torque on sphere due to frictional force about its centre must be zero. Hence frictional force on sphere is zero.

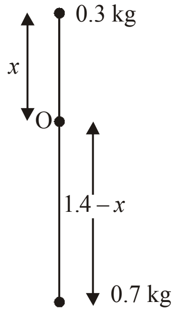
91. **(c)** The moment of inertia of the system about axis of rotation O is

$I=I_1+I_2$

$=0.3x^2+0.7\,(1.4-x)^2$

$=0.3x^2+0.7\,(1.96+x^2-2.8x)$

$=x^2+1.372-1.96x$



The work done in rotating the rod is converted into its rotational kinetic energy.

$$\therefore \quad W = \frac{1}{2}I\omega^2 = \frac{1}{2}[x^2 + 1.372 - 1.96x]\omega^2$$

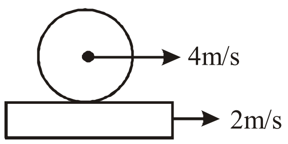
For work done to be minimum

$$\frac{dW}{dx} = 0$$

$$\Rightarrow \quad 2x - 1.96 = 0$$

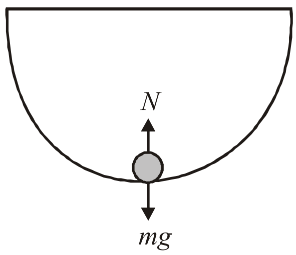
$$\Rightarrow \quad x = \frac{1.96}{2} = 0.98\,\text{m}$$

92. **(a)** For rolling, $v_A = 2$m/s
or $4 - 1.\omega = 2$



or $\omega = 2$ rad/sec (clockwise)

93. **(b)**



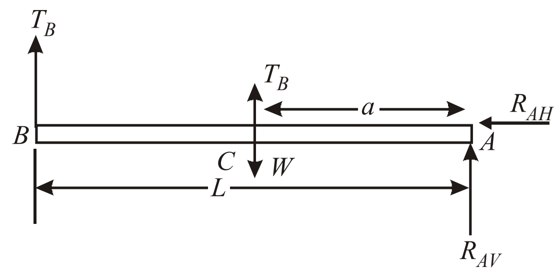
$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\omega = \frac{v}{r}\ ; \quad mgR = \frac{1}{2}mv^2 + \frac{1}{2}.\frac{2}{5}mr^2 \times \frac{v^2}{r^2}$$

$$gR = \frac{7}{10}mv^2 \Rightarrow v = \sqrt{\frac{10gR}{7}}$$

$$N - mg = \frac{mv^2}{R} \Rightarrow N = \frac{17mg}{7}$$

94. **(a)** FBD of rod will be as shown below



Summation of moments about A should be zero

$$\Sigma M_A = 0 = -T_B \times L - (T_B - W) \times a = 0$$

$$\therefore \quad T_B = \frac{Wa}{(L+a)}$$

Summation of moments about B should be zero

$$\Sigma M_B = 0 = (T_B - W) \times (L - a) + R_{AV} \times L = 0$$

$$\left(\frac{Wa}{(L+a)} - W\right) \times (L-a) + R_{AV} \times L = 0$$

$$R_{AV} \times L = -\left(\frac{Wa - WL - Wa}{(L+a)}\right) \times (L-a)$$

$$\therefore \quad R_{AV} = \frac{WL\,(L-a)}{(L+a) \times L} = W\,\frac{(L-a)}{(L+a)}$$

95. **(c)** $\alpha = \dfrac{\tau_{\text{axis}}}{I_{\text{axis}}} = \dfrac{Fx}{\left(\dfrac{Mr^2}{4} + Mx^2\right)}$

For maximum α,

$$\frac{d\alpha}{dx} = 0$$

$$\Rightarrow \quad \frac{r^2}{4} + x^2 - 2x^2 = 0 \Rightarrow x = \frac{r}{2}$$

96. **(c)** $\Sigma\tau_p = 0$

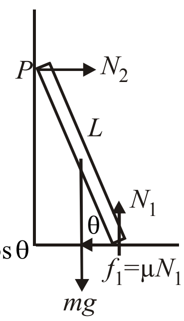
At equilibrium $\Sigma F = 0$,

$\Rightarrow N_1 = mg$

$$mg\frac{L}{2}\cos\theta + f_1 L\sin\theta = N_1 L\cos\theta$$

$$\frac{mg\cos\theta}{2}L + \mu mgL\sin\theta = mgL\cos\theta$$

$$\tan\theta = \frac{1}{2\mu}$$

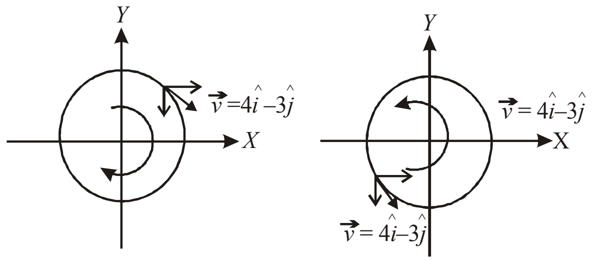


97. **(a)** $\alpha_A = \alpha_B$;

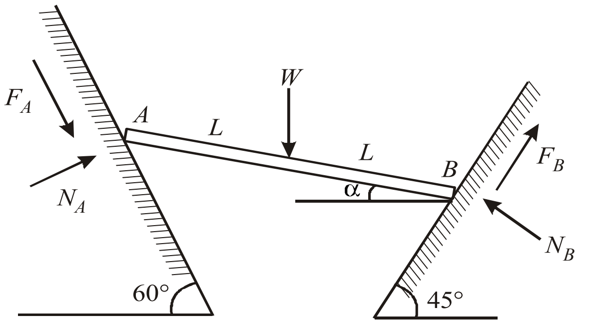
$$\frac{\tau_A}{I_A} = \frac{\tau_B}{I_B}$$

$$\frac{\tau_A}{mR^2} = \frac{\tau_B}{mR^2} \Rightarrow \tau_A = \tau_B$$

98. **(c)**



99. **(c)** Consider *FBD* of rod



Given $\phi = 15°$ $\therefore$ $\mu = \tan\phi = 0.268$

Applying equilibrium equations, we get

$\Sigma X = F_A \cos 60 + N_A \sin 60 + F_B \cos 45 - N_B \sin 45 = 0$

Also we know that

$F_A = 0.268\, N_A$ and $F_B = 0.268\, N_B$

Solving above equations we get

$N_A = 0.518\, \mathrm{N_B}$

$\Sigma Y = N_A \cos 60 - F_A \sin 60 + N_B \cos 45 + F_B \sin 45 = W$

Solving these equations we get

$N_B = 0.966\, W$ and $F = 0.259\, W$

Taking moment about *A* and equating it to zero, we get

$\Sigma M_A = (W \times L\cos\alpha) - (N_B \cos 45° \times 2L\cos\alpha) +$
$(N_B \sin 45° \times 2L\cos\alpha) - (F_B \sin 45° \times 2L\cos\alpha)$
$- (F_B \cos 45° \times 2L \times \sin\alpha) = 0$

By putting the values of known quantities in above equation we get $\alpha = 36.2°$

100. **(c)** Let speed of belt be v

Angular speeds of wheels

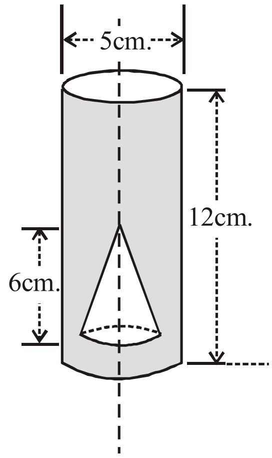
$$\omega_B = \frac{v}{2\pi R_B},\ \omega_A = \frac{v}{2\pi R_A};\ \frac{\omega_A}{\omega_B} = \frac{R_B}{R_A} = 2$$

101. **(b)** The given body = Solid cylinder – Right circular cone

As the body is symmetrical about *y*-axis, the center of gravity will lie on the *y*-axis i.e. $\bar{x} = 0$

For solid cylinder : Volume $= \pi r^2 h$

$$\text{or}\quad V_1 = \pi\left(\frac{5}{2}\right)^2 \times 12 = 75\pi\ \text{cm}^3$$



Position of *CG*, $y_1 = \dfrac{12}{2} = 6\,\text{cm.}$

For right circular cone : Volume $= \dfrac{1}{3}\pi r_1^2 h_1$

$$\text{or}\quad V_2 = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times 6 = \frac{9}{2}\pi\ \text{cm}^3$$

Position of *CG*,

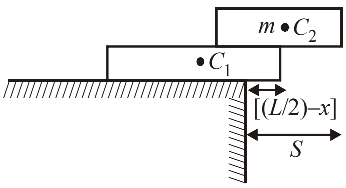
$$y_2 = \frac{1}{4}h_1 = \frac{1}{4}\times 6 = \frac{3}{2}\text{cm.}$$

$\therefore$ Position of *CG* of given body above its base

$$\bar{y} = \frac{V_1 y_1 - V_2 y_2}{V_1 - V_2} = \frac{75\pi \times 6 - \frac{9}{2}\pi \times \frac{3}{2}}{75\pi - \frac{9}{2}\pi}$$

$$= \frac{450 - 6.75}{75 - 4.5} = \frac{443.25}{70.5} = 6.3\text{cm.}$$

102. **(c)**



$$mx = m\left(\frac{L}{2} - x\right)$$

$$\Rightarrow\quad x = \frac{L}{4}\ ;$$

$$S = \frac{L}{4} + \frac{L}{2} = \frac{3L}{4}$$

103. **(d)** $mu = Mv_{cm} + mv_1$

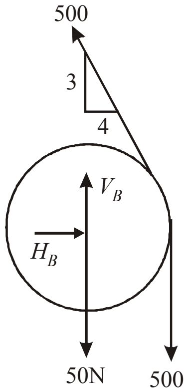
$mu - mv_1 = Mv_{cm}$

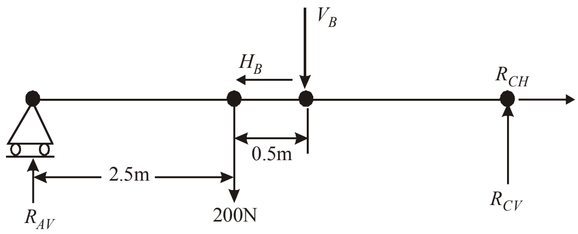
$mux = I\omega + mv_1 x$

$$mux = \frac{1}{12}ML^2\omega + mv_1 x\ ;$$

$$\frac{V_{cm}}{\omega} = \frac{L^2}{12x}$$

104. (d) Free body diagram for pulley and for the beam is shown in figure (a) and (b).





