CHAPTER › 08

Gravitation

KEY NOTES

- Each material object has a tendency to be attracted towards the earth. This tendency of an object is governed by few laws, namely Kepler's laws and gravitational law.

Kepler's Laws of Planetary Motion

There are three scientific laws describing the motion of planets around the sun as given below

- **Law of Orbits** All planets move in elliptical orbit with sun situated at one of the foci of the ellipse.
- **Law of Areas** The line that joins any planet to the sun sweeps equal areas in equal intervals of time. The closest point is called **perihelion** and the farthest point is called **aphelion**.

  The law of areas can be understood as a consequence of conservation of angular momentum, which is valid for any central force.
- A **central force** is such that the force on the planet is along the vector joining the sun and the planet.
- The area ΔA swept out by the planet of mass m in time interval Δt is given by

$$\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{v}\Delta \mathbf{t})$$

Hence, $$\frac{\Delta \mathbf{A}}{\Delta t} = \frac{1}{2}\frac{\mathbf{r} \times \mathbf{p}}{m} = \frac{\mathbf{L}}{2m} \qquad \left(\because \ \mathbf{v} = \frac{\mathbf{p}}{m}\right)$$

where, $\mathbf{v}$ is the velocity and $\mathbf{L}$ is the angular momentum equal to $(\mathbf{r} \times \mathbf{p})$.

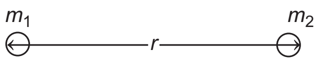
For a central force, which is directed along $\mathbf{r}$, L is a constant as the planet goes around. Hence, $\Delta \mathbf{A} / \Delta t$ is a constant.

- **Law of Periods** The square of the time period of revolution of a planet is proportional to the cube of semi-major axis of the ellipse traced out by the planet, i.e. $T^2 \propto a^3$

  where, $a =$ semi-major axis and $T =$ time period.

Universal Law of Gravitation

- It states that every body in the universe attract every other body with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them, i.e.



$$F \propto \frac{m_1 m_2}{r^2}$$

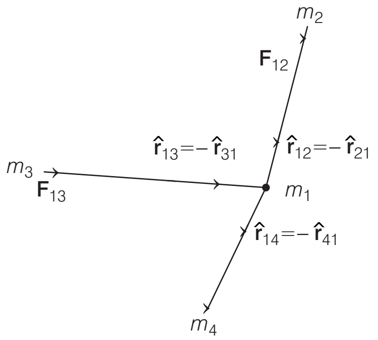
$$F = G\frac{m_1 m_2}{r^2}$$

or $$\mathbf{F} = -G\frac{m_1 m_2}{|\mathbf{r}|^3}\,\hat{\mathbf{r}}$$

where, G is the universal gravitational constant and its value is $6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$.

- The gravitational force is attractive in nature, i.e. the force $\mathbf{F}$ is along $-\mathbf{r}$.
- Gravitational force is a central as well as conservative force but the weakest force in nature.
- The universal law of gravitation refers to the point masses whereas we deal with the extended objects which have finite size.

If we have collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses as shown below



The total force on m_1 is

$$F_1 = \frac{Gm_2m_1}{r_{21}^2}\,\hat{\mathbf{r}}_{21} + \frac{Gm_3m_1}{r_{31}^2}\,\hat{\mathbf{r}}_{31} + \frac{Gm_4m_1}{r_{41}^2}\,\hat{\mathbf{r}}_{41}$$

- The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.
- The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.

Acceleration Due to Gravity

- The force of gravity acting on a body of unit mass placed near the surface of earth is known as acceleration due to gravity, which is denoted by g and given as $g = \frac{F}{m} = \frac{GM_E}{R_E^2}$.

  where, M_E and R_E are the mass and radius of the earth, respectively.
- **The value of g at height h from earth's surface is given by**

$$g(h) \approx g\left(1 - \frac{2h}{R_E}\right)$$

  Therefore, g decreases with height.
- **The value of g at depth d from earth's surface is given by**

$$g(d) = g\left(1 - \frac{d}{R_E}\right)$$

  Therefore, g decreases with depth and becomes zero at earth's centre.

Gravitational Potential

Gravitational potential at a point in the gravitational field is defined as the amount of work done in bringing a body of unit mass from infinity to that point without acceleration.

i.e. $$V = -\frac{W}{m}$$

Its SI unit is Jkg^{-1} and dimensional formula is $[M^0L^2T^{-2}]$.

Gravitational Potential Energy

- Gravitational potential energy of a body at a point is the amount of work done in bringing a given body from infinity to a point against the gravitational force.
- If m_1 and m_2 are two masses separated by infinite distance, then gravitational potential energy of the system when they are brought to a separation of r is given by

$$U = -\frac{Gm_1m_2}{r}$$

where, the negative sign indicates that the two bodies are attracting each other.

Escape Velocity

- The minimum speed required for an object to reach infinity (i.e. escape from the earth) is called escape velocity.

  It is given by

  Escape velocity $(v_e)_{\text{min}} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E}$

  Escape velocity at earth is 11.2 kms^{-1} and for the moon is 2.3 kms^{-1}, about five times smaller.

  This is the reason that moon has no atmosphere.

Earth Satellites

- Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the sun and hence Kepler's laws of planetary motion are equally applicable to them.
- Moon is the only natural satellite of the earth moving in a nearly circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.
- **Orbital velocity of a satellite** at a height h from surface of earth is given by $v_o = \sqrt{\frac{GM_E}{R_E + h}}$.

  When a satellite revolve near the earth's surface, i.e. $h = 0$, then its orbital velocity,

$$v_o = \sqrt{gR_E} \approx 7.92\ \text{km/h}$$

- **Time period of a satellite,**

$$T = \frac{2\pi(R_E + h)}{v_o} = \frac{2\pi(R_E + h)^{3/2}}{\sqrt{GM_E}}$$

  When a satellite revolve near the earth's surface, then

$$T_o = 2\pi\sqrt{\frac{R_E}{g}} \approx 85\ \text{min}$$

- **Kinetic energy of the satellite** in a circular orbit with speed v is $\text{KE} = \frac{GmM_E}{2(R_E + h)}$.

- **Potential energy** at distance $(R + h)$ from the centre of the earth is $PE = -\frac{GmM_E}{(R_E + h)}$.
- Total energy of a satellite = KE + PE $= \frac{GmM_E}{2(R_E + h)} - \frac{GmM_E}{(R_E + h)}$

$$= -\frac{GmM_E}{2(R_E + h)}.$$

- The total energy of an circularly orbiting satellite is thus negative, with the potential energy being negative but twice is magnitude of the positive kinetic energy.

Geo-Stationary and Polar Satellites

- Satellites in a circular orbit around the earth in the equilateral plane with $T = 24$ h are called **geo-stationary satellites**.

  For $T = 24$ h, the height of satellite above earth's surface is calculated from, $R_E + h = \left(\frac{T^2 GM_E}{4\pi^2}\right)^{1/3}$

  It (height h) comes out to be 35800 km, which is much larger than R_E.
- Low altitude ($h \approx 500$ to 800 km) satellites, which go around the poles of the earth in a north-south direction are called **polar satellites**, whereas the earth rotates around its axis in an east-west direction.

Weightlessness

- When a satellite, which does not produce its own gravity moves around the earth in a circular orbit under the action of gravity. The surface of satellite does not exert any force on the body, hence its apparent weight is zero.
- In other words, when a body is in free fall, it is said to be **weightless**, since there is no upward force on the body .

Mastering NCERT

MULTIPLE CHOICE QUESTIONS

TOPIC 1 ~ *Early Finding and Kepler's Laws*

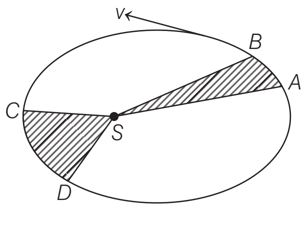
1 While going uphill more efforts are needed, it is due to the effect of

(a) attractive force of earth (b) repulsive force of earth
(c) magnetic force of earth (d) None of these

2 According to Kepler's law of planetary motion, path of planet around the sun is

(a) circular (b) elliptical
(c) parabolic (d) hyperbolic

3 A planet moves around the sun on an elliptical path as shown in the figure. If t_1 and t_2 are the time taken by the planet to cover equal area SAB and SCD respectively, then



(a) $t_1 > t_2$ (b) $t_1 < t_2$
(c) $t_1 = t_2$ (d) $t_1 = \sqrt{t_2}$

4 The law of areas can be understood as a consequence of

(a) conservation of angular momentum
(b) conservation of energy
(c) conservation of linear momentum
(d) Both (a) and (b)

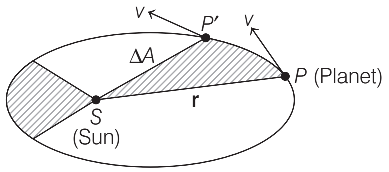
5 A central force is that force which acts along

(a) the vector perpendicular to sun
(b) the vector joining the sun and the planet
(c) the vector tangential to the orbit
(d) None of the above

6 The law of areas can be interpreted as

(a) $\frac{\Delta A}{\Delta t} = \frac{L}{2m}$ (b) $\frac{\Delta A}{\Delta t} = \frac{L}{m}$
(c) $\frac{\Delta A}{\Delta t} = 2(r \times p)$ (d) $\frac{\Delta A}{\Delta t} = \frac{2L}{m}$

7 Let a planet is moving around the sun in an elliptical orbit as shown below.



The area swept out by the planet of mass m in time interval Δt is given by

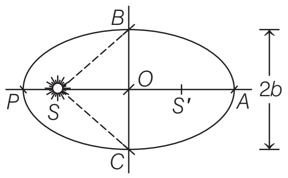
(a) $\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{v}\Delta t)$ (b) $\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{v})$

(c) $\Delta A = \frac{1}{2}(\mathbf{r} \times \mathbf{p})$ (d) All of these

8 In planetary motion, the areal velocity of position vector of a planet depends on angular velocity ω and the distance of the planet from sun r. So, the correct relation for areal velocity is

(a) $\frac{dA}{dt} \propto \omega r$ (b) $\frac{dA}{dt} \propto \omega^2 r$ (c) $\frac{dA}{dt} \propto \omega r^2$ (d) $\frac{dA}{dt} \propto \sqrt{\omega r}$

9 Let the speed of the planet at the perihelion P in above figure be v_P and the sun planet distance SP be r_P. The corresponding quantity at the aphelion A is (r_A, v_A). The mass of the planet is m_p.

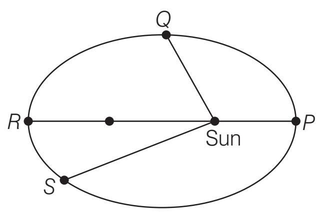


The magnitude of the angular momentum at P is

(a) $L_P = \frac{m_P r_P}{v_P}$ (b) $L_P = \frac{m_P v_P}{r_P}$

(c) $L_P = m_P r_P v_P$ (d) $L_P = m_P v_P$

10 A planet is moving around the sun in an elliptical path as shown in the figure. The linear speed of the planet is minimum at



(a) P (b) S

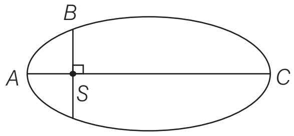
(c) Q (d) R

11 The maximum and minimum velocities of a planet are 3×10^4 ms^{-1} and 1×10^3 ms^{-1}, respectively. The minimum distance of the planet from the sun, if maximum distance is 4×10^4 km, is

(a) 4×10^3 km (b) 3×10^3 km

(c) $(4/3)\times 10^3$ km (d) 1×10^3 km

12 The kinetic energies of a planet in an elliptical orbit about the sun, at positions A, B and C are K_A, K_B and K_C, respectively. AC is the major axis and SB is perpendicular to AC at the position of the sun S as shown in the figure, then ***NEET 2018***



(a) $K_B < K_A < K_C$ (b) $K_A > K_B > K_C$

(c) $K_A < K_B < K_C$ (d) $K_B > K_A > K_C$

13 The time period of revolution T of planet in years (yr) is 84. The semi-major axis a in units of 10^{10} m is 287. The quotient (T^2/a^3) in units of 10^{-34} yr^{2}m^{-3} is approximately

(a) 3 units (b) 4 units (c) 2 units (d) 1 unit

14 According to Kepler's law of planetary motion, if T represents time period and r is orbital radius, then for two planets these are related as

(a) $\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{r_1}{r_2}\right)^2$ (b) $\left(\frac{T_1}{T_2}\right)^{3/2} = \frac{r_1}{r_2}$

(c) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$ (d) $\left(\frac{T_1}{T_2}\right) = \left(\frac{r_1}{r_2}\right)^{2/3}$

15 Assuming that earth and mars move in circular orbits around the sun, with the martian orbit being 1.52 times the orbital radius of the earth. The length of the martian year in days is

(a) $(1.52)^{2/3}\times 365$ (b) $(1.52)^{3/2}\times 365$

(c) $(1.52)^2\times 365$ (d) $(1.52)^3\times 365$

16 A planet of radius R has a time period of revolution T. Find the time period of a planet of radius $9R$. ***JIPMER 2019***

(a) $3\sqrt{3}\,T$ (b) $9T$ (c) $27T$ (d) $9\sqrt{3T}$

17 A geostationary satellite is orbiting the earth at a height of $5R$ above the surface of the earth and R being the radius of the earth. The time period of another satellite in hour at a height of $2R$ from the surface of the earth is ***CBSE AIPMT 2012***

(a) 5 h (b) 10 h (c) $6\sqrt{2}$ h (d) $6/\sqrt{2}$ h

18 Following table shows the data from measurement of planetary motions to confirm Kepler's law of periods.

Planet	a	T	Q
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	A
Earth	15.0	1.00	2.96
Mars	22.8	1.88	2.98

Here, a = semi-major axis in units of 10^{10} m for the elliptical orbits,

T = time-period of revolution of the planet in years (yr) and Q = the quotient (T^2/a^3) in units of 10^{-34} y^2m^{-3})

The missing term A in the given table is approximately equal to

(a) 3 units (b) 2 units
(c) 2.5 units (d) 3.5 units

19 Two satellites A and B revolve around the same planet in coplanar circular orbits lying in the same plane. Their periods of revolutions are 1 h and 8 h, respectively and the radius of the orbit of A is 10^4 km. The speed of B relative to A (in km/h) when they are close to each other, will be **AIIMS 2018**

(a) $3\pi \times 10^4$ (b) zero (c) $2\pi \times 10^4$ (d) $\pi \times 10^4$

20 If the sun and the earth carried huge amounts of opposite charges, then

(a) all three of Kepler's laws would still be valid
(b) only the third law will be valid
(c) the second law will change
(d) the first law will not be valid

TOPIC 2 ~ *Newton's Law of Gravitation*

21 Newton's law of gravitation is universal because

(a) it acts on all bodies in the universe
(b) it acts on all the masses at all distances and not affected by the medium
(c) it is a attractive force
(d) it acts only when bodies are in contact

22 Newton arrived at an universal law based on his observation of an apple falling from a tree. Newton's reasoning was that the moon revolving in an orbit of radius R_m was subjected to a centripetal acceleration due to earth's gravity whose magnitude is given by

(a) $a_m = \dfrac{v^2}{R_m}$ (b) $a_m = \dfrac{4\pi^2 R_m}{T^2}$
(c) $a_m = \dfrac{mv^2}{R_m}$ (d) Both (a) and (b)

23 On going from one planet to other planet, the value of G (gravitational constant)

(a) increases (b) decreases
(c) remains same (d) None of these

24 There have been suggestions that the value of the gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth,

(a) nothing will change
(b) it will become hotter after billions of years
(c) it will be going around strictly in closed orbits
(d) after sufficiently long time it will leave the solar system

25 Kepler's third law states that square of period of revolution T of a planet around the sun, is proportional to third power of average distance r between sun and planet i.e. $T^2 = kr^3$, where k is constant.

If the masses of sun and planet are M and m, respectively, then as per Newton's law of gravitation force of attraction between them is $F = \dfrac{GMm}{r^2}$, where G is gravitational constant.

The relation between G and k is described as **CBSE AIPMT 2015**

(a) $k = 1/G$ (b) $Gk = 4\pi^2$
(c) $GMk = 4\pi^2$ (d) $k = G$

26 A planet is revolving around the sun in a circular orbit with a radius r and the time period is T. If the force between the planet and star is proportional to $r^{-3/2}$, then the square of time period is proportional to **AIIMS 2018**

(a) $r^{3/2}$ (b) r^2
(c) r (d) $r^{5/2}$

27 If the law of gravitation, instead of being inverse square law, becomes an inverse cube law, then

(a) planets will have elliptical orbits
(b) Kepler's law of areas does not hold good
(c) projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic
(d) there will be no gravitational force inside a spherical shell of uniform density

28 The height at which the weight of a body becomes (1/16)th of its weight on the surface of the earth (radius R), is **CBSE AIPMT 2012**

(a) $5R$ (b) $15\,R$ (c) $3\,R$ (d) $4\,R$

29 If mass of a planet is 2% of mass of earth. The ratio of gravitational pull of earth on the planet and that of planet on the earth will be

(a) 1 : 20 (b) 2 : 5 (c) 1 : 1 (d) 1 : 50

30 Two equal point masses are separated by a distance d_1. The force of gravitation acting between them is F_1. If the separation is decreased to d_2, then the new force of gravitation F_2 is given by

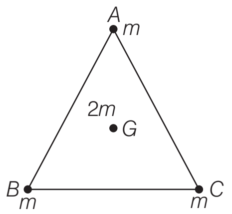
(a) $F_2 = F_1$ (b) $F_2 = F_1 \left(\frac{d_1}{d_2}\right)^2$

(c) $F_2 = F_1 \left(\frac{d_2}{d_1}\right)^2$ (d) $F_2 = F_1 \left(\frac{d_1}{d_2}\right)$

31 Two sphere of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) F (b) $\frac{F}{3}$ (c) $\frac{F}{9}$ (d) $3F$

32 Three equal masses of m kg each are fixed at the vertices of an equilateral triangle ABC and a mass $2m$ is placed at centroid G of the triangle as shown below.



