1 MARK

1. If the sequence $\{a_n\}$ is defined by
$a_1 = 2$, $a_{n+1} = a_n + 2n$ $(n \geq 1)$, then a_{100} equals
(1) 9900 (2) 9902
(3) 9904 (4) 10100

2. The sum of all integers between 50 and 350 which end in 1 is -
(1) 5880 (2) 5539 (3) 5208 (4) 4877

3. The number of terms in an A.P. (Arithmetic Progression) is even. The sums of the odd and even- numbered terms are 24 and 30 respectively. If the last terms exceeds the first by 10.5 the number of terms in the A.P. is -
(1) 20 (2) 18 (3) 12 (4) 8

4. Cars A and B travel the same distance. Car A travels half that distance at u miles per hour and half at v miles per hour. The average speed of Car A is x miles per hour and that of Car B is y miles per hour : Then we always have -
(1) $x \leq y$ (2) $x \geq y$ (3) $x = y$ (4) $x < y$

5. If a,b and c are in geometric progression (G.P.) with $1 < a < b < c$ and $n > 1$ is an integer then $\log_a n . \log_b n . \log_c n$ form a sequence -
(1) which is a G.P.
(2) which is an arithmetic progression (A.P.)
(3) in which the reciprocals of the terms form an A.P.
(4) in which the second and terms are the n^{th} powers of the first and second respectively

6. In a geometric series of positive terms the difference between the fifth and fourth terms is 576, and the difference between the second and first terms is 9. What is the sum of the first five terms of the series ?
(1) 1061 (2) 1023 (3) 1024 (4) 768

7. The sum of the first eighty positive odd integers substracted from the sum of the first eighty positive even integers is-
(1) 0 (2) 20 (3) 40 (4) 80

8. Let $a_1, a_2, ...$ and $b_1, b_2.....$ be arithmetic progressions such that $a_1 = 25$, $b_1 = 75$ and $a_{100} + b_{100} = 100$. Find the sum of the first one hundred terms of the progression $a_1 + b_1$, $a_2 + b_2......$
(1) 0 (2) 100 (3) 10,000 (4) 505,00

9. In the sequence of numbers 1,3,2,..... each term after the first two is equal to the term precending it minus the term preceding that. The sum of the first one hundred terms of the sequence is -
(1) 5 (2) 4 (3) 2 (4) 1

10. If the first term of an infinite geometric series is a positive integer, the common ratio is the reciprocal of a positive integer, and the sum of the series is 3, then the sum of the first two terms of the series is -
(1) 1/3 (2) 2/3 (3) 8/3 (4) 2

11. Let a geometric progression with n terms has first term one common ratio r and sum s, where r and s are not zero. The sum of the geometric progression formed by replacing each term of the original progression by its reciprocal is -
(1) $\frac{1}{s}$ (2) $\frac{1}{r^n s}$ (3) $\frac{s}{r^{n-1}}$ (4) $\frac{r^n}{s}$

12. If a_1, a_2, a_3 .... is a sequence of positive numbers such that $a_{n+2} = a_n a_{n+1}$ for all positive integers n then the sequence a_1, a_2, a_3 ...... is a geometric progression
(1) for all positive values of a_1 and a_2
(2) if and only if $a_1 = a_2$
(3) if and only if $a_1 = 13$
(4) if and only if $a_1 = a_2 = 1$

13. If $x \neq y$ and the sequences x, a_1, a_2, y and x, b_1, b_2, b_3, y each are in arithmetic progression, then $(a_2 - a_1)/(b_2 - b_1)$ equals -
(1) $\frac{2}{3}$ (2) $\frac{3}{4}$ (3) 1 (4) $\frac{4}{3}$

14. If the distinct non-zero numbers $x(y - z)$, $y(z-x)$, $z(x-y)$ form a geometric progression with common ratio r, then r satisfies the equation -
(1) $r^2 + r + 1 = 0$ (2) $r^2 - r + 1 = 0$
(3) $r^4 + r^2 - 1 = 0$ (4) $(r + 1)^4 + r = 0$