Pulley : $\Sigma F_V = 0 : V_B = 500 + 50 - (0.6 \times 500) = 250$ N

$\Sigma F_H = 0 : H_B = 500 \times 0.8 = 400$ N

Beam : $\Sigma F_H = 0$

$R_{CH} = 400$

$R_{CV} \times 5 = (250 \times 3) + (200 \times 2.5)$

i.e. $R_{CV} = 250$ N

$R_{AV} = 250 + 200 - 250 = 200$ N

$R_C = (400^2 + 250^2)^{1/2} = 471.69$ N

105. (b) The forces acting on the rod are :

(i) Weight W of the rod acting vertically downwards from centre of gravity.

(ii) Reaction R at A acting normally at A i.e. along AO.

(iii) Reaction R' at C acting at right angles to rod.

For equilibrium the three forces will be concurrent.

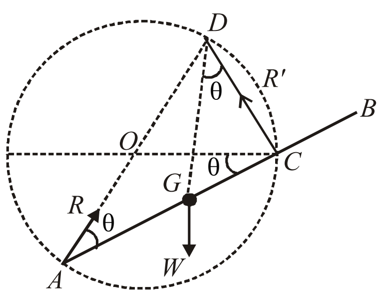
By geometry, $\angle OCA = \angle OAC = \angle GDA = \theta$

$AC = AD \cos\theta = 2r\cos\theta$, $DC = 2r\sin\theta$

and $AG = 1.5r$

In triangle GDC,

$$\tan\theta = \frac{GC}{DC} = \frac{AC - AG}{DC} \quad \text{or} \quad \frac{\sin\theta}{\cos\theta} = \frac{2r\cos\theta - 1.5r}{2r\sin\theta}$$



$\therefore\ 2\sin^2\theta = 2\cos^2\theta - 1.5\cos\theta$

or $1 - \cos^2\theta = \cos^2\theta - 0.75\cos\theta$

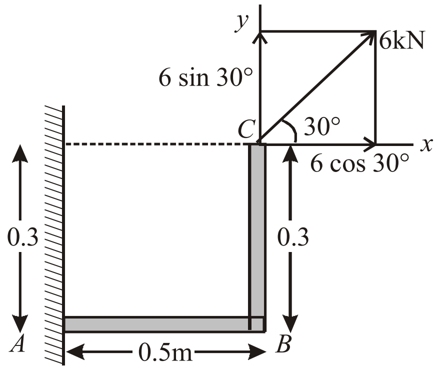
or $2\cos^2\theta - 0.75\cos\theta - 1 = 0$

Solving it, $\cos\theta = \dfrac{0.75 \pm \sqrt{(0.75)^2 + 4\times 2}}{4}$

$= \dfrac{0.75 \pm \sqrt{8.5625}}{4} = 0.92$ (using the +sign)

$\therefore\ \theta = \cos^{-1}(0.92) = 23°$

106. (a) The moment of force F (6kN) about point A is equal to the sum of the moment of its components about the same point A.



Moment of x-component of force is

$M_x = 6\cos 30° \times 0.3$

$= 6 \times \dfrac{\sqrt{3}}{2} \times 0.3 = 1.56$ kN-m clockwise

moment of y-component of force is

$M_y = 6\sin 30° \times 0.5$

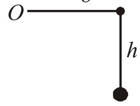
$= 6 \times \dfrac{1}{2} \times 0.5 = 1.50$ kN-m

Net moment about $A = M_y - M_x = (1.50 - 1.56)$ kNm

$= -0.06$ kNm

or 0.06 kNm clockwise.

107. (b)



Velocity at $B = \sqrt{2gh}$

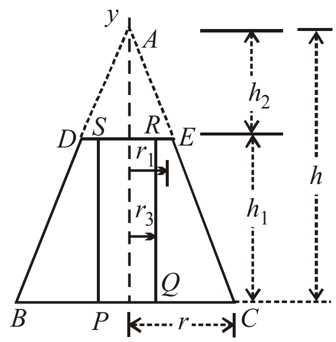
$\therefore$ Angular momentum $= m \times \sqrt{2gh} \times b$

108. (a) The given body can be considered as a right circular cone ABC from which a cone ADE and a cylindrical $PQRS$ have been cut out as shown.

Let the x-axis be along the base and y-axis as the axis of symmetry. For the cone ABC of height h is :

$$\frac{h_2}{r_1} = \frac{h}{r}$$

or $\dfrac{h-20}{5} = \dfrac{h}{10}$



$\Rightarrow 2h - 40 = h \Rightarrow h = 40$cm.

Volume $V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(10)^2 \times 40 = \frac{4000}{3}\pi \text{ cm}^3$

Position of CG on y-axis,

$$y_1 = \frac{1}{4} \times 40 = 10 \text{cm}.$$

For cone ADE, Volume

$$V_2 = \frac{1}{3}\pi r_1^2 h_2 = \frac{1}{3}\pi(5)^2 \times 20 = \frac{500}{3}\pi \text{ cm}^3$$

Position of CG on y-axis,

$$y_2 = 20 + \frac{1}{4} \times 20 = 25\text{cm}.$$

For cylindrical hole

Volume, $V_3 = \pi r_3^2 h_1 = \pi\left(\frac{5}{2}\right)^2 \times 20 = 125\,\pi \text{ cm}^3$

Position of CG on y-axis, $y_3 = \frac{h_1}{2} = \frac{20}{2} = 10$ cm.

The given body has a volume = $V_1 - V_2 - V_3$

$\therefore$ Position of CG of given body on y-axis

$$\bar{y} = \frac{\Sigma V_y}{\Sigma V} = \frac{V_1 y_1 - V_2 y_2 - V_3 y_3}{V_1 - V_2 - V_3}$$

$$= \frac{\frac{4000}{3}\pi \times 10 - \frac{500}{3}\pi \times 25 - 125\pi \times 10}{\frac{4000}{3}\pi - \frac{500}{3}\pi - 125\pi}$$

$$\bar{y} = \frac{40000 - 12500 - 3750}{4000 - 500 - 375} = \frac{23750}{3125} = 7.6\text{cm}.$$

109. (d) S_1 : For a rotating body, if ω is constant then $\alpha = 0 \Rightarrow \tau = 0$.

S_2 : A particle moving along x-axis with some velocity, at some distance from origin then its angular momentum not equal to zero.

S_3 : Direction continuously changes so it is not constant.

B COMPREHENSION TYPE

1. (c) As shown in the figure that all of the angular velocities are in the same direction, so we can regard ω_A, ω_B, and ω as the components of angular velocity along the rotation axis. Conservation of angular momentum then gives

$I_A\omega_A + I_B\omega_B = (I_A + I_B)\omega$

$$\omega = \frac{I_A\omega_A + I_B\omega_B}{I_A + I_B}$$

2. (b) The moment of inertia of the two disks are

$$I_A = \frac{1}{2}m_A r_A^2 = \frac{1}{2}(2.0 \text{ kg})(0.20 \text{ m})^2$$
$$= 0.040 \text{ kg.m}^2$$

$$I_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(4.0 \text{ kg})(0.10 \text{ m})^2$$
$$= 0.020 \text{ kg.m}^2$$

$$\omega = \frac{I_A\omega_A + I_B\omega_B}{I_A + I_B}$$

$$= \frac{(0.040 \text{ kg.m}^2)(50 \text{ rad/s}) + (0.020 \text{ kg.m}^2)(200 \text{ rad/s})}{0.040 \text{ kg.m}^2 + 0.020 \text{ kg.m}^2}$$

$= 100$ rad/s

3. (a) The initial kinetic energy is

$$K_1 = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2$$

$$= \frac{1}{2}(0.040 \text{ kg.m}^2)(50 \text{ rad/s})^2 + \frac{1}{2}(0.020 \text{ kg.m}^2)(200 \text{ rad/s})^2$$

The final kinetic energy is

$$K_2 = \frac{1}{2}(I_A + I_B)\omega_2 = \frac{1}{2}(0.040 \text{ kg.m}^2 + 0.020 \text{ kg.m}^2)(100 \text{ rad/s})^2 = 300 \text{ J}$$

4. (c) The acceleration of the centre of mass is $a_{C.M} = \frac{F}{2m}$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2}a_{C.M}t^2 = \frac{Ft^2}{4m}$$

5. (a), 6. (b)

Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m} \quad \text{or} \quad \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

or $x_1 + x_2 = \frac{Ft^2}{2m}$ .... (1)

Further, the extension of the spring is $x_1 - x_2$.

Therefore, $x_1 - x_2 = x_0$ .... (2)

From eq. (1) and eq. (2),

$$x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$$

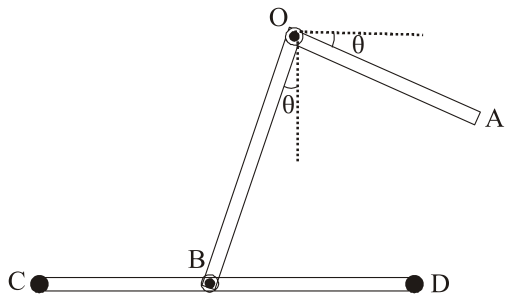
and $x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$

7. (a); 8. (d); 9. (a)

Taking origin at O

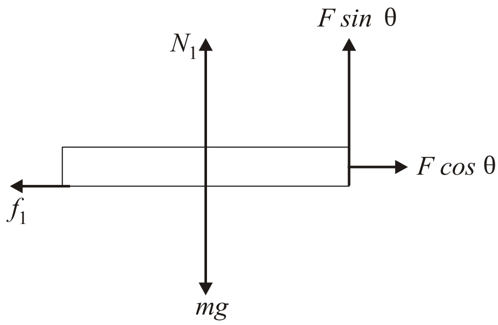
Coordinate of A = (ℓ cos θ, – ℓ sin θ)

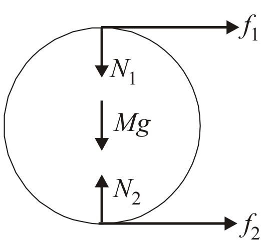
Coordinate of C = (– ℓ – ℓ sin θ, – ℓ cos θ)



10. (b); 11. (a); 12. (c)

Drawing the F.B. D of the plank and the cylinder.