(AG=BG=CG= 1m)

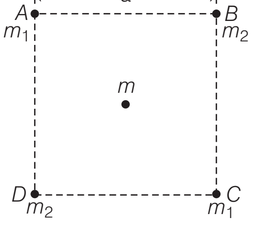
The force acting on a mass $2m$ placed at the centroid G of the triangle is

(a) $\frac{2Gm^2}{\sqrt{2}}$ (b) $2Gm^2$ (c) $6Gm^2$ (d) zero

33 Three uniform spheres of mass M and radius R each are kept in such a way that each touches the other two. The magnitude of the gravitational force on any of the spheres due to the other two is

(a) $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$ (b) $\frac{3}{2}\frac{GM^2}{R^2}$ (c) $\frac{\sqrt{3}\,GM^2}{R^2}$ (d) $\frac{\sqrt{3}}{2}\frac{GM^2}{R^2}$

34 A point mass m is placed at the centre of the square $ABCD$ of side a units as shown below.



The resultant gravitational force on mass m due to masses m_1 and m_2 placed on the vertices of square is

(a) $\frac{Gm_1m_2}{(a\sqrt{2})^2}$ (b) $\frac{2Gm\,(m_1 + m_2)}{a^2}$

(c) zero (d) $\frac{Gm\,(m_1 + m_2)}{(a\sqrt{2})^2}$

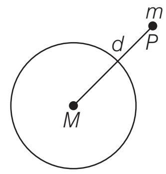
35 A mass M is split into two parts, m and $(M - m)$, which are then separated by a certain distance. What ratio of m / M maximizes the gravitational force between the two parts?

(a) 1/3 (b) 1/2 (c) 1/4 (d) 1/5

36 Two spherical bodies of masses m and $5\,m$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is ***CBSE AIPMT 2015***

(a) $2.5\,R$ (b) $4.5\,R$
(c) $7.5\,R$ (d) $1.5\,R$

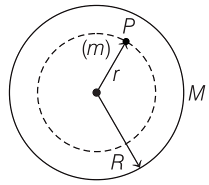
37 A point mass m is placed outside a hollow spherical shell of mass M and uniform density at a distance d from centre of the sphere as shown in figure. Gravitational force on point mass m at P is



(a) $\frac{GmM}{d^2}$ (b) zero

(c) $\frac{2\,GmM}{d^2}$ (d) Data insufficient

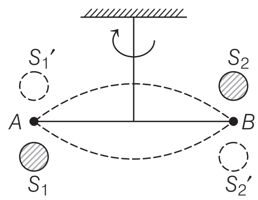
38 The force of attraction due to a hollow spherical shell of mass M, radius R and uniform density, on a point mass m situated inside it is



(a) $\frac{GmM}{r^2}$ (b) $\frac{Gm\,M}{R^2}$

(c) zero (d) Data insufficient

39 The figure shown is the schematic representation of Cavendish's experiment to determine the value of the gravitational constant. The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown.



The big spheres attract the near by small ones by a force which is

(a) equal and opposite
(b) equal but in same direction
(c) unequal and opposite
(d) None of the above

TOPIC 3 ~ Acceleration due to Gravity and its Variation

40 If g is the acceleration due to gravity at the surface of the earth. The force acting on the particle of mass m placed at the surface is
(a) mg
(b) $\frac{GmM_E}{R_E^2}$
(c) Data insufficient
(d) Both (a) and (b)

41 The weight of a body at the centre of earth is
(a) same as on the surface of earth
(b) same as on the poles
(c) same as on the equator
(d) None of the above

42 According to Newton's law of gravitation, let $\mathbf{F}_1$ and $\mathbf{F}_2$ be the forces between two masses m_1 and m_2 at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, where

$$\mathbf{F}_1 = -\mathbf{F}_2 = -\frac{\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

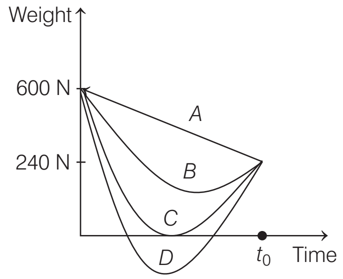
where, M_0 is a constant of dimension of mass, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$ and n is a number. In such a case,
(a) the acceleration due to gravity on the earth will be different for different objects
(b) None of the three laws of Kepler will be valid
(c) only the third law will be valid
(d) None of the above

43 If the mass of the earth is doubled and its radius halved, then new acceleration due to the gravity g' is
(a) $g' = 4g$ (b) $g' = 8g$
(c) $g' = g$ (d) $g' = 16g$

44 A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of radius of earth R will be
(a) $R/4$ (b) $R/2$
(c) $R/3$ (d) $R/8$

45 Suppose, the acceleration due to gravity at the earth's surface is 10 ms^{-2} and at the surface of mars it is 4.0 ms^{-2}. A 60 kg passenger goes from the earth to the mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky.

Which curve best represents the weight (net gravitational force) of the passenger as a function of time?



(a) A (b) B (c) C (d) D

46 If the mass of the sun is ten times smaller and gravitational constant G is ten times larger in magnitude. Then, for earth,
(a) walking on ground would become more easy
(b) acceleration due to gravity on the earth will not change
(c) raindrops will fall much slower
(d) airplanes will have to travel much faster

47 If one assumes that the gravitational force due to the earth decreases in proportion to the inverse square of the distance from the centre of the earth, then which of the following relation between a_m (centripetal acceleration of moon), R_m (distance of the moon from centre of the earth), g (acceleration due to gravity on the surface of the earth) and R_E (radius of earth) is correct?
(a) $\frac{g}{a_m} = \frac{R_m}{R_E}$ (b) $\frac{g}{a_m} = \left(\frac{R_m}{R_E}\right)^2$
(c) $\frac{g}{a_m} = \frac{R_E^{-2}}{R_m^{-2}}$ (d) Both (b) and (c)

48 The time period of the moon is $T = 27.3$ days and radius of orbit is $R_m = 3.84 \times 10^8$ m, the value of centripetal acceleration due to the earth's gravity is
(a) much smaller than the value of acceleration due to gravity g on the surface of the earth
(b) is equal to the value of acceleration due to gravity g on the surface of the earth
(c) much larger than the value of acceleration due to gravity g on the surface of the earth
(d) Either (a) or (b)

49 The value of acceleration due to gravity at earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to (Take, radius of earth $= 6.4 \times 10^6$ m) ***JEE Main 2019***

(a) 9.0×10^6 m (b) 2.6×10^6 m

(c) 6.4×10^6 m (d) 1.6×10^6 m

50 A body weigh's w N at the surface of the earth. Its weight at a height equals to half the radius of the earth will be

(a) $\frac{w}{2}$ (b) $\frac{2w}{3}$ (c) $\frac{4w}{9}$ (d) $\frac{8w}{27}$

51 At what height h above the surface of the earth, the value of g becomes $\left(\frac{g}{2}\right)$? (Take, $g =$ acceleration due to gravity at the surface of the earth and $R =$ radius of the earth)

(a) $3R$ (b) $\sqrt{2}R$

(c) $(\sqrt{2}-1)R$ (d) $\frac{1}{\sqrt{2}}R$

52 The radius of earth is R. Height of a point vertically above the earth's surface at which acceleration due to gravity becomes 1% of its value at the surface is

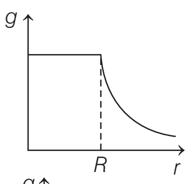
(a) $8R$ (b) $9R$ (c) $10R$ (d) $20R$

53 The weight of a body on the surface of the earth is 45 N. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth? ***AIIMS 2018***

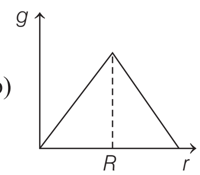
(a) 35 N (b) 20 N (c) 18 N (d) 40 N

54 Assuming the earth to have a constant density, point out which of the following curves show the variation of acceleration due to gravity from the centre of earth to the points far away from the surface of earth. ***NEET 2016, JEE Main 2017***

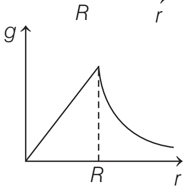
(a)



(b)

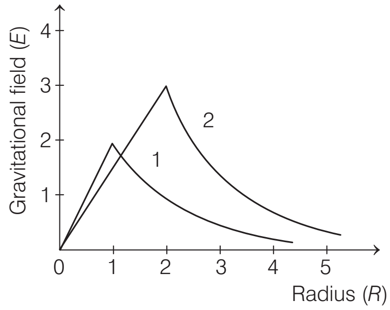


(c)



(d) None of these

55 Consider two solid spheres of radii $R_1 = 1$ m, $R_2 = 2$ m and masses M_1 and M_2, respectively. The gravitational field due to sphere 1 and 2 are shown. The value of $\frac{M_1}{M_2}$ is ***JEE Main 2020***



(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{1}{6}$ (d) $\frac{2}{3}$

56 Find acceleration due to gravity at a distance of 2000 km from the centre of earth.
(Given, $R_E = 6400$ km, $r = 2000$ km and $M_E = 6 \times 10^{24}$ kg) ***AIIMS 2019***

(a) 1.53 m/s^2 (b) 7.12 m/s^2

(c) 3.06 m/s^2 (d) 1.8 m/s^2

57 A body weighs 200 N on the surface of the earth. How much will it weigh half-way down to the centre of the earth ? ***NEET 2019***

(a) 200 N (b) 250 N

(c) 100 N (d) 150 N

58 A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance, if it is shifted to the equator is close to (Take, $g = 10\ ms^{-2}$ at the north pole and the radius of the earth $= 6400$ km) ***JEE Main 2020***

(a) 195.66 N (b) 195.32 N

(c) 194.66 N (d) 194.32 N

59 The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then , ***NEET 2017***

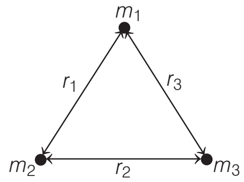
(a) $d = \frac{1}{2}$ km (b) $d = 1$ km

(c) $d = \frac{3}{2}$ km (d) $d = 2$ km

TOPIC 4 ~ *Gravitational Potential and Gravitational Potential Energy*

60 The gravitational potential energy of a system consisting two particles separated by a distance r is
(a) directly proportional to product of the masses of particles
(b) inversely proportional to the separation between them
(c) independent of distance r
(d) Both (a) and (b)

61 Gravitational potential energy of a system of particles as shown in the figure is



(a) $\frac{Gm_1m_2}{r_1}+\frac{Gm_2m_3}{r_3}+\frac{Gm_1m_3}{r_3}$
(b) $\left(\frac{-Gm_1m_2}{r_1}\right)+\left(\frac{-Gm_2m_3}{r_2}\right)+\left(\frac{-Gm_1m_3}{r_3}\right)$
(c) $\frac{-Gm_1m_2}{r_1}-\frac{Gm_2m_3}{r_2}+\frac{Gm_1m_3}{r_3}$
(d) $\frac{Gm_1m_2}{r_1}+\frac{Gm_2m_3}{r_2}-\frac{Gm_1m_3}{r_3}$

62 Two point masses m_1 and m_2 are separated by a distance r. The gravitational potential energy of the system is G_1. When the separation between the particles is doubled, the gravitational potential energy is G_2. Then, the ratio of $\frac{G_1}{G_2}$ is
(a) 1 (b) 2 (c) 3 (d) 4

63 The mass of the earth is 6×10^{24} kg and that of the moon is 7.4×10^{22} kg. The potential energy of the system is -7.79×10^{28} J. The mean distance between the earth and moon is ($G=6.67\times10^{-11}\ \text{Nm}^2\text{kg}^{-2}$)
(a) 3.8×10^8 m (b) 3.37×10^6 m
(c) 7.60×10^4 m (d) 1.9×10^2 m

64 The work done to raise a mass m from the surface of the earth to a height h, which is equal to the radius of the earth, is ***NEET 2019***
(a) $2mgR$ (b) $\frac{1}{2}mgR$ (c) $\frac{3}{2}mgR$ (d) mgR

65 The gravitational potential at a distance r from the centre of the earth $(r>R)$ is given by (consider, mass of the earth $=M_E$ and radius of the earth $=R$)
(a) $\frac{-GM_E}{R}$ (b) $\frac{GM_E}{R}$ (c) $\frac{-GM_E}{r}$ (d) $\frac{+GM_E}{r}$

66 At what height from the surface of earth, the gravitation potential and the value of g are -5.4×10^7 J kg^{-2} and 6.0 ms^{-2}, respectively? [Take, radius of earth is 6400 km]. ***NEET 2016***
(a) 1600 km (b) 1400 km
(c) 2000 km (d) 2600 km

67 Three particles each of mass m are kept at vertices of an equilateral triangle of side L. The gravitational potential at the centre due to these particles is
(a) $-\frac{3Gm}{L}$ (b) $\frac{-9\,Gm}{\sqrt{3}\,L}$
(c) $-\frac{3\sqrt{3}\,Gm}{L}$ (d) Both (b) and (c)

TOPIC 5 ~ *Escape Velocity and Satellites*

68 Gas escapes from the surface of a planet because it acquires an escape velocity. The escape velocity will depend on which of the following factors?
(a) Mass of the planet
(b) Mass of the particle escaping
(c) Temperature of the planet
(d) None of the above

69 Escape velocity of a body on the surface of earth is independent of
(a) mass
(b) radius of earth
(c) direction of projection of body
(d) Both (a) and (c)

70 An object is thrown from the surface of the moon. The escape speed for the object is
(a) $\sqrt{2g'R_m}$, where g' = acceleration due to gravity on the moon and R_m = radius of the moon
(b) $\sqrt{2g'R_E}$, where g' = acceleration due to gravity on the moon and R_E = radius of the earth
(c) $\sqrt{2gR_m}$, where g = acceleration due to gravity on the earth and R_m = radius of the moon
(d) None of the above

71 The escape velocity of a body from the earth is v_e. If the radius of earth contracts to $(1/4)$th of its value, keeping the mass of the earth constant, escape velocity will be
(a) doubled (b) halved
(c) tripled (d) unaltered

72 A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass $= 5.98 \times 10^{24}$ kg) have to be compressed to be a black hole? **CBSE AIPMT 2013**

(a) 10^{-2} m (b) 100 m
(c) 10^{-9} m (d) 10^{-6} m

73 An asteroid of mass m is approaching earth, initially at a distance 10 R_E with speed v_i. It hits earth with a speed v_f (R_E and M_E are radius and mass of earth), then

(a) $v_f^2 = v_i^2 + \frac{2Gm}{R_E}\left(1+\frac{1}{10}\right)$
(b) $v_f^2 = v_i^2 + \frac{2GM_E}{R_E}\left(1+\frac{1}{10}\right)$
(c) $v_f^2 = v_i^2 + \frac{2GM_E}{R_E}\left(1-\frac{1}{10}\right)$
(d) $v_f^2 = v_i^2 + \frac{2Gm}{R_E}\left(1-\frac{1}{10}\right)$

74 A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small as compared to the mass of the earth. Then, **CBSE AIPMT 2015**

(a) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(b) the total mechanical energy of S varies periodically with time
(c) the linear momentum of S remains constant in magnitude
(d) the acceleration of S is always directed towards the centre of the earth

75 Two stars of masses 3×10^{31} kg each and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take, gravitational constant, $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}) **JEE Main 2019**

(a) 2.8×10^5 m/s (b) 3.8×10^4 m/s
(c) 2.4×10^4 m/s (d) 1.4×10^5 m/s

76 If escape velocity on earth's surface is 11.1 km/h, then find the escape velocity on moon's surface. If mass of the moon is $\left(\frac{1}{81}\right)$ times of the mass of earth and radius of the moon is $\left(\frac{1}{4}\right)$ times radius of the earth. **JIPMER 2019**

(a) 2.46 km/h (b) 3.46 km/h
(c) 4.4 km/h (d) None of these

77 The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is **NEET 2016**

(a) $1:2\sqrt{2}$ (b) 1 : 4
(c) $1:\sqrt{2}$ (d) 1 : 2

78 The orbital angular momentum of a satellite revolving at a distance r from the centre is L. If the distance is increased to $16r$, then the new angular momentum will be

(a) $16\,L$ (b) $64\,L$
(c) $\frac{L}{4}$ (d) $4\,L$

79 Two satellites A and B have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A / T_B is **JEE Main 2019**

(a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) 1

80 The kinetic energy of the satellite in a circular orbit with speed v is given as

(a) $\text{KE} = \frac{-GmM_E}{2(R_E+h)}$ (b) $\text{KE} = \frac{GmM_E}{(R_E+h)}$
(c) $\text{KE} = \frac{GmM_E}{2(R_E+h)}$ (d) $\text{KE} = -\frac{1}{2}mv^2$

81 The radius of the orbit of a satellite is r and its kinetic energy is K. If the radius of the orbit is doubled, then the new kinetic energy K' is

(a) $2\,K$ (b) $\frac{K}{2}$
(c) $4\,K$ (d) Data insufficient

82 If the gravitational potential energy at infinity is assumed to be zero, the potential energy at distance $(R_E + h)$ from the centre of the earth is

(a) $\text{PE} = \frac{GmM_E}{(R_E+h)}$ (b) $\text{PE} = \frac{-GmM_E}{(R_E+h)}$
(c) $\text{PE} = mgh$ (d) $\text{PE} = \frac{-GmM_E}{2(R_E+h)}$

83 The potential energy of a satellite is given as

$$\text{PE} = \lambda\,(\text{KE})$$

where, PE = potential energy of the satellite
and KE = kinetic energy of the satellite.
The value of the constant λ is

(a) -2 (b) 2
(c) $-1/2$ (d) $+1/2$

84 A satellite is revolving in a circular orbit at a height h from the earth surface such that $h << R$, where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is *JEE Main 2019*

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$ (c) $\sqrt{2gR}$ (d) $\sqrt{gR}\,(\sqrt{2}-1)$

85 The time period of geo-stationary satellite is

(a) 6 h (b) 12 h (c) 24 h (d) 48 h

86 A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be JEE Main 2019

(a) in the same circular orbit of radius R
(b) in an elliptical orbit
(c) such that it escapes to infinity
(d) in a circular orbit of a different radius

SPECIAL TYPES QUESTIONS

I. Assertion and Reason

■ **Direction** (Q. Nos. 87-98) *In the following questions, a statement of Assertion is followed by a corresponding statement of Reason. Of the following statements, choose the correct one?*

(a) Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.
(c) Assertion is correct but Reason is incorrect.
(d) Assertion is incorrect but Reason is correct.