15. In a certain sequence of numbers, the first number is 1 and for all $n \geq 2$, the product of the first n numbers in the sequence is n^2. The sum of the third and the fifth numbers in the sequence is :-

(1) 25/9 (2) 31/15

(3) 61/16 (4) 576/225

16. If the sum of the first 10 terms and the sum of the first 100 terms of a given arithmetic progression are 100 and 10, respectively, then the sum of the first 110 terms is :-

(1) 90 (2) –90 (3) 110 (4) –100

2 MARKS

1. The three sides of a right triangle have integral lengths which form an arithmetic progression. One of the sides could have length

(1) 22 (2) 58

(3) 81 (4) 91

2. The number $\underbrace{11....11}_{1997}\ \underbrace{22....22}_{1998}\ 5$ equals

(1) $\underbrace{33....33}_{1997}\ 5^2$ (2) $\underbrace{33....33}_{1997}\ 5^2$

(3) $\underbrace{33....33}_{1997}\ 5^2$ (4) $\underbrace{33....33}_{1997}\ 5^2$

3. Five distinct 2-digit numbers are in a geometric progression. Find the middle term

(1) 24 (2) 36 (3) 45 (4) 54

4. Suppose x is a positive real number such that {x}. [x] and x are in the geometric progression. Find the least positive integer n such that $x^n > 100$. (Here [x] denotes the integer part of x and $\{x\} = x - [x]$)

(1) 9 (2) 8 (3) 10 (4) 11

5. The first term of a sequence is 2014. Each succeeding term is the sum of the cubes of the digits of the previous term. What is the 2014th term of the sequence ?

(1) 370 (2) 430 (3) 240 (4) 570

6. A sequence of positive $(a_1, a_2, ... a_n)$ is called good if $a_i = a_1 + a_2 + ... + a_{i-1}$ for all $2 \leq i \leq n$. What is the maximum possible value of n for a good sequence such that $a_n = 9216$?

(1) 12 (2) 14 (3) 16 (4) 10

7. What is the maximum possible value of k for which 2013 can be written as sum of k non consecutive positive integers ?

(1) 54 (2) 61 (3) 23 (4) 73

8. If a, b & c are positive real numbers such that a + b + c = 1 then minimum value of

$$\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$$ is

(1) 6 (2) 8 (3) 27 (4) 9

9. If $x^5 - x^3 + x = 5$ then minimum value of x^6 equals

(1) 9 (2) 11 (3) 13 (4) 15

10. Find the number of triplets (a, b, c) satisfying $a^4 + b^4 + c^4 = abc(a + b + c)$ where $a, b, c \in R$ and a is a prime factor of 374

(1) 3 (2) 2 (3) 4 (4) 5

11. Let $a_1, a_2, a_3 .... a_n$ are n distinct odd natural numbers not divisible by any prime greater than 5. If $\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+.....+\frac{1}{a_n}<\lambda$ then least positive integral value of λ is

(1) 3 (2) 2 (3) 4 (4) 1

1 MARK

1. Ans. (2)

Given : $a_1 = 2$ and

$a_{n+1} = a_n + 2n$

put $n = 1 \Rightarrow a_2 = a_1 + 2(1)$

put $n = 2 \Rightarrow a_3 = a_2 + 2(2)$

$n = 3 \Rightarrow a_4 = a_3 + 2(3)$

$\vdots \qquad \vdots$

$n = 98 \Rightarrow a_{99} = a_{98} + 2(98)$

$n = 99 \Rightarrow a_{100} = a_{99} + 2(99)$

On addition $\Rightarrow a_{100} = a_1 + 2(1+2+3+.....99)$

$$a_{100} = 2 + \frac{2(99)(99+1)}{2}$$

$a_{100} = 2 + 9900$

$a_{100} = 9902$

2. Ans. (1)

Integers between 50 and 60 which ends with 1:

$S = 51+61+71+81+.......+341$

common difference = 10

Let number of terms be n.