Equations of motion are

$F\cos\theta - f_1 = ma$ ....(1)

$F\sin\theta + N_1 = mg$ ....(2)

$f_1 + f_2 = MA$ .....(3)

$f_1R - f_2R = I\alpha$ .....(4)

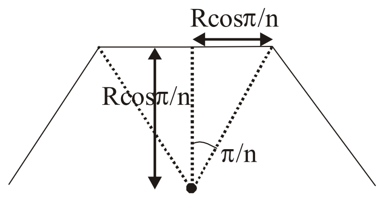
$A = R\alpha$ .....(5)

$$a = \frac{4F\cos\theta}{3M+8m} = \frac{4\times 55\times\frac{1}{2}}{[(3\times1)+(8\times1)]} = 10 \text{ m/s}^2$$

$$f_1 = \frac{3MF\cos\theta}{3M+8m} = \frac{3\times1\times55\times\frac{1}{2}}{3\times1+8\times1} = 7.5\ N$$

and $f_2 = \frac{MF\cos\theta}{3M+8m} = \frac{1\times55\times\frac{1}{2}}{3\times1+8\times1} = 2.5\ N$

13. (a)



$$I = \left[\frac{m\left(2R\sin\frac{\pi}{n}\right)^2}{12} + m.R^2\cos^2\frac{\pi}{n}\right] + MR^2$$

$$I = n.m.R^2\left[\frac{\sin^2\frac{\pi}{n}}{3} + \cos^2\frac{\pi}{n}\right] + MR^2.$$

14. (b) $I = nmR^2\left[\frac{\sin^2\frac{\pi}{n}}{3} + \cos^2\frac{\pi}{n}\right] + MR^2 + (M+mn)R^2$

$$= nmR^2\left[\frac{\sin^2\frac{\pi}{n}}{3} + \cos^2\frac{\pi}{n} + 1\right] + 2MR^2.$$

15. (d) Where $I = nmR^2\left[\frac{\sin^2\frac{\pi}{n}}{3} + \cos^2\frac{\pi}{n}\right] + MR^2$

$(M+nm)\ g\sin\theta - f = (M+nm)\ a$ ... (i)

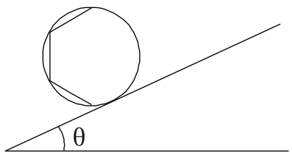
$fR = I\ a/R$ ... (ii)

From (i) and (ii)

$$f = \frac{(M+nm)g\sin\theta I}{[I+(M+nm)R^2]}$$

$$\mu\,(M+nm)\ g\sin\theta = \frac{(M+nm)g\sin\theta I}{[I+(M+nm)R^2]}$$

$$\mu = \frac{I \tan\theta}{I + (M + nm)R^2}$$



16. (d) $I = 4\left[\frac{2}{5}MR^2 + M(R\sqrt{2})^2\right]$

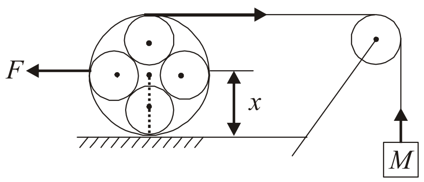
$$= 4MR^2\left[\frac{2}{5} + 2\right]$$

$$= \frac{4MR^2 \times 12}{5} = \frac{48MR^2}{5}.$$

17. (b) Let a be the acceleration of centre of mass

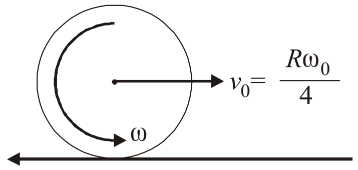
$Mg - T = 0$ ... (i)

$F.x = T.2x$ ... (ii)



18. (c) remain the same

19. (a) Let linear velocity of the disc will become zero after a time t_1. Then it starts moving in backward direction and at time t_2 it comes in pure rolling. When disc starts pure rolling its linear and angular velocities will become constant and friction will be zero.



At time t_1,

$$0 = \frac{R\omega_0}{4} - \mu g t_1$$

$$t_1 = \left[\frac{R\omega_0}{4\mu g}\right] \quad \text{... (i)}$$

At time t_2,

$v = \mu g\, t_2$

$\mu g\, t_2 = R\omega$

$$t_2 = \left[\frac{R\omega}{\mu g}\right] \quad \text{... (ii)}$$

$$\omega = \omega_0 - \frac{2\mu g}{R}(t_1 + t_2)$$

$$\omega = \omega_0 - \frac{2\mu g}{R}\left[\frac{R\omega_0}{4\mu g} + \frac{R\omega}{\mu g}\right]$$

$$\omega = \omega_0 - \frac{\omega_0}{2} - 2\omega$$

$$3\omega = \frac{\omega_0}{2} \text{ or, } \omega = \left[\frac{\omega_0}{6}\right]$$

Now from (ii), $t_2 = \frac{R\omega_0}{6\mu g}$

Maximum displacement of the disc in forward direction,

$$S = \frac{R\omega_0}{4} \times \frac{R\omega_0}{4\mu g} - \frac{1}{2}\left(\frac{R\omega_0}{4\mu g}\right)^2 \mu g$$

$$= \frac{R^2\omega_0^2}{\mu g}\left[\frac{1}{16} - \frac{1}{32}\right] = \frac{R^2\omega_0^2}{32\mu g}$$

The displacement of the disc when it starts pure rolling

$$= \frac{1}{32}\frac{R^2\omega_0^2}{\mu g} - \frac{1}{2}\mu g\left(\frac{R\omega_0}{6\mu g}\right)^2 = \frac{5}{32 \times 9}\frac{(R\omega_0)^2}{\mu g}$$

$$\therefore \frac{\omega_0 R}{6} \times t_3 = \frac{5}{32 \times 9}\frac{(\omega_0 R)^2}{\mu g}$$

$$\text{or, } t_3 = \frac{5}{48}\left(\frac{\omega_0 R}{\mu g}\right)$$

$$\text{Total time} = t_1 + t_2 + t_3 = \frac{25}{48}\frac{\omega_0 R}{\mu g}.$$

20. (b) Time after which disc starts pure rolling.

$$t = t_1 + t_2 = \frac{R\omega_0}{4\mu g} + \frac{R\omega_0}{6\mu g} = \frac{5R\omega_0}{12\mu g}$$

21. (c) Angular momentum of disc after it starts pure rolling,

$L = MvR + I\omega$

$$= \left[\frac{MR\omega_0 R}{6} + \frac{MR^2}{2}\frac{\omega_0}{6}\right]$$

$$= MR^2\omega_0\left[\frac{1}{6} + \frac{1}{12}\right] = \left(\frac{MR^2\omega_0}{4}\right)$$

22. (c) The power delivered is constant; the kinetic energy of the wheel is proportional to t and so the velocity is proportional to $\sqrt{t}$.

23. (d) Since $v = \frac{dx}{dt} \propto \sqrt{t}$

We get, $x \propto t^{3/2}$;

and acceleration $= a = \frac{dv}{dt} \propto t^{-1/2}$

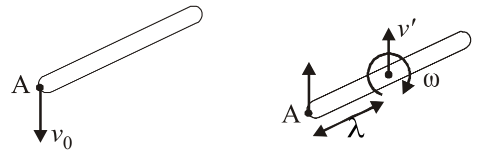
$\therefore$ friction, $f = ma \propto t^{-1/2}$

or, $f^3 x =$ constant.

24. (d) The required time is independent of μ.

25. **(b)** The force of impact at A is vertically upward.

26. **(d)**



$v_A = v' + \ell\omega\cos\theta$

$\because$ collision is elastic

$\therefore\ v_0 = v' + \ell\omega\cos\theta$

27. **(b)** Angular momentum $\vec{L} = \vec{r} \times m\vec{v}$

$|\vec{L}| = mv_0\ell\cos\theta.$

28. **(c)** The disc will stop (translation) at time

$$t = \frac{v_0}{\mu g}$$

Since, $\tau = I\alpha$

$$\mu Mg.R = \frac{MR^2}{2}.\alpha$$

$$\alpha = \frac{2\mu g}{R}$$

and $\omega = \omega_0 - \alpha t$

$\downarrow$

$$0 = \frac{2v_0}{R} - \frac{2\mu g}{R}.t \ \Rightarrow t = \frac{v_0}{\mu g}$$

Also at the same time, linear speed also ceases.

$v = v_0 - \mu g t \qquad 0 = v_0 - \mu g t$

$$t = \frac{v_0}{\mu g}$$

Thus it will not regain ω.

Since linear speed and angular speed becomes zero at the same time.

29. **(d)** At time $t = 4v_0/5\mu g$ the angular speed will be zero.

At this instant $v = v_0/5$.

Now when pure rolling starts, it will take a time $\left[\frac{2v_0}{35\mu g} + \frac{4v_0}{5\mu g}\right]$ from $t = 0$

Thus finally $\omega = \frac{v_0}{7R}$. Hence first ω will decrease from $\frac{2v_0}{R}$ to zero and then increase from zero to $\frac{v_0}{7R}$.

30. **(c)** COM : about point of contact

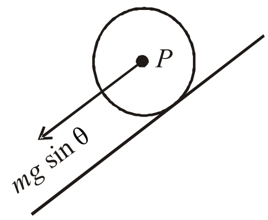
$$Mv_0R - \frac{2}{5}MR^2\left(\frac{2v_0}{R}\right) = \frac{7}{5}MvR;\ v = \frac{v_0}{7}$$

31. **(b)** Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$

32. **(d)** Average acceleration $= \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{v(-\hat{j}) - v\hat{j}}{t} = \frac{-2v\hat{j}}{t}$

33. **(d)** Let P be the point on sphere in contact with incline. Then P is also instantaneous centre of rotation

$$\therefore\ mg\sin\theta \times R = (I_{cm} + mR^2)\,\alpha = \frac{7}{5}mR^2\alpha$$



$$\therefore\ a_{cm} = R\alpha = \frac{5}{7}g\sin\theta = \frac{3g}{7}$$

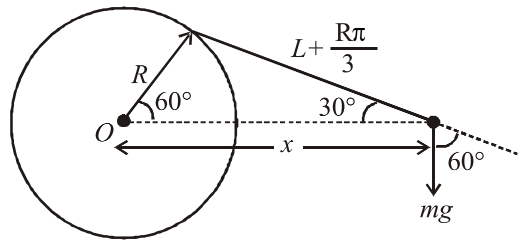
34. **(c)** Speed of point P is always zero.