87 **Assertion** The force of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the centre of the shell.

Reason Gravitational force caused by various regions of the shell have components along the line joining the point mass to the centre as well as along a direction perpendicular to this line.

88 **Assertion** The force of attraction due to a hollow spherical shell of uniform density, on a point mass situated inside it is zero.

Reason Various region of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

89 **Assertion** The measurement of G by Cavendish's experiment, combined with the knowledge of g and R_E enables one to estimates M_E.

Reason By Newton's second law, the value of g is given by the relation, $g = \frac{GM_E}{R_E^2}$.

90 **Assertion** As we go up the surface of the earth, we feel light weighed than on the surface of the earth.

Reason The acceleration due to gravity decreases on going up above the surface of the earth.

91 **Assertion** The escape speed for the moon is 2.3 kms^{-1} which is five times smaller than that for the earth.

Reason The escape speed depends on acceleration due to gravity on the moon and radius of the moon and both of them are smaller than that of earth.

92 **Assertion** Moon has no atmosphere.

Reason The escape speed for the moon is much smaller.

93 **Assertion** The escape velocity for a planet is $v_e = \sqrt{2gR}$. If the radius of the planet is doubled, the escape velocity becomes twice (i.e. $v_e' = 2\,v_e$)

Reason In the relation for escape velocity, $v_e = \sqrt{2gR}$, the acceleration due to gravity g is inversely proportional to radius of the planet.

Thus, $v_e \propto \frac{1}{\sqrt{R}}$.

94 **Assertion** The velocity of the satellite decreases as its height above earth's surface increases and is maximum near the surface of the earth.

Reason The velocity of the satellite is inversely proportional to square root of its height above earth's surface.

95 **Assertion** The total energy of the satellite is always negative irrespective of the nature of its orbit, i.e. elliptical or circular and it cannot be positive or zero.

Reason If the total energy is positive or zero, the satellite would leave its orbit.

96 **Assertion** The geo-stationary satellite goes around the earth in west-east direction.

Reason Geo-stationary satellites orbits around the earth in the equatorial plane with $T = 24$ h same as that of the rotation of the earth around its axis.

97 **Assertion** In the satellite, everything inside it is in a state of free fall.

Reason Every part and parcel of the satellite has zero acceleration.

98 **Assertion** An object is weightless when it is in free fall and this phenomenon is called weightlessness.

Reason In free fall, there is no upward force acting on the object.

II. Statement Based Questions

99 The scheme of motion put forward by Ptolemy in order to describe the observed motion of the planets is that

I. the planets are moving in circles with the centre of the circles themselves moving in larger circles.

II. the planets are moving in circles with the centre of circles themselves remaining stationary.

III. the planets are moving in an elliptical orbit with the centre of the ellipse moving in a circle.

Which of the following statement(s) is/are correct?

(a) Only I (b) Only II
(c) Only III (d) None of these

100 I. All celestial objects, stars, the sun and the planets, revolve around the earth.

II. The only motion that was thought to be possible for celestial objects was motion in a circle.

III. The star, the earth and the planets, all revolve around the sun.

Which of the following statement(s) is/are correct for a geocentric model?

(a) Only I (b) Only II
(c) Only III (d) Both I and II

101 I. The planets revolve around the sun as its centre.

II. The planets revolves around the earth and earth revolve around the sun as its centre.

III. The sun revolves around the earth as its centre.

IV. The planets revolve around the sun in elliptical orbit.

With reference to the 'heliocentric model' of planetary motions, which of the given statement(s) is/are correct?

(a) Only I (b) Only II
(c) Only III (d) Only IV

102 According to Newton's law of gravitation, the magnitude of the force **F** on point masses m_1 and m_2 (say) is given by $|\mathbf{F}| = \frac{Gm_1 m_2}{r^2}$.

I. This equation is not directly applicable for the gravitational force between extended object (like the earth) and a point mass.

II. This equation can be directly applied for gravitational force between an extended object and a point mass.

III. The equations can be applied to find the gravitational force between an extended object and a point mass provided r is the distance of the point mass from the geometric centre of the extended object.

IV. None of the above

Which of the following statement(s) about this equation is/are correct?

(a) Only I (b) Both I and II
(c) Both II and IV (d) Both II and III

103 I. The centripetal force is directed towards the centre of the earth.

II. The centripetal force is provided by the gravitational force.

III. The magnitude of the gravitational force acting on the satellite is

$F_{\text{gravitational}} = \frac{GmM_e}{(R_e + h)^2}$, where M_e is the mass of the earth and it is same as centripetal force.

Which of the given statements is correct for the centripetal force required by the satellite to remain in orbit?

(a) Only I (b) Only II
(c) Both I and II (d) I, II and III

104 The orbit of a geo-stationary satellite is circular. The time period of satellite depends on

I. mass of the satellite. II. mass of the earth.

III. radius of the orbit.

IV. height of the satellite from the surface of the earth.

Which of the following statement(s) is/are correct?

(a) Only I (b) Both I and II
(c) I, II and III (d) II, III and IV

105 I. A strip on earth's surface is visible from satellite in one cycle.

II. The whole earth can be viewed strip by strip during the entire day from the polar satellite.

III. These satellites can view polar and equatorial regions at close distances with good resolution.

Which of the given statements is correct about the polar satellites?

(a) Only I (b) Both I and II
(c) Both II and III (d) I, II and III

106 If the mass of the sun were ten times smaller and the universal gravitational constant were ten times larger in magnitude, which one of the following statement is incorrect? ***NEET 2018***

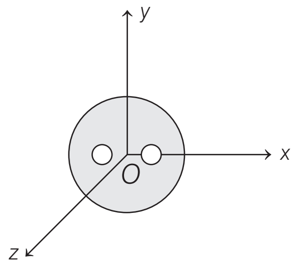
(a) Time period of a simple pendulum on the earth would decrease.

(b) Walking on the ground would become more difficult.

(c) Raindrops will fall faster.

(d) g on the earth will not change.

107 A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit with their centres at $A(-2, 0, 0)$ and $B(2, 0, 0)$ respectively are taken out of the solid leaving behind spherical cavities as shown in figure.



Which one of the following statement with respect to the given situation is incorrect?

(a) The gravitational force due to this object at the origin is zero.

(b) The gravitational force at the point $B(2, 0, 0)$ is zero.

(c) The gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$.

(d) The gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

108 Which of the following statement(s) is/are incorrect?

(a) Acceleration due to gravity decreases with altitude.

(b) Acceleration due to gravity increases with increasing depth (assume the earth to be a sphere of uniform density).

(c) Acceleration due to gravity increases with latitude.

(d) Acceleration due to gravity is independent of the mass of the object.

109 The radius and mass of earth are increased by 0.5%. Which one of the following statement is incorrect at the surface of the earth?

(a) g will increase.

(b) g will decrease.

(c) Escape velocity will remain unchanged.

(d) Potential energy will remain unchanged.

110 Which one of the following statement(s) is/are correct?

(a) A polar satellite goes around the earth's pole in north-south direction.

(b) A geo-stationary satellite goes around the earth in east-west direction.

(c) A geostationary satellite goes around the earth in north-south direction.

(d) A polar satellite goes around the earth in east-west direction.

111 Which of the following statement(s) is/are correct about satellites?

(a) A satellite cannot move in a stable orbit in a plane passing through the earth's centre.

(b) Geo-stationary satellites are launched in the equatorial plane.

(c) We can use just one geostationary satellite for global communication around the globe.

(d) The speed of satellite increases with an increase in the radius of its orbit.

112 Which one of the following statement(s) is/are correct?

(a) The energy required to rocket an orbiting satellite out of earth's gravitational influence is more than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

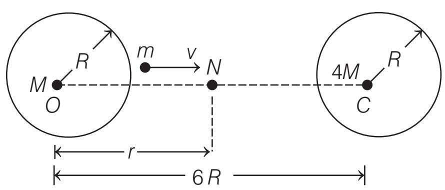
(b) If the potential energy is zero at infinity, the total energy of an orbiting satellite is negative of potential energy.

(c) The first artificial satellite sputnik I was launched in the year 2001.

(d) The time period of rotation of the SYNCOMS (Synchronous communications satellite) is 24 h.

III. Matching Type

113 Two uniform solid spheres of equal radii R but mass M and $4M$ have a centre-to-centre separation $6R$ as shown in figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. N is the point where net gravitational is zero.



With reference to the above situation, match the Column I (quantities) with Column II (mathematical expressions) and select the correct answer from the codes given below.

Column I		Column II
A. Distance r of neutral point N	1.	$\sqrt{\frac{3GM}{5R}}$
B. Minimum speed of the projectile, v_{min}, so that it reaches the surface of the second sphere.	2.	$2R$
C. The speed with which the projectile hits the seconds sphere, if projected with v_{min}.	3.	$\sqrt{\frac{27GM}{5R}}$

	A	B	C		A	B	C
(a)	1	2	3	(b)	3	2	1
(c)	2	1	3	(d)	3	1	2

114 Two satellites S_1 and S_2 revolve around a planet in coplanar circular orbits in the same sense. Their periods of revolutions are 1 h and 8 h, respectively. The radius of the orbit of S_1 is 10^4 km. With reference to the above situation, match the Column I (quantities) with Column II (approximate values) and select the correct answer from the codes given below.

Column I		Column II
A. Speed of S_2 in kmh^{-1}	1.	$\pi/3$
B. Speed of S_1 in kmh^{-1}	2.	$2\pi \times 10^4$
C. Velocity of S_2 relative to S_1 when S_2 is closest to S_1 in kmh^{-1}	3.	$\pi \times 10^4$
D. Angular speed of S_2 as observed by an astronaut in S_1 when S_2 is closest to S_1 in $radh^{-1}$	4.	$-\pi \times 10^4$

	A	B	C	D
(a)	3	2	4	1
(b)	2	3	4	1
(c)	2	3	1	4
(d)	4	2	1	3

115 A 400 kg satellite is in a circular orbit of radius 2 R_e about the earth, where R_e = radius of the earth. With reference to the above situation, match the Column I (quantities) with Column II (calculated values) and select the correct answer from the codes given below.

Column I		Column II
A. Energy required to transfer it to a circular orbit of radius $4R_e$.	1.	-6.26×10^9 J
B. Change in KE, ΔK (KE $\rightarrow$ Kinetic Energy)	2.	3.13×10^9 J
C. Change in potential energy, ΔPE	3.	-3.13×10^9 J

	A	B	C		A	B	C
(a)	2	3	1	(b)	1	2	3
(c)	3	2	1	(d)	2	1	3

116 A satellite of mass m revolving with a velocity v around the earth. With reference to the above situation, match the Column I (types of energy) with Column II (expression) and select the correct answer from the codes given below.

Column I		Column II
A. Kinetic energy of the satellite	1.	$-\frac{1}{2}mv^2$
B. Potential energy of the satellite	2.	$\frac{1}{2}mv^2$
C. Total energy of the satellite	3.	$-mv^2$

	A	B	C		A	B	C
(a)	1	2	3	(b)	2	3	1
(c)	3	2	1	(d)	3	1	2

117 Match the Column I (quantities) with Column II (approximate values/direction) and select the correct answer from the codes given below.

Column I		Column II
A. Direction of motion of polar satellite	1.	35800 km
B. Direction of motion of geo-stationary satellite	2.	West-East
C. Height above the surface of the earth for polar satellite	3.	512 km
D. Height above the surface of the earth for geostationary satellite	4.	North-South

	A	B	C	D
(a)	2	4	1	3
(b)	4	2	1	3
(c)	4	2	3	1
(d)	2	4	3	1

NCERT & NCERT Exemplar

MULTIPLE CHOICE QUESTIONS

NCERT

118 If one of the satellites of jupiter has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^8 m. The mass of Jupiter as compared to that of sun will be

(a) $M_J = \frac{1}{1000} M_S$ (b) $M_J = \frac{1}{10^5} M_S$

(c) $M_J = \frac{1}{10^8} M_S$ (d) $M_J = \frac{1}{10^6} M_S$

119 If zero of potential energy is at ∞, then total energy of satellite is equals to

(a) $-2K$ (b) $-3K$

(c) $-(1/2)K$ (d) $-K$

120 A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket is zero? Mass of the sun $= 2 \times 10^{30}$ kg, mass of the earth $= 6.0 \times 10^{24}$ kg and orbital radius $= 1.5 \times 10^{11}$ m. Neglect the effect of the other planets etc.

(a) 7.30×10^7 m (b) 2.6×10^8 m

(c) 5.30×10^8 m (d) 1.7×10^8 m

121 A saturn year is 29.5 times the earth year. How far is the saturn from the sun, if the earth is 1.5×10^8 km away from sun?

(a) 1.4×10^6 km (b) 1.4×10^7 km

(c) 1.4×10^8 km (d) 1.4×10^9 km

122 Weight of a body on surface of earth = 63 N. When this object is taken below earth surface at a height equals to half the radius, its weight becomes

(a) 28 N (b) 10 N

(c) 0 (d) 63 N

123 A rocket is fired vertically with a speed of 5 kms^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth?
Mass of the earth $= 6.0 \times 10^{24}$ kg, mean radius of earth $= 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ N-m^2/kg^2.

(a) 1600×10^6 km (b) 6.4×10^6 m

(c) 8×10^6 m (d) 9×10^7 m

124 The escape velocity of a projectile on earth's surface is 11.2 kms^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of sun and other planets.

(a) 31.7 kms^{-1} (b) 31.7 kmh^{-1}

(c) 108.7 kmskms^{-1} (d) 270.6 kms^{-1}

125 A satellite orbits the earth at a height of 400 km above the surface. How much energy must be expanded to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200 kg, mass of the earth $= 6.0 \times 10^{24}$ kg, radius of the earth $= 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ N-m^2/kg^2.

(a) 7×10^6 J (b) 8.5×10^9 J

(c) 10×10^9 J (d) 5.9×10^9 J

126 A spaceship is stationed on mars. How much energy must be expanded on the spaceship to rocket it out of the solar system? Mass of the spaceship = 1000 kg, mass of sun $= 2 \times 10^{30}$ kg, mass of mars $= 6.4 \times 10^{23}$ kg, radius of mars = 3395 km, radius of orbit of mars $= 2.28 \times 10^8$ km and $G = 6.67 \times 10^{-11}$ N-m^2/kg^2.

(a) 3.1×10^{11} J (b) 6.7×10^{11} J

(c) 3.1×10^{-11} J (d) 6.7×10^{-11} J

127 A rocket is fired vertically from the surface of mars with a speed of 2 kms^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars $= 6.4 \times 10^{23}$ kg, radius of mars = 3395 km and $G = 6.67 \times 10^{-11}$ N-m^2 kg^{-2}.