$\Rightarrow 341 = 51 + (n-1)10$

$(\because T_n = a+(n-1)d)$

$\Rightarrow 290 = (n-1)10 \Rightarrow n-1 = 29 \Rightarrow n = 30$

Sum = $\frac{n}{2}$(first term + last term)

$= \frac{30}{2}(51+341) = 15 \times (392) = 5880$

3. Ans. (4)

Let the number of terms be 2n and common difference be d

sum of odd numbered term = 24

$a_1 + a_3 + a_5 + ...+a_{2n-1} = 24$ ....(1)

sum of even number terms = 30

$a_2 + a_4 + a_5 + ...a_{2n} = 30$ ....(2)

(2) – (1)

$(a_2-a_1) + (a_4-a_3) + (a_6-a_5)+.........+(a_{2n}-a_{2n-1})=6$

$d + d + d + ...+d = 6$

$\Rightarrow nd = 6$

Also given : $a_{2n} = a_1 + 10.5$

$\Rightarrow a_1 + (2n-1)d = a_1 + 10.5$

$2nd - d = 10.5$

$\Rightarrow 2(6) - d = 10.5$

$\Rightarrow d = 12 - 10.5 = 1.5$

$\Rightarrow \boxed{d = \frac{3}{2}}$

$\because nd = 6 \Rightarrow n\left(\frac{3}{2}\right) = 6$

$\Rightarrow n = 4$

$\Rightarrow$ Number of terms = 2n = 8

4. Ans. (1)

$$S = \frac{1}{10} + \frac{2}{10^2} + \frac{3}{10^3} + ... + \frac{n}{10^n} + ...$$

$$\frac{S}{10} = \frac{1}{10^2} + \frac{2}{10^3} + ... \frac{n-1}{10^n} + ...$$

Subtracting :

$$\frac{9s}{10} = \frac{1}{10} + \frac{1}{10^2} + \frac{1}{10^3} + ...\infty$$

$$\frac{9s}{10} = \frac{\frac{1}{10}}{1 - \frac{1}{10}} = \frac{1}{9}$$

$\Rightarrow s = \frac{10}{81}$

5. Ans. (3)

$\because$ a, b, c are in G.P.

$b = ar, \; c = ar^2$

$$\Rightarrow \log_a n, \; \log_b n, \log_c n = \log_a n, \log_{ar} n, \log_{ar^2} n$$

$$= \frac{1}{\log_n a}, \frac{1}{\log_n ar}, \frac{1}{\log_n ar^2}$$

$$= \frac{1}{\log_n a}, \frac{1}{\log_n a + \log_n r}, \frac{1}{\log_n a + 2\log_n r}$$

The reciprocal of this sequence makes an A.P. with common difference $\log_n r$

6. Ans. (2)

Let the series be a, ar, ar^2, ...ar^{n-1}

Given : $T_5 - T_4 = 576$

$\Rightarrow$ $ar^4 - ar^3 = 576$ ....(1)

Also $T_2 - T_1 = 9$

$\Rightarrow$ $ar - a = 9$ ....(2)

(1) divided by (2) $\Rightarrow$ $\frac{ar^3(r-1)}{a(r-1)} = \frac{576}{9} = 64$

$\Rightarrow$ $r^3 = 64 \Rightarrow r = 4$

From (2) $\Rightarrow$ a(r–1) = 9 $\Rightarrow$ a = 3

Sum of first five terms = a + ar + ar^2 + ar^3 + ar^4

$= \frac{a(r^5-1)}{(r-1)} = \frac{3(4^5-1)}{4-1} = 1023$

7. Ans. (4)

$\underbrace{(2+4+6+8+...........)}_{80\text{ terms}} - \underbrace{(1+3+5+7+..........)}_{80\text{ terms}}$

= (2 – 1) + (4 – 3) + (6 – 5) +.........