35. **(b)** Distance covered $= S = \frac{h}{\sin 37^\circ} = \frac{5h}{3}$

$$a_{cm} = \frac{3g}{7}$$

$$\therefore\ \text{time } t = \sqrt{\frac{2s}{a_{cm}}} = \sqrt{\frac{70h}{9g}}$$

36. **(a), 37. (d), 38. (b).**



Since $u = \sqrt{2gL}$, the highest point to which the block shall reach is $\frac{u^2}{2g} = L$ distance above its initial position.

Hence at highest point, the thread has rotated by $\frac{\pi}{3} = 60^\circ$ and the block is at same horizontal level as centre of cylinder as shown.

$$x = \frac{R}{\sin 30^\circ} = 2R$$

$$T = mg\cos 60^\circ = \frac{mg}{2}$$

Tangential acceleration, $a_t = g\sin 60^\circ = \frac{\sqrt{3}g}{2}$

39. (a), 40. (b)

Some deceleration required:

$v^2 - v_0{}^2 = 2as$

$v_0 = 18$ km / hour = 5 m/s

$$a = \frac{v^2 - v_0^2}{2s} = \frac{0-25}{2\times 5} = -2.5 \text{ m/s}^2$$

$F = m\,a = -96 \times 2.5 = -240$ N

i.e., 120 N per wheel opposite to the direction of rotation

The brake pads apply a torque to the rim, but at a shorter distance from the axle than the outside diameter of the wheel (contact point with the road).

$\tau_{OD} = \tau_{\text{rim}}$

$$F_{p-r} = \frac{F_{OD} . R_{OD}}{R_{rim}} = \frac{120 \times 0.350}{0.325} = 129\ N$$

$F_{p\text{-}r} = \mu \times F_{\text{perpendicular}}$

$F_{\text{perpendicular}} = F_{p\text{-}r} / \mu = 129 / 0.5 = 258$ N

$F_{\text{perpendicular}}$ per pad = 129 N

The combination of handlebar lever and caliper assembly has a mechanical advantage of 10.

(Think of a lever with 20 units on one side of the fulcrum and 2 units, 1 per pad, on the other.)

F_{lever} per pad = $F_{\text{perpendicular}}$ / 10 = 12.9 N

However, as the lever acts on two pads, the total force which must be applied to the lever is 25.8 N.

C REASONING TYPE

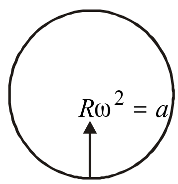
1. **(d)** For a disc rolling without slipping on a horizontal rough surface with uniform angular velocity, the acceleration of lowest point of disc is directed vertically upwards and is not zero (Due to translation part of rolling, acceleration of lowest point is zero. Due to rotational part of rolling, the tangential acceleration of lowest point is zero and centripetal acceleration is non-zero and upwards). Hence statement 1 is false.
2. **(d)** As x increases, the required component of reaction decreases to zero and then increases (with direction reversed). Hence statement-1 is false.
3. **(d)** The moment of inertia about both given axis shall be same if they are parallel. Hence statement-1 is false.
4. **(c)** The applied horizontal force F has tendency to rotate the cube in anticlockwise sense about centre of cube. Hence statement-2 is false.
5. **(a)** Radius of gyration of body is not a constant quantity. Its value changes with the change in location of the axis of rotation. Radius of gyration of a body about a given axis is given as

$$k = \frac{\sqrt{r_1^2 + r_2^2 \ldots\ldots\ldots\ldots r_n^2}}{n}$$

6. **(c)** The two bodies will move towards their common centre of mass. But the location of the centre of the mass will remain unchanged. Therefore, the centre of mass will remain at rest with respect to A as well as B.
7. **(b)** Rolling occurs only on account of friction which is tangential force capable of providing torque. When the inclined plane is perfectly smooth, it will simply slip under the effect of its own weight. Once the perfect rolling begins, force of friction becomes zero. Hence work done against friction is zero.
8. **(d)** Statement – 1 is false because, it is applicable only in rotational frame.
9. **(a)** In non-uniform circular motion acceleration vector makes some angle with radius hence it is not perpendicular to velocity vector.
10. **(d)** Statement – 1 is False, Statement – 2 is True. Friction force is not always zero and a point object cannot roll as a point object will translate.
11. **(d)** In non-uniform circular motion, particle's kinetic energy changes with time. By work-energy theorem, net work done on the particle is non-zero. In uniform circular motion, total force on the particle is centripetal in nature.
12. **(d)** The moment of inertia about both given axis shall be same if they are parallel. Hence statement-1 is false.
13. **(c)** $KE = KE_{CM} + \frac{1}{2}MV_{CM}^2$

KE in CM frame is least as $V_{CM} = 0$ in that frame.

14. **(d)** If speed is increasing there is a tangential acceleration. Net acceleration is not pointing towards centre.
15. **(d)** S-1 will be true only if both the axis in comparison are parallel to each other.
16. **(a)** While seen from CM rigid body appears in pure rotation around CM. So KE = $\frac{1}{2}I_{cm}\omega^2$.
17. **(b)** Both statements are true but statement-2 is not correct explanation for statement-1.
18. **(d)** Acceleration of lowest point on the disc is $\omega^2 R$

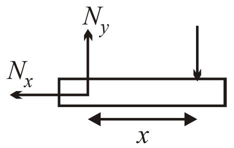


19. (d)

$\tau = Fx$

$$\alpha = \frac{Fx}{m\ell^2/3}$$

$$a_{cm} = \frac{\ell}{2}\times\alpha = \frac{3Fx}{2m\ell}$$



Component of reaction by hinge on the rod perpendicular to rod

$$F - N_y = M\times\frac{3Fx}{2M\ell} \Rightarrow N_y = F\left[1-\frac{3x}{2\ell}\right]$$

$\therefore$ as x increases, N_y decreases α increases.

20. (c) Torque about all points should be zero.

D MULTIPLE CORRECT CHOICE TYPE

1. (a, b, c, d) $I_z = \dfrac{m(\ell\sin\theta)^2}{12} + m\left(\dfrac{\ell}{2}\cos\theta\right)^2 + m\dfrac{(\ell\cos\theta)^2}{12}$

$$+m\left(\frac{\ell}{2}\sin\theta\right)^2 + \frac{m\ell^2}{12} + \frac{m\ell^2}{12}$$

$$= \frac{m\ell^2}{12} + \frac{m\ell^2}{4} + \frac{m\ell^2}{12} + \frac{m\ell^2}{12}$$

$$= \frac{m\ell^2}{2} \text{ (constant independent of } \theta)$$

[I_z will be maximum for any value of θ (Obviously)]

2. (a, d) $a_n = \dfrac{v^2}{R} = \omega^2 R$

3. (b) $dm = \rho(2\pi r)drL = \dfrac{M}{\pi(R_2^2 - R_1^2)}2\pi rdrL$

$$= \frac{2M}{(R_2^2 - R_1^2)}r\,dr$$

$$\therefore I = \frac{2M}{(R_2^2 - R_1^2)}\int_{R_1}^{R_2} r^3 dr = \frac{1}{2}M(R_1^2 + R_2^2)$$

4. (b) The external torque about O is contributed only by mg. The equation of motion is

$-mg\,(d\sin\theta) = I_0\alpha$ ...(1)

$$\Rightarrow I_0 = \frac{mL^2}{12} + md^2 = \frac{mL^2}{12} + m(R^2 - \frac{L^2}{4})$$

where θ is small, $\sin\theta \approx \theta$

$$\therefore \omega = \sqrt{\frac{\left(R^2 - \dfrac{L^2}{4}\right)^{1/2}}{\left(R^2 - \dfrac{L^2}{6}\right)}g}$$

5. (b, c) $\vec{\tau} = \dfrac{d\vec{L}}{dt} = \vec{C}\times\vec{L}$

$\therefore \dfrac{d\vec{L}}{dt}$ is $\perp$ to $\vec{C}$ and $\vec{L}$ both. Hence B is correct.

Further, $\vec{L}\,\vec{L} = L^2$

Differentiating w.r.t. time

$$\vec{L}.\frac{d\vec{L}}{dt} + \frac{d\vec{L}}{dt}\vec{L} = 2L\frac{dL}{dt}$$

$$\text{or, } 2\vec{L}.\frac{d\vec{L}}{dt} = 2L\frac{dL}{dt}$$

But $\vec{L} \perp \dfrac{d\vec{L}}{dt}$

$$\therefore \vec{L}.\frac{d\vec{L}}{dt} = 0 \Rightarrow \frac{dL}{dt} = 0$$

$\Rightarrow$ Magnitude of $\vec{L}$ does not change with time.

6. (a, d) Acceleration of the bead down the wire is $g\cos\theta$ and the length of wire is $2R\cos\theta$

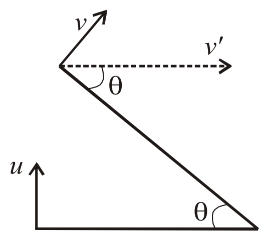
$$\therefore v = \sqrt{2as} = \sqrt{2(g\cos\theta)(2R\cos\theta)}$$

$$\Rightarrow v \propto \cos\theta$$

Further, $t = \dfrac{v}{a} = \dfrac{2\sqrt{gR}\cos\theta}{g\cos\theta} = 2\sqrt{\dfrac{R}{g}}$

i.e., t is independent of θ.

7. (a, d) As the block does not move, the ball moves along a circular path of radius ℓ. The centre of mass of the system always lies somewhere on the string. Let v = spseed of the ball when the string makes an angle θ with the horizontal.



$$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mg\ell\sin\theta$$

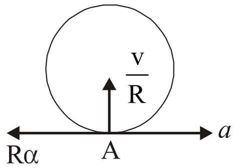
Horizontal component of $v = v' = v\sin\theta$

$= \sin\theta\sqrt{u^2 - 2g\ell\sin\theta}$

For v to be maximum, $dv/d\theta = 0$, which gives

$\sin\theta = u^2/3g\ell$

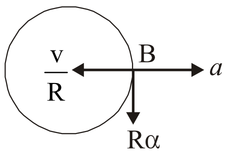
8. **(a, b, c, d)** At A :



for a rolling wheel, $a = R\alpha$ ∴

(a) is correct.