(a) 685 km (b) 785 km

(c) 495 km (d) 500 km

NCERT Exemplar

128 The earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the earth, the acceleration due to gravity

(a) will be directed towards the centre but not the same everywhere

(b) will have the same value everywhere but not directed towards the centre

(c) will be same everywhere in magnitude directed towards the centre

(d) Cannot be zero at any point

129 As observed from the earth, the sun appears to move in an approximate circular orbit. For the motion of another planet like mercury as observed from the earth, this would

(a) be similarly true
(b) not be true because the force between the earth and mercury is not inverse square law
(c) not be true because the major gravitational force on mercury is due to the sun
(d) not be true because mercury is influenced by forces other than gravitational forces

130 Different points in the earth are at slightly different distances from the sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the CM (centre of mass) causing translation and a net torque at the CM causing rotation around an axis through the CM. For the earth-sun system (approximating the earth as a uniform density sphere),
(a) the torque is zero
(b) the torque causes the earth to spin
(c) the rigid body result is not applicable since the earth is not even approximately a rigid body
(d) the torque causes the earth to move around the sun

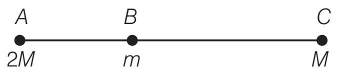
131 Satellites orbiting earth have finite life and sometimes debris of satellites fall to the earth. This is because broken part of satellite
(a) solar cells and batteries in satellites run out
(b) laws of gravitation predict a trajectory spiralling inwards
(c) of viscous forces causing the speed of satellite and hence height to gradually decrease
(d) of collisions with other satellites

132 Both the earth and the moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon
(a) will be elliptical
(b) will not be strictly elliptical because the total gravitational force on it is not central
(c) is not elliptical but will necessarily be a closed curve
(d) deviates considerably from being elliptical due to influence of planets other than the earth

133 In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They
(a) will not move around the sun, since they have very small masses compared to the sun
(b) will move in an irregular way because of their small masses and will drift away into outer space
(c) will move around the sun in closed orbits but not obey Kepler's laws
(d) will move in orbits like planets and obey Kepler's laws

134 Choose the incorrect option.
(a) Inertial mass is a measure of difficulty of accelerating a body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.
(b) The gravitational mass and inertial mass are equal is an experimental result.
(c) The acceleration due to gravity on the earth is same for all bodies is due to the equality of gravitational mass and inertial mass.
(d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot.

135 Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = 1/2(BC)$. m is much-much smaller than M and at time $t = 0$, they are all at rest as given in figure. At subsequent times before any collision takes place,



(a) m will remain at rest
(b) m will move towards M
(c) m will move towards $2M$
(d) m will have oscillatory motion

136 The centre of mass of an extended body on the surface of the earth and its centre of gravity
(a) are always at the same point for any size of the body
(b) are always at the same point only for spherical bodies
(c) can never be at the same point
(d) is close to each other for objects, say of sizes less than 100 m

137 The eccentricity of earth's orbit is 0.0167. The ratio of its maximum speed in its orbit to its minimum speed.
(a) 2.507 (b) 1.033
(c) 8.324 (d) 1.000

Answers

Mastering NCERT with MCQs

1 (a)	2 (b)	3 (c)	4 (a)	5 (b)	6 (a)	7 (a)	8 (c)	9 (c)	10 (d)
11 (c)	12 (b)	13 (a)	14 (c)	15 (b)	16 (c)	17 (c)	18 (a)	19 (d)	20 (a)
21 (b)	22 (d)	23 (c)	24 (d)	25 (c)	26 (d)	27 (c)	28 (c)	29 (c)	30 (b)
31 (a)	32 (d)	33 (a)	34 (c)	35 (b)	36 (c)	37 (a)	38 (c)	39 (a)	40 (d)
41 (d)	42 (a)	43 (b)	44 (b)	45 (c)	46 (d)	47 (d)	48 (a)	49 (b)	50 (c)
51 (c)	52 (b)	53 (b)	54 (c)	55 (c)	56 (c)	57 (c)	58 (b)	59 (d)	60 (d)
61 (b)	62 (b)	63 (a)	64 (b)	65 (c)	66 (d)	67 (d)	68 (a)	69 (d)	70 (a)
71 (a)	72 (a)	73 (c)	74 (d)	75 (a)	76 (a)	77 (a)	78 (d)	79 (d)	80 (c)
81 (b)	82 (b)	83 (a)	84 (d)	85 (c)	86 (b)				

Special Types Questions

87 (a)	88 (a)	89 (a)	90 (a)	91 (a)	92 (a)	93 (d)	94 (a)	95 (a)	96 (a)
97 (c)	98 (a)	99 (a)	100 (d)	101 (a)	102 (a)	103 (d)	104 (d)	105 (d)	106 (d)
107 (b)	108 (b)	109 (a)	110 (a)	111 (b)	112 (d)	113 (c)	114 (a)	115 (a)	116 (b)
117 (c)									

NCERT & NCERT Exemplar MCQs

118 (a)	119 (d)	120 (b)	121 (d)	122 (a)	123 (c)	124 (a)	125 (d)	126 (a)	127 (c)
128 (d)	129 (c)	130 (a)	131 (c)	132 (b)	133 (d)	134 (d)	135 (c)	136 (d)	137 (b)

Hints & Explanations

1 (*a*) Earth exerts an attractive force on all the material objects as it has a tendency to attract all of them towards it.

So, going uphill is a lot more tiring than going downhill as more efforts are required to work against the attractive force of earth.

3 (*c*) According to Kepler's second law of motion, the line that joins any planet to the sun sweeps equal area in equal intervals of time.

Given, areas *SAB* and *SCD* are equal,

so $$t_1 = t_2$$

4 (*a*) The law of areas (Kepler's second law) says that areal velocity is constant, i.e. the line joining the planet sweeps out equal areas in equal interval of time.

The area swept out by the planet of mass m in time interval Δt is given as

$$\Delta A = \frac{1}{2} r\,(vdt)$$

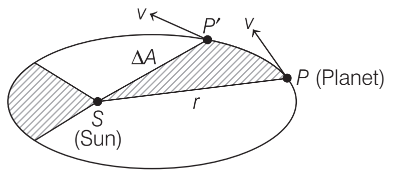
then areal velocity $= \dfrac{dA}{dt} = \dfrac{d}{dt}\left[\dfrac{1}{2} r(vdt)\right] = \dfrac{1}{2} rv$

$$= \frac{mvr}{2m} = \frac{L}{2m} = \text{constant}$$

i.e. angular momentum is conserved. So, we can say that the law of areas can be understood as a consequence of conservation of angular momentum.

7 (*a*) The area swept out by the given planet in time interval Δt is

$$\Delta A = \frac{1}{2} |\mathbf{r} \times \mathbf{PP'}|$$



$$\Rightarrow \quad \Delta A = \frac{1}{2} (\mathbf{r} \times \mathbf{v}\Delta t) \qquad (\because \text{arc } PP' = v\Delta t)$$

8 (*c*) From Kepler's second law, the areal velocity can be given as

$$\frac{dA}{dt} = \frac{L}{2m}$$

$$= \frac{I\omega}{2m} = \frac{mr^2\omega}{2m}$$

$$\Rightarrow \quad \frac{dA}{dt} \propto \omega r^2 \qquad \left[\begin{array}{l}\therefore L = I\omega \\ \text{and } I = mr^2\end{array}\right]$$

9 (*c*) For a particle, angular momentum is $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

So, magnitude of angular momentum $= |\mathbf{L}|$

$$\Rightarrow \quad |\mathbf{L}| = |\mathbf{r}|\,|\mathbf{p}| \sin\theta$$

where, θ = angle between vector $\mathbf{r}$ and $\mathbf{p}$ or $\mathbf{r}$ and $\mathbf{v}$.

$\Rightarrow \qquad |\mathbf{L}| = L = r\,mv \sin\theta \qquad (\because p = mv)$

or $\qquad L = mv\, r \sin\theta$

When $\theta = 90°$, then

$$L = mvr = \text{magnitude of angular momentum.}$$

Here, $\theta = 90°$ means **r** and **v** are mutually perpendicular.

So, for the planet at position P, $L_P = m_P r_P v_P$.

10 (*d*) By Kepler's second law of planetary motion, for a moving planet around the sun, angular momentum remains conserved.

i.e. $\qquad m_1 v_1 r_1 = m_2 v_2 r_2$

but $\qquad m_1 = m_2$

$\therefore \qquad v_1 r_1 = v_2 r_2$

$\Rightarrow \qquad \dfrac{v_1}{v_2} = \dfrac{r_2}{r_1}$

For points R and P, $\dfrac{v_R}{v_P} = \dfrac{r_P}{r_R}$

Since, $\qquad r_R > r_P$

$\therefore \qquad v_R < v_P$

Hence, linear speed of planet is minimum at R.

11 (*c*) From conservation of angular momentum,

$L = mvr = \text{constant}$ (for planet at all positions) ...(i)

Given, maximum velocity of planet,

$v_{max} = 3 \times 10^4 \text{ ms}^{-1}$

Minimum velocity of planet, $v_{min} = 1 \times 10^3 \text{ ms}^{-1}$

Maximum distance of the planet from the sun,

$$r_{max} = 4 \times 10^4 \text{ km}$$

and minimum distance of the planet from the sun,

$r_{min} = ?$

From Eq. (i), $m\,v_{max}\, r_{min} = m\, v_{min}\, r_{max}$

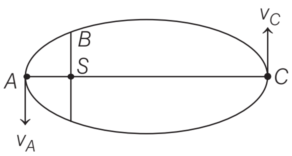
(at maximum distance, velocity is minimum and *vice-versa*)

$\Rightarrow \qquad v_{max} \cdot r_{min} = v_{min} \cdot r_{max}$

or $\qquad r_{min} = \dfrac{v_{min} \cdot r_{max}}{v_{max}} = \dfrac{1 \times 10^3 \times 4 \times 10^4}{3 \times 10^4}$

$$= \frac{4}{3} \times 10^3 \text{ km}$$

12 (*b*) According to the question,



The figure above shows an ellipse traced by a planet around the sun S. The closest point A is known as **perihelion** and the farthest point C is known as **aphelion**.

Since, as per the result of the Kepler's second law of area, the planet will move slowly (v_{min}) only when it is farthest from the sun and more rapidly (v_{max}) when it is nearest to the sun.

If velocities at point A, B and C are v_A, v_B and v_C respectively, thus $v_A = v_{max}$, $v_C = v_{min}$

Therefore, we can write

$$v_A > v_B > v_C \qquad \text{...(i)}$$

Kinetic energy (K) of the planet at any point is given as

$$K = \frac{1}{2} mv^2$$

Thus, the kinetic energies, at A, $K_A = \dfrac{1}{2} mv_A^2$

At B, $\qquad K_B = \dfrac{1}{2} mv_B^2$

and at C, $\qquad K_C = \dfrac{1}{2} mv_C^2$

From Eq. (i), we can write

$$K_A > K_B > K_C$$

13 (*a*) Given, time period of revolution, $T = 84$ yr,

Semi-major axis, $a = 287 \times 10^{10}$ m

Then, $\qquad \dfrac{T^2}{a^3} = \dfrac{(84)^2}{(287 \times 10^{10})^3} = 2.98 \times 10^{-34} \text{ yr}^2\text{m}^{-3}$

$$\approx 3.00 \times 10^{-34} \text{ yr}^2\text{m}^{-3}$$

Thus, the quotient (T^2/a^3) in units of 10^{-34} yr^{2}m^{-3} is approximately 3 units.

14 (*c*) From Kepler's third law of planetary motion,

$$T^2 \propto r^3 \quad \Rightarrow \quad \frac{T^2}{r^3} = \text{constant}$$

then for any two planets, the above equation becomes

$$\Rightarrow \qquad \frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \text{constant or } \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

15 (*b*) If time period for mass and earth are T_m and T_e, respectively. Then, according to Kepler's third law,

$$\frac{T_m^2}{T_e^2} = \frac{R_{ms}^3}{R_{es}^3}$$

where, R_{ms} is the mars-sun distance and R_{es} is the earth-sun distance.

$\therefore \qquad T_m = \left(\dfrac{R_{ms}}{R_{es}}\right)^{3/2} \cdot T_e \qquad \left[\because \dfrac{R_{ms}}{R_{es}} = 1.52\right]$

$\therefore \qquad T_m = (1.52)^{3/2} \times 365 \text{ days}$

16 (*c*) By Kepler's third law of planetary motion,

$$\frac{T_1}{T_2} = \left(\frac{a_1}{a_2}\right)^{3/2}$$

Given, $\qquad T_1 = T, a_1 = R$

$\qquad T_2 = ?\ a_2 = 9R$

$\therefore \qquad \dfrac{T}{T_2} = \left(\dfrac{R}{9R}\right)^{3/2}; \dfrac{T}{T_2} = \left[\left(\dfrac{1}{3}\right)^2\right]^{3/2}$

$$\frac{T}{T_2} = \frac{1}{27} \Rightarrow T_2 = 27T$$

$\therefore$ Time period of a planet of radius $9R$ is $27T$.

17 (*c*) From Kepler's third law, $T^2 \propto r^3$

where, T = time period of satellite and r = radius of elliptical orbit.

Here, $r_1 = R$ (radius of earth) $+ 5R = 6R$

and $r_2 = R + 2R = 3R$

Hence, $T_1^2 \propto r_1^3$ and $T_2^2 \propto r_2^3$

So, $$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3} = \frac{(3R)^3}{(6R)^3}$$

or $$\frac{T_2^2}{T_1^2} = \frac{1}{8} \Rightarrow T_2^2 = \frac{1}{8}T_1^2$$

$$T_2 = \frac{24}{2\sqrt{2}} = 6\sqrt{2}\text{h} \qquad [\because T_1 = 24 \text{ h}]$$

18 (*a*) Given, the quotient $Q = \dfrac{T^2}{a^3}$

For A, $a = 10.8$ units $= 10.8 \times 10^{10}$ m, $T = 0.615$ yr

$$\Rightarrow \quad Q = \frac{T^2}{a^3} = \frac{(0.615)^2}{(10.8)^3 \times 10^{30}} \frac{\text{y}^2}{\text{m}^3}$$

$$\Rightarrow \quad A = 3.00 \times 10^{-34} \text{y}^2\text{m}^{-3}$$

So, the missing term A is approximately equal to 3 units.

19 (*d*) If the time period and radius of satellites A and B are T_A, T_B and r_A, r_B, respectively. Then, from Kepler's law,

$$\frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3} \Rightarrow \frac{1^2}{8^2} = \frac{(10^4)^3}{r_B^3}$$

$$[\because T_A = 1\text{h}, T_2 = 8\text{h}, r_A = 10^4 \text{ km}, r_B = ?]$$

$$\Rightarrow \quad r_B^3 = 64 \times (10^4)^3$$

$$\therefore \quad r_B = 4 \times 10^4 \text{ km}$$

Speed of satellite A, $v_A = \dfrac{2\pi r_A}{T_A} = \dfrac{2\pi \times 10^4}{1}$

$$= 2\pi \times 10^4 \text{ km/h}$$

Speed of satellite B, $v_B = \dfrac{2\pi r_B}{T_B} = \dfrac{2\pi \times 4 \times 10^4}{8}$

$$= \pi \times 10^4 \text{ km/h}$$

The speed of B relative to A when they are close is

$$v_{BA} = v_A - v_B$$
$$= 2\pi \times 10^4 - \pi \times 10^4$$
$$= \pi \times 10^4 \text{ km/h}$$

20 (*a*) Due to huge amounts of opposite charges on the sun and the earth, electrostatic force of attraction will be produced. Gravitational force which is already exist between sun and earth is also attractive in nature. So, both forces will be added and obey inverse square law and are central forces. As both the forces are of same nature, hence all the three Kepler's laws will be valid.

22 (*d*) In a circular motion, the centripetal acceleration,

$$a = \omega^2 R \qquad \text{...(i)}$$

where, angular velocity, $\omega = \dfrac{v}{R}$ ...(ii)

From Eqs (i) and (ii), we get

$$a = \frac{v^2}{R_m} \qquad \text{...(iii)}$$

where, v = magnitude of velocity and R_m = radius of the moon.

Also, speed of moon, $v = \dfrac{2\pi R_m}{T}$, where T = time period of revolution.

On putting the above value of v in Eq. (iii), we get

$$a = \left(\frac{2\pi R_m}{T}\right)^2 \times \frac{1}{R_m} = \frac{4\pi^2 R_m}{T^2}$$

24 (*d*) We know that, gravitational force between the earth and the sun is

$$F_G = \frac{GMm}{r^2}$$

where, M is mass of the sun and m is mass of the earth. When G decreases with time, the gravitational force F_G will become weaker with time. As, F_G is changing with time, so the earth will not be revolving around the sun in a closed orbit strictly. Also, the radius may increase, since the attraction force is becoming weaker. Hence, after long time period, the earth will leave the solar system.