= 1 + 1 + 1 + 1 +.........80 times

= 80

8. Ans. (3)

a_1, a_2, a_3........Let coommon difference = d_1

b_1, b_2, b_3........Let coommon difference = d_2

Given $a_{100} + b_{100} = 100$

$\Rightarrow$ $a_1 + 99d_1 + b_1 + 99d_2 = 100$

($\because$ $a_1 = 25$ & $b_1 = 75$)

$\Rightarrow$ $99(d_1 + d_2) = 0 \Rightarrow d_1 + d_2 = 0$

$\Rightarrow$ $a_1 + b_1 = a_2 + b_2 = a_3 + b_3 = ... = a_{100} + b_{100} = 100$

$\Rightarrow$ $(a_1 + b_1) + (a_2 + b_2) = +.........+ (a_{100} + b_{100})$

$= (100)(100) = 10000$

9. Ans. (1)

$T_1 = 1$

$T_2 = 3$

$T_3 = 2$

$T_4 = 2–3 = –1$

$T_5 = T_4 - T_3 = –3$

$T_6 = T_5 - T_4 = –2$

$T_7 = T_6 - T_5 = –2 –(–3) = 1$

$T_8 = 3$

$T_9 = 2$

$T_{10} = –1$

Observe that pattern repeats after six terms

$T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = 0$

$\Rightarrow$ Sum of first 6 × (16) = 96 terms will be zero.

($\because$ sum of every six consecutive terms is zero)

Hence sum of last from terms = 1 + 3 + 2 – 1 = 5

10. Ans. (3)

Sum of infinite series = $\frac{a}{1-r} = 3$

$\Rightarrow$ a = 3(1–r)

$\because$ $a \in I$ and $r = \frac{1}{I}$

$\Rightarrow$ r = $\frac{1}{3}$

& a = $3\left(1-\frac{1}{3}\right) = 2$

sum of first two terms

= a + ar

= $2+\frac{2}{3} = \frac{8}{3}$

11. Ans. (3)

Given first term = a = 1

S = a + ar + ar^2 + ... + ar^{n-1}

S = $\frac{a(r^n-1)}{r-1}$ ...(1)

Let S' = $\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+.......+\frac{1}{ar^{n-1}}$

S' = $\left(\frac{1}{a}\right)\frac{\left(1-\frac{1}{r^n}\right)}{\left(1-\frac{1}{r}\right)} = \frac{1}{a}\left(\frac{r^n-1}{r-1}\right)\frac{1}{r^{n-1}}$

From (1)

S' = $\frac{1}{a}\left(\frac{S}{a}\right)\frac{1}{r^{n-1}} = \frac{S}{a^2r^{n-1}}$

$\because$ a = 1

$\Rightarrow S' = \frac{S}{r^{n-1}}$

12. Ans. (4)

Let a_1, a_1r, a_1r^2, ... are in G.P.

$a_{n+2} = a_n \times a_{n+1}$

$\Rightarrow$ $a_1r^{n+1} = a_1 \cdot r^{n-1} \times a_1r^n$

$\Rightarrow$ $a_1r^{n-2} = 1$

It is possible only

When $a_1 = r = 1$

So $a_1 = a_2 = 1$

13. Ans. (4)

Let common difference of the series

x, a_1, a_2, y be d_1

$\Rightarrow y = x + (4-1)d_1 = x + 3d_1$

$$\Rightarrow \boxed{d_1 = \frac{y-x}{3}}$$

Let common difference of the series

x, b, b_2, b_3, y be d_2

$\Rightarrow y = x + (5-1)d_2$

$$\Rightarrow \boxed{d_2 = \frac{y-x}{4}}$$

$$\frac{a_2-a_1}{b_2-b_1} = \frac{d_1}{d_2} = \frac{\frac{y-x}{3}}{\frac{y-x}{4}} = \frac{4}{3}$$

14. Ans. (1)

Let $x(y-z) = a$ ...(1)

$y(z-x) = ar$ ...(2)

and $z(x-y) = ar^2$ ...(3)

Adding (1) + (2) + (3)