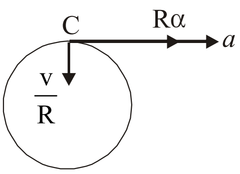
At B :



If $\frac{v^2}{R} = a$ then a_B may be vertically downwards

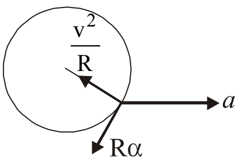
∴ (b) is correct.

At C :



∴ (c) is correct.

Consider this



∴ (d) is correct.

9. **(b, c)** The initial velocity of c.m. is upward. The acceleration of the c.m. is 'g' downward.

10. **(c, d)** The velocity of the disc when rolling begins can be obtained using the conservation of angular momentum principle about the point through which the friction force acts. So, the coefficient of friction has no bearing on final velocity. The work done by the force of friction will simply be change in kinetic energy.

11. **(a, b)** (a) $a_s = \frac{dv}{dt} = \frac{d}{dt}(\ell\omega) = \ell\alpha = \ell k\sin\theta$

(b) From $\frac{d\omega}{dt} = \alpha$,

$$d\omega = k\sin\theta\, dt = k\sin\theta\frac{dt}{d\theta}d\theta = \frac{k}{\omega}\sin\theta\, d\theta$$

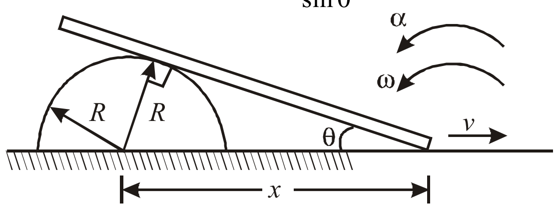
Then $\int_0^\omega \omega\, d\omega = k\int_0^\theta \sin\theta\, d\theta$

or $\omega^2 = 2k(1-\cos\theta)$ and

$a_n = \ell\omega^2 = 2k\ell(1-\cos\theta)$

12. **(a, c, d)** As no external torque is present, using angular momentum conservation about any point we can say that $\omega_f = \omega_i$.

13. **(c, d)** From the geometry, $x = \frac{R}{\sin\theta}$



Also, $\omega = -\frac{d\theta}{dt}$. Therefore,

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{R}{\sin\theta}\right)$$

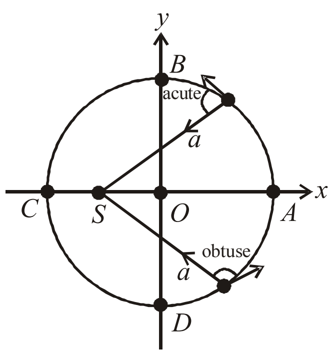
$$= \frac{-R\,(d\theta/dt)\cos\theta}{\sin^2\theta} = \frac{\omega R\cos\theta}{\sin^2\theta}$$

$$\omega = \frac{v\sin^2\theta}{R\sin\theta} = \frac{Rv}{x\sqrt{x^2-R^2}}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{Rv}{x\sqrt{x^2-R^2}}\right)$$

$$= \frac{-Rv^2(2x^2-R^2)}{x^2(x^2-R^2)^{3/2}}$$

14. **(a, c)**

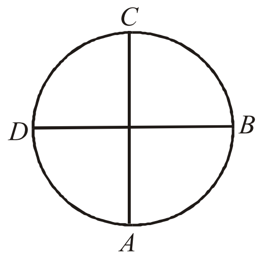


As the object moves from A to C via B the angle between acceleration vector and velocity vector decreases from 90° and then increases back to 90°. Since the angle between velocity and acceleration is acute, the object speeds up.

As the object moves from C to A via D the angle between acceleration vector and velocity vector increases from 90° and then decreases back to 90°. Since the angle between velocity and acceleration is obtuse, the object slows down.

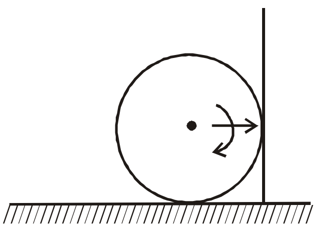
15. (a, c, d) Internal force does not change the motion of centre of mass.
If "g" varies, COG doesn't coincide with centre of mass.

16. (b, d) In the shown diagram at position A and C radial acceleration has its maximum values and there is no component of acceleration in tangential direction.



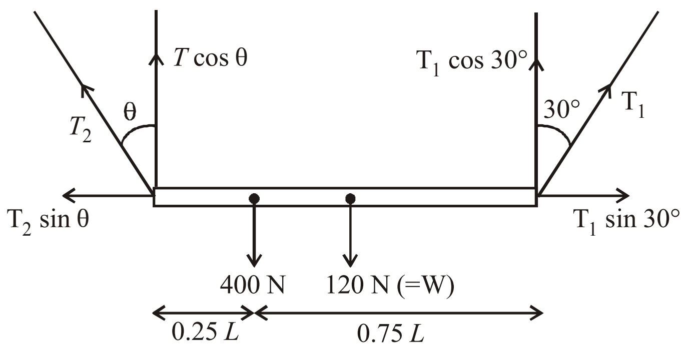
At position B and D tangential acceleration is maximum (g) and radial acceleration in some where between maximum and minimum.

17. (a, c, d)



Because the wall is frictionless, so there will be no torque during collision. Hence ω will not change instantaneously. But velocity will be same in magnitude and in opposite direction as collision is elastic with a fixed wall.
Hence, now the slipping will start and the friction present on the ground will reduce ω. After some time, pure rolling starts.

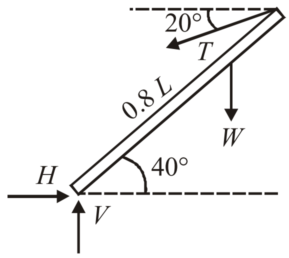
18. (a, b, d) Remembering that the weight acts at the center of gravity,



$\Sigma F_x = 0$ yields $T_1 \sin 30° - T_2 \sin\theta = 0$
or $T_2 \sin\theta = 0.50\, T_1$.
$\Sigma F_y = 0$ yields $T_1 \cos 30° + T_2 \cos\theta$
$- 120 - 400 = 0$
or $T_2 \cos\theta + 0.866\, T_1 = 520$ N.

Taking moments about the left edge and resolving T_1 into x and y components,
$\Sigma\tau = 0$ yields $LT_1 \cos 30° - (0.25\,L)(400)$
$- (0.5\,L)(120) = 0$
Dividing throughout by L and solving, we get $T_1 = 185$ N.
Substituting into our earlier equations, we get
$T_2 \sin\theta = 92.5$ N and $T_2 \cos\theta = 360$ N
Dividing the equations yields $\tan\theta = 0.257$,
or $\theta = 14.4°$.
Then $0.249\, T_2 = 92.5$, and $T_2 = 371$ N.
One can always check moment problem results by taking moments about another point, such as the right end of the bar for this problem.

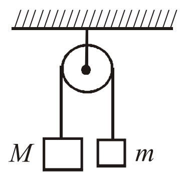
19. (a, b, c, d) Refer to figure.



Taking torques about the hinge,
$-W(0.8\,L\cos 40°) + (T\cos 20°)(L\sin 40°) -$
$(T\sin 20°)(L\cos 40°) = 0.$
This yields $T = 1.80\,W$.
From $\Sigma F_x = 0$, $H - T\cos 20° = 0$,
Giving $H = 1.69\,W$.
From $\Sigma F_y = 0$,
$V - W - T\sin 20° = 0$,
and so $V = 1.62\,W$.
If the beam has weight, then an additional $W/2$ must be added acting vertically downward through the center of the beam. The torque about the hinge will be $-(W/2)(0.5\,\mathrm{L}\cos 40°)$ due to the weight. With these additions to the equations above, we obtain $T = 2.35\,W$, from which $H = 2.21\,W$ and $V = 2.30\,W$.

20. (a, b, c, d) If larger mass comes down with acceleration a,

$$a_{cm} = \frac{aM - am}{M + m} = a\left(\frac{M - m}{M + m}\right)$$



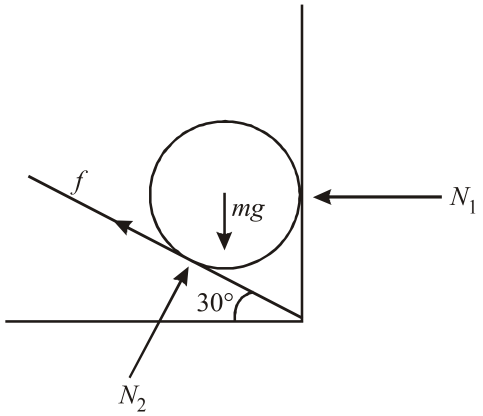
$\therefore$ COM is not at rest and net force on system

$$= (M + m)\left(\frac{M - m}{M + m}\right) a = (M - m)\, a$$

Velocity of COM changes continuously.

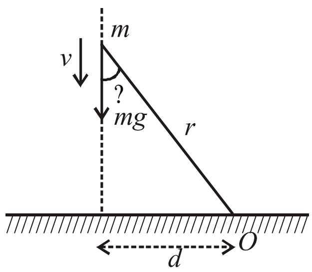
21. **(a, b)** The most convenient point to take the sum of torques is at the hinge. The torque equation is $+60L-(2L/3)(T_m \sin 12°)+250(L/2)=0$, leading to $T_m = 1335$ N.
Also $T\cos 12° - H = 0$, so $H = 1305$ N

22. **(a, b, c)** The possible forces are shown in figure.



If we take moments about an axis through the center of the sphere, only f can have a torque and $\Sigma\tau = 0$ implies $f = 0$.
Then $\Sigma F_y = 0$ yields
$N_2 \cos 30° = mg = (10\text{ kg})(9.8\text{ m/s}^2)$
$\Sigma F_x = 0$ yields $N_2 \sin 30° - N_1 = 0$,
or $N_1 = 56.5$ N, $N_2 = 113$ N

23. **(a, c, d)**



The magnitude of angular momentum of particle about $O = mvd$
Since speed v of particle increases, its angular momentum about O increases.
Magnitude of torque of gravitational force about $O = mgd \Rightarrow$ constant
Moment of inertia of particle about $O = mr^2$
Hence MI of particle about O decreases.