25 (*c*) As we know that, period of revolution,

$$T = \frac{2\pi r}{v} = 2\pi \frac{r}{\sqrt{GM/r}} = 2\pi \frac{r^{3/2}}{\sqrt{GM}}$$

$$\left[\because \text{Orbital speed}, v = \sqrt{\frac{GM}{r}}\right]$$

$$T^2 = \frac{4\pi^2}{GM} r^3 = kr^3$$

$$\therefore \quad k = \frac{4\pi^2}{GM}$$

$$\Rightarrow \quad GMk = 4\pi^2$$

26 (*d*) According to the question, force $= \dfrac{GMm}{r^{3/2}} = m\omega^2 r$

$$\Rightarrow \quad \frac{GMm}{r^{3/2}} = \frac{4\pi^2 mr}{T^2} \qquad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\Rightarrow \quad T^2 = \left(\frac{4\pi^2}{GM}\right) \cdot r^{5/2}$$

$$\Rightarrow \quad T^2 \propto r^{5/2}$$

27 (*c*) If the law of gravitation suddenly changes and becomes an inverse cube law, then the law of areas still holds. This is because the force will still be central force, due to which angular momentum will remain constant.

Also, for a planet of mass m revolving around the sun of mass M, we can write according to the question,

$$F = \frac{GMm}{a^3} = \frac{mv^2}{a}$$

$$[\because \text{Centripetal force, } m\omega^2 a = \frac{mv^2}{a}]$$

where, a is radius of orbiting planet.

Orbital speed, $v = \frac{\sqrt{GM}}{a} \Rightarrow v \propto \frac{1}{a}$

Time period of revolution of a planet,

$$T = \frac{2\pi r}{v}$$

On putting the value of orbital speed, we get

$$T = \frac{2\pi a}{\frac{\sqrt{GM}}{a}} = \frac{2\pi a^2}{\sqrt{GM}}$$

$$\Rightarrow \quad T^2 \propto a^4$$

Hence, the orbit of the planet around sun will not be elliptical because for elliptical orbit $T^2 \propto a^3$.

As force $F = \left(\frac{GM}{a^3}\right) m = g' m$, where $g' = \frac{GM}{a^3}$.

Since, g' = acceleration due to gravity is constant, hence path followed by a projectile will be approximately parabolic (as $T \propto a^2$).

28 (*c*) The force of gravity on a body at a height h above the surface of the earth is given by

$$F = \frac{GMm}{(R+h)^2}$$

where, m = mass of the body and M = mass of earth.

As we know that, weight of an object is the force (force of gravity) with which the earth attracts it.

Then, according to the question,

$$\frac{GMm}{(R+h)^2} = \frac{1}{16} \frac{GMm}{R^2}$$

where, $\frac{GM}{R^2}$ = gravitational acceleration.

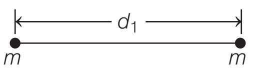
$$\frac{1}{(R+h)^2} = \frac{1}{16R^2}$$

or $$\frac{R}{R+h} = \frac{1}{4} \quad \text{or} \quad \frac{R+h}{R} = 4$$

$$h = 3R$$

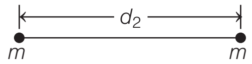
29 (*c*) The gravitational forces are mutually equal and opposite, hence the ratio of gravitational pull of the earth on the planet and that of planet on the earth will be 1 : 1.

30 (*b*) From Newton's law of gravitation, the two equal point masses are separated by a distance d_1 is shown below



Gravitational force, $F_1 = \frac{Gm \cdot m}{d_1^2} = \frac{Gm^2}{d_1^2}$ ...(i)

Similarly,



Gravitational force, $F_2 = \frac{Gm^2}{d_2^2}$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

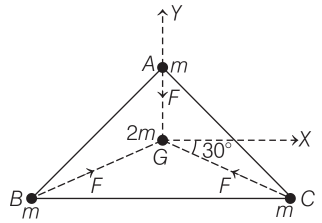
$$\frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^2$$

$$\therefore \quad F_2 = F_1 \left(\frac{d_1}{d_2}\right)^2$$

31 (*a*) Gravitational force does not depend on the medium between the masses. So, the liquid filled between the masses of specific gravity 3 will not have any effect on gravitational force.

So, it will remain same, i.e. F.

32 (*d*) The angle between GC and positive X-axis is 30° and so is the angle between GB and negative X-axis.



From figure, resolving the forces on $2m$ due to masses at B and C along X-axis.

$$\Rightarrow \quad |\mathbf{F}_{x\,(\text{net})}| = |F \cos 30° \,\hat{\mathbf{i}} - F \cos 30° \,\hat{\mathbf{i}}| = 0$$

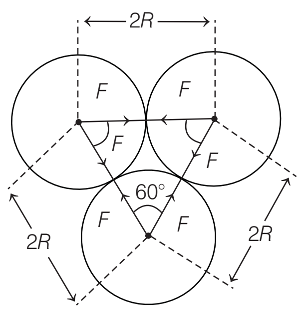
Similarly along Y-axis,

$$|\mathbf{F}_{y(\text{net})}| = |F\,\hat{\mathbf{j}} - (F \sin 30° + F \sin 30°)\,\hat{\mathbf{j}}|$$

$$= \left| F\,\hat{\mathbf{j}} - \left(\frac{F}{2} + \frac{F}{2}\right)\hat{\mathbf{j}} \right| = |F\hat{\mathbf{j}} - F\hat{\mathbf{j}}| = 0$$

F_{net} = Resultant force on mass at G due to masses at A, B and $C = \sqrt{F_{x\,(\text{net})}^2 + F_{y\,(\text{net})}^2} = 0$

33 (*a*) The figure of three uniform spheres of mass M and radius R is shown below



Force between any two spheres of mass M and radius R is

$$F = G \cdot \frac{M \cdot M}{(2R)^2} = \frac{GM^2}{4R^2} \quad \ldots\text{(i)}$$

Net force on any sphere due to other two is given by

$$F_{\text{net}} = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ} = \sqrt{3}F$$

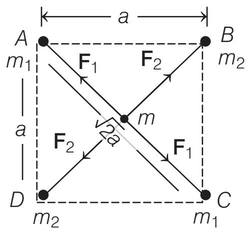
Putting the value of F from Eq. (i), we get

$$F_{\text{net}} = \frac{\sqrt{3}}{4} \frac{GM^2}{R^2}$$

34 (c) A point mass m is located at the centre of the square $ABCD$. Let gravitational force m and m_1 is

$$|\mathbf{F}_1| = \left[\frac{Gmm_1}{(a/\sqrt{2})^2}\right]$$

and between m and m_2 is $|\mathbf{F}_2| = \left[\frac{Gmm_2}{(a/\sqrt{2})^2}\right]$ as shown below



$\because$ Along BD there are two forces of same magnitude F_2 acting between m and m_2 (on B or D)

Net gravitational force along BD,

$$\mathbf{F}_{\text{net }(BD)} = \mathbf{F}_2 - \mathbf{F}_2 = 0$$

Similarly, net gravitational force along AC,

$$\mathbf{F}_{\text{net }(AC)} = \mathbf{F}_1 - \mathbf{F}_1 = 0$$

$\therefore$ Net force on m is $|\mathbf{F}_{\text{net}}| =$ zero

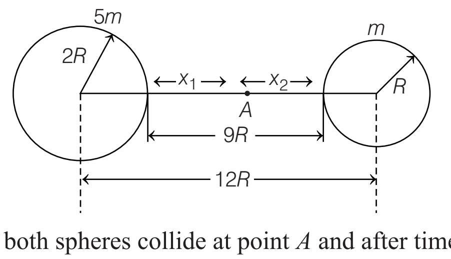
35 (b) Gravitational force between masses m and $(M - m)$ is

$$F = \frac{Gm(M-m)}{r^2}$$

For maximum force, $\frac{dF}{dm} = 0$

$$\Rightarrow \frac{d}{dm}\left(\frac{GmM}{r^2} - \frac{Gm^2}{r^2}\right) = 0 \Rightarrow M - 2m = 0 \Rightarrow \frac{m}{M} = \frac{1}{2}$$

36 (c) The figure of two spherical bodies of masses m and $5m$ and radii R and $2R$ is shown below



Let both spheres collide at point A and after time t, x_1 be the distance travelled by bigger sphere while x_2 be the distance travelled by smaller sphere.

$$\therefore \quad x_1 + x_2 = [12R - (R + 2R)] \quad \ldots\text{(i)}$$

Gravitational force between both spheres is same and given by

$$F = G \cdot \frac{5m \cdot m}{(12R)^2} \Rightarrow F = \frac{5Gm^2}{(12R)^2} \quad \ldots\text{(ii)}$$

Acceleration of bigger sphere, $a_1 = \frac{F}{5m} = \frac{Gm}{(12R)^2}$

[using Eq. (ii)]

$\therefore$ Distance travelled by bigger sphere,

$$x_1 = \frac{1}{2}a_1t^2 = \frac{1}{2} \cdot \frac{Gm}{(12R)^2} \cdot t^2 \quad \ldots\text{(iii)}$$

Similarly, acceleration of smaller sphere,

$$a_2 = \frac{F}{m} = \frac{5Gm}{(12R)^2}$$

$\therefore$ Distance travelled by smaller sphere,

$$x_2 = \frac{1}{2}a_2t^2 = \frac{1}{2} \cdot \frac{5Gm}{(12R)^2} \cdot t^2 \quad \ldots\text{(iv)}$$

Putting these values of x_1 and x_2 in Eq. (i), we have

$$\frac{1}{2} \cdot \frac{Gm}{(12R)^2} \cdot t^2 + \frac{1}{2} \cdot \frac{5Gm}{(12R)^2} \cdot t^2 = 9R$$

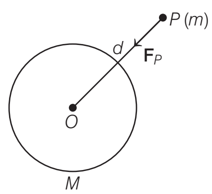
$$\Rightarrow \quad t^2 = \frac{3R(12R)^2}{Gm}$$

From Eq. (iv), we have

$$x_2 = \frac{1}{2} \cdot \frac{5Gm}{(12R)^2} \times \frac{3R(12R)^2}{Gm}$$

$$= \frac{15R}{2} = 7.5R$$

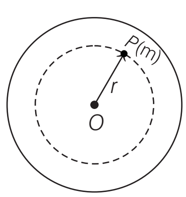
37 (a) For a point outside the spherical shell as shown below



According to Newton's gravitational law, gravitational force on point mass m at P is

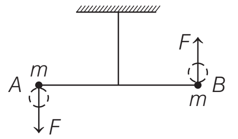
$$|\mathbf{F}_P| = \frac{GmM}{d^2}$$

38 (c) Net resultant force at point P inside a hollow spherical shell will be zero because the attractive forces due to various parts of the shell cancel out each other.



i.e. $\quad |\mathbf{F}_P \text{(net)}| =$ zero

39 (*a*) The force of attraction on small spheres due to big sphere are equal and opposite in direction as shown below.



There is no net force on the bar but only a torque which is nearly equal to F times the length of the bar, where F is the force of attraction between a big sphere and the neighbouring small sphere.

40 (*d*) The force acting on the particle of mass m at surface of the earth

$$F = mg \qquad \ldots(\text{i})$$

where, g = acceleration due to gravity at the earth's surface.

Also, $$g = \frac{GM_E}{R_E^2} \qquad \ldots(\text{ii})$$

Then, from Eqs. (i) and (ii), we get

$$\Rightarrow \quad F = mg = \frac{GmM_E}{R_E^2}$$

Hence, options (a) and (b) are correct.

41 (*d*) Gravitational acceleration (g) at the centre of earth is zero, hence weight of body ($w = mg$) at the centre of earth becomes zero.

42 (*a*) Given, $$\mathbf{F}_1 = -\mathbf{F}_2 = \frac{-\mathbf{r}_{12}}{r_{12}^3} GM_0^2 \left(\frac{m_1 m_2}{M_0^2}\right)^n$$

where, $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$

Acceleration due to gravity,

$$g = \frac{|F|}{\text{mass}} = \frac{GM_0^2 (m_1 m_2)^n}{r_{12}^2 (M_0)^{2n}} \times \frac{1}{(\text{mass})}$$

Since, g depends upon position vector $\mathbf{r}_{12}$, hence it will be different for different objects.

As g is not constant, hence constant of proportionality will not be constant in Kepler's third law.

Hence, Kepler's third law will not be valid.

As the force is of central nature, hence, first two Kepler's laws will be valid.

43 (*b*) As we know that, acceleration due to gravity is

$$g = \frac{GM}{R^2}$$

Given, $M' = 2M$ [$\because$ mass gets doubled]

$\Rightarrow$ $R' = (R/2)$ [$\because$ radius gets halved]

Then, acceleration becomes

$$\Rightarrow \quad g' = \frac{GM'}{R'^2} = \frac{G\,(2M)}{(R/2)^2} = \frac{8\,GM}{R^2}$$

$$\therefore \quad g' = 8g$$

Thus, the new acceleration due to gravity g' is 8 times that of g.

44 (*b*) It is given that, the density of planet is twice the density of earth and both of them have same acceleration due to gravity.

if g and g' are the acceleration due to gravity of earth and planet, then $g = g'$

$$\therefore \quad \frac{G}{R^2}\left[\frac{4}{3}\pi R^3 \rho\right] = \frac{G}{R'^2} \times \frac{4}{3}\pi R'^3 (2\rho)$$

$$\Rightarrow \quad R' = \frac{R}{2}$$

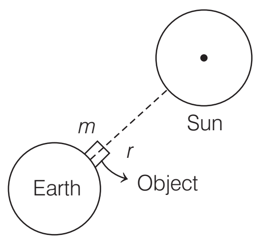
45 (*c*) Initially, the weight of the passenger at the earth's surface $w = mg = 60 \times 10 = 600$ N.

Finally, the weight of the passenger at the surface of the mass $= 60 \times 4 = 240$ N and during the flight in between somewhere its weight will be zero because at that point, gravitational pull of earth and mars will be equal.

Only the curve (c) represents the weight = 0.

So, (c) is correct option.

46 (*d*) Consider the given diagram,



Force on the object due to the earth,

$$F = \frac{G' M_E m}{R^2} = \frac{10 G M_E m}{R^2} \qquad (\because G' = 10\,G)$$

$$= 10\left(\frac{GM_E m}{R^2}\right) = (10g)\,m = 10\,mg \qquad \ldots(\text{i})$$

$$\left(\because g = \frac{GM_E}{R^2}\right)$$

Now, force on the object due to the sun, $F' = \dfrac{GM_s' m}{r^2}$

$$= \frac{G(M_s)m}{10 r^2} \qquad \left(\because M_s' = \frac{M_s}{10}\right)$$

As $r >> R$ (radius of the earth)

$\Rightarrow$ F' will be very small, so the effect of the sun will be neglected.

Now, as $g' = 10\,g$

Hence, weight of person $= mg' = 10\,mg$ [from Eq. (i)]

i.e. gravity pull on the person will increase.

Due to it, walking on ground would become more difficult.

Escape velocity v_c is proportional to g,

i.e. $v_c \propto g$

As, $g' > g \Rightarrow v_c' > v_c$

Hence, rain drops will fall much faster.

To overcome the increased gravitational force of the earth, the airplanes will have to travel much faster.

47 (*d*) Since, centripetal acceleration of moon,

$$a_m \propto \frac{1}{R_m^2} \quad \ldots(i)$$

Acceleration due to gravity,

$$g \propto \frac{1}{R_E^2} \quad \ldots(ii)$$

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{g}{a_m} = \frac{R_m^2}{R_E^2} = \left(\frac{R_m}{R_E}\right)^2 = \frac{R_E^{-2}}{R_m^{-2}}$$

Hence, options (b) and (c) are correct.

48 (*a*) Given, time period of the moon, $T = 27.3$ days

Radius of orbit, $R_m = 3.84 \times 10^8$ m

Centripetal acceleration is given as

$$a = \frac{4\pi^2 R_m}{T^2} \qquad \left[\because a = \omega^2 R = \left(\frac{2\pi^2}{T}\right) R_m\right]$$

$$= \frac{4 \times \left(\frac{22}{7}\right)^2 \times (3.84 \times 10^8)}{(27.3 \times 24 \times 60 \times 60)^2}$$

$$\approx 0.0027 \text{ ms}^{-2}$$

From the above value we can say that, centripetal acceleration of the moon due to earth's gravity is much smaller than the value of acceleration due to gravity g on the surface of the earth.

49 (*b*) Given that at some height h, acceleration due to gravity,

$$g_h = 4.9 \text{ m/s}^2 \approx \frac{g}{2} \quad \ldots (i)$$

$\therefore$ The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$$\left(1 + \frac{h}{R_e}\right) = \sqrt{\frac{g}{g_h}} = \sqrt{2} \qquad \text{[from Eq. (i)]}$$

$$\therefore \quad \frac{h}{R_e} = \sqrt{2} - 1$$

or $\quad h = 0.414 \times R_e$

$h = 0.414 \times 6400$ km

($\because$ given, radius of earth, $R_e = 6400 \text{ km} = 6.4 \times 10^6$ m)

or $\quad h = 2649.6 \text{ km} = 2.6 \times 10^6$ m

Thus, at 2.6×10^6 m above the earth's surface, the acceleration due to gravity decreases to 4.9 m/s^2.