$\Rightarrow xy - xz + yz - yx + zx - zy = a + ar + ar^2$

$\Rightarrow a + ar + ar^2 = 0$

$\Rightarrow 1 + r + r^2 = 0$

15. Ans. (3)

$T_1.T_2.T_3............T_n = n^2$ ...(1)

$\Rightarrow T_1.T_2.T_3............T_{n-1} = (n-1)^2$ ...(2)

$$\frac{(1)}{(2)} \Rightarrow \frac{T_1.T_2.T_3......T_n}{T_1.T_2.T_3......T_{n-1}} = \frac{n^2}{(n-1)^2}$$

$$\Rightarrow T_n = \frac{n^2}{(n-1)^2}$$

put $n = 3 \Rightarrow T_3 = \frac{3^2}{(3-1)^2} = \frac{9}{4}$

put $n = 5 \Rightarrow T_5 = \frac{5^2}{(5-1)^2} = \frac{25}{16}$

$$T_3 + T_5 = \frac{9}{4} + \frac{25}{16} = \frac{36+25}{16} = \frac{61}{16}$$

16. Ans. (3)

Sum of first 10 terms = $S_{10} = 100$

$$\Rightarrow \frac{10}{2}(2a+9d) = 100$$

$\Rightarrow 2a + 9d = 20$ ...(1)

$\Rightarrow 2a = 20 - 9d$

and also given: $S_{100} = 10$

$$\Rightarrow \frac{100}{2}(2a+99d) = 10$$

$\Rightarrow 5(20 - 9d + 99d) = 1$

$\Rightarrow 5(20 + 90d) = 1$

$\Rightarrow 20 + 90d = 0.20$

$\Rightarrow 90d = 0.2 - 20 = -19.8$

$\Rightarrow d = -0.22$

$$S_{110} = \frac{110}{2}[2a+109d]$$

$= 55\ [20 - 9d + 109d]$

$= 55\ [20 + 100d]$

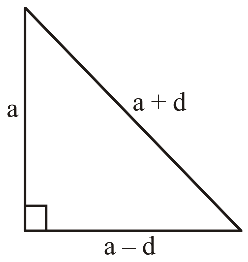
$S_{110} = 55(20-22) = 55(-2)$

$S_{100} = -110$

2 MARK

1. **Ans. (3)**

Let the length of the sides be a-d, a, a+d.
($\because$ sides are in A.P.)



$\Rightarrow$ $a^2 + (a–d)^2 = (a+d)^2$

$\Rightarrow$ $2a^2 + d^2 - 2ad = a^2 + d^2 + 2ad$

$\Rightarrow$ $a^2 = 4ad \Rightarrow a = 4d$

$\Rightarrow$ sides are 4d, 3d and 5d

$\Rightarrow$ sides are either a multiple of 3 or 4 or 5.

Hence option(3) is only possibility

2. **Ans. (1)**

$\underbrace{11.....11}_{1997}\ \underbrace{22.....22}_{1998}\ 5 = 5 + 2.10 + 2.10^2 + ... + 2.10^{1998} + 1.10^{1999} + 10^{2000} + ... + 10^{3995}$

$$= 5+2.10\frac{\left(10^{1998}-1\right)}{10-1}+\frac{10^{1999}\left(10^{1997}-1\right)}{10-1}$$

$$= 5+\frac{2.10^{1999}-20}{9}+\frac{10^{3996}-10^{1999}}{9}$$

$$= \frac{45+2.10^{1999}-10^{1999}-20+10^{3996}}{9}$$

$$= \frac{10^{3996}+10^{1999}+25}{9}$$

$$= \frac{\left(10^{1998}\right)^2+\left(5\right)^2+2.\left(10^{1998}\right).5}{9}$$

$$= \frac{\left(10^{1998}+5\right)^2}{9}$$

$$= \left(\frac{\underbrace{10000...............5}_{1998}}{3}\right)^2$$

$$= \frac{\left(333...........5\right)^2}{1997}$$

. **Ans. (2)**

Consider the terms as a, ar, ar^2, ar^3, ar^4, assuming all are ordered in ascending order. All are 2 digit numbers

$a \geq 10$ and $ar^4 \leq 99$

$\Rightarrow$ $r^4 \leq \frac{99}{10} \Rightarrow r \leq (9.9)^{\frac{1}{4}}$

$\Rightarrow$ $r < 2$

For all the numbers to be integers, the ideal combination would be the power of $\left(\frac{3}{2}\right)$.