Angular velocity of particle about $O = \dfrac{v\sin\theta}{r}$

$\because$ v and sin θ increases and r decreases
$\therefore$ angular velocity of particle about O increases.

24. **(a, d)** From $\omega - \omega_0 = \alpha t$ and $\tau = I\alpha$

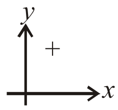
$$t = \frac{(\omega-\omega_0)\,I_{\text{total}}}{\tau}$$

where $I_{\text{total}} = I_{\text{rotor}} + I_{\text{samples}} = I_{\text{rotor}} + 4\text{m}_{\text{sample}}r^2$
$= 0.06 + 4 \times 0.012 \times (0.1)^2 = 0.0605\text{ kg m}^2$
and $\omega = 18{,}000$ (rpm) / 60 (seconds/minute) $\times 2\pi$ (radians/revolution) $= 1885\text{ s}^{-1}$

$$t = \frac{1885 \times 0.0605}{0.25} = 456.2\text{ s}$$

Force on the sample:
$F_{\text{central}} = m\,\omega^2 r = 0.012 \times (1885)^2 \times 0.1$
$= 4264$ N.

25. **(a, b)** The system of coplanar-unlike forces are shown above in figure. Sign convention used :



for forces ; 

for moments

Let R be the resultant of given system of forces and R_x and R_y be the horizontal and vertical components of resultant force.

$\therefore\ R = \sqrt{R_x^2 + R_y^2}$

Also, $R_x = \Sigma F_x$ and $R_y = \Sigma F_y$
Resolving all force in xx' direction with proper sign convention
$\Sigma F_x = (-)\,1\cos 90° - 1.5\cos 60° - 1\cos 45° - 0.5\cos 30° = -1.890$ kN
Resolving all force in yy' direction with shown sign convention
$\Sigma F_y = (-)\,1\sin 90° - 1.5\sin 60° - 1\sin 45° - 0.5\sin 30°$
$= -3.265$ kN
Putting value in eq. (1) we get

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-1.89)^2 + (-3.265)^2}$$

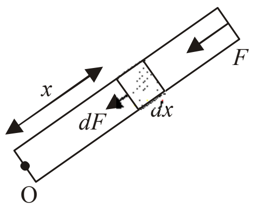
$R = \pm 3.77$ kN
As the forces in the system are directed downwards (–ve direction) so the resultant R will also be negative i.e.
The resultant force is given by : $R = -3.77$ kN.

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-3.265}{-1.890} \text{ or } \theta = 59.93° \approx 60°$$

To find position of resultant force on beam use
$R.d_\perp = \Sigma M_P$
where R = resultant force = (–) 3.77 kN
d = distance between point of application of resultant force about P.
ΣM_P = sum of the moments of all forces about P.
$\therefore \Sigma M_P = (1 \times \sin 90° \times 0) + (1.5 \times \sin 60° \times 4) + (1.0 \times \sin 45° \times 8) + (0.5 \times \sin 30° \times 12) = 13.852$ kN-m
as $R_y = R\sin\theta$
$\therefore\ R_y d = R\sin\theta\, d$
$\Sigma M_P = R\sin\theta\, d$
So, $13.852 = 3.77\sin 60°.d$

$$d = \frac{13.852}{3.265} = 3.67\text{m from } P$$

26. (a) The force acting on the mass of liquid dm of length dx at a distance x from the axis of rotation O.



$$dF = (dm)\, x\, \omega^2$$

$$\therefore \quad dF = \frac{M}{L} dx \times x\omega^2$$

where $\frac{M}{L}$ is mass of liquid in unit length.

$\therefore$ The force acting at the other end is for the whole liquid in tube.

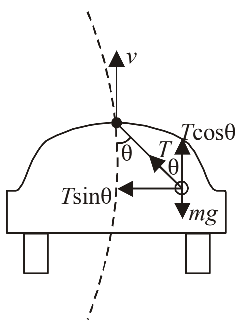
$$F = \int_0^L \frac{M}{L}\omega^2 x\, dx = \frac{M}{L}\omega^2 \int_0^L x\, dx$$

$$= \frac{M}{L}\omega^2 \left[\frac{x^2}{2}\right]_0^L = \frac{M}{L}\omega^2\left[\frac{L^2}{2} - 0\right] = \frac{ML\omega^2}{2}$$

27. (c) When the car is moving in a circular horizontal track of radius 10 m with a constant speed, then the bob is also undergoing a circular motion. The bob is under the influence of two forces.

(i) T (tension in the rod)

(ii) mg (weight of the bob)



Resolving tension, we get

$$T\cos\theta = mg \qquad \text{... (i)}$$

$$\text{And } T\sin\theta = \frac{mv^2}{r} \qquad \text{... (ii)}$$

(Here $T\sin\theta$ is producing the necessary centripetal force for circular motion)

Dividing (ii) by (i), we get

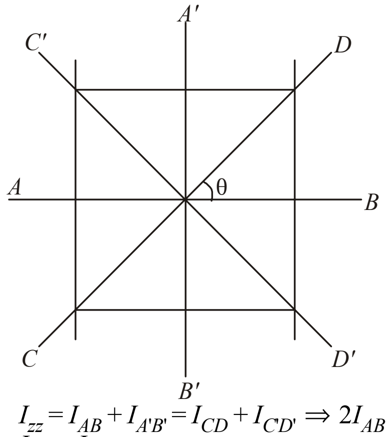
$$\tan\theta = \frac{v^2}{rg} = \frac{10\times 10}{10\times 10} = 1$$

$$\Rightarrow \quad \theta = 45^\circ$$

28. (a) $A'B' \perp AB$ and $C'D' \perp CD$

From symmetry $I_{AB} = I_{A'B'}$ and $I_{CD} = I_{CD'}$

From theorem of perpendicular axes,



$$I_{zz} = I_{AB} + I_{A'B'} = I_{CD} + I_{CD'} \Rightarrow 2I_{AB} = 2I_{CD}$$

$$\therefore \quad I_{AB} = I_{CD}$$

29. (a, b, c) $\vec{\tau} = \frac{\vec{dL}}{dt}$

Given that

$$\vec{\tau} = \vec{A} \times \vec{L} \Rightarrow \frac{\vec{dL}}{dt} = \vec{A} \times \vec{L}$$

From cross-product rule, $\frac{\vec{dL}}{dt}$ is always perpendicular to the plane containing $\vec{A}$ and $\vec{L}$.

By the dot product definition

$$\vec{L}.\vec{L} = L^2$$

Differentiating with respect to time

$$\vec{L}\cdot\frac{\vec{dL}}{dt} + \vec{L}\cdot\frac{\vec{dL}}{dt} = 2L\frac{dL}{dt}$$

$$\Rightarrow \quad 2\vec{L}\cdot\frac{\vec{dL}}{dt} = 2L\frac{dL}{dt}$$

Since, $\frac{\vec{dL}}{dt}$ i.e. $\vec{\tau}$ is perpendicular to $\vec{L}$

$$\therefore \quad \vec{L}\cdot\frac{\vec{dL}}{dt} = 0$$

$$\Rightarrow \quad \frac{dL}{dt} = 0$$

$\Rightarrow \quad L$ = constant

Thus, the magnitude of L always remains constant.

As $\vec{A}$ is a constant vector and it is always perpendicular to $\vec{\tau}$,

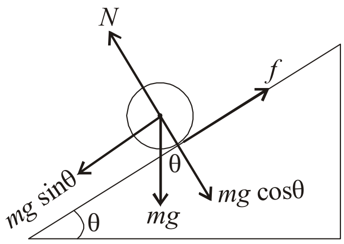
Also, $\vec{L}$ is perpendicular to $\vec{A}$

$\because \quad \vec{L} \perp \vec{A}$

$\therefore \quad \vec{L}.\vec{A} = 0$

Thus, it can be concluded that component of $\vec{L}$ along $\vec{A}$ is zero i.e., always constant.

30. **(c, d)** As shown in the figure, the component of weight $mg \sin\theta$ tends to slide the point of contact (of the cylinder with inclined plane) along its direction. The sliding friction acts in the opposite direction to oppose this relative motion. Because of frictional force the cylinder rolls.



Thus frictional force adds rotation but hinders translational motion.
Applying $F_{net} = ma$ along the direction of inclined plane,
we get $mg \sin\theta - f = ma_c$,
where a_c = acceleration of centre of mass of the cylinder
$\therefore \quad f = mg\sin\theta - ma_c \qquad \ldots(i)$

But $a_c = \dfrac{g\sin\theta}{1+\dfrac{I_c}{mR^2}} = \dfrac{g\sin\theta}{1+\dfrac{mR^2/2}{mR^2}} = \dfrac{2}{3}g\sin\theta$ ... (ii)

From (i) and (ii), $f = \dfrac{mg\sin\theta}{3}$

If θ is reduced, frictional force is reduced.

31. **(a, c)** $\vec{\omega}$ is perpendicular to the plane of circular motion and $\vec{v}$ lies in this plane.
$\vec{\alpha}$ is perpendicular to the plane of circular motion and $\vec{a}_c$ also lies in this plane
$\vec{v}$ is tangential and $\vec{a}_c$ is radial hence perpendicular.

32. **(a)** $\sum \vec{F}_{ext} = \dfrac{d\,\vec{p}_{system}}{dt}$

Given $\sum \vec{F}_{ext} = 0 \;\Rightarrow\; \vec{p}_{system} = \text{Constant}$

Due to internal forces acting in the system, the kinetic and potential energy may change with time.
Also zero external force may create a torque if the line of action of forces are along different direction. Thus the torque will change the angular momentum of the system.

E MATRIX-MATCH TYPE

1. **A-s; B-r; C-p; D-q**

$a_n = \dfrac{v^2}{16} = 9t^2 \Rightarrow v = 12\,t$

$\dfrac{dv}{dt} = 12$

Tangential force $m.\dfrac{dv}{dt} = \dfrac{3}{2}\sqrt{16} = 6\text{N}$

Total force $= \sqrt{6^2 + \left(\dfrac{mv^2}{R}\right)^2} = \sqrt{6^2 + \left(\dfrac{9}{2}\right)^2} = 7.5\,\text{N}.$

Power $= F_T.v = 6 \times 3\sqrt{16} = 72$ watt.