50 (*c*) Weight of the body at the surface of the earth, $w = mg$.

Weight of the body at any point w.r.t. earth's surface

= Mass of the body × Acceleration due to gravity at the point

$$\Rightarrow \quad w' = mg' \quad \ldots(i)$$

The gravitational acceleration experienced by that point for height h above the surface of the earth is

$$g' = g\left(\frac{R_E}{R_E + h}\right)^2$$

$\because$ According to the question, $h = \left(\frac{R_E}{2}\right)$

$$\therefore \quad g' = g\left[\frac{R_E}{\frac{3R_E}{2}}\right]^2 = \frac{4}{9} g \quad \ldots(ii)$$

On substituting Eq. (ii) in Eq. (i), we get

$$w' = \frac{4\,mg}{9} = \frac{4}{9} w$$

51 (*c*) The acceleration due to gravity for height h above the surface of the earth is

$$g' = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{g}{2} = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{2} = \left(\frac{R}{R+h}\right)^2$$

or $\quad (R+h)^2 = 2R^2$

or $\quad R + h = \sqrt{2}R$

$$\Rightarrow \quad h = (\sqrt{2} - 1) R$$

52 (*b*) Given, $g' = \left(\frac{1}{100}\right) g$ or $g'/g = \frac{1}{100}$

For height h above the surface of the earth,

$$g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{g'}{g} = \left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \left(\frac{1}{100}\right) = \left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{R}{R+h} = \frac{1}{10}$$

$$\therefore \quad h = 10R - R = 9R$$

53 (*b*) Given, height, $h = \frac{R_E}{2}$

Acceleration due to gravity at altitude h is given by

$$g' = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2} = \frac{g}{\left(1 + \frac{R_E/2}{R_E}\right)^2}$$

$$= \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{g}{\left(\frac{3}{2}\right)^2} = \frac{4g}{9} \quad \ldots(i)$$

Weight of the body at the earth's surface,

$$w = mg = 45 \text{ N} \quad \ldots(ii)$$

Then, weight of the body at altitude h is

$$w' = mg' = \frac{4}{9} mg \quad \ldots(iii)$$

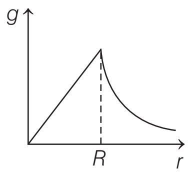
From Eqs. (ii) and (iii), we get

$$w' = \frac{4}{9} \times 45 = 20 \text{ N}$$

54 (*c*) Assuming earth of radius R, then at a point inside such that $r < R$, acceleration due to gravity, $g \propto r$ and at a point outside such that $r > R$,
Acceleration due to gravity,

$$g \propto \frac{1}{r^2}$$

So, the variation of g with r can be represented by

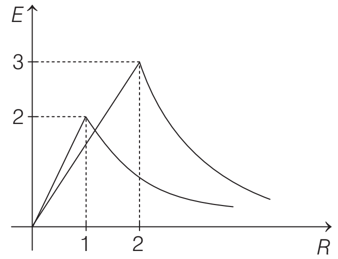


Hence, option (c) is correct.

55 (*c*) Gravitational field of a solid sphere is maximum at its surface ($r = R$) and its value at surface,

$$E = \frac{GM}{R^2}$$

From graph given in the question,



We can observe that

$$E_1 = \frac{GM_1}{(1)^2} = 2 \text{ and } E_2 = \frac{GM_2}{(2)^2} = 3$$

or $\quad GM_1 = 2 \quad$ ...(i)

and $\quad GM_2 = 12 \quad$ ...(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{M_1}{M_2} = \frac{2}{12} = \frac{1}{6}$$

56 (*c*) Given, radius of earth, $R_E = 6400\,\text{km}$

Distance from centre of earth, $r = 2000\,\text{km}$

Distance from the surface of earth,

$$d = R_E - r = 6400 - 2000$$
$$= 4400\,\text{km} = 4.4 \times 10^6\ \text{m}$$

If g' be the acceleration due to gravity below the surface of earth at a depth d, then

$$g' = g\left(1 - \frac{d}{R_E}\right)$$
$$= 9.8\left(1 - \frac{4.4 \times 10^6}{6.4 \times 10^6}\right) \quad [\because g = 9.8\,\text{m/s}^2]$$
$$= 9.8 \times 0.3125 = 3.06\,\text{m/s}^2$$

57 (*c*) Given, weight of the body, $w = mg = 200\,\text{N}$,

where m is the mass of the body and g ($\approx 10\,\text{m/s}^2$) is acceleration due to gravity at the surface of the earth. Since, mass m remains constant irrespective of the position of the body on the earth. However, g is not constant and its value at a depth d below the earth's surface is given as

$$g' = g\left(1 - \frac{d}{R}\right) \quad \text{... (i)}$$

where, R is the radius of the earth.

Multiplying m on both sides of Eq. (i), we get

$$mg' = mg\left(1 - \frac{d}{R}\right)$$

Thus, the weight of the body at half way down $\left(\text{i.e. } d = \frac{R}{2}\right)$ to the centre of the earth is

$$mg' = 200 \times \left(1 - \frac{R/2}{R}\right)$$
$$= 200\left(1 - \frac{1}{2}\right) = 200 \times \frac{1}{2}$$
$$= 100\ \text{N}$$

$\therefore$ The body will weigh 100 N half-way down to the centre of the earth.

58 (*b*) Acceleration due to gravity at poles,

$$g_p = 10\,\text{ms}^{-2}$$

Weight of box at poles,

$$w_p = 196\,\text{N}$$

So, mass of box,

$$m = \frac{w_p}{g_p} = 19.6\,\text{kg}$$

Now, due to rotation of earth acceleration due to gravity at equator,

$$g_e = g_p - R\omega^2 \quad \text{...(i)}$$

Here, $\quad g_p = 10\ \text{ms}^{-2}$, $R = 6400 \times 10^3\text{m}$,

$$\omega = \frac{2\pi}{T} \text{ and } T = 24 \times 3600\,\text{s}$$

Now, substituting the given values in Eq. (i), we get

$$g_e = g_p - 0.034$$
$$= 10 - 0.034$$
$$= 9.966\ \text{ms}^{-2}$$

So, weight of box at equator,

$$w_e = g_e \times m = 9.966 \times 19.6$$
$$= 195.32\,\text{N}$$

59 (*d*) Acceleration due to gravity at height h above earth's surface,

$$g_h = g\left(\frac{R}{R + h}\right)^2$$

$$= g\left(\frac{1}{1+\frac{h}{R}}\right)^2 = g\left(1+\frac{h}{R}\right)^{-2}$$

$$= g\left(1-\frac{2h}{R}\right)$$

Acceleration at depth d below earth's surface,

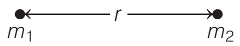
$$g_d = g\left(1-\frac{d}{R}\right)$$

Given, when $h = 1\,\text{km}$, so $g_d = g_h$

or $\quad g\left(1-\frac{d}{R}\right) = g\left(1-\frac{2h}{R}\right)$

$\Rightarrow \qquad d = 2h$ or $d = 2$ km

60 (d) Two point masses m_1 and m_2 are separated by a distance r is shown as



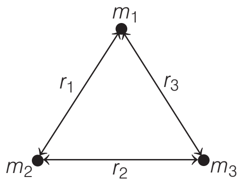
Gravitational potential energy (U) of the above system is given as

$$U = -\frac{Gm_1m_2}{r}$$

i.e. gravitational potential energy $\propto m_1m_2$ and gravitational potential energy $\propto \frac{1}{r}$ or gravitational potential energy is directly proportional to the product of the masses of particles and inversely proportional to the separation between them.

61 (b) For a system of particles, all possible pairs are taken and total gravitational potential energy is the algebraic sum of the potential energies due to each pair, applying the principle of superposition.

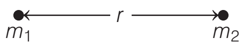
Total gravitational potential energy



$$= \frac{-Gm_1m_2}{r_1} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$$

$$= \left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1m_3}{r_3}\right)$$

62 (b) Two point masses m_1 and m_2 are separated by a distance r is shown as



As,we know $G_1 = \frac{-Gm_1m_2}{r}$ $\quad$ ...(i)

When separation between the particle is doubled, i.e. $2r$, then

$$G_2 = -\frac{Gm_1m_2}{2r} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{G_1}{G_2} = 2$$

63 (a) Given, potential energy of system,
$U = -7.79 \times 10^{28}$ J

Mass of the earth, $M_E = 6 \times 10^{24}$ kg

Mass of the moon, $M_m = 7.4 \times 10^{22}$ kg

The potential energy of the earth-moon system is

$$U = -\frac{GM_E \cdot M_m}{r}$$

where, r is the mean distance between earth and moon

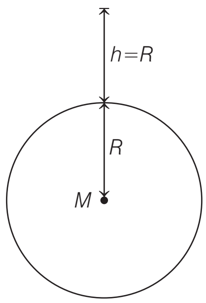
$$\therefore \quad r = -\frac{GM_E\,M_m}{U}$$

$$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.4 \times 10^{22}}{-7.79 \times 10^{28}}$$

$$= 3.8 \times 10^8 \text{ m}$$

64 (b) Amount of work done in moving the given body from one point to another against the gravitational force is equal to the change in potential energy of the body.

Consider a point mass m at a height $h(= R)$ above the surface of the earth as shown in the figure



As we know, the potential energy of body of mass m on the surface of earth is $V_1 = -\frac{GMm}{R}$ $\quad$ ... (i)

where, G = gravitational constant,

M = mass of earth and R = radius of earth.

When the mass is raised to a height h from the surface of the earth, then the potential energy of the body becomes

$$V_2 = -\frac{GMm}{(R+h)}$$

Here, $h = R$ (given)

$$\Rightarrow \quad V_2 = -\frac{GMm}{2R} \quad \text{... (ii)}$$

Thus, the change in potential energy, $\Delta V = V_2 - V_1$

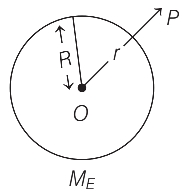
Substituting the values from Eqs. (i) and (ii), we get

$$\Delta V = -\frac{GMm}{2R} + \frac{GMm}{R}$$

$$= \frac{GMm}{2R} = \frac{gR^2m}{2R} \qquad \left[\because g = \frac{GM}{R^2}\right]$$

$$= \frac{mgR}{2}$$

65 (*c*) Consider a point P at a distance $r (> R)$ from the centre of the earth (mass M_E and radius R) is as shown below



The gravitational potential at point P is

$$V_g(r) = -\frac{GM_E}{r}$$

66 (*d*) Gravitational potential at some height h from the surface of the earth is given by

$$V_g = -\frac{GM}{R+h} \quad \ldots(i)$$

and acceleration due to gravity at some height h from the earth surface can be given as

$$g' = \frac{GM}{(R+h)^2} \quad \ldots(ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\Rightarrow \quad \frac{|V_g|}{g'} = R + h \quad (iii)$$

Given, $V_g = -5.4 \times 10^7$ J kg^{-2}, $g' = 6.0$ ms^{-2}

and radius of earth, $R = 6400$ km.

Substituting these values in Eq. (iii), we get

$$\frac{5.4 \times 10^7}{6.0} = R + h$$

$$\Rightarrow \quad 9 \times 10^6 = R + h$$

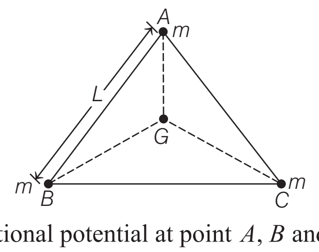
$$h = 9 \times 10^6 - R$$

$$h = 9 \times 10^6 - 6.4 \times 10^6$$

$$\Rightarrow \quad h = 2.6 \times 10^6 \text{ m}$$

$$\Rightarrow \quad h = 2600 \text{ km}$$

67 (*d*) Consider three particles A, B and C of equal mass m kept at vertices of an equilateral triangle of side L is shown as



If gravitational potential at point A, B and C are V_A, V_B and V_C respectively, then total gravitational potential at centre, i.e. centroid of triangle G,

$$V_T = V_A + V_B + V_C$$

$$\Rightarrow \quad V_T = -\frac{Gm}{AG} - \frac{Gm}{BG} - \frac{Gm}{CG} \qquad \left[\because V = -\frac{GM}{r}\right]$$

Also, $AG = BG = CG = \frac{2}{3} \times \left(\frac{\sqrt{3}\,L}{2}\right) = \frac{\sqrt{3}\,L}{3}$

$$\Rightarrow \quad V_T = -Gm\left(\frac{1}{AG} + \frac{1}{BG} + \frac{1}{CG}\right)$$

$$= -Gm\left(\frac{3 \times 3}{\sqrt{3}L}\right) = -\frac{9\,Gm}{\sqrt{3}\,L} \quad \text{or} \quad \frac{-3\sqrt{3}\,Gm}{L}$$

Hence, options (b) and (c) are correct.

68 (*a*) As we know that, escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$ ...(i)

where, M is mass of planet.

So, on the basis of Eq. (i), it can be said that escape velocity will depend upon the mass of the planet (M).

69 (*d*) Escape velocity on the surface of earth is given by

$$v = \sqrt{2gR_e}$$

i.e. $\quad v \propto \sqrt{R_e}$

Hence, escape velocity does not depend on the mass and direction of projection of body, it depends on the radius of earth.

Hence, options (a) and (c) are correct.

71 (*a*) Given, escape velocity on the surface of earth,

$$v_e = \sqrt{\frac{2GM_E}{R_E}}$$

where, $M_E =$ mass of the earth and $R_E =$ radius of the earth.

Now according to the question, radius of earth,

$$R' = R_E/4$$

$$\Rightarrow \quad v'_e = \sqrt{\frac{2GM_E}{R'}} = \sqrt{4\left(\frac{2\,GM_E}{R_E}\right)} = 2\sqrt{\frac{2GM_E}{R_E}}$$

or $\quad v'_e = 2\,v_e$

Hence, the escape velocity will be doubled.

72 (*a*) A black hole is a super dense planetary material formed due to the continued compression.

If R be the approximate radius of super compressed earth such that it behave as a black hole.

In this case,

$v_e = c$, where c is velocity of light (3×10^8 m/s).

$$v_e = \sqrt{\frac{2GM}{R}} = c$$

$$\Rightarrow \quad R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{9 \times 10^{16}}$$

$$= 8.8 \times 10^{-3} \approx 10^{-2} \text{ m}$$

73 (*c*) The kinetic energy of an asteroid of mass m and speed v_i is

$$K_i = \frac{1}{2}mv_i^2 \quad \ldots(i)$$

The potential energy of asteroid at a distance 10 R_E is

$$U_i = -\frac{GM_E m}{10R_E} \quad \text{...(ii)}$$

where, M_E is mass of earth.

$\therefore$ Initial energy of the asteroid is

$$E_i = K_i + U_i = \frac{1}{2}mv_i^2 - \frac{GM_E m}{10R_E}$$

As it hits earth with a speed of v_f (R_E and M_E are radius and mass of earth), then

Final energy of the asteroid is

$$E_f = \frac{1}{2}mv_f^2 - \frac{GM_E m}{R_E}$$

According to law of conservation of energy,

$$E_i = E_f$$

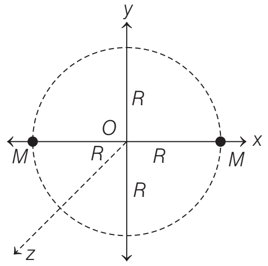
$$\frac{1}{2}mv_i^2 - \frac{GM_E m}{10R_E} = \frac{1}{2}mv_f^2 - \frac{GM_E m}{R_E}$$

$$\Rightarrow \quad v_f^2 - \frac{2GM_E}{R_E} = v_i^2 - \frac{2GM_E}{10R_E}$$

$$\Rightarrow \quad v_f^2 = v_i^2 + \frac{2GM_E}{R_E}\left(1 - \frac{1}{10}\right)$$

74 (*d*) As we know that, force on satellite is only gravitational force which will always be towards the centre of earth. Thus, the acceleration of *S* is always directed towards the centre of the earth.

75 (*a*) Let us assume that stars are moving in *x y*-plane with origin as their centre of mass as shown in the figure below



According to question,

mass of each star, $M = 3 \times 10^{31}$ kg

and diameter of circle, $2R = 2 \times 10^{11}$ m

$$\Rightarrow \quad R = 10^{11} \text{ m}$$

Potential energy of meteorite at O, origin $\hat{\mathbf{j}}$ is,

$$U_{\text{total}} = -\frac{2\,GMm}{r}$$

If v is the velocity of meteorite at O then

Kinetic energy K of the meteorite is

$$K = \frac{1}{2}mv^2$$

To escape from this dual star system, total mechanical energy of the meteorite at infinite distance from stars must be at least zero.

By conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{2\,GMm}{R} = 0$$

$$\Rightarrow v^2 = \frac{4\,GM}{R} = \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}$$

$$\Rightarrow \quad v = 2.83 \times 10^5 \text{ m/s}$$

76 (*a*) Given, escape velocity on the surface of earth, $v_e = 11.1$ km/h.