$t_1 = a = 16$

$t_2 = ar = 16.\frac{3}{2} = 24$

$t_3 = ar^2 = 16 \times \left(\frac{3}{2}\right)^2 = 36$

$t_4 = ar^3 = 16 \times \left(\frac{3}{2}\right)^3 = 54$

$t_5 = ar^4 = 16.\left(\frac{3}{2}\right)^4 = 81$

Hence middle term = 36

4. **Ans. (3)**

$\because$ {x}, [x], x are in G.P.
Let these numbers be a, ar, ar^2
Also as {x} = x – [x]

$\therefore$ $a = ar^2 - ar$

$\Rightarrow$ $r^2 - r - 1 = 0$

$\Rightarrow$ $r = \frac{1\pm\sqrt{5}}{2}$, $\therefore$ $r = \frac{1+\sqrt{5}}{2}$

$\because$ ar = [x] = integer = K (lets say)

$\Rightarrow$ $a = \frac{K}{r} = \frac{2K}{\sqrt{5}+1} = \frac{K\left(\sqrt{5}-1\right)}{2}$

$\because$ a = {x} which lies in [0, 1)

$\therefore$ $0 < \frac{K\left(\sqrt{5}-1\right)}{2} < 1$

$\Rightarrow$ $0 < K < \frac{\sqrt{5}+1}{2}$

$\Rightarrow$ K = 1 (as K is an integer)

$x^n = (ar^2)^n = \left(\frac{\sqrt{5}+1}{2}\right)^n > 100$

$$\left(\because \frac{\sqrt{5}+1}{2} = \frac{2.23+1}{2} = 1.6\right)$$

$\therefore$ we require $(1.6)^n > 100$

$\Rightarrow$ $n \log_{10} 1.6 > \log_2 100$

$$\Rightarrow \quad n\left(\log\frac{16}{10}\right) > 2$$

$\Rightarrow$ $n(\log 16 - \log 10) > 2$

$\Rightarrow$ $n(4 \log 2 - 1) > 2$

$\Rightarrow$ $n(0.204) > 2$

$\Rightarrow$ $n_{min} = 10$

5. Ans. (1)

Second term of the sequence

$= 2^3 + 0^3 + 1^3 + 4^3 = 73$

third term $= 7^3 + 3^3 = 370$

fourth term $= 3^3 + 7^3 + 0^3 = 370$

.... and so on.

So the 2014th term of the sequence will be 370.

6. Ans. (1)

$a_2 = a_1$

$a_3 = a_1 + a_2 = 2a_1$

$a_4 = a_1 + a_2 + a_3 = 4a_1$

$a_5 = a_1 + a_2 + a_3 + a_4 = 8a_1$

$\Rightarrow$ $a_6 = 16a_1 = 2^4a$, $a_7 = 2^5a_1$, $a_8 = 2^6a$ .... and so on.

Given

$a_n = 9216 = 2^{10} \times 9$

$\Rightarrow$ maximum possible value of n is 12. Because a_{13} will have power of 2 more than 10.

7. Ans. (2)

$a + (a + 1) + (a + 2) + ...$K-terms $= 2013$

$$\frac{K}{2}\left(2a + (K-1)\right) = 2013$$

$\Rightarrow$ $K(2a + K - 1) = 4026$

$(2a + K - 1)K = 4026.1$

$= (61.11.3).2$

$= (61.11.2).3$

$= (11.3.2).61$

K can be 61 max.