Average power $= \dfrac{72}{2} = 36$ watt.

2. **A-s; B-r; C-q; D-p**

$f = \dfrac{mg\sin\theta}{\left(1+\dfrac{mr^2}{I}\right)}$

For ring $I = mr^2$

$f = \dfrac{mg\sin\theta}{2}$

For hollow sphere $I = \dfrac{2}{3}mr^2$

$f = \dfrac{mg\sin\theta}{5/2} = \dfrac{mg\sin\theta}{2.5}$

For solid cylinder, $f = \dfrac{mg\sin\theta}{3}$;

For solid sphere, $f = \dfrac{mg\sin\theta}{3.5}$

3. **A-r, s; B-p, q; C-s; D-p, q, r, s**
4. **A-q; B-p; C-s; D-r**
5. **A-p, q, r; B-p, q, r; C-p, q, r; D-s**
6. **A-q, r, s; B-q, r, s; C-q, r, s; D-p, q**
7. **A-s; B-p, s; C-q; D-r**

(A) Constant magnitude $\Rightarrow \dfrac{mv^2}{R}$ = net force

(B) $\vec{N} + \vec{W} + \vec{f} \neq 0, \vec{N} \neq 0$

(C) Total reaction force = $\vec{N} + \vec{f}$ (D)
When motion is along vertical

$N = \dfrac{mv^2}{R}, \;\; \vec{f} + \vec{W} = 0$

8. **A- p, r; B-q, r; C-p, r, s; D-p, r, s**

Assume friction to be absent and horizontal force F is applied at a distance x above centre

$\therefore \quad a = \dfrac{F}{m}$ .... (1)

and $Fx = \dfrac{mR^2}{2}\alpha$

or $R\alpha = \dfrac{2Fx}{mR}$ .... (2)

If $a = R\alpha$ then from eq. (1) and (2) $x = \dfrac{R}{2}$

The friction force will be zero and $a = \dfrac{F}{m}$

If $a > R\alpha$ or $x < \dfrac{R}{2}$, friction force is towards left and

$a \neq \dfrac{F}{m}$

If $a < R\alpha$ or $x > \dfrac{R}{2}$, friction force is towards right and

$a \neq \dfrac{F}{m}$

9. **A-p, s; B-q, s; C-r, s; D-s**

(A) $u < \sqrt{2g\ell}$, Hence string will never get horizontal

$\therefore$ (p) and (s) are true

(B) $\sqrt{2g\ell} < u < \sqrt{5g\ell}$, At one point tension will be zero.

$\therefore$ (q) and (s) are true

(C) $T_{\max} = 3mg$. Hence string will just become horizontal

$\therefore$ (r) and (s) are true

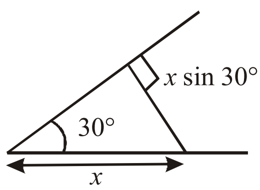
(D) $T_{\max} > 6mg$

$\therefore$ The object will undergo complete vertical circular motion.

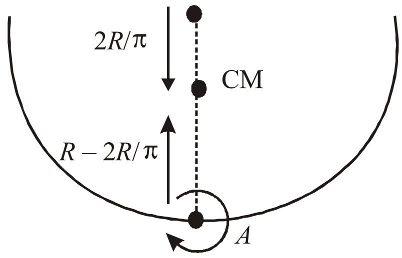
10. **A-q; B-p; C-s; D-r**

(A) $I = \int dm\left(\dfrac{x}{2}\right)^2$

$$= I = \int \frac{dm}{4}x^2 = \frac{m}{4\ell}\int x^2 dx = \frac{m}{4\ell}\left(\frac{x^3}{3}\right)_0^\ell = \frac{m\ell^2}{12}$$



(B) $MR^2 = I_{cm} + M\left(\dfrac{2R}{\pi}\right)^2$



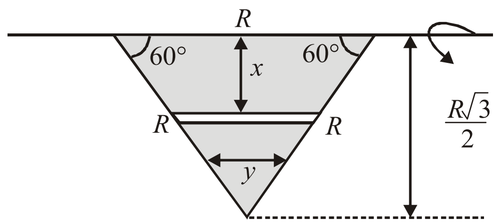
$$I_A = I_{cm} + M\left(R - \frac{2R}{\pi}\right)^2$$

$$I_A - MR^2 = MR^2\left(\left(1 - \frac{2}{\pi}\right)^2 - \left(\frac{2}{\pi}\right)^2\right)$$

$$= MR^2\left[1 - \frac{4}{\pi}\right]$$

or $I_A = MR^2\left[2 - \dfrac{4}{22/7}\right] = \dfrac{8MR^2}{11}$

(C) $\dfrac{\left(\dfrac{R\sqrt{3}}{2} - x\right)}{\dfrac{R\sqrt{3}}{2}} = \dfrac{y}{R}$



$$I = \int (dm)\, x^2 = \int \sigma\,(dx)\, yx^2 = \sigma\int\left(R - \frac{2x}{\sqrt{3}}\right)x^2 dx$$

$$= \frac{M}{\left(\dfrac{\sqrt{3}R^2}{4}\right)}\left[R\frac{x^3}{3} - \frac{2}{\sqrt{3}}\frac{x^4}{4}\right]_0^{\frac{R\sqrt{3}}{2}} = \frac{MR^2}{8}$$

(D) $\dfrac{1}{2}M\,(2R)^2 - \left[\dfrac{1}{2}\dfrac{M}{4}R^2 + \dfrac{M}{4}R^2\right]$

$$2MR^2 - \frac{3}{8}MR^2 = \frac{13}{8}MR^2$$

11. **A-p, q; B-r, s; C-p, r; D-r, s**

(A) $F_{\text{ext}} = M_{\text{accln}}$

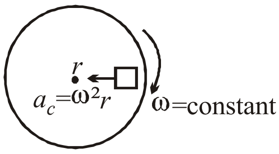
$F_{\text{ext}} = 0 \Rightarrow a_{\text{cm}} = 0$

(C) V_{cm} must be constant and may not be zero.

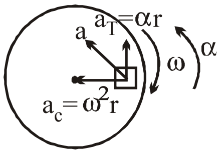
(D) Nothing can be said for sure

12. A-r; B-s; C-q; D-p

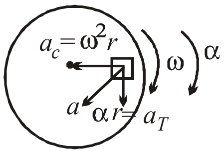
Case - A :



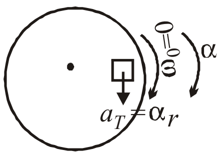
Case B :



Case C :



Case D :



13. A-p; B-r, s; C-q; D-r

$\vec{v} \quad 3\hat{i}\ , a_t = 0,\ L \quad mvr_{\perp} \quad (mr^2)\omega$;

$\omega \quad \dfrac{12}{\sqrt{9t^2 \quad 16}}$

distance = $|\vec{r}| \quad \sqrt{(3t)^2 \quad (4)^2}$;

$\dfrac{d}{dt}$(distance) $\left(\dfrac{3t}{\sqrt{(3t)^2 \quad (4)^2}}\right)$

this is increasing function with time (t).

14. A-q, r; B-p, s; C-p, s; D-p, s

Centre of mass lies where concentration of mass is more.

(A) $y > 0, x < 0$ (B) $y < 0, x = 0$

(C) $x = 0, y < 0$ (D) $x = 0, y < 0$

15. A-q, s; B-p; C-p, q, r, s; D-q, s

$a \quad \dfrac{g\sin\theta}{1 \quad \dfrac{k^2}{r^2}}$; $\dfrac{k^2}{r^2} \uparrow a \downarrow T \uparrow v \downarrow k^2/r^2$

Solid sphere	2/5
Hollow cylinder	1
Hollow sphere	2/3
Ring	1

16. A-s; B-s; C-p; D-p

(B) $\tau \quad fR \quad \dfrac{mg\sin\theta}{\left(\dfrac{mR^2}{I} \quad 1\right)}$

If $\dfrac{mR^2}{I}$ is minimum t will be maximum.

$\Rightarrow$ I is $mR^2 \Rightarrow \tau$ is maximum for ring.

(C) $v = \sqrt{2\dfrac{g\sin\theta}{\left(1 \quad \dfrac{I}{mR^2}\right)}s}$; v is maximum when $\dfrac{I}{mR^2}$ minimum. $\Rightarrow v$ is minimum for solid sphere

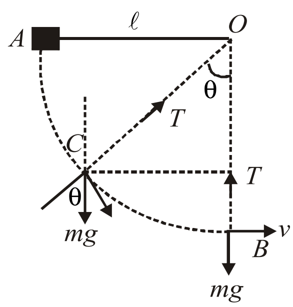
(D) $t \quad \sqrt{\dfrac{2s}{g\sin\theta}\left(1 \quad \dfrac{I}{mR^2}\right)}$; For $t_{\min}$, $\left(\dfrac{I}{mr^2}\right)$ should be min.

17. A-r; B-q; C-s

At 'A ' : v = 0, no centripetal acceleration. So acceleration is downward (Due to mg)

At B : T and mg both are vertical so acceleration is vertically upward (centripetal acceleration)

At C : $T - \text{mg}\cos\theta = \dfrac{\text{mv}^2}{\ell}$ ...... (1)



$\text{mg}\,\ell\cos\theta = \dfrac{1}{2}\text{mv}^2$ .... (2)

From (1) and (2),

$T - \text{mg}\cos\theta = 2\text{mg}\cos\theta$

$T = 3\text{mg}\cos\theta$

If $T\cos\theta = mg$ then vertical component of acceleration will become zero.

$(3mg\cos\theta)\cos\theta = mg$

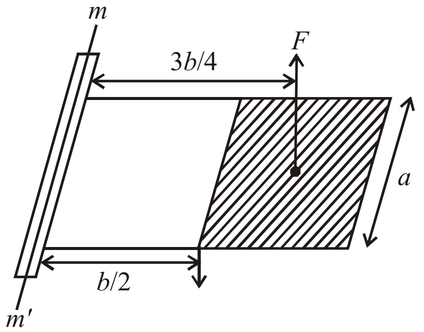
$\therefore\ \cos\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \tan\theta \quad \sqrt{2}$

So, at $\theta = \tan^{-1}(\sqrt{2})$ acceleration has only horizontal component.