Escape velocity on the surface of the earth,

$$v_e = \sqrt{2gR_E}$$

or

$$v_e = \sqrt{\frac{2GM_E}{R_E}}$$

Mass of moon, $M_m = \dfrac{M_E}{81}$

Radius of moon, $R_m = \dfrac{R_E}{4}$

$\therefore$ Escape velocity on the surface of moon,

$$v_m = \sqrt{\frac{2G\,M_m}{R_m}} = \sqrt{\frac{2G\,\frac{M_E}{81}}{\frac{R_E}{4}}}$$

$$= \frac{2}{9}\sqrt{\frac{2GM_E}{R_E}} = \frac{2}{9}v_e$$

$$= \frac{2}{9} \times 11.1 = 2.46 \text{ km/h}$$

77 (*a*) Since, the escape velocity of earth can be given as

$$v_e = \sqrt{2gR}$$

$$\Rightarrow \quad v_e = R\sqrt{\frac{8}{3}\pi G\rho} \quad (\rho = \text{density of earth}) \text{...(i)}$$

As it is given that the radius and mean density of planet are twice as that of earth. So, escape velocity at planet will be

$$v_p = 2R\sqrt{\frac{8}{3}\pi G2\rho} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{v_e}{v_p} = \frac{R\sqrt{\frac{8}{3}\pi G\rho}}{2R\sqrt{\frac{8}{3}\pi G2\rho}} \Rightarrow \frac{v_e}{v_p} = \frac{1}{2\sqrt{2}} = 1 : 2\sqrt{2}$$

78 (*d*) $\because$ Angular momentum,

$$L = mvr = m\sqrt{\frac{GM}{r}}\,r = m\sqrt{GMr}$$

$$\left(\because v = \sqrt{\frac{GM}{r}}\right)$$

$$\therefore \quad L \propto \sqrt{r}$$

If r is increased to $16r$, then new angular momentum,

$$L' \propto \sqrt{16r} \quad \Rightarrow \quad L' = 4L$$

79 (*d*) Orbital speed of a satellite in a circular orbit is

$$v_0 = \sqrt{\left(\frac{GM}{r_0}\right)}$$

where r_0 is the radius of the circular orbit.

So, kinetic energies of satellites A and B are

$$T_A = \frac{1}{2} m_A v_{OA}^2 = \frac{GMm}{2R}$$

$$T_B = \frac{1}{2} m_B v_{OB}^2 = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R}$$

So, ratio of their kinetic energies is

$$\frac{T_A}{T_B} = 1$$

80 (*c*) KE of satellite, $K = \frac{1}{2} m v_o^2$

$$= \frac{1}{2} m \left(\sqrt{\frac{GM_E}{(R_E + h)}}\right)^2 = \frac{1}{2} \frac{GmM_E}{(R_E + h)}$$

$$\left[\because \text{ orbital velocity of satellite, } v_o = \sqrt{\frac{GM_E}{(R_E + h)}}\right]$$

81 (*b*) KE of a satellite, $K = \frac{GmM_E}{2(R_E + h)}$.

$$\Rightarrow \quad K = \frac{GmM_E}{2r} \quad [\because R_E + h = r(\text{let})]$$

$$\text{Since,} \quad K \propto \frac{1}{r}$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{r_2}{r_1}$$

$$\Rightarrow \quad \frac{K}{K_2} = \left(\frac{2r}{r}\right)$$

(since, radius is doubled and $K_1 = K$)

$$\therefore \quad K_2 = \frac{K}{2}$$

82 (*b*) Gravitational potential energy of a body of mass m at a distance r from the centre of the earth is

$$= -\frac{GmM_E}{r}$$

$$\because \quad r = R_E + h$$

$$\therefore \quad \text{PE} = -\frac{GmM_E}{(R_E + h)}$$

83 (*a*) PE of satellite $= \frac{-GmM_E}{(R_E + h)}$ ...(i)

KE of satellite $= +\frac{1}{2} \frac{GmM_E}{(R_E + h)}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\text{PE} = -2\,\text{KE}$$

$$\Rightarrow \quad \lambda = -2$$

84 (*d*) Orbital velocity of the satellite is given as,

$$v_O = \sqrt{\frac{GM}{R + h}}$$

Since, $R >> h$

$$\therefore \quad v_O = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad \left[\because g = \frac{GM}{R^2}\right]$$

Escape velocity of the satellite,

$$v_e = \sqrt{\frac{2GM}{R + h}} = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Since, we know that in order to escape the earth's gravitational field a satellite must get escape velocity.

$\therefore$ Change in velocity,

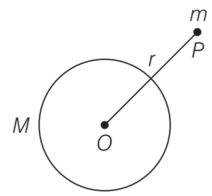
$$\Delta v = v_e - v_O$$
$$= \sqrt{gR}\,(\sqrt{2} - 1)$$

86 (*b*) According to the given condition in the question, after collision the mass of combined system is doubled. Also, this system would be displaced from its circular orbit.

So, the combined system revolves around centre of mass of 'system + earth' under action of a central force.

Hence, orbit must be elliptical.

87 (*a*) For point outside the spherical shell, the gravitational force on point mass placed at that point P is just as if the entire mass of the shell is situated at the centre O, i.e. as if a point mass M is placed at centre O.

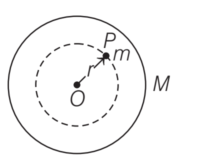


Gravitational force have two components along the line joining the centre of masses as well as along the direction perpendicular to this line.

Only gravitational forces due to various regions of the shell along the line joining P and O remains while components perpendicular to OP cancel out.

Therefore Assertion and Reason are correct and Reason is the correct explanation of Assertion.

88 (*a*) A point P (mass m) is situated inside the hollow spherical shell is shown in the figure.



For point P inside the hollow spherical shell,

$$F_P = \text{zero}$$

The force of gravitation due to various region of the spherical shell be attractive and hence from symmetry it can be seen that these forces cancel each other completely.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

89 (*a*) Henry-Cavendish experiment helped to determine the value of G ($G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}).

From the value of g (acceleration due to gravity on the surface of the earth) and R_E (radius of the earth) using the relation $g = \frac{GM_E}{R_E^2}$ (by using Newton's law), the mass of the earth (M_E) can be estimated

$$\Rightarrow \quad M_E = \frac{gR_E^2}{G}$$

where, $g = 9.8\,\text{ms}^{-2}$,

$R_E = 6400 \times 10^3$ m

and $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

90 (a) Since, acceleration due to gravity decreases above the surface of the earth and weight is directly proportional to the acceleration due to gravity,
so as we go up, we feel light weighted than on the surface of the earth.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

91 (a) Escape speed for moon $= \sqrt{2g'R_m}$, where g' and R_m both are much smaller than corresponding values on the earth, hence on substituting the values.

Escape speed on the moon comes out to be 2.3 kms^{-1} as calculated below

$$\because \quad g' = g/6 \text{ and } R_m = 1760\text{ km}$$

$$\Rightarrow \text{Escape speed} = \sqrt{2 \times \frac{9.8}{6} \times 1760 \times 10^3}\ \text{ms}^{-1}$$

$$= 2.3\ \text{kms}^{-1}$$

$$\because \quad (v_i)_{\min}\ (\text{earth}) = 11.2\ \text{kms}^{-1}$$

$$\text{and} \quad (v_i)_{\min}\ (\text{moon}) = 2.3\ \text{kms}^{-1}$$

Thus, escape speed for the moon is five times smaller than that of earth.

Therefore, Assertion and Reason are correct and Reason is the correct explainetion of Assertion.

92 (a) The escape speed for the moon is much smaller and hence any gas molecule formed having thermal velocity larger than escape speed will escape the gravitational pull of the moon.
So, moon has no atmosphere.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

93 (d) Escape velocity, $v_e = \sqrt{2gR}$, where $g = \frac{GM}{R^2}$.

$$\Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}}$$

$$\text{i.e.} \quad v_e \propto \frac{1}{\sqrt{R}}$$

So, if radius is doubled, i.e. $R' = 2R$

$$v'_e = \sqrt{\frac{2GM}{(2R)}} = \frac{1}{\sqrt{2}}\sqrt{\frac{2GM}{R}} = \frac{v_e}{\sqrt{2}}$$

Therefore, Assertion is incorrect but Reason is correct.

94 (a) Orbital velocity of satellite, $v_o = \sqrt{\frac{GM_E}{(R_E + h)}}$

$$\Rightarrow \quad v_o \propto \frac{1}{\sqrt{R_E + h}}$$

Thus, v_o is maximum near the surface of the earth for $h = 0$.

$$(v_o)_{\max} = \sqrt{\frac{GM_E}{R_E}}$$

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

95 (a) Total energy of a satellite is always negative irrespective of the nature of its orbit. It indicates that the satellite is bound to the earth. At infinity, the potential energy and kinetic energy of satellite is zero.
Hence, total energy at infinity is zero, therefore only negative energy of satellite is possible when it is revolved around the earth.
If it is positive or zero, the satellite would leave its definite orbit and escape to infinity.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

96 (a) The geo-stationary satellite goes around the earth in west-east direction.

It is because it orbits around earth in the equatorial plane with a time period of 24 h same as that of rotation of the earth around its axis.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

97 (c) In a satellite around the earth, every part and parcel of the satellite has an acceleration towards the centre of the earth which is exactly the value of earth's acceleration due to gravity at that position.
Thus, in the satellite, everything inside it is in a state of free fall.
Therefore, Assertion is correct but Reason is incorrect.

98 (a) An object is weightless when it is in free fall as during free fall, there is no upward force acting on the body and this phenomenon is called weightlessness.

Therefore, Assertion and Reason are correct and Reason is the correct explanation of Assertion.

99 (a) The pattern of motion of the planets was put forward by Ptolemy. According to his scheme of motion, the planets are moving in circles with the centre of the circles themselves moving in larger circles.

So, statement I is correct but II and III are incorrect.

100 (d) In a 'geocentric model' all celestial objects, stars, the sun and the planets, all revolve around the earth. The only possible motion for celestial objects was motion in a circle.

So, statements I and II are correct but III is incorrect.

101 (a) With reference to the 'heliocentric model' of planetary motions, the planets revolve around the sun as its centre.

So, statement I is correct but II, III and IV are incorrect.

102 (a) For the gravitational force between an extended object (like the earth) and a point mass, the Newton's universal law of gravitation is not directly applicable.

So, statement I is correct but II and III are incorrect.

104 (d) Time period of satellite $= \dfrac{2\pi\,(R_E + h)^{3/2}}{\sqrt{GM_E}}$

From the above equation, it is evident that the time period of a satellite depends on mass of the earth (M_E), radius of the orbit ($r = R_E + h$) and height of the satellite from the surface of the earth (h).

So, statements II, III and IV are correct but I is incorrect.

106 (d) Let the original mass of sun was M_s and gravitational constant G.

According to the question,

New mass of sun, $M_s' = \dfrac{M_s}{10}$

New gravitational constant, $G' = 10G$

As, the acceleration due to gravity is given as

$$g = \frac{GM_E}{R^2} \quad \ldots\text{(i)}$$

where, M_E is the mass of earth and R is the radius of the earth.

Now, new acceleration due to gravity,

$$g' = \frac{G'M_E}{R^2} = \frac{10M_EG}{R^2} \quad \ldots\text{(ii)}$$

$\therefore \quad g' = 10g \quad$ [from Eqs. (i) and (ii)]

This means the acceleration due to gravity has been increased. Hence, force of gravity acting on a body placed on the surface of the earth increases.

Due to this, rain drops will fall faster and

walking on ground would become more difficult.

As, time period of the simple pendulum is

$$T = 2\pi\sqrt{\frac{l}{g}} \text{ or } T \propto \frac{1}{\sqrt{g}}$$

Thus, time period of the pendulum also decreases with the increase in g.

Thus, the statement given in option (d) is incorrect, rest are correct.

107 (b) Since, cavities are symmetrical w.r.t. O. So, the gravitational force at the centre is zero,

but gravitational force at the point $B(2,0,0)$ is not zero due to lack of symmetry.

The radius of the circle $z^2 + y^2 = 36$ is 6. For all points for $r \geq 6$, the body behaves such that whole of its mass is concentrated at the centre. So the gravitational potential is same.

Above is true for $z^2 + y^2 = 4$ as well.

Thus, the statement given in option (b) is incorrect, rest are correct.

108 (b) Acceleration due to gravity at altitude h,

$$g_h = \frac{g}{(1 + h/R)^2} \approx g\left(1 - \frac{2h}{R}\right)$$

At depth d, $g_d = g\left(1 - \dfrac{d}{R}\right)$

In both cases with increase in h and d, g decreases.

At latitude ϕ, $\quad g_\phi = g - \omega^2 R\cos^2\phi$

As ϕ increases, g_ϕ increases.

Also, we can conclude from the formulae, that it is independent of mass.

Thus, the statement given in option (b) is incorrect, rest are correct.

109 (a) As we know that,

$$g = \frac{GM}{R^2} \Rightarrow v_e = \sqrt{\frac{2GM}{R}}$$

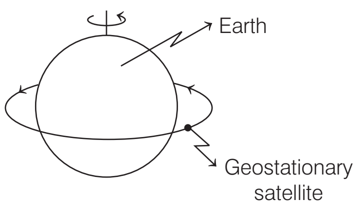
and $$U = \frac{-GMm}{R} \Rightarrow g \propto \frac{M}{R^2}$$

$$v_e \propto \sqrt{\frac{M}{R}} \text{ and } U \propto \frac{M}{R}$$

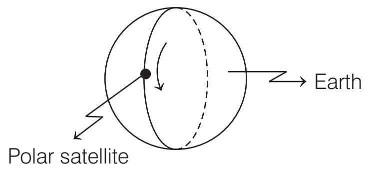
If both mass and radius are increased by 0.5%, then v_e and U remains unchanged whereas g decrease by 0.5%.

Thus, the statement given in option (a) is incorrect, rest are correct.

110 (a) A geo-stationary satellite is having same sense of rotation as that of earth, i.e. west-east direction as shown below



A polar satellite goes around the earth's pole in north-south direction as shown below.



Thus, the statement given in option (a) is correct, rest are incorrect.

111 (b) For stable orbit, plane of orbit of satellite must pass through the centre of earth.

Geo-stationary satellites are launched in the equatorial plane.

We need more than one satellite for global communication.

Orbital speed of satellite, $v_o = \sqrt{\dfrac{GM_e}{r}}$

So, orbital speed of satellite decrease with the increase in the radius of its orbit.

Thus, the statement given in option (b) is correct, rest are incorrect.

112 (*d*) The statement given in option (d) is correct, rest are incorrect and these can be corrected as

The energy required to rocket an orbiting satellite out of earth's gravitational influence is less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.

If the potential energy is zero at infinity, the total energy of an orbiting satellite is negative of its kinetic energy.

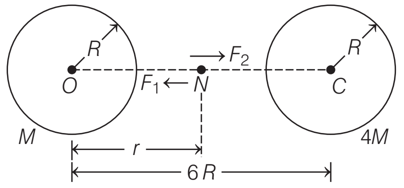
The first artificial satellite was launched by Soviet scientists in the year 1957.

113 (*c*)

A. The projectile is acted upon by two mutually opposing gravitational forces of the two spheres.

Hence, there must be a point on the line OC, where $F_{ext} = 0$, i.e. net external force due to gravitational attraction force vanishes. The point is called neutral point N. Let it be at a distance r from O.

$\Rightarrow \quad ON = r$



At N, $\quad F_1 = F_2$

$$\Rightarrow \quad \frac{GmM}{r^2} = \frac{Gm(4M)}{(6R-r)^2} \Rightarrow r = 2R \text{ or } -6R$$

The neutral point $r = -6R$ is not relevant.

Thus, $ON = r = 2R$.

B. To project the projectile with minimum speed from M. It is sufficient to project the particle with a speed which would enable it to reach N. After N, projectile will be attracted by 4 M.

At the neutral point, speed approaches zero,

i.e. $\quad v_N = 0$

Total energy of the particle at $N = E_N$

$\Rightarrow \quad E_N =$ GPE due to $M +$ GPE due to $4M$

$$E_N = \frac{-GmM}{2R} - \frac{4GMm}{4R} \quad \ldots\text{(i)}$$

$$\left[\because \text{GPE due to } M = -\frac{GmM}{2R}; r = 2R\right.$$

$$\left.\text{GPE due to } 4M = -\frac{4GMm}{4R}; r = 4R\right]$$

From the principle of conservation of mechanical energy,

$$\frac{1}{2}mv^2 - \frac{GmM}{R} - \frac{4GMm}{5R} = -\frac{GMm}{2R} - \frac{4GMm}{4R}$$

$$\Rightarrow \quad v = \sqrt{\frac{3GM}{5R}}$$

$$\Rightarrow \quad v_{min} = \sqrt{\frac{3GM}{5R}}$$

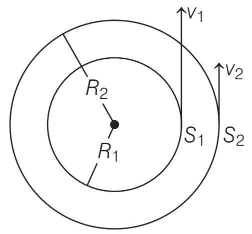
Therefore, minimum speed of the projectile with which it reaches the surface of second sphere is $\sqrt{\frac{3GM}{5R}}$.