8. Ans. (2)

Given $a + b + c = 1$

$\Rightarrow$ $a = 1 - b - c$

$\Rightarrow$ $a + 1 = 1 + 1 - b - c$

$\Rightarrow$ $a + 1 = (1 - b) + (1 - c)$ ...(1)

Applying A.M. $\geq$ G.M. m(1–b) and (1–c)

$$\Rightarrow \quad \frac{(1-b)+(1-c)}{2} \geq \sqrt{(1-b)(1-c)}$$

From (1)

$\Rightarrow$ $1 + a \geq 2\sqrt{(1-b)(1-c)}$ ....(2)

similarly $1 + b \geq 2\sqrt{(1-a)(1-c)}$ ....(3)

and $1 + c \geq 2\sqrt{(1-a)(1-b)}$ ....(4)

On multiplying

$(1 + a)(1 + b)(1 + c) \geq 8(1 - a)(1 - b)(1 - c)$

$$\Rightarrow \quad \frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)} \geq 8$$

9. Ans. ()

$x - x^3 + x^5 = 5$

G.P. with first term x and common ratio $-x^2$.

Applying formula for sum of G.P.

$$\frac{x\left(1-\left(-x^2\right)^3\right)}{1-\left(-x^2\right)} = 5$$

$\Rightarrow$ $x(1 + x^6) = 5(1 + x^2)$

$$\Rightarrow \quad 1 + x^6 = 5\left(x + \frac{1}{x}\right)$$

$$\left(\begin{array}{ll}\because \quad \text{if } x > 0, & x + \frac{1}{x} \geq 2 \\ \text{and} \quad \text{if } x < 0, & x + \frac{1}{x} \leq -2\end{array}\right)$$

$$\Rightarrow \quad 1 + x^6 = 5\left(x + \frac{1}{x}\right) \geq 5.2 \quad \text{or}$$

$$1 + x^6 = 5\left(x + \frac{1}{x}\right) \leq 5.(-2)$$

$\Rightarrow$ $1 + x^6 \geq 10$ or $1 + x^6 \leq -10$

10. Ans. (1)

$$\because \quad \frac{a^4+b^4+c^4}{3} \geq \left(\frac{a+b+c}{3}\right)^4$$

$$\frac{a^4+b^4+c^4}{3} \geq \left(\frac{a+b+c}{3}\right)\left(\frac{a+b+c}{3}\right)^3$$

$$\because \quad \frac{a+b+c}{3} \geq (abc)^{1/3}$$

$$\Rightarrow \quad \frac{a^4+b^4+c^4}{3} \geq \left(\frac{a+b+c}{3}\right)\left(\left(abc\right)^{1/3}\right)^3$$

$$\Rightarrow \frac{a^4+b^4+c^4}{3} \geq \left(\frac{a+b+c}{3}\right)abc$$

equality occurs when a = b = c.

374 = 17 × 11 × 2

∵ a is a prime factor of 374

⇒ a can be 2, 11 or 17

possible triplets = (2,2,2), (11,11,11) or (17,17,17)

⇒ Number of triplets = 3

11. Ans. (2)

$a_i = 5^{n_1}3^{n_2}$ (∵ a_i are odd natural numbers)

$$\Rightarrow \frac{1}{a_i} = \frac{1}{5^{n_1} \times 3^{n_2}} = 5^{-n_1}3^{-n_2}$$

$$\sum \frac{1}{a_i} = \sum_{n_2=0}^{\infty}\sum_{n_1=0}^{\infty} 5^{-n_1}.3^{-n_2}$$

$$= \sum_{n_2=0}^{\infty} 3^{-n_2} \sum_{n_1=0}^{\infty} 5^{-n_1}$$

$$= \frac{1}{1-\left(\frac{1}{3}\right)} . \frac{1}{1-\frac{1}{5}} = \frac{3}{2} . \frac{5}{4} = \frac{15}{8}$$

(∵ $a + ar + ar^2 \ldots\ldots\ldots\infty = \frac{a}{1-r}$)

$$\Rightarrow \sum \frac{1}{a_i} = \frac{15}{8} < 2 \Rightarrow \lambda = 2$$