F NUMERIC/INTEGER ANSWER TYPE

1. **1**

Since the plate is held horizontal therefore net torque acting on the plate is zero.



$$\Rightarrow \quad Mg \times \frac{b}{2} = F \times \frac{3b}{4} \qquad \text{... (i)}$$

$$F = n\frac{dp}{dt}(\text{Area}) = n \times (2mv) \times a \times \frac{b}{2} \qquad \text{... (ii)}$$

From (i) and (ii)

$$Mg \times \frac{b}{2} = n \times (2mv) \times a \times \frac{b}{2} \times \frac{3b}{4}$$

$$\Rightarrow \quad 3 \times 10 = 100 \times 2 \times 0.01 \times v \times 1 \times \frac{3 \times 2}{4}$$

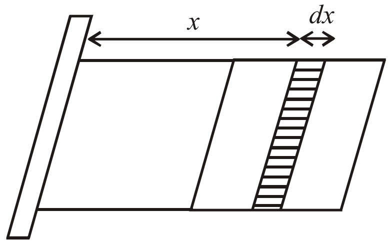
$$\Rightarrow \quad v = 10 \text{ m/s} \quad \Rightarrow \quad 0.1v \quad 1 \text{ m/s}$$

Alternatively :

Torque due to weight of plate $\tau_1 = Mg \times \frac{b}{2}$

torque due to small element shown dotted in the figure

$$= x \times dF = x\left[n\frac{dp}{dt}a\,dx\right]$$



$\therefore$ Total torque

$$\tau_2 = \int_{b/2}^{b} n(2mv)\,ax\,dx = 2mnv\frac{a}{2}\left[b^2 - \frac{b^2}{4}\right]$$

$$= 2nmv\,\frac{a}{2} \times \frac{3b^2}{4}$$

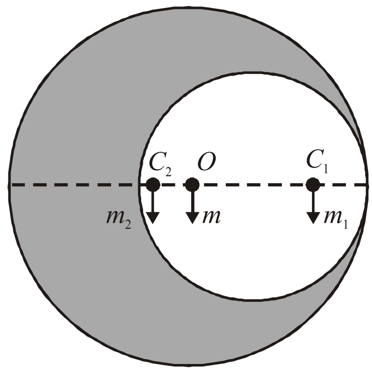
As $\tau_1 = \tau_2$

$$Mg\frac{b}{2} \quad 2nmv\frac{a}{2}\frac{3b^2}{4} \Rightarrow Mg = 2nm\frac{a}{2} \times \frac{3b}{2} \times v$$

$$\Rightarrow \quad v = \frac{4Mg}{2nma \times 3b} \quad \frac{4 \times 3 \times 10}{2 \times 100 \times 0.01 \times 1 \times 3 \times 2} = 10 \text{ m/s}$$

$\therefore \quad 0.1v \quad 1$ m/s

2. **9**



Area of whole plate = $\pi\,(56/2)^2 = 784\,\pi$ sq. cm.
Area of cutout portion = $\pi\,(42/2)^2 = 441\,\pi$ sq. cm. ;
Area of remaining portion = $784\pi - 441\pi = 343\,\pi$ cm^2;
As mass area.

$$\therefore \quad \frac{\text{mass of cutout portion}}{\text{mass of remaining portion}} \quad \frac{m_1}{m_2} \quad \frac{441\pi}{343\pi} \quad \frac{9}{7}$$

Let C_2 be centre of mass of remaining portion and C_1 be centre of mass of cutout portion.
O is centre of mass of the whole disc.;
$OC_1 = r_1 = 28 - 21 = 7$ cm.
$OC_2 = r_2 = ?$;
Equating moments of masses about O,

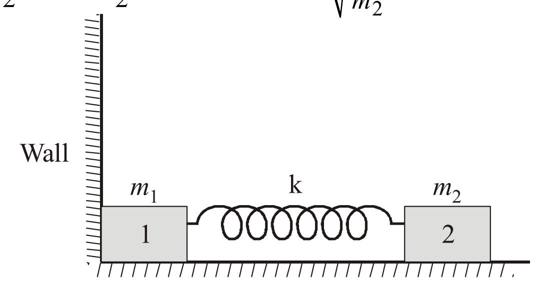
we get $m_2 \times r_2 = m_1 \times r_1 \;\Rightarrow\; r_2 = \frac{m_1}{m_2} \times r_1 = \frac{9}{7} \times 7 \quad 9$

$\therefore$ Centre of mass of remaining portion is at 9 cm to the left of centre of disc.

3. **0.02**

When block m_1 breaks off the wall, length of spring is unstretched. Therefore, KE of block m_2 = P.E. of compression

$$\frac{1}{2}m_2v_2^2 \quad \frac{1}{2}k\,x^2\;;\;\text{ or }\; v_2 \quad x\sqrt{\frac{k}{m_2}}\,.$$



Co-ordinates of centre of mass are given by

$$x \quad \frac{m_1x_1 \quad m_2x_2}{m_1 \quad m_2}$$

$$\frac{dx}{dt} \quad \frac{m_1}{m_1 \quad m_2}\left(\frac{dx_1}{dt}\right) \quad \frac{m_2}{m_1 \quad m_2}\left(\frac{dx_2}{dt}\right)$$

To start with, $x_1 = 0$, $\qquad \therefore \frac{dx_1}{dt} \quad 0$

$$\frac{dx}{dt} \quad \frac{m_2}{m_1 \quad m_2}\left(\frac{dx_2}{dt}\right) \quad \frac{m_2 v_2}{m_1 \quad m_2} \quad \frac{m_2}{m_1 \quad m_2} x\sqrt{\frac{k}{m_2}}$$

$$x\sqrt{km_2}/(m_1 \quad m_2) \quad \frac{(0.1\sqrt{0.2\times 2})}{(1 \quad 2)} = \mathbf{0.02\ m/s.}$$

4. 0.2

Cue exerts a force on sphere, so there will be change in momentum and change in angular momentum. Let the time of collision be Δt and force exerted by cue during the collision is F.

$\therefore F\Delta t = \Delta p$ or $F\Delta t = MV_{cm}$ ....(1)

[here impulse by frictional force is supposed to be zero since $f << F$]

Now take linear momentum about horizontal surface so that change in linear momentum by frictional force becomes zero.

$\therefore L_i = 0, L_f = I\omega + MV_{cm}R$

Since pure rolling is taking place, $\omega \quad \frac{V_{cm}}{R}$ ....(2)

$\therefore L_f = 2/5MR^2\omega + MV_{cm}R \quad \frac{2}{5}MV_{cm}R \quad MV_{cm}R$

$$L_f \quad \frac{7}{5}MV_{cm}R$$

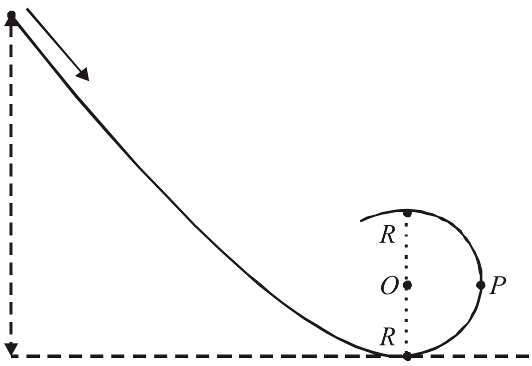
$(F\Delta t) \times (h') = L_f - L_i \Rightarrow (F\Delta t)h = 7/5\ MV_{cm}R$ ...(3)

h = vertical height from horizontal surface

From (1) & (3), $h' \quad \frac{7}{5}\frac{MV_{cm}R}{MV_{cm}} \quad \frac{7}{5}R \quad R \quad \frac{2}{5}R$

Hence $h = 2/5\ R = \frac{2}{5}\times 0.5 =$ **0.2 cm** above centre of sphere.

5. 10



Suppose m is the mass of the ball of radius r.

On reaching P, the net height through which the ball descends is $8R - R = 7R$, (from the fig. shown)

$\therefore$ Decrease in P.E. of ball $= mg \times 7R$;

This appears as total KE of ball at P.

Thus $mg \times 7R$ = KE of translation + KE of rotation

$$\frac{1}{2}mv^2 \quad \frac{1}{2}I\omega^2 \quad \frac{1}{2}mv^2 \quad \frac{1}{2}\times\left(\frac{2}{5}mr^2\right)\omega^2 \quad \frac{7}{10}mv^2$$

$\therefore v^2 = 10\,gR$ (Where $v = r\omega$ & r is radius of solid ball)

The horizontal force acting on the ball,

F_h = centripetal force towards O

$$\frac{mv^2}{R} \quad \frac{m(10gR)}{R} \quad 10\,mg = (10 \times 0.1 \times 10)\,\text{N} = 10\,\text{N}$$

6. 2

Applying law of conservation of energy at point D and point A.

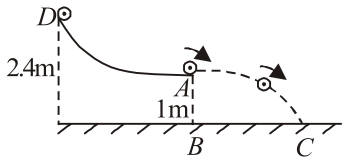
P.E. at D = P.E. at A + $(\text{K.E.})_T$ + $(\text{K.E.})_R$

where $(\text{K.E.})_T$ = Translational K.E.

$(\text{K.E.})_R$ = Rotational K.E.

$$mg(2.4) = mg(1) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Since the case is of rolling without slipping.



$\therefore \quad v = r\omega$

$\therefore \quad \omega = \frac{v}{r}$ where r is the radius of the sphere

Also, $\quad I = \frac{2}{5}mr^2$

$\Rightarrow \quad v = 4.43$ m/s

After point A, the body takes a parabolic path.

The vertical motion parameters of parabolic motion will be

$u_y = 0 \qquad s_y = 1\text{m}$

$a_y = 9.8\ \text{m/s}^2 \qquad t_y = ?$

$$s = ut \quad \frac{1}{2}at^2$$

$$1 = 4.9\,t_y^2$$

$$t_y = \frac{1}{\sqrt{4.9}} \quad 0.45\,\text{sec}$$

Applying this time in horizontal motion of parabolic path,

$BC = 4.43 \times 0.45 =$ **2 m**