C. Let v_f be the speed with which projectile hits the second sphere. Applying principle of conservation of energy, we get

$$\Rightarrow \quad v_f = \sqrt{\frac{27\,GM}{5R}}$$

Hence, $A \rightarrow 2, B \rightarrow 1$ and $C \rightarrow 3$.

114 (*a*) Let the mass of the planet be M, that of S_1 be m_1 and of S_2 be m_2. Let the radius of the orbit of S_1 be R_1 ($= 10^4$ km) and of S_2 be R_2. Let v_1 and v_2 be the linear speeds of S_1 and S_2 with respect to the planet. The figure shows the situation.



If the period of revolutions of satellites S_1 and S_2 are T_1 (1h) and T_2 (8h), respectively.

As the square of the time period is proportional to the cube of the radius,

$$\left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{8\text{ h}}{1\text{ h}}\right)^2 = 64$$

$$\Rightarrow \quad \frac{R_2}{R_1} = 4 \Rightarrow R_2 = 4R_1 = 4 \times 10^4 \text{ km}$$

Now, the time-period of S_1 is 1 h. So, $\frac{2\pi R_1}{v_1} = 1$

$$\Rightarrow \text{Speed of } S_1,\ v_1 = \frac{2\pi R_1}{1} = 2\pi \times 10^4 \text{ km h}^{-1} \ldots\text{(i)}$$

Similarly, speed of

$$S_2,\ v_2 = \frac{2\pi R_2}{8} = \pi \times 10^4 \text{ kmh}^{-1} \quad \ldots\text{(ii)}$$

At the closest separation, they are moving in the same direction. Hence, the velocity of S_2 with respect to S_1 is

$$v_2 - v_1 = \pi \times 10^4 \text{ km h}^{-1} - 2\pi \times 10^4 \text{ kmh}^{-1}$$

$$= -\pi \times 10^4 \text{ km h}^{-1} \quad \ldots\text{(iii)}$$

As seen from S_1, the satellite S_2 is at a distance $R_2 - R_1 = 3 \times 10^4$ km at the closest separation. Also, it is moving at $\pi \times 10^4$ km h^{-1} in a direction perpendicular to the line joining them. Thus, the angular speed of S_2 as observed by S_1 is

$$\omega = \frac{v}{r} = \frac{\pi \times 10^4 \text{ km h}^{-1}}{3 \times 10^4 \text{ km}} = \frac{\pi}{3} \text{ rad h}^{-1}$$

Hence, A $\rightarrow$ 3, B $\rightarrow$ 2, C $\rightarrow$ 4 and D $\rightarrow$ 1.

115 (*a*)

A. Initially, energy of satellite $E_i = -\frac{GM_e m}{4R_e}$

Finally, energy of satellite , $E_f = \frac{-GM_e m}{8R_e}$

Change in total energy, $\Delta E = E_f - E_i$

$$= \left(-\frac{GM_e m}{8R_e}\right) - \left(-\frac{GM_e m}{4R_e}\right)$$

$$= \frac{GM_e m}{8R_e} \text{ or } \frac{gR_e m}{8} \qquad \left[\because g = \frac{GM}{R_e^2}\right]$$

Thus, $\Delta E = \frac{gR_e m}{8} = \frac{9.8 \times 400 \times 6.37 \times 10^6}{8}$

$[\because g = 9.8 \text{ ms}^2, m = 400 \text{ kg}, R = 6.37 \times 10^6$ m]

$$= 3.13 \times 10^9 \text{ J}$$

B. The kinetic energy is reduced and change in KE is just negative of ΔE.

$\Rightarrow \quad \Delta K = K_f - K_i = -3.13 \times 10^9$ J

C. The change in potential energy is twice the change in the total energy namely,

$$\Delta \text{PE} = \text{PE}_f - \text{PE}_i = -6.26 \times 10^9 \text{ J}$$

Hence, A $\rightarrow$ 2, B $\rightarrow$ 3 and C $\rightarrow$ 1.

116 (*b*)

A. If the velocity of satellite is v and mass m, then

$$\text{KE} = \frac{1}{2} mv^2 \qquad \text{...(i)}$$

B. Since, potential energy of the satellite

= $-$ 2 kinetic energy of satellite

$\Rightarrow \quad \text{PE} = -mv^2 \qquad$ ...(ii)

C. Also, total energy = KE + PE = $\frac{1}{2} mv^2 - mv^2$

$$= -\frac{1}{2} mv^2$$

Hence, A $\rightarrow$ 2, B $\rightarrow$ 3 and C $\rightarrow$ 1.

118 (*a*) Given, orbital period of Jupiter's satellite

$T = 1.769 \text{ days} = 1.528 \times 10^5$ s

Radius of orbit, $r = 4.22 \times 10^8$ m

$$G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$$

Mass of the sun, $M_S = 2 \times 10^{30}$ kg

Let mass of the Jupiter be M_J.

$\because$ Centripetal force = Gravitation force

$$\frac{mv^2}{r} = \frac{GM_J m}{r^2} \text{ or } v^2 = \frac{GM_J}{r}$$

$$(r\omega)^2 = \frac{GM_J}{r} \qquad (\because v = r\omega)$$

$$\therefore \quad \omega^2 = \frac{GM_J}{r^3}$$

But $\omega = \frac{2\pi}{T}$, where T is the time period.

$$\therefore \quad \left(\frac{2\pi}{T}\right)^2 = \frac{GM_J}{r^3}$$

or $\quad M_J = \frac{4\pi^2 r^3}{T^2 G} = \frac{4 \times (3.14)^2 \times (4.22 \times 10^8)^3}{(1.528 \times 10^5)^2 \times 6.67 \times 10^{-11}}$

$$= 1.9 \times 10^{27} \text{ kg} \approx 2 \times 10^{27} \text{ kg}$$

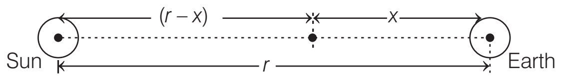
Now, $\quad \frac{M_J}{M_S} = \frac{2 \times 10^{27}}{2 \times 10^{30}} \approx \frac{1}{1000} \Rightarrow M_J = \frac{1}{1000} M_S$

119 (*d*) Since, potential energy is equal to twice the negative of kinetic energy of satellite.

Total energy of satellite = Potential energy + Kinetic energy.

$$E = V + K = -2K + K = -K$$

120 (*b*) Consider r is the distance between sun and earth and rocket is at distance x from the earth as shown below



Let gravitational force acting on the rocket be zero at a distance x from earth's centre and the mass of the rocket be m.

Gravitational force between rocket and the sun

= Gravitational force between rocket and the earth

$$\Rightarrow \quad \frac{GM_s m}{(r-x)^2} = \frac{GM_E m}{x^2} \Rightarrow \frac{M_s}{(r-x)^2} = \frac{M_E}{x^2}$$

Given, $M_s = 2 \times 10^{30}$ kg and $M_E = 6.0 \times 10^{24}$ kg

$$\Rightarrow \quad \frac{(r-x)^2}{x^2} = \frac{M_s}{M_E} = \frac{2 \times 10^{30}}{6.0 \times 10^{24}}$$

(On putting values of M_s and M_E)

$$\Rightarrow \quad \frac{(r-x)^2}{x^2} = \frac{10^6}{3}$$

Taking square root on both sides, we get

or $\quad \frac{r-x}{x} = \frac{10^3}{\sqrt{3}} \Rightarrow \frac{r}{x} - 1 = \frac{10^3 \times \sqrt{3}}{3}$

or $\quad x = \frac{3}{1735} \times r = \frac{3 \times 1.5 \times 10^{11}}{1735}$

$$= 2.594 \times 10^8 \text{ m}$$

$$x = 2.6 \times 10^8 \text{ m}$$

121 (d) Earth year, $T_E = 1\,\text{yr}$

Saturn year, $T_S = 29.5\,\text{yr}$

Radius of earth's orbit, $R_E = 1.5 \times 10^8$ km

Radius of saturn's orbit, $R_S = ?$

According to Kepler's planetary law of period,

$$T^2 \propto R^3 \Rightarrow \frac{T_E^2}{T_S^2} = \frac{R_E^3}{R_S^3}$$

$$\left(\frac{1}{29.5}\right)^2 = \left(\frac{1.5 \times 10^8}{R_S}\right)^3$$

$$R_S^3 = (29.5)^2 \times (1.5 \times 10^8)^3$$

$$= 2.947 \times 10^{27}\ \text{km}$$

$$\Rightarrow \quad R_S = 1.43 \times 10^9\ \text{km}$$

122 (a) Given, height, $h = \frac{R_E}{2}$

Acceleration due to gravity at altitude h is given by

$$g' = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2}$$

$$= \frac{g}{\left(1 + \frac{R_E/2}{R_E}\right)^2} = \frac{g}{\left(1 + \frac{1}{2}\right)^2} = \frac{g}{(3/2)^2} = \frac{4}{9} g$$

Weight of the body at earth's surface

$$w = mg = 63\ \text{N}$$

Weight of the body at altitude $h = R_E / 2$,

$$w' = mg' = \frac{4}{9} mg = \frac{4}{9} \times 63 = 28\ \text{N}$$

123 (c) Let a rocket of mass m be fired vertically with a speed v and it reach at height h from earth's surface.

$$\text{KE of the rocket} = \frac{1}{2} mv^2$$

PE of the rocket at earth's surface, $V_0 = -\frac{GM_E m}{R_E}$

PE of the rocket at height h from earth's surface,

$$V_h = -\frac{GM_E m}{(R_E + h)}$$

$\therefore$ Increase in PE $(\Delta V) = V_h - V_0$

$$\Delta V = GM_E m \times \frac{h}{R_E (R_E + h)} \quad \ldots\text{(i)}$$

But $\quad GM_E = gR_E^2 \quad \ldots$ (ii)

From Eqs. (i) and (ii)

$$\therefore \quad \Delta V = \frac{gR_E^2 mh}{R_E^2\left(1 + \frac{h}{R_E}\right)} = \frac{mgh}{\left(1 + \frac{h}{R_E}\right)}$$

According to law of conservation of energy,

KE of the rocket = Increase in PE

$$\frac{1}{2} mv^2 = \frac{mgh}{\left(1 + \frac{h}{R_E}\right)} \Rightarrow h = \left(\frac{v^2 R_E}{2gR_E - v^2}\right)$$

On putting the values, we get

$$h = 1600 \times 10^3\ \text{m} = 1.6 \times 10^6\ \text{m} = 1600\ \text{km}$$

$\therefore$ Distance from the centre of earth,

$$r = R_E + h = 6.4 \times 10^6 + 1.6 \times 10^6 = 8 \times 10^6\ \text{m}$$

124 (a) Initial KE of the body $= \frac{1}{2} mv^2$

Initial gravitational PE of the body $= -\frac{GM_E m}{R_E}$

At very far away from earth's surface,

KE of the body $= \frac{1}{2} mv'^2$

Gravitational PE of the body = 0

Then from conservation of energy

$$\therefore \quad \frac{1}{2} mv'^2 = \frac{1}{2} mv^2 - \frac{GM_E m}{R_E} \quad \ldots\text{(i)}$$

If v_e is the escape velocity, then

$$\frac{1}{2} mv_e^2 = \frac{GM_E m}{R_E} \quad \ldots\text{(ii)}$$

Substituting values from Eq. (ii) in Eq. (i), we get

$$\frac{1}{2} mv'^2 = \frac{1}{2} mv^2 - \frac{1}{2} mv_e^2$$

$$v'^2 = v^2 - v_e^2 = (3v_e)^2 - v_e^2 = 8v_e^2$$

$$[\because v = 3v_e]$$

$$v' = 2\sqrt{2} v_E = 2 \times 1.414 \times 11.2\ \text{kms}^{-1}$$

$$= 31.68\ \text{kms}^{-1} \simeq 31.7\ \text{kms}^{-1}$$

125 (d) Energy required to send a satellite out of earth's gravitational influence is called its binding energy.

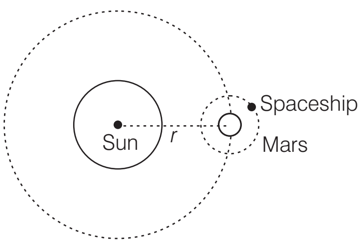
Given, mass of the satellite = 200 kg

Binding energy of a satellite $= \frac{GM_E m}{2(R_E + h)}$

$$= \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 200}{2(6.4 \times 10^6 + 0.4 \times 10^6)}$$

$$= 5.9 \times 10^9\ \text{J}$$

126 (a) Spaceship is present in gravitational field of the sun as well as in the gravitational field of the mars.



$\therefore$ Total potential energy of the spaceship due to sun as well as mars

$$= \left(\frac{-GM_s m}{r}\right) + \left(\frac{-GM_m m}{R_m}\right)$$

[where, M_s is mass of sun and M_m is mass of mars]

$$= -Gm\left[\frac{M_s}{r} + \frac{M_m}{R_m}\right]$$

Potential energy of the spaceship outside the solar system = 0

$\therefore$ Energy imparted to the spaceship to rocket it out of solar system

$$= 0 - \left[-Gm\left(\frac{M_s}{r} + \frac{M_m}{R_m}\right)\right]$$

$$= Gm\left[\frac{M_s}{r} + \frac{M_m}{R_m}\right]$$

$$= 6.67 \times 10^{-11} \times 1000\left[\frac{2 \times 10^{30}}{2.28 \times 10^{11}} + \frac{6.4 \times 10^{23}}{3.395 \times 10^{6}}\right]$$

$$\approx 3.1 \times 10^{11}\ \text{J}$$

127 (c) Given, speed of rocket = 2 kms^{-1}

As 20% of KE is lost due to martian atmospheric resistance.

$\therefore$ Total KE available $= \frac{1}{2}mv^2 \times \frac{80}{100} = \frac{2}{5}mv^2$

Let the rocket be reached at height h from the surface of mars

$$\text{Increase in PE} = \frac{GM_m m}{(R_m + h)} - \left(\frac{-GM_m m}{R_m}\right)$$

$$= GM_m m \times \frac{h}{R_m(R_m + h)}$$

$$\Rightarrow \quad GM_m m \times \frac{h}{R_m(R_m + h)} = \frac{2}{5}mv^2$$

(law of conservation of energy)

Solving it, we get

$$\frac{R_m + h}{h} = \frac{5GM_m}{2R_m v^2}$$

$$\frac{R_m}{h} + 1 = \frac{5GM_m}{R_m v^2} \Rightarrow \frac{R_m}{h} = \frac{5GM_m}{2R_m v^2} - 1$$

On putting values, we get

$$h = \frac{R_m}{6.85862} \approx 495\ \text{km}$$

128 (d) If the earth is considered as a sphere of different density, in that case value of g will be different at different points and cannot be zero at any point.

129 (c) As observed from the earth, the sun appears to move in an approximate circular orbit. As, the gravitational force of attraction between the earth and the sun always follows inverse square law. However, due to relative motion between the earth and mercury, the orbit of mercury, as observed from the earth will not be approximately circular, since the major gravitational force on mercury is due to the sun.

Hence, it revolve around sun and not around earth.

130 (a) As the earth is revolving around the sun in a circular motion due to gravitational attraction. The force of attraction will be of radial nature, i.e. angle between position vector **r** and force **F** is zero. So, torque $= |\tau| = |\mathbf{r} \times \mathbf{F}| = rF\sin 0° = 0$

131 (c) As the total energy of the earth satellite bounded system is negative $\left(\text{i.e.} \frac{-GM}{2a}\right)$, where a is radius of the satellite and M is mass of the earth. So, due to the viscous force acting on satellite, energy decreases continuously and radius of the orbit or height gradually decreases.

Therefore, debris of satellites fall to the earth.

132 (b) As observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. Therefore, net force on the moon is the resultant of these two forces. Hence on observing from the sun, the orbit of the moon will not be strictly elliptical because total gravitational force on the moon is not central.

133 (d) Asteroids are also being acted upon by central gravitational forces, hence they are moving in circular orbits like planets and obey Kepler's laws.

134 (d) Gravitational mass of proton is equivalent to its inertial mass and is independent of presence of neighbouring heavy objects.

135 (c) Particles of masses $2M, m$ and M are respectively at points A, B and C as shown below

Force on B due to A, $F_{BA} = \frac{G(2Mm)}{(AB)^2}$ towards BA

Force on B due to C, $F_{BC} = \frac{GMm}{(BC)^2}$ towards BC

As, $\quad BC = 2AB$

$$\Rightarrow \quad F_{BC} = \frac{GMm}{(2AB)^2} = \frac{GMm}{4(AB)^2} < F_{BA}$$

Hence, m will move towards BA, (i.e., $2M$).

136 (d) For small objects, say of sizes less than 100 m centre of mass is very close with the centre of gravity of the body. But when the size of object increases, its weight changes and its CM and CG become far from each other.

137 (b) Given, $e = 0.0167$

Ratio of maximum speed to minimum speed is

$$\frac{v_{\max}}{v_{\min}} = \frac{1+e}{1-e}$$

$$= \frac{1 + 0.0167}{1 - 0.0167} = 1.033$$